“无中微子双贝塔衰变”研讨会
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Nuclear double-β decay studied by self-consistent QRPA approach

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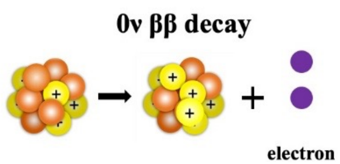
Outline

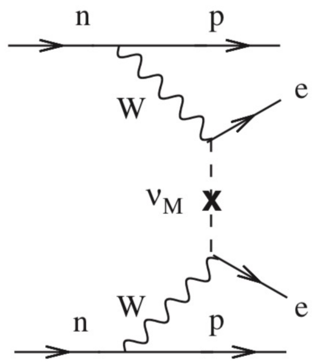
- **Introduction**
- **Theoretical Framework**
- **$2\nu2\beta$ NME calculated by Skyrme QRPA**
- **$2\nu2\beta$ NME calculated by relativistic QRPA**
- **Summary and Perspective**

Nuclear double beta decays

- **0ν ββ-decay**

Furry, Phys. Rev. 56, 1184 (1939)





$${}^{A}_{Z}X \rightarrow {}_{Z+2}^{\;\;\;A}Y + 2e^{-}$$

$$\mathrm{T}_{1/2} > 10^{26}\mathrm{yr}$$

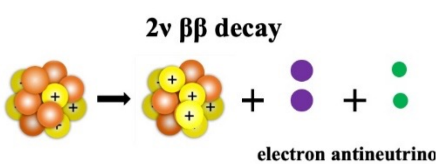
GERDA Collaboration, Phys. Rev. Lett. 125, 252502 (2020)

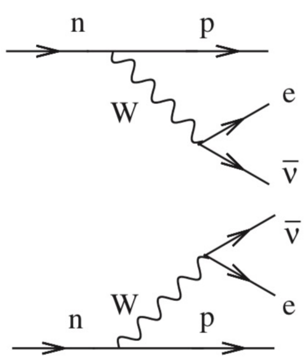
- Neutrino
  Majorana or Dirac nature?
- Neutrino Mass
- Lepton number conservation

Avignone, et al., RMP 80, 481(2008)

- **2ν ββ-decay**

M. G. Mayer, Phys. Rev. 48, 512 (1935)





- Experimentally observed
- Benchmark nuclear models for further study of 0ν ββ-decay

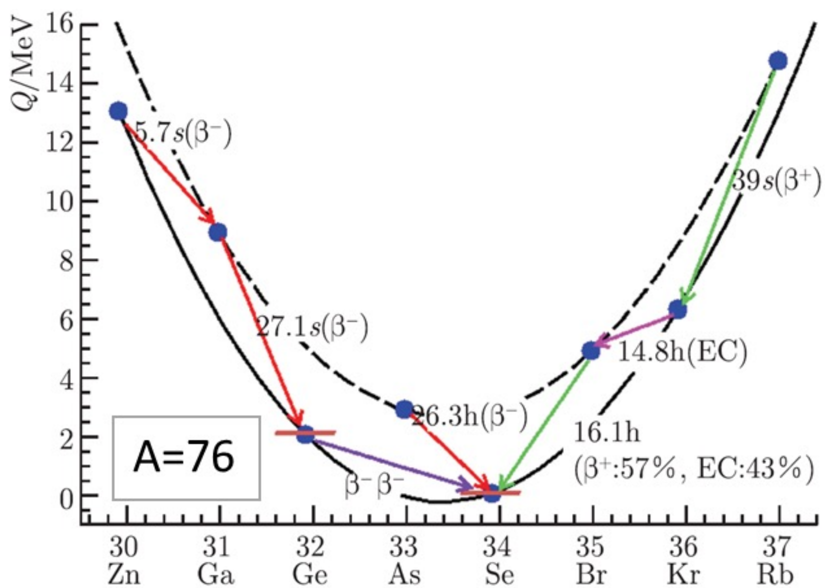
second order process

$${}^{A}_{Z}X \rightarrow {}_{Z+2}^{\;\;\;A}Y + 2e^{-} + 2\bar{\nu}_e \qquad \mathrm{T}_{1/2}{\sim}10^{18-24}\mathrm{yr}$$

A. S. Barabash, Phys. Rev. C 81, 035501 (2010)

Candidate nuclei for ββ-decay

- **ββ-decay candidates:**
  - ✓ even-even nucleus (Z,A)
  - ✓ pairing forces make it more bound than its (Z+1, A) neighbor, but less so than the (Z+2,A) nuclide.



<无中微子双贝塔衰变实验> 科学出版社(2020)

Haxton, PPNP 12, 409 (1984)

$\beta^-\beta^-$ transition	$\beta^-\beta^-$ transition
$^{46}Ca \to {}^{46}Ti$	$^{134}Xe \to {}^{134}Ba$
$^{48}Ca \to {}^{48}Ti$	$^{136}Xe \to {}^{136}Ba$
$^{70}Zn \to {}^{70}Ge$	$^{142}Ce \to {}^{142}Nd$
$^{76}Ge \to {}^{76}Se$	$^{146}Nd \to {}^{146}Sm$
$^{80}Se \to {}^{80}Kr$	$^{148}Nd \to {}^{148}Sm$
$^{82}Se \to {}^{82}Kr$	$^{150}Nd \to {}^{150}Sm$
$^{86}Kr \to {}^{86}Sr$	$^{154}Sm \to {}^{154}Gd$
$^{94}Zr \to {}^{94}Mo$	$^{160}Gd \to {}^{160}Dy$
$^{96}Zr \to {}^{96}Mo$	$^{170}Er \to {}^{170}Yb$
$^{98}Mo \to {}^{98}Ru$	$^{176}Yb \to {}^{176}Hf$
$^{100}Mo \to {}^{100}Ru$	$^{186}W \to {}^{186}Os$
$^{104}Ru \to {}^{104}Pd$	$^{192}Os \to {}^{192}Pt$
$^{110}Pd \to {}^{110}Cd$	$^{198}Pt \to {}^{198}Hg$
$^{114}Cd \to {}^{114}Sn$	$^{204}Hg \to {}^{204}Pb$
$^{116}Cd \to {}^{116}Sn$	$^{226}Ra \to {}^{226}Th$
$^{122}Sn \to {}^{122}Te$	$^{232}Th \to {}^{232}U$
$^{124}Sn \to {}^{124}Te$	$^{238}U \to {}^{238}Pu$
$^{128}Te \to {}^{128}Xe$	$^{244}Pu \to {}^{244}Cm$
$^{130}Te \to {}^{130}Xe$	$^{248}Cm \to {}^{248}Cf$

Experimentally observed $2\nu2\beta$

Nuclear Matrix Element (NME) of 2ν ββ decay

- **2ν ββ decay half-life and NMEs**

$$\left[T^{2\nu}_{1/2}\right]^{-1} = G^{2\nu} g_A^4 \left|M^{2\nu}_{\mathrm{GT}}\right|^2$$

✓ $G^{2\nu}(Q_{\beta\beta}, Z)$: phase space factor
kinetics calculation

J. Suhonen and O. Civitarese, Phys. Rep. 300, 123(1998)
J. Kotila and F. Iachello, Phys. Rev. C 85, 034316 (2012)
S. Stoica and M. Mirea, Phys. Rev. C 88, 037303(2013)

- majority nuclei : < 1%
- ^{96}Zr ^{100}Mo ^{116}Cd: ~ 4%-6%

A. S. Barabash, Phys. Rev. C 81, 035501 (2010)
A. S. Barabash, Nucl. Phys. A 935, 52 (2015)

✓ Nuclear Matrix Element (NME)

Lowest-order transition operators:
Fermi: forbidden by isospin conservation
Gamow-Teller (GT): allowed

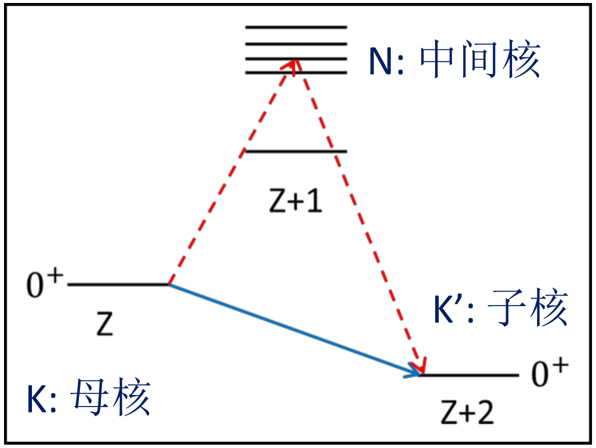
$$M^{2\nu}_{\mathrm{GT}} = \sum_{N_1 N_2} \frac{\langle \psi_{K'} || \hat{O}^{-}_{\mathrm{GT}} || \psi_{N_2} \rangle \langle \psi_{N_2} | \psi_{N_1} \rangle \langle \psi_{N_1} || \hat{O}^{-}_{\mathrm{GT}} || \psi_K \rangle}{E^*_N + M_N - (M_K + M_{K'})/2}$$

"Experimental" NMEs

TABLE II. Half-life and nuclear matrix element values for two-neutrino double-β decay (see Sec. IV).

Isotope	$T_{1/2}(2\nu)$ (years)	$M^{2\nu}$
^{48}Ca	$4.4^{+0.6}_{-0.5} \times 10^{19}$	$0.0238^{+0.0015}_{-0.0017}$
^{76}Ge	$(1.5 \pm 0.1) \times 10^{21}$	$0.0716^{+0.0025}_{-0.0023}$
^{82}Se	$(0.92 \pm 0.07) \times 10^{20}$	$0.0503^{+0.0020}_{-0.0018}$
^{96}Zr	$(2.3 \pm 0.2) \times 10^{19}$	$0.0491^{+0.0023}_{-0.0020}$
^{100}Mo	$(7.1 \pm 0.4) \times 10^{18}$	$0.1258^{+0.0037}_{-0.0034}$
^{100}Mo-^{100}Ru(0^+_1)	$5.9^{+0.8}_{-0.6} \times 10^{20}$	$0.1017^{+0.0056}_{-0.0063}$
^{116}Cd	$(2.8 \pm 0.2) \times 10^{19}$	$0.0695^{+0.0025}_{-0.0024}$
^{128}Te	$(1.9 \pm 0.4) \times 10^{24}$	$0.0249^{+0.0031}_{-0.0023}$
^{130}Te	$(6.8^{+1.2}_{-1.1}) \times 10^{20}$	$0.0175^{+0.0016}_{-0.0014}$
^{150}Nd	$(8.2 \pm 0.9) \times 10^{18}$	$0.0320^{+0.0018}_{-0.0017}$
^{150}Nd-^{150}Sm(0^+_1)	$1.33^{+0.45}_{-0.26} \times 10^{20}$	$0.0250^{+0.0029}_{-0.0034}$
^{238}U	$(2.0 \pm 0.6) \times 10^{21}$	$0.0271^{+0.0053}_{-0.0033}$
^{130}Ba; ECEC(2ν)	$(2.2 \pm 0.5) \times 10^{21}$	$0.105^{+0.014}_{-0.010}$

A. S. Barabash, Phys. Rev. C 81, 035501 (2010)



Nuclear Models for 2νββ-decay

- Theoretical model in 2νββ:
  - ✓ Shell Model

    E. Caurier, F. Nowacki, and A. Poves, Phys. Lett. B 711, 62 (2012)

    R. A. Senkov and M. Horoi, Phys. Rev. C 90, 051301R (2014)

    B. A. Brown, D. L. Fang, and M. Horoi, Phys. Rev. C 92, 041301 (2015)

    H.-T. Li, and Z.-Z. Ren, Phys. Rev. C 96, 065503 (2017)
  - ✓ QRPA

    J. Suhonen and O. Civitarese, Phys. Rep. 300, 123 (1998)

    A. Faessler and F. Simkovic, J. Phys. G 24, 2139 (1998)

    R. Alvarez-Rodriguez, P. Sarriguren, et al. Phys. Rev. C 70, 064309 (2004)

    M. T. Mustonen and J. Engel, Phys. Rev. C 87, 064302 (2013)
  - ✓ Projected HFB

    B. M. Dixit, P. K. Rath, and P. K. Raina, Phys. Rev. C 65, 034311 (2002)

    R. Chandra, J. Singh, et al., Eur. Phys. J. A 23, 223 (2005)

    *Angular momentum projection for deformation
  - ✓ Interacting Boson Model

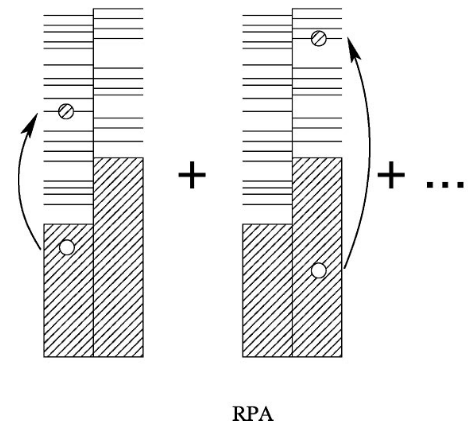
    J. Barea, J. Kotila, F. Iachello, Phys. Rev. C 91, 034304 (2015)

Quasiparticle Random Phase Approximation (QRPA)

QRPA: widely used for the description of spin-isospin excitations

- **The RPA excited state is generated by**

$$Q_\nu^\dagger = \sum_{mi} X_{mi}^\nu a_m^\dagger a_i - \sum_{mi} Y_{mi} a_i^\dagger a_m$$

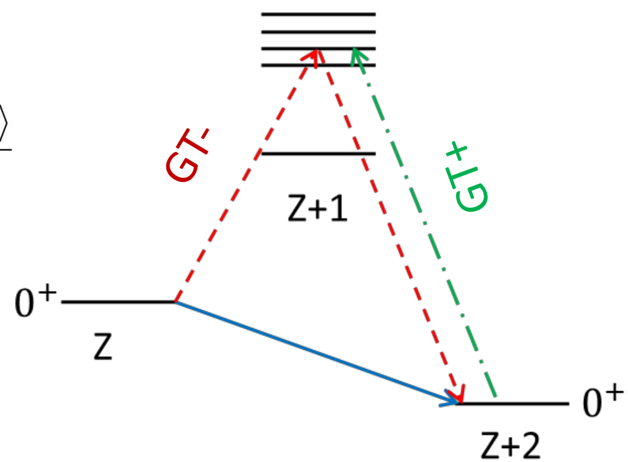


✓ **Full 1p1h configuration space ⇒ almost whole nuclear chart**

✓ **No closure approximation when calculating NME of 2νββ**

$$M_{\mathrm{GT}}^{2\nu} = \sum_{N_1 N_2} \frac{\langle \psi_{K'} || \hat{O}_{\mathrm{GT}}^- || \psi_{N_2} \rangle \langle \psi_{N_2} | \psi_{N_1} \rangle \langle \psi_{N_1} || \hat{O}_{\mathrm{GT}}^- || \psi_K \rangle}{E_N^* + M_N - (M_K + M_{K'})/2}$$

$$\langle 1^+ \left|\left| \hat{O}_{\mathrm{GT}}^- \right|\right| 0^+ \rangle = - \langle 0^+ \left|\left| \hat{O}_{\mathrm{GT}}^+ \right|\right| 1^+ \rangle^*$$



Quasiparticle Random Phase Approximation (QRPA)

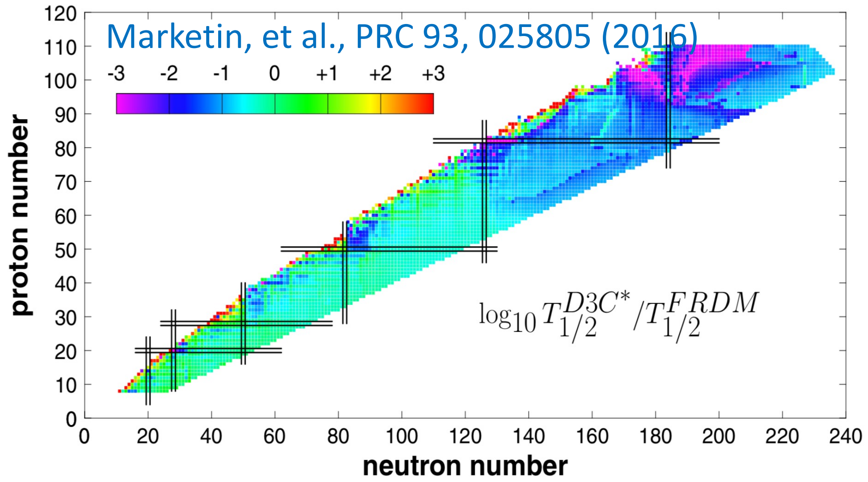
- **Self-consistent QRPA approach:**

the same interaction is used for ground state and excited states calculation

$$h_{kk'} = \frac{\delta E_{HF}}{\delta \rho_{kk'}}, \qquad V_{res.}^{mi,nj} = \frac{\delta^2 E_{HF}}{\delta \rho_{mi} \delta \rho_{nj}}, \qquad \text{where} \qquad E_{HF} = \langle \Phi \,|\, H_{eff} \,|\, \Phi \rangle.$$

