Nuclear matrix elements for neutrinoless double beta decay

Dong-Liang Fang
Institute of Modern Physics, Chinese Academy of Sciences

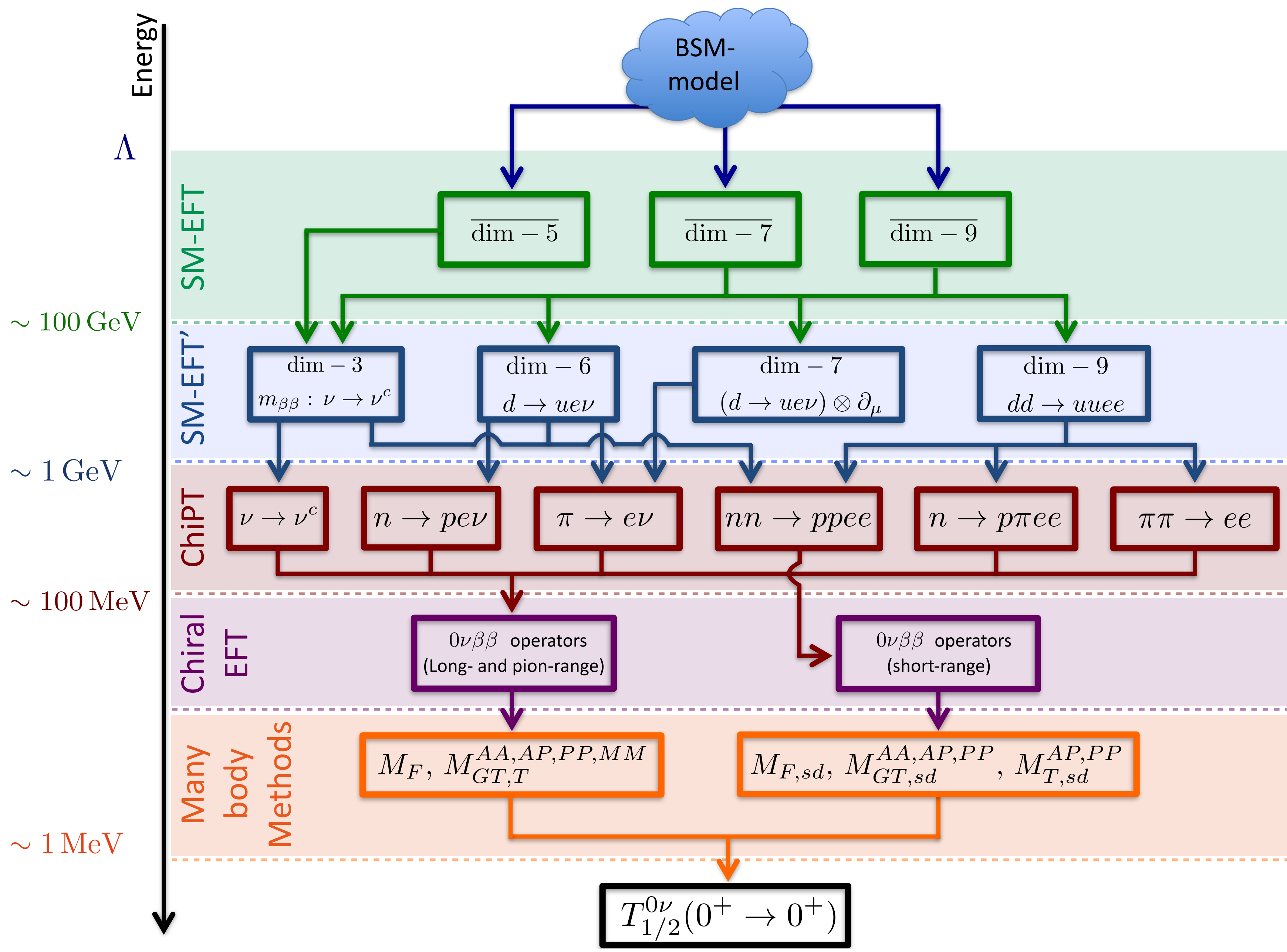
Outline

- Background
- Theoretical approaches and results
- Attempts of measuring the NME
- Conclusions and Outlook

Background

- Theoretical descriptions of 0νββ from new physics to nuclear physics

Cirigliano 18'



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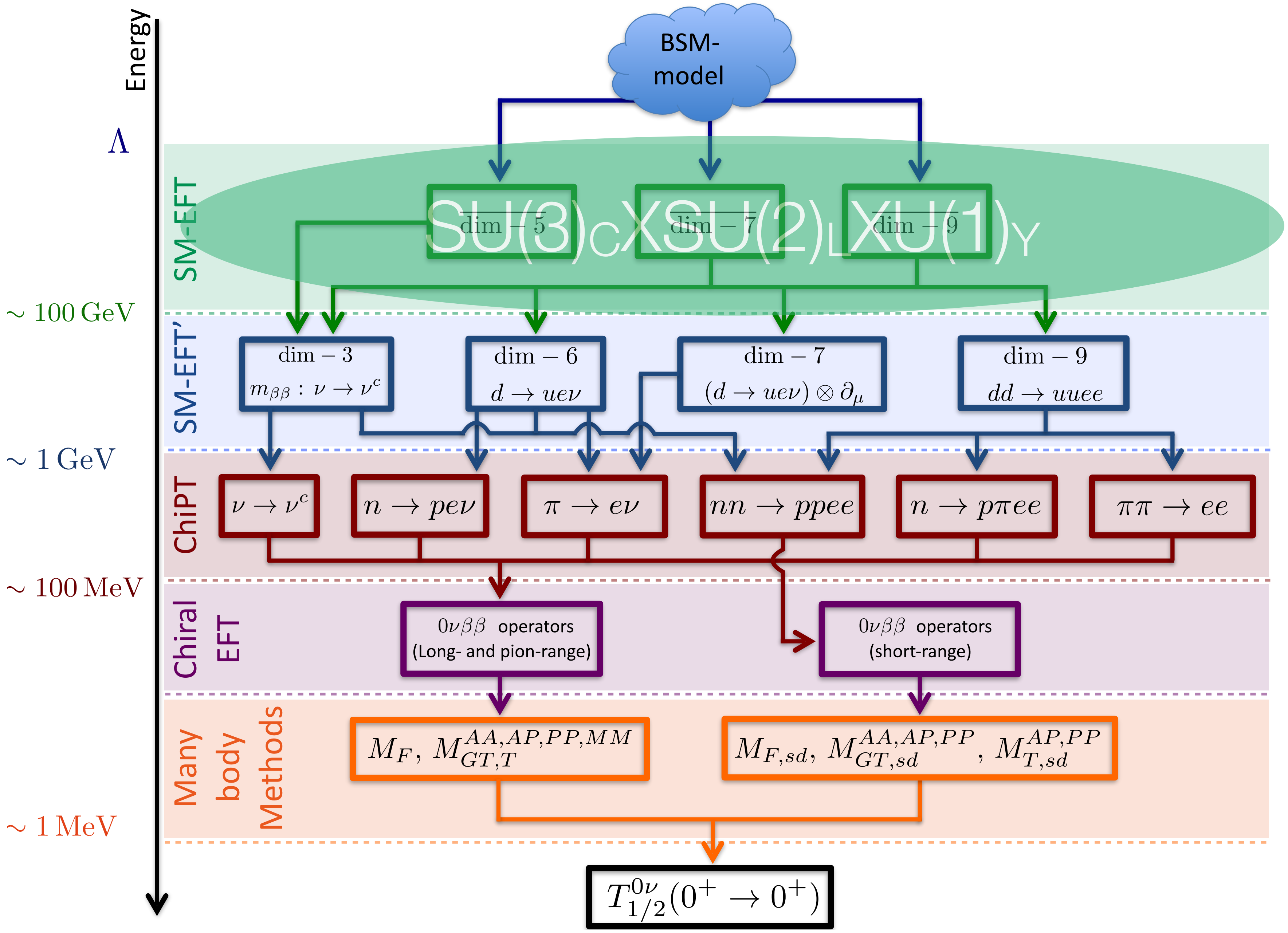

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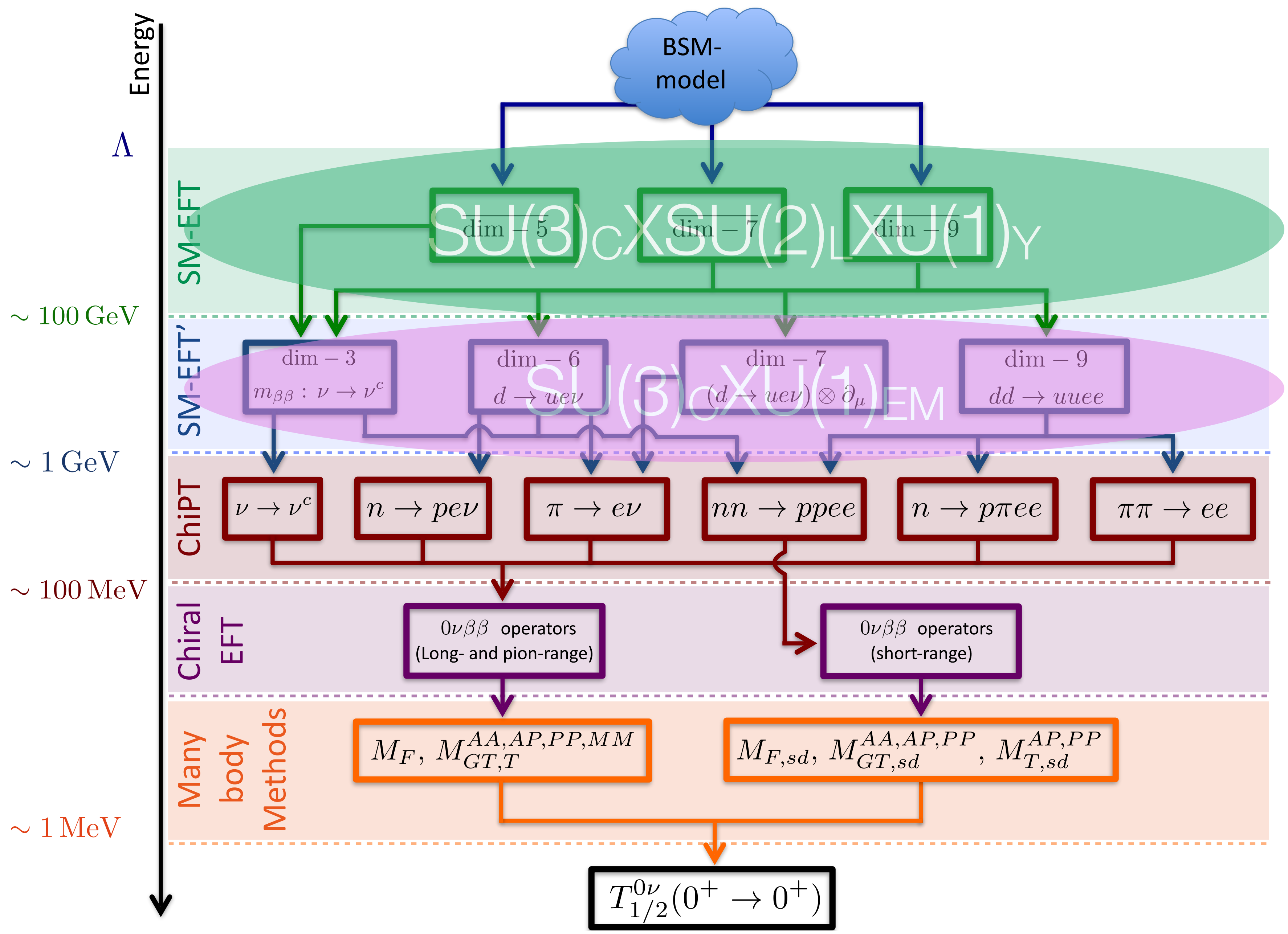

