

“无中微子双贝塔衰变”研讨会 2021年5月19-23日 珠海



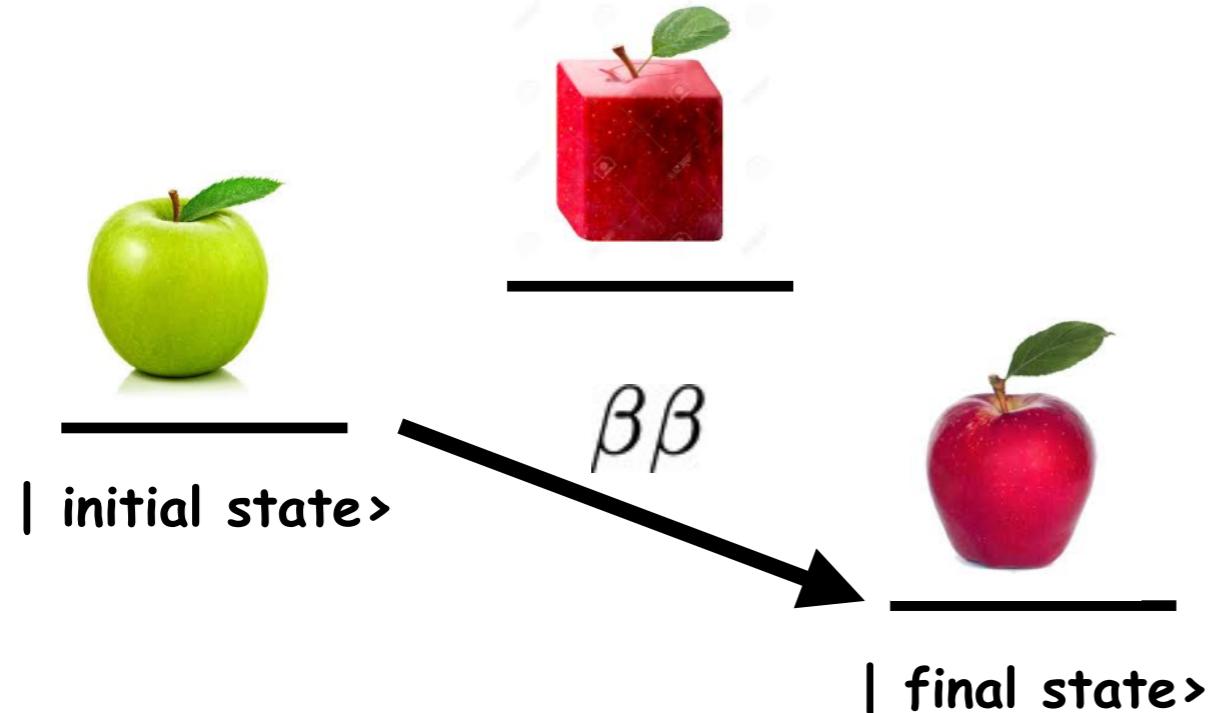
# $0\nu\beta\beta$ 衰变矩阵元的从头计算

尧江明  
中山大学物理与天文学院  
School of Physics and Astronomy  
Sun Yat-sen University

2021年5月21日



# Modeling of the NME for nuclear $0\nu\beta\beta$ decays



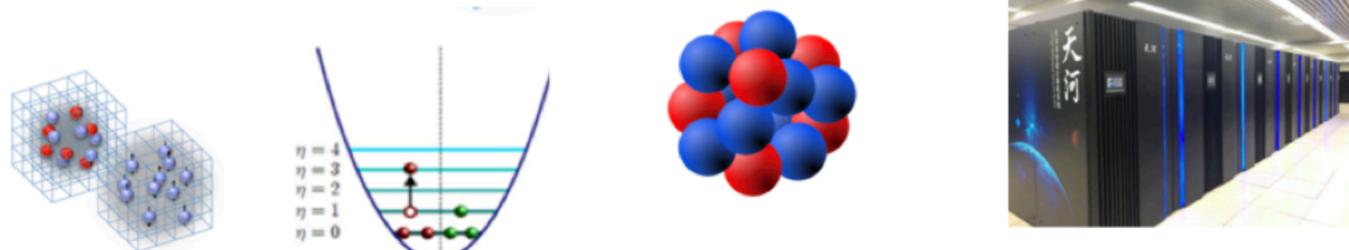
$$[T_{1/2}^{0\nu}]^{-1} = g_A^4 G_{0\nu} \left| \frac{\langle m_{\beta\beta} \rangle}{m_e} \right|^2 |M^{0\nu}|^2$$

Neutrino mass  $m_{\beta\beta} = \sum_k U_{ek}^2 m_k$

NME  $M^{0\nu} = \langle \Psi_F | \hat{O}^{0\nu} | \Psi_I \rangle$

- Nuclear many-body calculations (**challenge**)

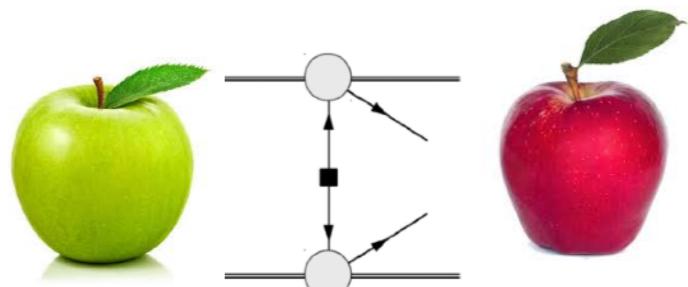
原子核波函数：量子多体计算



北京超级云计算中心  
BEIJING SUPER CLOUD COMPUTING CENTER

- Lepton-number-violating (LNV) mechanism**

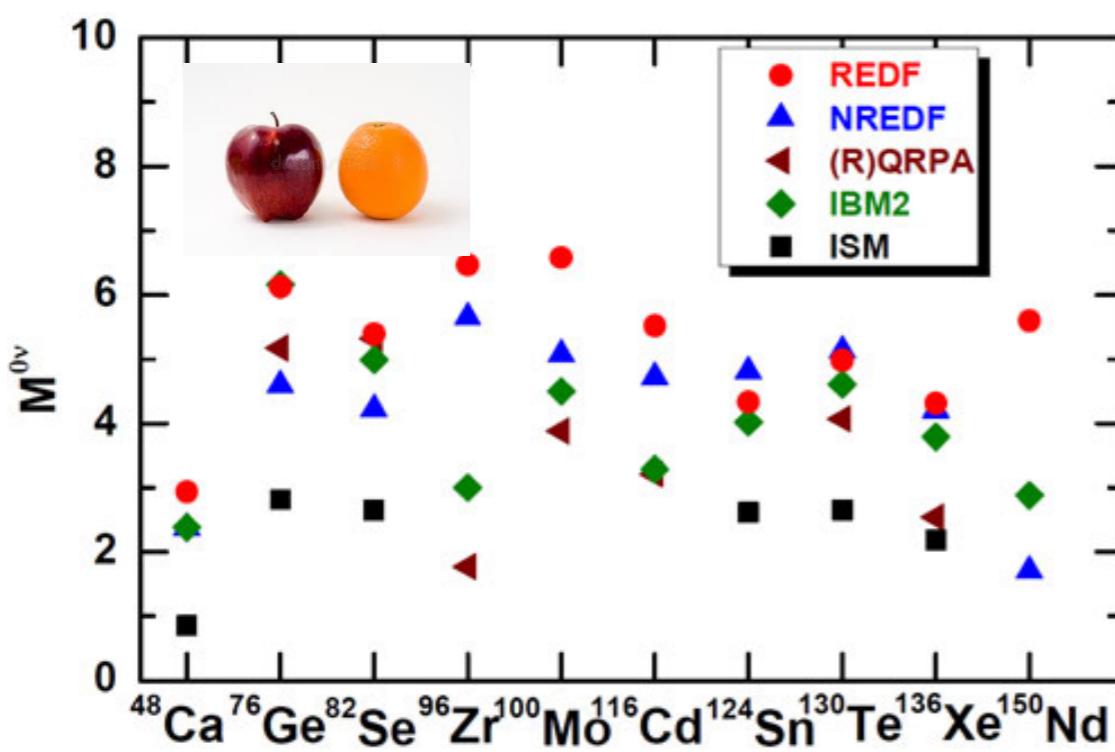
Low-energy effective operators  
in the “standard” mechanism



