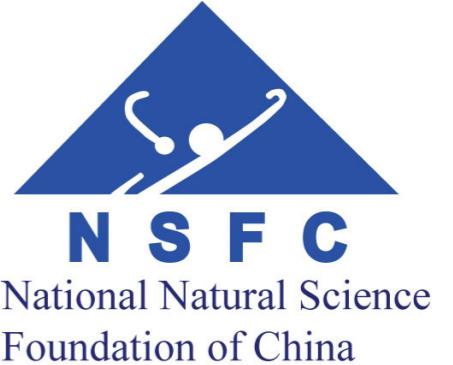




四川大學
SICHUAN UNIVERSITY

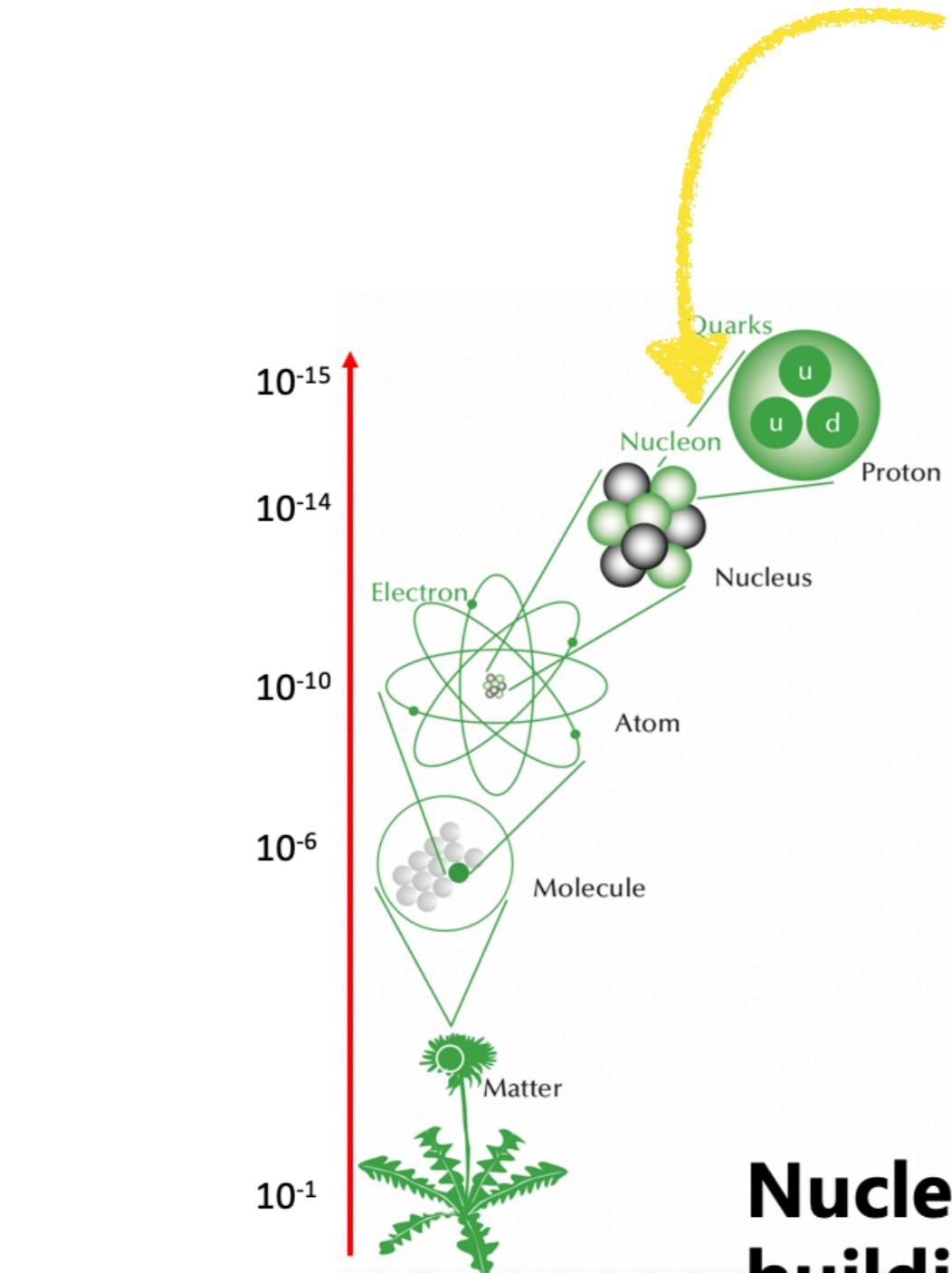


Chiral nuclear effective field theory

— Renormalization and power counting

龙炳蔚
四川大学

粒子物理与核物理结合，大有可为



IUPAC Periodic Table of the Elements

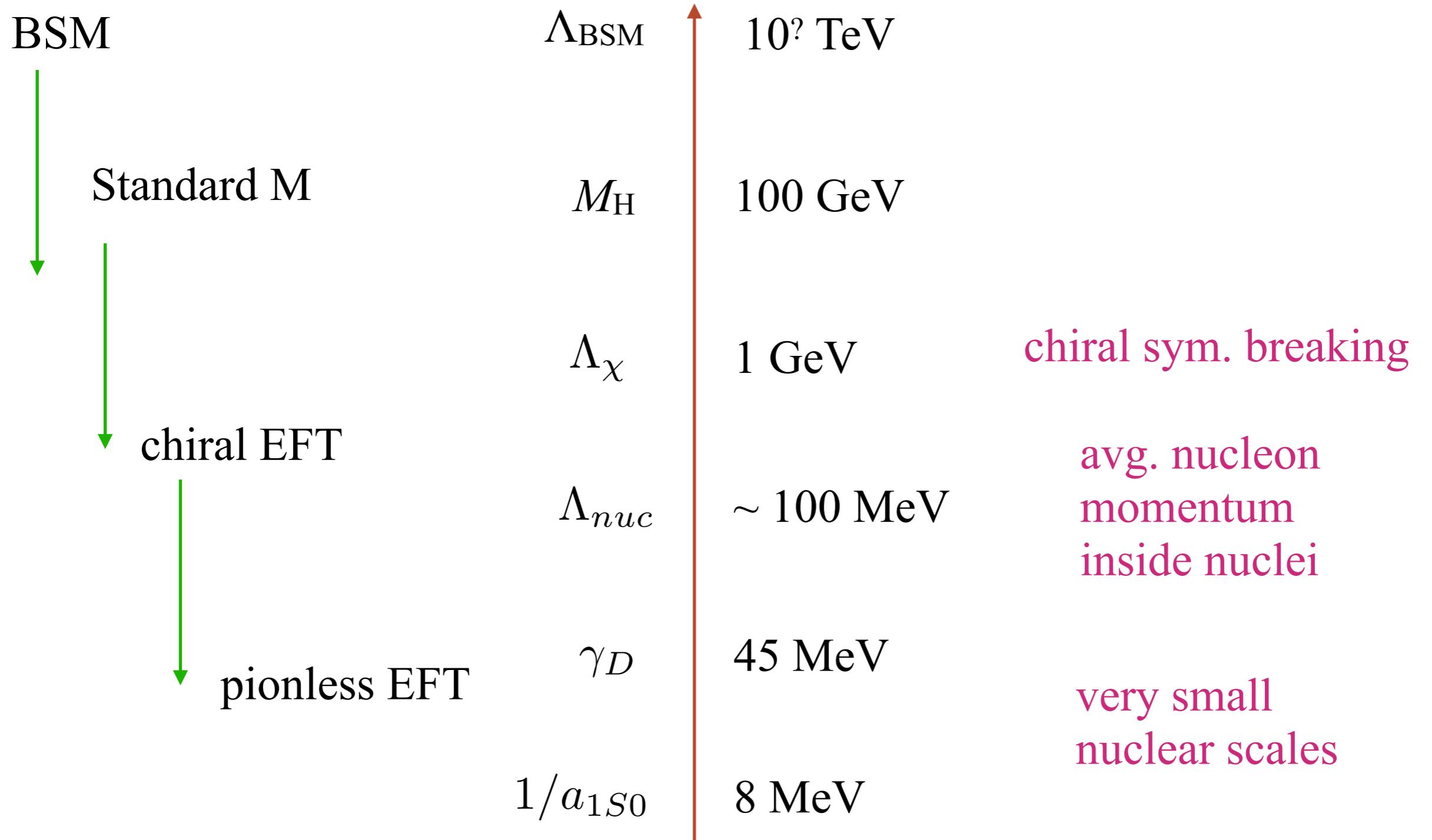
n	Key: atomic number Symbol name protonic mass standard atomic weight	13	14	15	16	17	18
4	22 Ti titanium 47.90 23 V vanadium 50.942 24 Cr chromium 51.986 25 Mn manganese 54.938 26 Fe iron 55.845(2) 27 Co cobalt 58.935 28 Ni nickel 58.693 29 Cu copper 63.546(2) 30 Zn zinc 65.402 31 Ga gallium 69.723 32 Ge germanium 72.630(2) 33 As arsenic 74.922 34 Se selenium 78.971(2) 35 Br bromine 79.909(2) 36 Kr krypton 83.798(2)	5 B boron 10.808 6 C carbon 12.000 7 N nitrogen 14.000 8 O oxygen 16.000 9 F fluorine 18.998 10 Ne neon 20.190	10 He helium 4.003				
5	40 Zr zirconium 91.224(2) 41 Nb niobium 92.908 42 Mo molybdenum 95.965 43 Tc technetium 97.070(2) 44 Ru ruthenium 101.070(2) 45 Rh rhodium 102.91 46 Pd palladium 106.42 47 Ag silver 107.87 48 Cd cadmium 112.41 49 In indium 114.82 50 Sn tin 118.71 51 Sb antimony 121.76 52 Te tellurium 127.600 53 I iodine 126.90 54 Xe xenon 131.29	11 Na sodium 22.989 28.08 30 Si silicon 28.084 28.08 31 P phosphorus 30.974 30.974 32 S sulfur 32.066 32.066 33 Cl chlorine 35.457 35.457 34 Ar argon 39.902 39.902					
6	72 Hf hafnium 178.492(2) 73 Ta tantalum 180.95 74 W tungsten 183.84 75 Re rhenium 186.21 76 Os osmium 190.230(2) 77 Ir iridium 192.22 78 Pt platinum 195.08 79 Au gold 196.97 80 Hg mercury 200.59 81 Tl thallium 204.6(2) 82 Pb lead 207.2 83 Bi bismuth 208.98 84 Po polonium 210.00 85 At astatine 212.00 86 Rn radon 222.00	12 Mg magnesium 24.320 24.320 13 Al aluminum 26.982 26.982 14 Si silicon 28.084 28.084 15 P phosphorus 30.974 30.974 16 S sulfur 32.066 32.066 17 Cl chlorine 35.457 35.457 18 Ar argon 39.902 39.902					