✓ Successful applications in β-decay and electron-capture calculations:

- β-decay based on relativistic density functional



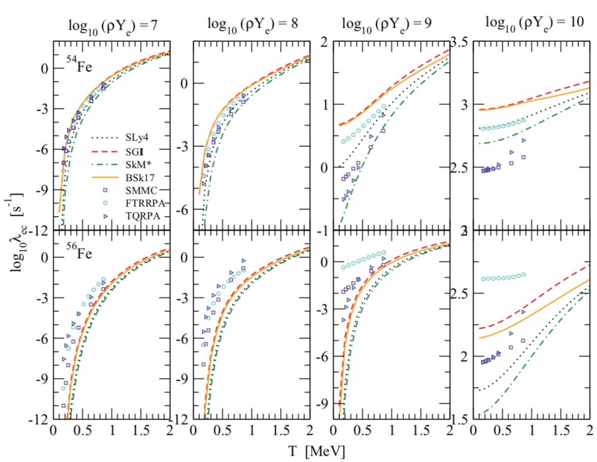
✓ β-decay half-lives for 5409 nuclei

✓ self-consistent

✓ Similar accuracy as FRDM+QRPA

- Electron capture based on Skyrme density functional

Fantina, et al., PRC 86, 035805 (2012)



✓ Electron capture rates

✓ self-consistent

✓ Good agreement with shell model

Quasiparticle Random Phase Approximation (QRPA)

- **NME of 2νββ studied by Skyrme QRPA**
  - ✓ Deformation effect on the NME

    P. Sarriguren, Phys. Rev. C 86, 034335 (2012)

    D. N. Nicolas and P. Sarriguren, Phys. Rev. C 91, 024317 (2015)
  - ✓ Single- and low-lying-states dominance hypotheses (SSDH/LLDH)

    O. Moreno, et al., J. Phys. G 36, 015106 (2009)

    P. Sarriguren, O. Moreno, and E.Moya de Guerra, Adv. HEP (2016)
  - ✓ Attempt to remove the uncertainty of g_A and the strength of IS pairing

    J. Terasaki, and Y. Iwata, Phys. Rev. C 100, 034325 (2019)
- **NME of 2νββ studied by Relativistic QRPA**
  - ✓ RMF-BCS+RQRPA with parameter set NL1:
    only NME values for 6 nuclei are reported

    Conti, Krmptoic and Carlson, Proceedings of Science, XXXIV BWNP 126 (2011)

$2\nu\beta\beta$: SSDH/LLDH

- **Single- and low-lying-states dominance hypotheses (SSDH/LLDH):**

The decay rate of the two-neutrino $2\nu\beta\beta$ decay to the final ground state is determined by virtual single-β-decay transitions via the ground state/low-lying states of the intermediate nucleus. J. Abad, et al., Ann. Fis. A 80, 9 (1984)

SSDH: $$M_{\rm GT}^{2\nu} \approx \frac{\langle 0^{+(f)}_{\rm g.s.}||\hat{O}^{-}_{\rm GT}||1^{+(N)}_{\rm g.s.}\rangle\langle 1^{+(N)}_{\rm g.s.}||\hat{O}^{-}_{\rm GT}||0^{+(i)}_{\rm g.s.}\rangle}{M_N-(M_i+M_f)/2}$$

LLDH: $$M_{\rm GT}^{2\nu} \approx \sum_{n\in\{\rm LL\}} \frac{\langle 0^{+(f)}_{\rm g.s.}||\hat{O}^{-}_{\rm GT}||1^{+(N)}_{n}\rangle\langle 1^{+(N)}_{n}||\hat{O}^{-}_{\rm GT}||0^{+(i)}_{\rm g.s.}\rangle}{E_n^{*(N)}+M_N-(M_i+M_f)/2}$$

✓ Avoid the tremendous summation.

✓ Directly calculate the NME from the β^- or EC experimental data.

H. Akimune, et al., Phys. Lett. B 394, 23 (1997)

LBNL collaboration calculate the $2\nu\beta\beta$ half-life of ^{100}Mo(g.s.) -> ^{100}Ru(g.s.) with the EC of ^{100}Tc -> ^{100}Mo and the β^- of ^{100}Tc -> ^{100}Ru. They get $(9.7 \pm 4.9)\times 10^{18}$ yr. [$(7.1 \pm 0.4)\times 10^{18}$ yr, Barabash2010]

A. Garcia, et al., Phys. Rev. C 47, 2910 (1993)

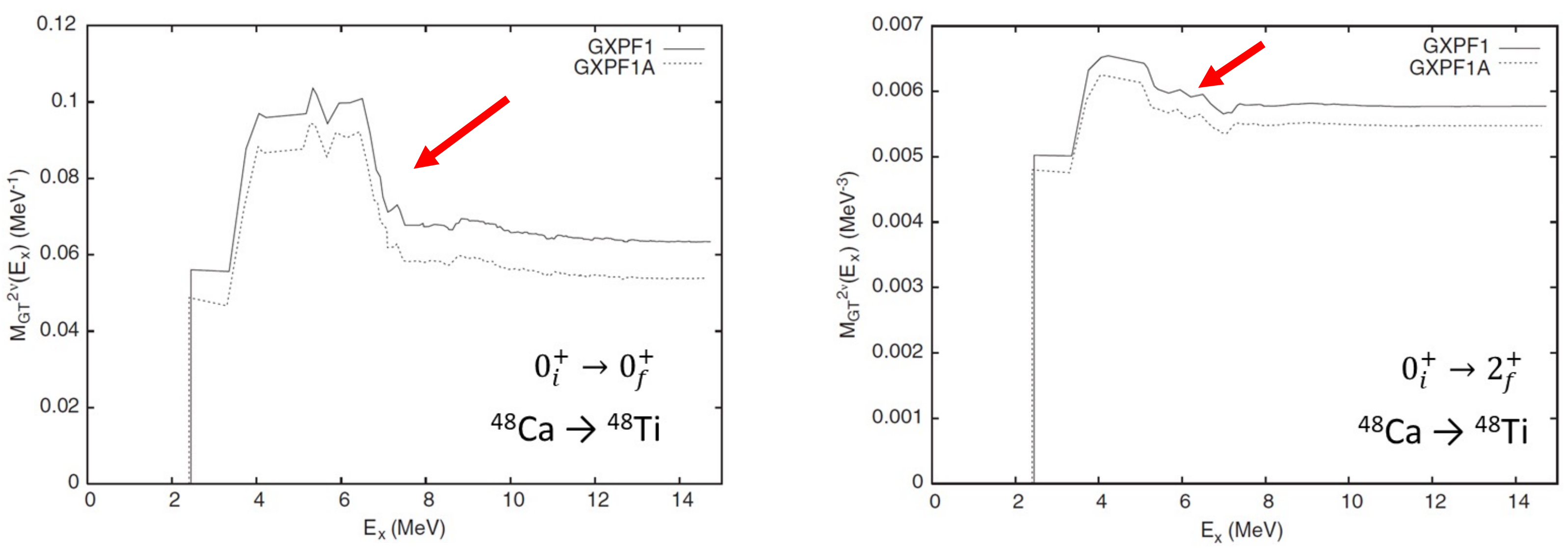
2νββ: SSDH/LLDH

- **Mechanism of SSDH/LLDH:**

  1. only g.s. or low-lying state(s) H. Nakada, T. Seba, and K. Muto, Nucl. Phys. A 607, 235 (1996)
  H. Ejiri and H. Toki, J. Phys. Soc. Japan 65, 7 (1996)

  2. cancellation between higher lying states
  F. Simkovic, A. Smetana, and P. Vogel, Phys. Rev. C 98, 064325 (2018)

✓ Shell model M. Horoi, S. Stoica, and B. A. Brown, Phys. Rev. C 75, 034303 (2007)



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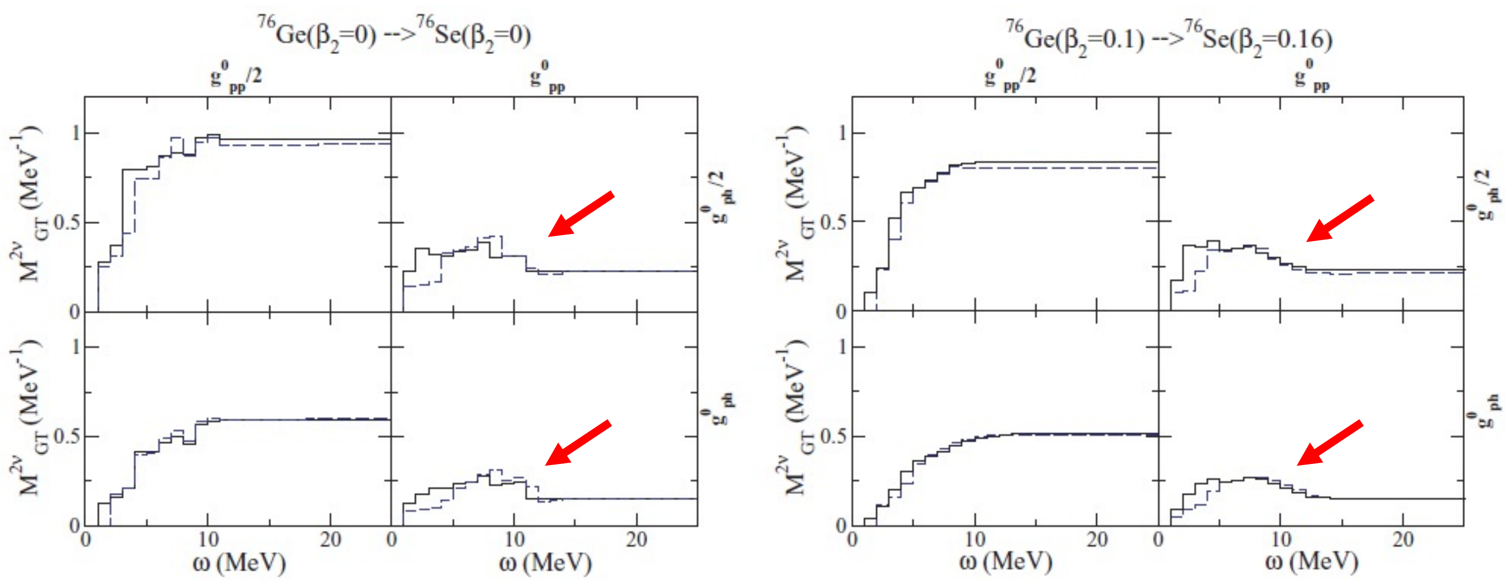
H. Ejiri and H. Toki, J. Phys. Soc. Japan 65, 7 (1996)

2. cancellation between higher lying states
F. Simkovic, A. Smetana, and P. Vogel, Phys. Rev. C 98, 064325 (2018)

✓ Spherical and deformed QRPA D. L. Fang, et al., Phys. Rev. C 81, 037303 (2010)



*Expt: $M^{2\nu}_{\mathrm{GT}} = 0.14\ \mathrm{MeV}^{-1}$

2νββ: SSDH/LLDH

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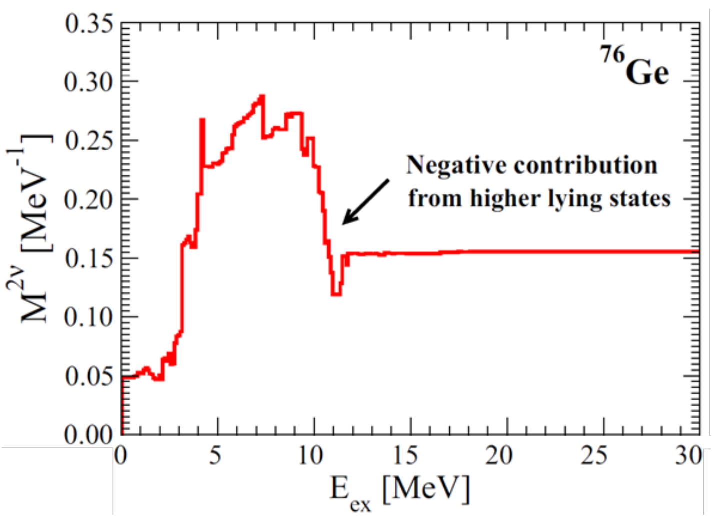
H. Ejiri and H. Toki, J. Phys. Soc. Japan 65, 7 (1996)

2. cancellation between higher lying states
F. Simkovic, A. Smetana, and P. Vogel, Phys. Rev. C 98, 064325 (2018)

✓ A natural question:

Why do the high-lying states give negative contributions such that the SSDH/LLDH is valid?



Aim of this work

- Within the framework of HFB + QRPA based on Skyrme density functional
  - ✓ To calculate the NMEs of $2\nu\beta\beta$ systematically
  - ✓ To study the dependence of isoscalar pairing strength, and determine the proper values by comparing with exp. data
  - ✓ To reveal the mechanism of SSDH/LLDH by understanding the cancellation from higher lying states
- Within the framework of RHB + QRPA based on relativistic density functional
  - ✓ To calculate the NMEs of $2\nu\beta\beta$ systematically for the first time
  - ✓ To study the dependence of isoscalar pairing strength, and determine the proper values by comparing with exp. data

Outline

- **Introduction**
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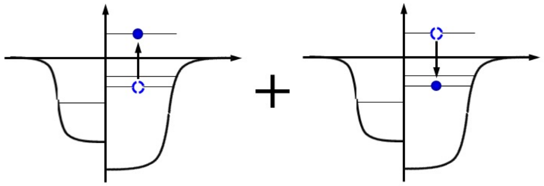
Formalism: QRPA

- Charge exchange QRPA in canonical basis

$$\begin{bmatrix} A & B \\ -B^* & -A^* \end{bmatrix} \begin{bmatrix} X^{nJ} \\ Y^{nJ} \end{bmatrix} = \Omega^{nJ} \begin{bmatrix} X^{nJ} \\ Y^{nJ} \end{bmatrix}$$



X term   Y term

$$\begin{aligned} A_{ll',kk'} = & (H^{11}_{lk}\delta_{l'k'} + H^{11}_{l'k'}\delta_{kl}) \\ & + \Big[(u_l u_{l'} u_{k'} u_k + v_l v_{l'} v_{k'} v_k)\langle ll'|V|kk'\rangle_J^{pp} \\ & + (u_l v_{l'} v_{k'} u_k + u_{l'} v_l v_k u_{k'})\langle lk'|V|l'k\rangle_J^{ph} \end{aligned}$$

$l,\ k$ for proton
l', k' for neutron

$$\begin{aligned} B_{ll',kk'} = & -(u_k u_{k'} v_l v_{l'} + u_l u_{l'} v_k v_{k'})\langle ll'|V|kk'\rangle_J^{pp} \\ & + (u_k u_{l'} v_{k'} v_l + u_{k'} u_l v_k v_{l'})\langle lk'|V|l'k\rangle_J^{ph} \end{aligned}$$

* B = 0 for QTDA

✓ Transition amplitude:

$$\langle nJ||\hat{O}^-||0\rangle = \sum_{pn} -\langle j_p||\hat{O}^-||j_n\rangle [X^{nJ}_{pn} v_n u_p + Y^{nJ}_{pn} v_p u_n]$$

$$\langle nJ||\hat{O}^+||0\rangle = \sum_{pn} (-)^{j_p+j_n+J}\langle j_n||\hat{O}^+||j_p\rangle [X^{nJ}_{pn} v_p u_n + Y^{nJ}_{pn} v_n u_p]$$

Formalism: Isoscalar pairing

- Isoscalar pairing

  1. No experimental constraint on isoscalar pairing
  2. Important for $B(\mathrm{GT}^{-})$, β-decay half-lives and $M_{\mathrm{GT}}^{2\nu}$.