# Nuclear Matrix Elements of $0\nu\beta\beta$ at the Crossroads



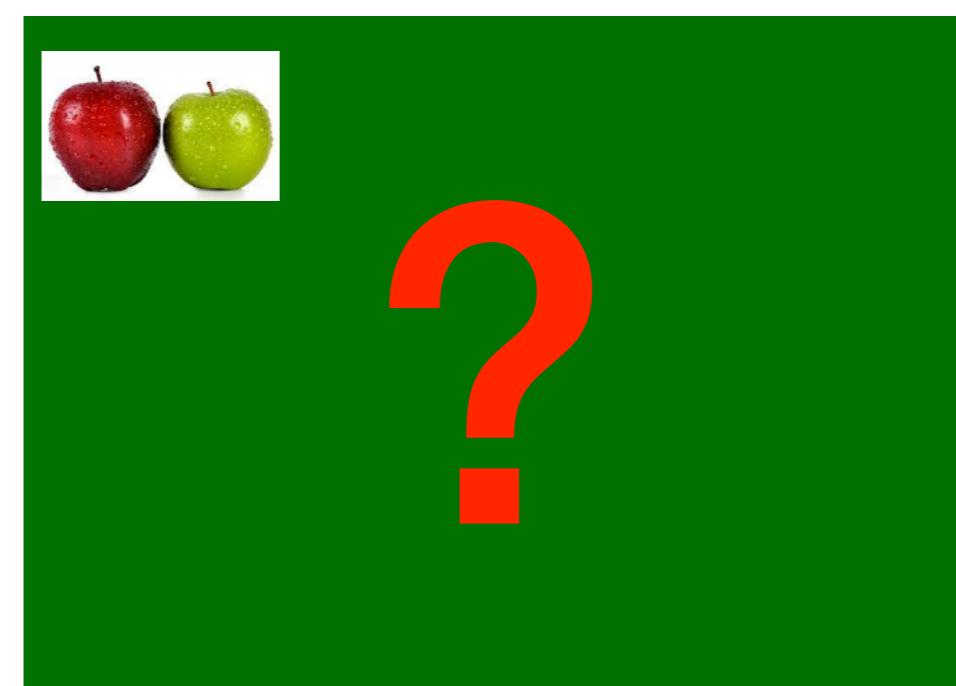
**comparing apples to oranges?**



参看房栋梁的报告

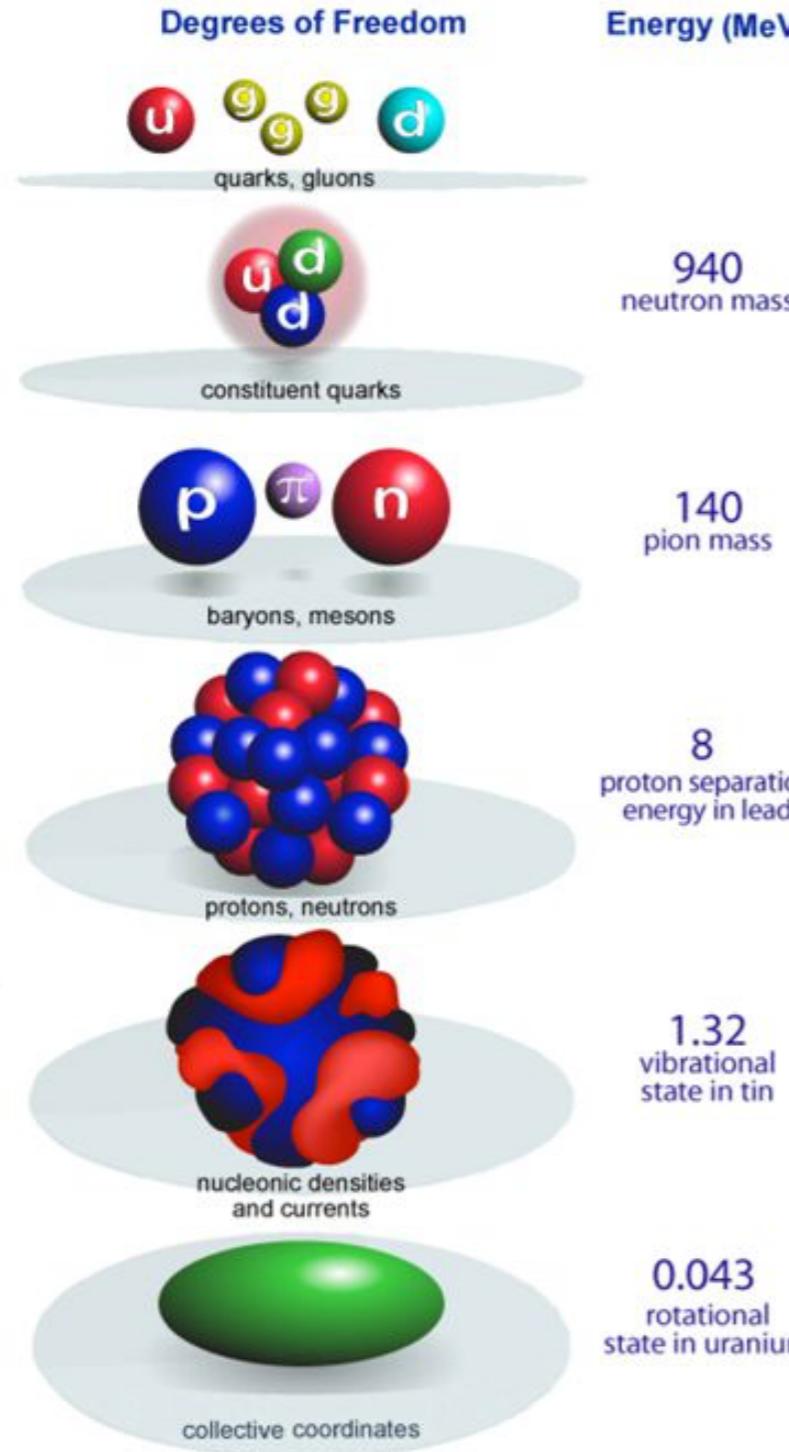


**comparing apples to apples**

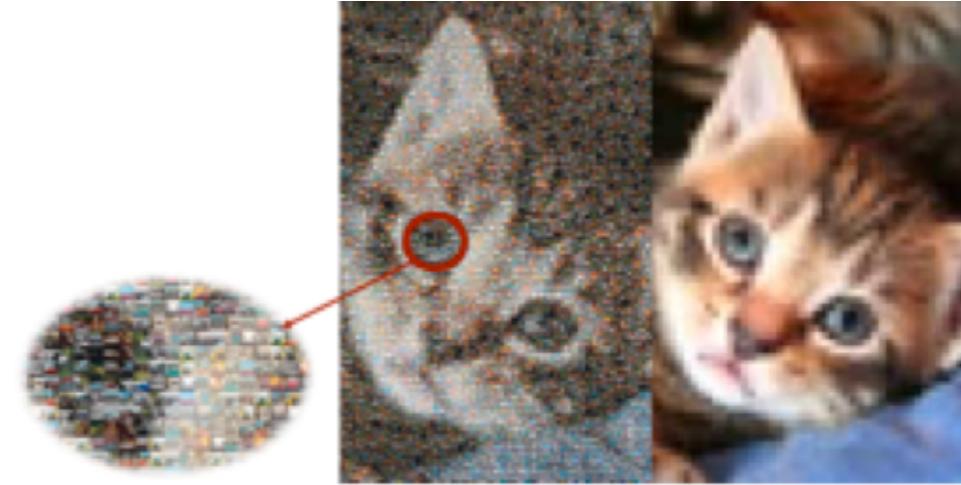


# How to modeling atomic nuclei?

*Physics of Hadrons*



**multi-faceted nuclei**

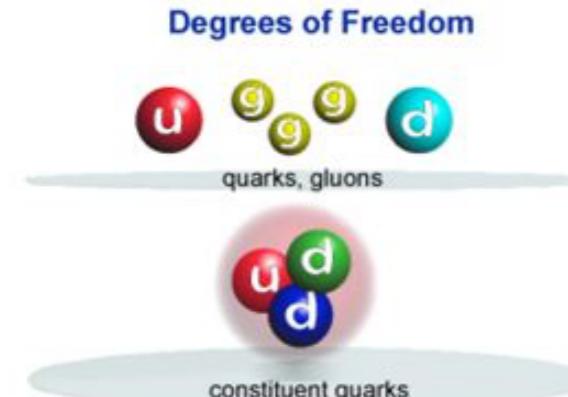


**Image with different resolutions**

**RG equivalent (unitary transformation)**

# How to modeling atomic nuclei?

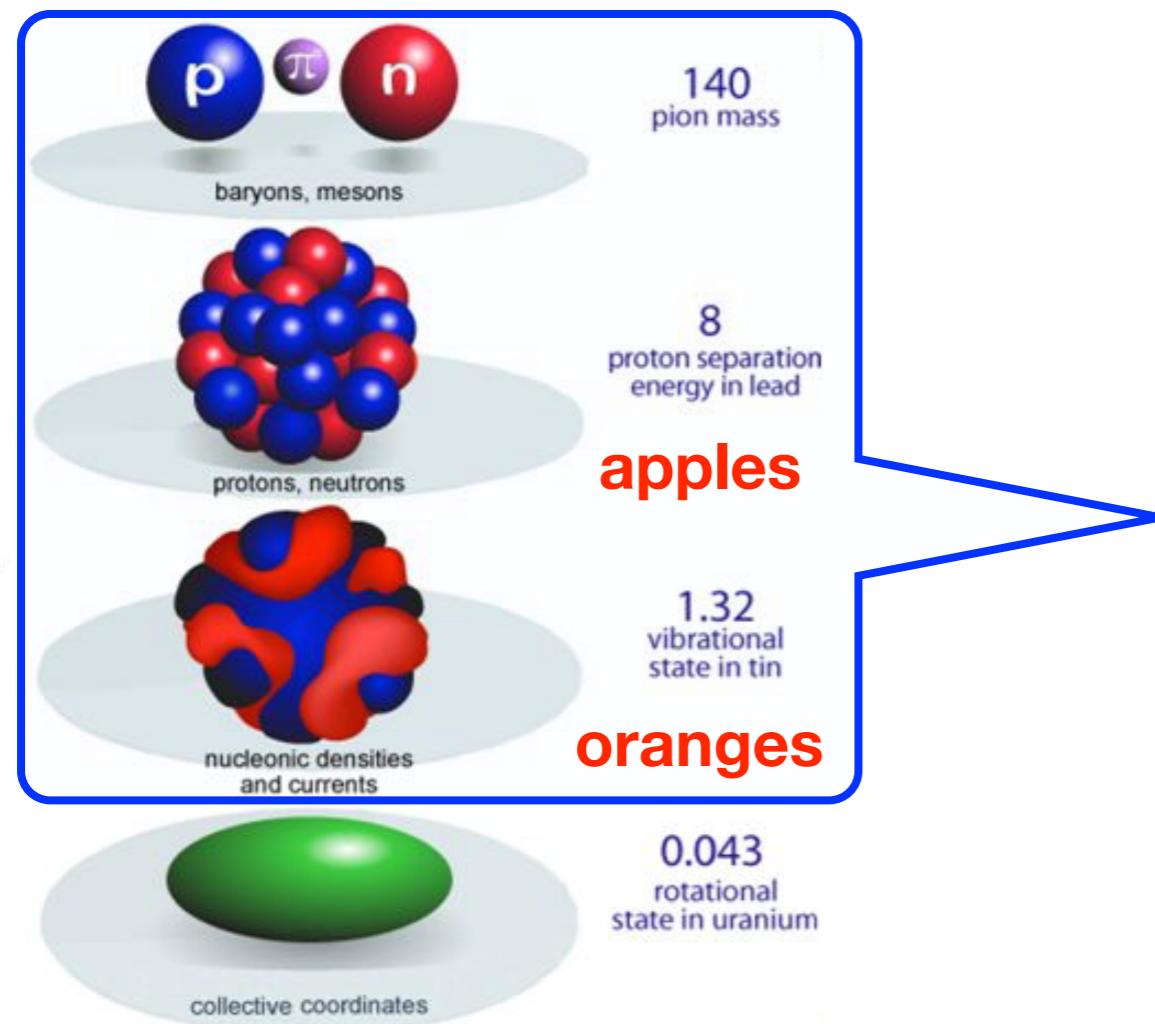
*Physics of Hadrons*



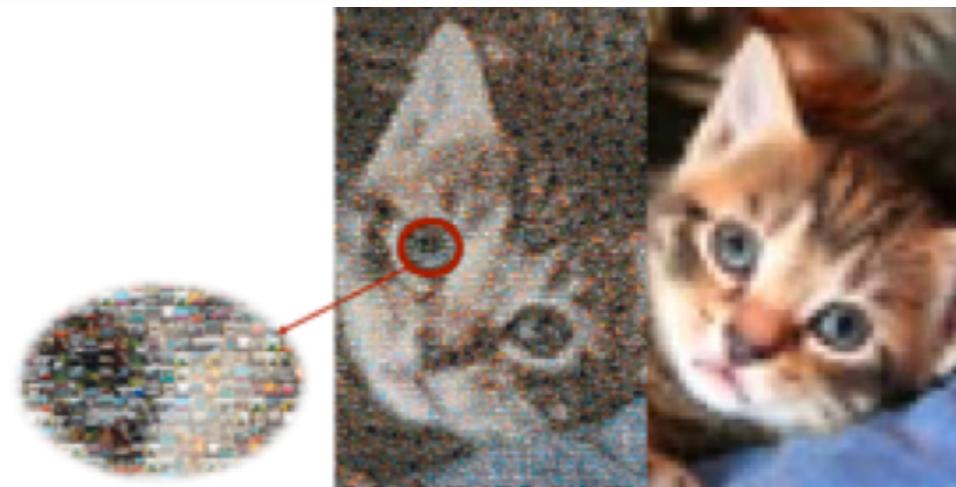
**Energy (MeV)**

940  
neutron mass

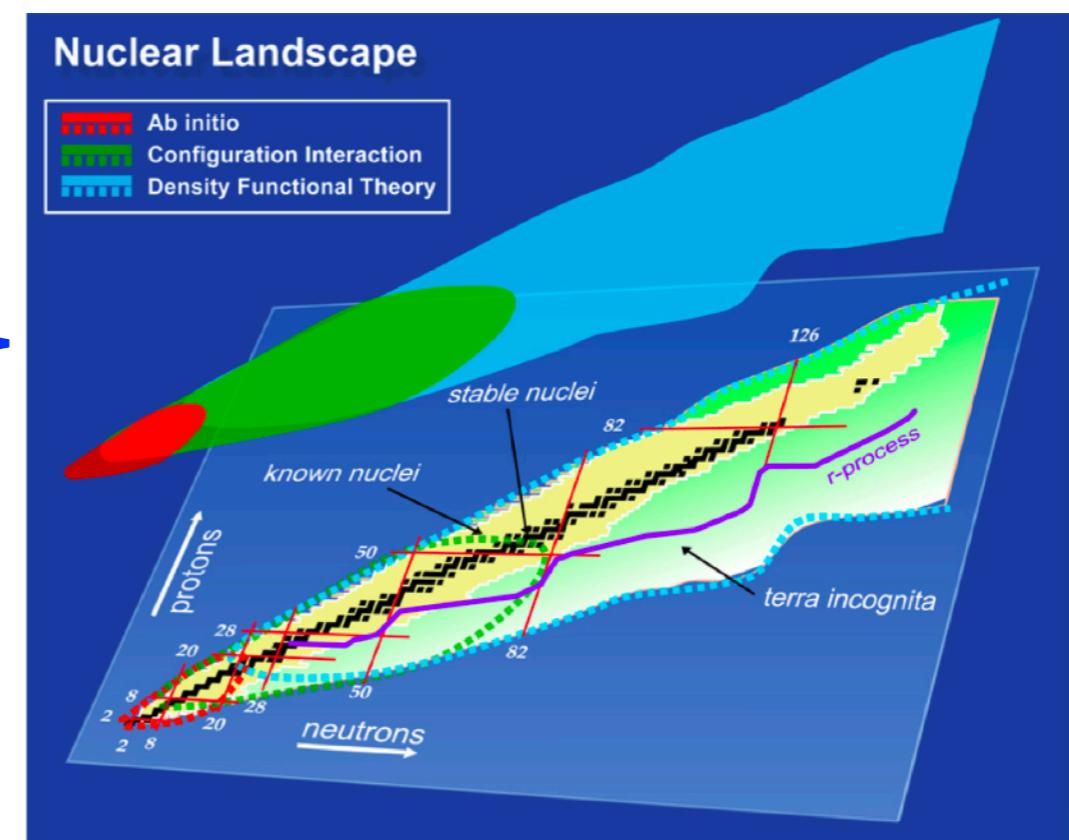
*Physics of Nuclei*



**multi-faceted nuclei**



**Image with different resolutions**



**NOT RG equivalent !**

The Frontiers of Nuclear Science: A Long-Range Plan, 2007.



# Ab initio modeling of nuclear $0\nu\beta\beta$ decays

Our goal is to provide **ab initio calculations of the NMEs (personally)**:

- in nuclear many-body methods with **controllable approximations**
- using **nuclear interactions and weak transition operators derived consistently** from an (chiral) EFT
- with the feature of **order-by-order convergence**.

Clarifications (Three Not Necessaries):

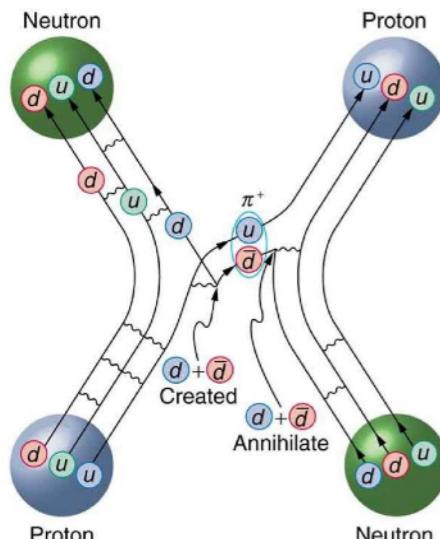
- **Nuclear many-body methods** not necessary to be full configuration-interaction
- **Nuclear force** not necessary to be derived directly from QCD in terms of (q,g)
- **LNV transition operator** not necessary to be derived from a fundamental theory (if any)

This talk will provide a brief overview of

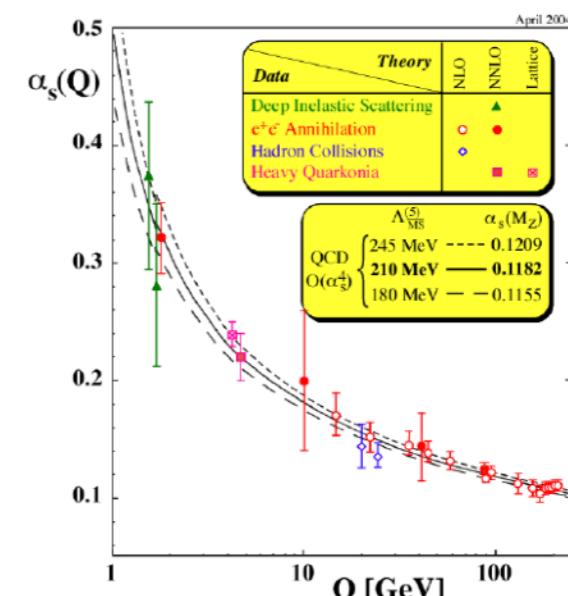
- Advances in ab initio modeling of atomic nuclei (related to  $0\nu\beta\beta$  decay)
- Advances in the determination of leading-order contact transition operators in the “standard” mechanism

# Modeling atomic nuclei from first principles?

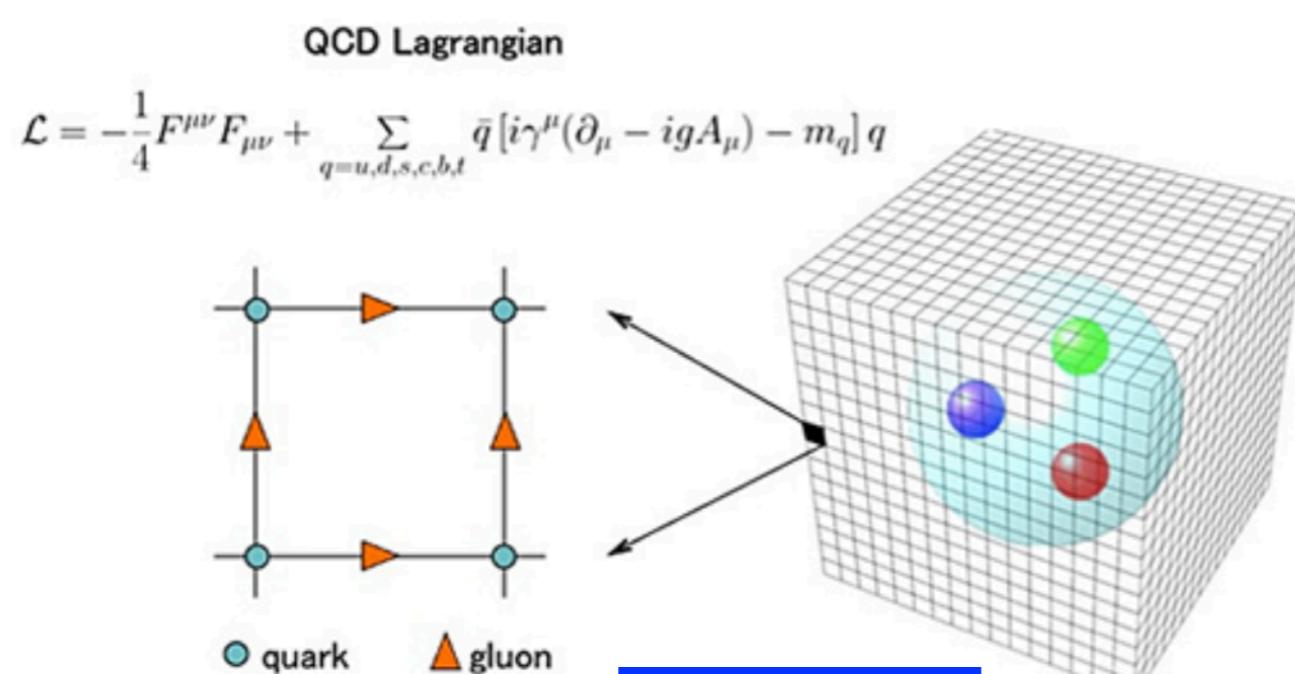
- Construction of nuclear force directly from QCD (**difficult**)



**Quark and gluons:**  
 Non-perturbative nature of  
 strong interaction in the  
 low-energy regime relevant  
 to nuclear physics

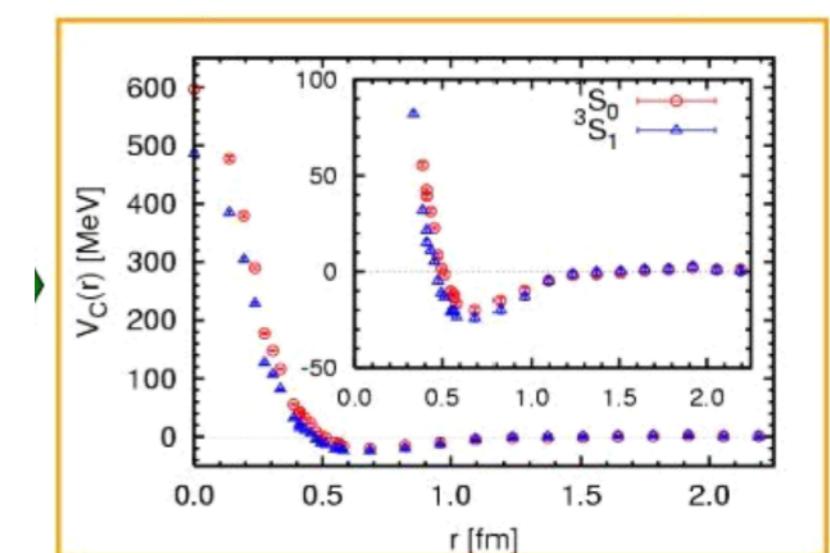


- Nuclear force from Lattice QCD (**infancy**)



参看冯旭的报告

Computation challenge at physical pion mass



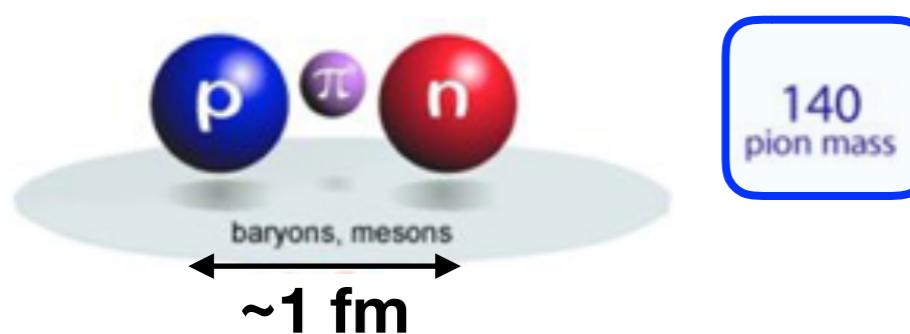
Ishii-Aoki-Hatsuda,  
 PRL99(2007)022001

Workshop on neutrinoless double beta decay

# Modeling atomic nuclei from first principles?

- Nuclear force from the chiral EFT

d.o.f.: nucleons and pions



Weinberg's power counting:

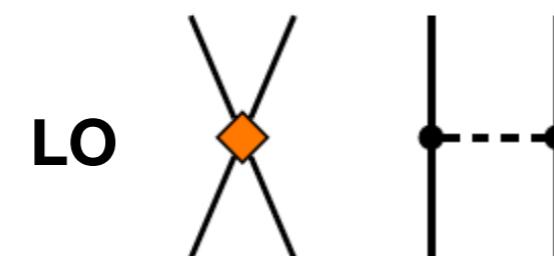
$$(Q/\Lambda_\chi)^\nu$$

chiral-symmetry-breaking  
hard scale (~700 MeV)

soft scale associated with external  
momenta, pion mass (~140 MeV)

S. Weinberg, PLB251, 288 (1990)

S. Weinberg, NPB 363, 3 (1991)



Nuclear structure and reaction observables

*Ab initio* many-body frameworks

Renormalization group methods

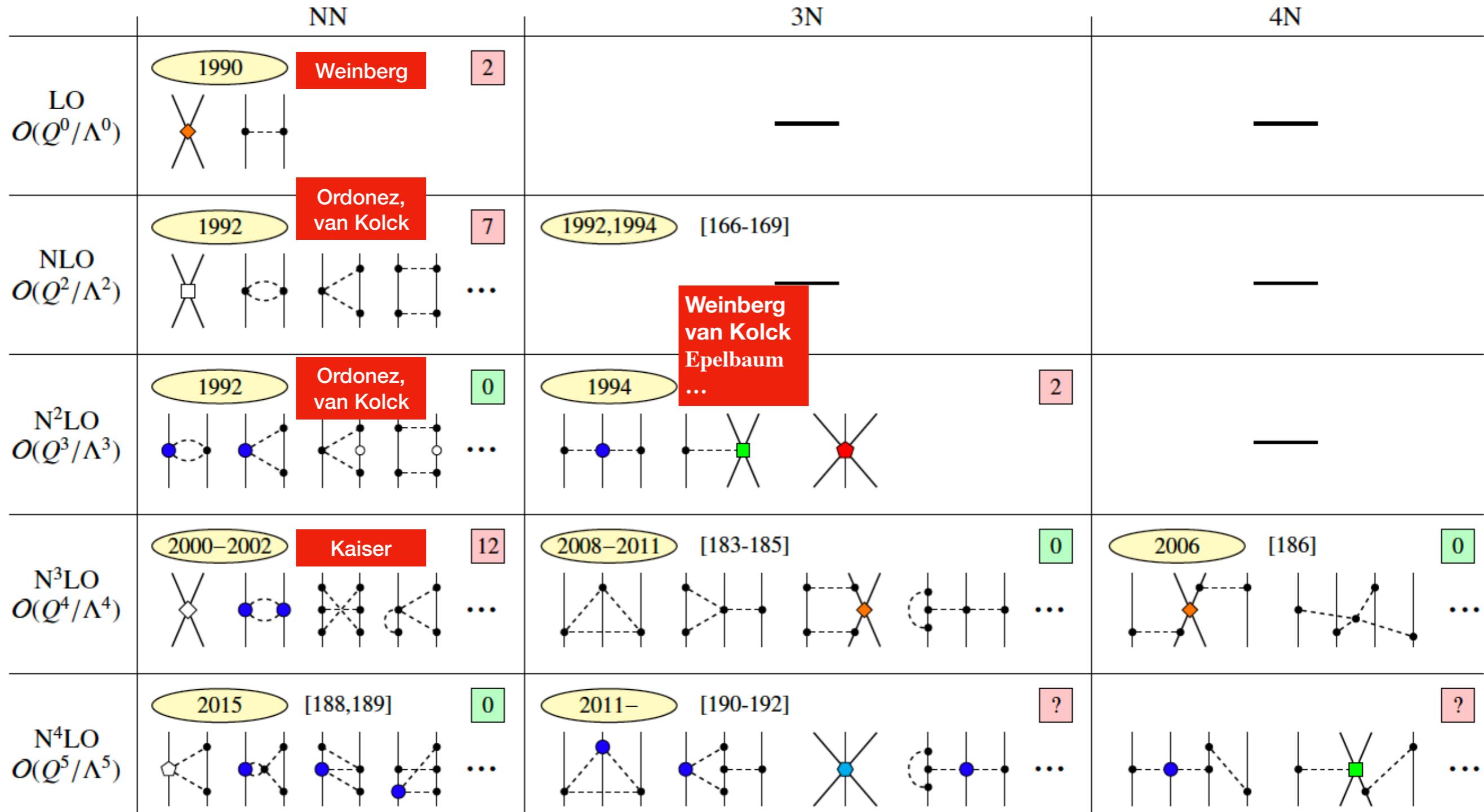
Chiral effective field theory  
nuclear interactions

Quantum chromodynamics

Lattice QCD

K. Hebeler, Phys. Rep. 890, 1 (2020)

# Nuclear force from chiral EFT



# ab initio many-body frameworks

- **Quantum Monte Carlo methods**

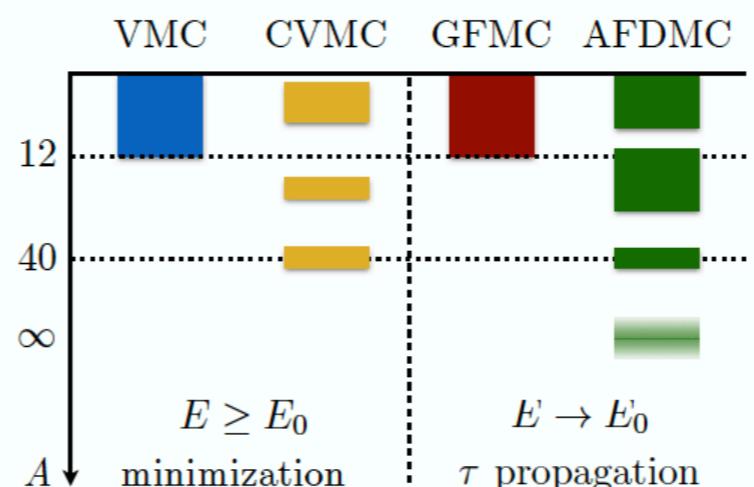
Pieper, S.C.; Wiringa, R.B. (2001)

J. Carlson et al., RMP 87, 1067 (2015)

Variational Monte Carlo (VMC)

Green's function Monte Carlo (GFMC)

Auxiliary-field diffusion Monte Carlo (AFDMC)



credit: D. Lonardoni

- **Lattice effective field theory (LEFT)**

D. Lee, Prog. Part. Nucl. Phys. 63, 117 (2009)

- **No-core shell model (NCSM)**

Barrett, Navrátil, Vary, Prog. Part. Nucl. Phys. 69, 131 (2013)

- **Self-consistent Green's function (SCGF)**

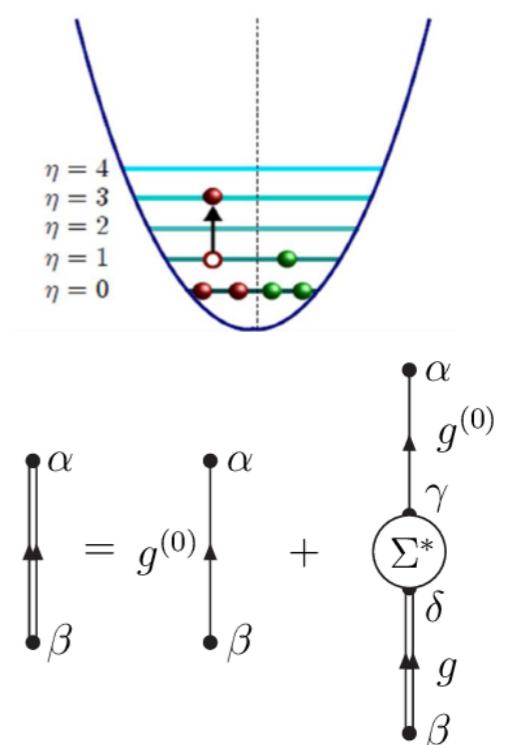
V. Somà, Frontiers in Physics 8, 340 (2020)