7	104 Rf rutherfordium 267.00 105 Db dubnium 269.00 106 Sg seaborgium 272.00 107 Bh berkelium 274.00 108 Hs hassium 277.00 109 Mt meitnerium 281.00 110 Ds darmstadtium 285.00 111 Rg roentgenium 289.00 112 Cn copernicium 294.00 113 Nh nilssonium 299.00 114 Fl fermium 304.00 115 Mc mendelevium 307.00 116 Lv livermorium 313.00 117 Ts tsenium 315.00 118 Og oganesson 319.00	19 K potassium 39.098 39.098 20 Ca calcium 40.078 40.078 21 Sc scandium 45.067 45.067 22 Ti titanium 47.90 47.90 23 V vanadium 50.942 50.942 24 Cr chromium 51.986 51.986 25 Mn manganese 54.938 54.938 26 Fe iron 55.845(2) 55.845(2) 27 Co cobalt 58.935 58.935 28 Ni nickel 58.693 58.693 29 Cu copper 63.546(2) 63.546(2) 30 Zn zinc 65.402 65.402 31 Ga gallium 69.723 69.723 32 Ge germanium 72.630(2) 72.630(2) 33 As arsenic 74.922 74.922 34 Se selenium 78.971(2) 78.971(2) 35 Br bromine 79.909(2) 79.909(2) 36 Kr krypton 83.798(2) 83.798(2) 37 Rb rubidium 84.751 38 Sr strontium 87.621 39 Y yttrium 88.905 39.902 40 Zr zirconium 91.224(2) 41 Nb niobium 92.908 42 Mo molybdenum 95.965 43 Tc technetium 97.070(2) 44 Ru ruthenium 101.070(2) 45 Rh rhodium 102.91 46 Pd palladium 106.42 47 Ag silver 107.87 48 Cd cadmium 112.41 49 In indium 114.82 50 Sn tin 118.71 51 Sb antimony 121.76 52 Te tellurium 127.600 53 I iodine 126.90 54 Xe xenon 131.29 55 Cs cesium 132.910(2) 56 Ba barium 137.340 57 La lanthanum 138.91 58 Ce cerium 140.12 59 Pr praseodymium 140.91 60 Nd neodymium 144.24 61 Pm promethium 146.942(2) 62 Sm samarium 151.96 63 Eu europium 152.20(3) 64 Gd gadolinium 157.98 65 Tb terbium 158.93 66 Dy dysprosium 162.50 67 Ho holmium 164.93 68 Er erbium 167.26 69 Tm thulium 168.90 70 Yb ytterbium 170.95 71 Lu lutetium 174.97					

For notes and updates to this table, see www.iupac.org. This version is dated 1 December 2018.
Copyright © 2018 IUPAC, the International Union of Pure and Applied Chemistry.

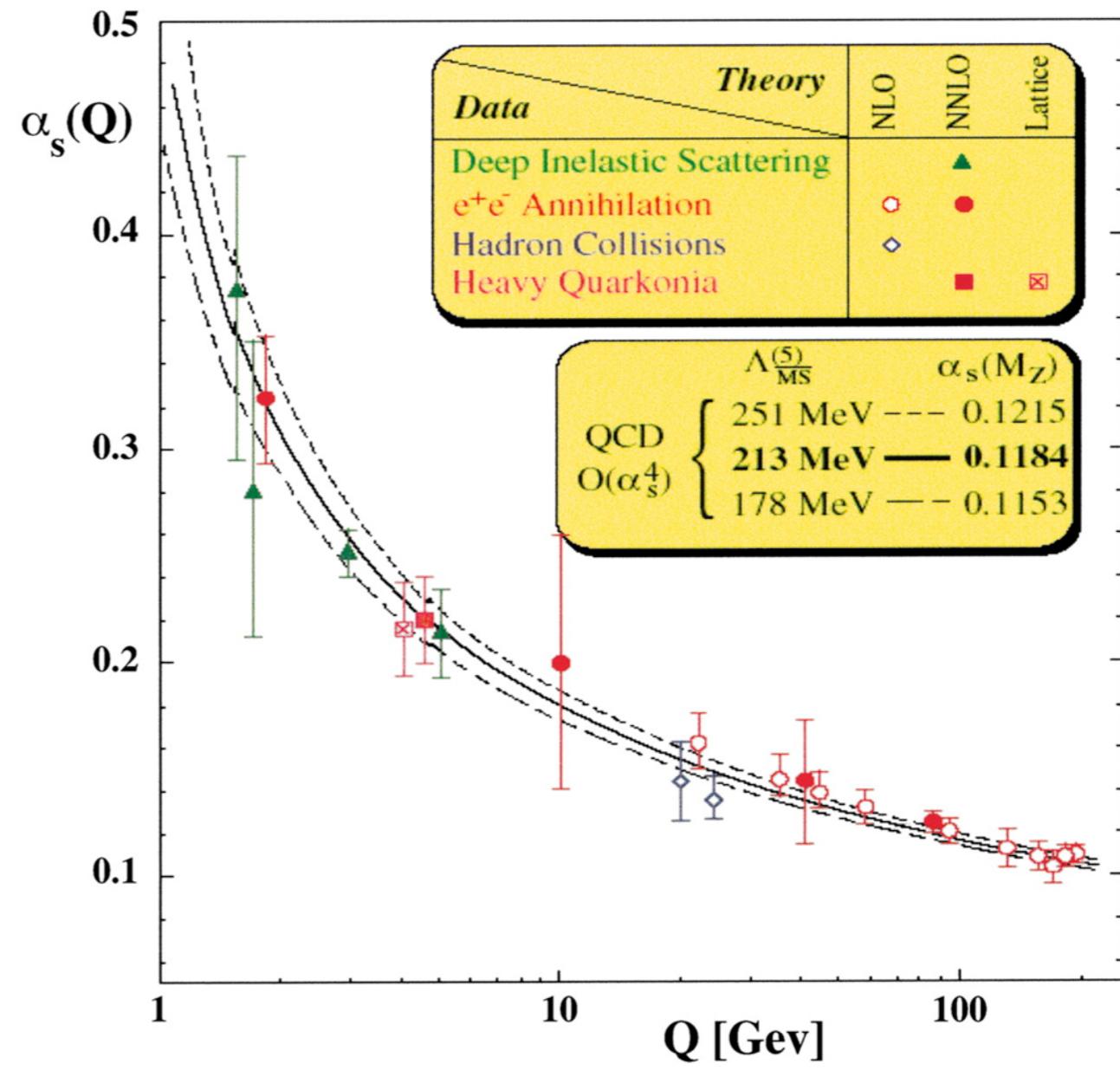


Nucleons are the essential building blocks of Matter!