M. K. Cheoun, et al., Nucl. Phys A 561, 74 (1993) C. L. Bai, et al., Phys. Rev. C 90, 054335 (2014)

Y. F. Niu, et al., Phys. Lett. B 780, 325 (2018)

$$V^{pp}(\boldsymbol{r}_1,\boldsymbol{r}_2)=\left(t_0'+\frac{t_3'}{6}\rho^{\gamma'}(\boldsymbol{R})\right)\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)$$

$$\boxed{\begin{aligned}&\langle ab|V^{pp}(1-P_rP_\sigma P_\tau)|cd\rangle\\ =&\langle T=0,S=1|V^{pp}\frac{1+P_\sigma}{2}|T=0,S=1\rangle \quad \text{(IS)}\\ &+\langle T=1,S=0|V^{pp}\frac{1-P_\sigma}{2}|T=1,S=0\rangle \quad \text{(IV)}\end{aligned}}$$

$$V_{T=0}^{pp}(\boldsymbol{r}_1,\boldsymbol{r}_2)=\left(t_0'+\frac{t_3'}{6}\rho^{\gamma'}(\frac{\boldsymbol{r}_1+\boldsymbol{r}_2}{2})\right)\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)f_{\mathrm{IS}}\frac{3+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2}{4} \qquad (f_{\mathrm{IS}}\text{ is free})$$

$$V_{T=1}^{pp}(\boldsymbol{r}_1,\boldsymbol{r}_2)=\left(t_0'+\frac{t_3'}{6}\rho^{\gamma'}(\frac{\boldsymbol{r}_1+\boldsymbol{r}_2}{2})\right)\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\frac{1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2}{4}$$

Formalism: Overlap factor

- **Overlap factor**

$$M_{\mathrm{GT}}^{2\nu} = \sum_{n_i n_f} \frac{\langle 0_{\mathrm{g.s.}}^{+(f)} || \hat{O}_{\mathrm{GT}}^{-} || 1_{n_f}^{+} \rangle \boxed{\langle 1_{n_f}^{+} | 1_{n_i}^{+} \rangle} \langle 1_{n_i}^{+} || \hat{O}_{\mathrm{GT}}^{-} || 0_{\mathrm{g.s.}}^{+(i)} \rangle}{E_n^{*(N)} + M_N - (M_f + M_i)/2} \qquad \langle 1_{n_f}^{+} || \hat{O}_{\mathrm{GT}}^{+} || 0_{\mathrm{g.s.}}^{+(f)} \rangle$$

$|1_{n_i}^{+}\rangle$ and $|1_{n_f}^{+}\rangle$ calculated from initial and final nucleus are not normal orthogonal.

$$\langle 1_{n_f}^{+} | 1_{n_i}^{+} \rangle = a_{n_i n_f} \langle \mathrm{HFB}_{\mathrm{can}.f} | \mathrm{HFB}_{\mathrm{can}.i} \rangle$$

$$\langle \mathrm{HFB}_{\mathrm{can}.f} | \mathrm{HFB}_{\mathrm{can}.i} \rangle \approx \prod_{k>0} (u_k^{(i)} u_k^{(f)} + v_k^{(i)} v_k^{(f)})$$

$$a_{n_i n_f} = \sum_{k_i k_i'} \sum_{k_f k_f'} C_{k_f k_i} C_{k_f' k_i'} (X_{k_i k_i'}^{n_i} X_{k_f k_f'}^{n_f} - Y_{k_i k_i'}^{n_i} Y_{k_f k_f'}^{n_f}) \cdot (u_{k_i}^{(i)} u_{k_f}^{(f)} + v_{k_i}^{(i)} v_{k_f}^{(f)})(u_{k_i'}^{(i)} u_{k_f'}^{(f)} + v_{k_i'}^{(i)} v_{k_f'}^{(f)})$$

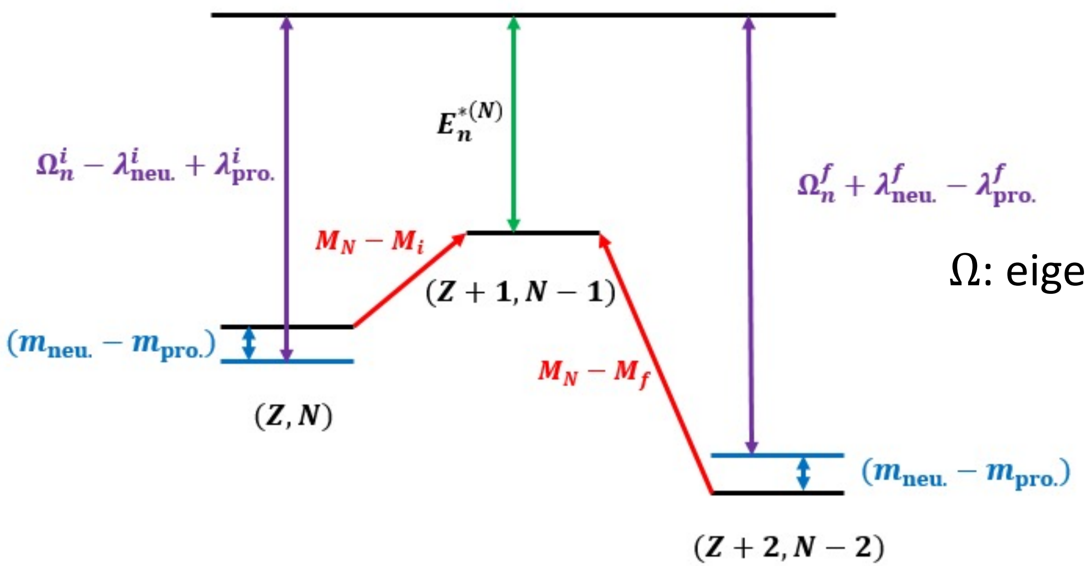
$C_{k_f k_i}$: overlap between s.p. wavefunctions

Formalism: Energy denominator

- Energy denominator $= E_n^{*(N)} + M_N + \frac{M_i - M_f}{2} - M_i = E_n^{*(N)} + M_N - \frac{M_i + M_f}{2}$

$$M_{\mathrm{GT}}^{2\nu} = \sum_{n_i n_f} \frac{\langle 0_{\mathrm{g.s.}}^{+(f)} || \hat{O}_{\mathrm{GT}}^{-} || 1_{n_f}^{+} \rangle \langle 1_{n_f}^{+} | 1_{n_i}^{+} \rangle \langle 1_{n_i}^{+} || \hat{O}_{\mathrm{GT}}^{-} || 0_{\mathrm{g.s.}}^{+(i)} \rangle}{\underline{E_n^{*(N)}} + M_N - (M_f + M_i)/2}$$

$$= \sum_{n_i n_f} \frac{\langle 0_{\mathrm{g.s.}}^{+(f)} || \hat{O}_{\mathrm{GT}}^{-} || 1_{n_f}^{+} \rangle \langle 1_{n_f}^{+} | 1_{n_i}^{+} \rangle \langle 1_{n_i}^{+} || \hat{O}_{\mathrm{GT}}^{-} || 0_{\mathrm{g.s.}}^{+(i)} \rangle}{\frac{1}{2}\left[(\Omega_n^i - \lambda_{\mathrm{neu.}}^i + \lambda_{\mathrm{pro.}}^i) + (\Omega_n^f + \lambda_{\mathrm{neu.}}^f - \lambda_{\mathrm{pro.}}^f) \right]}$$



Ω: eigenvalue of QRPA

$$E_n^{*(N)} = \frac{1}{2}\left(\left[(\Omega_n^i - \lambda_{\mathrm{neu.}}^i + \lambda_{\mathrm{pro.}}^i) + (\Omega_n^f + \lambda_{\mathrm{neu.}}^f - \lambda_{\mathrm{pro.}}^f) - (2M_N - M_i - M_f) \right] \right)$$

Outline

- Introduction
- Theoretical Framework
- **$2\nu 2\beta$ NME calculated by Skyrme QRPA**
- $2\nu 2\beta$ NME calculated by relativistic QRPA
- Summary and Perspective

Numerical details

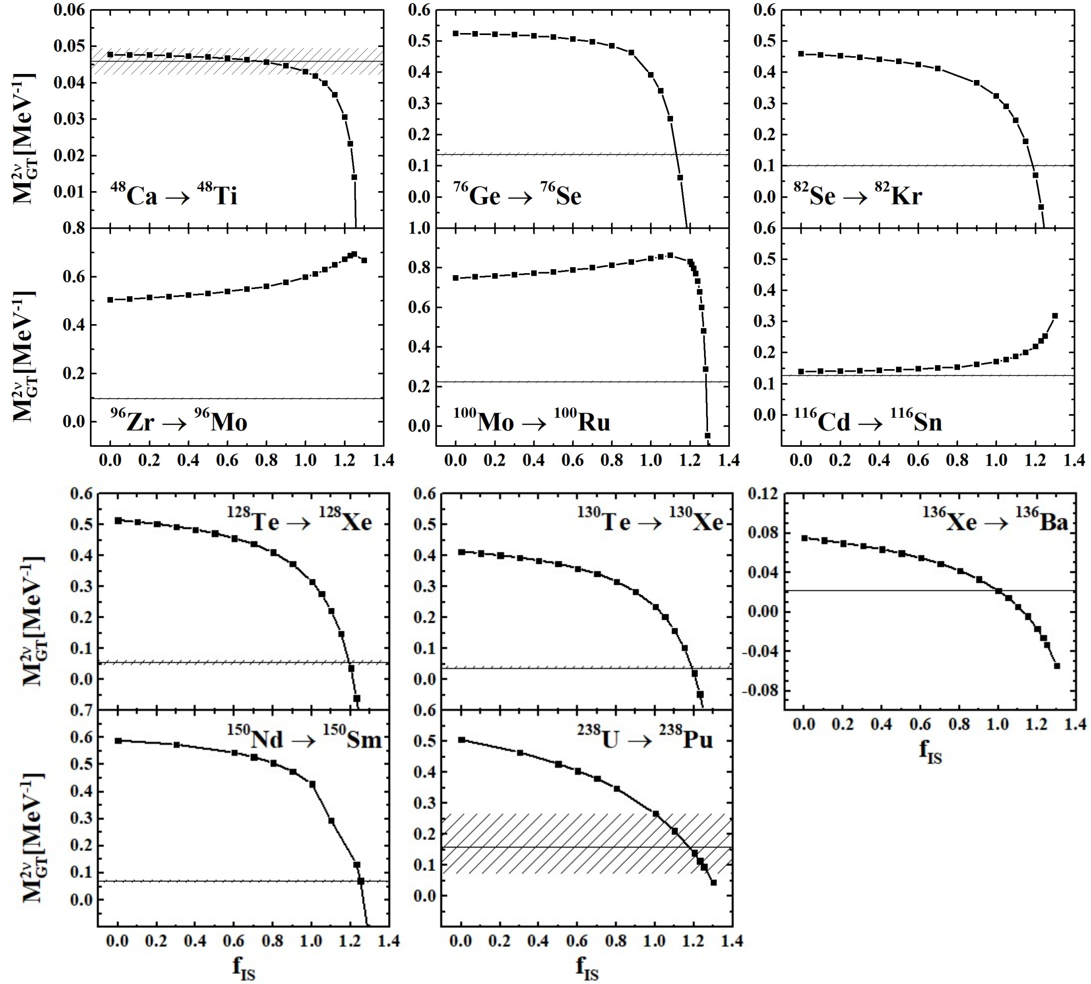
- HFB:
  1. Skyrme interaction: SkO'
  2. Surface pairing, fit the experimental mean pairing gap
  3. Diffuseness parameter of pairing window: 0.1 MeV
  4. $E_{\mathrm{cut}} = 80.0\ \mathrm{MeV}$ (Quasiparticle energy)
  5. $j_{\mathrm{max}} = 21/2$

- QRPA:
  1. Single-particle energy: $\varepsilon_{\mathrm{s.p.}} < 60.0\ \mathrm{MeV}$
  2. $\left|u_p v_n\right| > 10^{-3}$ and $\left|u_n v_p\right| > 10^{-4}$

- $M_{\mathrm{GT}}^{2\nu}$
  1. For each QRPA state, the ph configuration with $\left|X_{ph}^2 - Y_{ph}^2\right| > 10^{-6}$ is considered.

NME of 2νββ

- **Dependence of NME on isoscalar pairing strength**

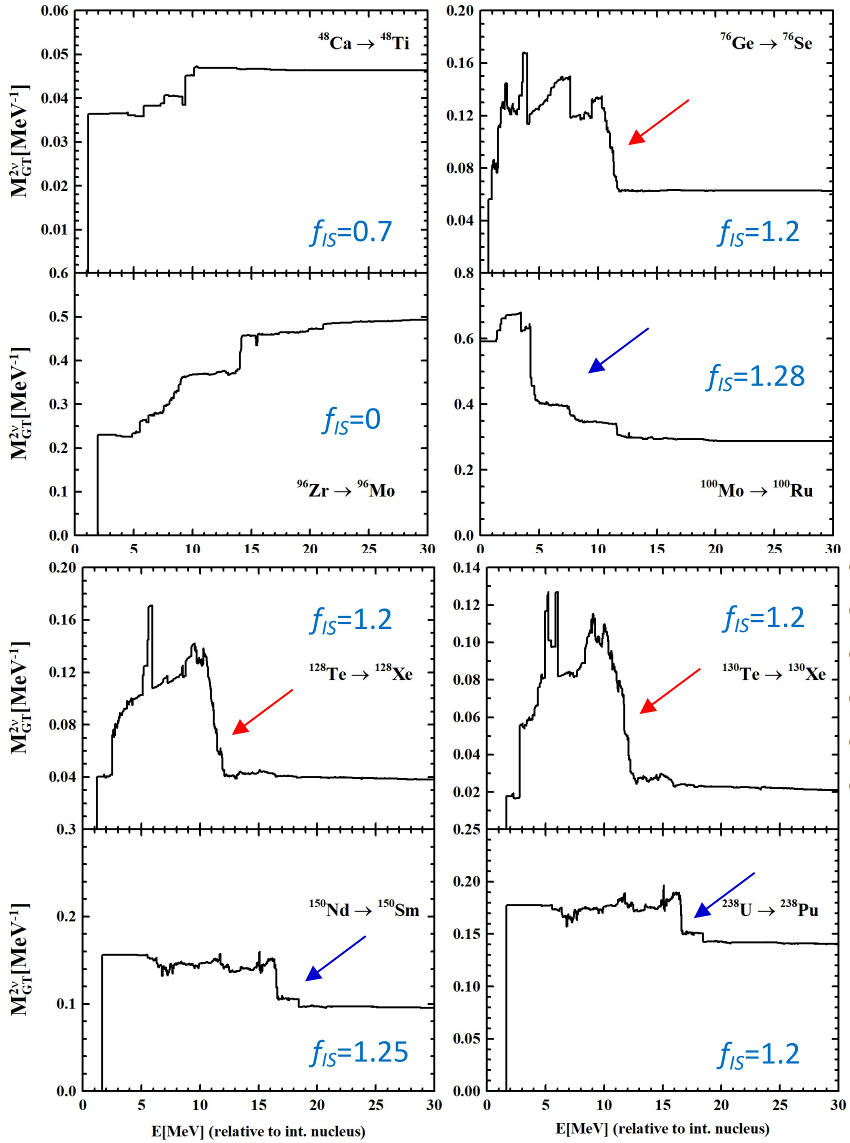


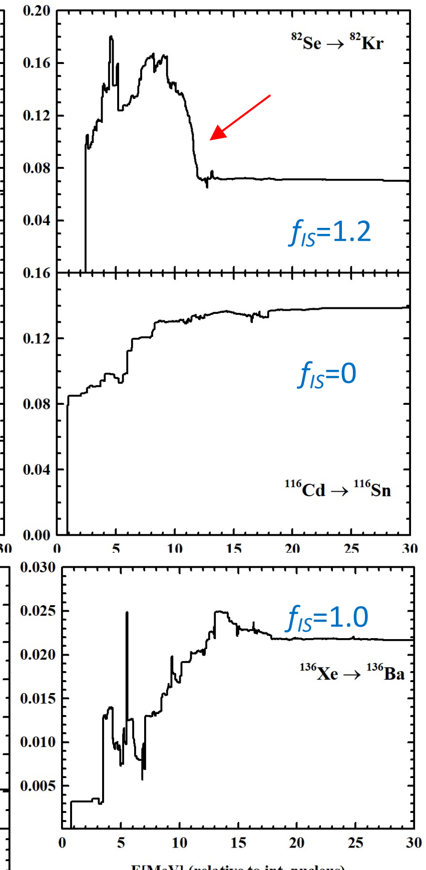
Exp. data from A. S. Barabash, Nucl. Phys. A 935, 52 (2015)