- **Coupled cluster (CC)**

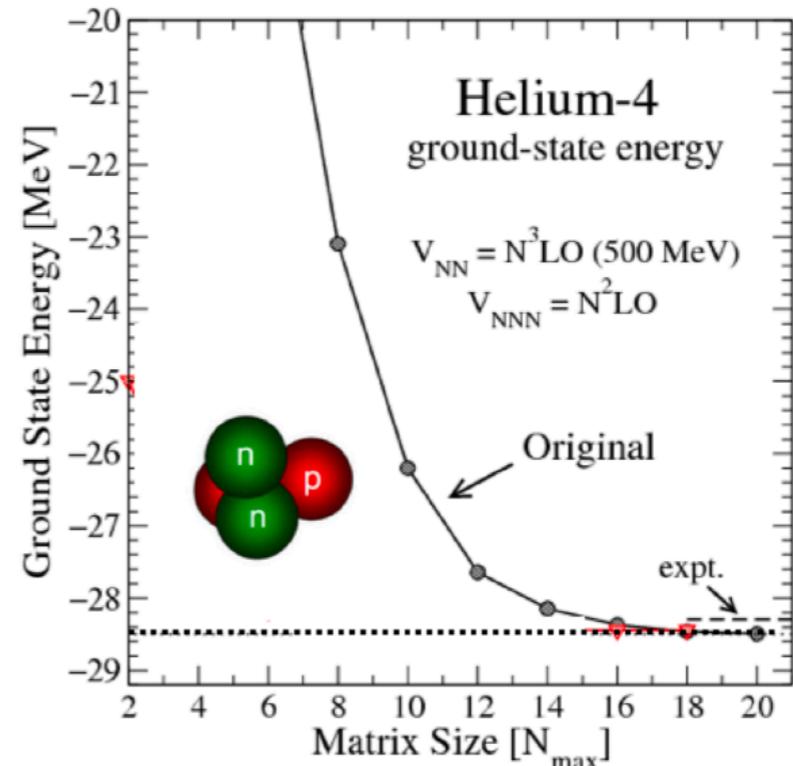
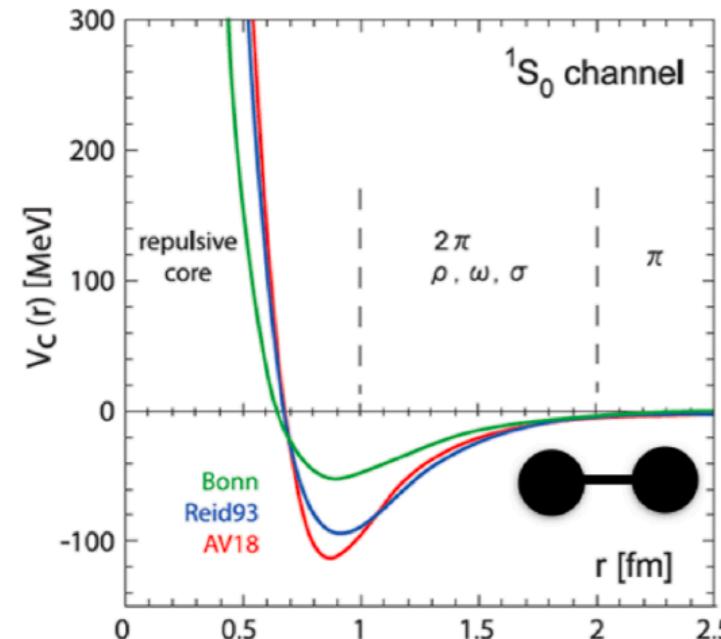
G. Hagen, T. Papenbrock, M. Hjorth-Jensen, and D. J. Dean, Rep. Prog. Phys. 77, 096302 (2014)

- **In-medium similarity renormalization group (IM-SRG)**

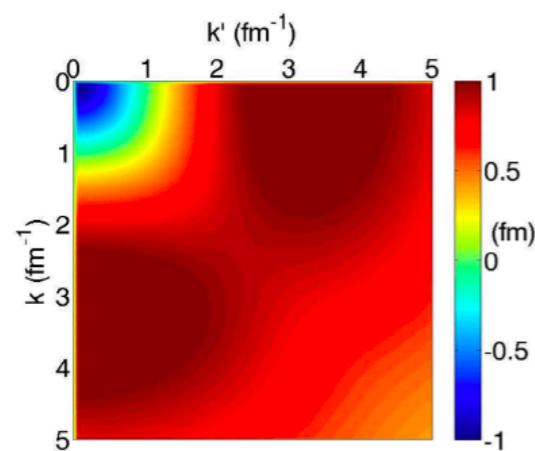
H. Hergert, S. K. Bogner, T. D. Morris, A. Schwenk, and K. Tsukiyama, Phys. Rep. 621, 165 (2016)



# Realistic nuclear force: challenge



$$V_{\ell=0}(k, k') = \int d^3r j_0(kr) V(r) j_0(k'r)$$



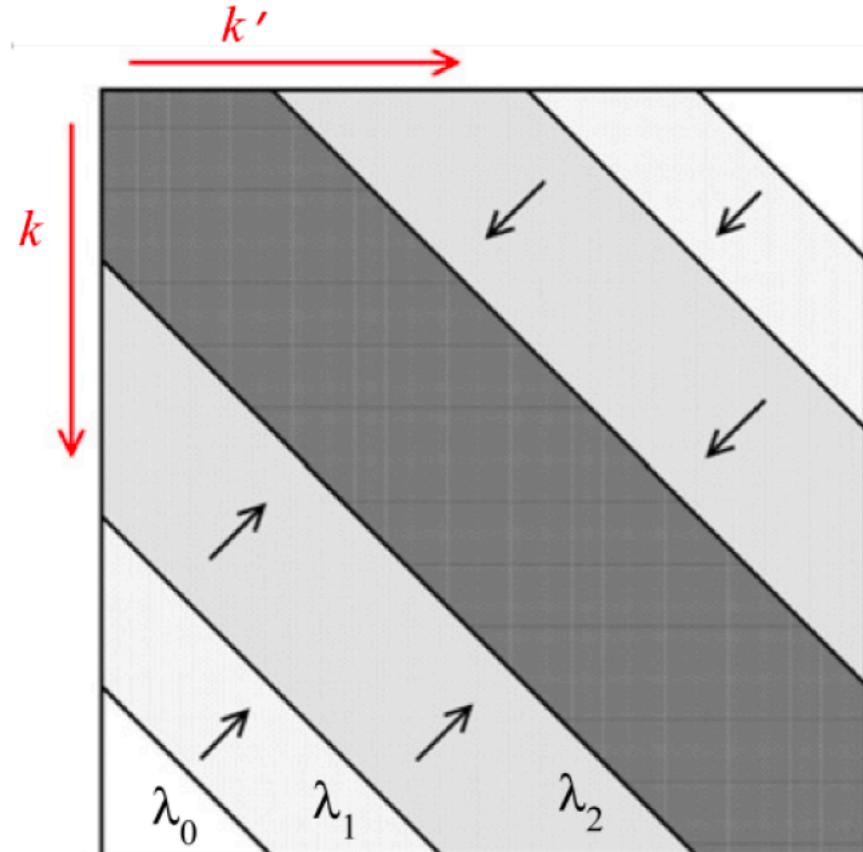
S. Bogner et al., PPNP (2010)

- Repulsive core & strong tensor force => low and high  $k$  modes strongly coupled by the interaction
- non-perturbative, poorly convergent basis expansions (cutoff  $\Lambda$ , No. of s.p. states  $D$ )

$$\text{Dim}(H) \sim \frac{D!}{(D-A)!A!}, \quad D \sim \Lambda^3 A \quad A \sim R^3$$

For  $\Lambda = 4.0 \text{ fm}^{-1}$ ,  $A = 16$ ,  $\text{Dim}(H) \sim 10^{14}$ .

# Realistic nuclear force: SRG



The flow parameter  $s$  is usually replaced with  $\lambda = s^{-1/4}$  with units of  $\text{fm}^{-1}$ .

S. K. Bogner, R. J. Furnstahl, and  
R. J. Perry (2007)

- Apply unitary transformations to Hamiltonian

$$H_s = U_s H U_s^\dagger \equiv T_{\text{rel}} + V_s \quad (1)$$

- Flow equation

$$\frac{dH_s}{ds} = [\eta_s, H_s], \quad (2)$$

where the generator  $\eta_s$  is chosen to diagonalize  $H(s)$  in the eigenbasis of  $T_{\text{rel}}$ ,

$$\eta_s = [T_{\text{rel}}, H_s] \quad (3)$$

$$\begin{aligned} \frac{dV_s(k, k')}{ds} &= -(k^2 - k'^2)V_s(k, k') \\ &+ \frac{2}{\pi} \int_0^\infty q^2 dq (k^2 + k'^2 - 2q^2)V_s(k, q)V_s(q, k') \end{aligned}$$

# Realistic nuclear force: SRG

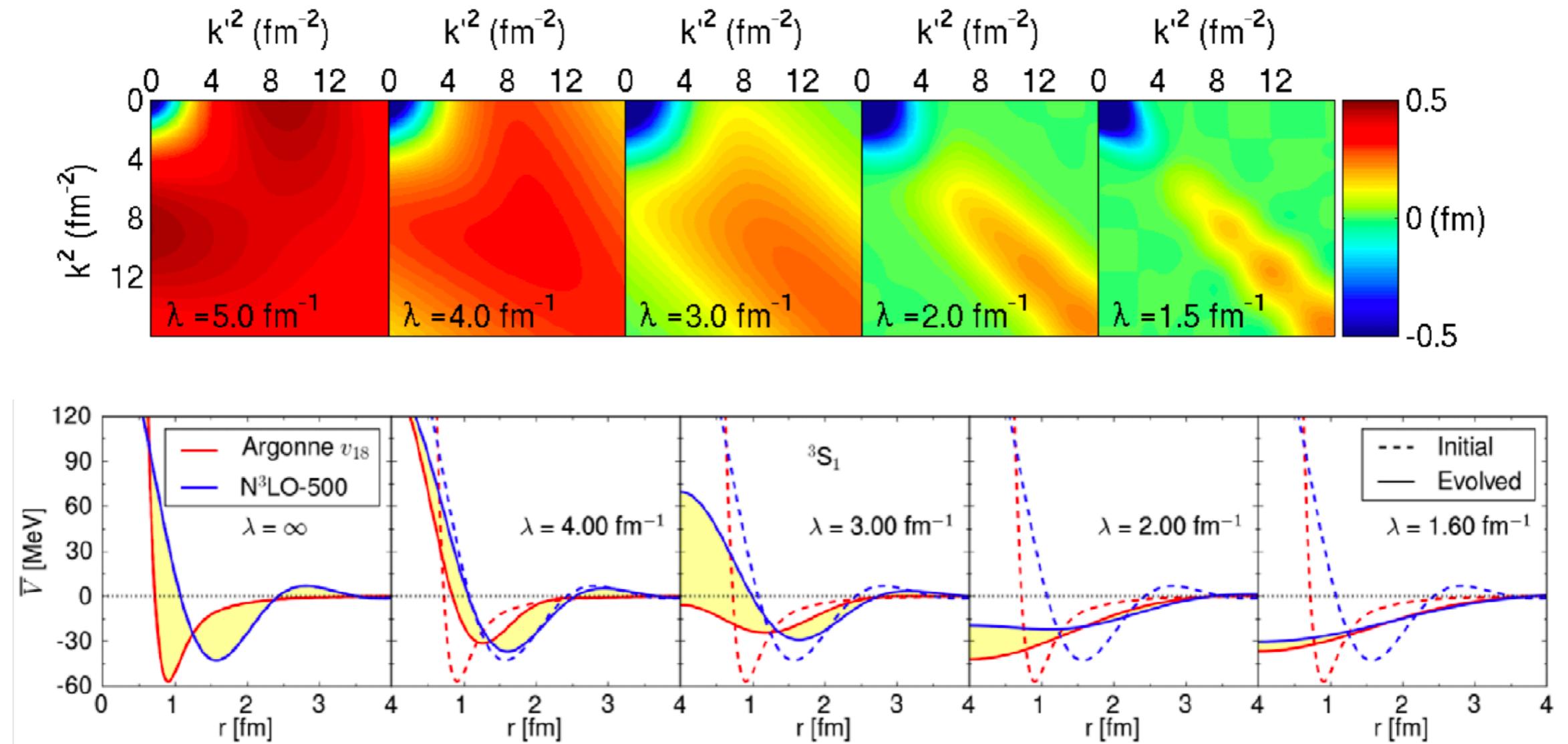


Figure: Local projection of AV18 and  $N^3\text{LO}(500 \text{ MeV})$  potentials  $V(r)$  in  $^3S_1$  channel.

- “Hard core” disappears in the softened interactions
- S. K. Bogner et al. (2010); Wendt et al. (2012)

# NCSM: exponential growth of the model space

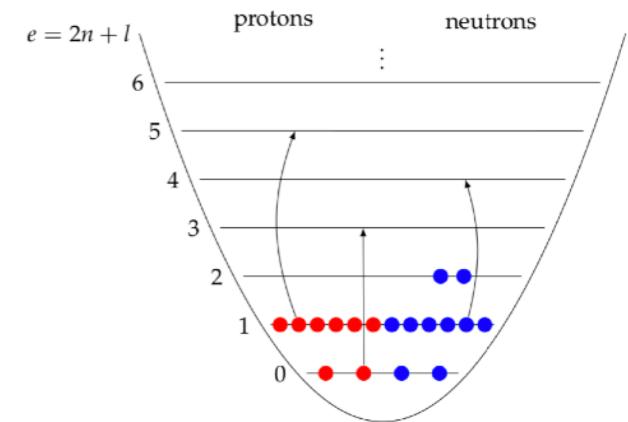
## ■ The A-body Schroedinger equation

$$H|\Psi\rangle = E|\Psi\rangle,$$

## ■ The wave function is expanded in terms of many-body basis states

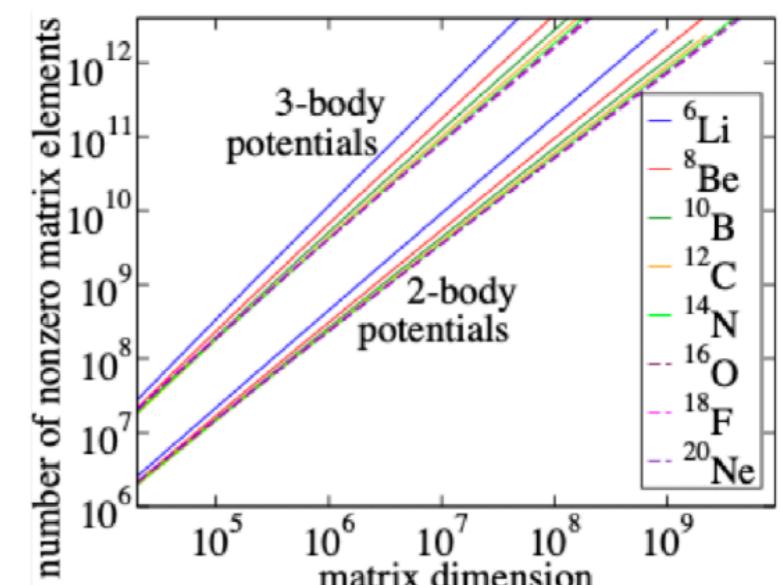
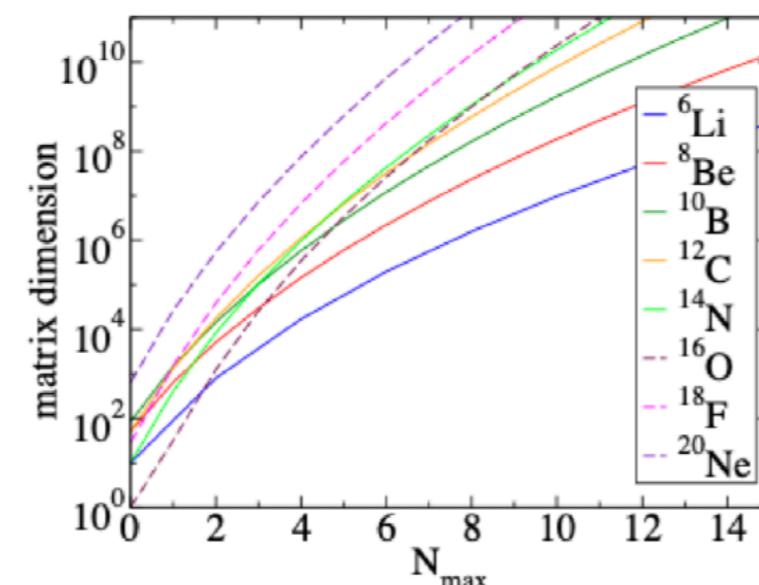
$$|\Psi\rangle = \sum_{\mu} c_{\mu} |\Phi_{\mu}\rangle,$$

where  $c_{\mu}$  is to be determined from the diagonalization of the  $H$ .  $|\Phi_{\mu}\rangle$  is a Slater Determinant of single-particle states occupied by the nucleons.



Dimension:

$$D \sim \begin{pmatrix} \Omega_{\pi} \\ N_{\pi} \end{pmatrix} \begin{pmatrix} \Omega_{\nu} \\ N_{\nu} \end{pmatrix}$$



**Computation challenge**

from: C. Yang, H. M. Aktulga, P. Maris, E. Ng, J. Vary, Proceedings of NTSE-2013

# In-medium similarity renormalization group (IMSRG)

- A set of continuous **unitary transformations** onto the Hamiltonian

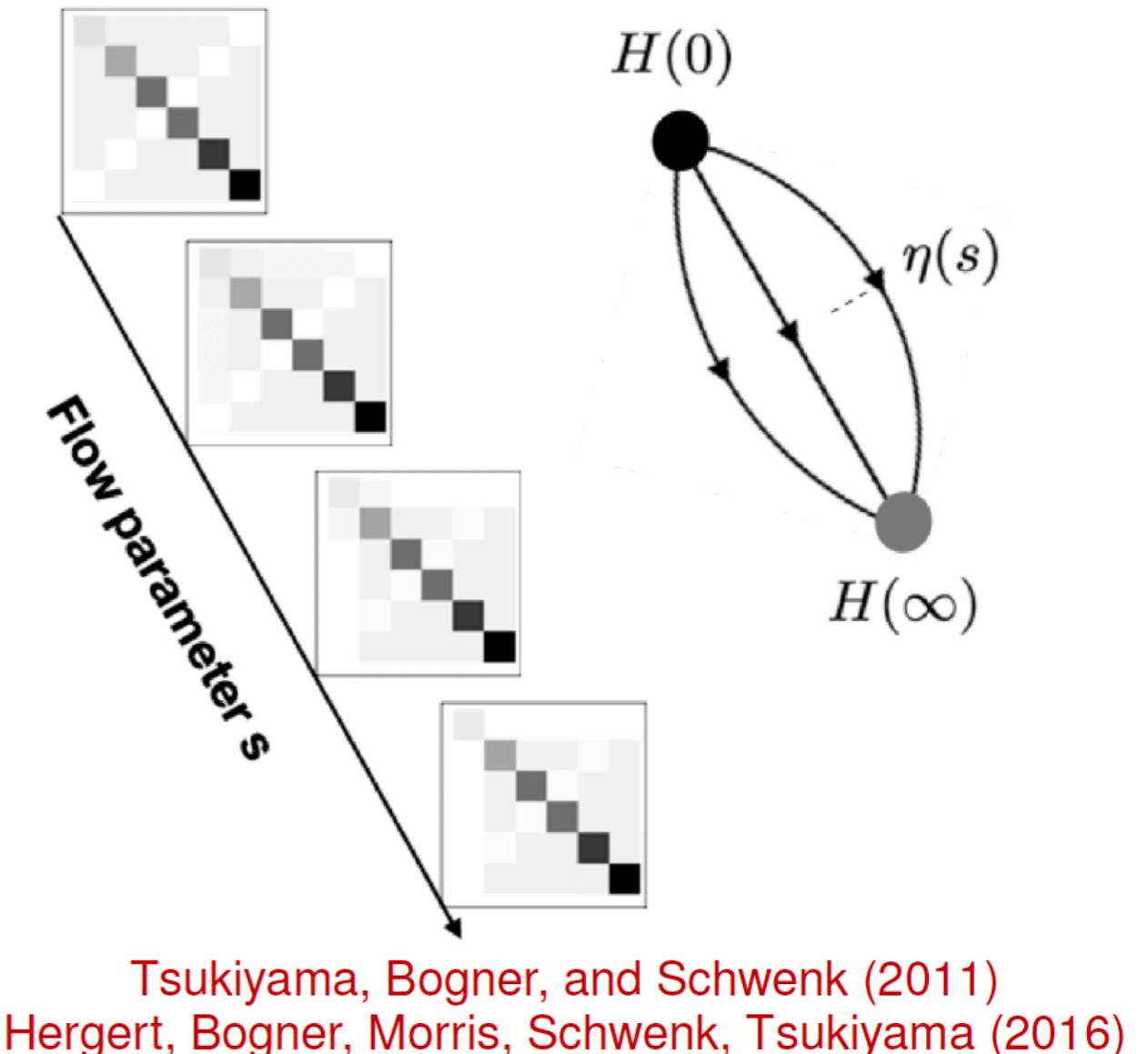
$$H(s) = U(s)H_0U^\dagger(s)$$

- Flow equation for the Hamiltonian

$$\frac{dH(s)}{ds} = [\eta(s), H(s)]$$

where the  $\eta(s) = \frac{dU(s)}{ds}U^\dagger(s)$  is the so-called generator chosen to decouple a given **reference state** from its excitations.