BSM nuclear physics



Perturbative QCD N/A



Nucl-ex/th is here!

What is EFT - a classical example

multipole expansions in electrodynamics

Charge distribution



Separation of scales

Symmetries of E&M $\longrightarrow R \gg r_0$ $V = \frac{q}{R} + \frac{d_i R_i}{R^3} + \frac{Q_{ij} R_i R_j}{R^5} + \dots$
(Without knowing $\rho(R)$)

Low energy
constants
(LECs)

$$\left. \begin{aligned} |d_i| &\sim qr_0 \\ |Q_{ij}| &\sim qr_0^2 \end{aligned} \right\}$$

Power counting
Based on dimensional
analysis

Systematic low-energy
approximation

$$V = \frac{q}{R} \left[1 + \mathcal{O}\left(\frac{r_0}{R}\right) + \mathcal{O}\left(\frac{r_0^2}{R^2}\right) + \dots \right]$$

Much more nontrivial in quantum systems!

Chiral EFT

- Low-energy approximation of QCD, expansion in Q/M_{hi}
 Q : small external momenta
 M_{hi} : EFT breakdown scale (~ 500 MeV?)

$$\mathcal{M} = \sum_n \left(\frac{Q}{M_{hi}} \right)^n \mathcal{F}_n \left(\frac{Q}{M_{lo}} \right)$$

Q : generic external momenta,
 $M_{hi} = \Lambda_{SB}, m_\rho, \dots \sim 1\text{GeV}$
 $M_{lo} = m_\pi, f_\pi \sim 100\text{MeV}$

Systematic approximation
→ able to estimate theoretical errors

Chiral EFT

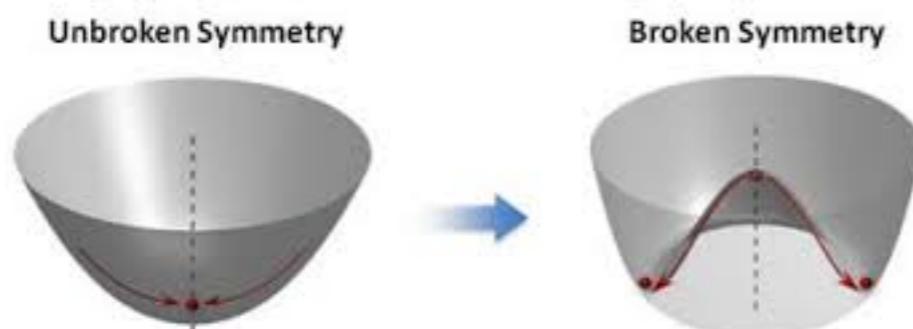
- Includes all symmetries of QCD, especially (approximate) chiral symmetry and its spontaneous breaking

$$\mathcal{L}_{\text{QCD}} = \sum_{f=u,d,s} \bar{q}_f (iD - m_f) q_f - \frac{1}{4} G_{a\mu\nu} G_a^{\mu\nu}$$

$$q_L \equiv \begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix} \mapsto \left(\text{SU}(3)_L \right) \begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix} \quad q_R \equiv \begin{pmatrix} u_R \\ d_R \\ s_R \end{pmatrix} \mapsto \left(\text{SU}(3)_R \right) \begin{pmatrix} u_R \\ d_R \\ s_R \end{pmatrix}$$

- Only two flavors used in our work

Lagrangian invariant when $m_f \rightarrow 0$, but broken by QCD ground state

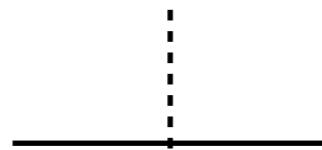


→ **chiral symmetry nonlinearly realized by hadronic Dofs**

CCWZ; Weinberg; ...

Chiral EFT

- The most general Lagrangian has infinitely many parameters



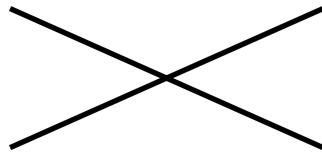
$$-\frac{g_A}{2f_\pi} N^\dagger \tau_a \vec{\sigma} \cdot \vec{\nabla} \pi_a N$$



$$-\frac{1}{4f_\pi^2} N^\dagger \epsilon_{abc} \tau_a \pi_b \dot{\pi}_c N$$

...

- Short-range interactions: large numbers of $4N$ operators



$$\begin{aligned}
 & -C^{(s)}(N^T P_i^{(s)} N)^\dagger (N^T P_i^{(s)} N) - C_2^{(s)} \left[(N^T P_i^{(s)} N)^\dagger (N^T P_i^{(s')} \overleftrightarrow{\nabla}^2 N) + h.c. \right] \quad \dots \\
 & -C^{(ss')}(N^T P_i^{(s)} N)^\dagger (N^T P_i^{(s')} N) \quad \dots \quad P_i^{(1S_0)} = \frac{(i\sigma_2)(i\tau_2\tau_i)}{2\sqrt{2}} \\
 & \mathbf{s, s'} = {}^1\mathbf{S}_0, {}^3\mathbf{S}_1, {}^3\mathbf{P}_0, \dots \quad P_i^{(3S_1)} = \frac{(i\sigma_2\sigma_i)(i\tau_2)}{2\sqrt{2}} \\
 & \qquad \qquad \qquad P_i^{(3D_1)} = \left(\overleftrightarrow{\nabla}_i \overleftrightarrow{\nabla}_j - \frac{\delta_{ij}}{n} \overleftrightarrow{\nabla}^2 \right) P_j^{(3S_1)}
 \end{aligned}$$

**NN contact
pot. in mom.
space**

$$V_{1S0} = c_0^{1S0} + c_2^{1S0}(p^2 + p'^2) + \dots$$

$$V_{3P0} = c_0^{3P0} pp' + \dots$$

Naive dim. analysis (NDA):

$$c_2^{1S0} \sim c_0^{3P0} \sim \frac{c_0^{1S0}}{M_{hi}^2}$$

Power counting: long-range physics

Typical size of external momenta: $Q \sim m_\pi$

$$\text{Diagram: } \overline{\text{---}} \text{---} \mid \text{---} \sim \frac{1}{f_\pi^2} \frac{Q^2}{m_\pi^2 + Q^2} \sim \frac{1}{f_\pi^2}$$
$$\text{Diagram: } \overline{\text{---}} \text{---} \mid \text{---} \text{---} \mid \text{---} \sim \frac{1}{f_\pi^2} \frac{m_N}{4\pi f_\pi} \frac{Q}{a_l f_\pi}$$

NN reducible

- Focus on loop momenta \sim external momenta Q
- Pion line or photon line $\sim 1/Q^2$, nucleon line in **irreducible** diagrams $\sim 1/Q$
- Nucleon line in **reducible** diagrams $\sim m_N/Q^2$
 - ⇒ Explain why we solve the Schrodinger eqn
 - ⇒ Explain why nuclei bound
- Strength of OPE $\sim a_l f_\pi$ (numerical factor $a_l \sim 1$ for small l , $a_l \gg 1$ for large l by centrifugal suppression)