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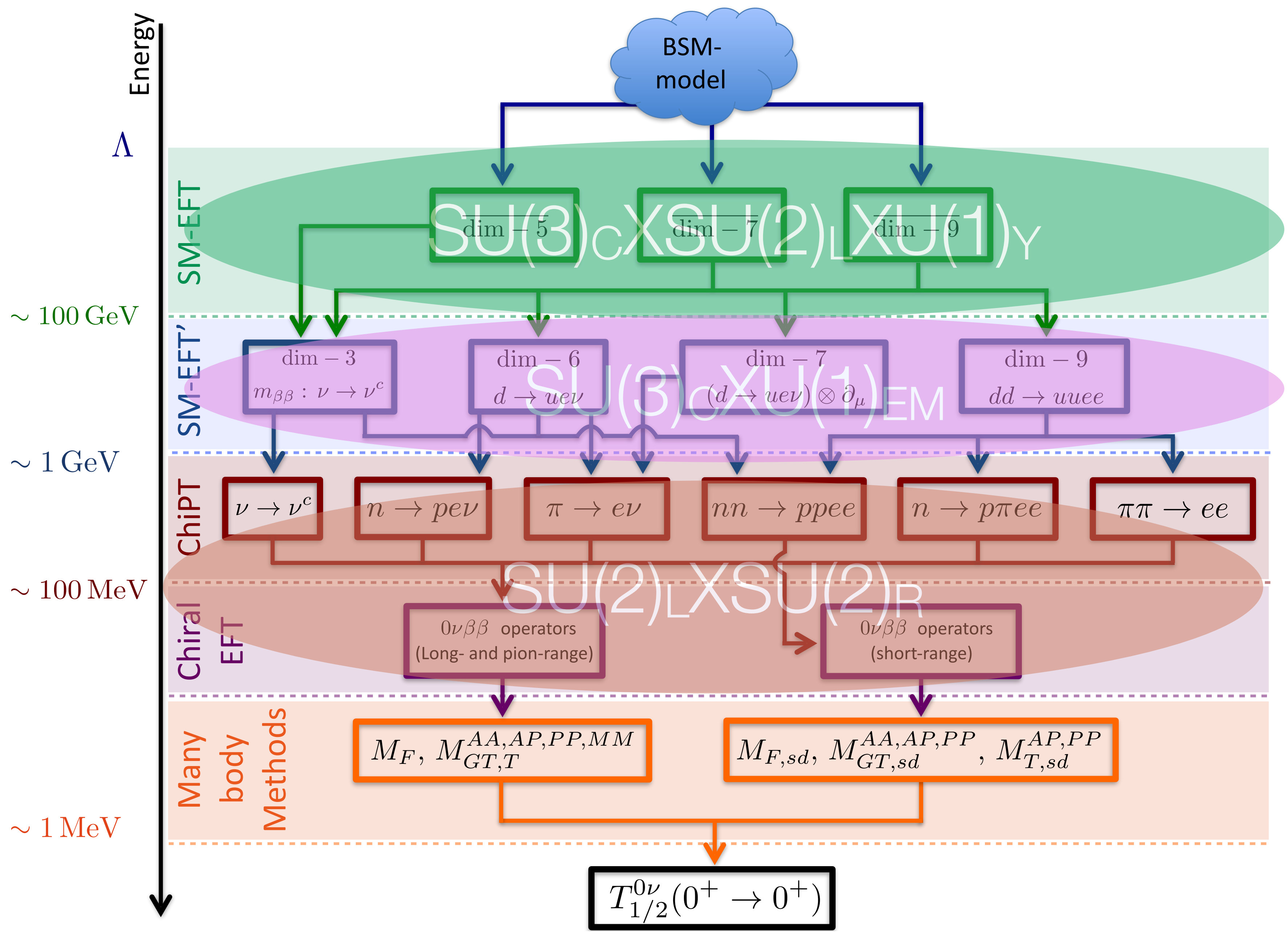

Background

- The master formula for decay width ($0^+ \rightarrow 0^+$): **Cirigliano 18'**

$$\left(T^{0\nu}_{1/2}\right)^{-1} = g_A^4\Big\{G_{01}\ \left(|\mathcal{A}_\nu|^2+|\mathcal{A}_R|^2\right) - 2(G_{01}-G_{04})\mathrm{Re}\mathcal{A}_\nu^*\mathcal{A}_R + 4G_{02}\,|\mathcal{A}_E|^2$$
$$+2G_{04}\left[|\mathcal{A}_{m_e}|^2 + \mathrm{Re}\left(\mathcal{A}_{m_e}^*(\mathcal{A}_\nu+\mathcal{A}_R)\right)\right]$$
$$-2G_{03}\,\mathrm{Re}\left[(\mathcal{A}_\nu+\mathcal{A}_R)\mathcal{A}_E^* + 2\mathcal{A}_{m_e}\mathcal{A}_E^*\right]$$
$$+G_{09}\,|\mathcal{A}_M|^2 + G_{06}\,\mathrm{Re}\left[(\mathcal{A}_\nu-\mathcal{A}_R)\mathcal{A}_M^*\right]\Big\}\,.$$

- Here A's are combinations of the ββ decay NMEs and LECs

- G's are the phase space factors and are trivial for numerical calculations

Background

$$\mathcal{A}_\nu = \frac{m_{\beta\beta}}{m_e}\mathcal{M}_\nu^{(3)} + \frac{m_N}{m_e}\mathcal{M}_\nu^{(6)} + \frac{m_N^2}{m_e v}\mathcal{M}_\nu^{(9)} \quad \mathcal{A}_M = \frac{m_N}{m_e}\mathcal{M}_M^{(6)} + \frac{m_N^2}{m_e v}\mathcal{M}_M^{(9)}$$

$$\mathcal{A}_E = \mathcal{M}_{E,L}^{(6)} + \mathcal{M}_{E,R}^{(6)} \quad \mathcal{A}_{m_e} = \mathcal{M}_{m_e,L}^{(6)} + \mathcal{M}_{m_e,R}^{(6)} \quad \mathcal{A}_R = \frac{m_N^2}{m_e v}\mathcal{M}_R^{(9)}$$

- M's here are the combinations of NMEs, for the neutrino mass mechanism, we have M_F, M_{GT} and M_T

$$\mathcal{M}_\nu^{(3)} = -V_{ud}^2\left(-\frac{1}{g_A^2}M_F + \mathcal{M}_{GT} + \mathcal{M}_T + 2\frac{m_\pi^2\, g_\nu^{NN}}{g_A^2}M_{F,\,sd}\right),$$

$$\begin{aligned}\mathcal{M}_\nu^{(9)} = &-\frac{1}{2m_N^2}C_{\pi\pi\,\mathrm{L}}^{(9)}\left(\frac{1}{2}M_{GT,sd}^{AP} + M_{GT,sd}^{PP} + \frac{1}{2}M_{T,sd}^{AP} + M_{T,sd}^{PP}\right)\\ &+\frac{m_\pi^2}{2m_N^2}C_{\pi N\,\mathrm{L}}^{(9)}\left(M_{GT,sd}^{AP} + M_{T,sd}^{AP}\right) - \frac{2}{g_A^2}\frac{m_\pi^2}{m_N^2}C_{NN\,\mathrm{L}}^{(9)}M_{F,\,sd},\end{aligned}$$

$$\mathcal{M}_R^{(9)} = \mathcal{M}_\nu^{(9)}\big|_{L\to R}$$

$$(T_{1/2}^{0\nu})^{-1} = g_A^4 G_{01}(|\mathrm{A}_\nu|^2 + |\mathrm{A}_R|^2)$$

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$$(T_{1/2}^{0\nu})^{-1} = g_A^4 G_{01}(|\mathrm{A}_\nu|^2 + |\mathrm{A}_R|^2)$$

Background

- M_F, M_{GT} and M_T are the long range Fermi, Gamow-Teller and tensor part we are familiar with

$$\mathcal{M}_{GT} = M_{GT}^{AA} + M_{GT}^{AP} + M_{GT}^{PP} + M_{GT}^{MM} \qquad \mathcal{M}_{T} = M_{T}^{AP} + M_{T}^{PP} + M_{T}^{MM}$$

- Where $M_I^K = \langle f | \dfrac{2R}{\pi} \displaystyle\int h_I^K(q) j_I(qr) \frac{q dq}{q + E_N} \mathcal{O}_I | i \rangle$

- Short range NMEs are similar $M_{I,sd}^K = \langle f | \dfrac{2R}{\pi} \displaystyle\int h_I^K(q) j_I(qr) \frac{q^2 dq}{q + E_N} \mathcal{O}_I | i \rangle$

- All these *M*’s can be expressed in 15 NMEs

$$M_F \quad M_{GT}^{AA} \quad M_{GT}^{AP} \quad M_{GT}^{PP} \quad M_{GT}^{MM} \quad M_{T}^{AA} \quad M_{T}^{AP} \quad M_{T}^{PP} \quad M_{T}^{MM}$$

$$M_{F,sd} \quad M_{GT,sd}^{AA} \quad M_{GT,sd}^{AP} \quad M_{GT,sd}^{PP} \quad M_{T,sd}^{AP} \quad M_{T,sd}^{PP}$$

Background

Stefanik 18'

