



北京大学

“无中微子双贝塔衰变”研讨会

2021年5月19日 - 23日 珠海

## Toward a unified description of nuclei

Pengwei Zhao (赵鹏巍)

Peking University

# Nuclide Chart

## stable nuclei

mass measured

## observed

## **prediction**

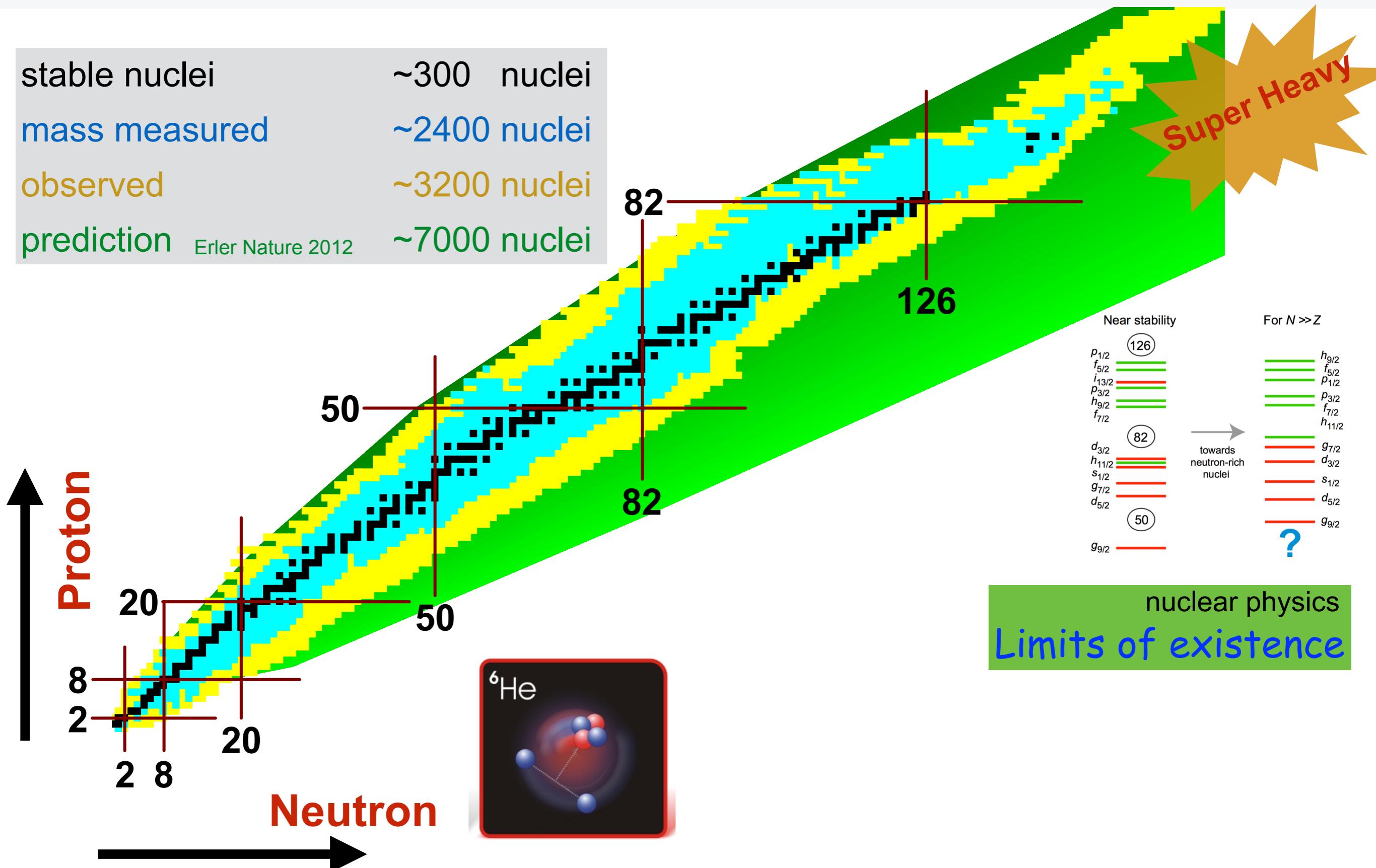
Erler Nature 2012

~300 nuclei

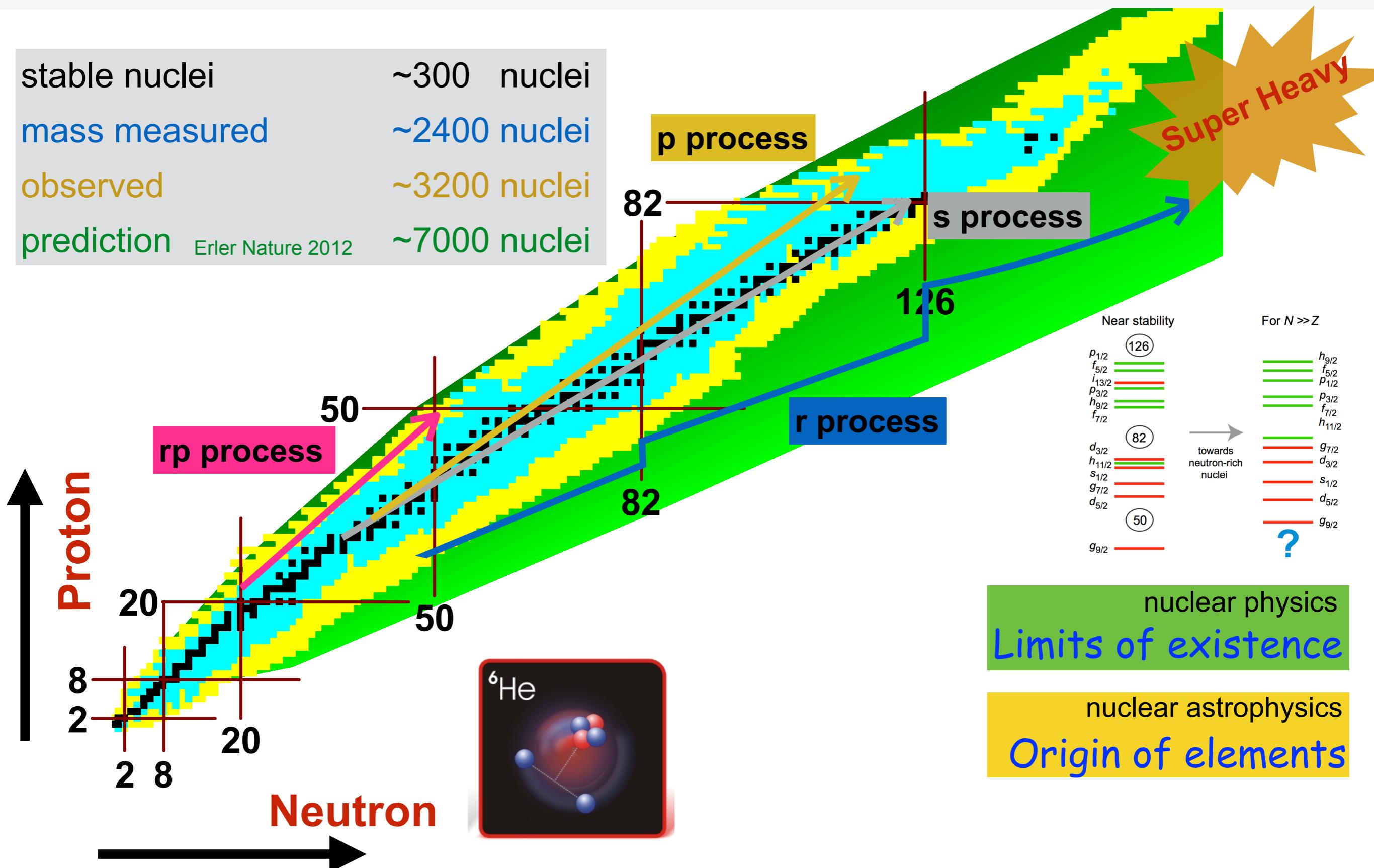
~2400 nuclei

~3200 nuclei

~7000 nuclei



# Nuclide Chart



# Nuclide Chart

## stable nuclei

## mass measured

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## **prediction**

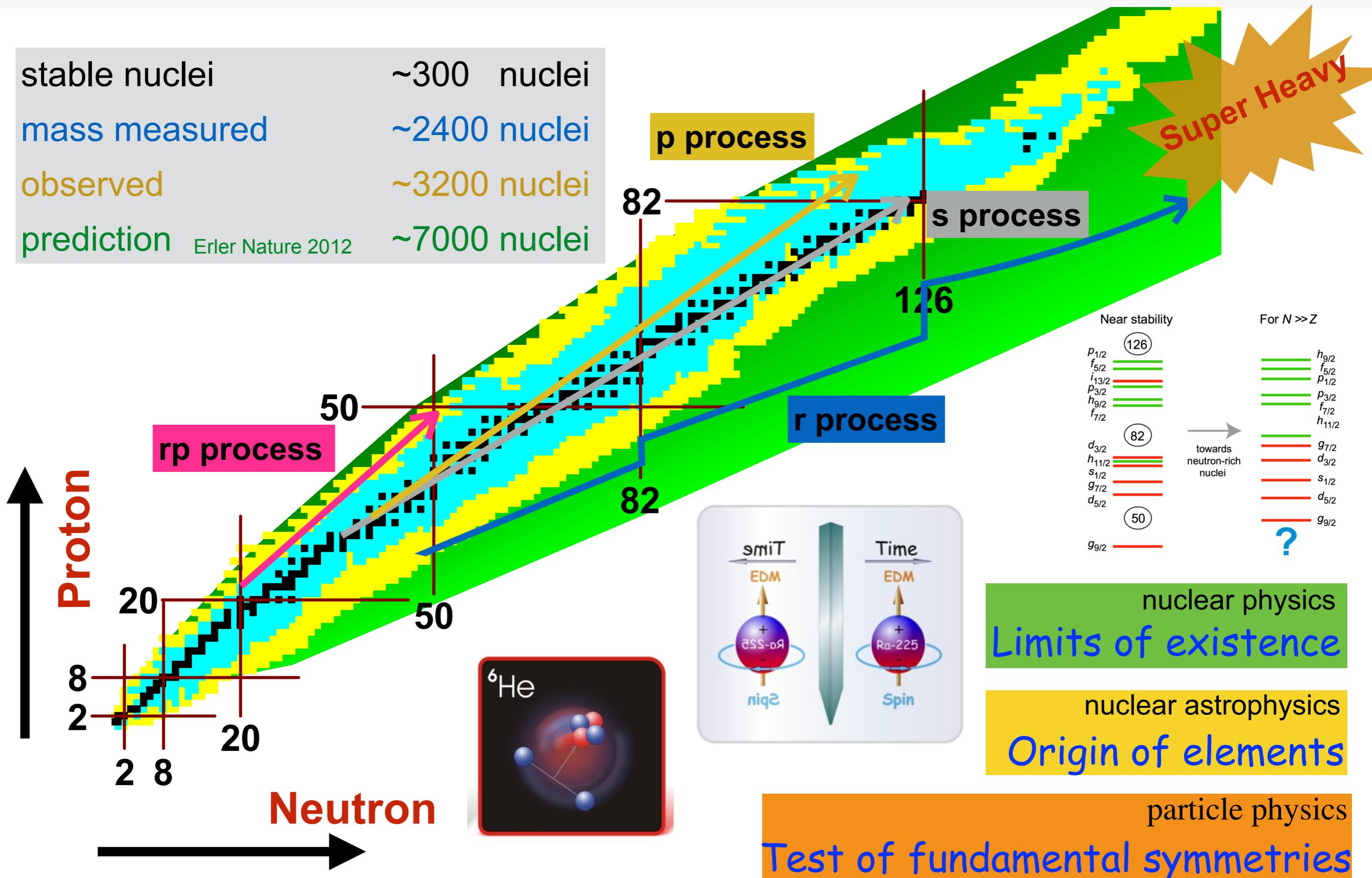
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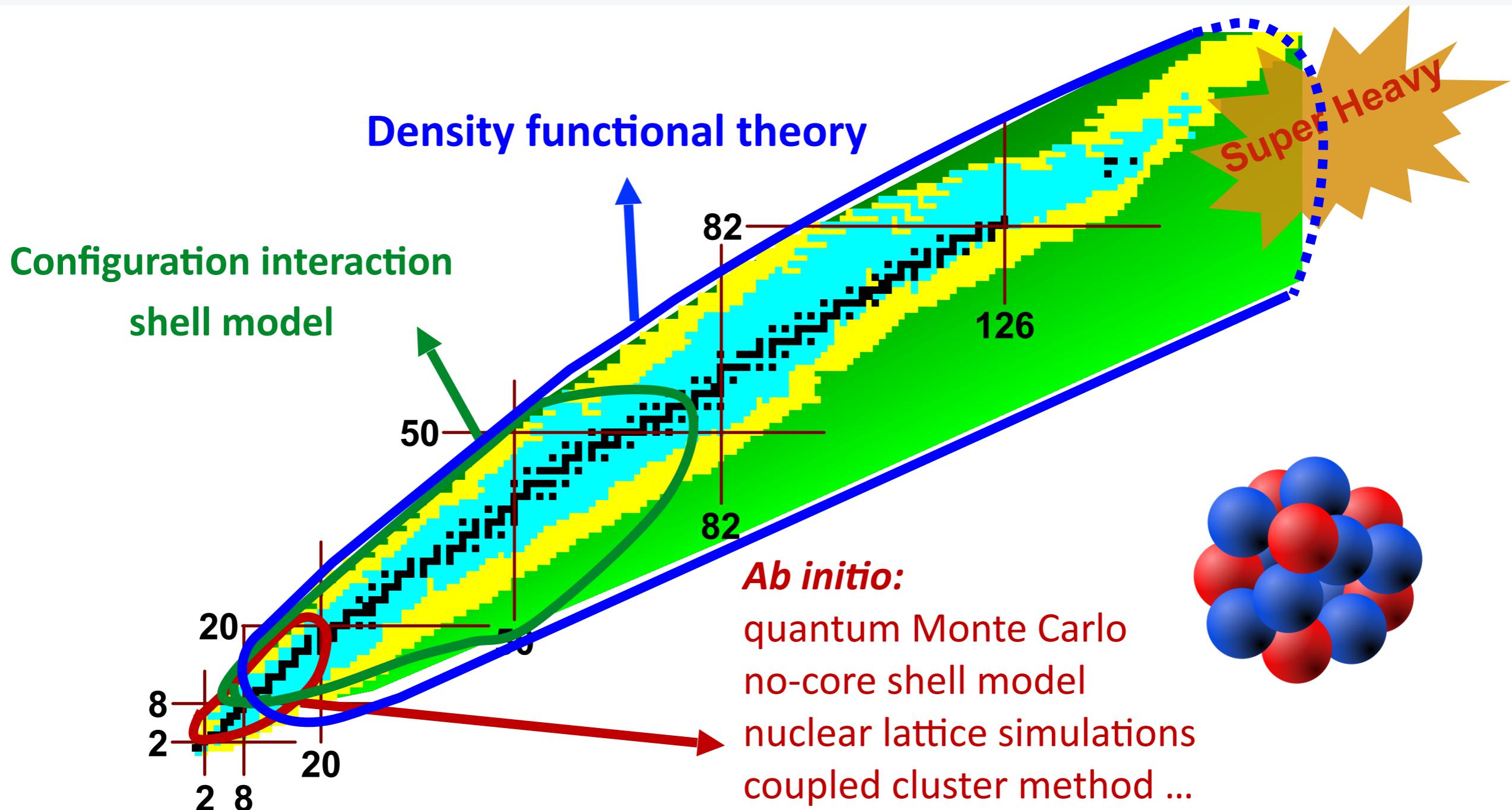
~7000 nuclei



# Toward a unified description of nuclei

- a global approach which is applicable for the whole nuclear chart (light & heavy; neutron-deficient & neutron-rich)
- a consistent treatment for the ground-state and spectroscopic properties (low spin & high spin)
- a unified description of nuclear structure and nuclear reaction
- a microscopic formalism based directly on realistic nuclear interactions
- ...

# State-of-the-art theories for nuclei



It would be interesting to investigate the intersections between different theories for a unified and comprehensive description of nuclei.

# Density functional theory

The many-body problem is mapped onto an one-body problem

## Hohenberg-Kohn Theorem

The **exact ground-state energy** of a quantum mechanical many-body system is a **universal functional** of the **local density**.

## Kohn-Sham DFT

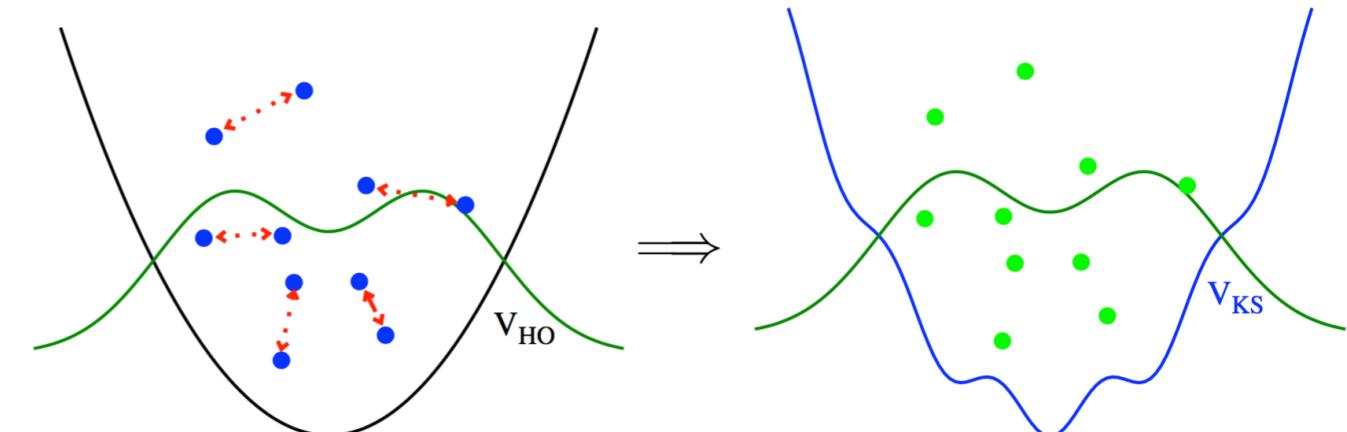


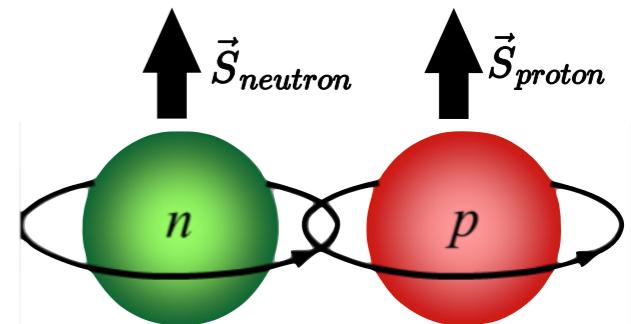
Figure from Drut PPNP 2010

$$E[\rho] \Rightarrow \hat{h} = \frac{\delta E}{\delta \rho} \Rightarrow \hat{h}\varphi_i = \varepsilon_i\varphi_i \Rightarrow \rho = \sum_{i=1}^A |\varphi_i|^2$$

The practical usefulness of the Kohn-Sham theory depends entirely on whether an **Accurate Energy Density Functional** can be found!