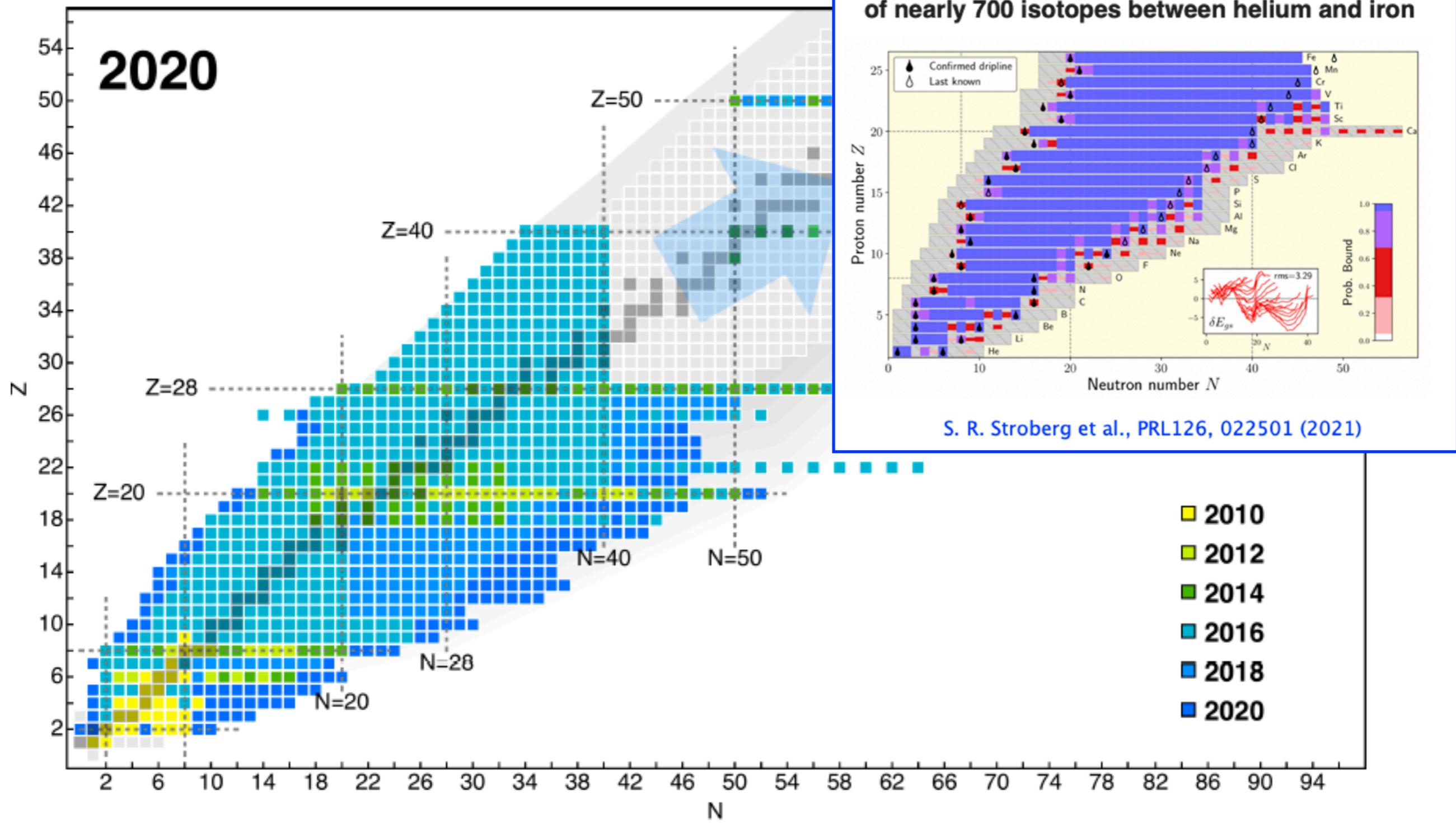
- Computation complexity scales **polynomially** with nuclear size



Not necessary to construct the H matrix elements in many-body basis !

# Achievements of ab initio calculations for nuclei

With the implementation of the SRG and IMSRG,



H. Hergert, Front. Phys. 8, 379 (2020)

# ab initio calculations of nuclear single-beta decay

## g<sub>A</sub> quenching in GT transition

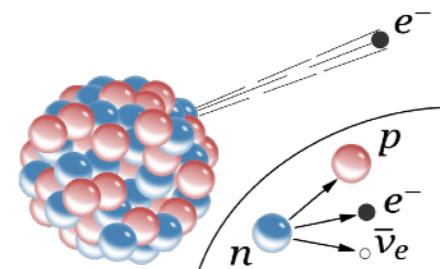
参看王龙军的报告

### Discrepancy between experimental and theoretical β-decay rates resolved from first principles

P. Gysbers, G. Hagen , J. D. Holt, G. R. Jansen, T. D. Morris, P. Navrátil, T. Papenbrock, S. Quaglioni, A. Schwenk, S. R. Stroberg & K. A. Wendt

Nature Physics 15, 428–431(2019) | Cite this article

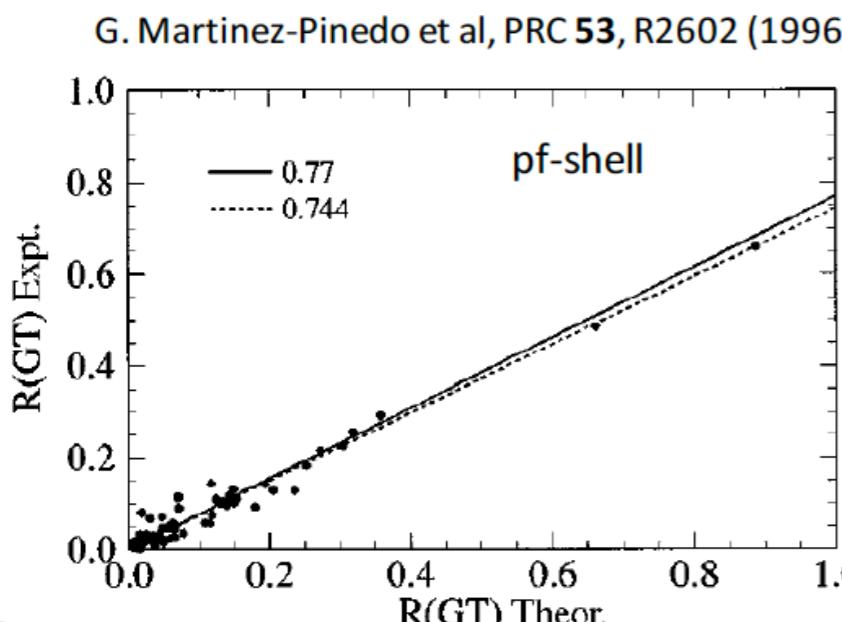
Two-body currents+  
many-body correlations



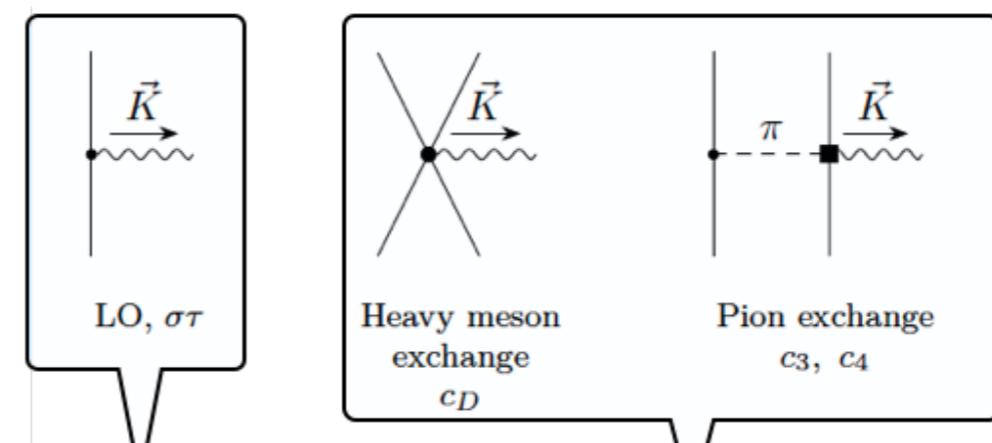
- The half-life of single-beta decay

$$t_{1/2} = \frac{\kappa}{f_0(B_F + B_{GT})},$$

$$B_F = \frac{g_V^2}{2J_i + 1} |M_F|^2, \quad B_{GT} = \frac{g_A^2}{2J_i + 1} |M_{GT}|^2$$



- charge-changing axial-vector current



$$\vec{J}^A(\vec{K}) = \sum_j i g_A \sigma_j \tau_j^\pm e^{i \vec{K} \cdot \vec{r}_j}.$$

2B currents

Park, T.-S. et al. Phys. Rev. C 67, 055206 (2003)

- GT transition operator

$$O_{GT} = O_{\sigma\tau}^{1b} + O_{2BC}^{2b}.$$

# ab initio calculations of nuclear single-beta decay

## g<sub>A</sub> quenching in GT transition

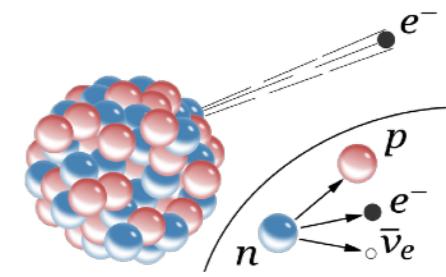
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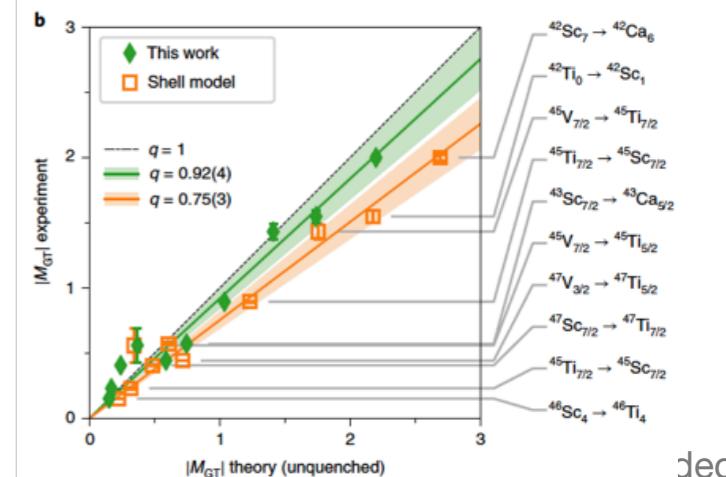
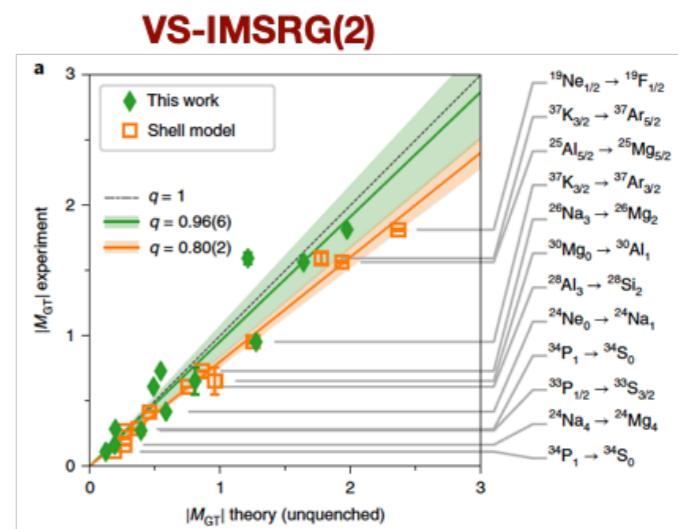
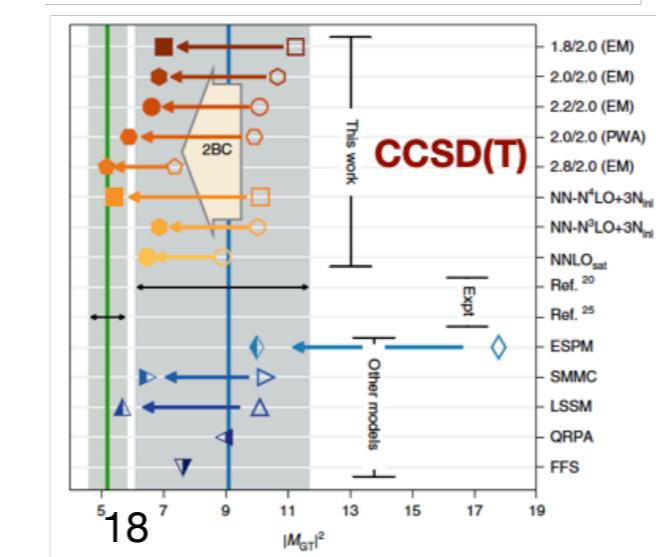
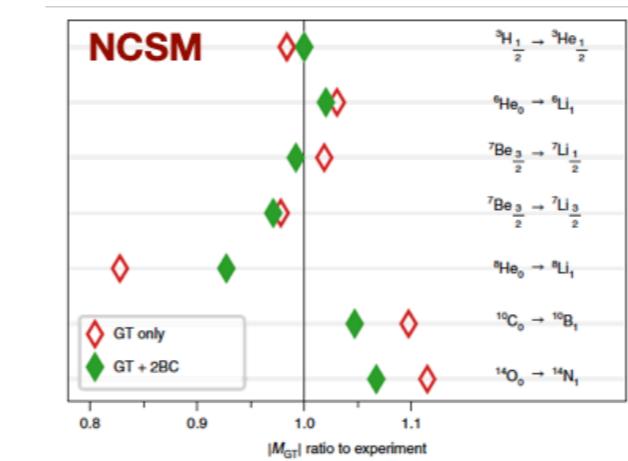
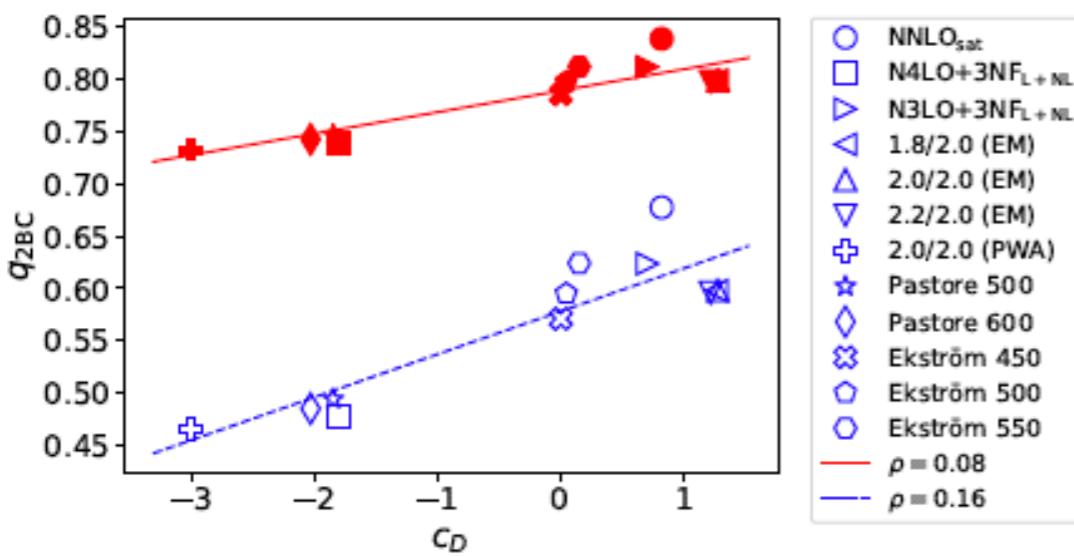
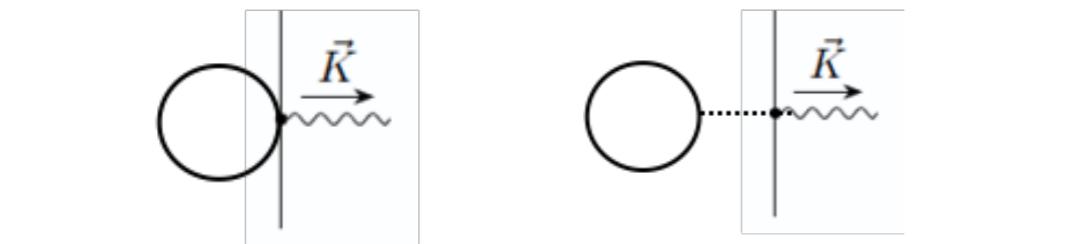
Nature Physics 15, 428–431(2019) | Cite this article

Two-body currents+  
many-body correlations



#### • Intuitive picture

Normal-ordering the 2BC w.r.t nuclear matter of two diff. density rho.



decay

# Advances in ab initio modeling of $0\nu\beta\beta$ -decay NME

Ab initio calculations of  $0\nu\beta\beta$ -decay candidate nuclei and corresponding NME of the decays

In-medium similarity renormalization group (IMSRG)+Generator coordinate method (GCM)

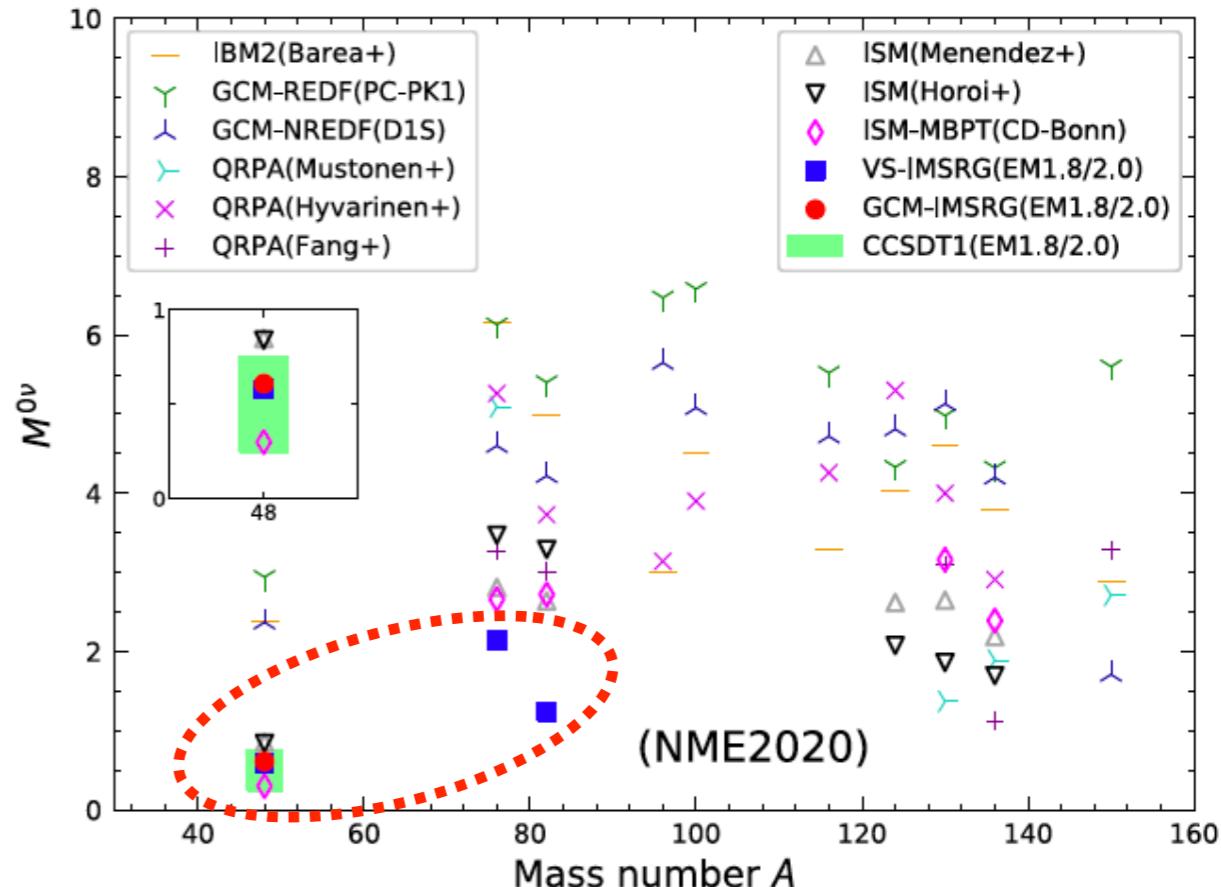
JMY et al., PRL124, 232501 (2020)

Valence-space IMSRG+ interacting-shell-model (ISM)

A. Belley et al., PRL126, 042502 (2021)

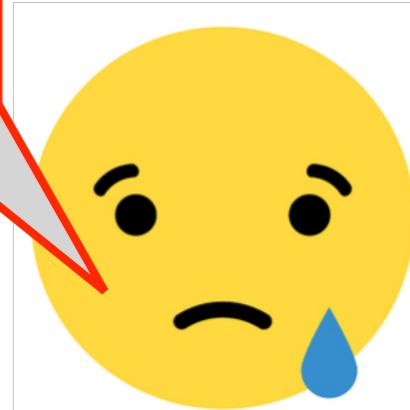
Coupled cluster (CC)

S. Novario et al., PRL126, 182502 (2021)



JMY, Science Bulletin (2021)

The NMEs by the three ab-initio methods consistently **smaller** than other phenomenological methods.