Running sum of NME

- **Running sum of NME as a function of excitation energy of intermediate states**

$$E_n^{*(N)} = \frac{1}{2}\Big(\big[(\Omega_n^i - \lambda_{\text{neu.}}^i + \lambda_{\text{pro.}}^i) + (\Omega_n^f + \lambda_{\text{neu.}}^f - \lambda_{\text{pro.}}^f) - (2M_N - M_i - M_f)\big] \Big)$$





f_{IS} is taken at the value that reproduces exp. data (or close)

Isoscalar pairing strength and SSDH/LLDH

Nucleus	^{48}Ca	^{76}Ge	^{82}Se	^{96}Zr	^{100}Mo
Expt. $M^{2\nu}_{\mathrm{GT}}$ (MeV^{-1})	0.046±0.004	0.136±0.007	0.100±0.005	0.097±0.005	0.223±0.006
Theo. $M^{2\nu}_{\mathrm{GT}}$ (MeV^{-1})	0.046	0.062	0.070	0.139	0.288
Theo. $M^{2\nu}_{\mathrm{GT}}$(SSD) (MeV^{-1})	0.035	0.054	0.015	0.230	0.570
Theo. $M^{2\nu}_{\mathrm{GT}}$(LLD) (MeV^{-1})	0.036	0.125	0.098	0.234	0.407
f_{IS}	0.70	1.20	1.20	0.00	1.28

Nucleus	^{116}Cd	^{128}Te	^{130}Te	^{136}Xe	^{150}Nd	^{238}U
Expt. $M^{2\nu}_{\mathrm{GT}}$ (MeV^{-1})	0.127±0.004	0.056±0.007	0.037±0.005	0.022±0.001	0.070±0.005	$0.157^{+0.109}_{-0.085}$
Theo. $M^{2\nu}_{\mathrm{GT}}$ (MeV^{-1})	0.139	0.037	0.021	0.022	0.071	0.140
Theo. SSD	<0.001	0.040	0.018	<0.001	0.156	0.177
Theo. LLD	0.096	0.102	0.116	0.007	0.156	0.177
f_{IS}	0.00	1.20	1.20	1.00	1.25	1.20

Exp. data from A. S. Barabash, Nucl. Phys. A 935, 52 (2015)

*$g_A = 1.273$ is used to calculate the expt. $M^{2\nu}_{\mathrm{GT}}$

The upper limit of energy of LLD is 5MeV, except for ^{82}Se (2 states only).

✓ SSD nuclei: ^{48}Ca, ^{76}Ge, ^{128}Te, ^{130}Te, ^{238}U

✓ LLD nuclei: ^{82}Se, ^{100}Mo, ^{116}Cd

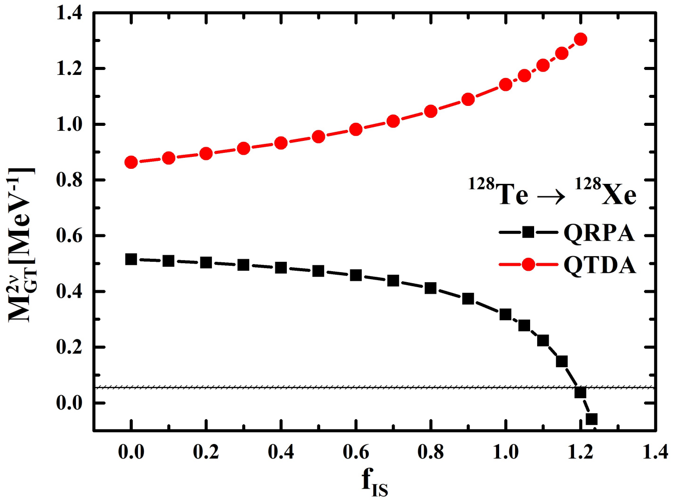
SSDH mechanism: ground-state correlation

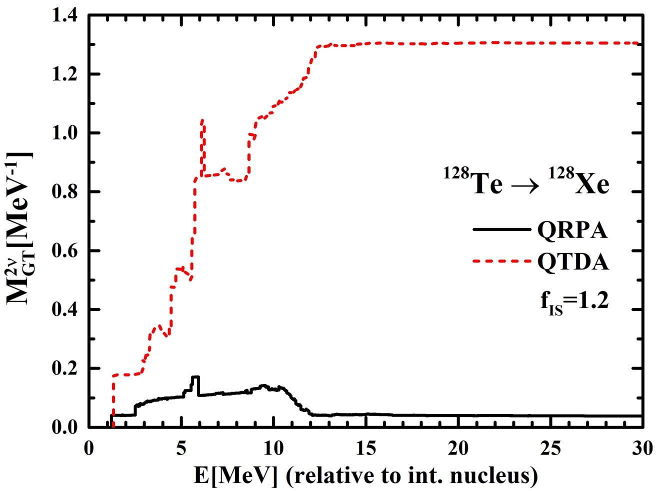
- **Single-state dominance** $^{128}\mathrm{Te} \rightarrow {}^{128}\mathrm{Xe}$

$f_{IS} = 1.2$

$M_{\mathrm{GT}}^{2\nu}$ [MeV^{-1}]	Expt.	Theo. (All 1^+)	Theo. (1_1^+)
^{128}Te	0.056	0.037	0.040

- **QRPA vs. QTDA**





With the inclusion of ground-state correlation Y term:

- ✓ NME is decreased.
- ✓ With the increase of f_{IS}, NME decreases monotonously
- ✓ Negative contributions appear in 10.0-12.5MeV of int. nucleus at f_{IS}=1.2.

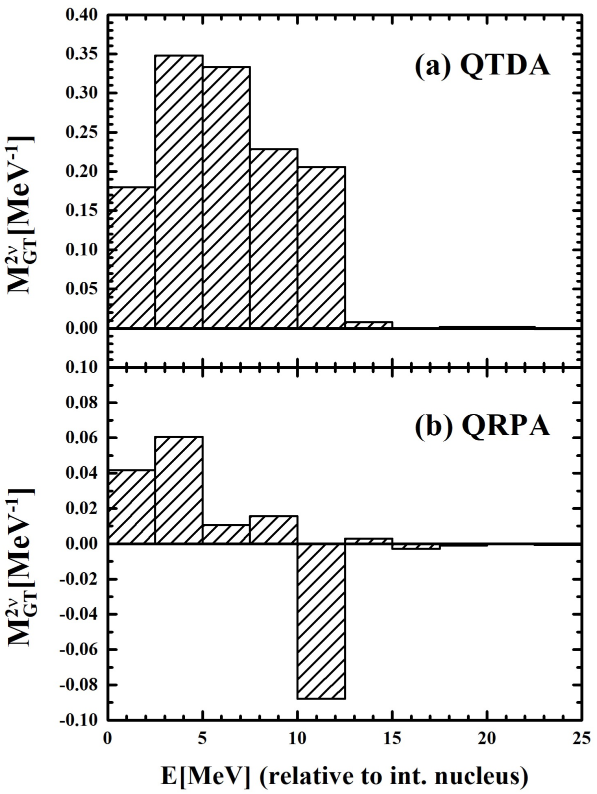
SSDH mechanism: ground-state correlation

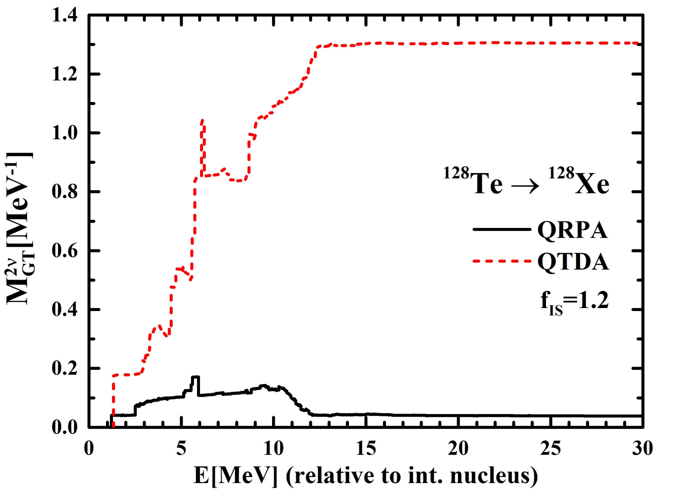
- **Single-state dominance** $^{128}\mathrm{Te} \rightarrow {}^{128}\mathrm{Xe}$

f_{IS} = 1.2

$M^{2\nu}_{\mathrm{GT}}$ [MeV^{-1}]	Expt.	Theo. (All 1^+)	Theo. (1^+_1)
^{128}Te	0.056	0.037	0.040

- **QRPA vs. QTDA**





SSDH mechanism

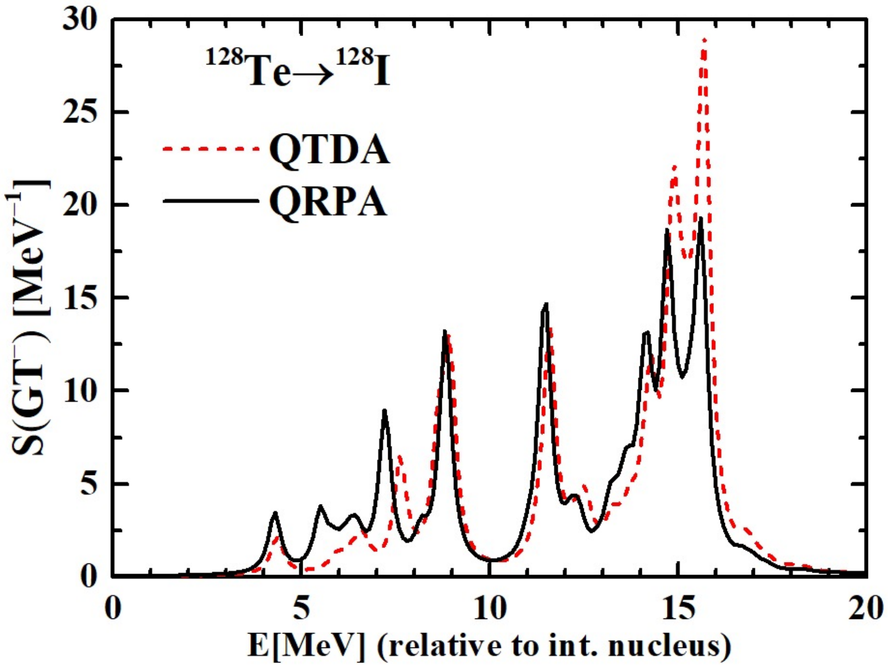
Negative contributions of higher lying states caused by ground-state correlation Y term

SSDH mechanism: ground-state correlation

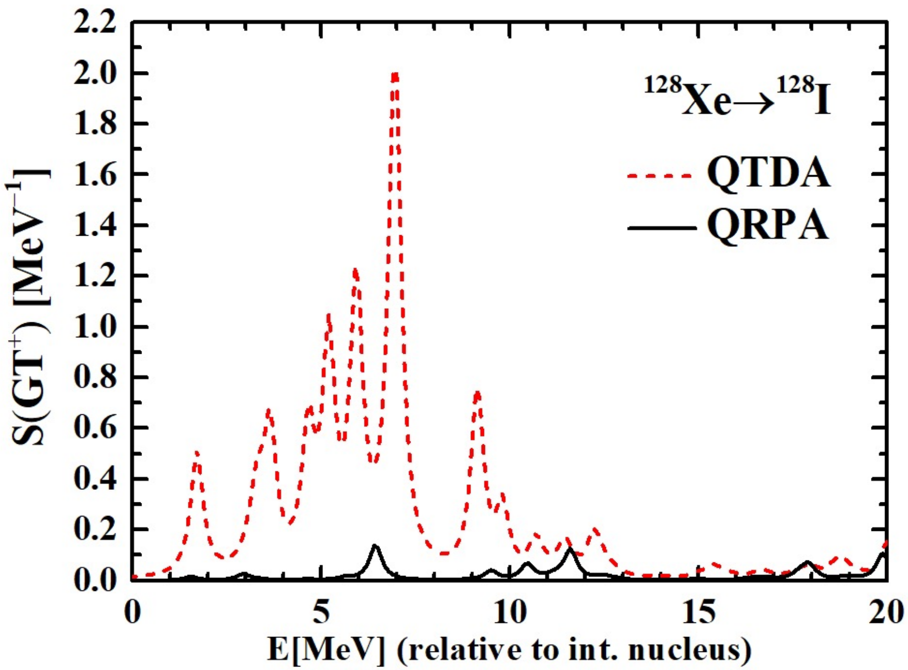
- **How does ground-state correlation influence the NME?**

$$M^{2\nu}_{\mathrm{GT}} = \sum_{n_i n_f} \frac{\langle 0^{+(f)}_{\mathrm{g.s.}} || \hat{O}^{-}_{\mathrm{GT}} || 1^{+}_{n_f} \rangle \langle 1^{+}_{n_f} | 1^{+}_{n_i} \rangle \langle 1^{+}_{n_i} || \hat{O}^{-}_{\mathrm{GT}} || 0^{+(i)}_{\mathrm{g.s.}} \rangle}{E^{*(N)}_{n} + M_N - (M_f + M_i)/2}$$

$$\langle 1^{+}_{n_f} || \hat{O}^{+}_{\mathrm{GT}} || 0^{+(f)}_{\mathrm{g.s.}} \rangle$$



$B(\mathrm{GT}^-)$ of QRPA and QTDA are similar.



$B(\mathrm{GT}^+)$ of QRPA are quite small.

Ground-state correlation Y_{ph} mainly influences the GT^+ transition.

SSDH mechanism: ground-state correlation

- **How does ground-state correlation make negative contribution?**

QRPA Eq: $-BX = (A+\Omega)Y$

$$\breve{B}_{ll',kk'} = -\left(u_k u_{k'} v_l v_{l'} + u_l u_{l'} v_k v_{k'}\right)\langle ll'|V|kk'\rangle_J^{pp} \quad \text{attractive}$$
$$+\left(u_k u_{l'} v_{k'} v_l + u_{k'} u_l v_k v_{l'}\right)\langle lk'|V|l'k\rangle_J^{ph} \quad \text{repulsive}$$

✓ Both $(A+\Omega)$ and B are positive.

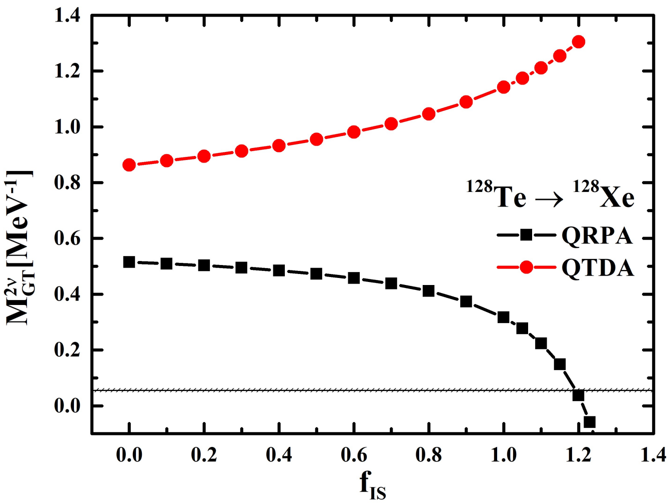
➡ X and Y are of opposite sign.