# Density functional theory for nuclei

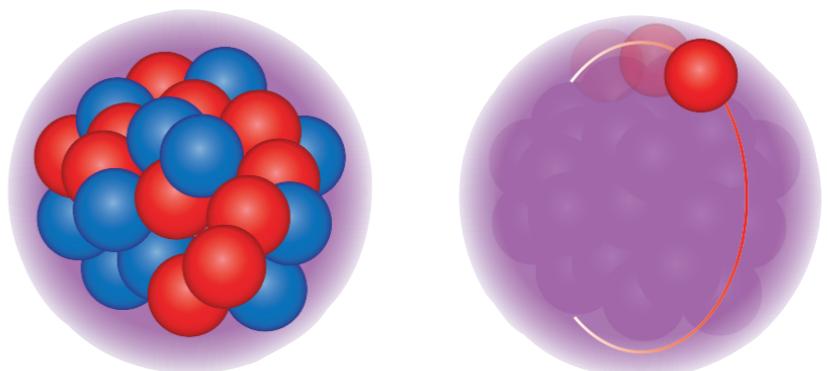
✓ The nuclear force is **complicated**



✓ More degrees of freedom: **spin, isospin, pairing, ...**

✓ Nuclei are **self-bound systems**

DFT for the **intrinsic density**



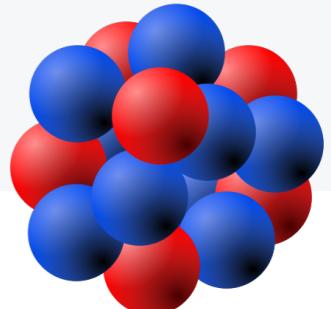
✓ At present, all successful functionals are **phenomenological**

not connected to any NN- or NNN-interaction

✓ Adjust to properties of **nuclear matter and/or finite nuclei**, and

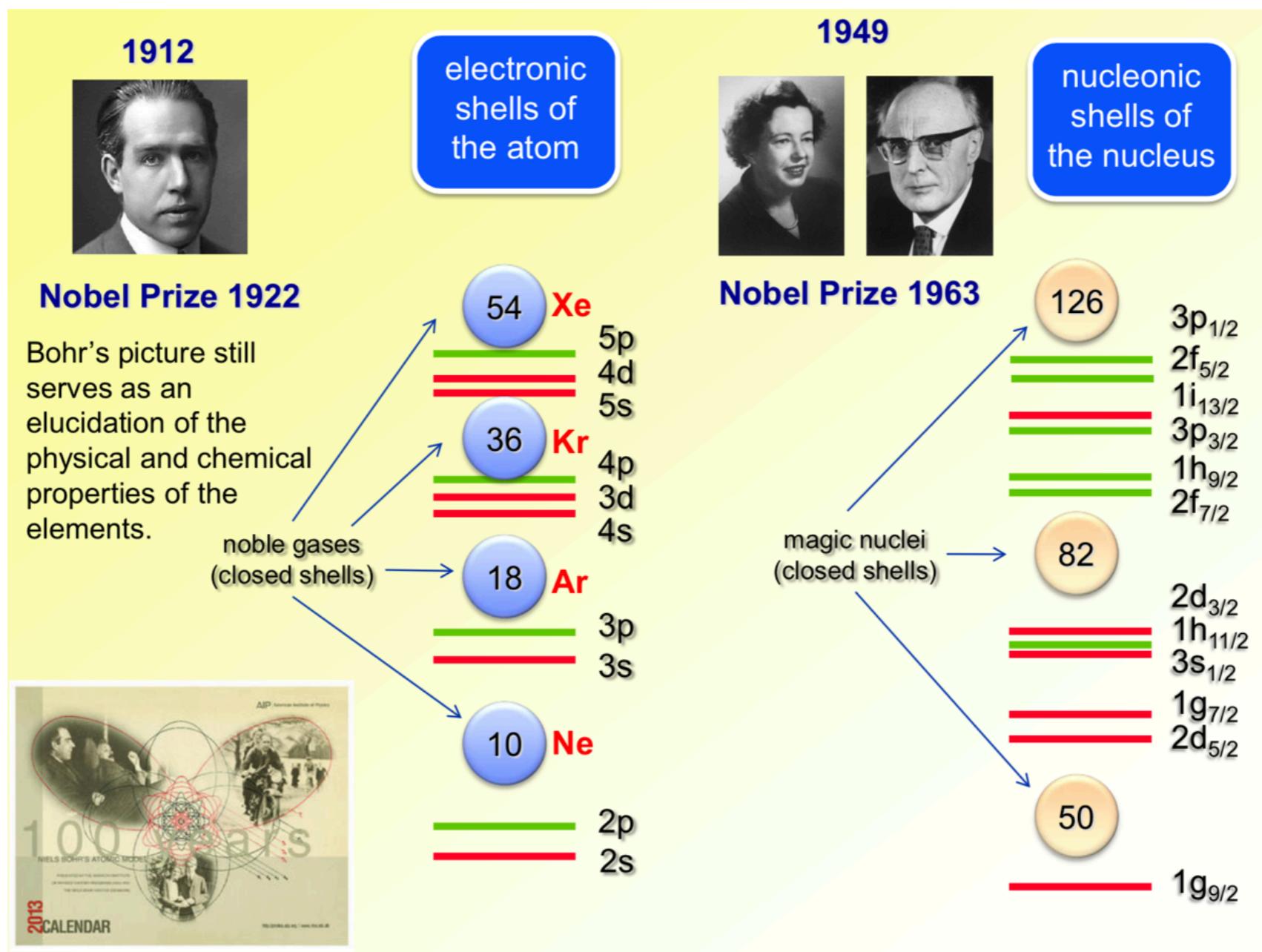
(in future) to **ab-initio results**

# Relativistic vs Nonrelativistic



- ✓ No relativistic kinematics necessary
- ✓ Large spin-orbit splitting

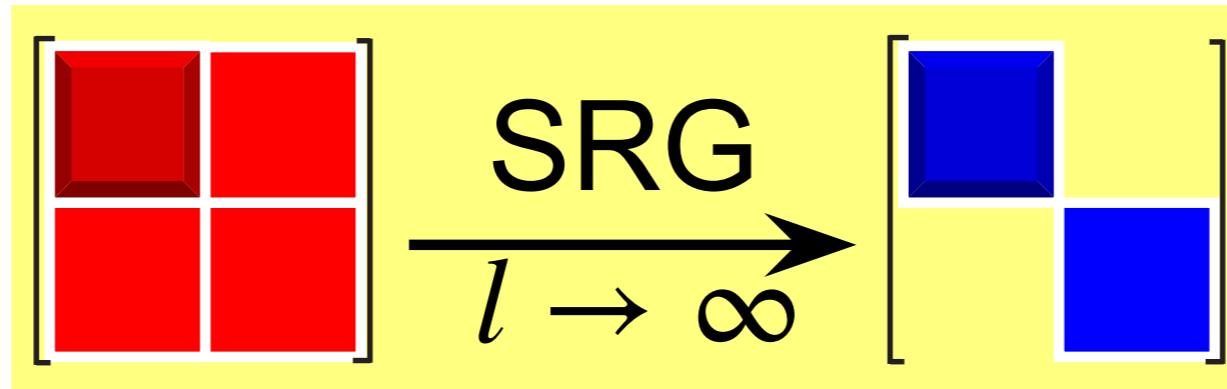
$$\sqrt{p_F^2 + m_N^2} = m_N \sqrt{1 + 0.075}$$



# Similarity Renormalization Group

## Relativistic Kohn-Sham Equation: Dirac Equation

$$\begin{pmatrix} m + V + S & \sigma \cdot p \\ \sigma \cdot p & -m + V - S \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix} = \varepsilon \begin{pmatrix} f \\ g \end{pmatrix}$$



$$\frac{dH(l)}{dl} = [\eta(l), H(l)], \quad \eta(l) = [\beta, H(l)]$$

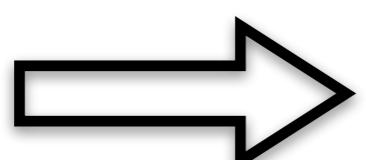
Bylev, Pirner, Phys. Lett. B 428, 329 (1998)

# Flow equation

$$\frac{dH(l)}{dl} = [\eta(l), H(l)], \quad \eta(l) = [\beta, H(l)]$$

Expansion with nucleon mass

$$\frac{1}{M}H(l) = \sum_{k=0}^{\infty} \frac{1}{M^k} H_k(l)$$

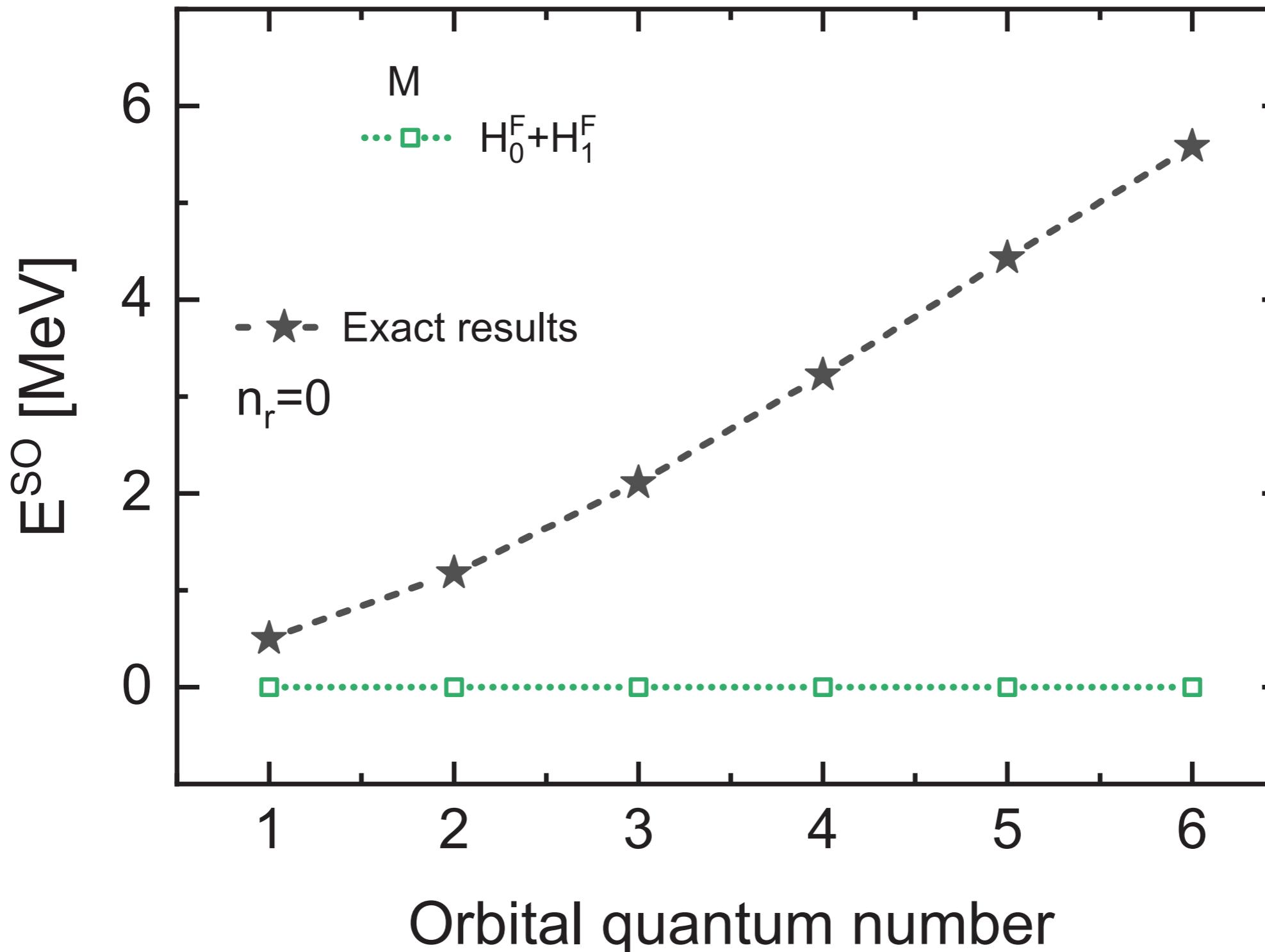


$$H(\infty) = \begin{pmatrix} H^F + M & 0 \\ 0 & H^D - M \end{pmatrix}$$

Bylev, Pirner, Phys. Lett. B 428, 329 (1998)  
Guo, Phys. Rev. C 85, 021302(R) (2012)

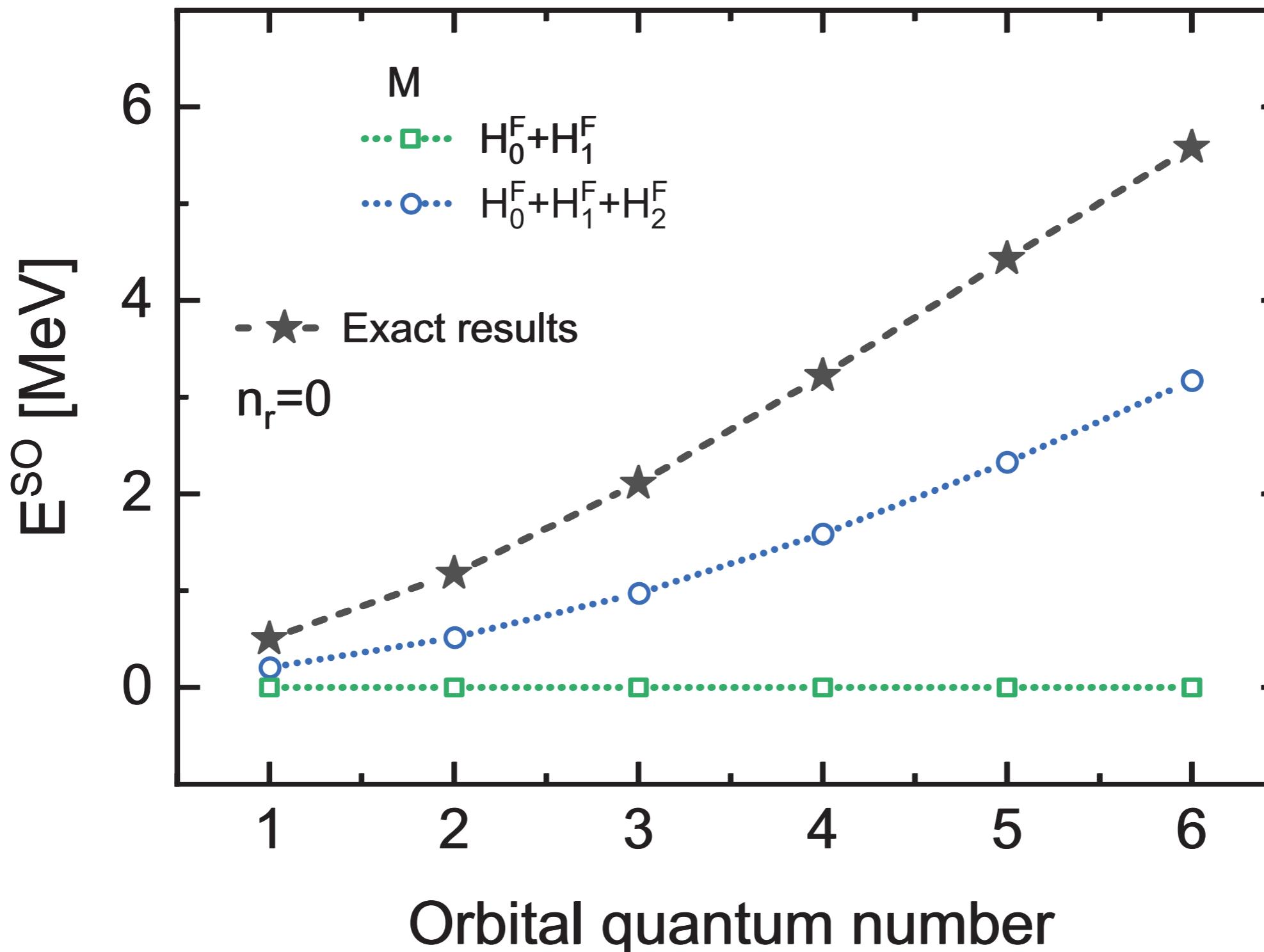
# Spin-orbit Splitting Energy

Too slow convergence!



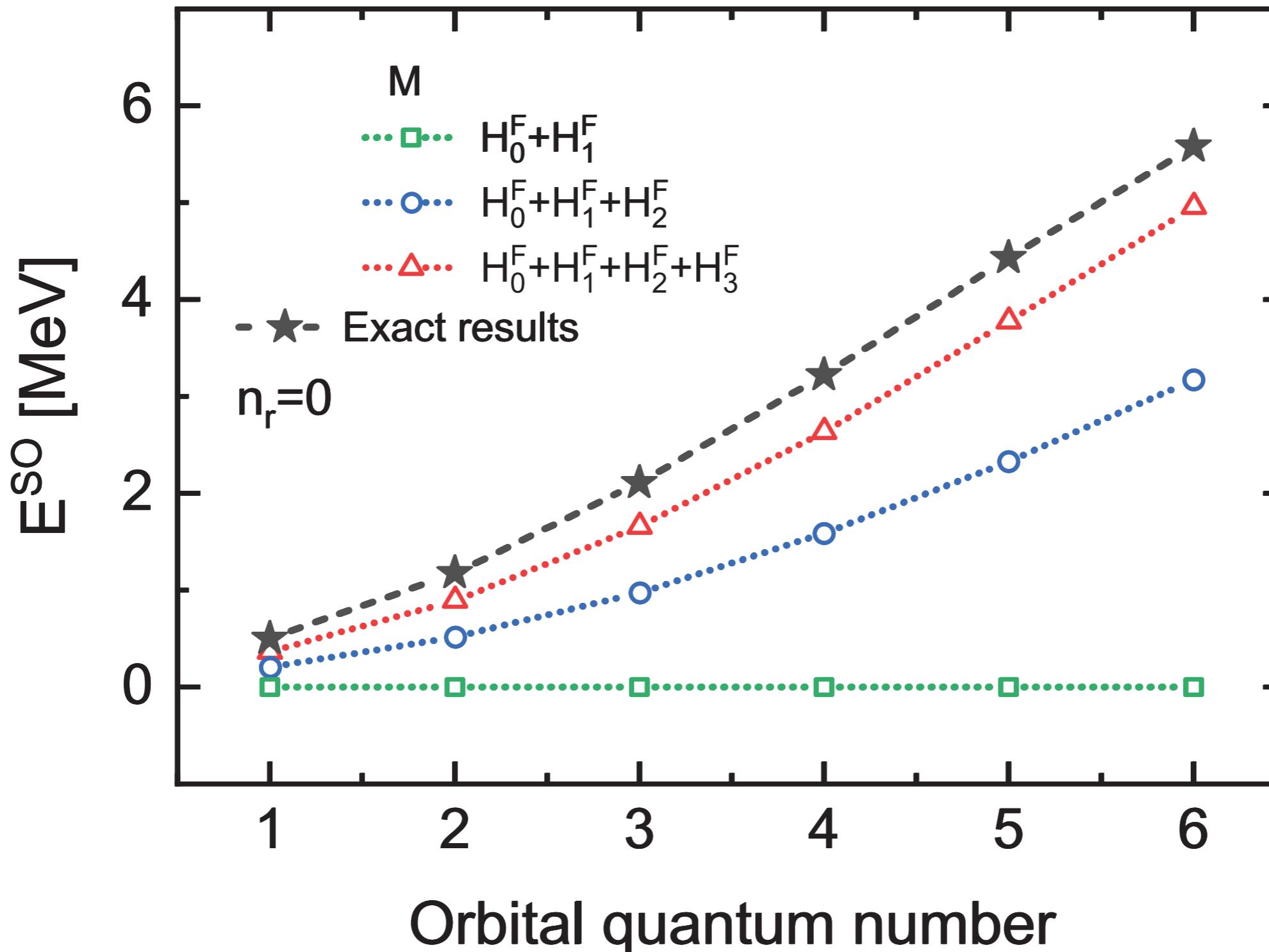
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# Medium effects

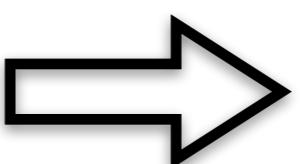
$$\begin{pmatrix} m + V + S & \sigma \cdot p \\ \sigma \cdot p & -m + V - S \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix} = \varepsilon \begin{pmatrix} f \\ g \end{pmatrix}$$