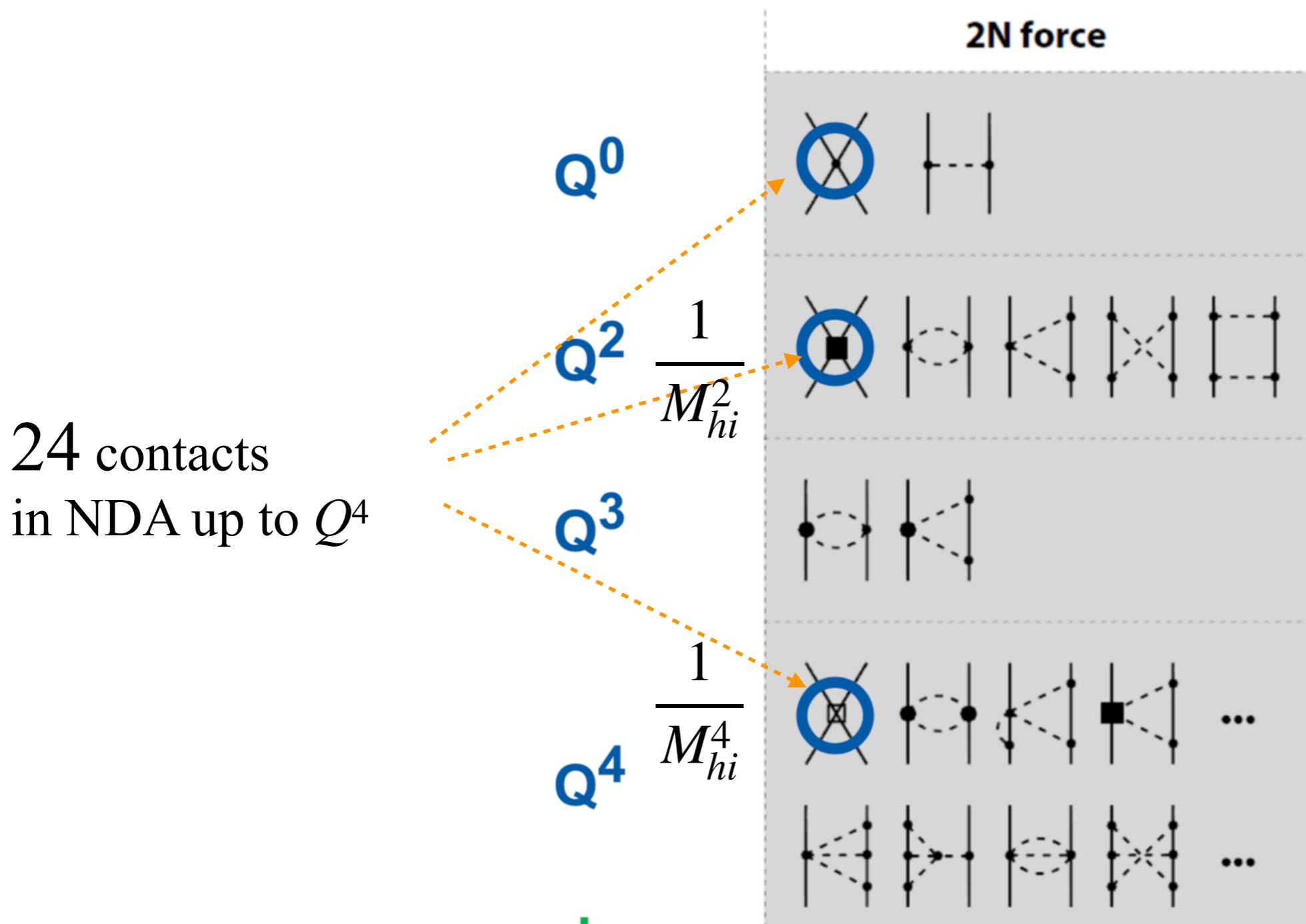
Power counting: short-range physics

The diagram shows a black dot representing a contact interaction vertex, with four external lines meeting at it. To its right is the text "Renormalizing" above a horizontal orange double-headed arrow. To the right of the arrow is a box diagram consisting of two horizontal lines and two vertical lines connecting them. To the right of the box is a renormalization relation:

$$\boxed{\quad} \sim \frac{1}{f_\pi^2} \frac{m_N}{4\pi f_\pi} \frac{Q}{a_l f_\pi}$$

- Strength of OPE $a_l f_\pi$ may have impact on contacts through renormalization
- Coexistence of $a_l f_\pi$ and M_{hi} makes NDA no longer reliable
- Operators gaining large anomalous dimension through nuclear dynamics → “irrelevant” operators become relevant

Need to re-examine contact operators

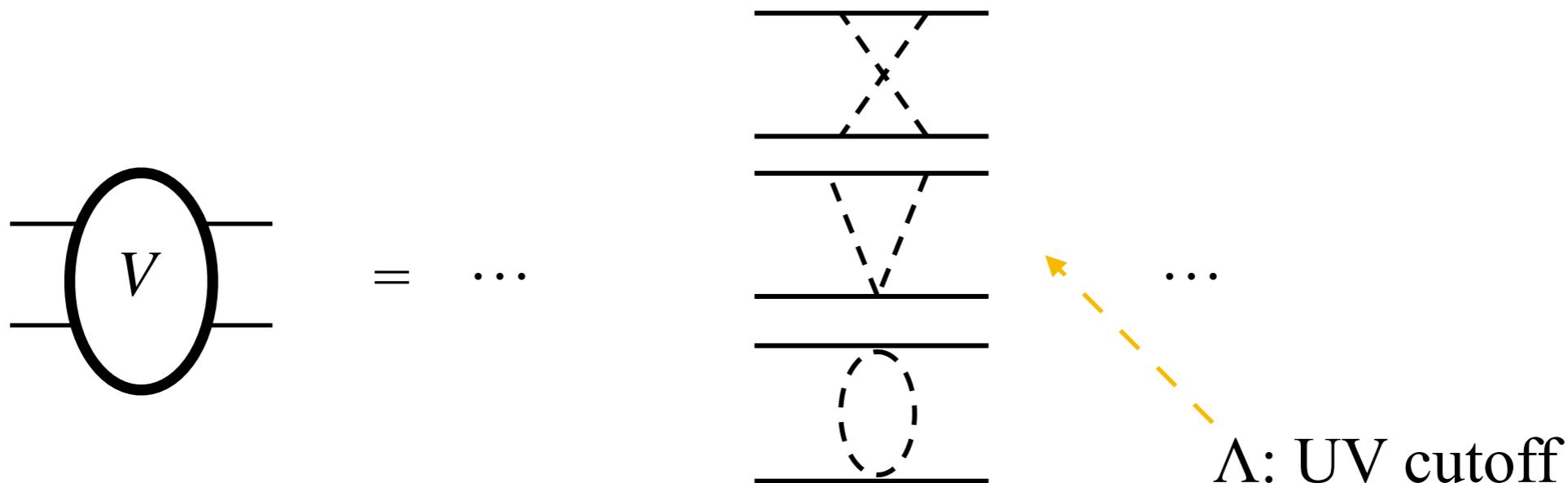


Counting short-range operators

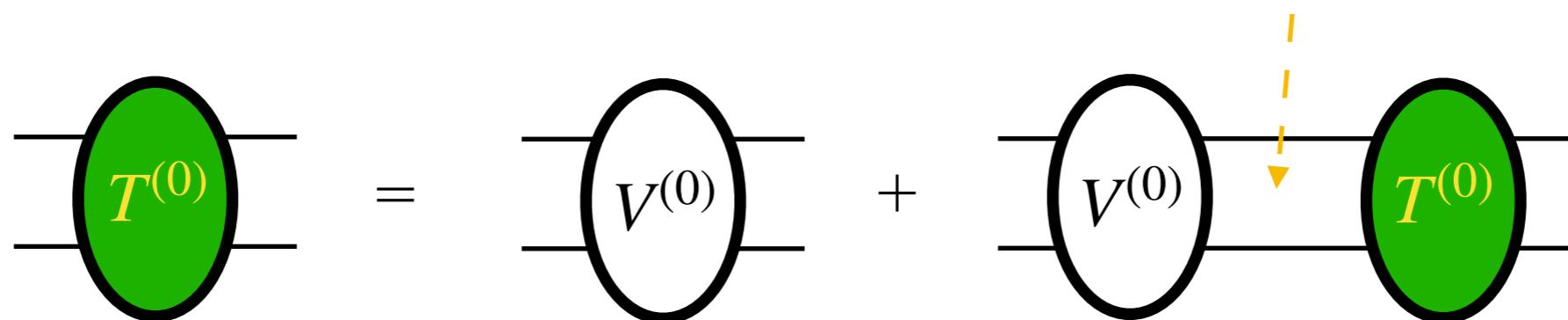
- At any given order in a power counting scheme, there must be enough operators to satisfy renormalization group invariance
 - ⇒ explicit checking UV cutoff independence
 - ⇒ model independence
- In terms of Wilson's RG
 1. Assume O be irrelevant (statement of PC)
 2. Run RGE
 3. Will O stay irrelevant in EFT?