# Advances in ab initio modeling of $0\nu\beta\beta$ -decay NME

Benchmark calculations for light nuclei for which (quasi)-exact solution is possible

**cross-checking among different models**

✓ Quantum Monte Carlo vs **shell model**

X. Wang et al., PLB 798, 134974 (2019)

✓ NCSM vs IMSRG

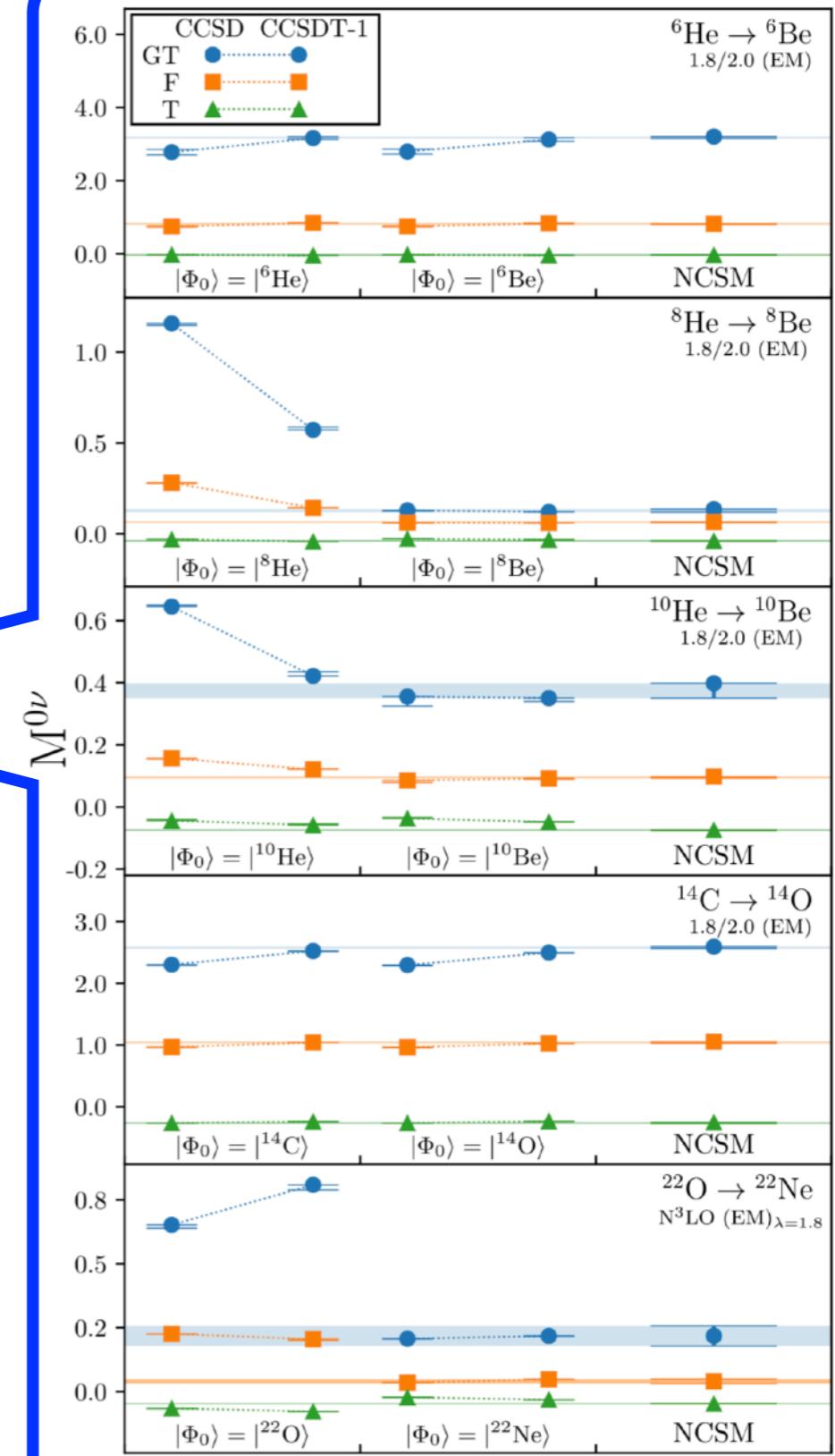
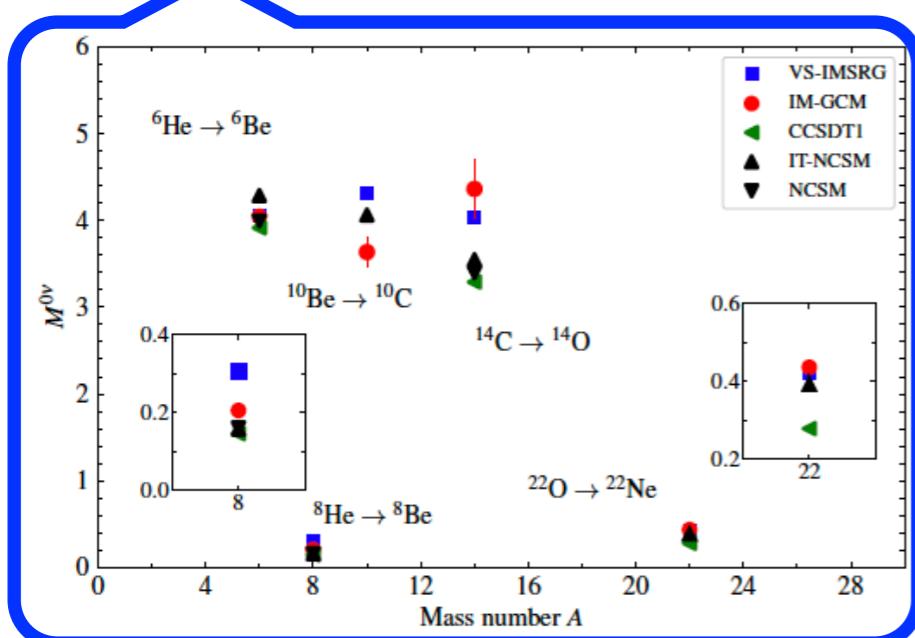
R. A. M. Basili et al., PRC102, 014302 (2020).

✓ NCSM vs CC

S. Novario et al., PRL126, 182502 (2021)

✓ NCSM vs IT-NCSM vs CC vs VS-IMSRG vs IM-GCM

JMY et al., PRC103, 014315 (2021)



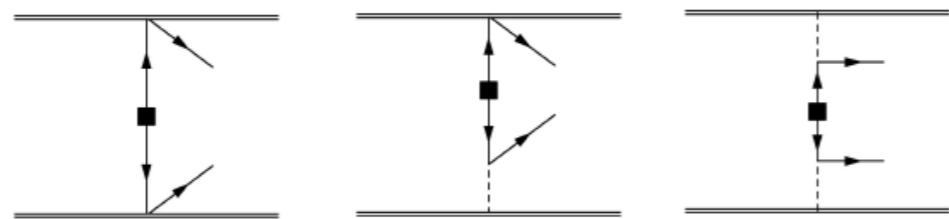
# Advances in ab initio modeling of $0\nu\beta\beta$ -decay NME

- The  $0\nu\beta\beta$ -decay in chiral EFT based on the “standard” mechanism of light Majorana neutrino exchange      [V. Cirigliano et al., PRC97, 065501 \(2018\)](#)

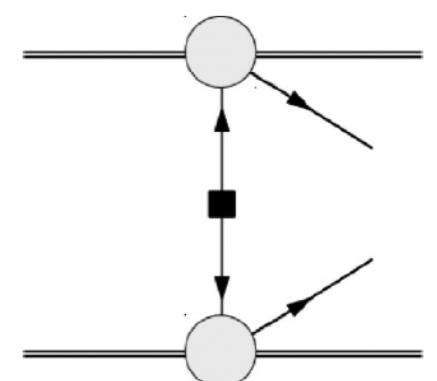
## ✓ Chiral expansion of neutrino potentials

$$V_\nu = \sum_{a \neq b} V_{\nu,0}^{(a,b)} + V_{\nu,2}^{(a,b)} + \dots.$$

**LO**



Pions and neutrinos integrated out



$$\begin{aligned} V_{\nu,0}^{(a,b)} &= \tau^{(a)+} \tau^{(b)+} \frac{1}{\mathbf{q}^2} \left\{ 1 - g_A^2 \right. \\ &\times \left. \left[ \boldsymbol{\sigma}^{(a)} \cdot \boldsymbol{\sigma}^{(b)} - \boldsymbol{\sigma}^{(a)} \cdot \mathbf{q} \boldsymbol{\sigma}^{(b)} \cdot \mathbf{q} \frac{2m_\pi^2 + \mathbf{q}^2}{(\mathbf{q}^2 + m_\pi^2)^2} \right] \right\} \end{aligned}$$

# Advances in ab initio modeling of $0\nu\beta\beta$ -decay NME

## ✓ Chiral expansion of neutrino potentials

V. Cirigliano et al., PRC97, 065501 (2018)

$$V_\nu = \sum_{a \neq b} (V_{\nu,0}^{(a,b)} + V_{\nu,2}^{(a,b)} + \dots).$$

**N<sup>2</sup>LO**

- N<sup>2</sup>LO contributions to single-nucleon currents are usually taken into account by introducing **dipole form factors**,

$$V_{\nu,0}^{(a,b)} = \tau^{(a)+} \tau^{(b)+} \frac{1}{\mathbf{q}^2} g_A^2 \{ h_F(\mathbf{q}^2)/g_A^2 - \sigma^{(a)} \cdot \sigma^{(b)} h_{GT}(\mathbf{q}^2) \\ - S^{(ab)} h_T(\mathbf{q}^2) \},$$

dipole form factors

$$g_V(q) = g_V \left(1 + \frac{q^2}{\Lambda_V^2}\right)^{-2}, \quad g_A(q) = g_A \left(1 + \frac{q^2}{\Lambda_A^2}\right)^{-2},$$

$$g_M(q) = (1 + \kappa_1)g_V(q), \quad g_P(q) = -\frac{2m_N g_A(q)}{q^2 + m_\pi^2}.$$

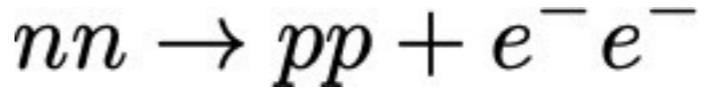
- Genuine N<sup>2</sup>LO contributions from loops corrections to the LO diagram (induce **short-range neutrino potential**) are NOT considered yet

$$V_{\nu,2}^{(a,b)} = \tau^{(a)+} \tau^{(b)+} \times \left( \mathcal{V}_{VV}^{(a,b)} + \mathcal{V}_{AA}^{(a,b)} + \tilde{\mathcal{V}}_{AA}^{(a,b)} \ln \frac{m_\pi^2}{\mu_{us}^2} + \mathcal{V}_{CT}^{(a,b)} \right). \quad \text{CT at N<sup>2</sup>LO}$$

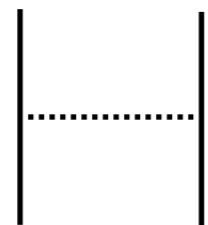
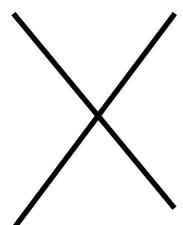
$$\mathcal{V}_{CT}^{(a,b)} = \frac{g_A^2}{(4\pi F_\pi)^2} \frac{\sigma^{(a)} \cdot \mathbf{q} \sigma^{(b)} \cdot \mathbf{q}}{m_\pi^2} \left[ \frac{5}{6} g_\nu^{\pi\pi} \frac{\hat{q}}{(1+\hat{q})^2} - g_\nu^{\pi N} \frac{1}{1+\hat{q}} \right] = \frac{2g_\nu^{NN}}{(4\pi F_\pi)^2} \mathbf{1}^{(a)} \times \mathbf{1}^{(b)}$$

# Advances in ab initio modeling of $0\nu\beta\beta$ -decay NME

## ► Transition amplitude of the process (LO)



V. Cirigliano et al., PRL120, 202001 (2018); PRC97,065501 (2019)



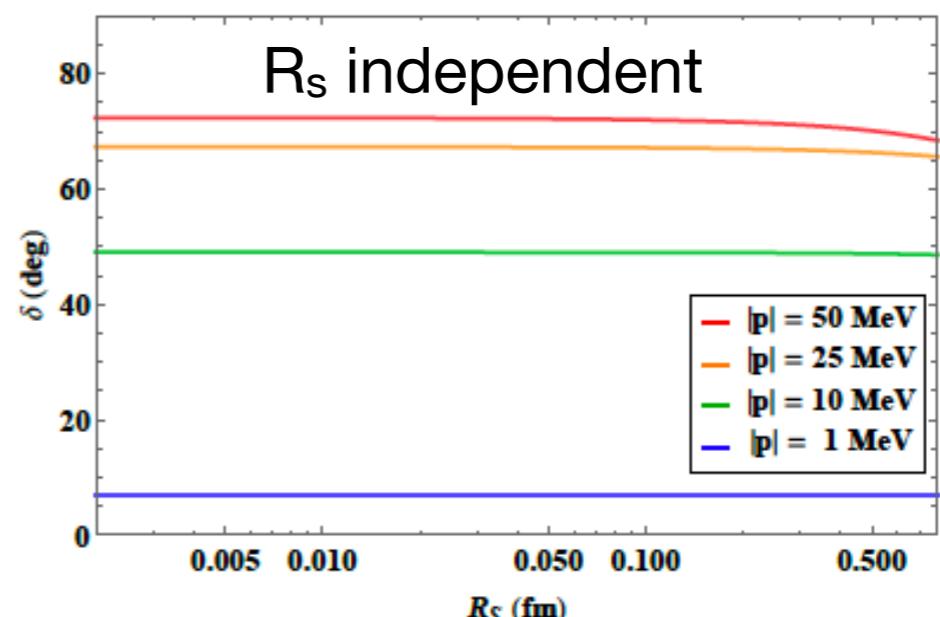
$$V_0(\mathbf{q}) = \tilde{C} + V_\pi(\mathbf{q}),$$

$$V_\pi(\mathbf{q}) = -\frac{g_A^2}{4F_\pi^2} \frac{m_\pi^2}{\mathbf{q}^2 + m_\pi^2},$$

The contact nuclear potential is regularized as

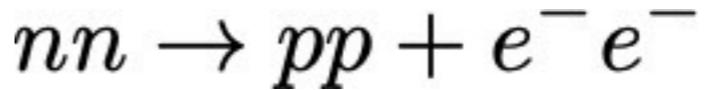
$$\tilde{C}\delta^{(3)}(\mathbf{r}) \rightarrow \frac{\tilde{C}(R_S)}{(\sqrt{\pi}R_S)^3} \exp\left(-\frac{r^2}{R_S^2}\right) \equiv \tilde{C}(R_S)\delta_{R_S}^{(3)}(\mathbf{r}),$$

The LEC  $\tilde{C}(R_S)$  is adjusted to reproduce the np-scattering length for a given  $R_S$ .

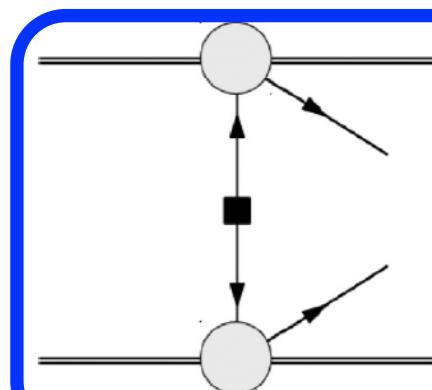


# Advances in ab initio modeling of $0\nu\beta\beta$ -decay NME

## ► Transition amplitude of the process (LO)



V. Cirigliano et al., PRL120, 202001 (2018); PRC97,065501 (2019)



$$V_{\nu,0}(\mathbf{q}) = \tau^{(1)} + \tau^{(2)} + \frac{1}{\mathbf{q}^2} \left( 1 - g_A^2 \boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{\sigma}^{(2)} \right. \\ \left. + g_A^2 \boldsymbol{\sigma}^{(1)} \cdot \mathbf{q} \boldsymbol{\sigma}^{(2)} \cdot \mathbf{q} \frac{2m_\pi^2 + \mathbf{q}^2}{(\mathbf{q}^2 + m_\pi^2)^2} \right),$$



$$\mathcal{A}_\nu(E, E') = -\langle \Psi_{pp}(E') | V_{\nu L}^{1S_0} | \Psi_{nn}(E) \rangle,$$

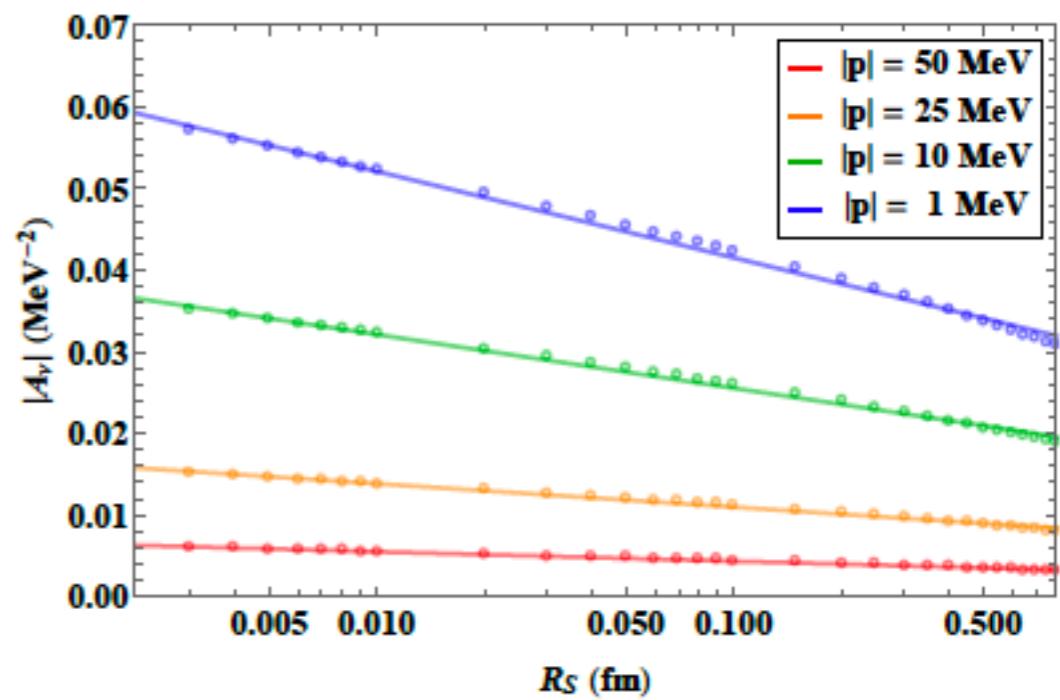
$$E = \mathbf{p}^2/m_n \text{ and } E' = \mathbf{p}'^2/m_p$$

$$E' = E + 2(m_n - m_p - m_e)$$

$$|\mathbf{p}'| = \sqrt{\mathbf{p}^2 + 2m_N(m_n - m_p - m_e)},$$

The transition amplitude is regulator dependent!