Expansion with effective  
nucleon mass

$$\tilde{M}(r) = M + S(r)$$

$$\frac{1}{\tilde{M}} H(l) = \sum_{k=0}^{\infty} \frac{1}{\tilde{M}^k} H_k(l)$$

$$\frac{dH(l)}{dl} = [\eta(l), H(l)], \quad \eta(l) = [\beta, H(l)]$$

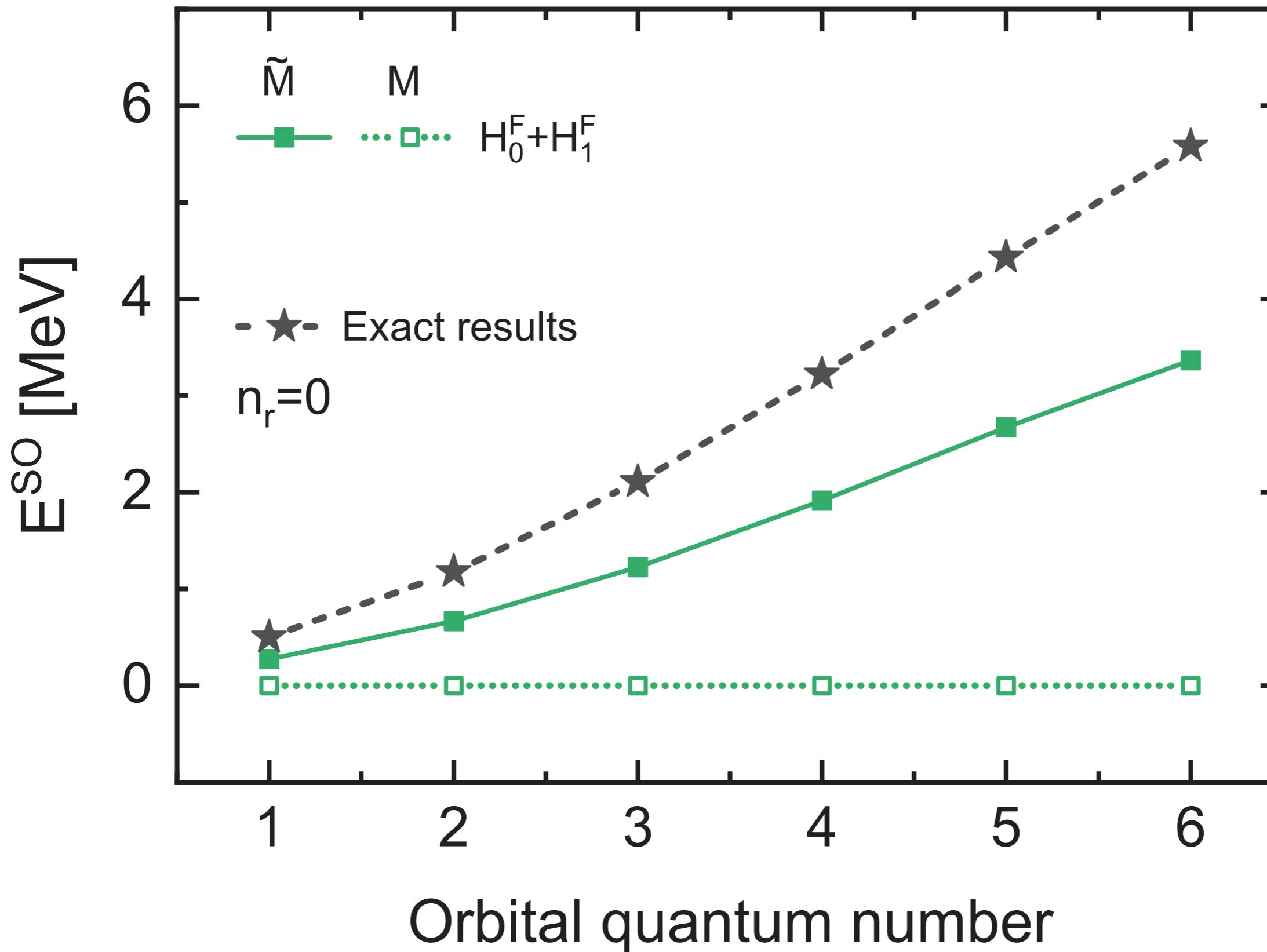


$$H(\infty) = \begin{pmatrix} H^F + M & 0 \\ 0 & H^D - M \end{pmatrix}$$

Ren, PWZ, Phys. Rev. C, 100, 044322 (2019)

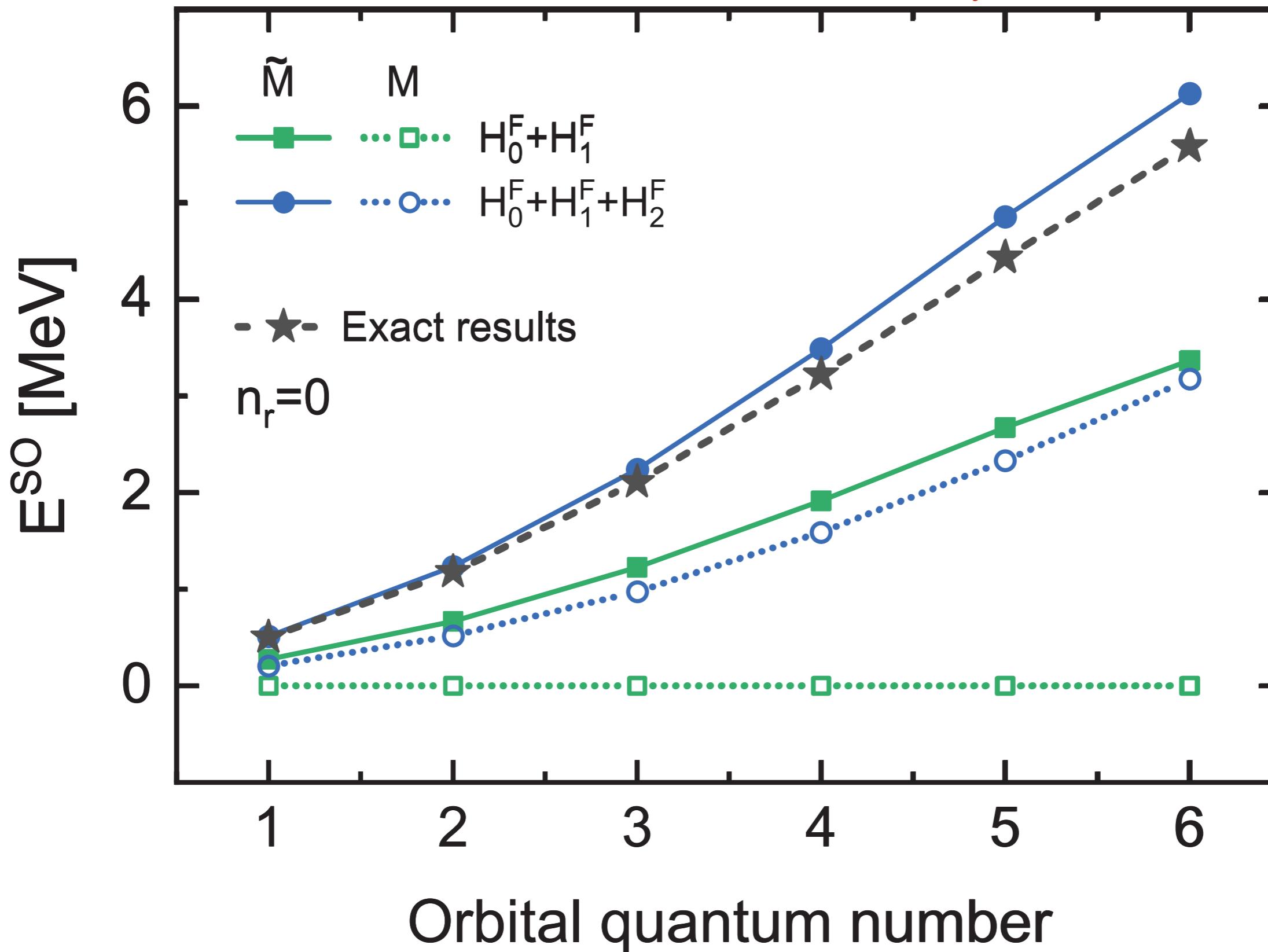
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Ren, PWZ, Phys. Rev. C, 100, 044322 (2019)



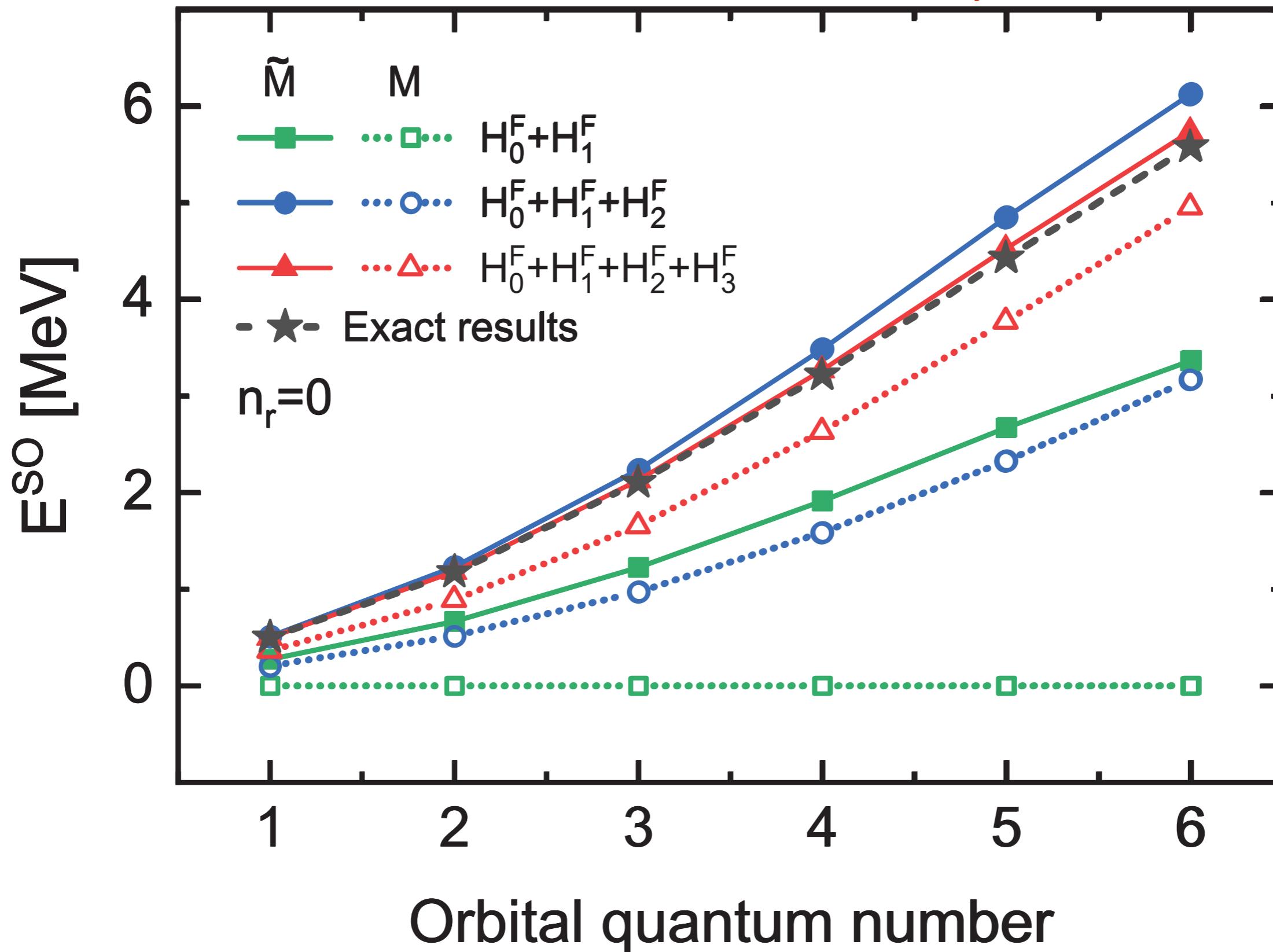
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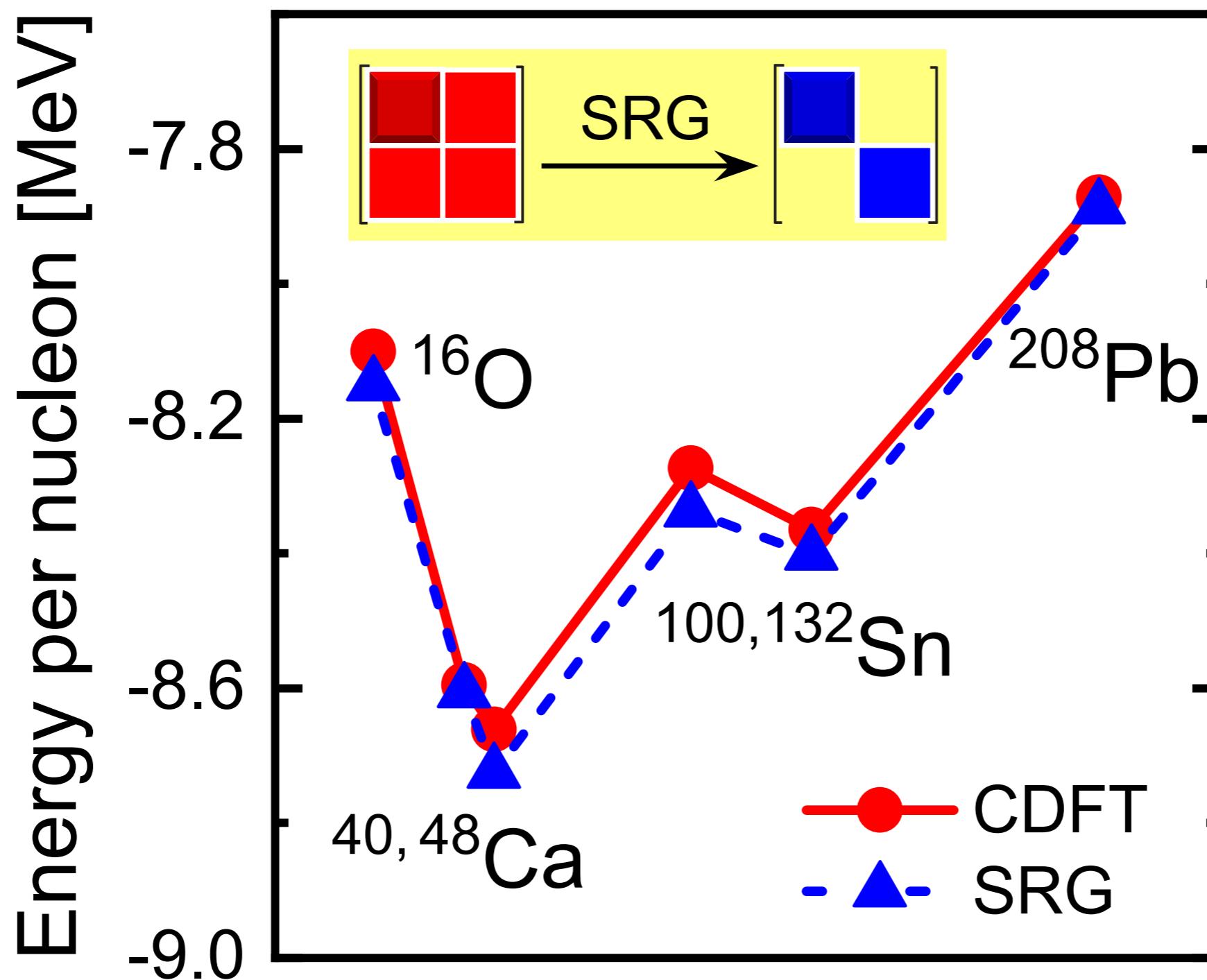


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Ren, PWZ, Phys. Rev. C, 100, 044322 (2019)

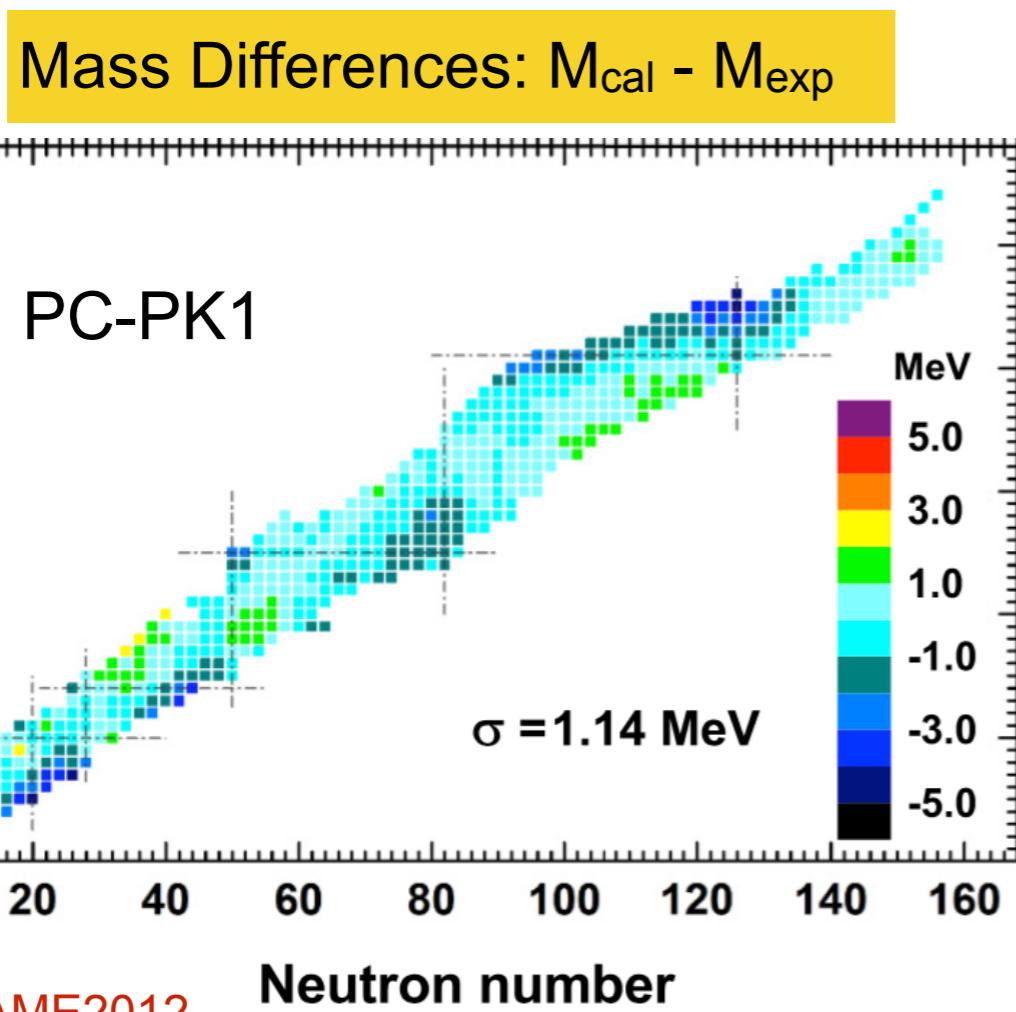


# Toward a bridge between relativistic and nonrelativistic DFT



# Covariant density functional: PC-PK1

~10 parameters fitted to 60 spherical nuclei ...

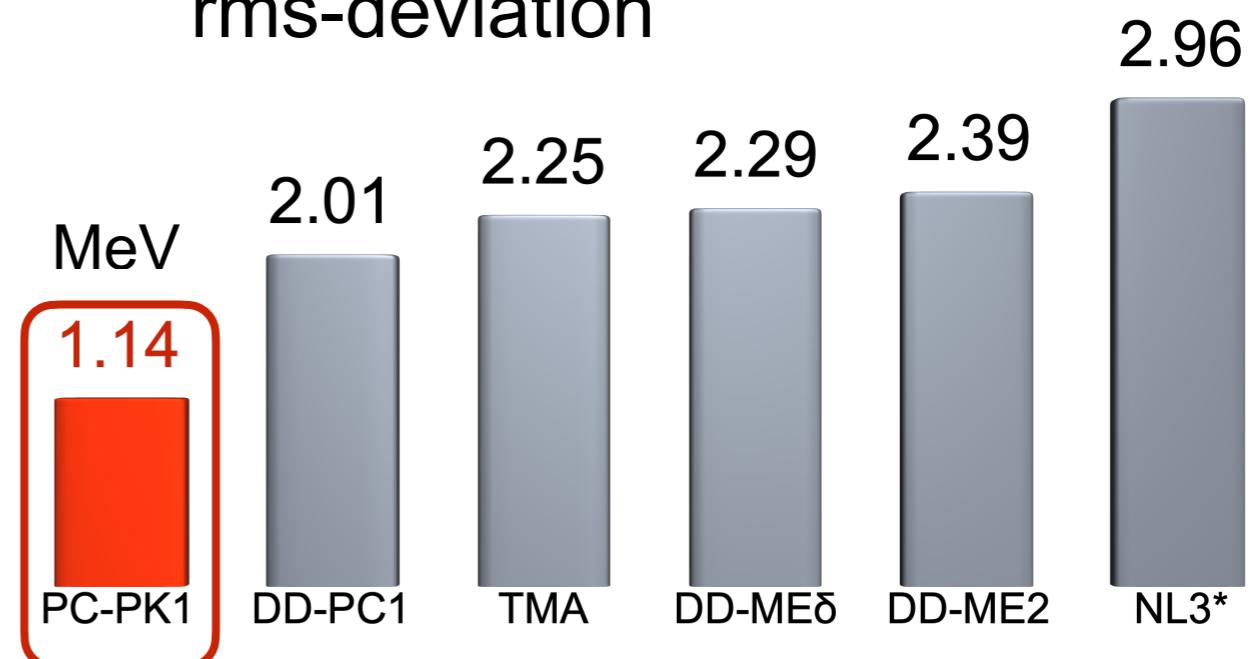


Data from AME2012

PWZ, Li, Yao, Meng, PRC 82, 054319 (2010)

Lu, Li, Li, Yao, Meng PRC 91, 027304 (2015)

rms-deviation



Agbemava PRC 2014  
Geng PTP 2005

Best density-functional description for nuclear masses so far!