What cutoff?

- Potentials: two-nucleon irreducible diagrams



- Lippmann-Schwinger eqn (equivalent to Schrödinger eqn)

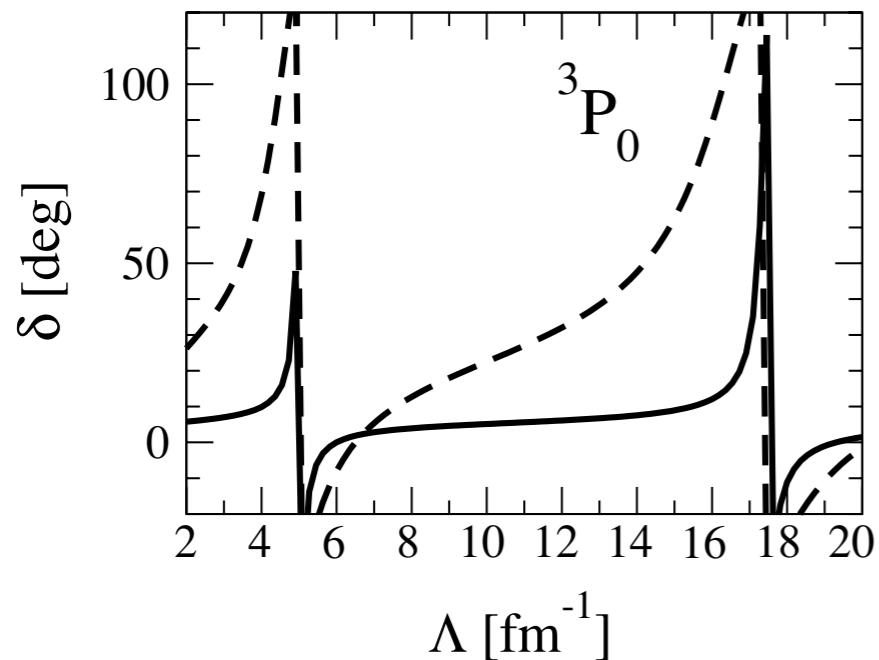


Renormalizing singular attraction

Nogga, Timmerman & van Kolck (2005)

$$C_{3P0} \vec{p} \cdot \vec{p}' \sim \frac{Q^2}{m_{hi}^2} \quad C_{3P0} \vec{p} \cdot \vec{p}' \sim \frac{Q^2}{m_{lo}^2}$$

Phase shifts vs. Λ



Solid: $T_{\text{lab}} = 10$ MeV, dashed: 50 MeV

$O(Q^2)$

WPC

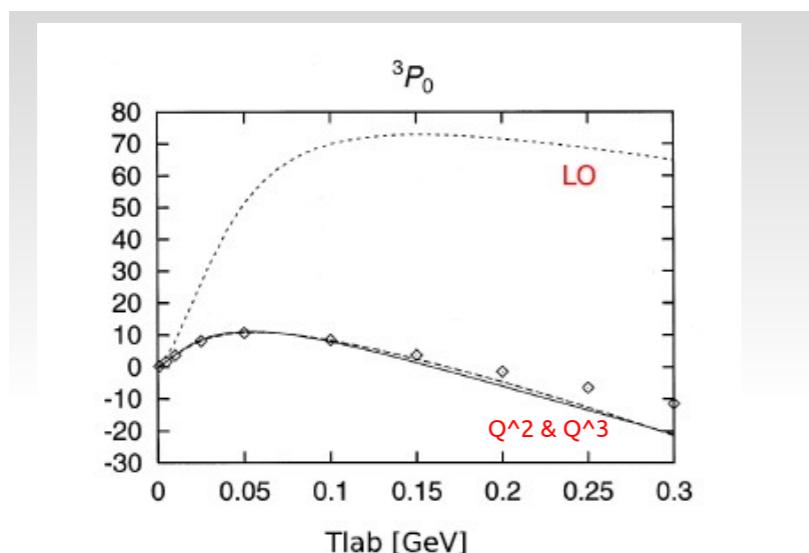
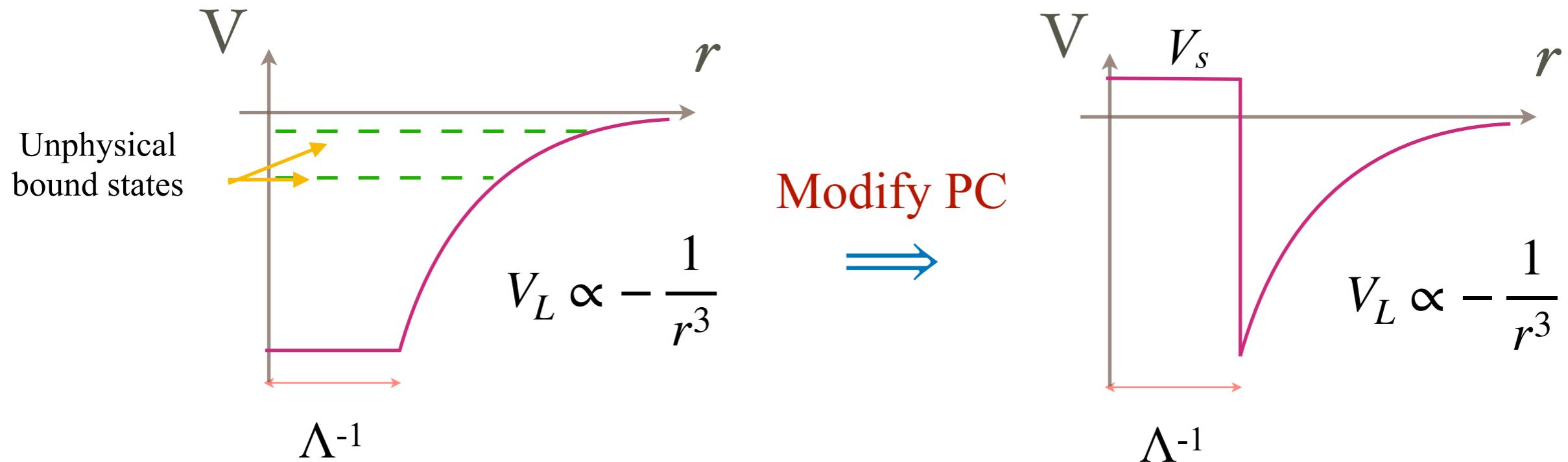
$O(1)$

RG inv. counting

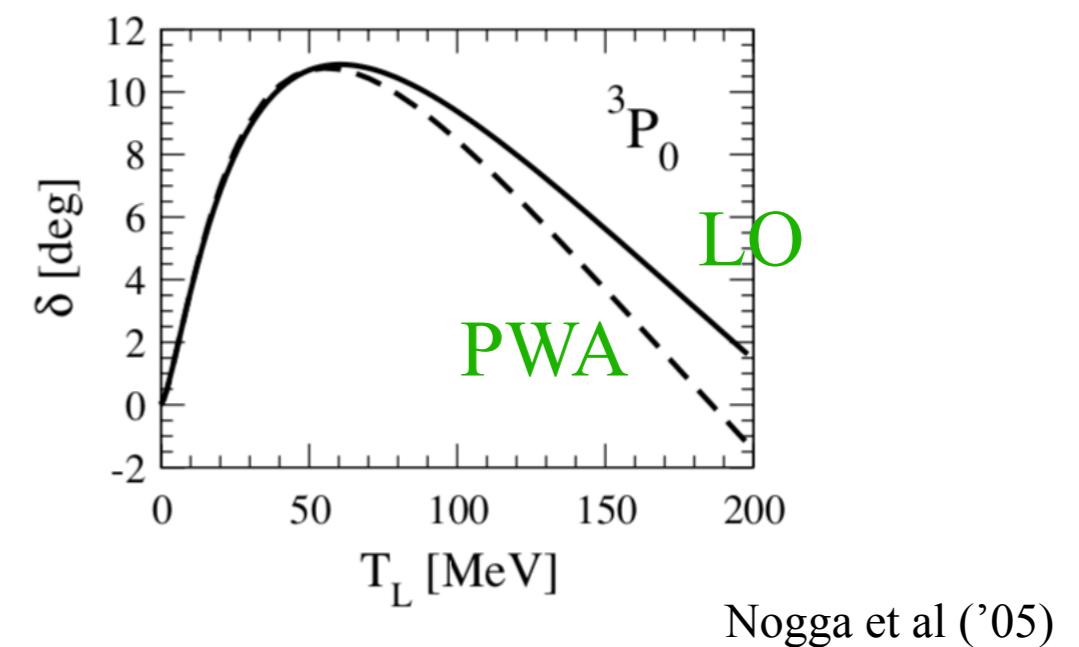
- Contacts needed at LO in attractive triplet channels: 3P2 - 3F2, 3D2, 3D3 ...