- A comparison with LR symmetric model in traditional treatment where left- and right-handed neutrino are treated equally (short range mechanism neglected)

$$[T_{1/2}^{0\nu}]^{-1} = g_A^4 |M_{GT}|^2 \left\{ C_{mm} \left(\frac{|m_{\beta\beta}|}{m_e} \right)^2 + C_{m\lambda} \frac{|m_{\beta\beta}|}{m_e} \langle\lambda\rangle \cos\psi_1 + C_{m\eta} \frac{|m_{\beta\beta}|}{m_e} \langle\eta\rangle \cos\psi_2 + C_{\lambda\lambda} \langle\lambda\rangle^2 + C_{\eta\eta} \langle\eta\rangle^2 + C_{\lambda\eta} \langle\lambda\rangle\langle\eta\rangle \cos(\psi_1 - \psi_2) \right\}$$

- Where

$$C_{mm} = (1 - \chi_F + \chi_T)^2 G_{01},$$

$$C_{m\lambda} = -(1 - \chi_F + \chi_T)[\chi_{2-} G_{03} - \chi_{1+} G_{04}],$$

$$C_{m\eta} = (1 - \chi_F + \chi_T)[\chi_{2+} G_{03} - \chi_{1-} G_{04} - \chi_P G_{05} + \chi_R G_{06}],$$

$$C_{\lambda\lambda} = \chi_{2-}^2 G_{02} + \tfrac{1}{9} \chi_{1+}^2 G_{011} - \tfrac{2}{9} \chi_{1+} \chi_{2-} G_{010},$$

$$C_{\eta\eta} = \chi_{2+}^2 G_{02} + \tfrac{1}{9} \chi_{1-}^2 G_{011} - \tfrac{2}{9} \chi_{1-} \chi_{2+} G_{010} + \chi_P^2 G_{08} - \chi_P \chi_R G_{07} + \chi_R^2 G_{09},$$

$$C_{\lambda\eta} = -2\big[\chi_{2-}\chi_{2+} G_{02} - \tfrac{1}{9}(\chi_{1+}\chi_{2+} + \chi_{2-}\chi_{1-}) G_{010} + \tfrac{1}{9} \chi_{1+}\chi_{1-} G_{011}\big].$$

Background

- The rich structures for these NMEs are simulated

$$\chi_{1\pm} = \chi_{qGT} - 6\chi_{qT} \pm 3\chi_{qF}, \quad \chi_{2\pm} = \chi_{GT\omega} + \chi_{T\omega} \pm \chi_{F\omega} - \frac{1}{9}\chi_{1\mp}.$$

- These are terms from the helicity exchange terms in neutrino propagator

$$M_{\omega F,\omega GT,\omega T} = \sum \langle A_f \| h_{\omega F,\omega GT,\omega T}(r_-) \mathcal{O}_{F,GT,T} \| A_i \rangle$$

$$M_{qF,qGT,qT} = \sum \langle A_f \| h_{qF,qGT,qT}(r_-) \mathcal{O}_{F,GT,T} \| A_i \rangle$$

- And also time-space components and recoil terms

$$M_P = \sum i \langle A_f \| h_P(r_-) \tau_r^+ \tau_s^+ \frac{(\mathbf{r}_- \times \mathbf{r}_+)}{R^2} \cdot \vec{\sigma}_r \| A_i \rangle$$

$$M_R = \sum \langle A_f \| [h_{RG}(r_-) \mathcal{O}_{GT} + h_{RT}(r_-) \mathcal{O}_T] \| A_i \rangle$$

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Stefanik 18'

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$$[T^{0\nu}_{1/2}]^{-1} = g_A^4 |M_{GT}|^2 \Bigg\{ C_{mm} \left(\frac{|m_{\beta\beta}|}{m_e}\right)^2 + C_{m\lambda} \frac{|m_{\beta\beta}|}{m_e} \langle\lambda\rangle \cos\psi_1 + C_{m\eta} \frac{|m_{\beta\beta}|}{m_e} \langle\eta\rangle \cos\psi_2 + C_{\lambda\lambda} \langle\lambda\rangle^2 + C_{\eta\eta} \langle\eta\rangle^2 + C_{\lambda\eta} \langle\lambda\rangle\langle\eta\rangle \cos(\psi_1 - \psi_2) \Bigg\}$$

- An comparison with SMEFT

$$\left(T^{0\nu}_{1/2}\right)^{-1} = g_A^4 \Bigg\{ G_{01} \left(|\mathcal{A}_\nu|^2 + |\cancel{\mathcal{A}_R}|^2\right) - 2(G_{01} - G_{04}) \mathrm{Re}\mathcal{A}_\nu^* \cancel{\mathcal{A}_R} + 4G_{02} |\mathcal{A}_E|^2 + 2G_{04} \left[|\mathcal{A}_{m_e}|^2 + \mathrm{Re}\left(\mathcal{A}_{m_e}^* (\mathcal{A}_\nu + \cancel{\mathcal{A}_R})\right)\right] - 2G_{03} \mathrm{Re}\left[(\mathcal{A}_\nu + \cancel{\mathcal{A}_R})\mathcal{A}_E^* + 2\mathcal{A}_{m_e}\mathcal{A}_E^*\right] + G_{09} |\mathcal{A}_M|^2 + G_{06} \mathrm{Re}\left[(\mathcal{A}_\nu - \cancel{\mathcal{A}_R})\mathcal{A}_M^*\right] \Bigg\}.$$

Background

Cirigliano 17', Hyvarinen15', Barea 15', Horoi 18'

- NME correspondence in different references

NMEs	Ref. [76, 84, 85]	Ref. [83]	Ref. [32]
M_F	M_F	M_F	$M_{F,F\omega,Fq}$
M_{GT}^{AA}	M_{GT}^{AA}	M_{GT}^{AA}	$M_{GT\omega,GTq}$
M_{GT}^{AP}	M_{GT}^{AP}	M_{GT}^{AP}	$4\frac{m_e}{B}M_{GT\pi\nu}+\frac{1}{3}M_{GT2\pi}$
M_{GT}^{PP}	M_{GT}^{PP}	M_{GT}^{PP}	$-\frac{1}{6}M_{GT2\pi}$
M_{GT}^{MM}	$r_M^2 M_{GT}^{MM}$	M_{GT}^{MM}	$r_M\frac{g_M}{2g_A g_V R_A m_N}M_R=\frac{g_M^2}{6g_A^2 R_A m_N}M_{GT'}$
M_T^{AA}	✗	✗	✗
M_T^{AP}	M_T^{AP}	M_T^{AP}	$4\frac{m_e}{B}M_{T\pi\nu}+\frac{1}{3}M_{T2\pi}$
M_T^{PP}	M_T^{PP}	M_T^{PP}	$-\frac{1}{6}M_{T2\pi}$
M_T^{MM}	$r_M^2 M_T^{MM}$	M_T^{MM}	$-\frac{g_M^2}{12g_A^2 R_A m_N}M_T'$
$M_{F,sd}$	$\frac{m_e m_N}{m_\pi^2}M_{F,sd}$	$\frac{m_e m_N}{m_\pi^2}M_{F,sd}$	$\frac{m_e m_N}{m_\pi^2}M_{FN}=\frac{m_N}{R_A m_\pi^2}M_F'$
$M_{GT,sd}^{AA}$	$\frac{m_e m_N}{m_\pi^2}M_{GT,sd}^{AA}$	$\frac{m_e m_N}{m_\pi^2}M_{GT,sd}^{AA}$	$\frac{m_e m_N}{m_\pi^2}M_{GTN}=\frac{m_N}{R_A m_\pi^2}M_{GT}'$
$M_{GT,sd}^{AP}$	$\frac{m_e m_N}{m_\pi^2}M_{GT,sd}^{AP}$	$\frac{m_e m_N}{m_\pi^2}M_{GT,sd}^{AP}$	$\frac{2}{3}M_{GT1\pi}$
$M_{GT,sd}^{PP}$	$\frac{m_e m_N}{m_\pi^2}M_{GT,sd}^{PP}$	$\frac{m_e m_N}{m_\pi^2}M_{GT,sd}^{PP}$	$\frac{1}{6}(M_{GT2\pi}-2M_{GT1\pi})$
$M_{T,sd}^{AP}$	$\frac{m_e m_N}{m_\pi^2}M_{T,sd}^{AP}$	$\frac{m_e m_N}{m_\pi^2}M_{T,sd}^{AP}$	$\frac{2}{3}M_{T1\pi}$
$M_{T,sd}^{PP}$	$\frac{m_e m_N}{m_\pi^2}M_{T,sd}^{PP}$	$\frac{m_e m_N}{m_\pi^2}M_{T,sd}^{PP}$	$\frac{1}{6}(M_{T2\pi}-2M_{T1\pi})$

Background

- A more precise derivation of decay half-lives and angular correlations has also been done including short-range dim-9 operators beyond these approximations **Deppisch 20'**

$$\frac{d^2\Gamma}{dE_1 d\cos\theta} = Cw(E_1)(a(E_1) + b(E_1)\cos\theta)$$

- With

$$a(E_1) = f_{11+}^{(0)}\left|\sum_{I=1}^{3}\epsilon_I^L\mathcal{M}_I + \epsilon_\nu\mathcal{M}_\nu\right|^2 + f_{11+}^{(0)}\left|\sum_{I=1}^{3}\epsilon_I^R\mathcal{M}_I\right|^2 + \frac{1}{16}f_{66}^{(0)}\left|\sum_{I=4}^{5}\epsilon_I\mathcal{M}_I\right|^2$$
$$+ f_{11-}^{(0)} \times 2\mathrm{Re}\left[\left(\sum_{I=1}^{3}\epsilon_I^L\mathcal{M}_I + \epsilon_\nu\mathcal{M}_\nu\right)\left(\sum_{I=1}^{3}\epsilon_I^R\mathcal{M}_I\right)^*\right]$$
$$+ \frac{1}{4}f_{16}^{(0)} \times 2\mathrm{Re}\left[\left(\sum_{I=1}^{3}\epsilon_I^L\mathcal{M}_I - \sum_{I=1}^{3}\epsilon_I^R\mathcal{M}_I + \epsilon_\nu\mathcal{M}_\nu\right)\left(\sum_{I=4}^{5}\epsilon_I\mathcal{M}_I\right)^*\right]$$

$$b(E_1) = f_{11+}^{(1)}\left|\sum_{I=1}^{3}\epsilon_I^L\mathcal{M}_I + \epsilon_\nu\mathcal{M}_\nu\right|^2 + f_{11+}^{(1)}\left|\sum_{I=1}^{3}\epsilon_I^R\mathcal{M}_I\right|^2 + \frac{1}{16}f_{66}^{(1)}\left|\sum_{I=4}^{5}\epsilon_I\mathcal{M}_I\right|^2$$