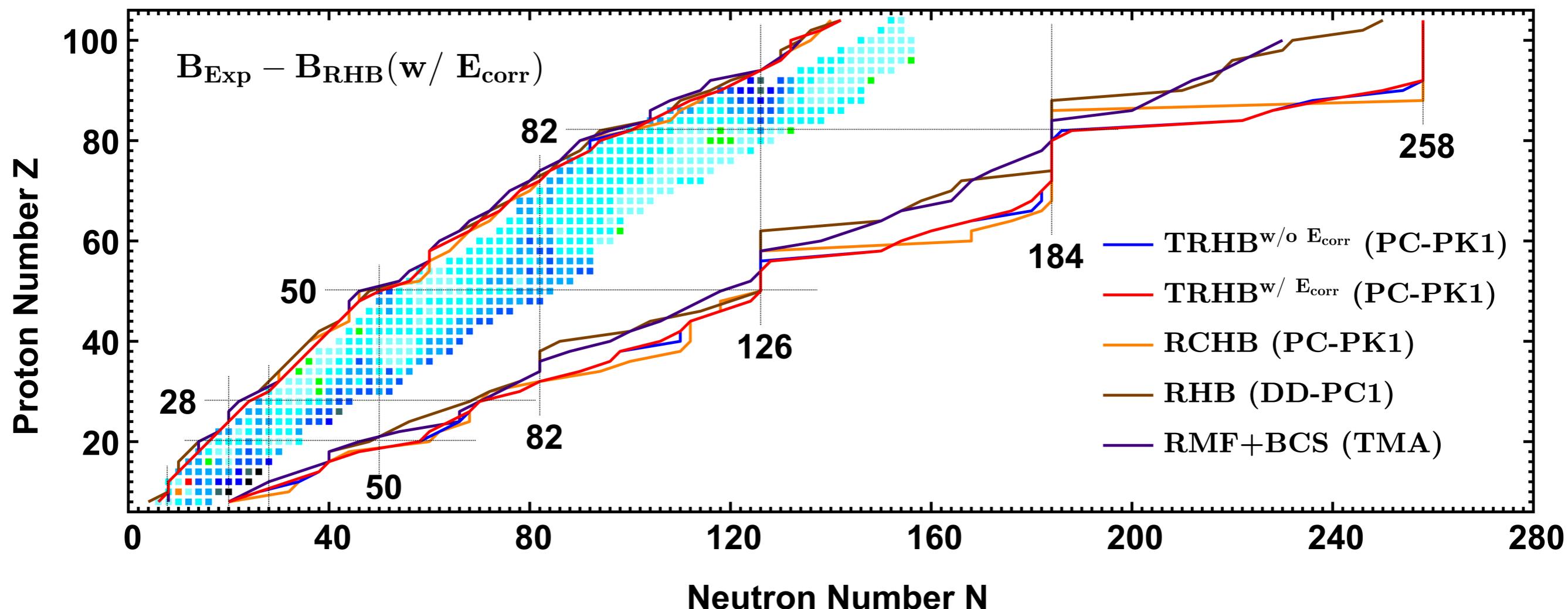
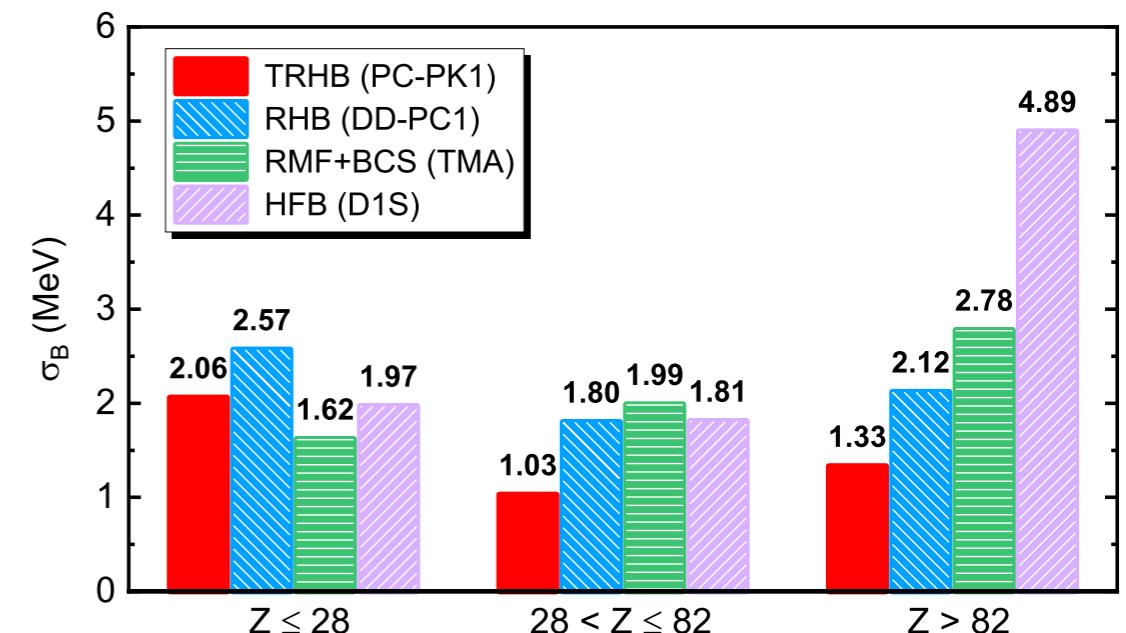
# Outline

- Nuclear landscape  
(light and heavy nuclei; neutron-deficient and neutron-rich nuclei)
- Nuclear spectroscopy  
(ground and excited states; low- and high- angular momentum states)
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(nuclear structure and reaction)
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(towards a microscopic formalism based on bare nuclear interactions)
- Summary

# Nuclear landscape

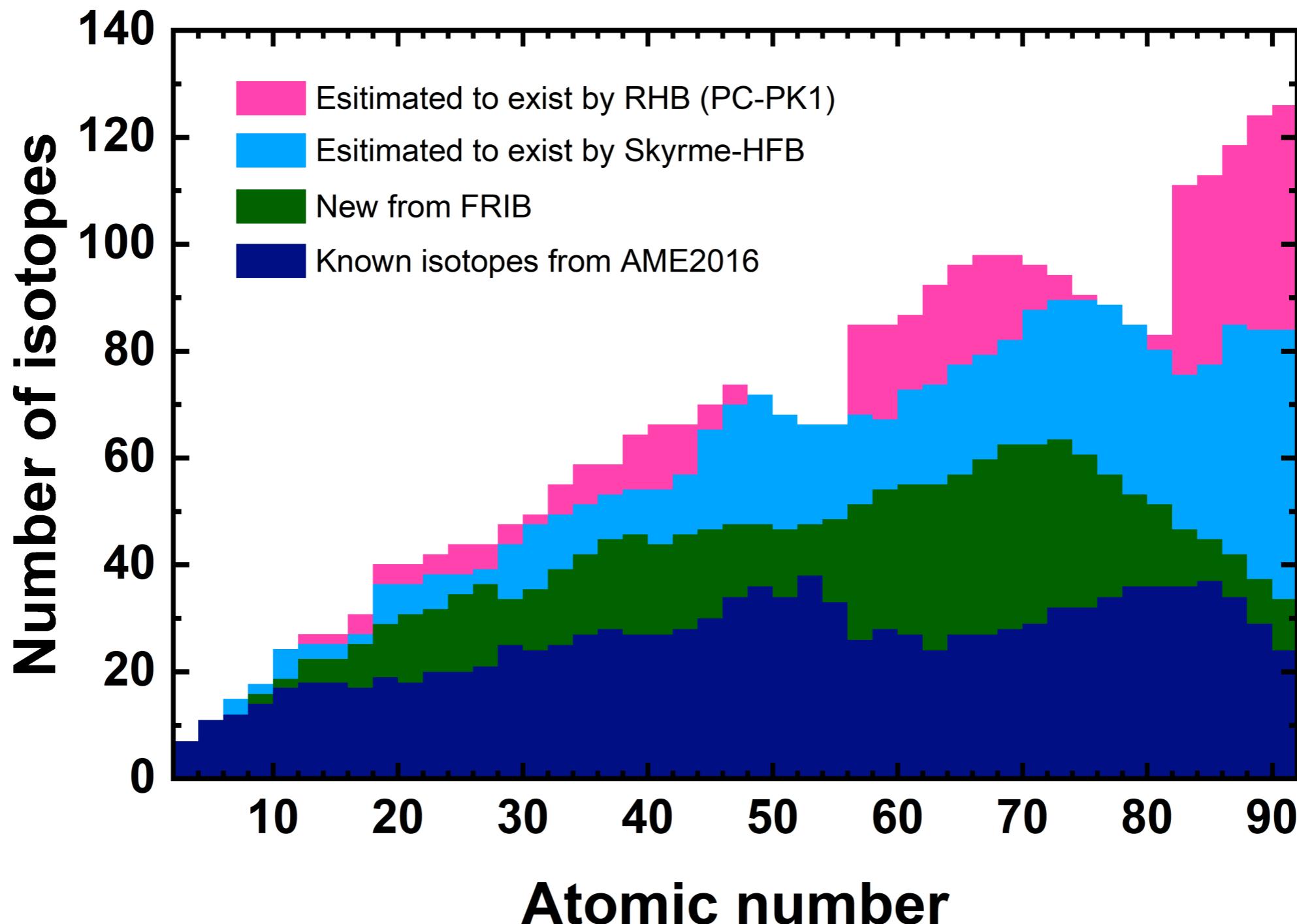
## Triaxiality + Dynamical correlations

Yang, Wang, PWZ, Li, Submitted



# What will the new facilities bring us?

More isotopes ...



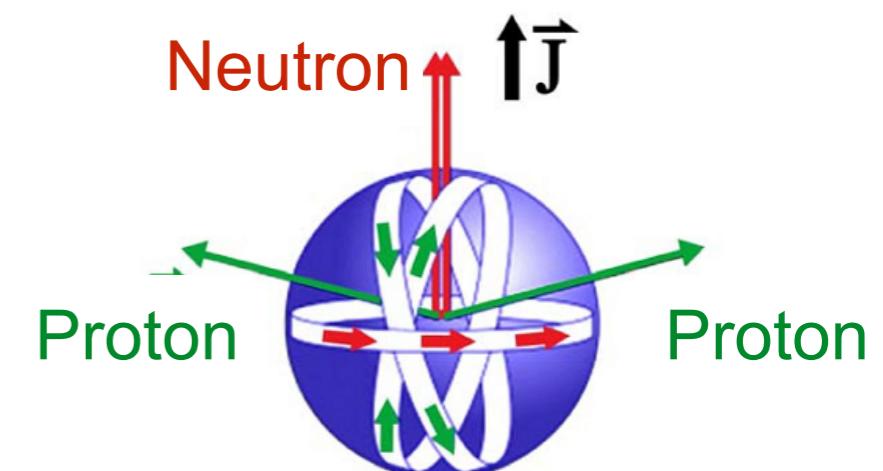
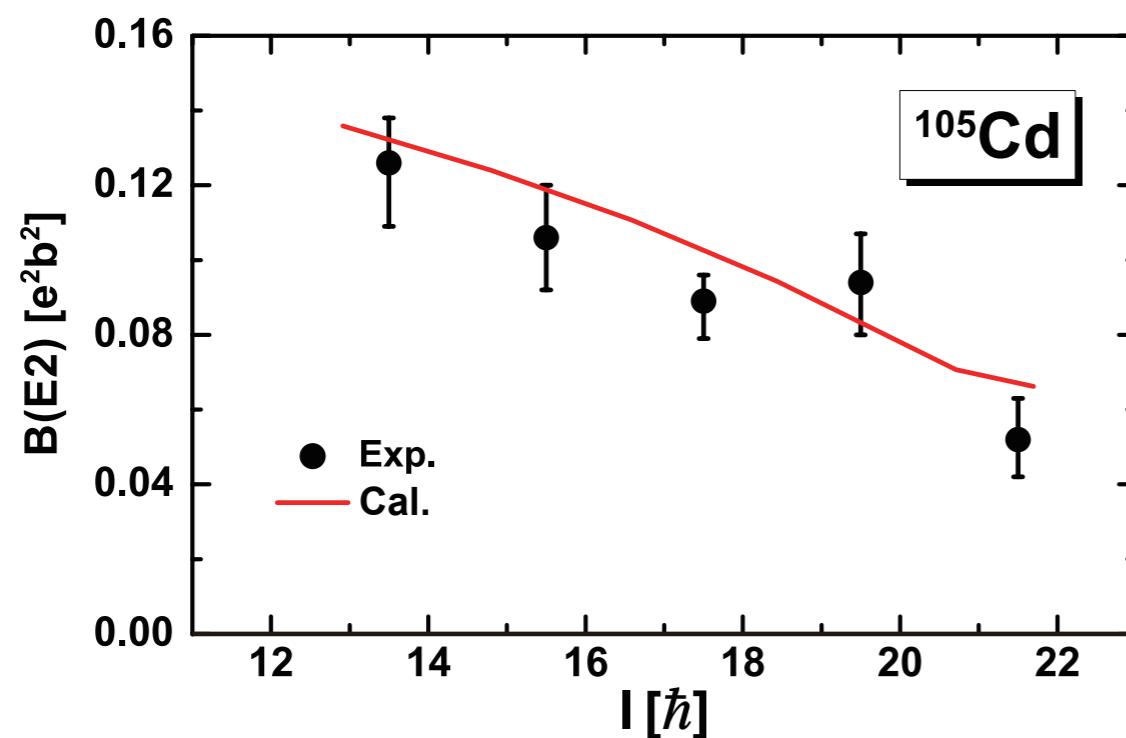
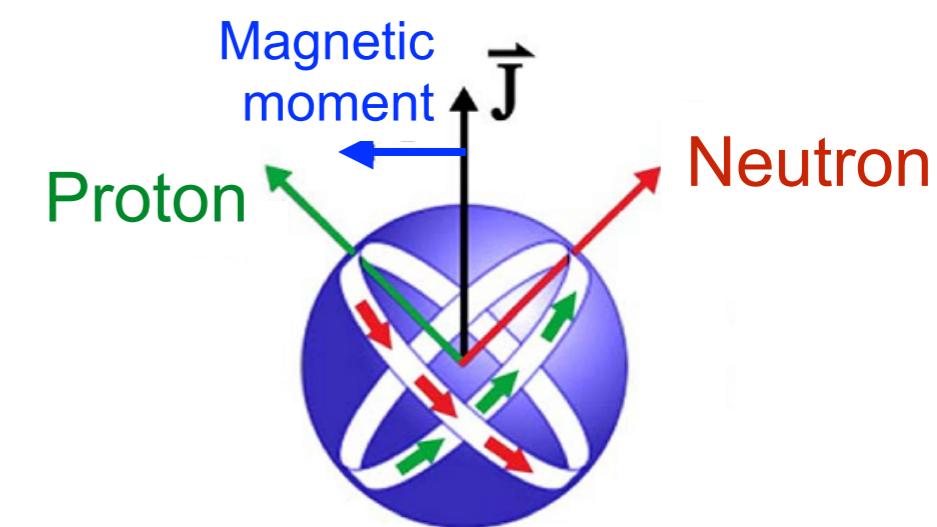
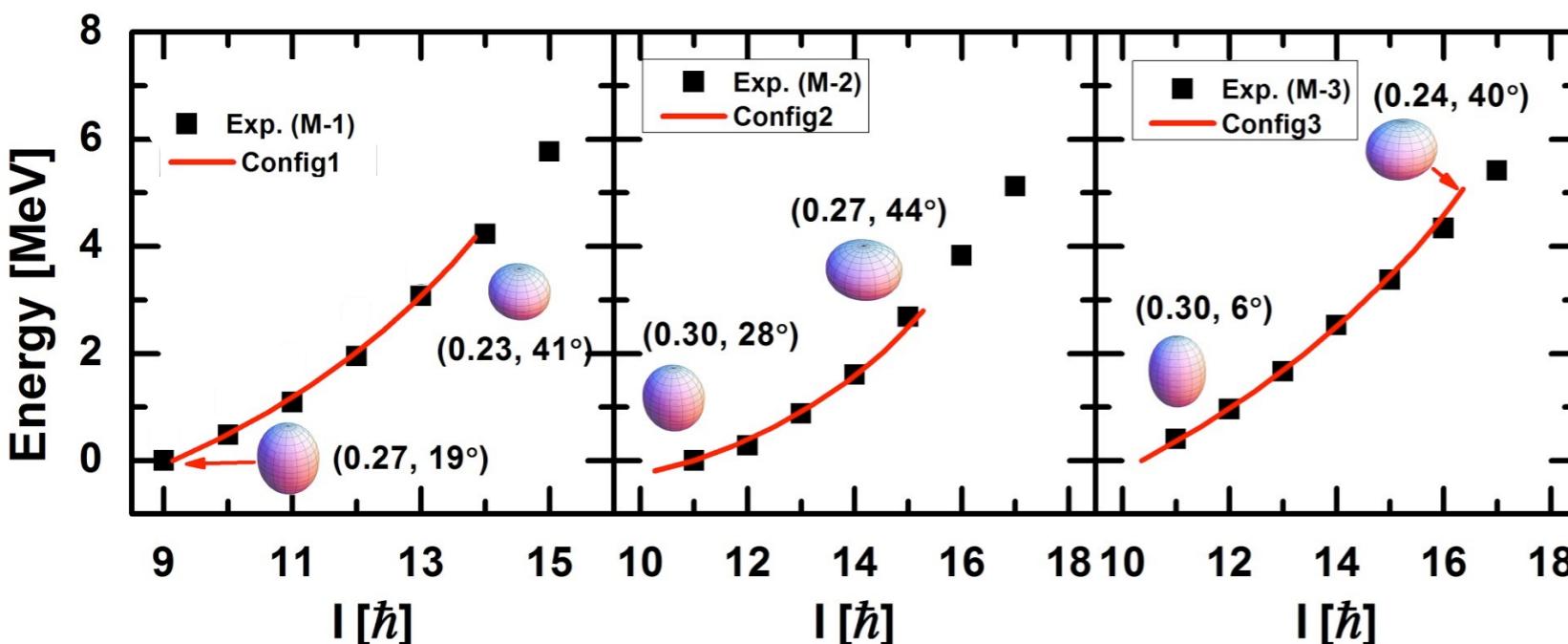
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# Magnetic and Anti-magnetic Rotation