Renormalizing singular attraction

Beane et al ('01)
 Pavon Valderrama & Ruiz Arriola ('05 ~ '07)
 Nogga et al ('05)



Epelbaum et al, NPA 671, 295



Nogga et al ('05)

Modified power counting for chiral nuclear forces

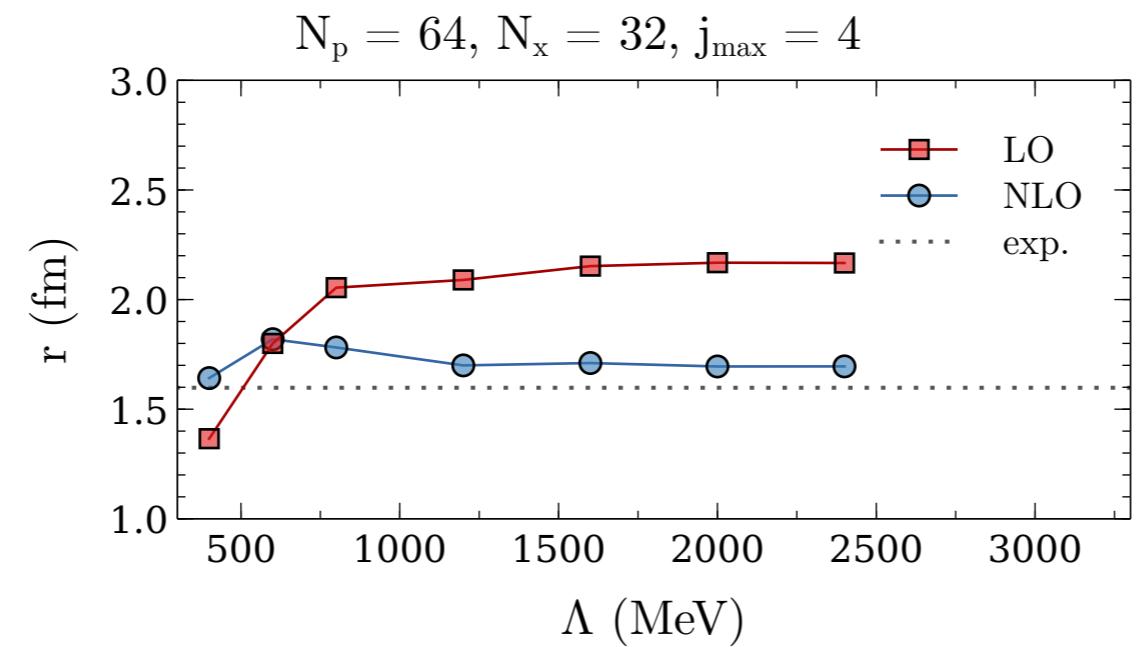
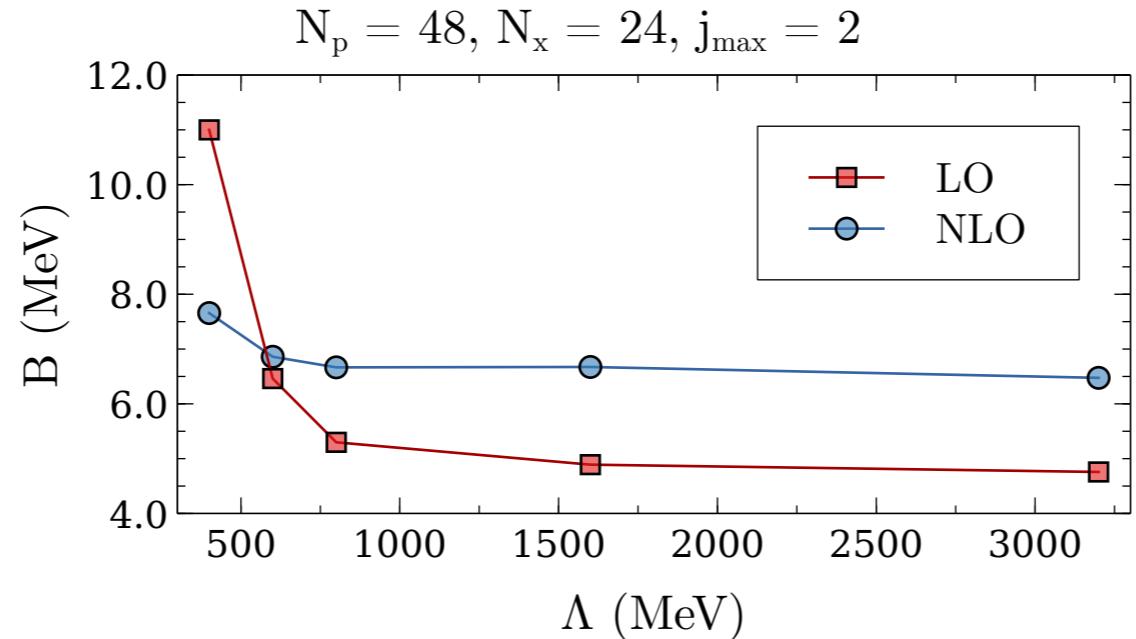
Nogga et al. '05

BwL & Yang '11, '12

Wu & BwL '19

- LO : (C + OPE) for 1S0, 3S1, **3P0** (perturbative OPE for most waves)
- NLO : **Q² C.T.** for 1S0; OPE for 1P1, 3P1, 3P2...
- N2LO: (**Q⁴ C.T.** + TPE) for 1S0; (Q² C.T. + TPE) for 3S1-3D1 and 3P0
- N3LO:

Triton BE and charge radius^{Preliminary}



$n\bar{d}$ elastic scattering

Preliminary

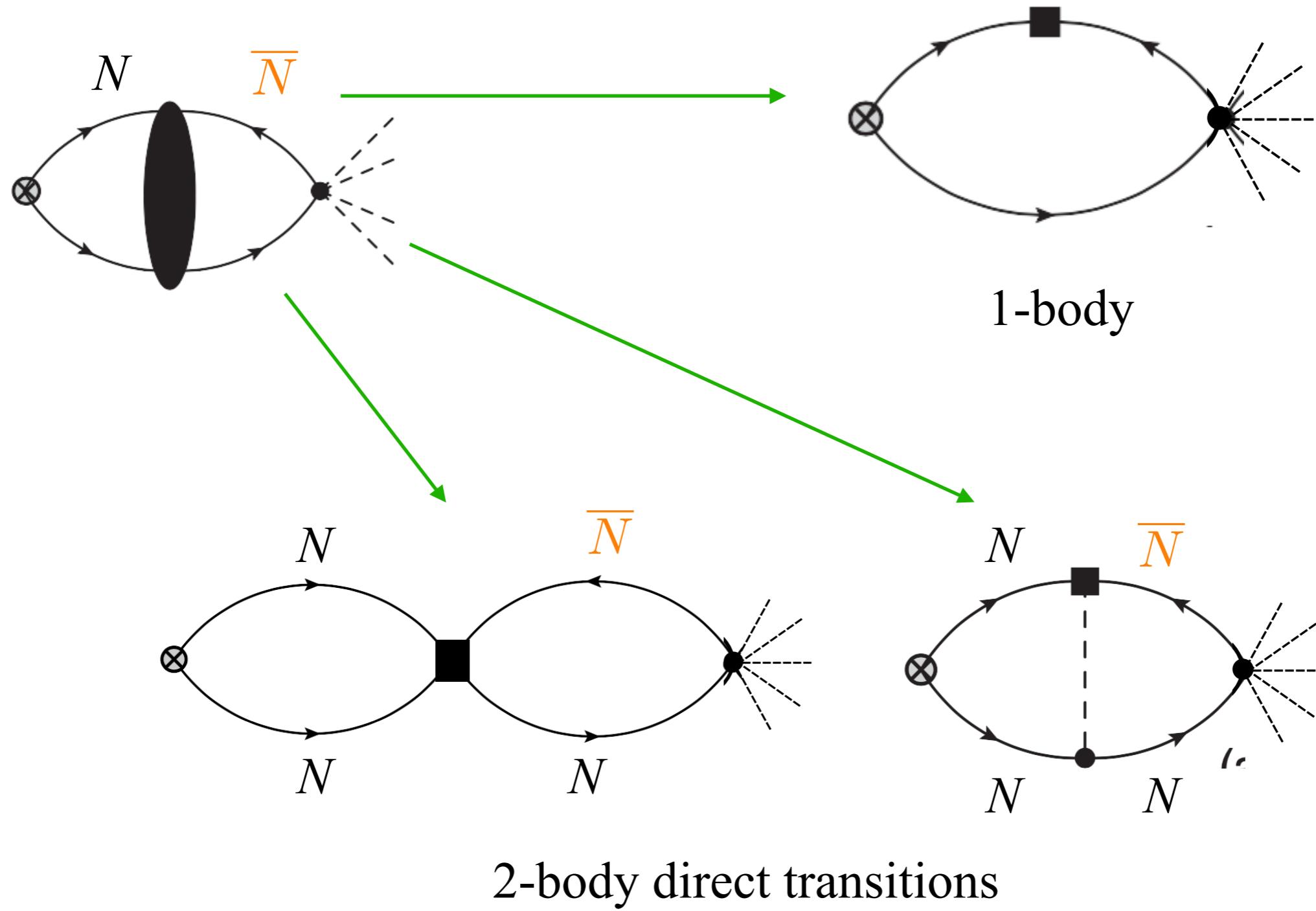
Phase shift in degrees ($j^\pi = 1/2^+$, S wave)

T _{lab}	1 MeV	2 MeV	3 MeV
LO	-20.2	-28.3	-37
NLO	-22.2	-33.1	-40.4
AV14	-17.8	-28	-34.9

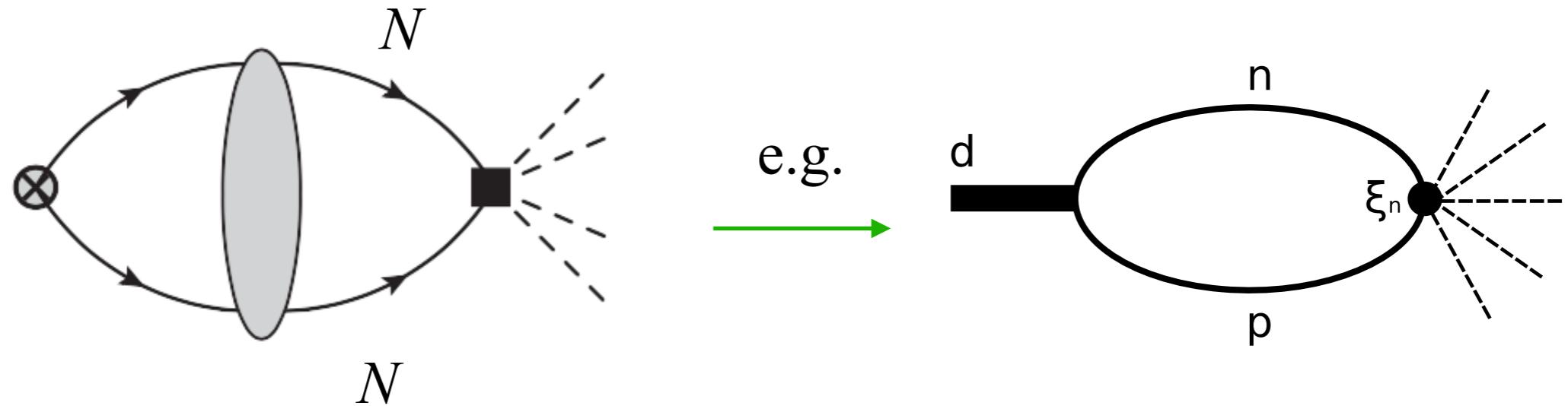
Deuteron decay by $NN \rightarrow NN\bar{N}$

Oosterhof, BWL, de Vries, Timmermans & van Kolck
PRL 122 (2019) 17, 172501

neutron-antineutron osc.



Direct NN annihilation



- To what extent can we disentangle these mechanisms?

Finally...

Oosterhof, BwL, de Vries, Timmermans & van Kolck
PRL 122 (2019) 17, 172501

$$R_d \equiv \Gamma_d^{-1} / \tau_{n\bar{n}}^2$$

$$R_d = - \left[\frac{m_N}{\kappa} \text{Im} a_{\bar{n}p} (1 + \underset{\text{NN range}}{0.40} + \underset{\text{pion}}{0.20} - \underset{\text{NN} \leftrightarrow N\bar{N} \text{ w/ unknown } B_0}{-0.13 \pm 0.4}) \right]^{-1}$$
$$= (1.1 \pm 0.3) \times 10^{22} \text{ s}^{-1}.$$

- Perturbative pion allows for analytic expression
- Loosely bound neutron helps sensitivity (nuclei with neutron halo?)
- B_0 gives largest uncertainty
- W/ nonperturbative pion EFT, unknown LECs may have smaller impact

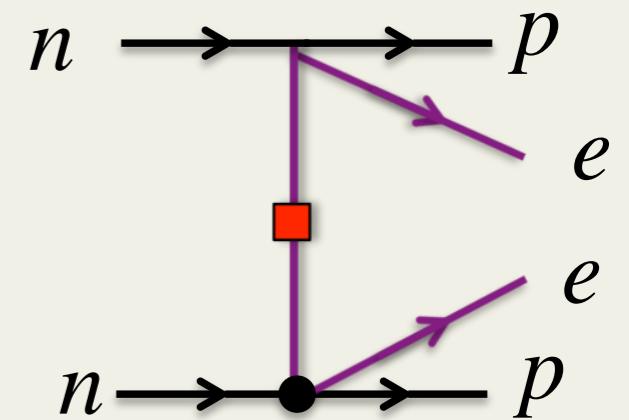
Chiral effective field theory

$\sim \text{GeV}$ $L = L_{QCD} + L_{Fermi} - m_{\beta\beta} \nu_L^T C \nu_L$ light quarks and gluons + electrons + neutrinos

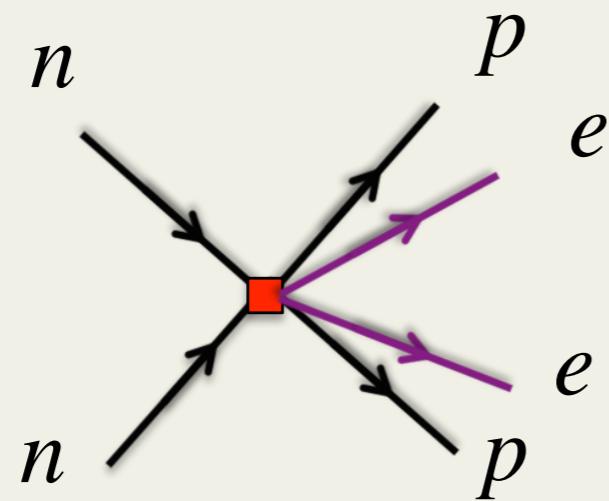
$\sim 100 \text{ MeV}$ Neutrinos are still degrees of freedom in the low-energy EFT

LO interaction : $\nu_L \longleftrightarrow \nu_L \sim m_{\beta\beta}$

Leads to long-range $n n \rightarrow p p + e e$ $\sim \frac{m_{\beta\beta}}{q^2}$
 $q \sim k_F \sim m_\pi$