✓ B get large with the increase of f_{IS}

➡ $|Y|$ will approach to X

➡ GT transition amplitude becomes smaller

➡ NME decreases with increasing f_{IS}



P. Vogel and M. R. Zirnbauer, Phys. Rev. Lett. 57, 3148(1986)

O. Civitarese, Amand Faessler and T. Tomoda, Phys. Lett. B 194, 11(1987)

SSDH mechanism: ground-state correlation

- **How does ground-state correlation make negative contribution?**

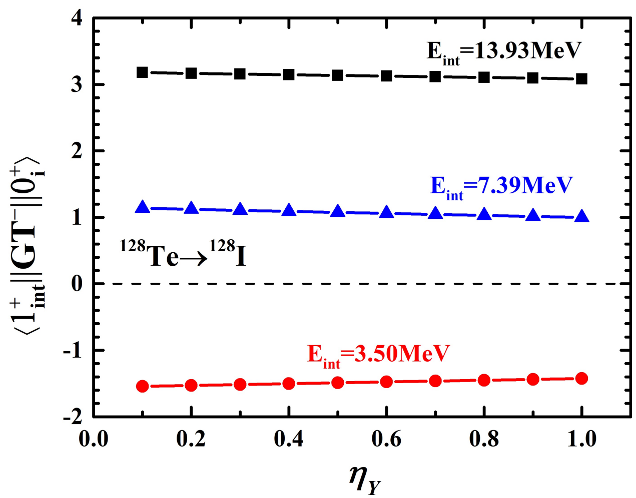
$$\langle nJ||\hat{O}^{+}||0\rangle = \sum_{pn}(-)^{j_p+j_n+J}\langle j_n||\hat{O}^{+}||j_p\rangle[X_{pn}^{nJ}v_p u_n + \eta_Y Y_{pn}^{nJ}v_n u_p]$$

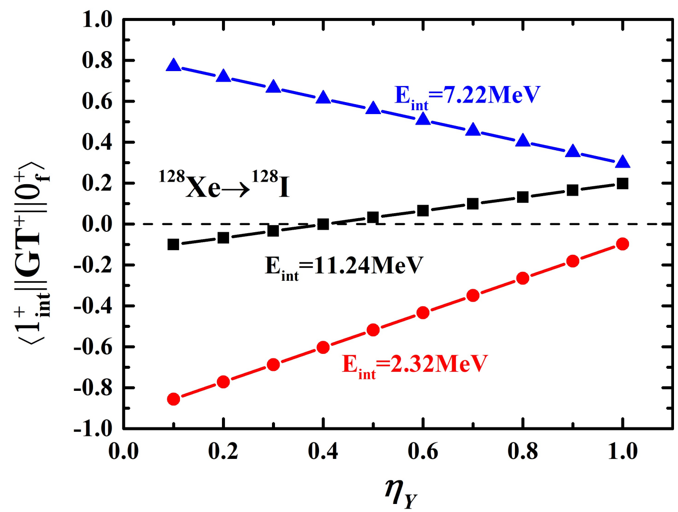
✓ At large f_{IS}:

$|Y|$ is increased ⟶ negative contribution of higher-lying states

↓ ↑

The sign of GT+ amplitude will change for higher-lying states





✓ $\langle 1^+_{\mathrm{int}}||\mathrm{GT}^-||0^+_{\mathrm{i}}\rangle$ is almost independent from Y_{ph}

✓ $\langle 1^+_{\mathrm{int}}||\mathrm{GT}^+||0^+_{\mathrm{f}}\rangle$ is sensitive to Y_{ph}, and its sign changes for higher-lying states.

SSDH mechanism: ground-state correlation

- **Why does ground-state correlation play its role through GT+?**

$$\langle J||\hat{O}^-||0\rangle = \sum_{pn} -\langle j_p||\hat{O}^-||j_n\rangle [X^{nJ}_{pn} v_n u_p + Y^{nJ}_{pn} v_p u_n]$$

$$\langle J||\hat{O}^+||0\rangle = \sum_{pn} (-)^{j_p+j_n+J} \langle j_n||\hat{O}^+||j_p\rangle [X^{nJ}_{pn} v_p u_n + Y^{nJ}_{pn} v_n u_p]$$

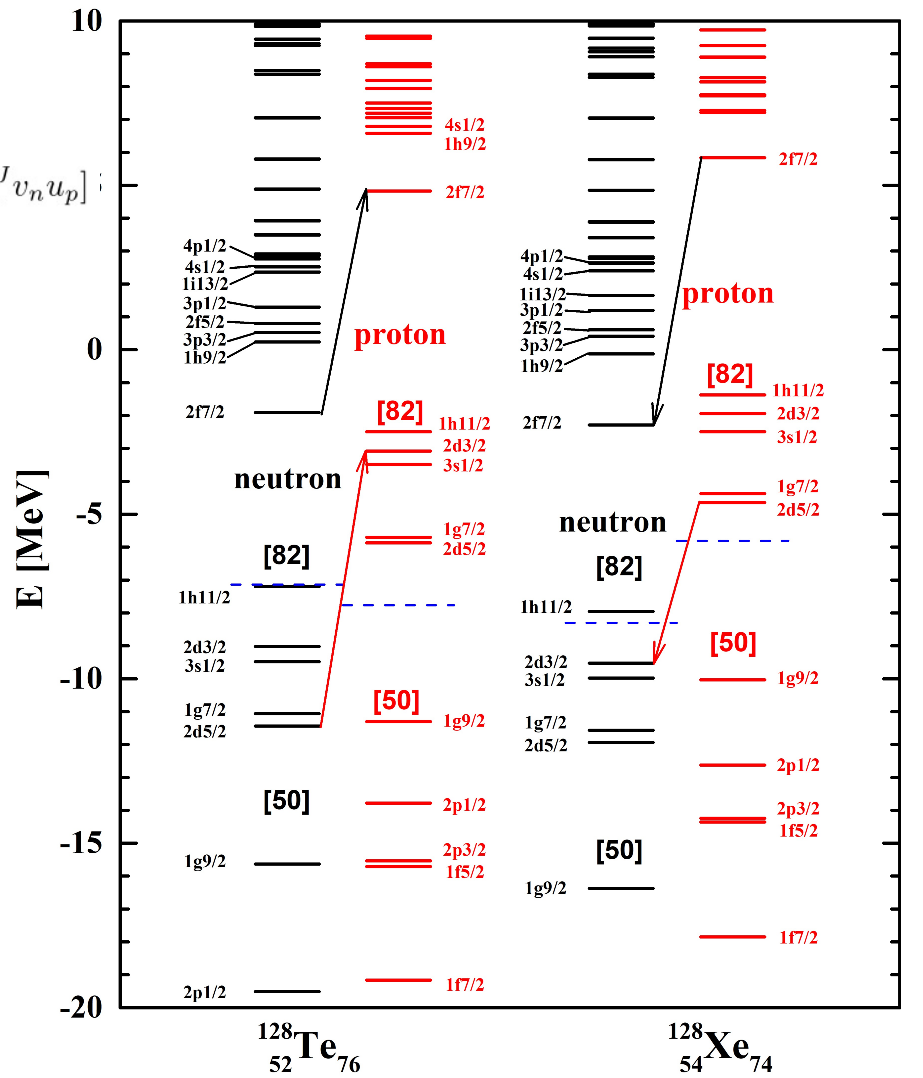
^{128}Te: neutron-rich nucleus

✓ GT+ transitions: completely blocked at mean-field level

✓ pairing correlation: unblock the transition through X term

✓ Ground-state correlation: block/unblock the transition through Y term

➡ GT+ is sensitive to ground-state correlation



Outline

- Introduction
- Theoretical Framework
- $2\nu2\beta$ NME calculated by Skyrme QRPA
- **$2\nu2\beta$ NME calculated by relativistic QRPA**
- Summary and Perspective

Numerical details

- RHB:

  1. Density dependent meson-exchange interaction: DD-ME1 DD-ME2

  2. Gogny pairing force D1S

  3. Harmonic oscillator basis N_{max}=20

- QRPA:

  1. Single-particle energy in Fermi sea: $\varepsilon_{\mathrm{s.p.}} < 200.0\ \mathrm{MeV}$

  2. Single-particle energy in Dirac sea: $\varepsilon_{\mathrm{s.p.}} \succ -2000.0\ \mathrm{MeV}$

  2. $|u_p v_n| > 10^{-2}$ and $|u_n v_p| > 10^{-2}$

- $M_{\mathrm{GT}}^{2\nu}$

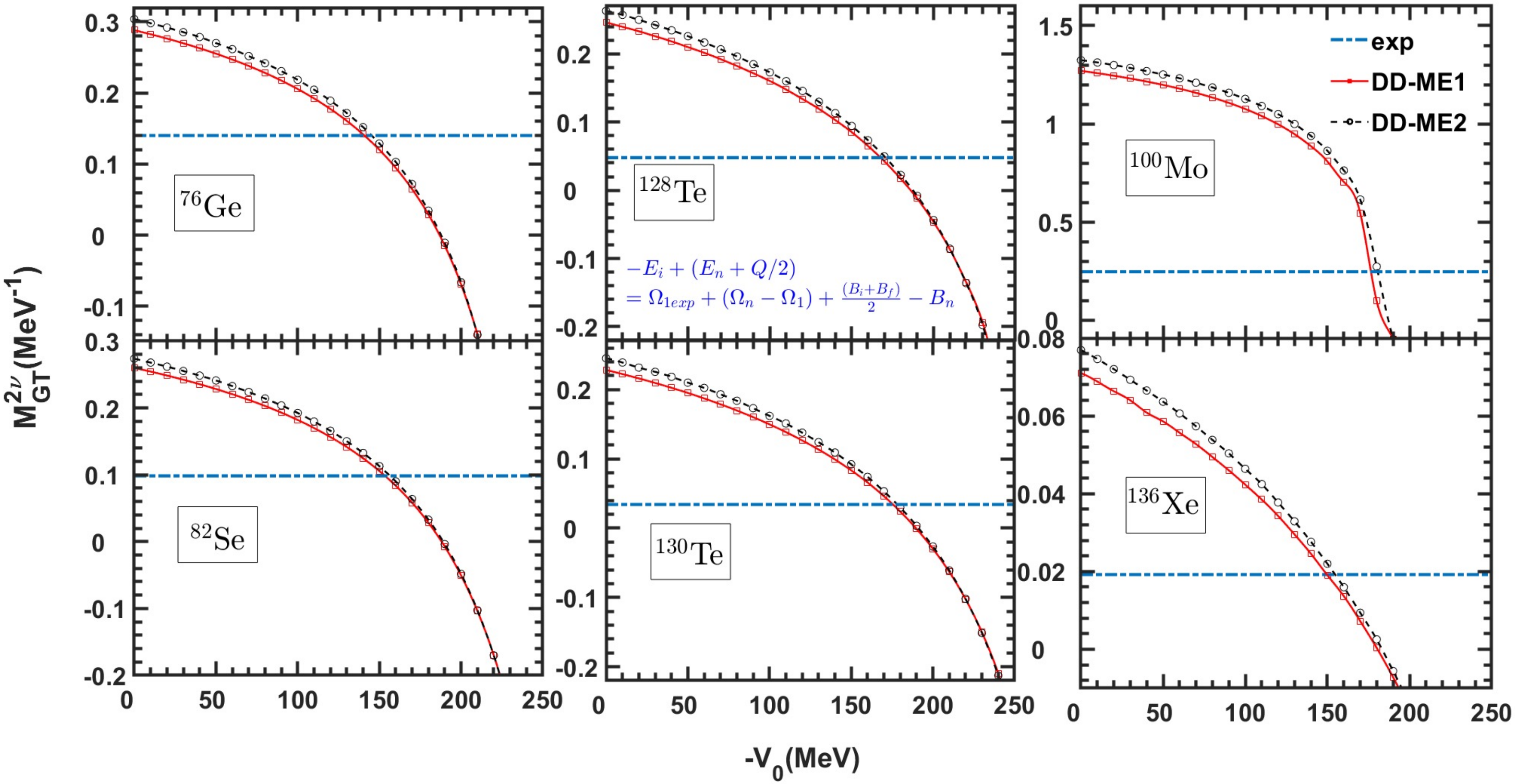
  1. For each QRPA state, the ph configuration with $\left|X_{ph}^2 - Y_{ph}^2\right| > 10^{-6}$ is considered.

  2. Single-particle wavefunctions of initial and final state are assumed to be the same

$$\langle \psi_\alpha^f | \psi_\beta^i \rangle = \left\langle - \left| a_\alpha^f {a_\beta^i}^+ \right| - \right\rangle = \delta_{\alpha\beta}$$

NME of $2\nu\beta\beta$

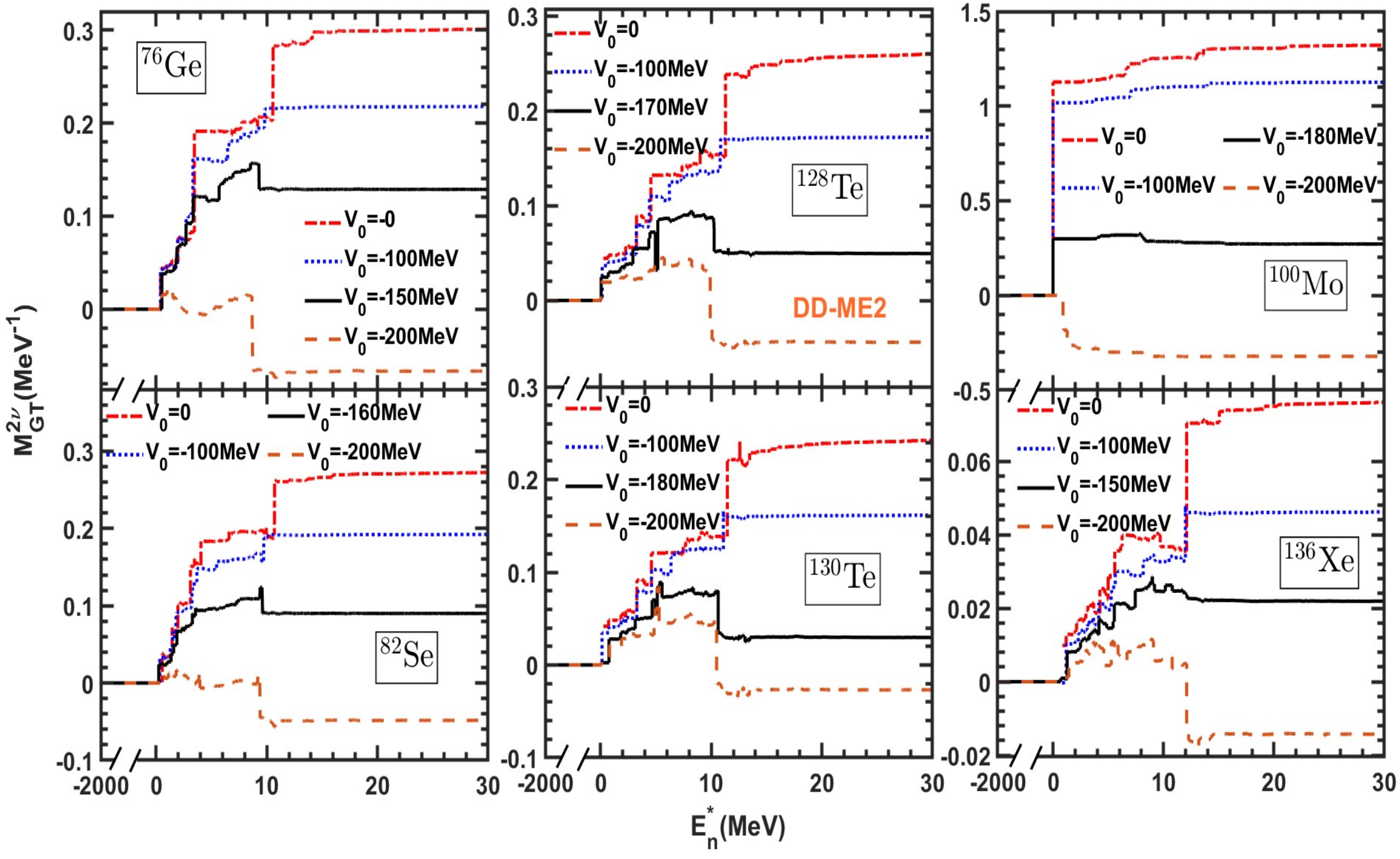
- Dependence of NME on isoscalar pairing strength



Running sum of NME

- Running sum of NME as a function of excitation energy of intermediate states

$$E_n^{*(N)} = \frac{1}{2}\Big(\left[(\Omega_n^i - \lambda_{\text{neu.}}^i + \lambda_{\text{pro.}}^i) + (\Omega_n^f + \lambda_{\text{neu.}}^f - \lambda_{\text{pro.}}^f) - (2M_N - M_i - M_f) \right] \Big)$$



Isoscalar pairing strength

- The isoscalar pairing strengths determined by experimental NME values

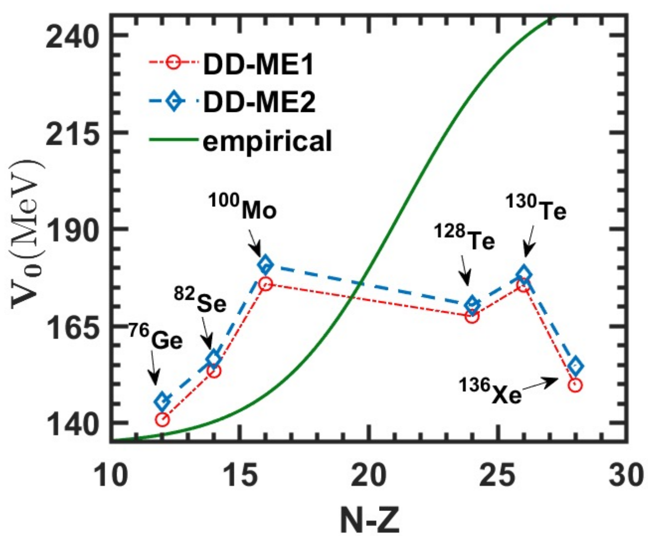
Isotope	$M^{2\nu}_{\mathrm{exp}}(\mathrm{MeV}^{-1})$	$V_0^{\mathrm{exp}}(\mathrm{MeV})$	
		DD-ME1	DD-ME2
^{76}Ge	0.140	-140.81	-145.27
^{82}Se	0.0984	-154.40	-156.50
^{100}Mo	0.246	-175.82	-180.65
^{128}Te	0.0478	-167.48	-170.35
^{130}Te	0.0342	-175.52	-178.20
^{136}Xe	0.0192	-149.69	-154.71

- The empirical isoscalar pairing strength formula proposed by fitting β-decay half-lives

$$V_0 = V_L + \frac{V_D}{1 + e^{a+b(N-Z)}},$$

Z. M. Niu et al. Phys. Lett. B 723, 172 (2013)

This formula is not good for 2ν2β decay



Outline

- **Introduction**
- **Theoretical Framework**
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- **$2\nu2\beta$ NME calculated by relativistic QRPA**
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Summary and Perspectives

With the spherical Skyrme QRPA model:

1. 11 observed $2\nu\beta\beta$ NME are calculated. The isoscalar pairing strengths are suggested.