## Tilted axis cranking CDFT

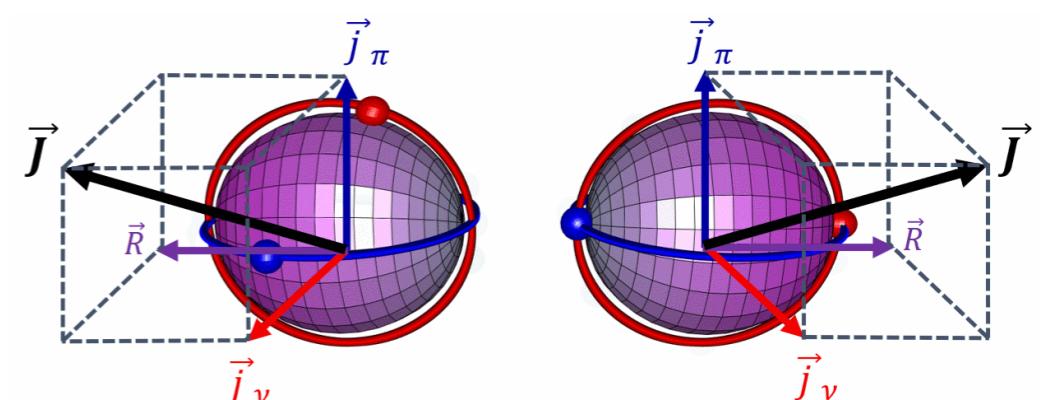
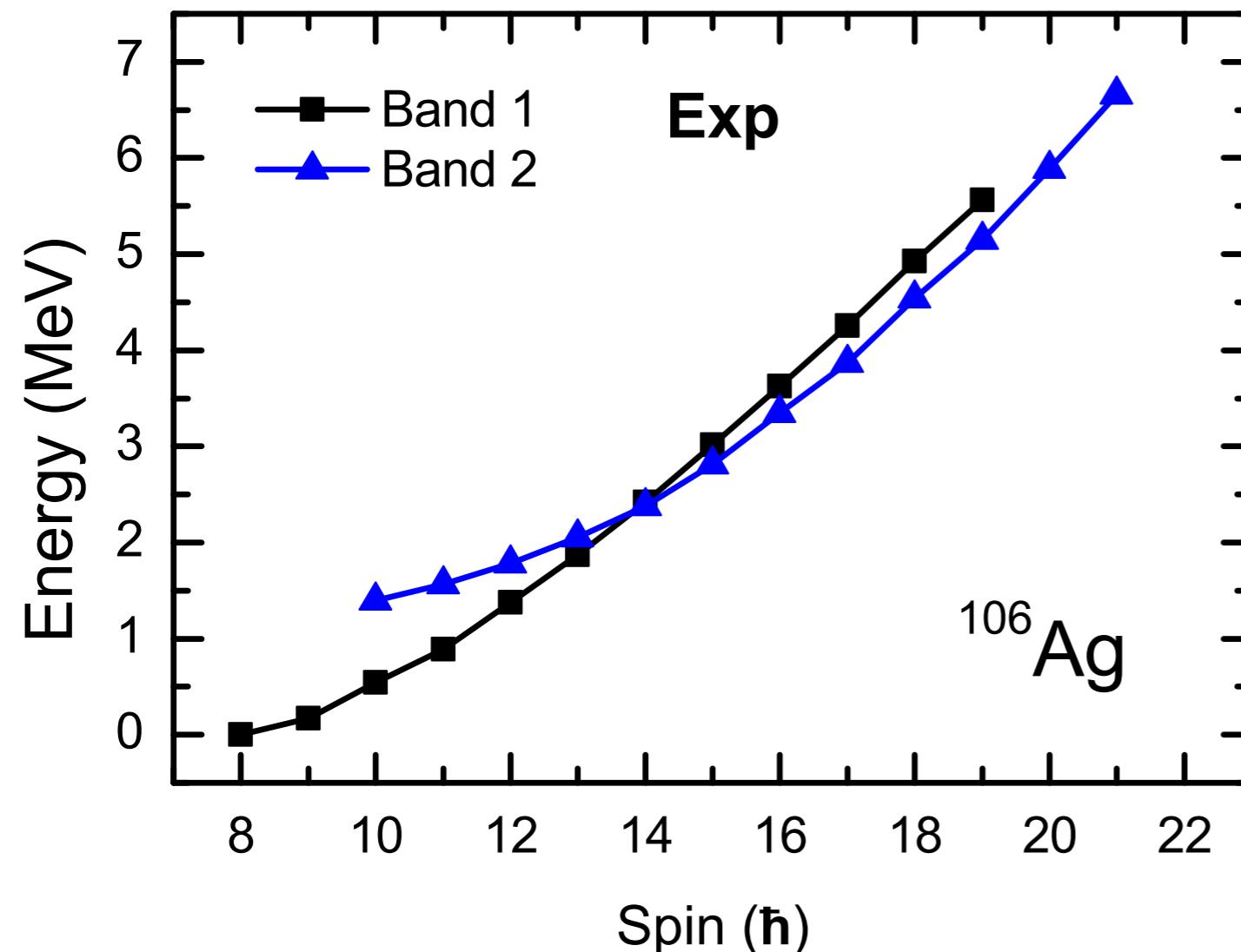
PWZ, Peng, Liang, Ring, Meng, PRL 107, 122501 (2011)  
PWZ, Zhang, Peng, Liang, Ring, Meng, PLB 699, 181 (2011)



# Chiral conundrum in $^{106}\text{Ag}$

## Experimental observations in 2007: Energy Spectrum

Joshi, et al., PRL 98, 102501 (2007)



Frauendorf and Meng, Nucl. Phys. A 617, 131 (1997)

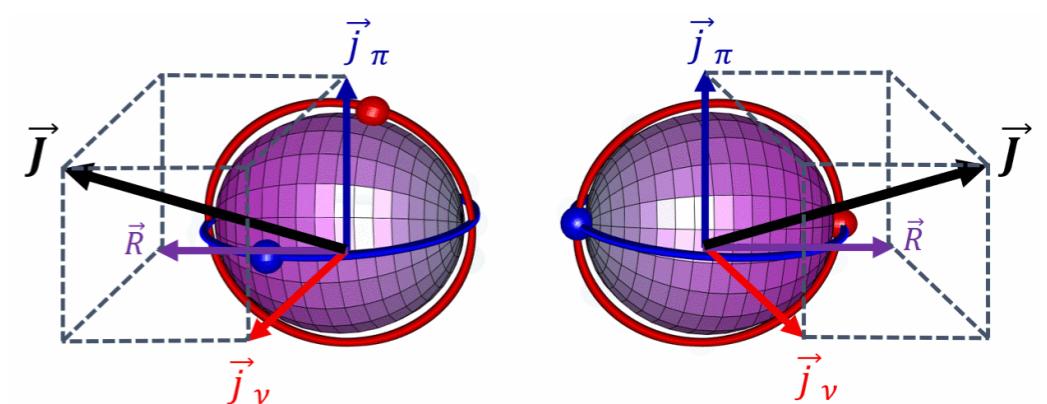
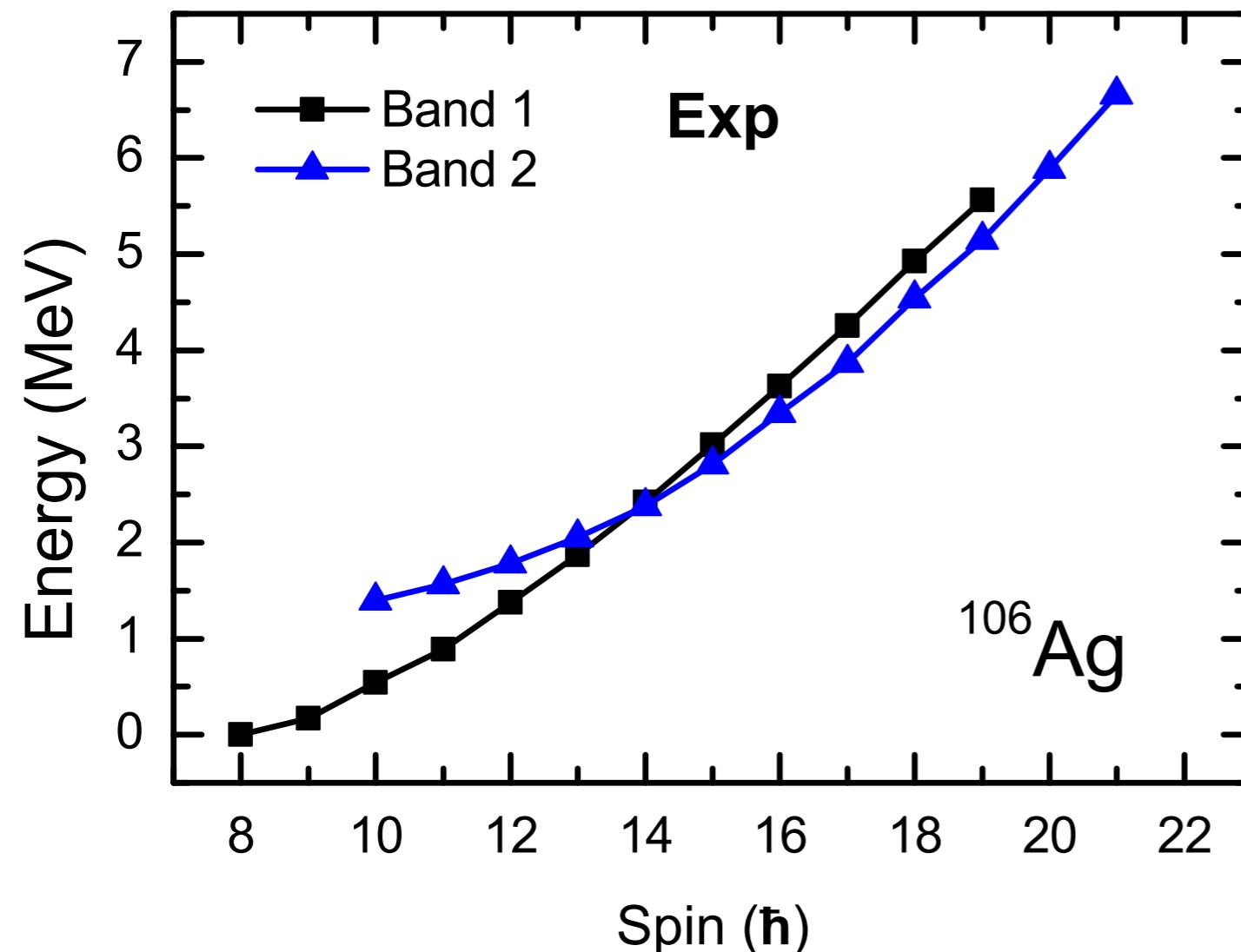
A pair of strongly coupled bands observed

Chiral bands? But why crossing?

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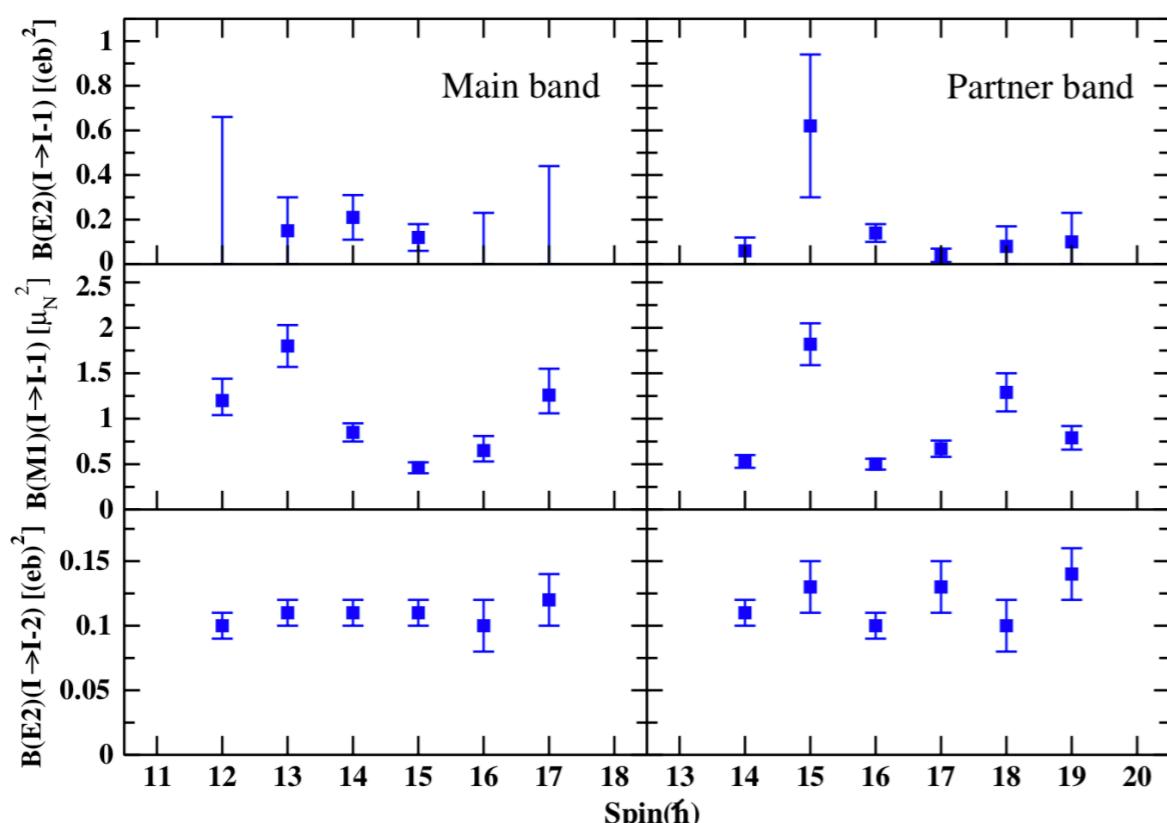
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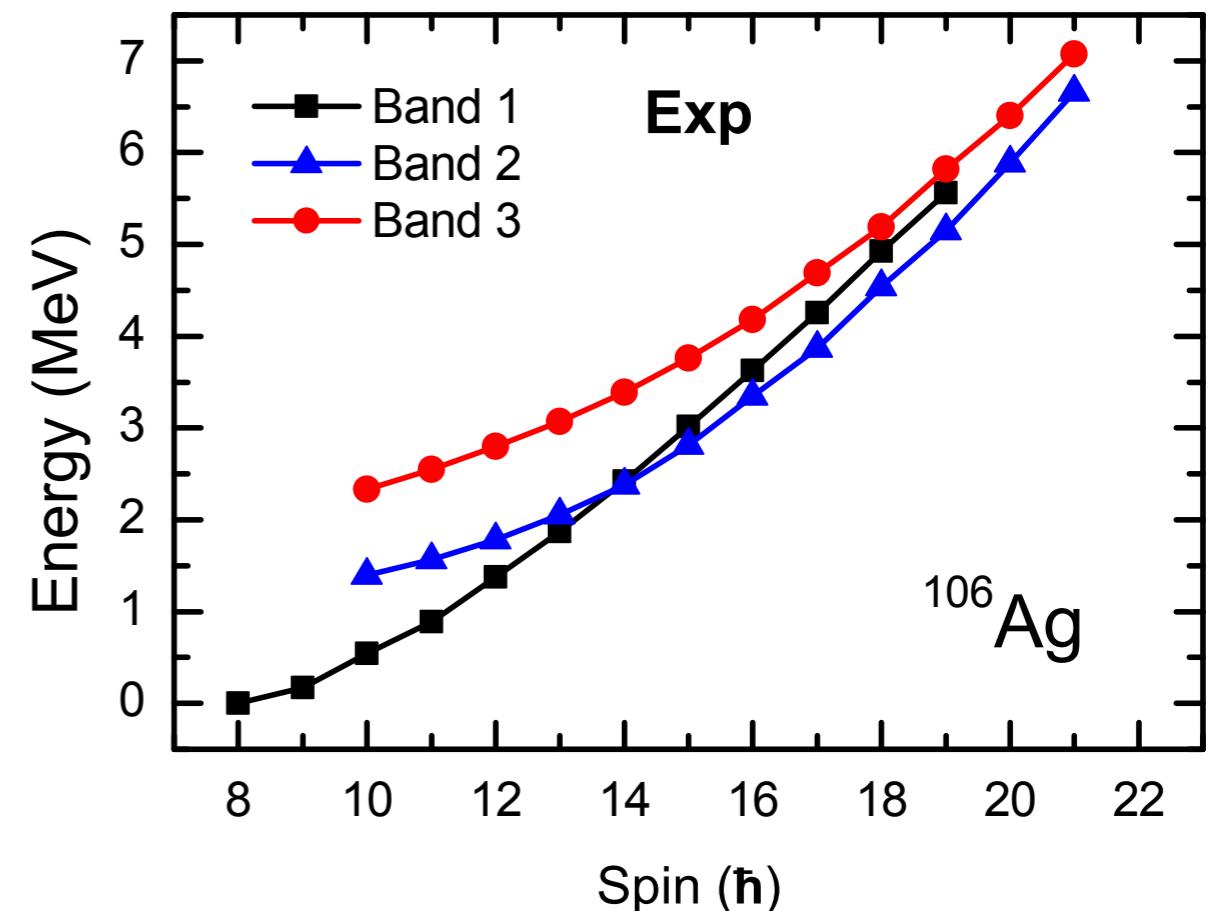
# Chiral conundrum in $^{106}\text{Ag}$

## Experimental observations in 2014: Transition strength

Rather, et al., PRL 112, 202503 (2014)



Lieder, et al., PRL 112, 202502 (2014)



$B(\text{M1})$  and  $B(\text{E2})$  values were measured in both experiments

A third band is reported in Lieder's experiment

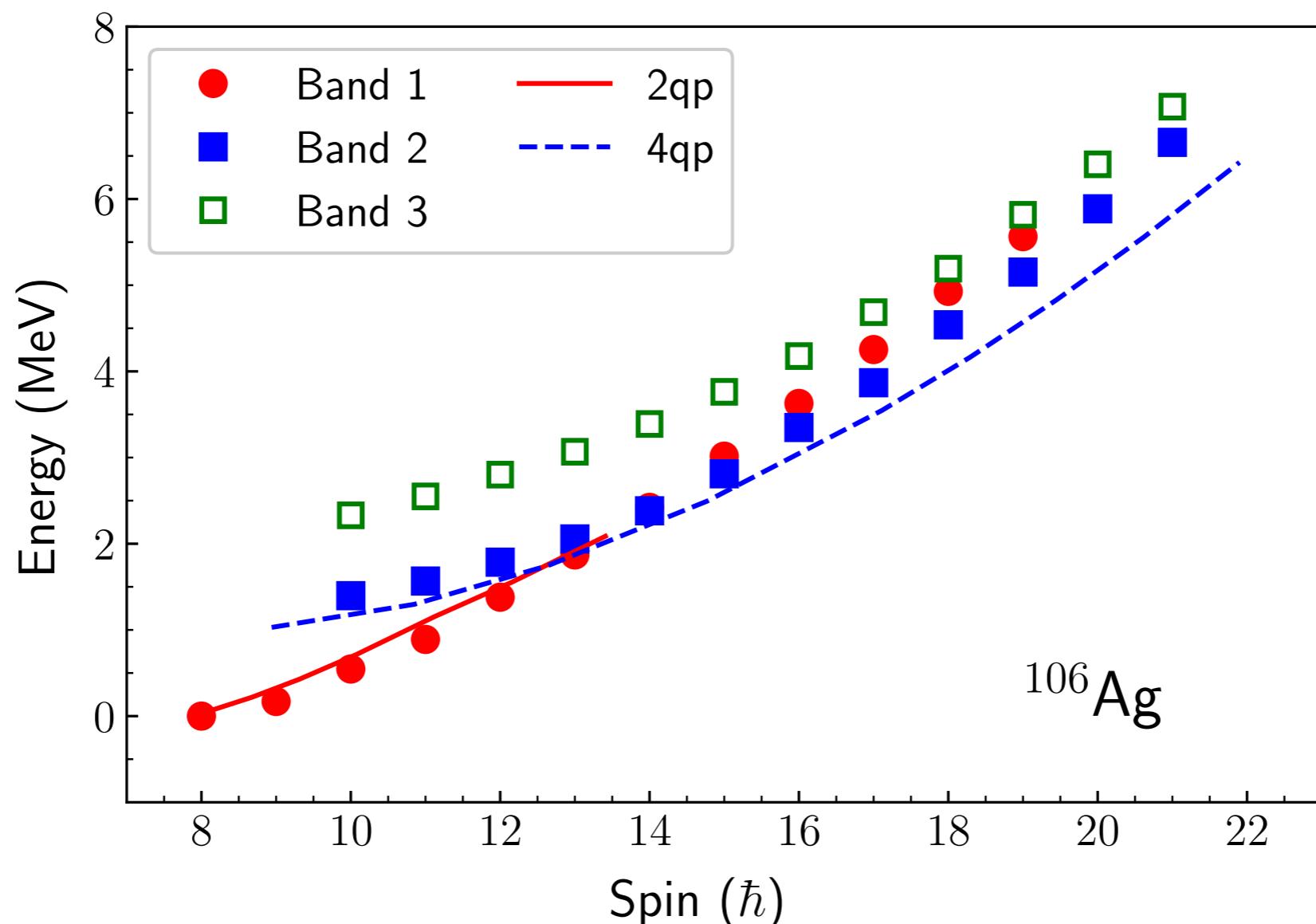
Chiral bands ? But why crossing ? Why three bands ?

# Chiral conundrum in $^{106}\text{Ag}$

## TAC-CDFT calculations

PWZ PLB 773,1 (2017)

PWZ, Wang, Chen PRC 99, 054319 (2019)



2qp

$\pi g_{9/2} \otimes \nu h_{11/2}$

4qp

$\pi g_{9/2} \otimes \nu h_{11/2}(gd)^2$

# (C)DFT and Shell Model

## (C)DFT

✓ Universal density functionals

Symmetry broken

Single config. fruitful physics

No Configuration mixing

✓ Applicable for almost all nuclei

✗ No spectroscopic properties

## Shell Model

✗ Non-universal effective interactions

No symmetry broken

Single config. little physics

Configuration mixing

✗ intractable for deformed heavy nuclei

✓ spectroscopy from multi config.