'Hard' neutrino exchange ($E, |\vec{p}| > \Lambda_\chi$) \rightarrow short-range operators

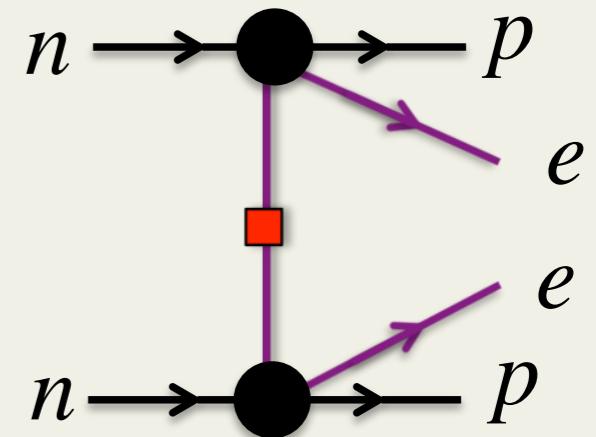


Expected at N²LO

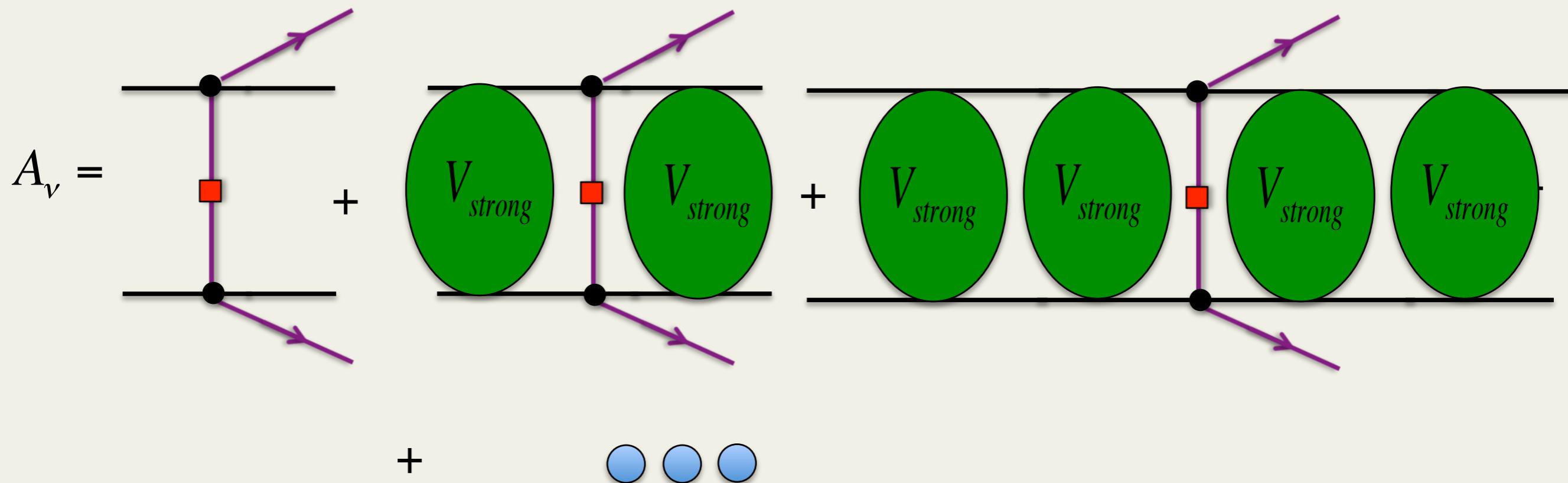
$$\sim \frac{m_{\beta\beta}}{\Lambda_\chi^2}$$

The neutrino amplitude

- At LO the ‘standard’ mechanism is long-range



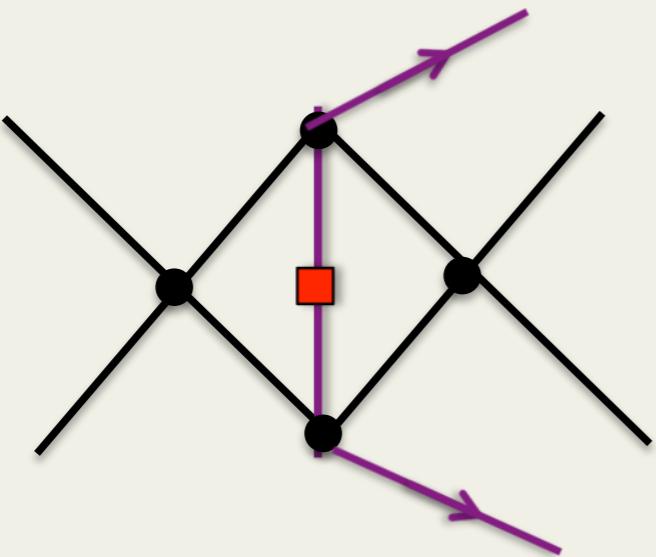
$$V_\nu = (2G_F^2 m_{\beta\beta}) \tau_1^+ \tau_2^+ \frac{1}{\vec{q}^2} \left[1 - g_A^2 \left(\vec{\sigma}_1 \cdot \vec{\sigma}_2 - \vec{\sigma}_1 \cdot \vec{q} \vec{\sigma}_2 \cdot \vec{q} \frac{2m_\pi^2 + \vec{q}^2}{(m_\pi^2 + \vec{q}^2)^2} \right) \right] \otimes \bar{e}_L e_L^c$$



Non-perturbative renormalization

Kaplan et al '98

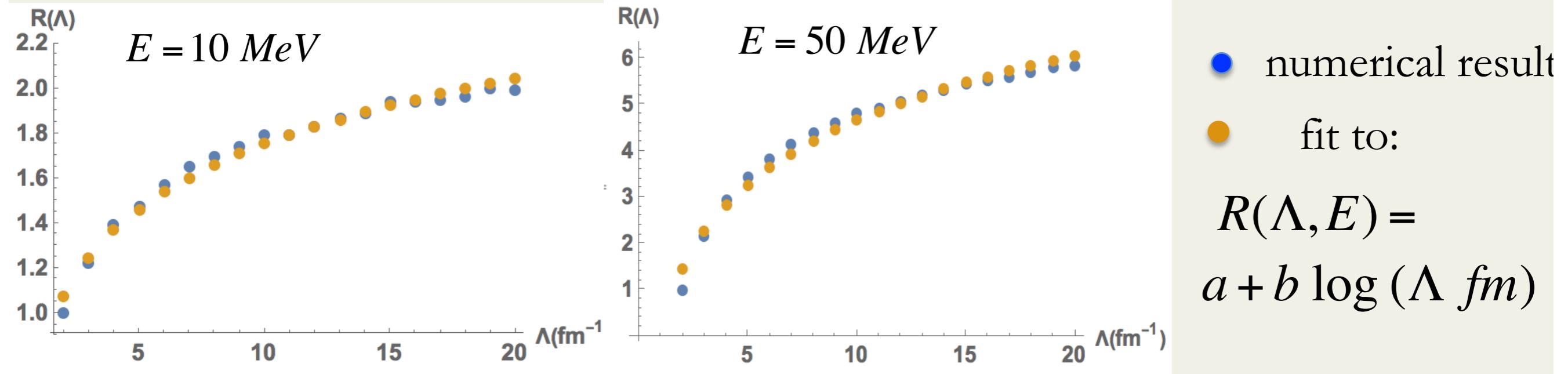
Can show analytically (dim-reg) two-loop diagram with two C_0 is UV divergent



$$\sim (1 + 2g_A^2) \left(\frac{m_N C_0}{4\pi} \right)^2 \left(\frac{1}{\epsilon} + \log \frac{\mu^2}{p^2} \right)$$

Confirmed numerically for $nn \rightarrow pp + ee$

$$R(\Lambda, E) = \frac{A_\nu(\Lambda, E)}{A_\nu(\Lambda = 2 \text{ fm}^{-1}, E)}$$



Summary

- Chiral EFT has infinite number of LECs
→ power counting is crucial
- NDA good for counting long-range physics, but unreliable for short-range interactions
- RG analysis (UV cutoff independence) can be used as guideline to test PC for short-range physics