2. Comparison between QTDA and QRPA is implemented. The ground-state correlation in QRPA largely suppresses the NME.

3. The cancellation mechanism of SSDH is studied. The isoscalar pairing could enlarge the ground state correlation so as to change the sign of GT+ transition amplitude.

With the spherical Relativistic QRPA model:

1. 6 observed $2\nu\beta\beta$ NME are calculated. The isoscalar pairing strengths are suggested, and compared with empirical formulas for β decay.

2. SSDH/LDSH nuclei are summarized.

Summary and Perspectives

Perspective

- Within QRPA approach
  - ✓ 0ν ββ-decay matrix elements
  - ✓ Deformation effect

- Going beyond QRPA : QRPA+QPVC
  - ✓ 2ν ββ-decay matrix elements
  - ✓ 0ν ββ-decay matrix elements

Acknowledgement

Collaborators:

LZU: 吕万里   胡志成

IMP: 房栋梁

Sichuan Uni. : 白春林

Thank you!

Numerical details

- Mean pairing gap:

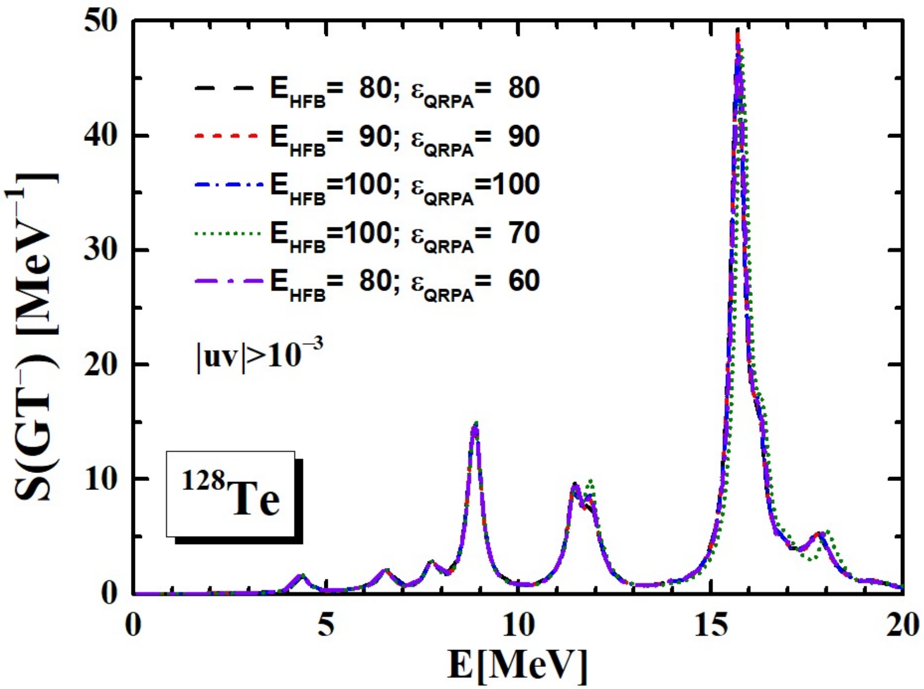
Nucleus	^{48}Ca	^{48}Ti	^{76}Ge	^{76}Se	^{82}Se	^{82}Kr
Δ_n MeV	—	1.56	1.54	1.71	1.54	1.64
Δ_p MeV	—	1.90	1.57	1.75	1.42	1.72

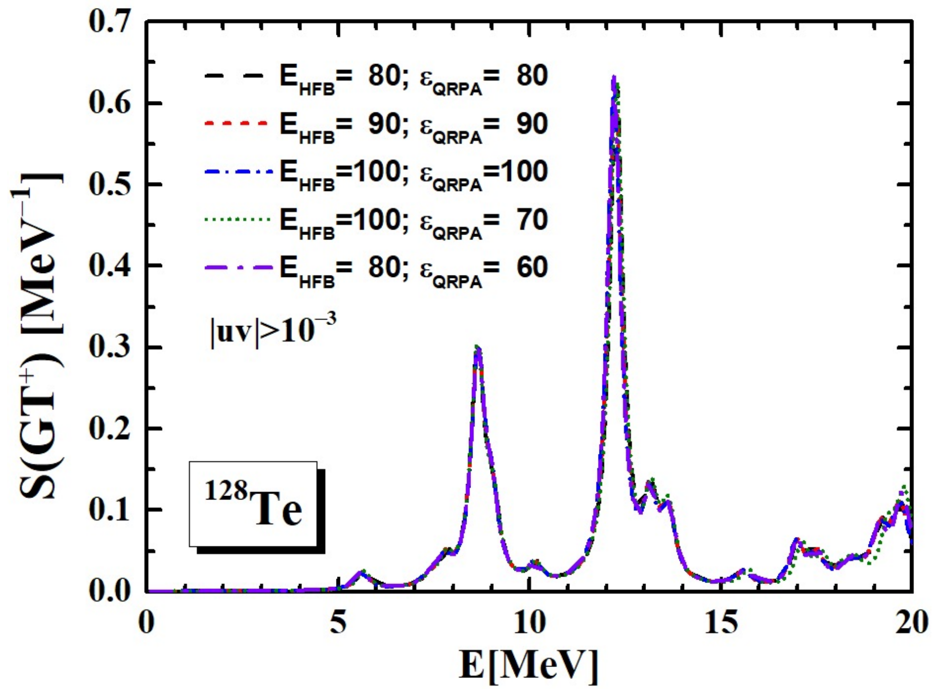
Nucleus	^{96}Zr	^{96}Mo	^{100}Mo	^{100}Ru	^{116}Cd	^{116}Sn
Δ_n MeV	0.85	1.03	1.36	1.30	1.37	1.21
Δ_p MeV	1.54	1.53	1.60	1.55	1.46	—

Nucleus	^{128}Te	^{128}Xe	^{130}Te	^{130}Xe	^{136}Xe	^{136}Ba
Δ_n MeV	1.28	1.26	1.18	1.25	—	1.03
Δ_p MeV	1.13	1.32	1.06	1.31	1.01	1.27

Numerical details

- Convergence check:

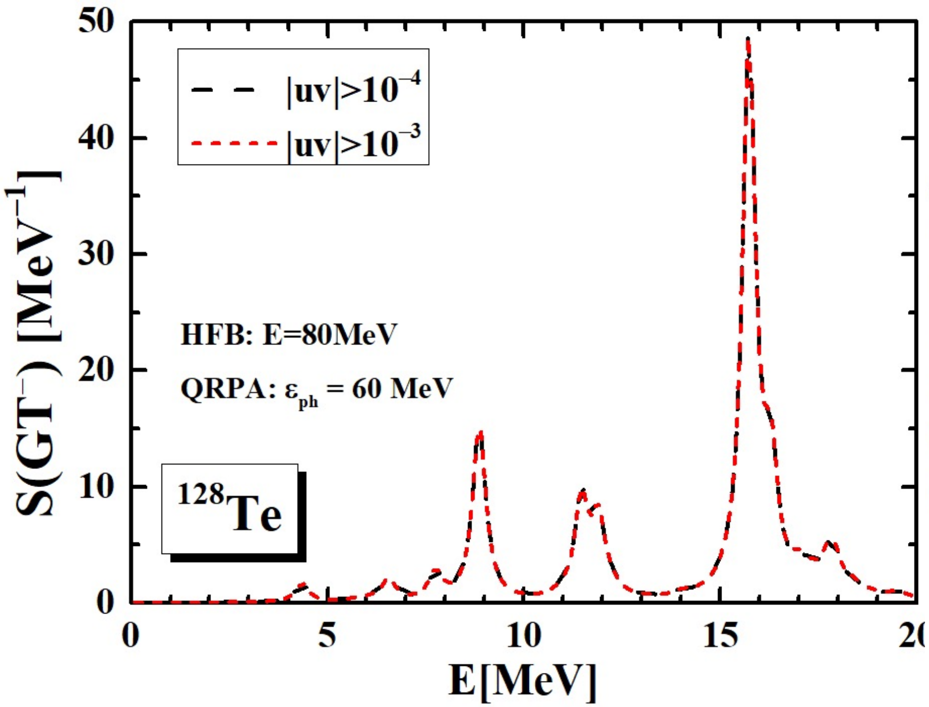


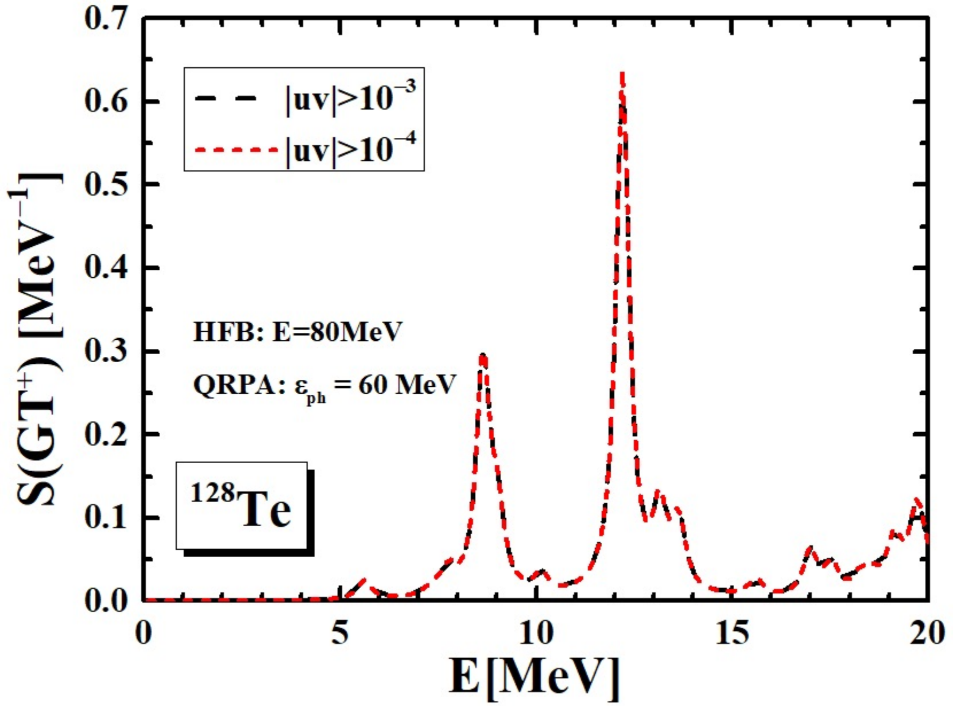


$E_{\mathrm{HFB}} = 80\mathrm{MeV}$; $\varepsilon_{\mathrm{HF}} = 60\mathrm{MeV}$ is stable enough.

Numerical details

- Convergence check:

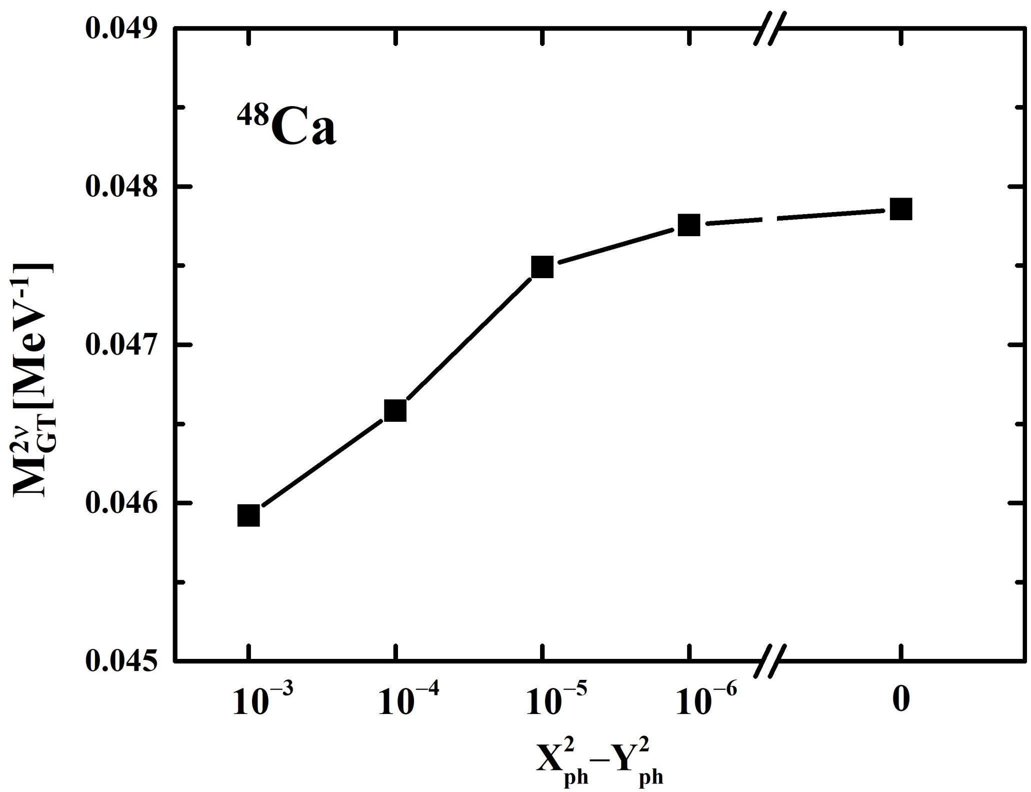




The occupation amplitude cut-off in QRPA $|uv| > 10^{-4}$ is stable enough.

Numerical details

- Convergence check:



For each QRPA state, the ph configuration with $|X_{ph}^2 - Y_{ph}^2| > 10^{-6}$ is considered.