Background

- In above derivation, extra currents with their form factors are derived

$$\langle p|\bar{u}(1\pm\gamma_5)d|n\rangle = \bar{N}\tau^+[F_S(q^2)\pm F_{P'}(q^2)\gamma_5]N'$$

$$\langle p|\bar{u}\sigma^{\mu\nu}(1\pm\gamma_5)d|n\rangle = \bar{N}\tau^+\left[J^{\mu\nu}\pm\frac{i}{2}\epsilon^{\mu\nu\rho\sigma}J_{\rho\sigma}\right]N'$$

$$J^{\mu\nu} = F_{T_1}(q^2)\sigma^{\mu\nu} + i\frac{F_{T_2}(q^2)}{m_p}(\gamma^\mu q^\nu - \gamma^\nu q^\mu) + \frac{F_{T_3}(q^2)}{m_p^2}(\sigma^{\mu\rho}q_\rho q^\nu - \sigma^{\nu\rho}q_\rho q^\mu).$$

- We have much complicated structure for NMEs

$$\mathcal{M}_1 = g_S^2\mathcal{M}_F \pm \frac{g_{P'}^2}{12}(\mathcal{M}_{\mathrm{GT}}^{\prime P'P'} + \mathcal{M}_T^{\prime P'P'})$$

$$\mathcal{M}_4 = \mp i\left[g_A g_{T_1}\mathcal{M}_{\mathrm{GT}}^{AT_1} - \frac{g_P g_{T_1}}{12}(\mathcal{M}_{\mathrm{GT}}^{\prime PT_1} + \mathcal{M}_T^{\prime PT_1})\right]$$

$$\mathcal{M}_3 = g_V^2\mathcal{M}_F + \frac{(g_V+g_W)^2}{12}(-2\mathcal{M}_{\mathrm{GT}}^{\prime WW} + \mathcal{M}_T^{\prime WW}) \mp\left[g_A^2\mathcal{M}_{\mathrm{GT}}^{AA} - \frac{g_A g_P}{6}(\mathcal{M}_{\mathrm{GT}}^{\prime AP} + \mathcal{M}_T^{\prime AP}) + \frac{g_P^2}{48}(\mathcal{M}_{\mathrm{GT}}^{\prime\prime PP} + \mathcal{M}_T^{\prime\prime PP})\right].$$

$$\mathcal{M}_2 = -2g_{T_1}^2\mathcal{M}_{\mathrm{GT}}^{T_1T_1}$$

$$\mathcal{M}_5 = g_V g_S\mathcal{M}_F \pm\left[\frac{g_A g_{P'}}{12}(\tilde{\mathcal{M}}_{\mathrm{GT}}^{AP'} + \tilde{\mathcal{M}}_T^{AP'}) - \frac{g_P g_{P'}}{24}(\mathcal{M}_{\mathrm{GT}}^{\prime q_0PP'} + \mathcal{M}_T^{\prime q_0PP'})\right].$$

Approaches

- Modern nuclear structure calculations relay on our understanding of nuclear force and many-body correlations

- For the nuclear force used in many-body approaches:

    - Effective nuclear force — derived from bare nucleon force and softened by certain methods

    - Phenomenological force — starting with certain symmetries and the parameters are fitted by nuclear properties

Approaches

- Most traditional methods used in double beta decay calculations are based on phenomenological forces
  - Shell Model (configuration interaction)
  - DFT based on relativistic and non-relativistic mean-field
  - GCM based on DFT
  - QRPA based on DFT or phenomenological mean-field
- Geometric models without explicit inclusions of nuclear forces: pSU(3), IBM etc.

Results

- The light neutrino mass mechanism has been in last decade well investigated although the new LO terms haven't been included

- It is impossible to give a complete list

  - SM: renormalization of operator; larger model space

  Caurier 12', Horoi 13', Menendez 14', Iwata 16', Menendez 18', Coraggio 20'

  - QRPA: isospin symmetry restoration

  Mustonen 13', Simkovic13', Hyvarinen 15', Fang 18'

  - IBM: ISR

  Barea 13', Barea15'

  - PHFB

  Sahu 15', Rath 19', Wang 21'

  - DFT+GCM: relativity

  Vaquero 13', Song14', Yao 15', Song17', Jiao 17'

Results

- Compared to light neutrino mass mechanism, there are less on heavy neutrino mass
  - SM: renormalization of operator; larger model space

Horoi 13', Menendez 18'

  - QRPA: isospin symmetry restoration

Hyvarinen 15', Fang 18'

  - IBM: ISR

Barea15'

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Rath 19'

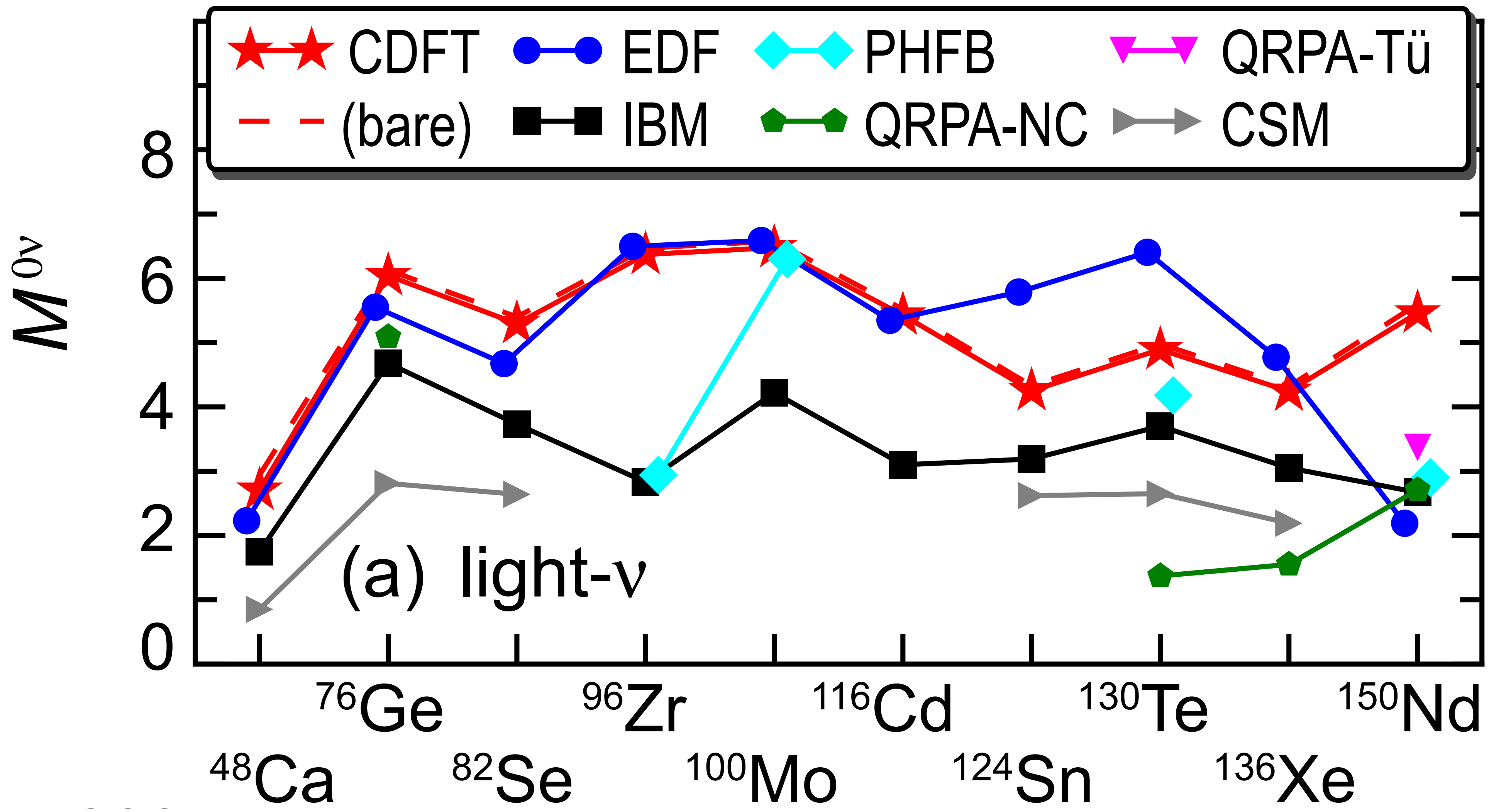
  - DFT+GCM: relativity

Song17'

Results

- Deviations from different methods

Song 17'

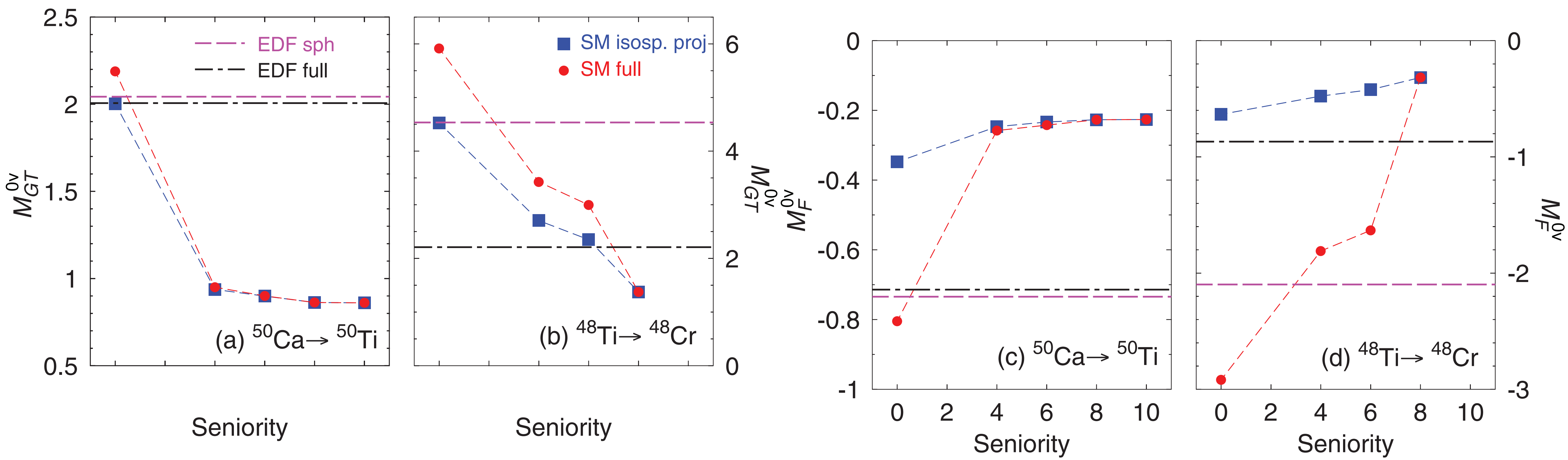


- Originating from various sources

Results

- Comparative studies between SM and EDF

Menendez 14'