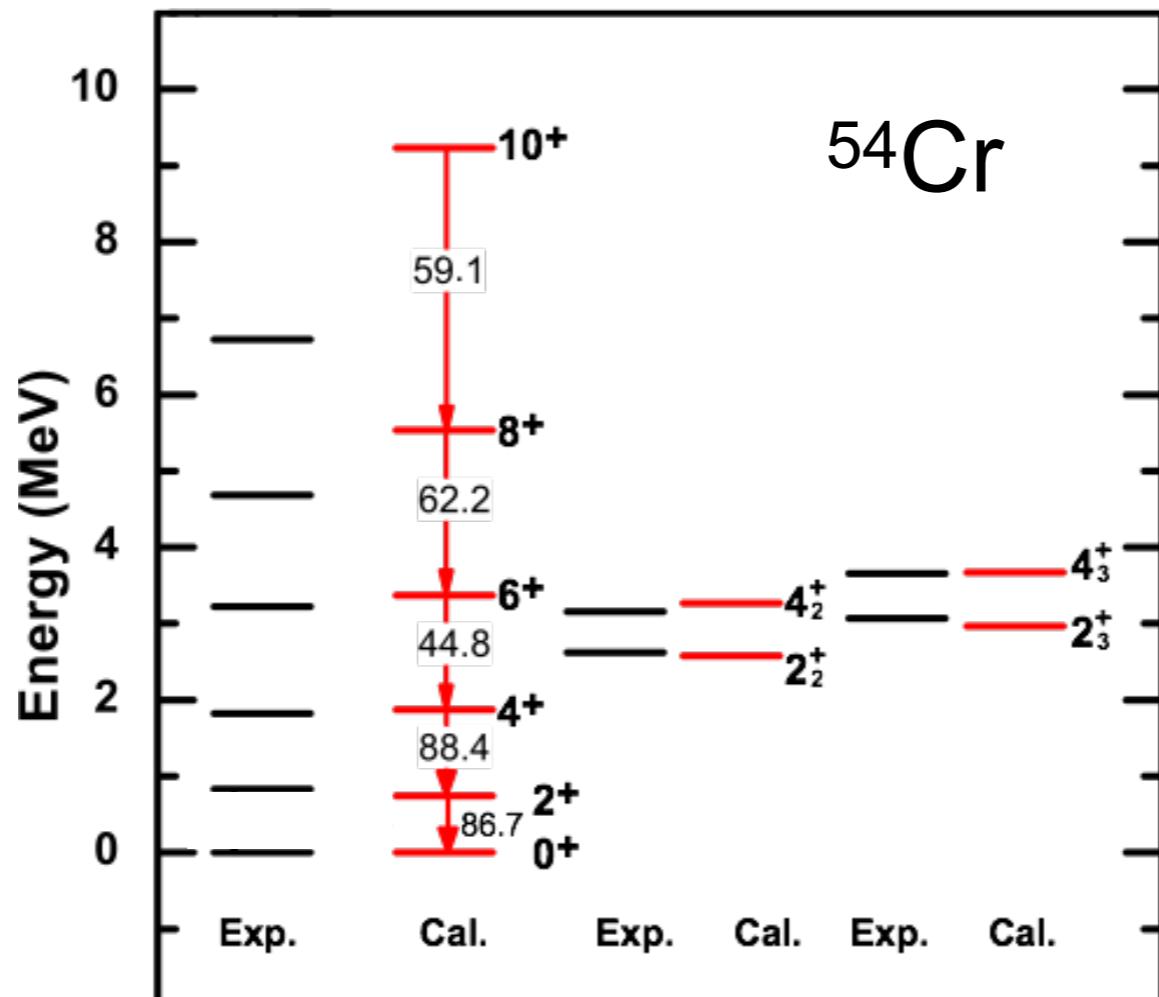
a theory combining the advantages  
from both approaches



# Configuration Interaction Projected DFT (CI-PDFT)

PWZ, Ring, Meng, PRC 94 (2016) 041301(R)

$0\nu\beta\beta$  ?



*Next step: time-odd interaction; beyond 2-qp configurations;*

Towards neutron-rich nuclei ?

Towards triaxial nuclei ?

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# Time-dependent density functional theory

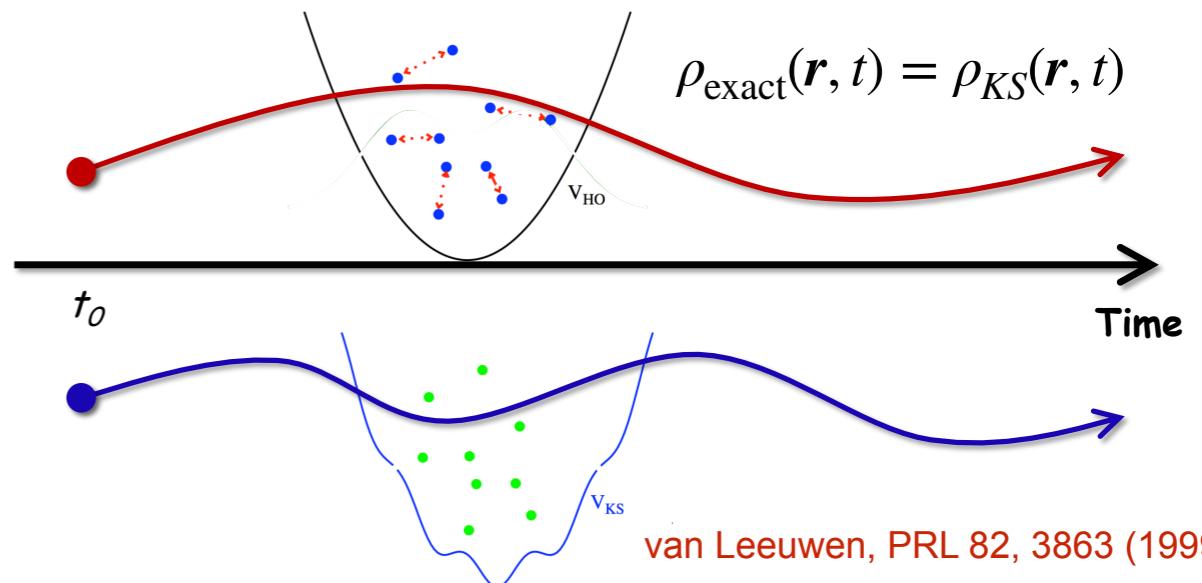
The many-body problem is mapped onto a one-body problem

## Runge-Gross Theorem

There is a **unique mapping** between the **time dependent external potential** and the **density**, for many body systems evolving from a **given initial state**.

Runge and Gross, PRL 52, 997 (1984)

## Time-dependent Kohn-Sham DFT



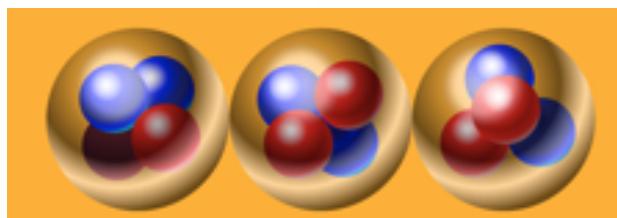
van Leeuwen, PRL 82, 3863 (1999)

$$i\hbar \frac{\partial \phi_i(\mathbf{r}, t)}{\partial t} = \left[ -\frac{\hbar^2}{2m} \nabla^2 + v_{\text{KS}}[\rho(\mathbf{r}, t)] \right] \phi_i(\mathbf{r}, t)$$

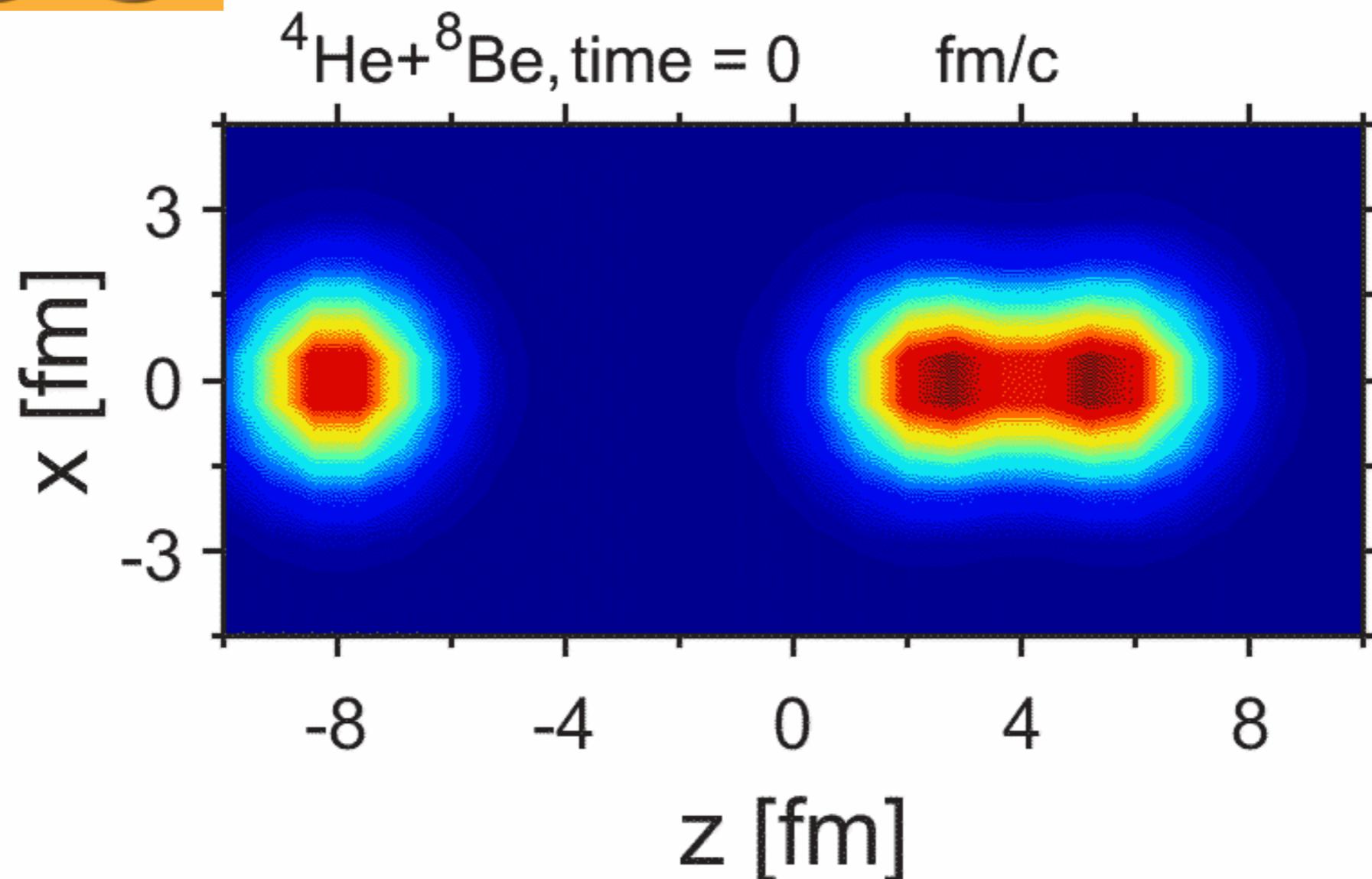
$$\rho(\mathbf{r}, t) = \sum_i^N |\phi_i(\mathbf{r}, t)|^2$$



# Resonant scattering of ${}^4\text{He} + {}^8\text{Be}$

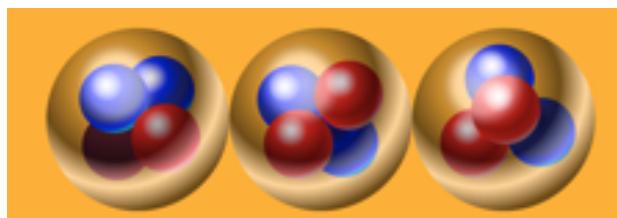


Ren, PWZ, Meng, PLB 801, 135194 (2020)

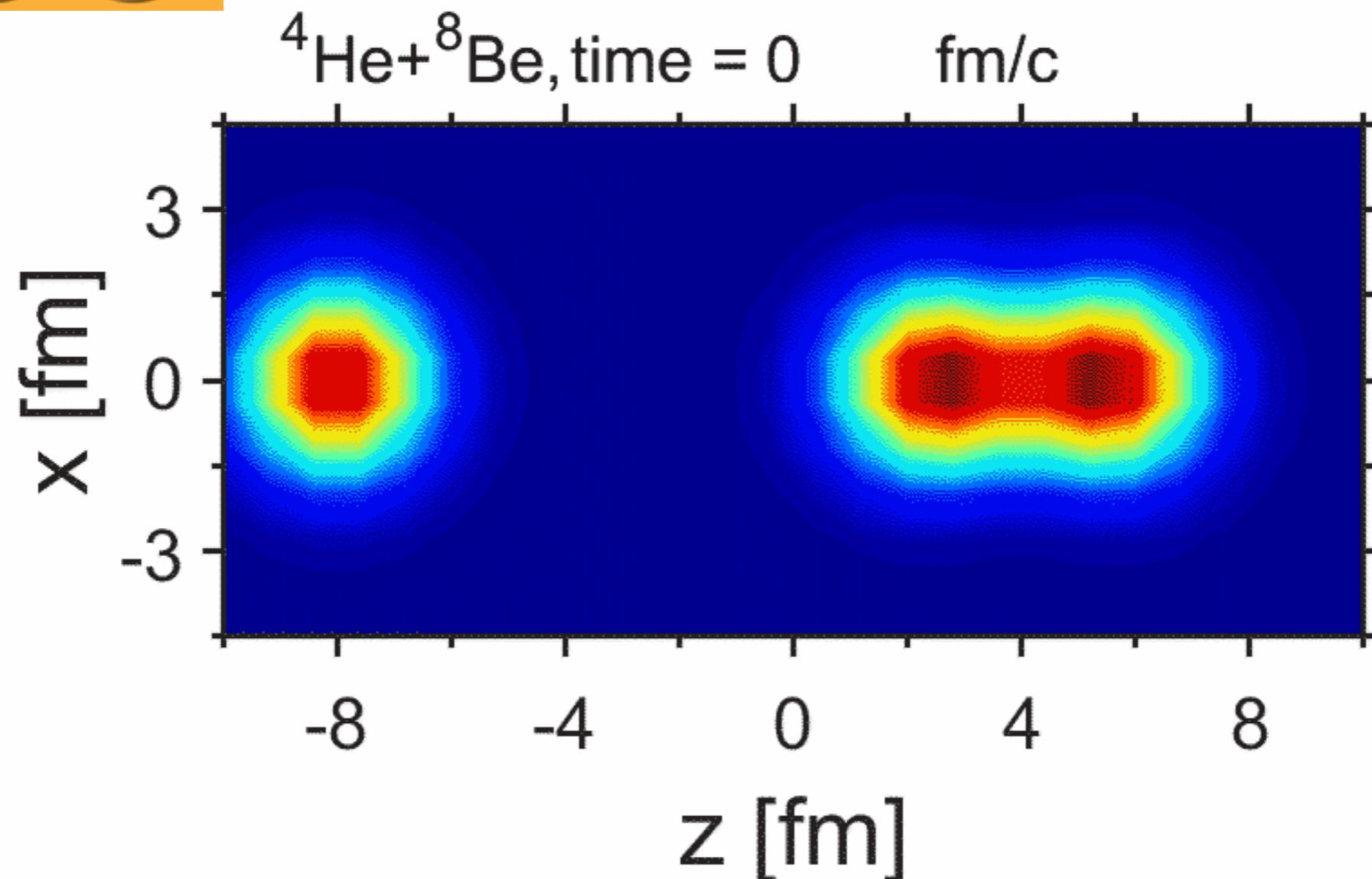


The linear-chain states are generated, and then evolve to a triangular configuration, and finally to a more compact shape.

# Resonant scattering of ${}^4\text{He} + {}^8\text{Be}$

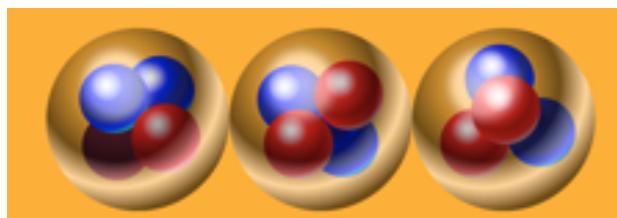


Ren, PWZ, Meng, PLB 801, 135194 (2020)

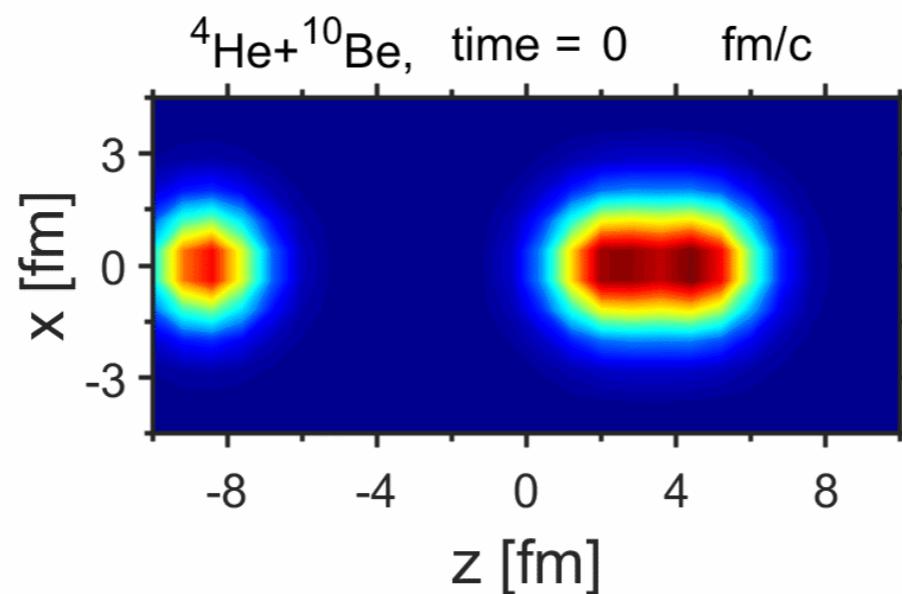
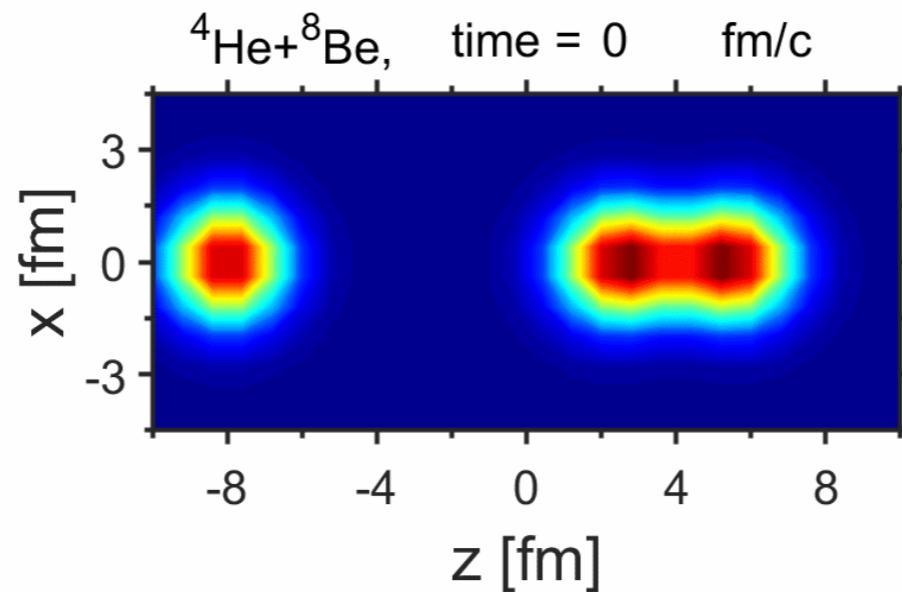


The linear-chain states are generated, and then evolve to a triangular configuration, and finally to a more compact shape.