Potential application in 0νββ decay

- **Nuclear Matrix Element (NME) of 0νββ**

$$M_K^{(0\nu)} = \sum_{J^\pi, k_1, k_2, J'} \sum_{pp'nn'} (-1)^{j_n + j_{p'} + J + J'} \sqrt{2J'+1}$$
$$\times \begin{Bmatrix} j_p & j_n & J \\ j_{n'} & j_{p'} & J' \end{Bmatrix} (pp' : J' || \mathcal{O}_K || nn' : J')$$
$$\times (0_f^+ \| [c_{p'}^\dagger \tilde{c}_{n'}]_J \| J_{k_1}^\pi) \langle J_{k_1}^\pi | J_{k_2}^\pi \rangle (J_{k_2}^\pi \| [c_p^\dagger \tilde{c}_n]_J \| 0_i^+)$$

where

$$\mathcal{O}_{\mathrm{GT}} = h_{\mathrm{GT}}(r, E_k) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$$

$$h_K(r_{mn}, E_a) = \frac{2}{\pi} R_A \int dq \frac{q h_K(q^2)}{q + E_a - (E_i + E_f)/2} j_0(q r_{mn})$$

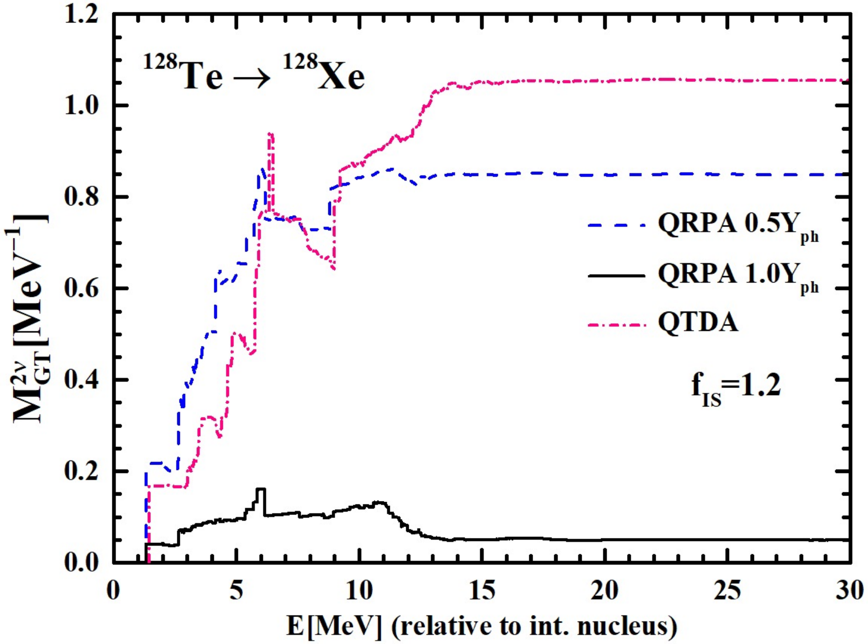
SSDH mechanism: ground-state correlation

- **How do ground-state correlation make negative contribution?**

$$\langle nJ||\hat{O}^{+}||0\rangle = \sum_{pn}(-)^{j_p+j_n+J}\langle j_n||\hat{O}^{+}||j_p\rangle[X^{nJ}_{pn}v_p u_n + \eta_Y Y^{nJ}_{pn} v_n u_p]$$

✓ At large f_{IS}:

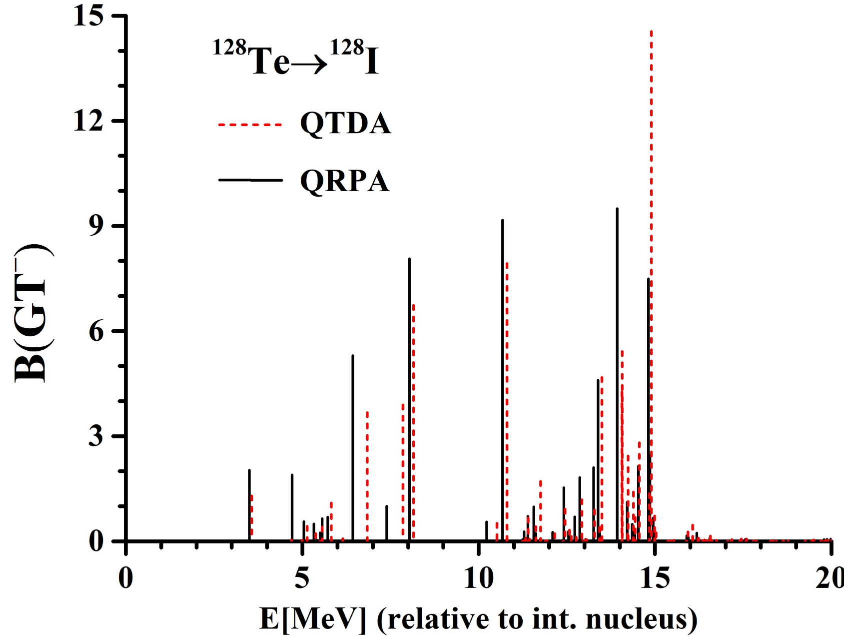
$|Y|$ is close to X ➡ negative contribution of higher-lying states

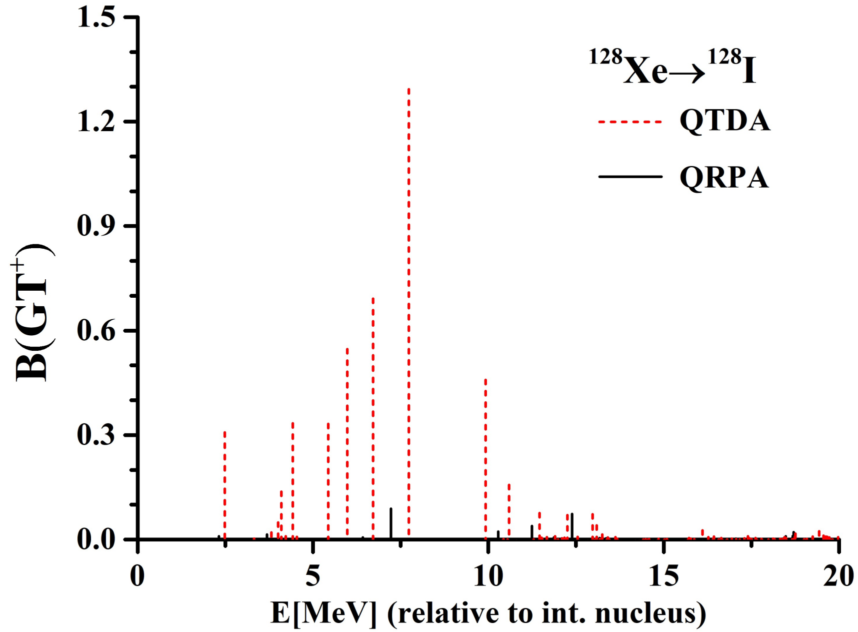


SSDH mechanism: ground-state correlation

- **How do ground-state correlation influence the NME?**

$$M_{\mathrm{GT}}^{2\nu} = \sum_{n_i n_f} \frac{\langle 0_{\mathrm{g.s.}}^{+(f)} || \hat{O}_{\mathrm{GT}}^{-} || 1_{n_f}^{+} \rangle \langle 1_{n_f}^{+} | 1_{n_i}^{+} \rangle \langle 1_{n_i}^{+} || \hat{O}_{\mathrm{GT}}^{-} || \langle 0_{\mathrm{g.s.}}^{+(i)} \rangle}{E_n^{*(N)} + M_N - (M_f + M_i)/2} \qquad \langle 1_{n_f}^{+} || \hat{O}_{\mathrm{GT}}^{+} || 0_{\mathrm{g.s.}}^{+(f)} \rangle$$





$B(\mathrm{GT}^{-})$ of QRPA and QTDA are similar.

$B(\mathrm{GT}^{+})$ of QRPA are quite small.

Ground-state correlation Y_{ph} mainly influences the GT^{+} transition.

SSDH mechanism: ground-state correlation

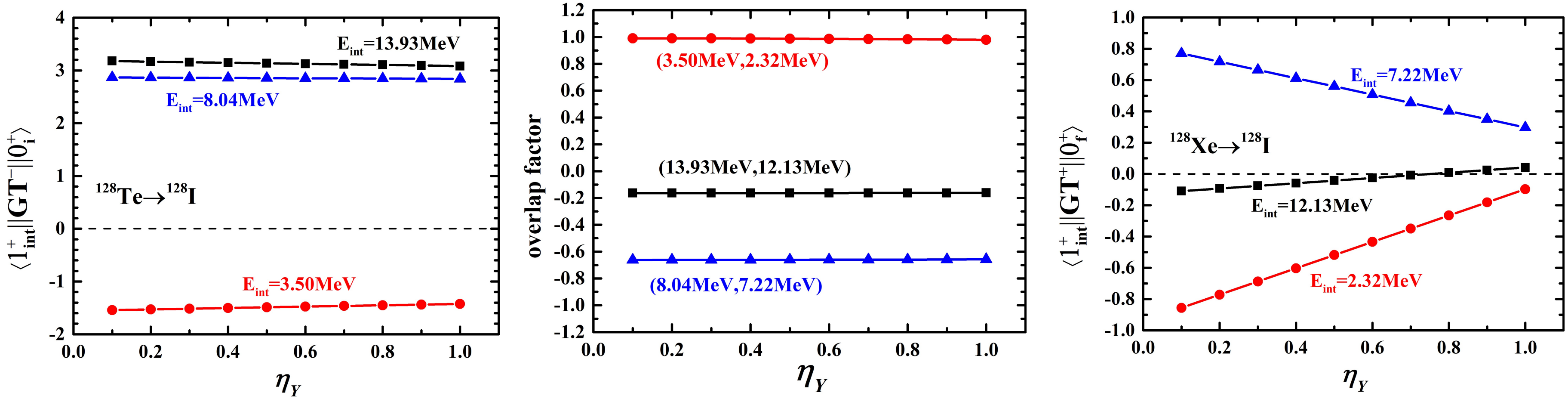
- **How do ground-state correlation make negative contribution?**

$$\langle nJ||\hat{O}^{+}||0\rangle = \sum_{pn}(-)^{j_p+j_n+J}\langle j_n||\hat{O}^{+}||j_p\rangle[X_{pn}^{nJ}v_p u_n + \eta_Y Y_{pn}^{nJ} v_n u_p]$$

✓ At large f_{IS}:

$|Y|$ is increased ⟶ negative contribution of higher-lying states

The sign of GT+ amplitude will change for higher-lying states



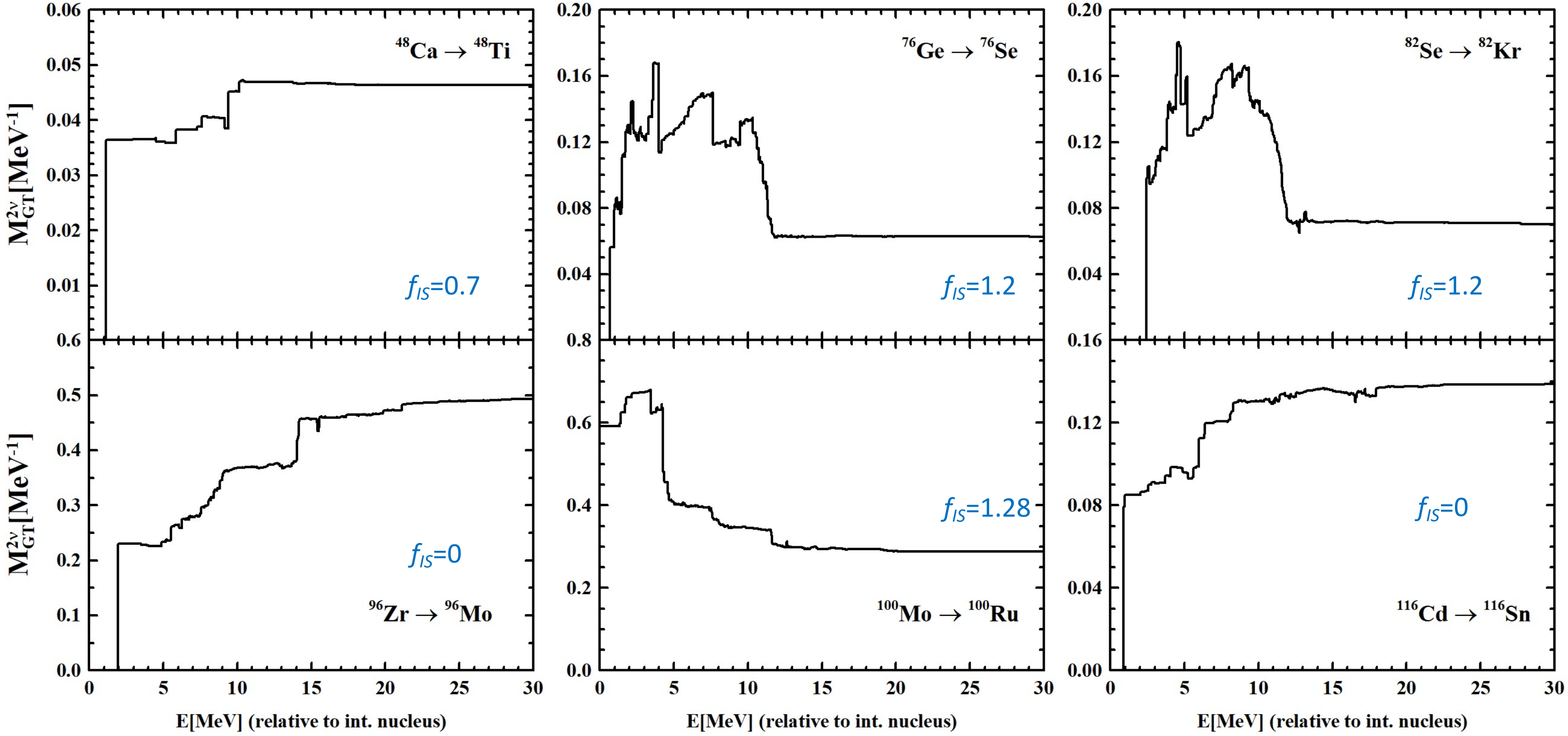
✓ $\langle 1^+_{\mathrm{int}}||\mathrm{GT}^-||0^+_{\mathrm{i}}\rangle$ is almost independent from Y_{ph}

✓ $\langle 1^+_{\mathrm{int}}||\mathrm{GT}^+||0^+_{\mathrm{f}}\rangle$ is sensitive to Y_{ph}, and its sign changes for higher-lying states.

Running sum of NME

- **Running sum of NME as a function of excitation energy of intermediate states**

$$E_n^{*(N)} = \frac{1}{2}\Big(\big[(\Omega_n^i - \lambda_{\text{neu.}}^i + \lambda_{\text{pro.}}^i) + (\Omega_n^f + \lambda_{\text{neu.}}^f - \lambda_{\text{pro.}}^f) - (2M_N - M_i - M_f)\big] \Big)$$



f_{IS} is taken at the value that reproduces exp. data (or close)

Running sum of NME

- **Running sum of NME as a function of excitation energy of intermediate states**

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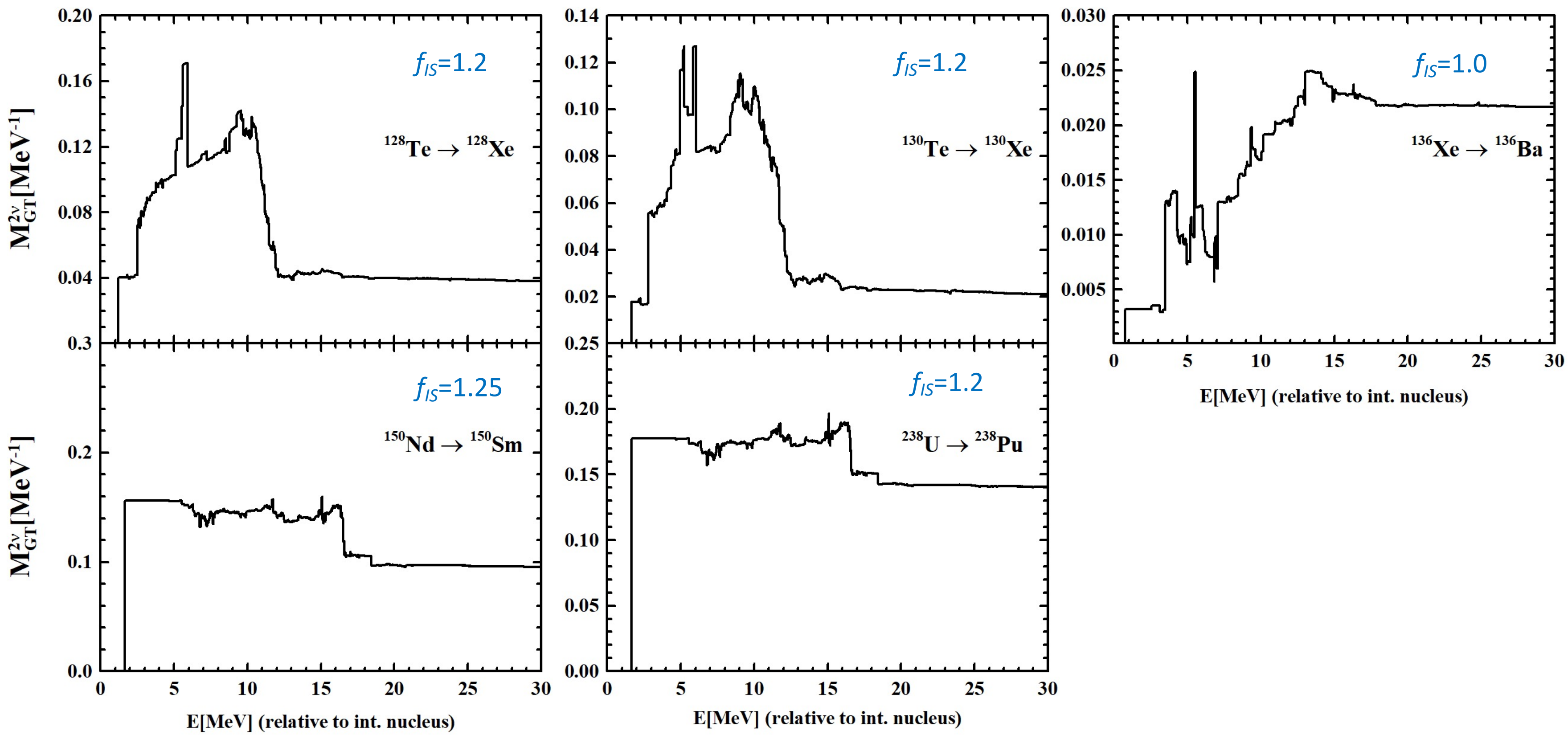


f_{IS} is taken at the value that reproduces exp. data (or close)