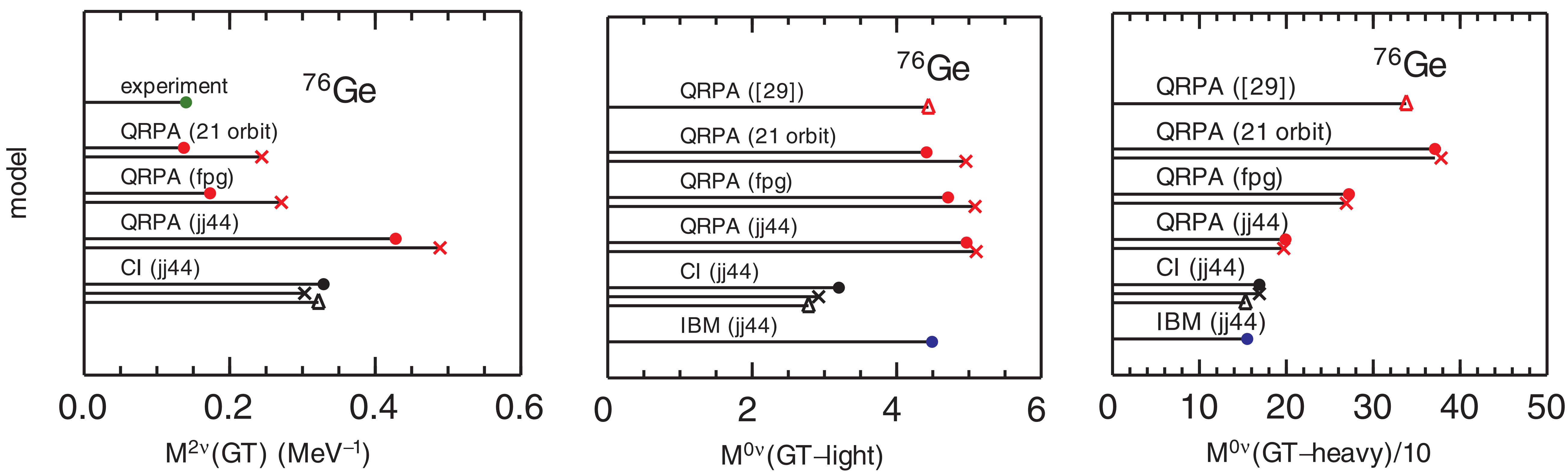
- They come out with the conclusion, SM and EDF are similar at some level when seniority is 0 for SM and only spherical shape are assumed for EDF

Results

Brown 15'

$$M^{0\nu} = [3.0(3)][1.2(2)][0.97(3)][1.12(7)] = 3.9(8)$$

$$M^{0N} = [155(10)][1.65(25)][0.80(20)][1.13(13)] = 232(80)$$



- comparative studies between SM and QRPA and estimations of errors

Results

Horoi 13', Simkovic 17', Singh 19', Sarkar 20', Ahmed 20'

- Even less are for the traditional LR symmetric models

NTMEs		^{94}Zr	^{96}Zr	^{100}Mo	^{110}Pd	^{128}Te	^{130}Te	^{150}Nd
$\overline{M}_{\omega F}$		0.569	0.443	1.004	1.102	0.587	0.642	0.456
$\Delta\overline{M}_{\omega F}$		0.066	0.050	0.130	0.150	0.061	0.081	0.071
$\overline{M}_{qF}$		0.627	0.470	1.115	1.259	0.699	0.779	0.567
$\Delta\overline{M}_{qF}$		0.058	0.055	0.156	0.185	0.065	0.114	0.094
$\overline{M}_{\omega GT}$		-3.119	-2.303	-4.985	-5.618	-2.849	-3.140	-2.134
$\Delta\overline{M}_{\omega GT}$		0.312	0.230	0.516	0.596	0.335	0.360	0.324
$\overline{M}_{qGT}$		-3.841	-2.799	-6.081	-7.068	-3.541	-3.969	-2.819
$\Delta\overline{M}_{qGT}$		0.318	0.183	0.483	0.591	0.325	0.455	0.398
$\overline{M}_{qT}$		0.021	0.050	0.050	0.065	0.189	0.084	0.033
$\Delta\overline{M}_{qT}$		0.065	0.024	0.067	0.073	0.015	0.005	0.011
$\overline{M}_{P}$		2.382	2.296	3.966	4.731	1.091	1.474	0.260
$\Delta\overline{M}_{P}$		0.207	0.121	0.245	0.241	0.156	0.073	0.106
$\overline{M}_{R}$		-2.274	-1.874	-3.832	-4.474	-2.541	-2.686	-1.801
$\Delta\overline{M}_{R}$		0.664	0.542	1.097	1.279	0.753	0.758	0.545
NTMEs with g_A=1.254 in pnQRPA by (a) Muto *et al.*[35] and (b) Šimkovic *et al.*[36]								
$M_{\omega F}$	(a)			-1.218		-1.047	-0.867	-1.630
	(b)		-1.117	-2.076	-2.015		-1.410	
M_{qF}	(a)			-1.161		-1.054	-0.860	-1.592
	(b)		-0.804	-1.588	-1.565		-0.995	
$M_{\omega GT}$	(a)			1.330		3.011	2.442	4.206
	(b)		2.088	4.159	4.436		3.091	
M_{qGT}	(a)			-1.145		1.999	1.526	2.485
	(b)		1.026	2.389	2.878		1.746	
M_{qT}	(a)			-0.823		-0.583	-0.574	-1.148
	(b)		-0.200	-0.329	-0.281		-0.252	
M_{P}	(a)			1.182		-0.483	-0.387	0.998
M_{R}	(a)			4.528		4.371	3.736	7.005

Results

Tomoda 88', Fang 21'

- If LR symmetric model dominates 0νββ decay, the decay to 2+ may be faster than decay to 0+ or comparable

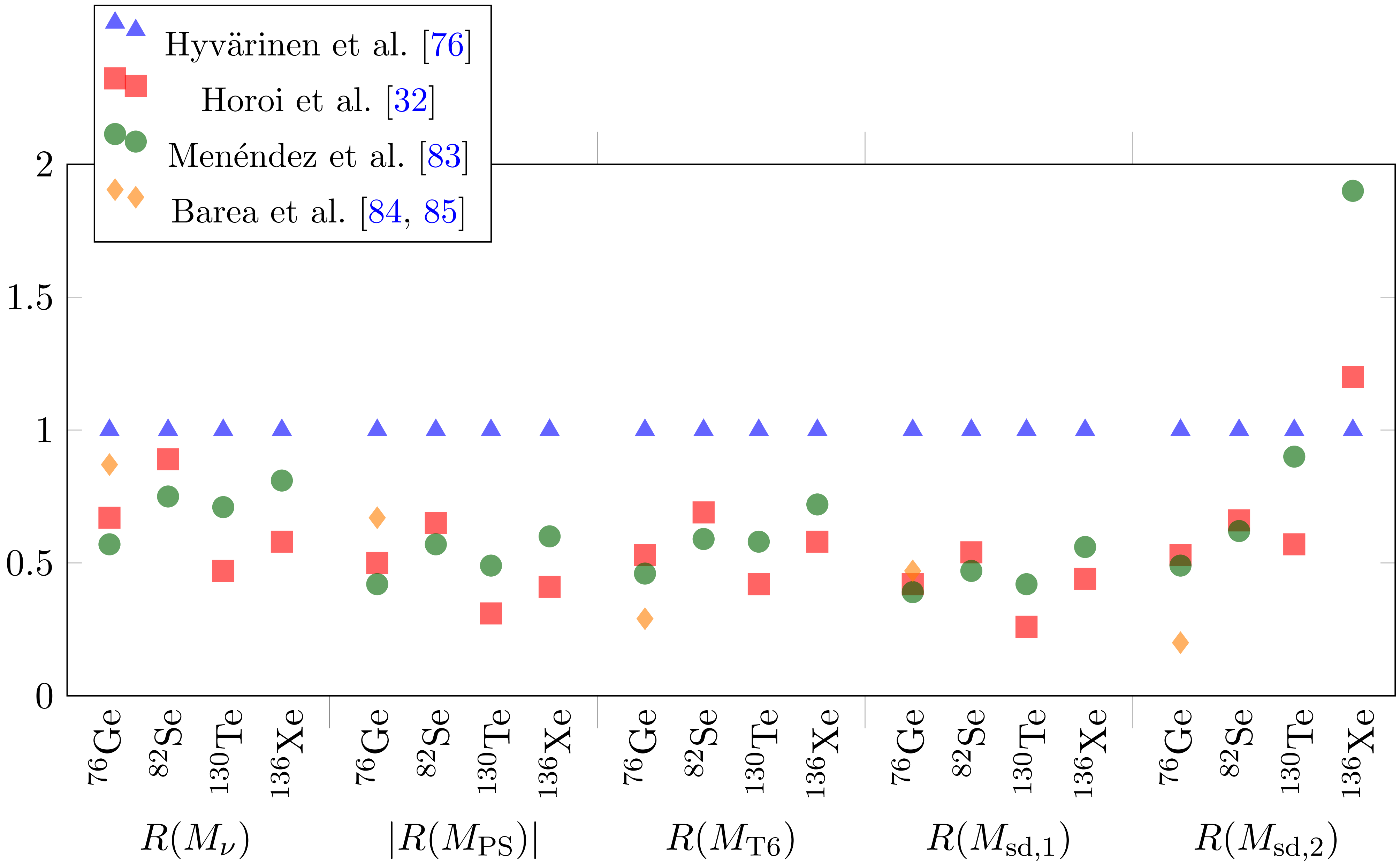
	M_1	M_2	M_3	M_4	M_5	M_λ	M_η	M_6	M_7	M'_η
PHFB[14]	0.151	0.027	-0.002	-0.049	-0.004	0.002	0.061	0.074	0.042	0.001
Baseline	0.705	-0.253	-0.046	-0.153	-0.048	0.150	0.469	0.527	-1.270	1.519
$N_{max} = 5$	0.629	-0.208	-0.014	-0.124	-0.069	0.151	0.438	0.661	-1.369	1.688
$N_{max} = 7$	0.640	-0.256	-0.048	-0.145	-0.063	0.121	0.439	0.643	-1.251	1.564
w/o src	0.701	-0.234	-0.049	-0.154	-0.051	0.128	0.451	0.485	-1.182	1.410
Argonne src	0.705	-0.250	-0.046	-0.153	-0.048	0.149	0.467	0.519	-1.261	1.505
L.O.	0.749	-0.347	-0.051	-0.154	-0.041	0.228	0.540	0.823	-1.756	2.152
w/o $F(q^2)$	0.695	-0.241	-0.047	-0.154	-0.050	0.136	0.457	0.529	-1.272	1.521
Closure Energy	0.696	-0.267	-0.043	-0.144	-0.041	0.177	0.472	0.522	-1.247	1.493
$g_{pp}^{T=0} = 0$	0.611	-0.169	-0.054	-0.161	-0.065	0.029	0.376	0.540	-1.240	1.496
$g_{pp}^{T=1} = 0$	0.795	-0.246	-0.034	-0.156	-0.034	0.206	0.516	0.501	-1.437	1.665
$g_A = 0.75$	0.695	-0.241	-0.047	-0.154	-0.050	0.008	0.317	0.529	-1.272	1.249