Needs a counter term (contact operator) at LO in order to ensure renormalizability.

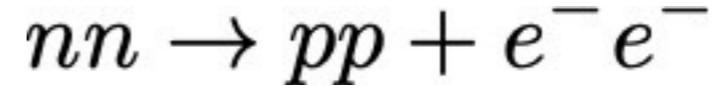


Lines fitted to  $\mathcal{A}_\nu = a + b \ln R_S$   
logarithmic dependence on  $R_S$

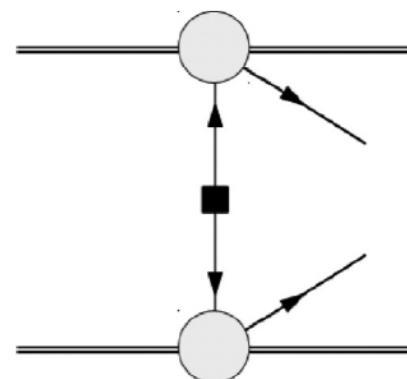
Violation of power counting? (龙炳蔚的报告)

# Advances in ab initio modeling of $0\nu\beta\beta$ -decay NME

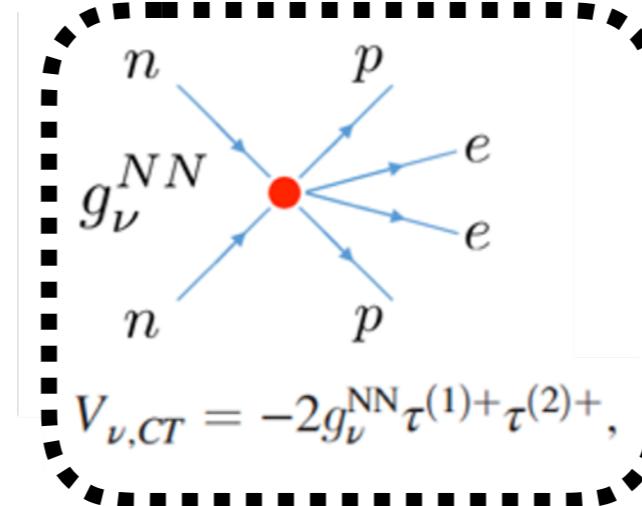
► Necessary of introducing a contact term at LO



V. Cirigliano et al., PRL120, 202001 (2018); PRC97,065501 (2019)



+



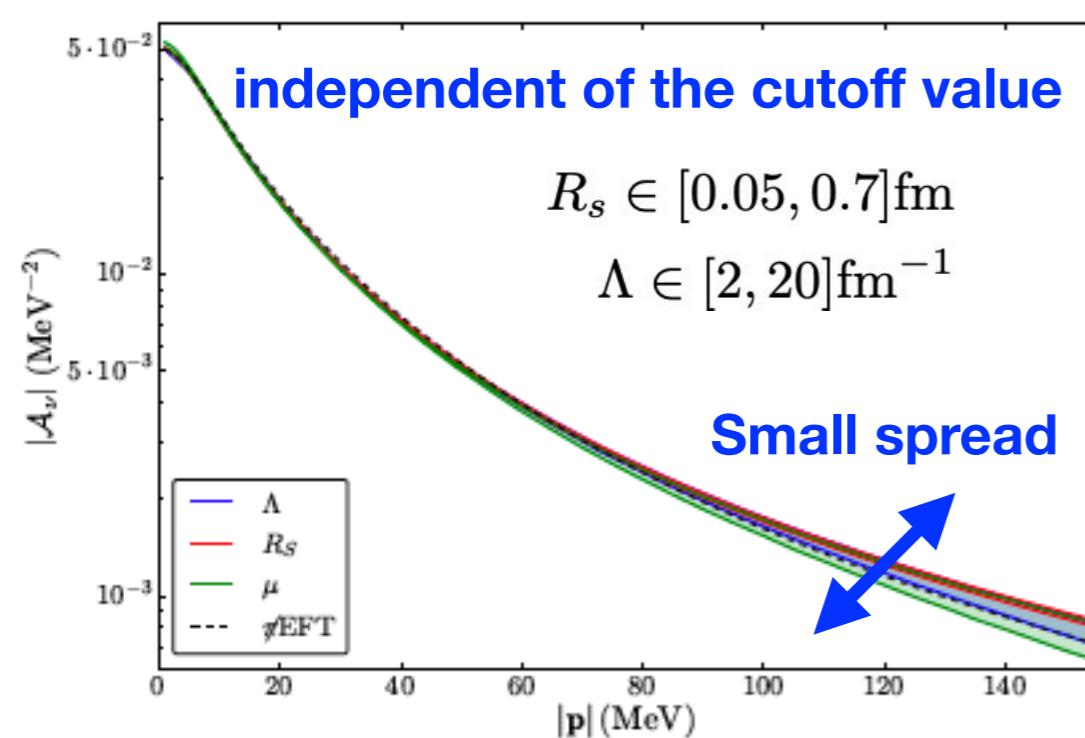
An missing piece



Total transition amplitude

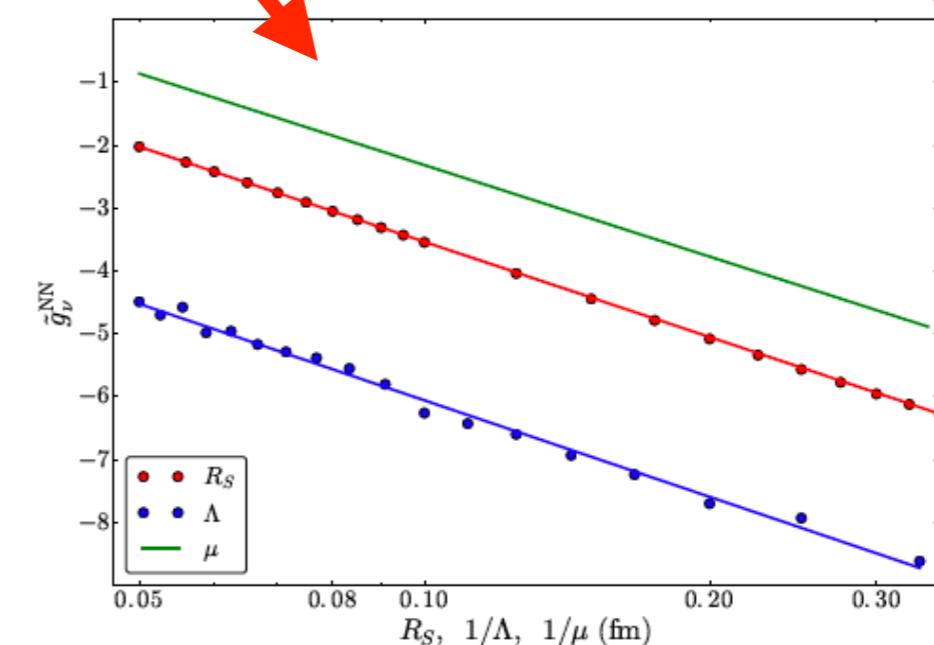
$$\mathcal{A}(p, p') = \mathcal{A}_L(p, p') - 2g\mathcal{A}_S(p, p').$$

Fitted to an arbitrary value



$\boxed{\mathcal{A}_{\nu}(|p| = 1 \text{ MeV}, |p'| = 38 \text{ MeV}) e^{-i(\delta_{1S_0}(E) + \delta_{1S_0}(E'))} = -0.05 \text{ MeV}^{-2}}$

LEC



# Determination of the leading-order contact operator

◆ The LEC should be fitted to **data** or the LD+SD **amplitude by Lattice QCD**

Light-Neutrino Exchange and Long-Distance Contributions to  $0\nu2\beta$   
Decays: An Exploratory Study on  $\pi\pi \rightarrow ee$

Xu Feng, Lu-Chang Jin, Xin-Yu Tuo, and Shi-Cheng Xia  
Phys. Rev. Lett. **122**, 022001 – Published 15 January 2019

参看冯旭的报告

$$\text{LQCD: } \left. \frac{\mathcal{A}(\pi\pi \rightarrow ee)}{F_\pi^2 T_{\text{lept}}} \right|_{m_\pi=140 \text{ MeV}} = 1.820(6)$$

$$T_{\text{lept}} = 4G_F^2 V_{ud}^2 m_{\beta\beta} \bar{u}_L(p_1) u_L^c(p_2).$$

discrepancy might be from

- lattice artifacts and finite-volume effects
- LO chiral expansion error

Path from Lattice QCD to the Short-Distance Contribution to  
 $0\nu\beta\beta$  Decay with a Light Majorana Neutrino

Zohreh Davoudi and Saurabh V. Kadam

Phys. Rev. Lett. **126**, 152003 (2021) – Published 16 April 2021

Providing a framework to match the total transition amplitude of the  $nn \rightarrow ppe-e-$   
process from the calculations of both **lattice QCD** and **chiral effective field theory**.

# Advances in ab initio modeling of $0\nu\beta\beta$ -decay NME

## ► Determination of the LEC for the contact term

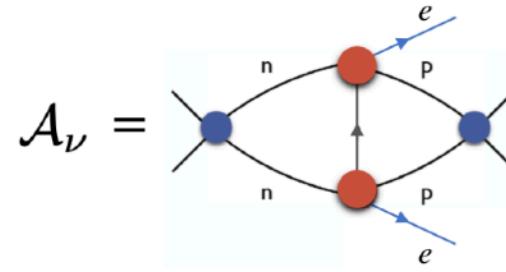
### Toward Complete Leading-Order Predictions for Neutrinoless Double $\beta$ Decay

Vincenzo Cirigliano, Wouter Dekens, Jordy de Vries, Martin Hoferichter, and Emar Mereghetti

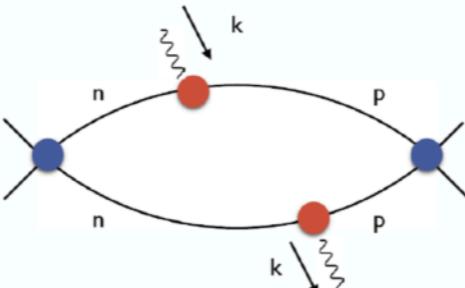
Phys. Rev. Lett. **126**, 172002 (2021) – Published 30 April 2021

- **Cottingham formula** [W.N. Cottingham, Ann. Phys. 25, 424 \(1963\)](#)

$$\mathcal{A}_\nu \propto \int \frac{d^4 k}{(2\pi)^4} \frac{g_{\mu\nu}}{k^2 + i\epsilon} \int d^4 x e^{ik \cdot x} \langle pp | T\{ j_w^\mu(x) j_w^\nu(0) \} | nn \rangle$$



$$\propto \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 + i\epsilon}$$



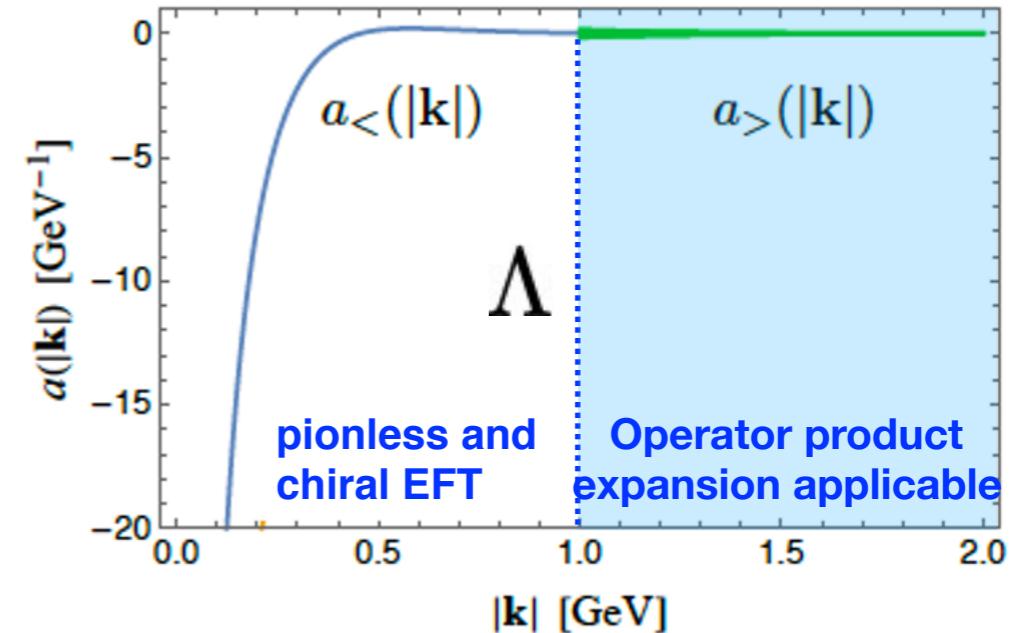
forward Compton amplitude

- **Synthetic datum**

$$\begin{aligned} \mathcal{A}_\nu(|\mathbf{p}|, |\mathbf{p}'|) \times e^{-i(\delta_1 s_0(|\mathbf{p}|) + \delta_1 s_0(|\mathbf{p}'|))} &= - \left( 2.271 - 0.075 \tilde{\mathcal{C}}_1(4M_\pi) \right) \times 10^{-2} \text{ MeV}^{-2} \\ |\mathbf{p}| = 25 \text{ MeV} \quad (|\mathbf{p}'| = 30 \text{ MeV}) &= -1.95(5) \tilde{\mathcal{C}}_1 \times 10^{-2} \text{ MeV}^{-2}, \end{aligned}$$

Uncertainty from the estimate of the **inelastic** contributions

The amplitude is observable and thus scheme independent.



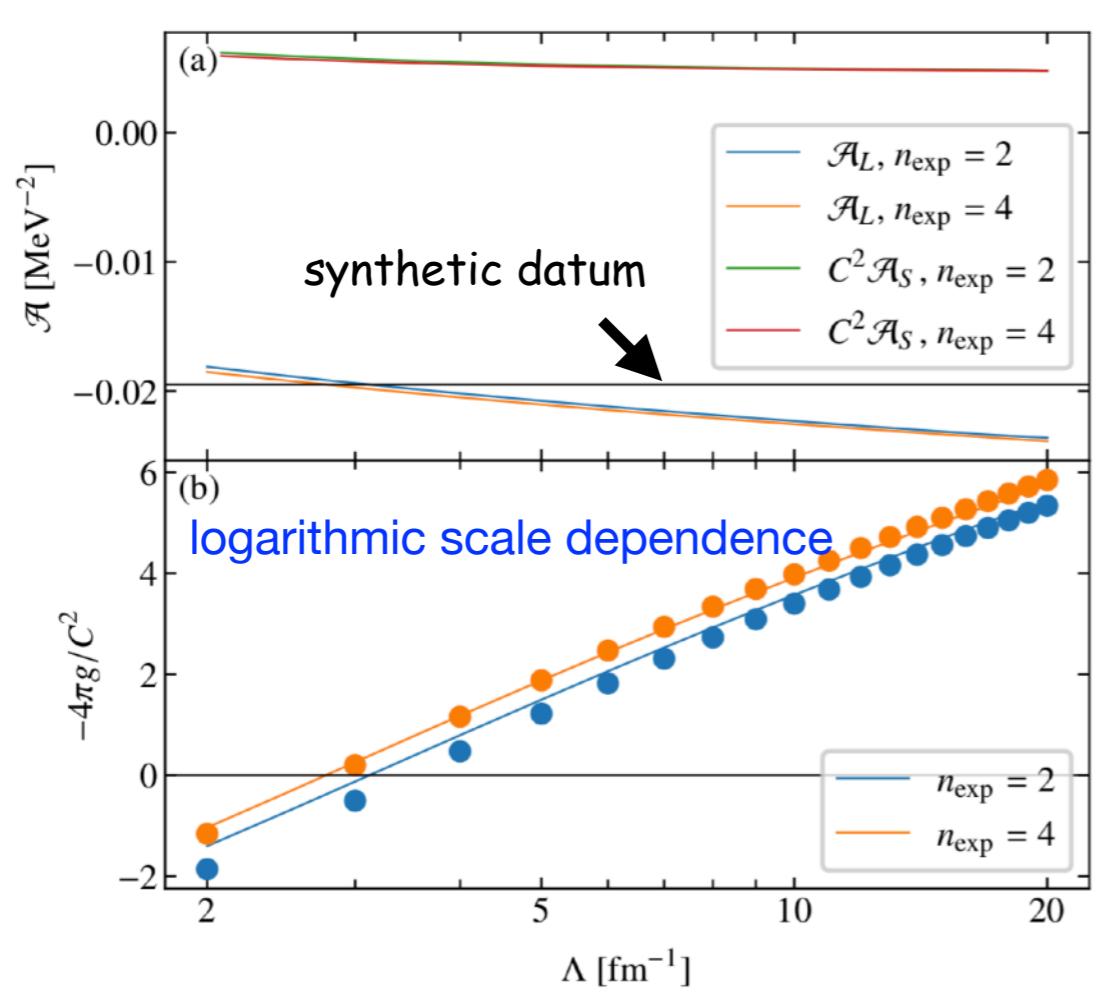
$$\begin{aligned} \mathcal{A}_\nu^{\text{full}} &= \int_0^\infty d|\mathbf{k}| a^{\text{full}}(|\mathbf{k}|) = \mathcal{A}^< + \mathcal{A}^>, \\ \mathcal{A}^< &= \int_0^\Lambda d|\mathbf{k}| a_<(|\mathbf{k}|), \\ \mathcal{A}^> &= \int_\Lambda^\infty d|\mathbf{k}| a_>(|\mathbf{k}|), \end{aligned}$$

# Determination of the leading-order contact operator

## ► Contribution of the contact term to the NME of finite nuclei

R. Wirth, JMY, H. Hergert, arXiv:2105.05415 [nucl-th]

$$\mathcal{A}(p, p') = \mathcal{A}_L(p, p') - 2g\mathcal{A}_S(p, p').$$



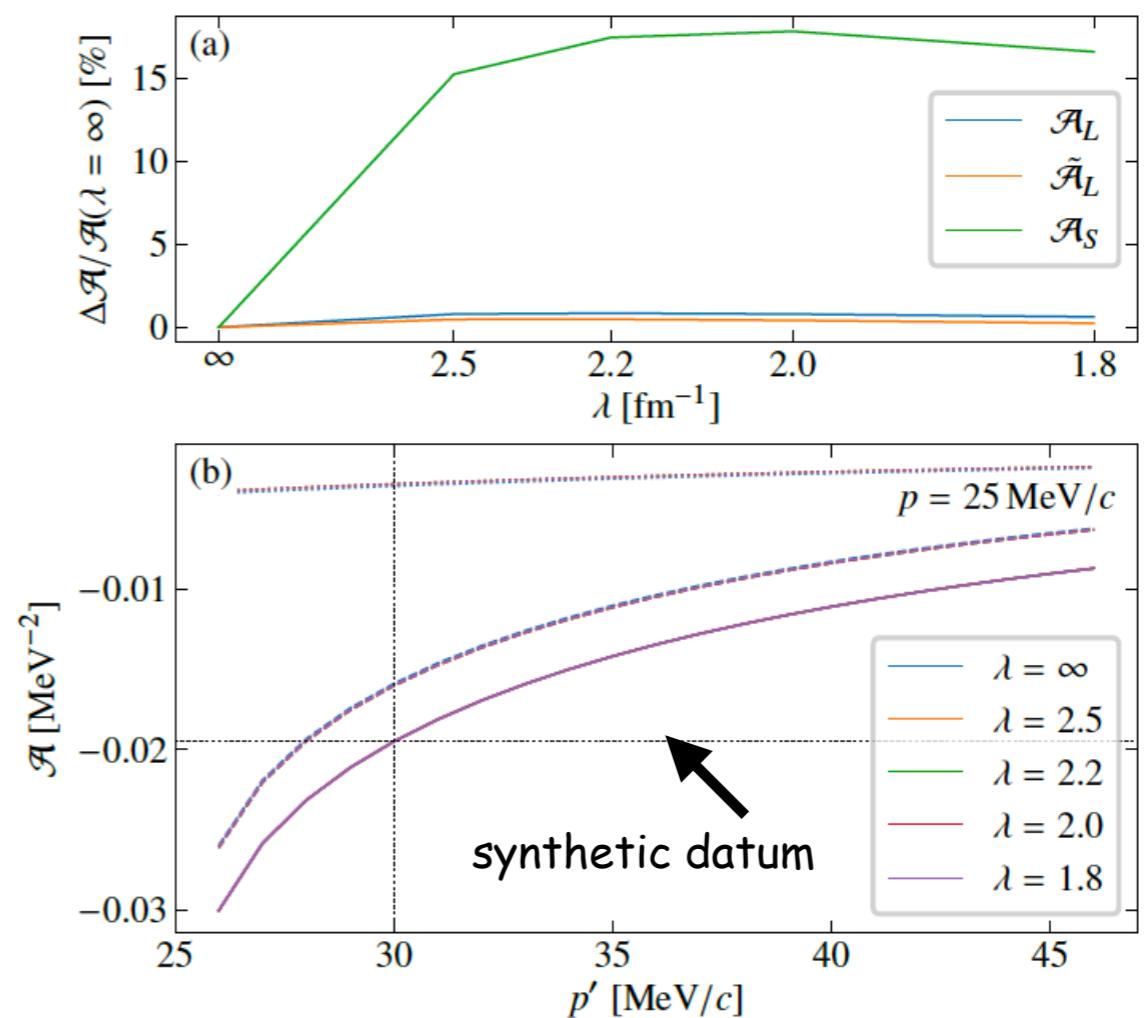
The dimensionless LEC  $C$  is adjusted to reproduce the neutron-proton scattering length  $a_{np} = -23.74$  fm.