Isoscalar pairing strength and SSDH/LLDH

Nucleus	^{48}Ca	^{76}Ge	^{82}Se	^{96}Zr	^{100}Mo
Expt. $M_{\rm GT}^{2\nu}$ (MeV^{-1})	0.046±0.004	0.136±0.007	0.100±0.005	0.097±0.005	0.223±0.006
Theo. $M_{\rm GT}^{2\nu}$ (MeV^{-1})	0.046	0.062	0.070	0.139	0.288
Theo. $M_{\rm GT}^{2\nu}$(SSD) (MeV^{-1})	0.035	0.054	0.015	0.230	0.570 (0.21)
Theo. $M_{\rm GT}^{2\nu}$(LLD) (MeV^{-1})	0.036	0.125	0.098	0.234	0.407
$f_{\rm IS}$	0.70	1.20	1.20	0.00	1.28

Nucleus	^{116}Cd	^{128}Te	^{130}Te	^{136}Xe	^{150}Nd	^{238}U
Expt. $M_{\rm GT}^{2\nu}$ (MeV^{-1})	0.127±0.004	0.056±0.007	0.037±0.005	0.022±0.001	0.070±0.005	$0.157^{+0.109}_{-0.085}$
Theo. $M_{\rm GT}^{2\nu}$ (MeV^{-1})	0.139	0.037	0.021	0.022	0.071	0.140
Theo. SSD	(0.14) <0.001	(0.024) 0.040	0.018	<0.001	0.156	0.177
Theo. LLD	0.096	0.102	0.116	0.007	0.156	0.177
$f_{\rm IS}$	0.00	1.20	1.20	1.00	1.25	1.20

Exp. data from A. S. Barabash, Nucl. Phys. A 935, 52 (2015)

*$g_A = 1.273$ is used to calculate the expt. $M_{\rm GT}^{2\nu}$

The upper limit of energy of LLD is 5MeV, except for ^{82}Se (2 states only).

✓ SSD nuclei: ^{48}Ca, ^{76}Ge, ^{128}Te, ^{130}Te, ^{238}U

✓ LLD nuclei: ^{82}Se, ^{100}Mo, ^{116}Cd

Exp. data of SSD from O. Moreno, et al., J. Phys. G 36, 015106 (2009)

SSDH mechanism: ground-state correlation

- **Why does ground-state correlation play its role through GT+?**

$$\langle J||\hat{O}^-||0\rangle = \sum_{pn} -\langle j_p||\hat{O}^-||j_n\rangle[X^{nJ}_{pn} v_n u_p + Y^{nJ}_{pn} v_p u_n]$$

$$\langle J||\hat{O}^+||0\rangle = \sum_{pn} (-)^{j_p+j_n+J}\langle j_n||\hat{O}^+||j_p\rangle[X^{nJ}_{pn} v_p u_n + Y^{nJ}_{pn} v_n u_p]$$

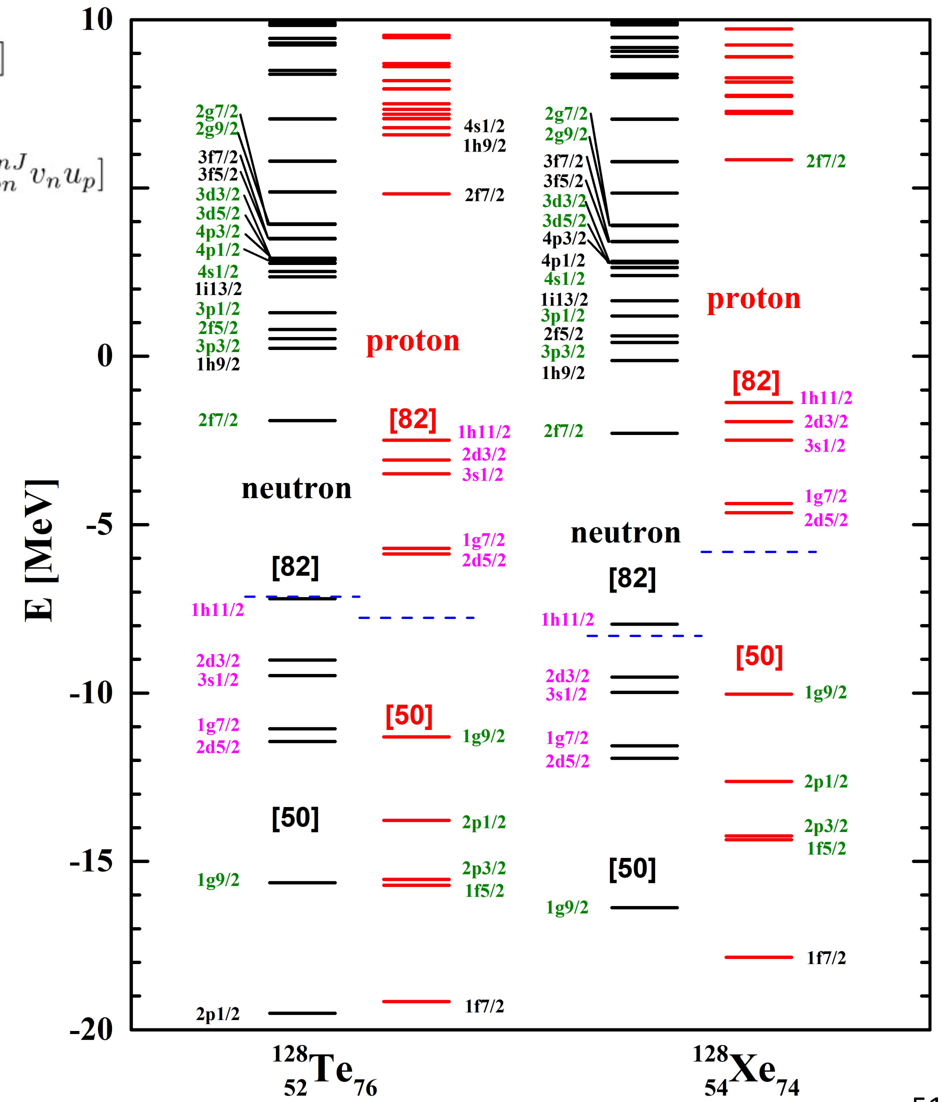
^{128}Te: neutron-rich nucleus

✓ GT+ transitions: completely blocked at mean-field level

✓ pairing correlation: unblock the transition through X term

✓ Ground-state correlation: block/unblock the transition through Y term

➡ GT+ is sensitive to ground-state correlation



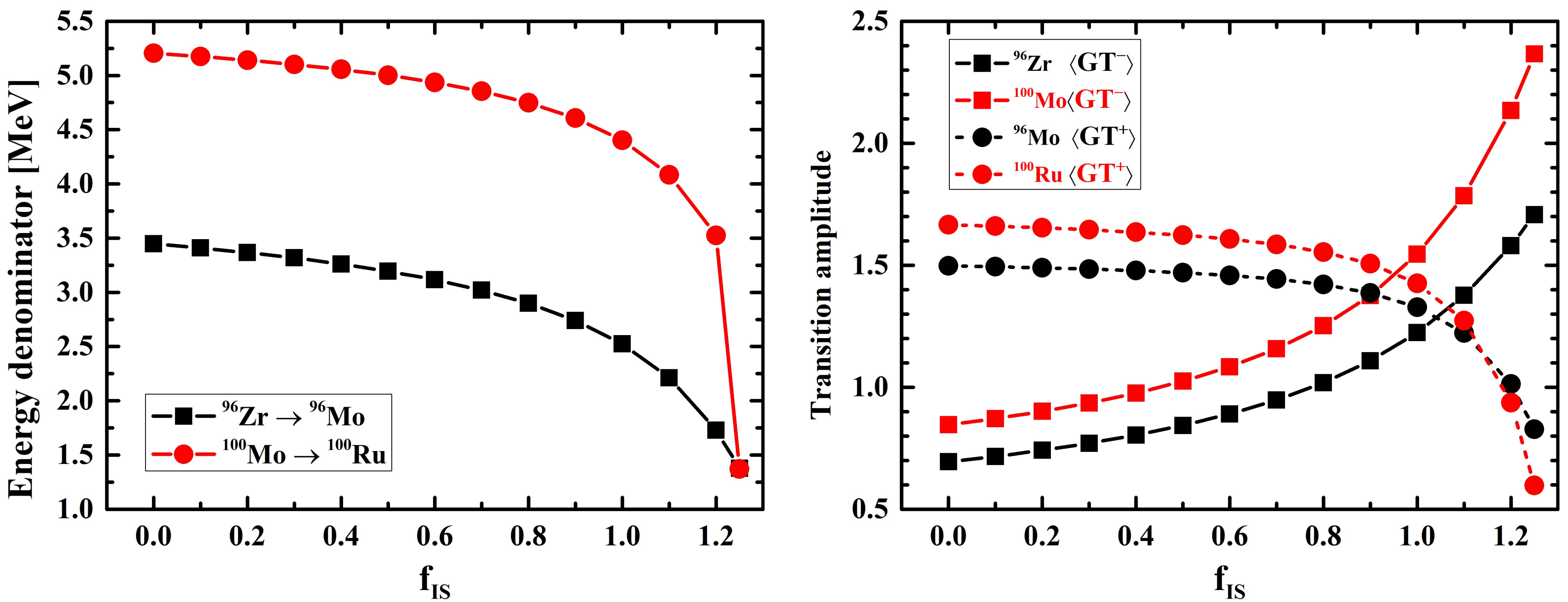
SSDH/LSDH

Nucleus	^{76}Ge	^{82}Se	^{100}Mo	^{128}Te	^{130}Te	^{136}Xe
Expt.$M^{2\nu}$(MeV^{-1})	0.140	0.0984	0.246	0.0478	0.0342	0.0192
Theo.$M^{2\nu}$(MeV^{-1})	0.141	0.0993	0.210	0.0498	0.0343	0.0191
Theo.$M^{2\nu}$(SSD)(MeV^{-1})	0.0388	0.254	0.243	0.0241	0.00271	$<$0.001
Theo.$M^{2\nu}$(LLD)(MeV^{-1})	0.124	0.102	0.257	0.0322	0.0715	0.0139
V_0(MeV)	-145.27	-156.50	-180.65	-170.00	-180.20	-154.71

The upper limit of energy of LLD is 5MeV, except for ^{130}Te (E<1.8 MeV)

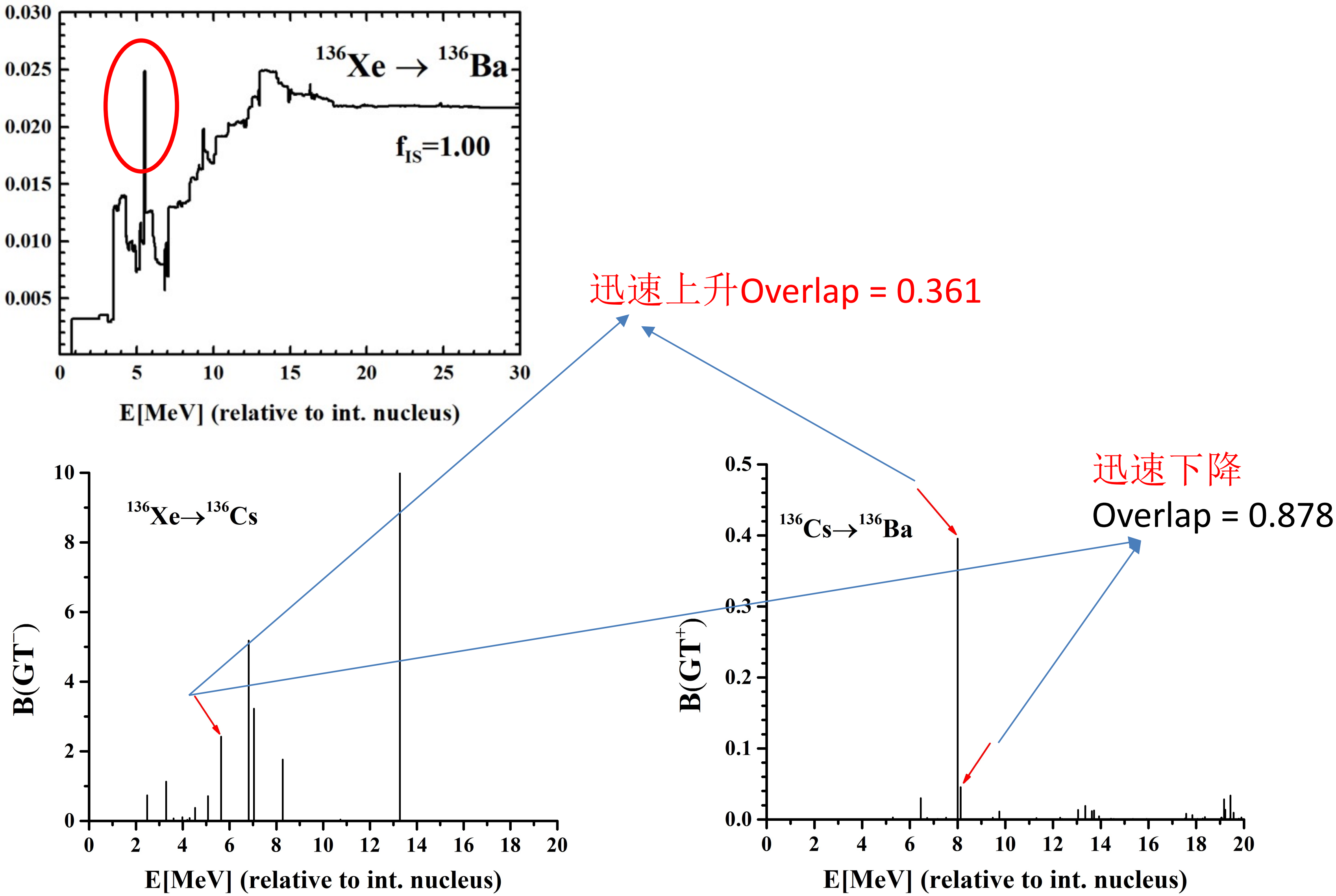
- ✓ SSD nuclei: ^{100}Mo
- ✓ LLD nuclei: ^{76}Ge, ^{82}Se, ^{128}Te, ^{130}Te ,^{136}Xe

NME vs. f_{IS} : ^{96}Zr and ^{100}Mo



$2\nu\beta\beta$ through the 1st intermediate state. The similar case appears in ^{116}Cd.

^{100}Mo当f_{IS}超过1.25后下降主要是因为更高能级的抵消。
^{116}Cd 的图没有画，因为中间核前几个态对矩阵元贡献近似为0


0.030
0.025
0.020
0.015
0.010
0.005
0
5
10
15
20
25
30
136Xe → 136Ba
fIS=1.00
E[MeV] (relative to int. nucleus)
迅速上升Overlap = 0.361
迅速下降
Overlap = 0.878
136Xe→136Cs
B(GT−)
E[MeV] (relative to int. nucleus)
136Cs→136Ba
B(GT+)
E[MeV] (relative to int. nucleus)