# Adding more neutrons: ${}^4\text{He} + {}^{10}\text{Be}$

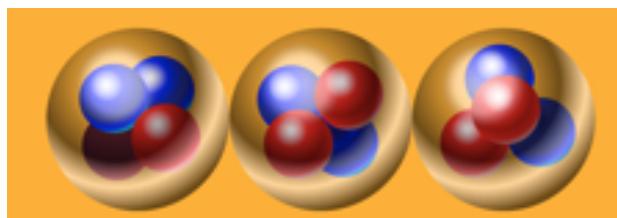


Ren, PWZ, Meng, PLB 801, 135194 (2020)

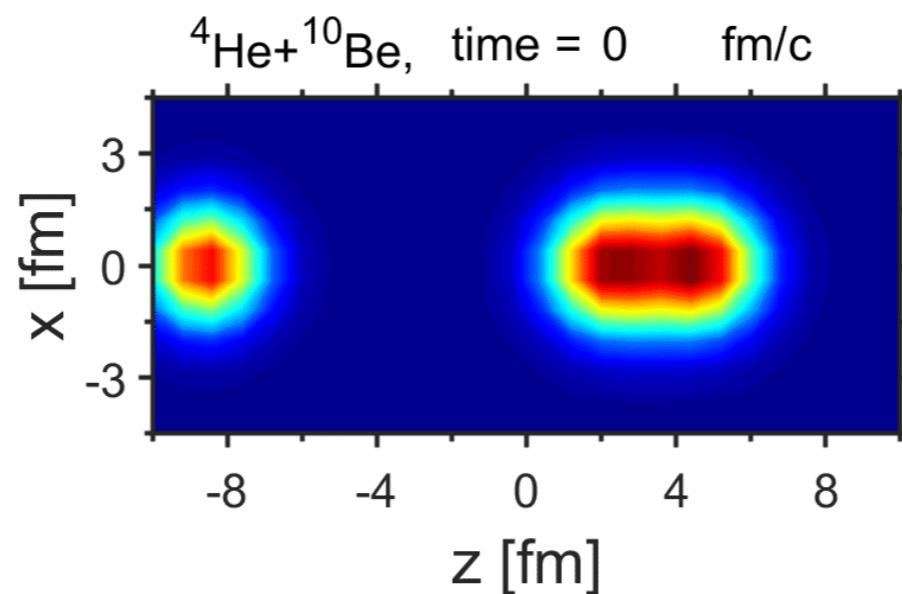
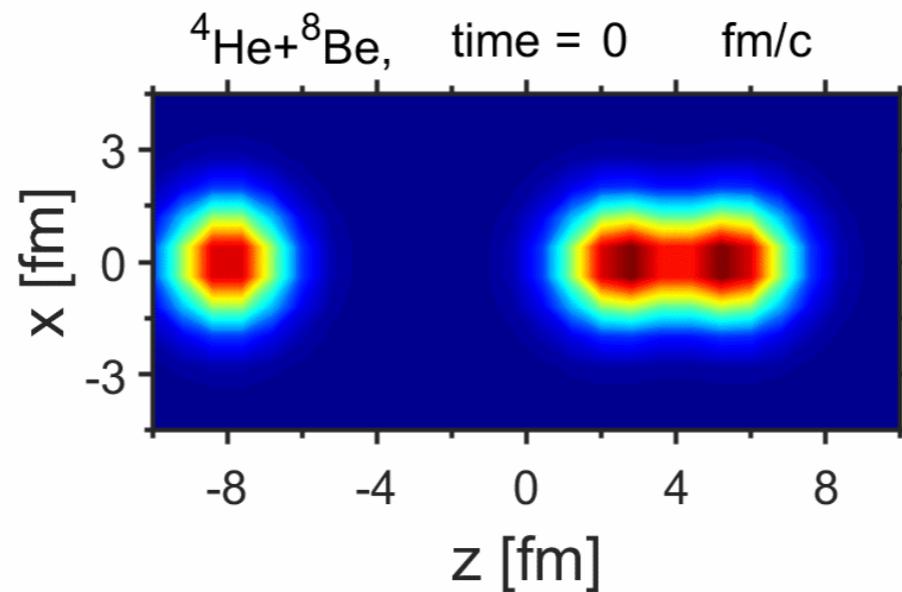


- ✓ The metastable linear chains can be formed in  ${}^4\text{He} + {}^8\text{Be}$  and  ${}^4\text{He} + {}^{10}\text{Be}$  collisions.
- ✓ During the time evolution of the linear-chain configuration, moving clusters can be found.

# Adding more neutrons: ${}^4\text{He} + {}^{10}\text{Be}$

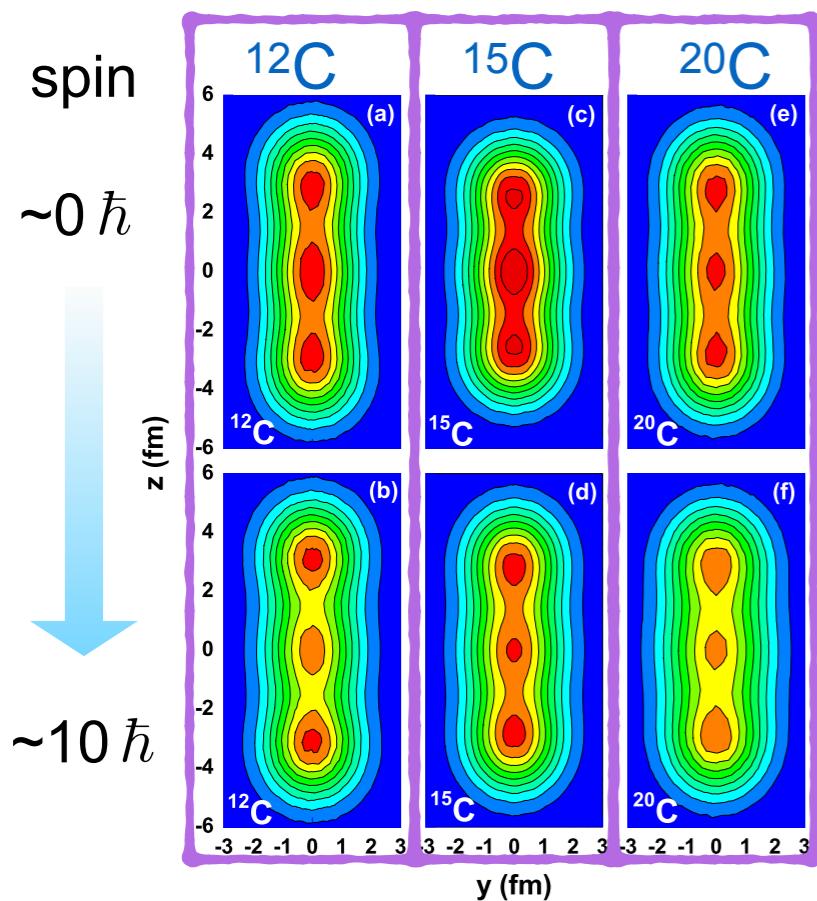


Ren, PWZ, Meng, PLB 801, 135194 (2020)



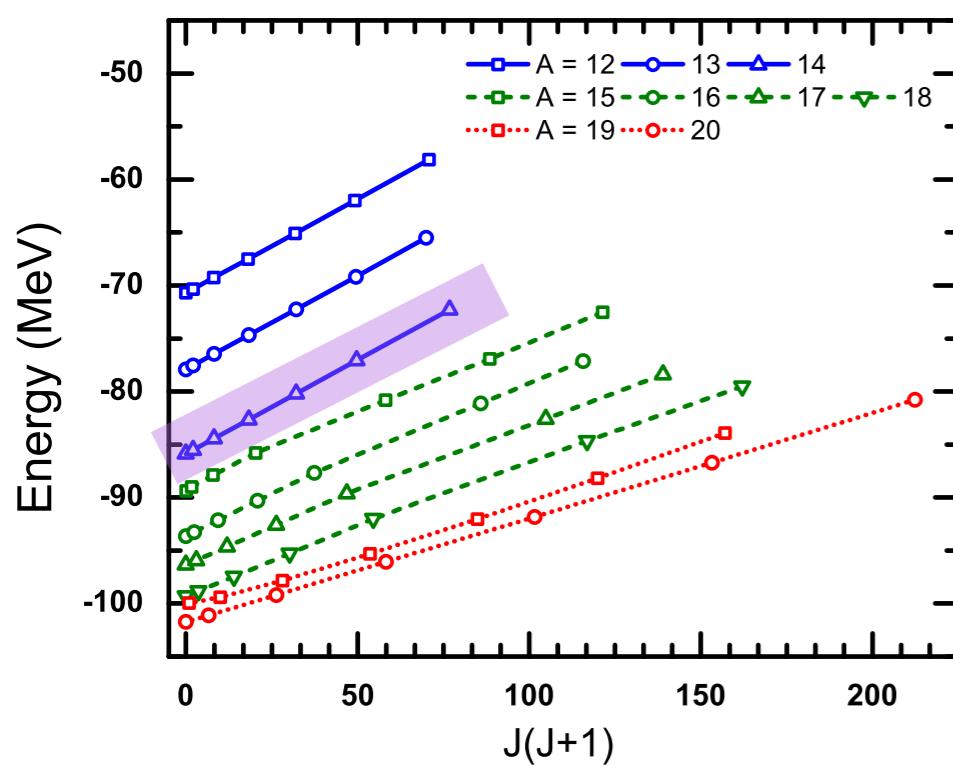
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# Statistical calculations

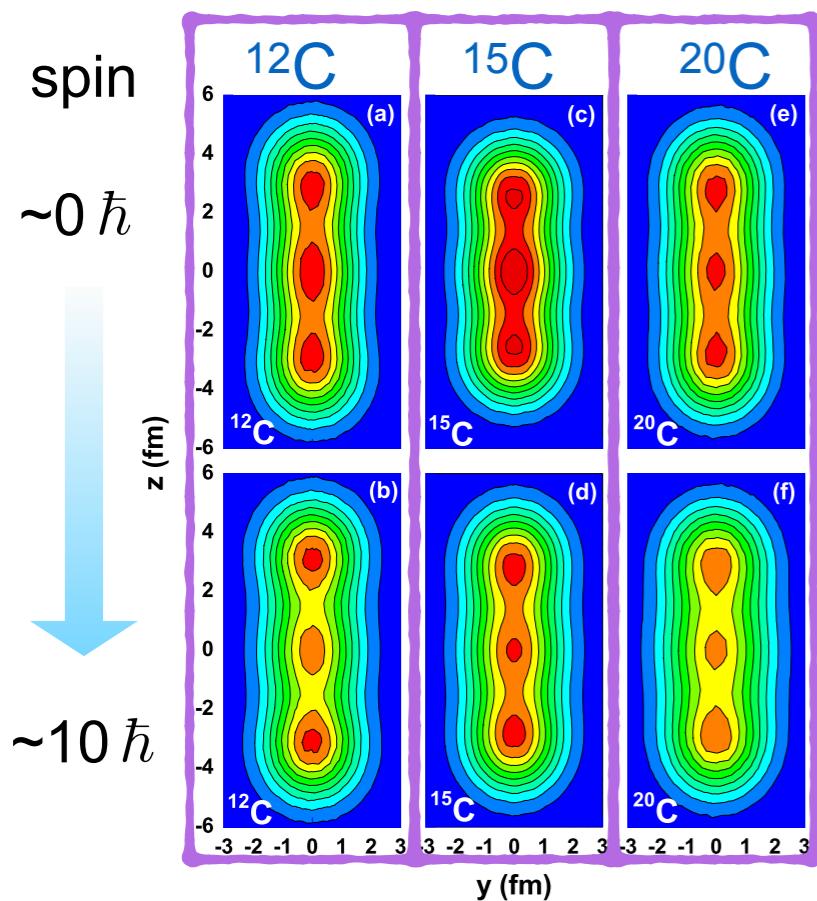


Rod-like shapes could be realized in nuclei with large total spin and isospin.

PWZ, Itagaki, Meng, PRL 115, 022501 (2015)

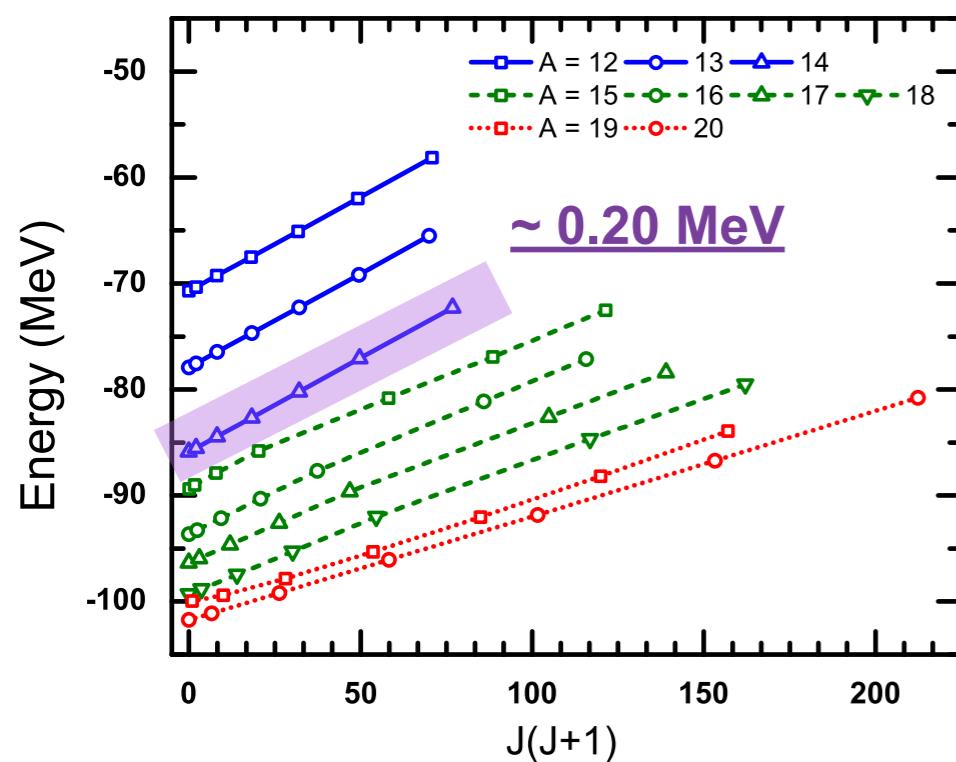


# Statistical calculations



Rod-like shapes could be realized in nuclei with large total spin and isospin.

PWZ, Itagaki, Meng, PRL 115, 022501 (2015)



Exp @RIKEN

where  $\mathfrak{I}$  is the moment of inertia of the nucleus. The linearity allows us to interpret the levels as a rotational band, and the low  $\hbar^2/2\mathfrak{I} = 0.19$  MeV implies the nucleus could be strongly deformed, consistent with the interpretation of an LCCS. Although we ob-

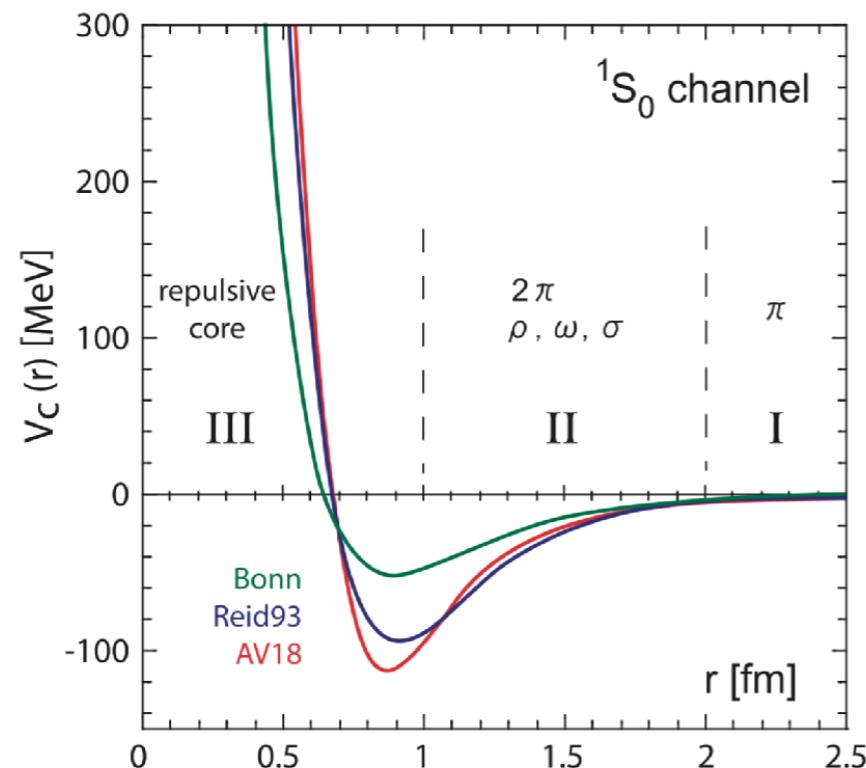
Yamaguchi et al., PLB 766 (2017) 11–16

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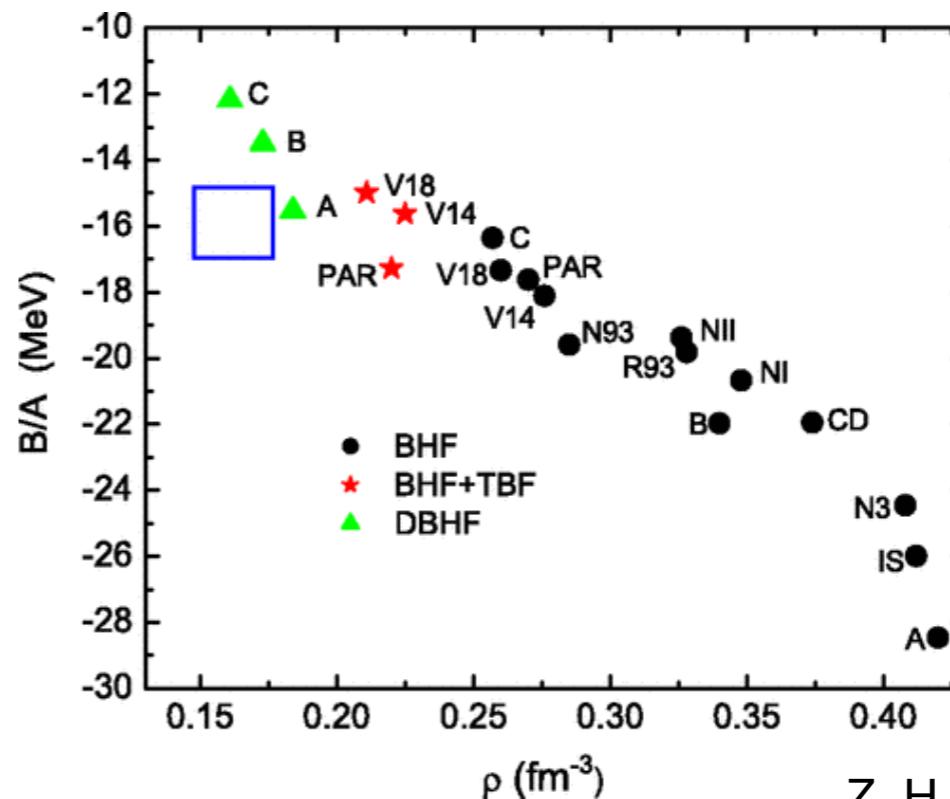
# Complicated nuclear force

NN force



S. Aoki 2008

Nuclear matter saturation



Z. H. Li 2006

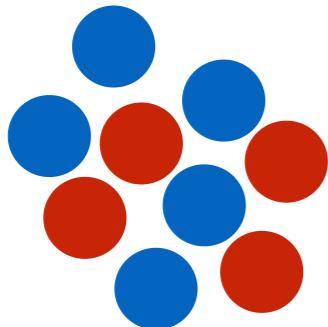
Short-range repulsive; Three-body force; Covariance

## NN forces from EFTs of QCD

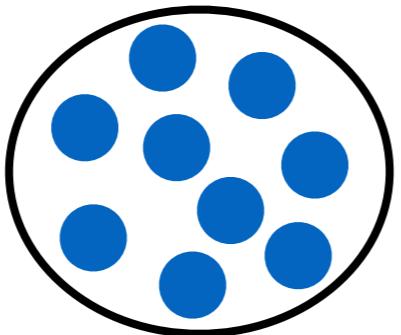


# Neutron Drops

Why study neutron drops? (A pure simple toy model?)



Self-bound



Confined

Pudliner, et al., PRL 76, 2416 (1996)  
Gandolfi, et al., PRL 106, 012501 (2011)  
Maris, et al., PRC 87, 054318 (2013)  
Potter, et al., PLB 739, 445 (2014)  
PWZ and Gandolfi, PRC 94 (2016) 041302(R)  
Shen, et al., PLB 778, 344 (2018)

.....