SRG scale

$$H_s = U(s)H U^\dagger(s)$$

$$\frac{dH_s}{ds} = [\eta(s), H_s],$$

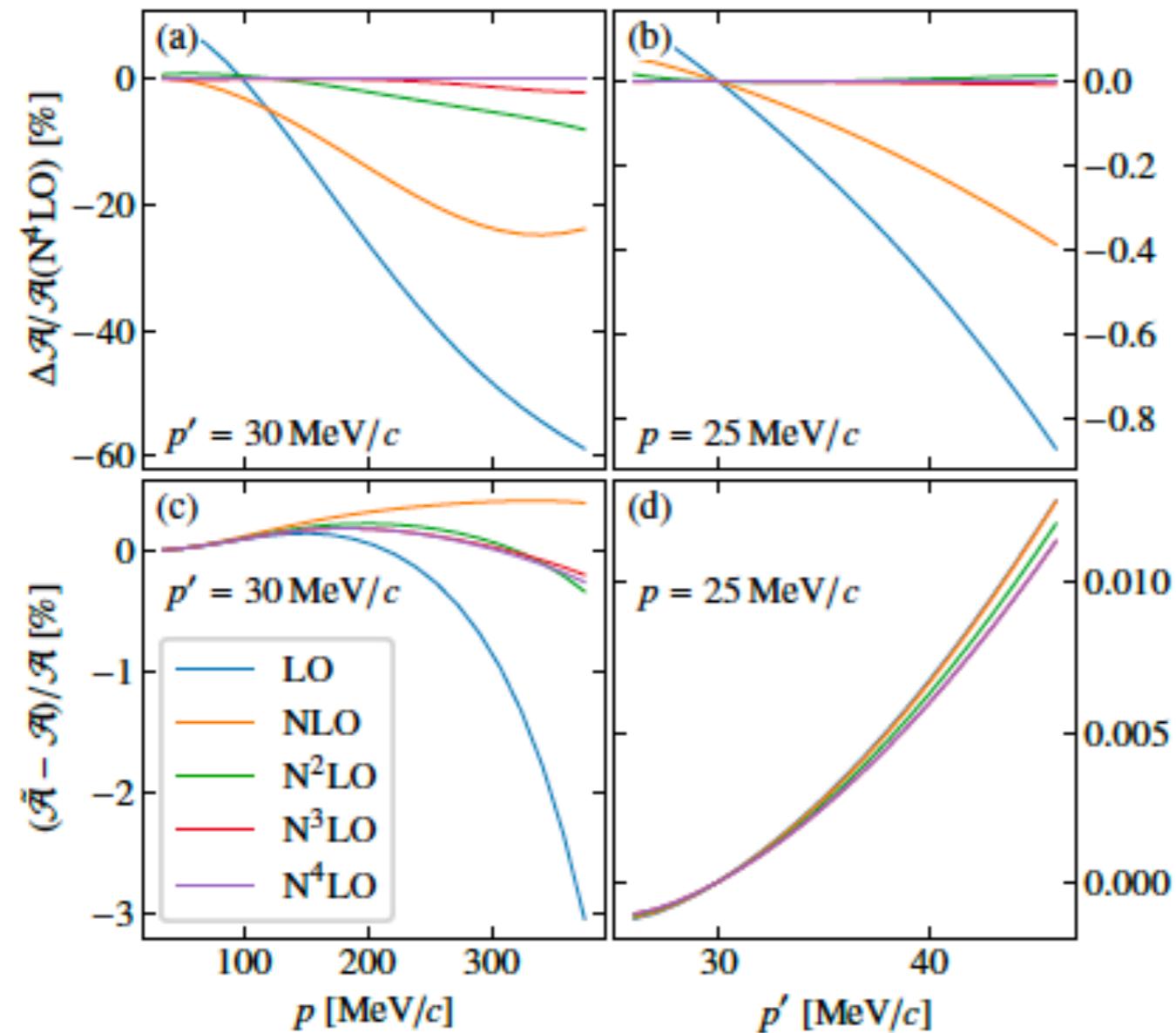
$$\lambda \equiv s^{-1/4}$$



# Determination of the leading-order contact operator

## ► Contribution of the contact term to the NME of finite nuclei

R. Wirth, JMY, H. Hergert, arXiv:2105.05415 [nucl-th]

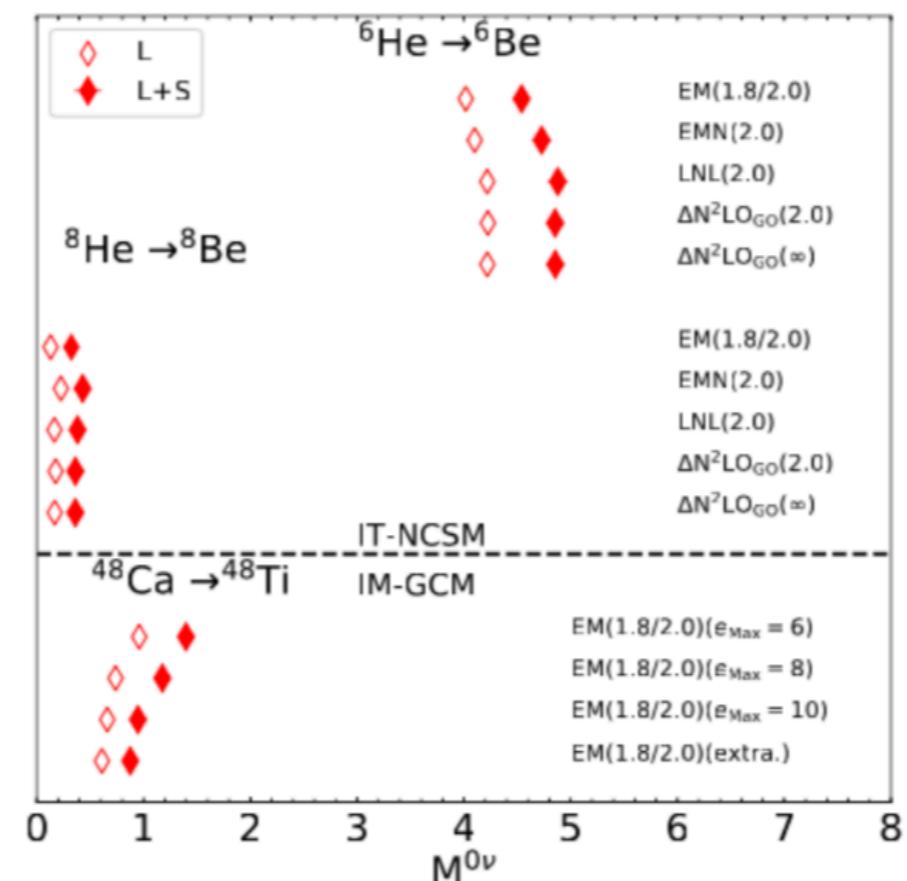
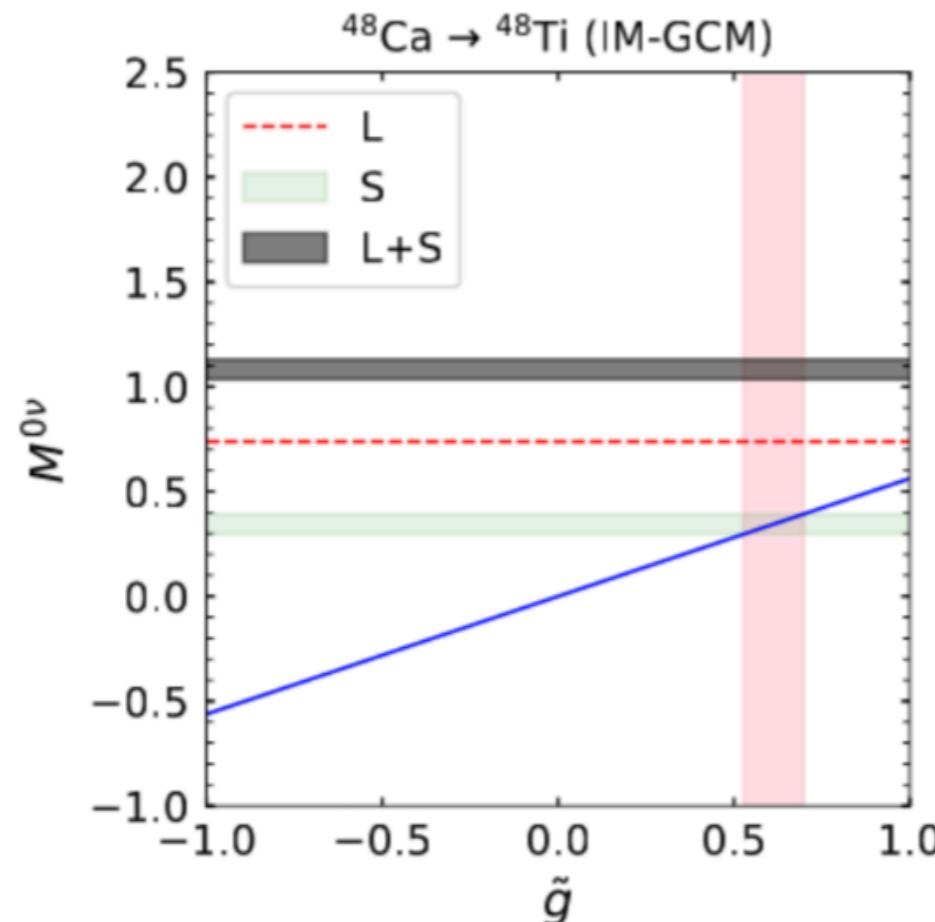


- Chiral expansion order of the nuclear interaction (not transition operator)
- LO and N<sup>2</sup>LO (partial) neutrino potential

# Determination of the leading-order contact operator

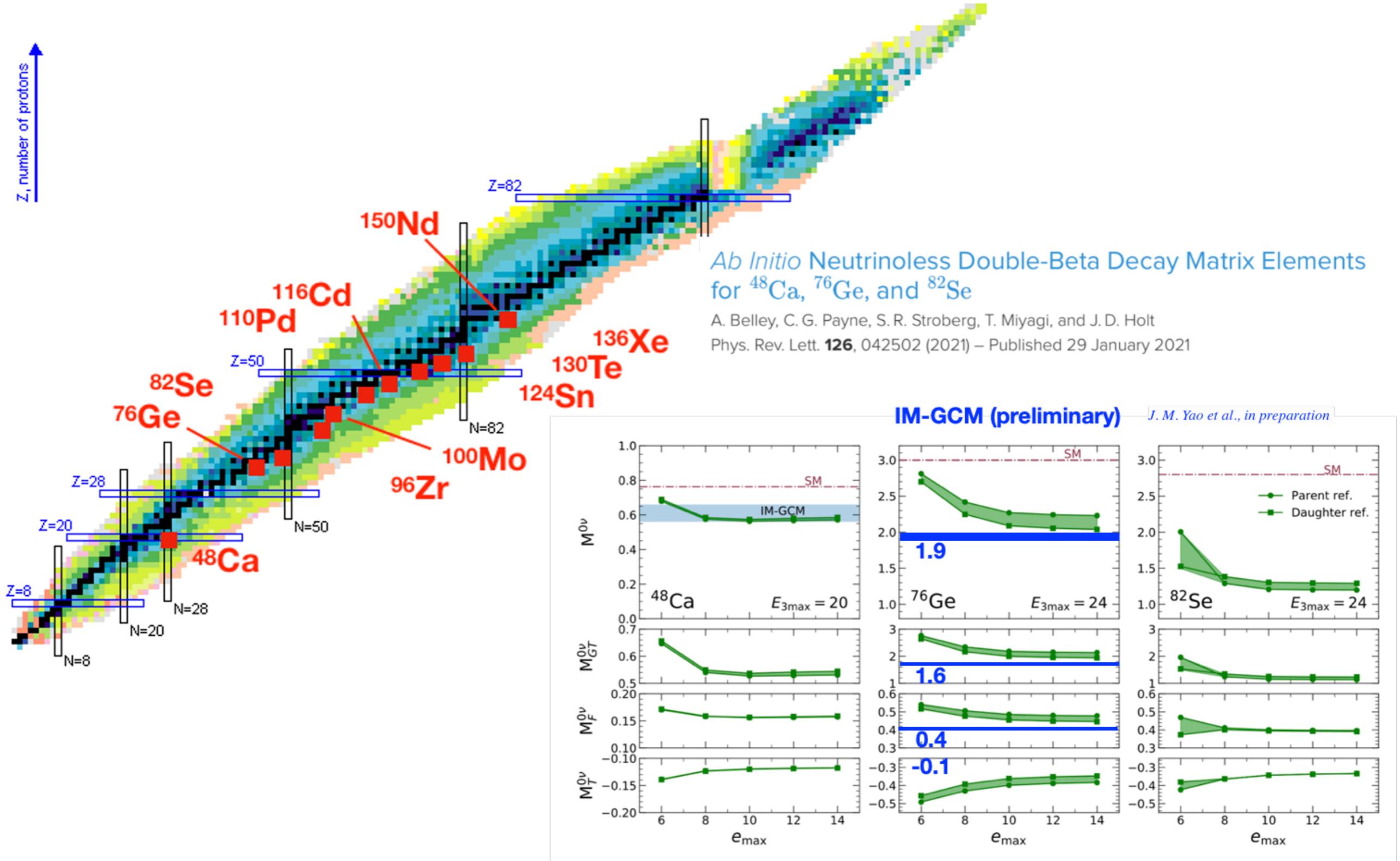
## ► The contribution of the contact term to the NME

R. Wirth, JMY, H. Hergert, arXiv:2105.05415 [nucl-th]

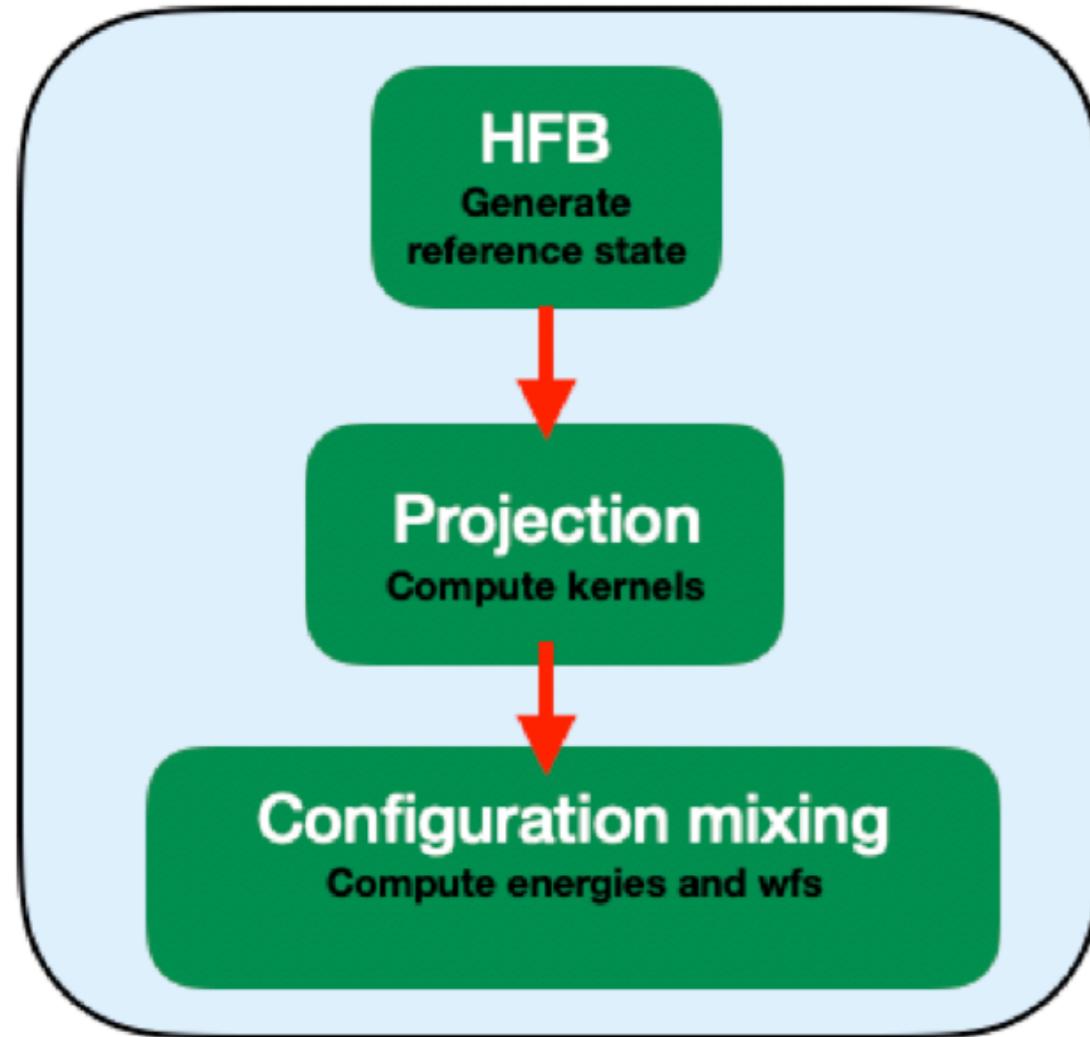


- The contact term **enhances the NME for  ${}^{48}\text{Ca}$  by 43(7)%**, the uncertainty is propagated only from the synthetic datum.
- An important positive message for planning and interpreting future experiments.

# Extension to heavier $0\nu\beta\beta$ candidates



# Generator coordinate method (GCM) in a nutshell



- The trial wave function of a GCM state

$$|\Phi^{JNZ\dots}\rangle = \sum_Q F_Q^{JNZ} \hat{P}^J \hat{P}^N \hat{P}^Z \dots |\Phi_Q\rangle$$

$|\Phi_Q\rangle$  are a set of HFB wave functions from constraint calculations,  $Q$  is the so-called generator coordinate.

- The mixing weight  $F_Q^{JNZ}$  is determined from the Hill-Wheeler-Griffin equation:

$$\sum_{Q'} \left[ H^{JNZ}(Q, Q') - E^J N^{JNZ}(Q, Q') \right] F_{Q'}^{JNZ} = 0$$

## Features (pros) of GCM

- The Hilbert space in which the  $H$  will be diagonalized is defined by the  $Q$ .  
**Many-body correlations are controlled by the  $Q$**
- The  $Q$  is chosen as (collective) degrees of freedom relevant to the physics.
- Dimension of the space in GCM is generally much smaller than full CI calculations.