- Orders of magnitude larger with QRPA calculations

Results

Cirigliano 17'

- Not so many studies of NMEs for mechanism in SMEFT frame, but we are on the edge for the booming



Results

Horoi 18', Deppisch 20'

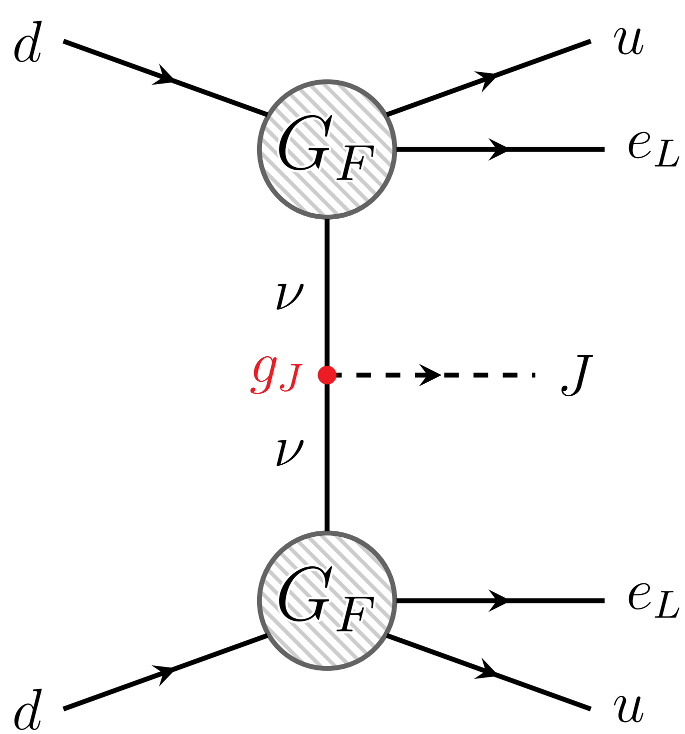
Isotope	$\mathcal{M}_F$	$\mathcal{M}_{GT}^{AA}$	$\mathcal{M}_{GT}^{AT_1}$	$\mathcal{M}_{GT}^{T_1T_1}$	$\mathcal{M}'^{WW}_{GT}$	$\mathcal{M}'^{WW}_{T}$	$\mathcal{M}'^{AP}_{GT}$	$\mathcal{M}'^{AP}_{T}$	$\mathcal{M}'^{PT_1}_{GT}$	$\mathcal{M}'^{PT_1}_{T}$	$\mathcal{M}'^{P'P'}_{GT}$	$\mathcal{M}'^{P'P'}_{T}$	$\mathcal{M}''^{PP}_{GT}$	$\mathcal{M}''^{PP}_{T}$
^{76}Ge	−48.89	170.0	174.3	173.5	−2.945	−6.541	2.110	−1.310	2.255	−1.183	0.798	−0.271	0.028	−0.022
^{82}Se	−41.22	140.7	144.3	143.6	−2.456	−6.206	1.758	−1.249	1.878	−1.183	0.660	−0.259	0.024	−0.021
^{96}Zr	−35.31	124.3	128.5	128.8	−3.116	5.436	1.523	1.090	1.652	0.984	0.613	0.228	0.020	0.019
^{100}Mo	−51.96	181.9	188.1	188.6	−4.590	8.055	2.273	1.590	2.464	1.128	0.910	0.317	0.029	0.027
^{110}Pd	−43.52	151.2	156.5	157.0	−3.945	6.816	1.892	1.356	2.055	1.223	0.762	0.271	0.024	0.023
^{116}Cd	−32.45	110.5	114.6	115.2	−3.069	4.222	1.374	0.843	1.497	0.760	0.565	0.169	0.017	0.015
^{124}Sn	−33.19	104.2	106.7	106.1	−1.701	−3.655	1.321	−0.723	1.407	−0.651	0.489	−0.146	0.018	−0.012
^{128}Te	−41.82	131.7	134.9	134.1	−2.439	−4.519	1.667	−0.890	1.776	−1.433	0.617	−0.178	0.023	−0.015
^{130}Te	−38.05	119.7	122.6	121.9	−1.951	−4.105	1.514	−0.807	1.613	−0.726	0.561	−0.160	0.021	−0.014
^{134}Xe	−39.45	124.7	127.8	127.2	−2.111	−4.191	1.564	−0.823	1.669	−0.741	0.585	−0.163	0.021	−0.014
^{136}Xe	−29.83	94.18	96.56	96.09	−1.625	−3.158	1.177	−0.620	1.257	−0.558	0.442	−0.123	0.016	−0.011
^{148}Nd	−31.71	103.0	106.0	105.8	−2.145	2.557	1.346	0.510	1.445	0.460	0.508	0.104	0.018	0.009
^{150}Nd	−30.18	100.0	103.2	103.1	−2.230	2.955	1.292	0.581	1.392	0.523	0.497	0.116	0.017	0.010
^{154}Sm	−31.83	107.1	110.7	110.9	−2.618	3.397	1.356	0.668	1.467	0.601	0.536	0.135	0.018	0.012
^{160}Gd	−41.43	142.9	148.0	148.6	−3.808	5.231	1.776	1.023	1.931	0.920	0.722	0.205	0.023	0.018
^{198}Pt	−31.87	104.4	108.4	109.0	−2.992	3.172	1.334	0.626	1.454	0.564	0.546	0.119	0.017	0.011
^{232}Th	−44.04	154.2	159.7	160.3	−4.116	6.146	1.900	1.185	2.067	1.063	0.783	0.230	0.024	0.021
^{238}U	−52.48	183.1	189.7	190.5	−4.981	7.206	2.255	1.393	2.456	1.251	0.932	0.272	0.029	0.024

- IBM results for short range dim-9 contributions under SMEFT frame

Results

- Mechanism not included in current SMEFT frame- the majoron mechanisms

Rath16', Capedello 19'



Nuclei	g_A	$\overline{M}^{(\chi)}_{m_\nu}$ Case I	$\overline{M}^{(\chi)}_{m_\nu}$ Case II	$\overline{M}^{(\chi)}_{\mathrm{CR}}$ Case I	$\overline{M}^{(\chi)}_{\mathrm{CR}}$ Case II	$\overline{M}^{(\chi)}_{\mathrm{CR}}$ [16]	$M^{(\chi)}_{\omega^2} \times 10^3$ Case I	$M^{(\chi)}_{\omega^2} \times 10^3$ Case II	$M^{(\chi)}_{\omega^2}$ $\times 10^{3\pm1}$ [16]
^{94}Zr	1.254	3.873 ± 0.373	4.071 ± 0.246	0.158 ± 0.015	0.165 ± 0.010		4.429 ± 0.560	4.500 ± 0.562	
	1.0	4.322 ± 0.421	4.550 ± 0.270	0.198 ± 0.018	0.207 ± 0.012		4.782 ± 0.557	4.860 ± 0.557	
^{96}Zr	1.254	2.857 ± 0.264	3.021 ± 0.119	0.115 ± 0.010	0.121 ± 0.004		3.198 ± 0.240	3.256 ± 0.229	
	1.0	3.204 ± 0.307	3.393 ± 0.141	0.144 ± 0.013	0.152 ± 0.006		3.414 ± 0.299	3.478 ± 0.290	
^{100}Mo	1.254	6.250 ± 0.638	6.575 ± 0.452	0.246 ± 0.024	0.258 ± 0.016	0.16	6.386 ± 0.709	6.499 ± 0.711	$\sim$1.0
	1.0	7.035 ± 0.746	7.410 ± 0.538	0.308 ± 0.029	0.324 ± 0.020		6.923 ± 0.851	7.047 ± 0.856	
^{128}Te	1.254	3.612 ± 0.395	3.810 ± 0.286	0.130 ± 0.014	0.137 ± 0.010	0.14	3.732 ± 0.456	3.795 ± 0.457	$\sim$1.0
	1.0	4.088 ± 0.450	4.316 ± 0.321	0.163 ± 0.018	0.172 ± 0.013		4.161 ± 0.518	4.230 ± 0.519	
^{130}Te	1.254	4.046 ± 0.497	4.254 ± 0.406	0.143 ± 0.016	0.151 ± 0.012	0.12	4.330 ± 0.892	4.395 ± 0.908	$\sim$1.0
	1.0	4.569 ± 0.568	4.808 ± 0.461	0.180 ± 0.020	0.189 ± 0.016		4.819 ± 1.003	4.890 ± 1.021	
^{150}Nd	1.254	2.826 ± 0.430	2.957 ± 0.408	0.094 ± 0.014	0.099 ± 0.013	0.15	3.042 ± 0.496	3.081 ± 0.508	$\sim$1.0
	1.0	3.193 ± 0.492	3.345 ± 0.466	0.118 ± 0.017	0.124 ± 0.016		3.332 ± 0.572	3.375 ± 0.586	

$$\left[T_{1/2}^{(0\nu\chi)}(0^+ \to 0^+)\right]^{-1} = \left|\langle g_\alpha \rangle\right|^m G_\alpha^{(\chi)} \left|M_\alpha^{(\chi)}\right|^2$$