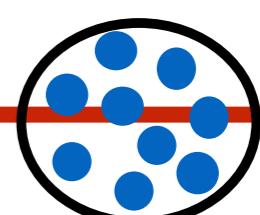
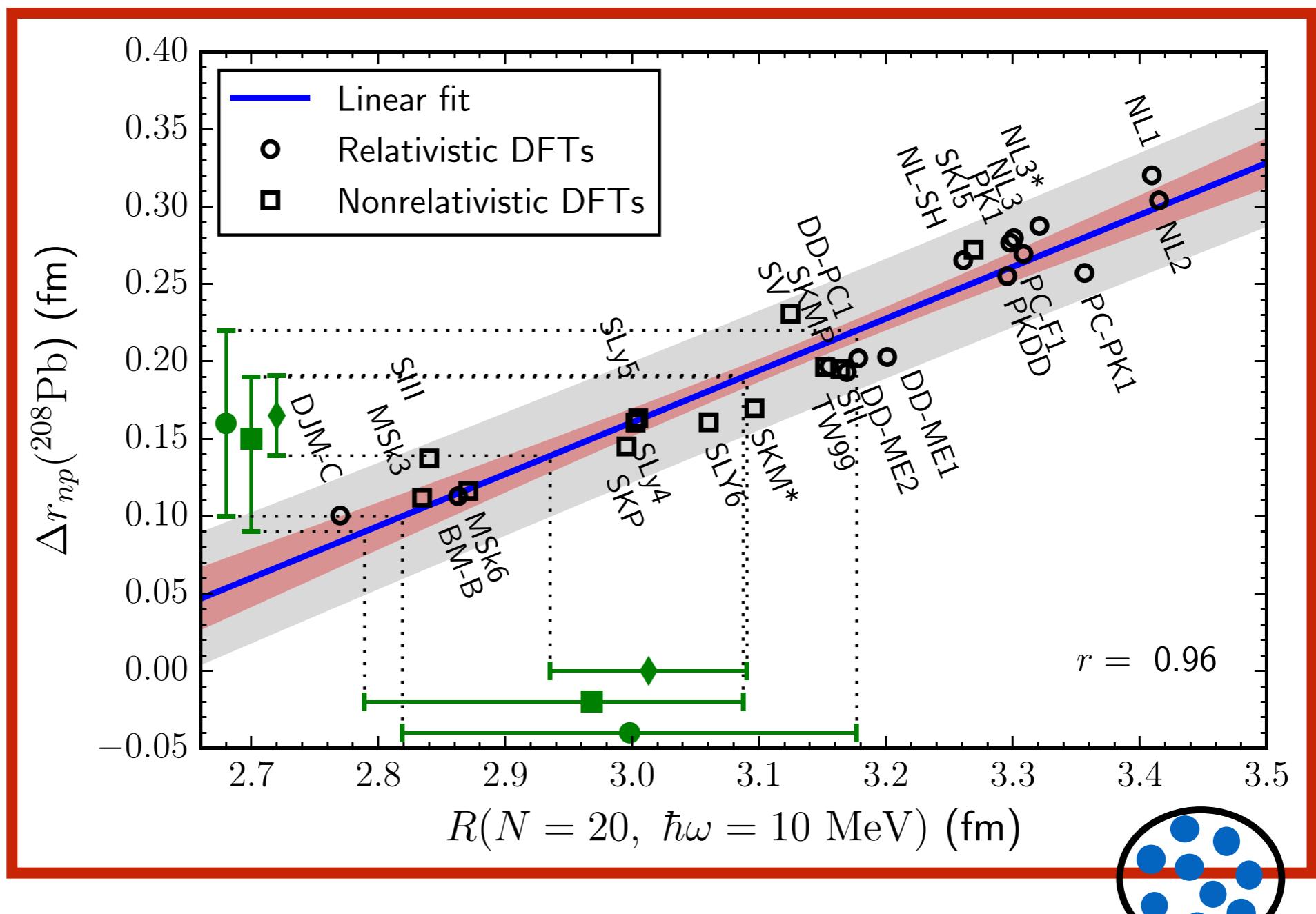
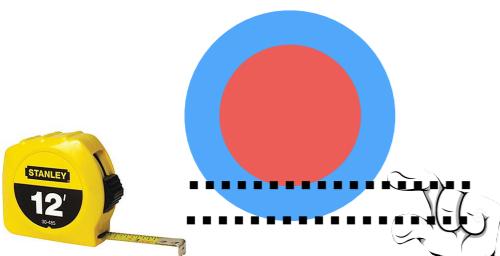
1. benchmark various nuclear many-body methods
2. calibrate nuclear interactions / energy density functionals
3. model neutron-rich nuclei and neutron stars
4. predict few neutron resonances
5. connections with the skin thickness and symmetry energy

# Neutron drop as a “toy” neutron-rich nucleus

## Neutron skin thickness

PWZ and Gandolfi PRC 94 (2016) 041302(R)

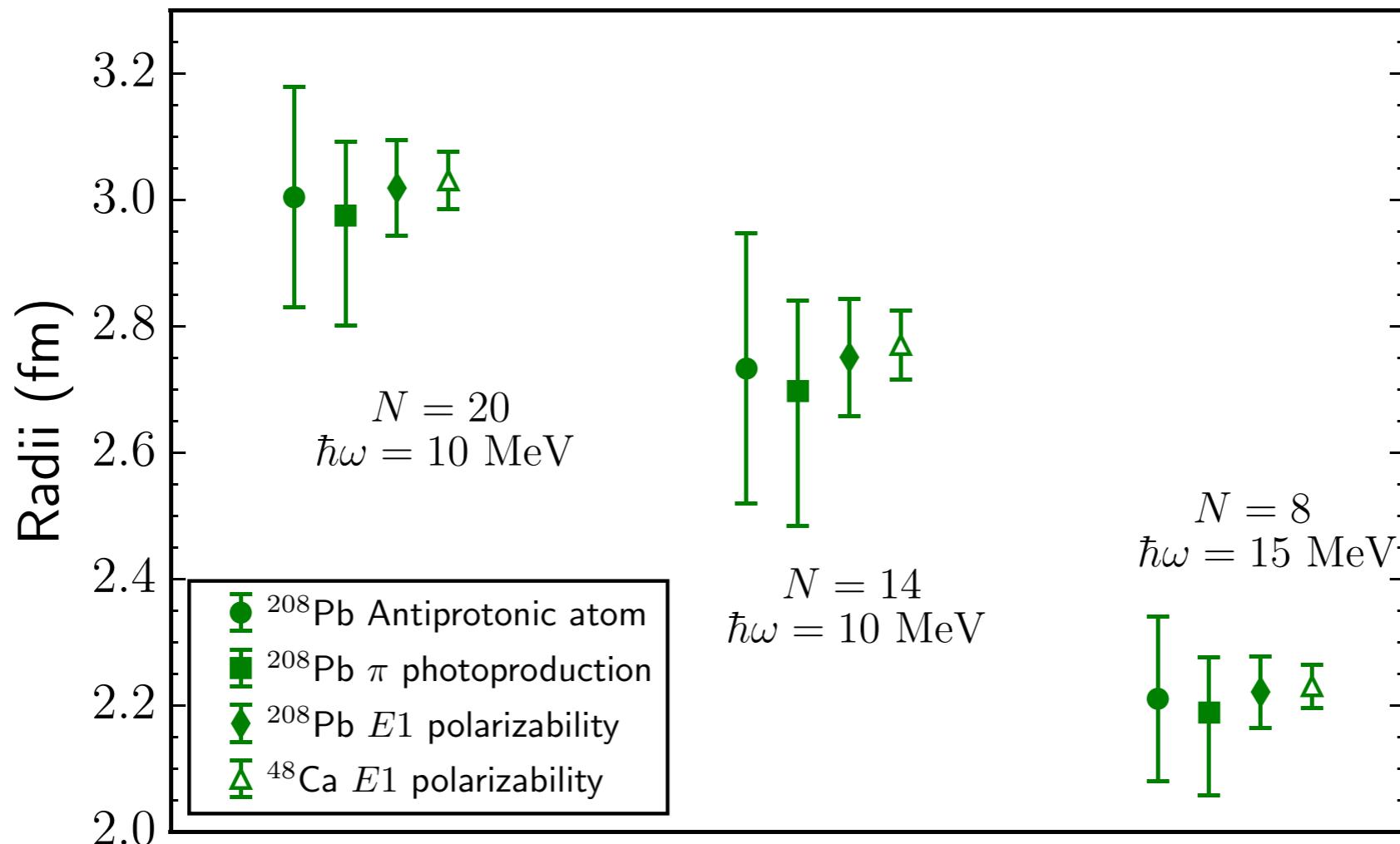
$$\Delta r_{np} = r_n - r_p$$



# Constraining three-body forces

Radii: QMC calculations with various Hamiltonians, compared to what is extracted from “experiments”.

PWZ and Gandolfi PRC 94 (2016) 041302(R)

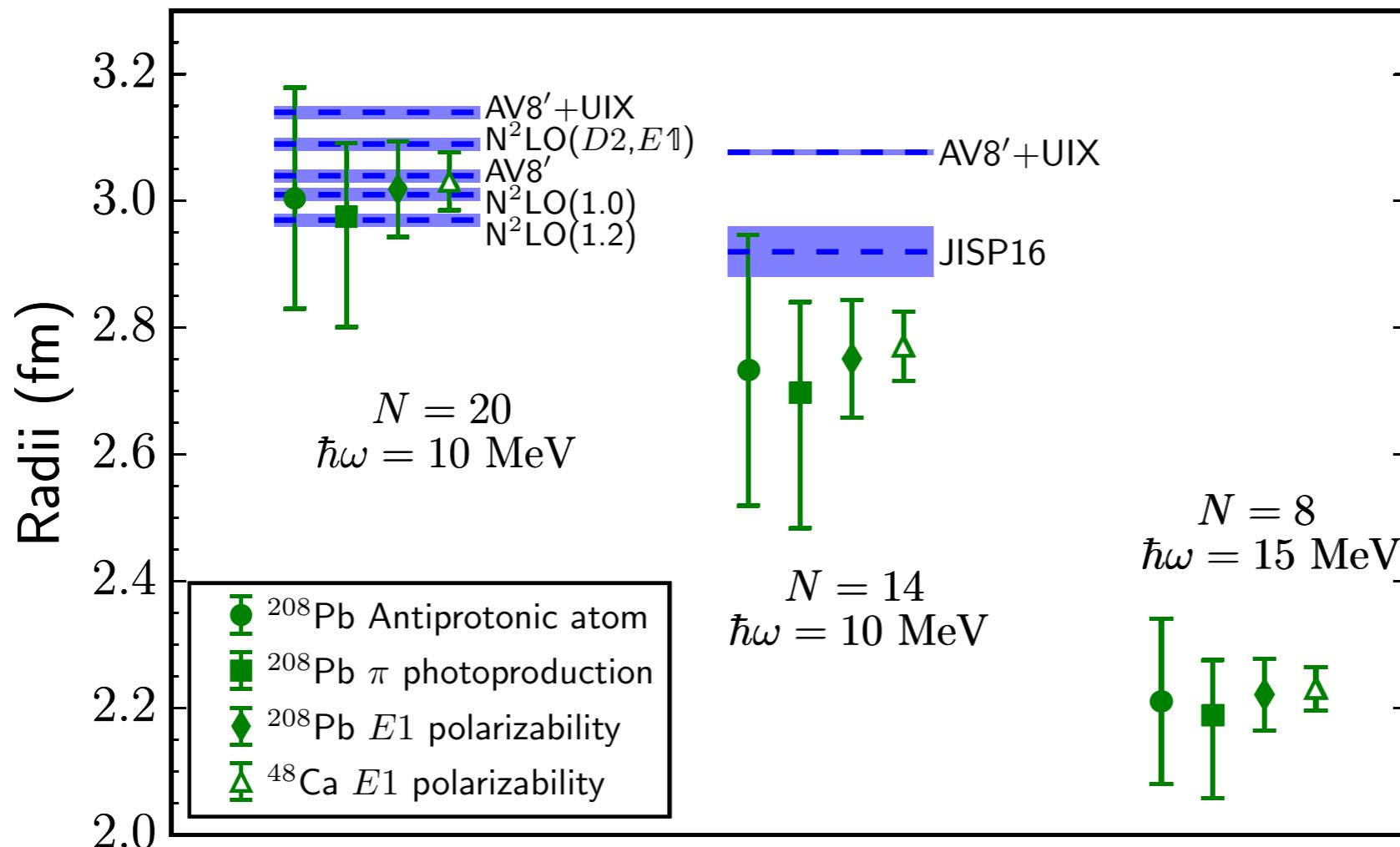


With the development of the high-accuracy measurements of neutron skin thickness, the radii obtained for neutron drops will provide a useful constraint for realistic three neutron forces.

# Constraining three-body forces

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PWZ and Gandolfi PRC 94 (2016) 041302(R)

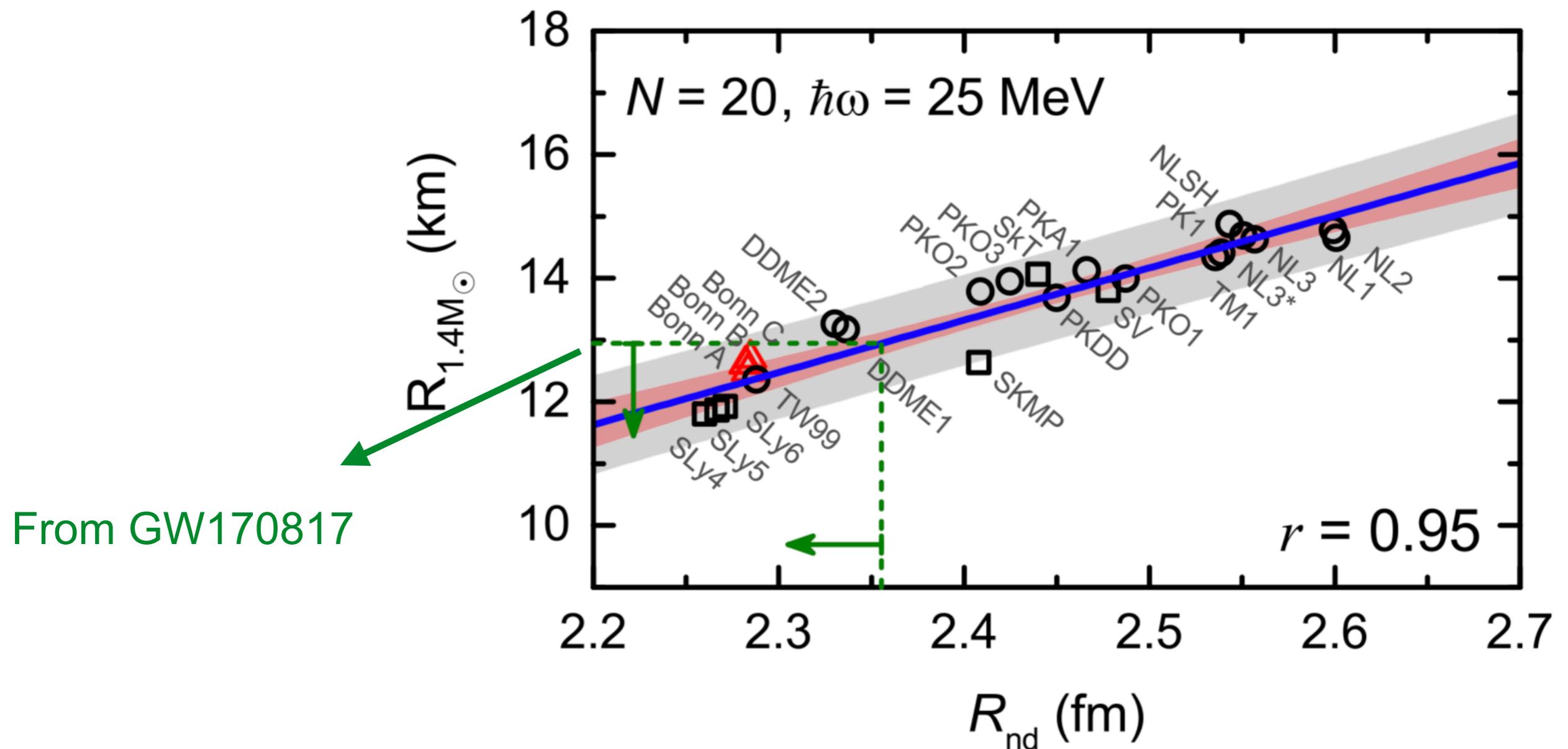


With the development of the high-accuracy measurements of neutron skin thickness, the radii obtained for neutron drops will provide a useful constraint for realistic three neutron forces.

# Neutron drop as a “toy” neutron star

Going to high densities ...

Tong, PWZ, Meng, PRC 101, 035802 (2020)



The strong correlation can be used to constrain the radius of the neutron drop

# Outline

- Nuclear landscape  
(light and heavy nuclei; neutron-deficient and neutron-rich nuclei)
- Nuclear spectroscopy  
(ground and excited states; low- and high- angular momentum states)
- Nuclear dynamics  
(nuclear structure and reaction)
- Nuclear interactions  
(towards a microscopic formalism based on bare nuclear interactions)
- Summary

# Summary

Covariant density functional theory is being improved and extended for a unified description of nuclei.

- Nuclear landscape
  - a systematic study with PC-PK1
  - triaxiality + dynamical correlations!
- Nuclear spectroscopy
  - a new tool for global studies of nuclear spectroscopy: CI-PDFT
  - merits of (C)DFT and Shell Model preserved
- Nuclear dynamics
  - time-dependent CDFT has been developed
  - pave the way for a unified description of nuclear structure and reaction
- Nuclear interaction
  - neutron drop, neutron star, and neutron skin are found to be strongly correlated
  - likely to have an enduring impact on the understanding of multi-neutron interactions

# Collaborations

## Beijing

Jie Meng

Jing Peng

Zhengxue Ren

Yakun Wang

Shuangquan Zhang

## Munich

Qibo Chen

Peter Ring

## Chongqing

Zhipan Li

## Kyoto

Naoyuki Itagaki

## Los Alamos

Stefano Gandolfi

...

Thank you for your attention!

... thank you for your attention

13	BSk19	BSk20	BSk21	BSk18	$\alpha$	1/12	1/12	1/12	0.3
$t_0$ [MeV fm <sup>3</sup> ]	-4115.21	-4056.04	-3961.39	-1837.96	$\beta$	1/3	1/6	1/2	1.0
$t_1$ [MeV fm <sup>5</sup> ]	403.072	438.219	396.131	428.880	$\gamma$	1/12	1/12	1/12	1.0
$t_2$ [MeV fm <sup>5</sup> ]	0	0	0	-3.23704	$f_n^+$	1.00	1.00	1.00	1.00
$t_3$ [MeV fm <sup>3+3\alpha</sup> ]	23670.4	23256.6	22588.2	11528.9	$f_n^-$	1.05	1.06	1.05	1.06
$t_4$ [MeV fm <sup>5+3\beta</sup> ]	-60.0	-100.000	-100.000	-400.000	$f_p^+$	1.10	1.09	1.07	1.04
$t_5$ [MeV fm <sup>5+3\gamma</sup> ]	-90.0	-120.000	-150.000	-400.000	$f_p^-$	1.17	1.16	1.13	1.09
$x_0$	0.398848	0.569613	0.885231	0.421290	$\epsilon_\Lambda$ [MeV]	16.0	16.0	16.0	16.0
$x_1$	-0.137960	-0.392047	0.0648452	-0.907175	$V_W$ [MeV]	-2.00	-2.10	-1.80	-2.10
$t_2 x_2$ [MeV fm <sup>5</sup> ]	-1055.55	-1147.64	-1390.38	-186.837	$\lambda$	250	280	280	340
$x_3$	0.375201	0.614276	1.03928	0.683926	$V'_W$ [MeV]	1.16	0.96	0.96	0.74
$x_4$	-6.0	-3.00000	2.00000	-2.00000	$A_0$	24	24	24	28
$x_5$	-13.0	-11.0000	-11.0000	-2.00000					
$W_0$ [MeV fm <sup>5</sup> ]	110.802	110.228	109.622	138.904					

3 density dependence

5 pairing properties

4 Wigner term

5 rotational correction

$b$ (MeV)	0.80
$c$	10
$d$ (MeV)	3.4
$l$	17
$\beta_2^0$	0.1

Gorieli et al, (2010)

13+3+5+4+5 = 30 parameters