# Optimization of GCM

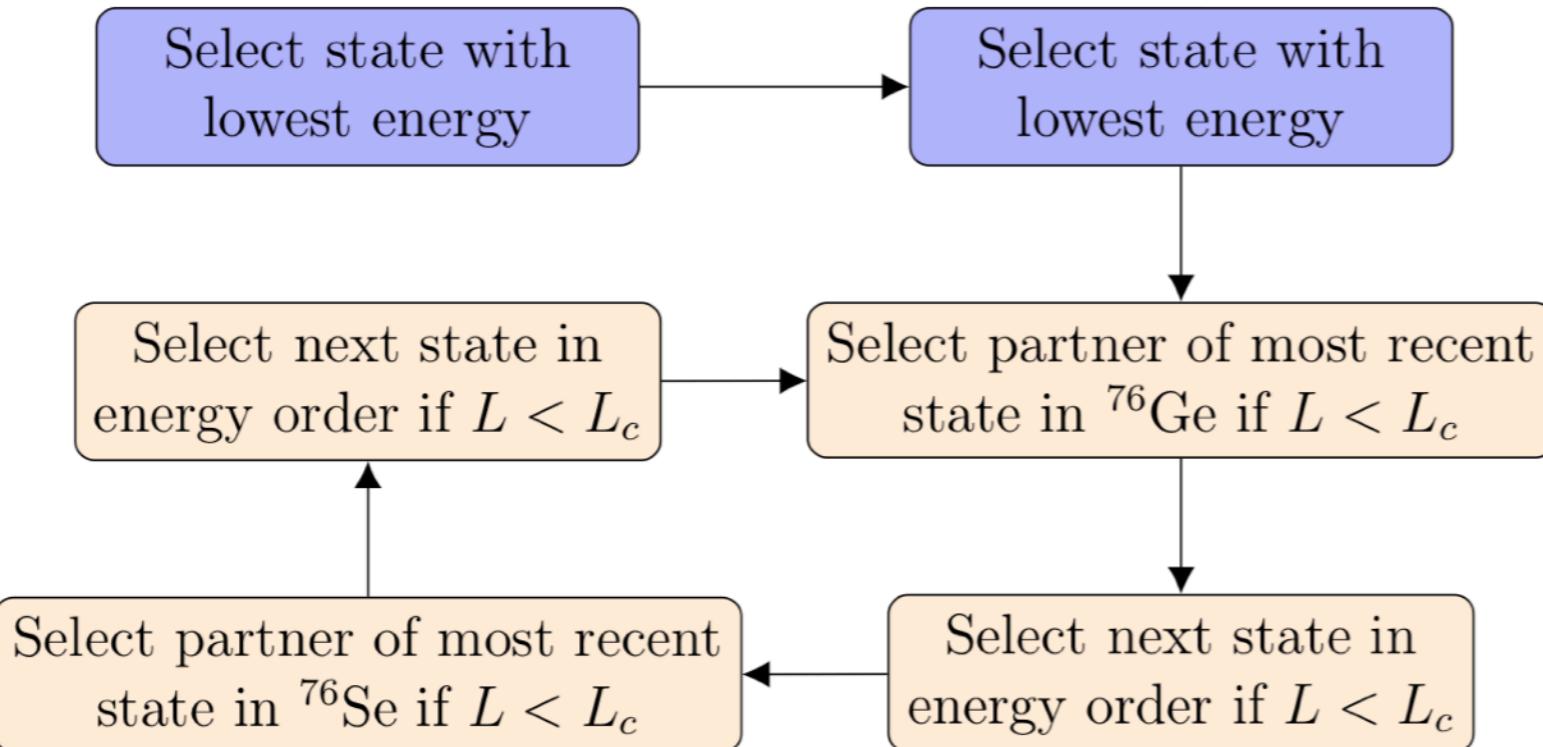
## “dimensionality curse” in GCM

A.M. Romero, J. Engel, JMY, arXiv:2105.03471 [nucl-th]

$N$  dimensional collective space  $Q = (q_1, q_2, \dots, q_N)$

- energy-transition-orthogonality procedure (ENTROP)

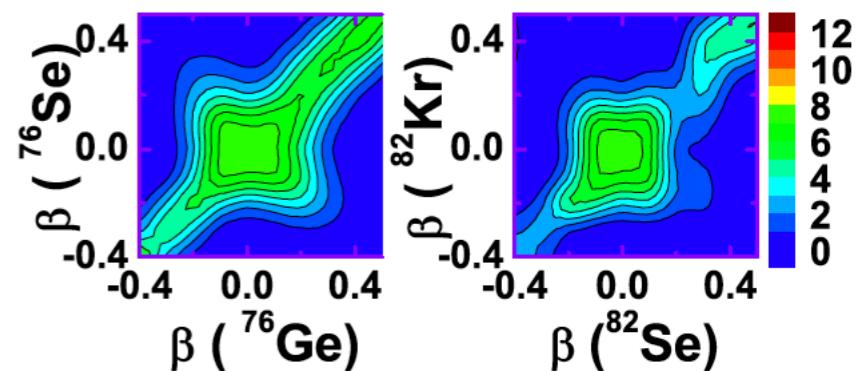
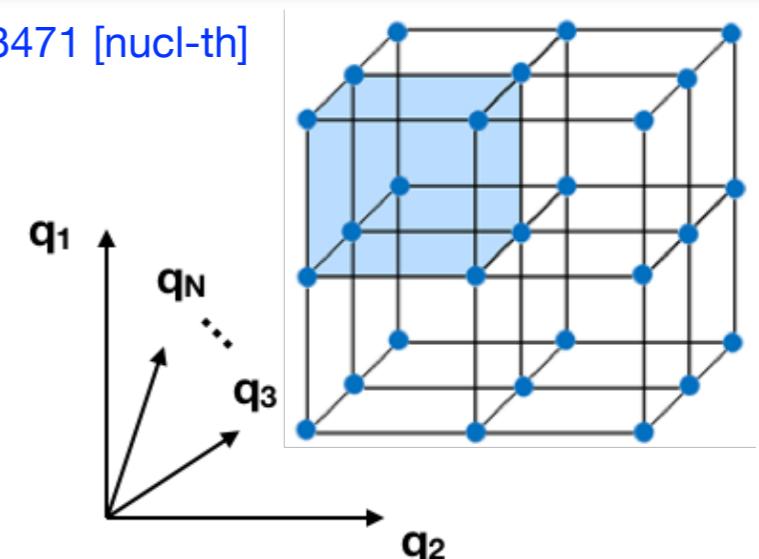
$^{76}\text{Ge}$        $^{76}\text{Se}$



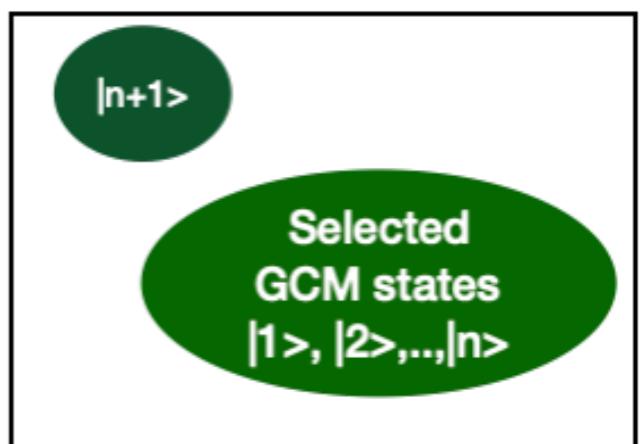
$$L = \frac{\langle n+1 | P^{(n)} | n+1 \rangle}{\langle n+1 | n+1 \rangle}$$

$$P^{(n)} |n+1\rangle = \sum_{i=1}^n \alpha_i^{(n)} |i\rangle,$$

**L: a measure if the model space is complete or not.**



JMY, L. S. Song, K. Hagino, P. Ring, and J. Meng PRC91, 024316 (2015)



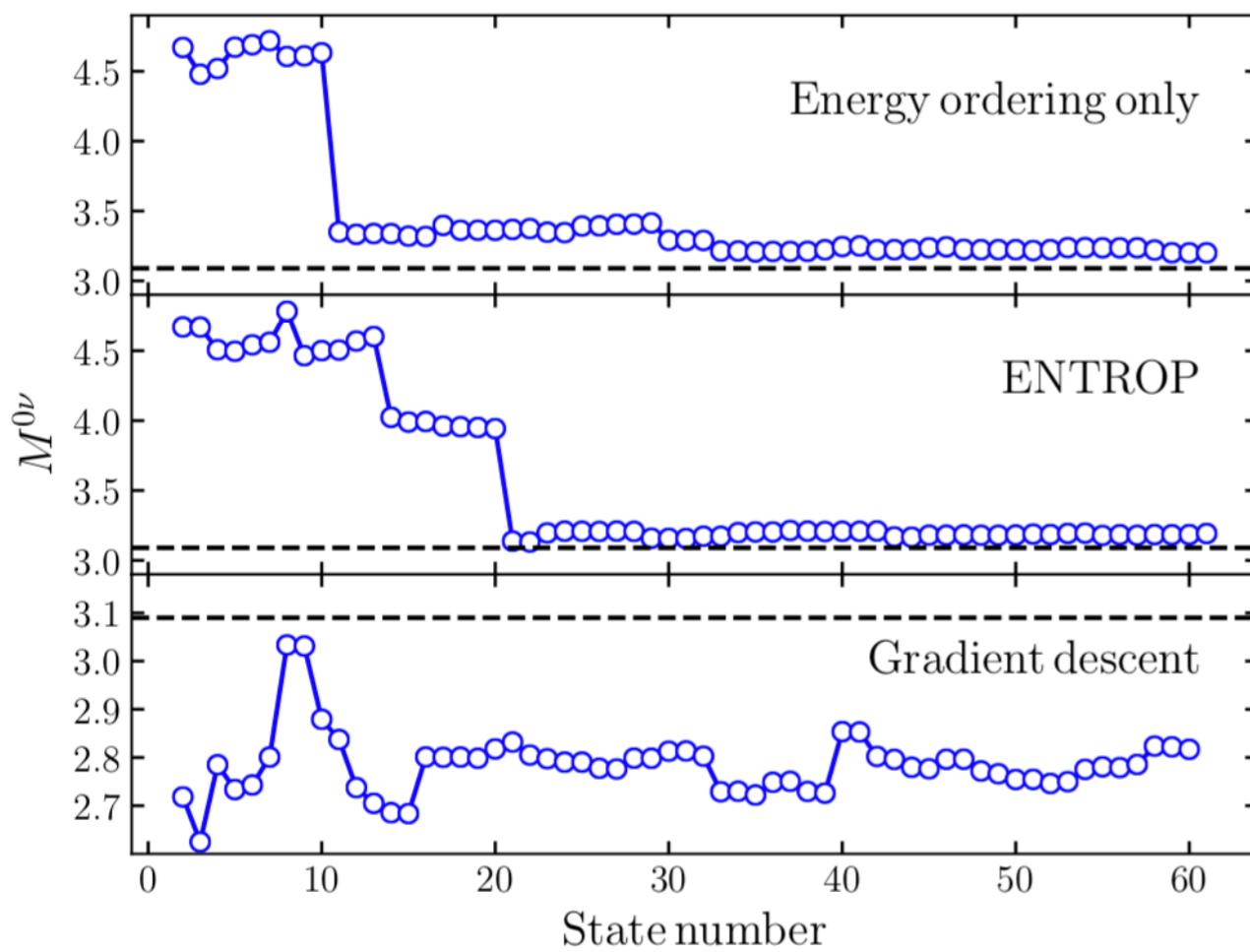
# Optimization of GCM

“dimensionality curse” in GCM

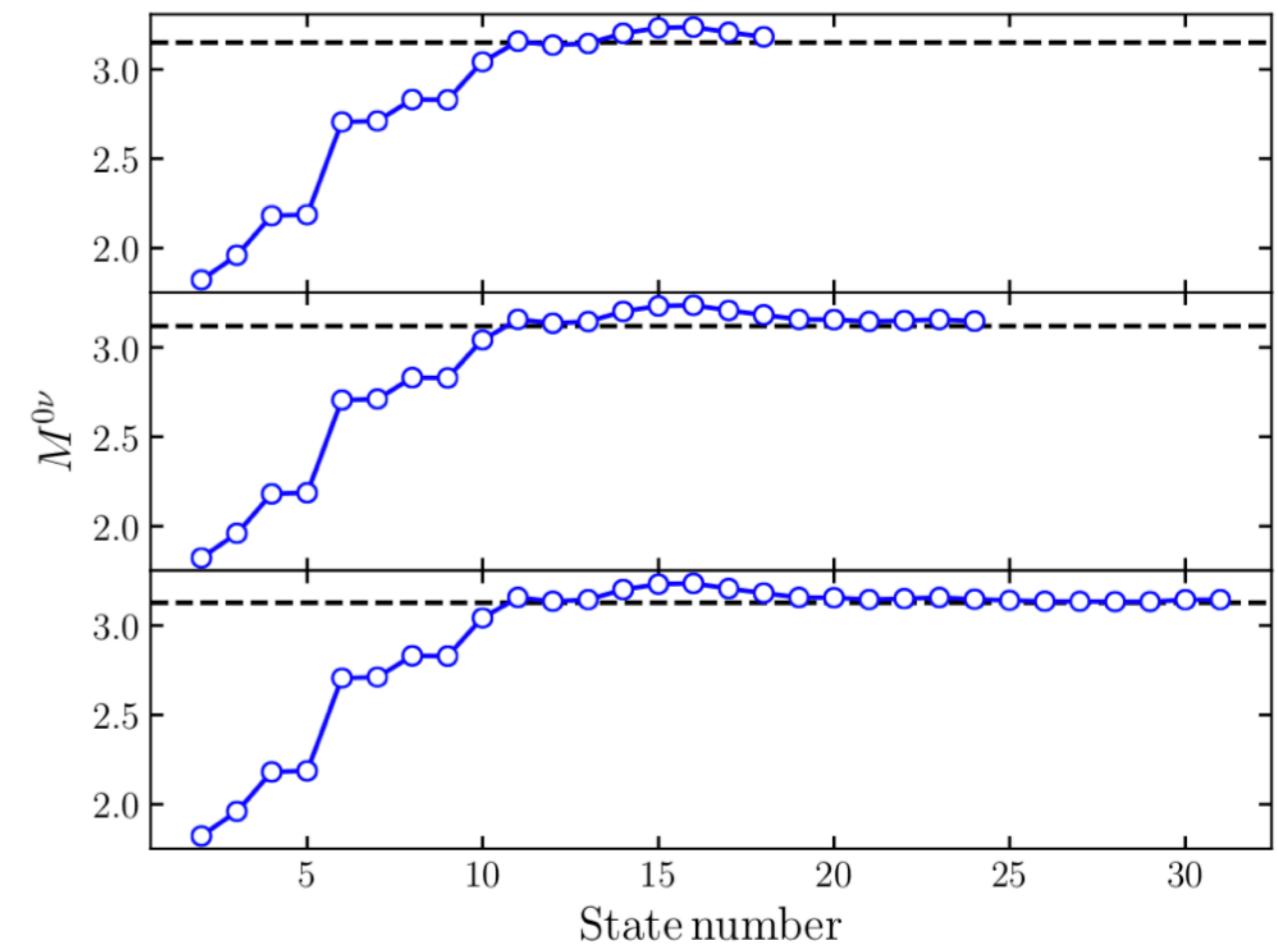
A.M. Romero, J. Engel, JMY, arXiv:2105.03471 [nucl-th]

$N$  dimensional collective space  $Q = (q_1, q_2, \dots, q_N)$

- energy-transition-orthogonality procedure (ENTROP)



GCM with shell-model interaction GCN2850



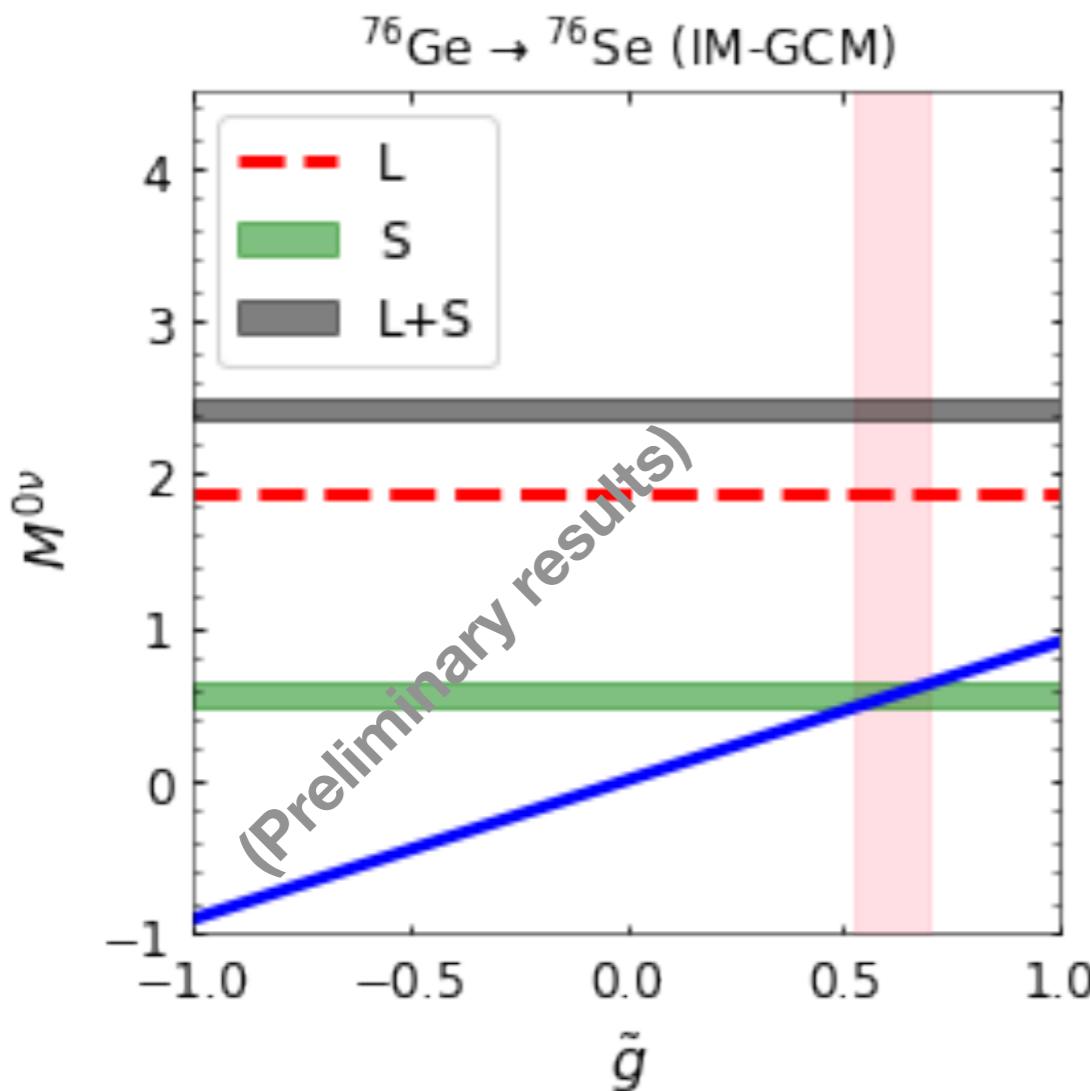
IM-GCM with a chiral nuclear force  
(eMax=6)

# Determination of the leading-order contact operator

- The contribution of the contact term to the NME

JMY et al., in preparation

$$e_{\text{Max}} = 8, \hbar\omega = 12 \text{ MeV}$$



The contact term **enhances**  
the NME for  $^{76}\text{Ge}$  by 29(5)% !



# Summary and outlook

## ◆ Summary

- Experimental searches of  $0\nu\beta\beta$  decay are pushing up to tonne-scale detectors with the half-life sensitivity up to  $10^{28}$  years.
- Significant advances in ab initio modeling of atomic nuclei.

From light to medium-mass nuclei,  
close-shell to open-shell nuclei,  
spherical to deformed nuclei.
- Ab initio calculation of the NMEs of candidate nuclei with both long- and short-range operators are possible.
  - ✓ The leading-order short-range operator generally enhances the NME in the ab initio calculations using a chiral nuclear force with low-energy scale regulator.
  - ✓ Ready to compute the NME of heavier candidate nuclei.

# Summary and outlook

## ◆ Outlook (TODO LIST)

- **Standard mechanism:** trans. operators derived consist. from EFT)
- **Other mechanisms:** Left-Right mixing, etc.
- **Uncertainty Quantification:** Truncation error in both nuclear interactions and many-body methods, IMSRG(3)
- **Building an emulator for the NME:** Machine learning?



Missing many pieces?



# Collaborators and acknowledgement

## Collaborators

- N. Li, C.F. Jiao, Sun Yat-sen University, China
- Z.P.Li, L. J. Wang, Southwest University, China
- X.Y. Wu, Jiangxi Normal University, China
- J. Meng, L. S. Song, Peking University, China
- P. Ring, Technical University of Munich, Germany
- **R. Wirth, H. Hergert**, Michigan State University, USA
- **J. Engel, A. Marquez Romero**, UNC-CH, USA
- **A. Belley, T. Miyagi, C. G. Payne, J. D. Holt**, TRIUMF, Canada
- **B. Bally, Tomás R. Rodríguez**, Universidad Autónoma de Madrid, Spain
- and more ...



**Thank you for your attention  
And enjoy sunshine in Zhuhai!**