- However, not no much attention has been paid

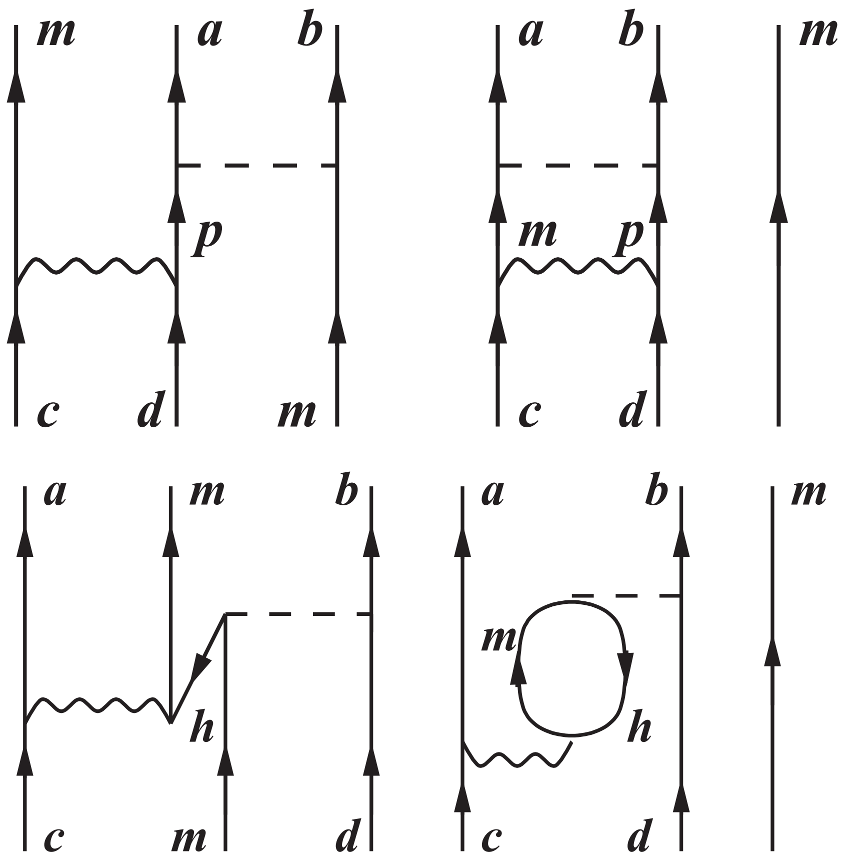
Results

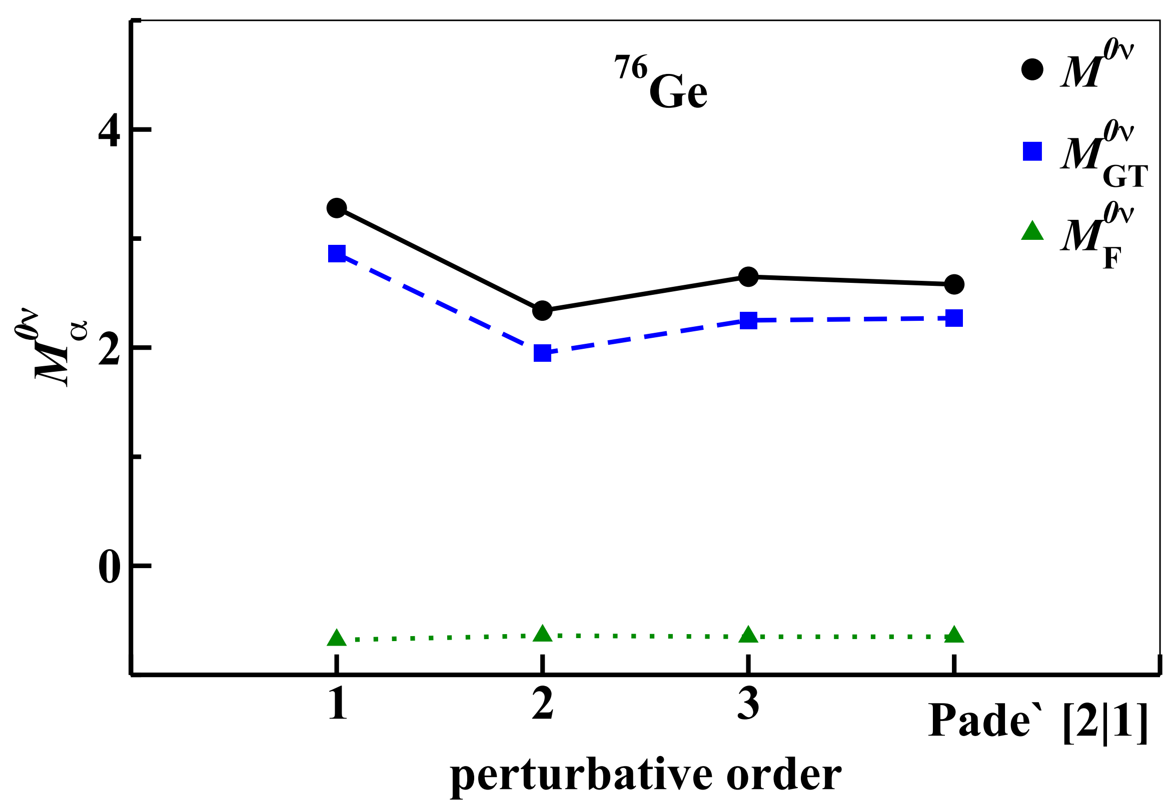
- Corrections to double beta decay operators
  - Contributions from chiral two-body currents

Menendez 11', Engel 14', Wang 18'

  - Modifications of operators in shell model

Coraggio 20'

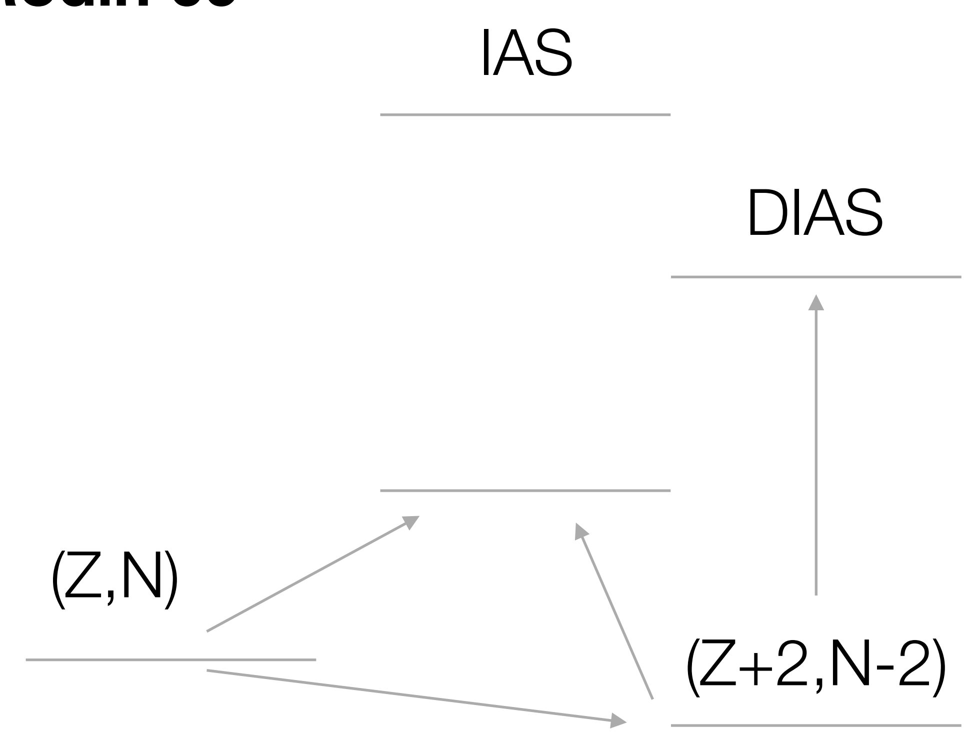




NME from experiments

- Are there any observables which can be related to the NMEs?
- Early attempts are to relate the Fermi NME with double Fermi transition or coulomb excitations
- $$M_F^{0\nu} \approx -\frac{2}{e^2}\,\bar{\omega}_{\mathrm{IAS}}\langle 0_f|\hat{T}^-|\mathrm{IAS}\rangle\langle \mathrm{IAS}|\hat{T}^-|0_i\rangle$$

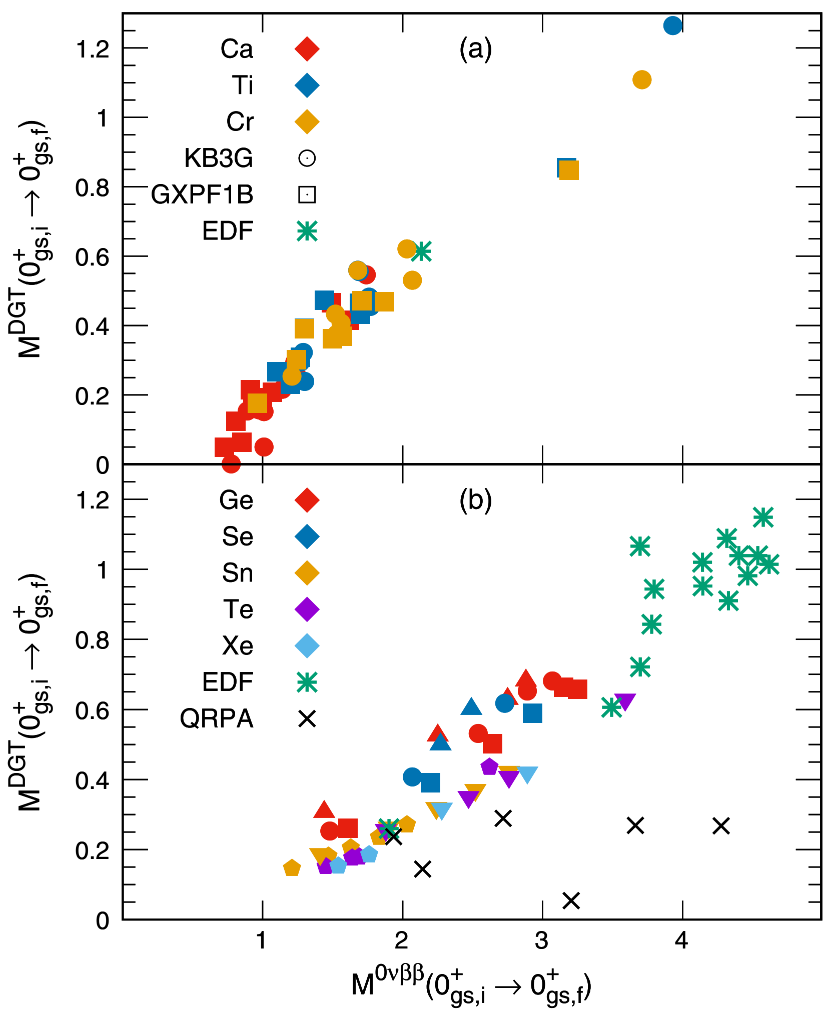
Rodin 09'

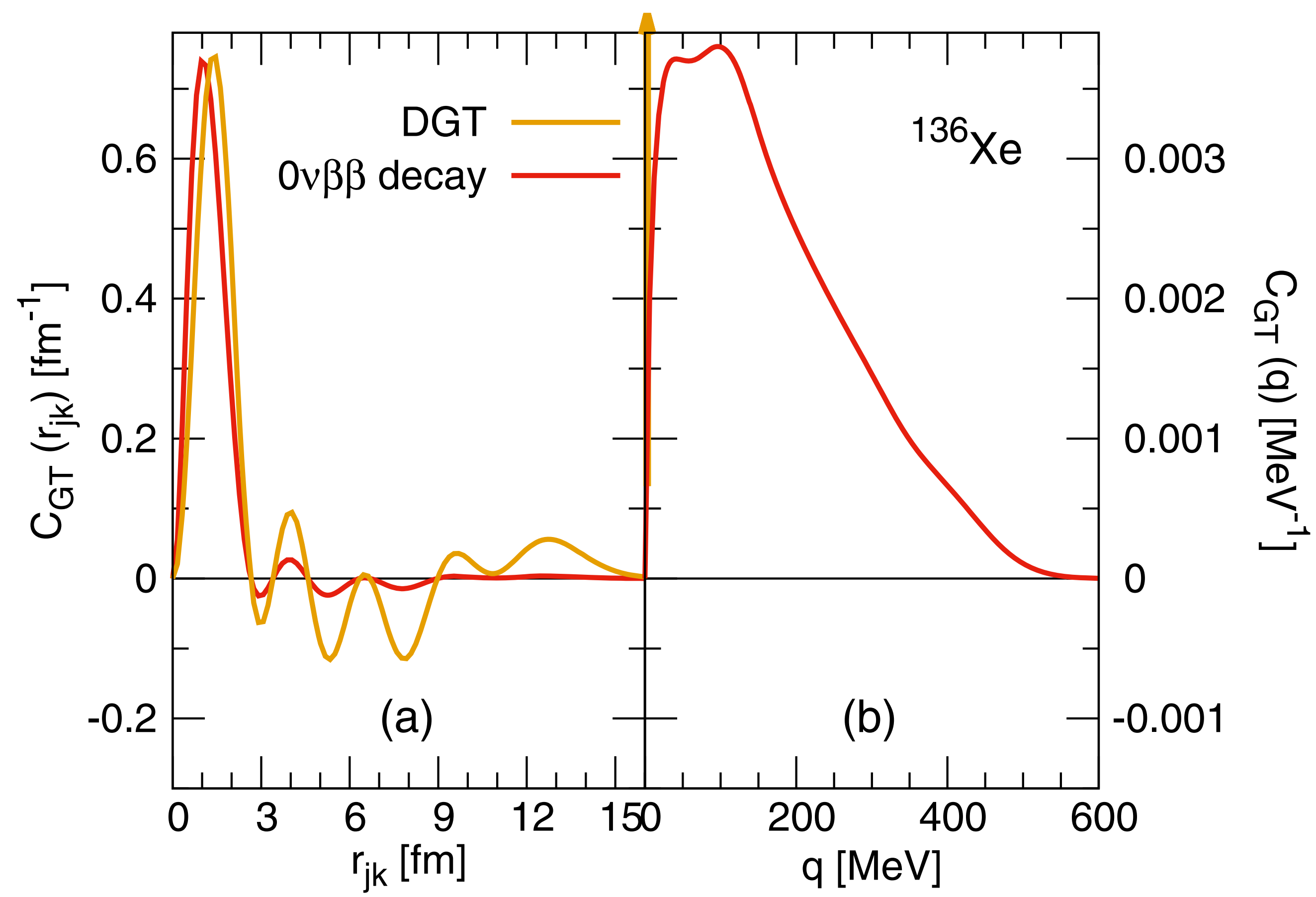


NME from experiments

- Recently, the measurement of DGT for determinations of double beta decay matrix elements are proposed

Shimizu 18'

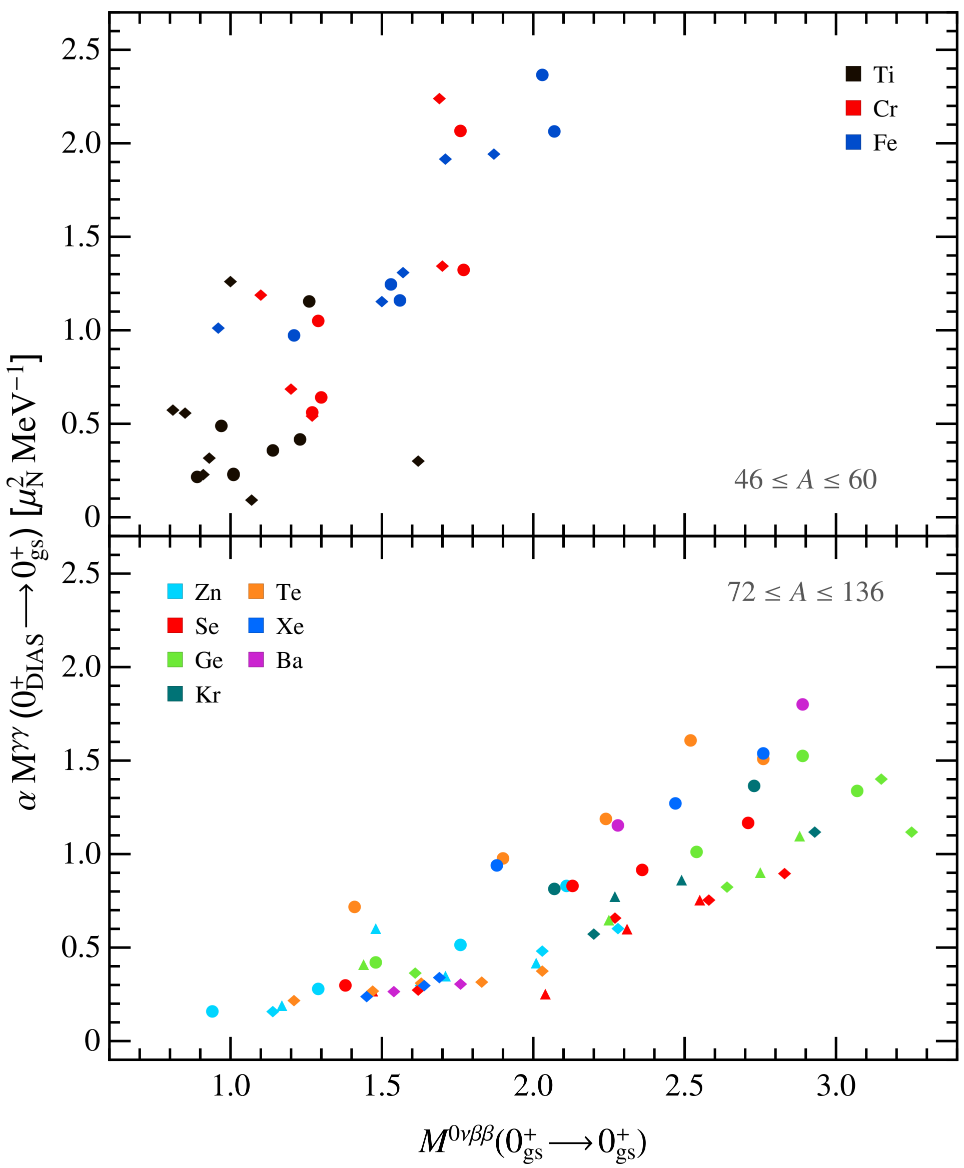


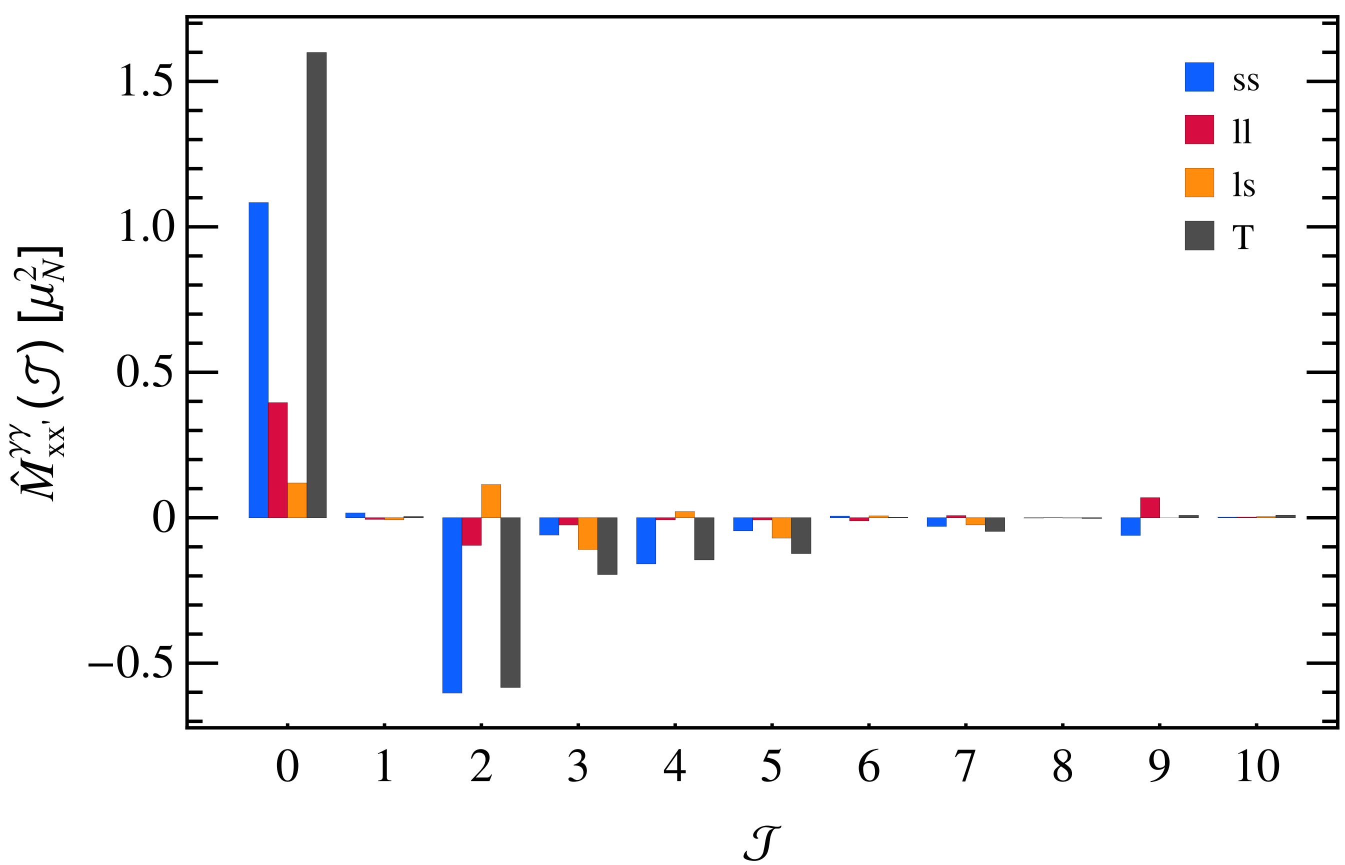


- What they found in shell model calculations,

NMEs from experiments

- The idea of EM transitions from DIAS to ground states has been formulated with shell model recently **Romeo 21'**

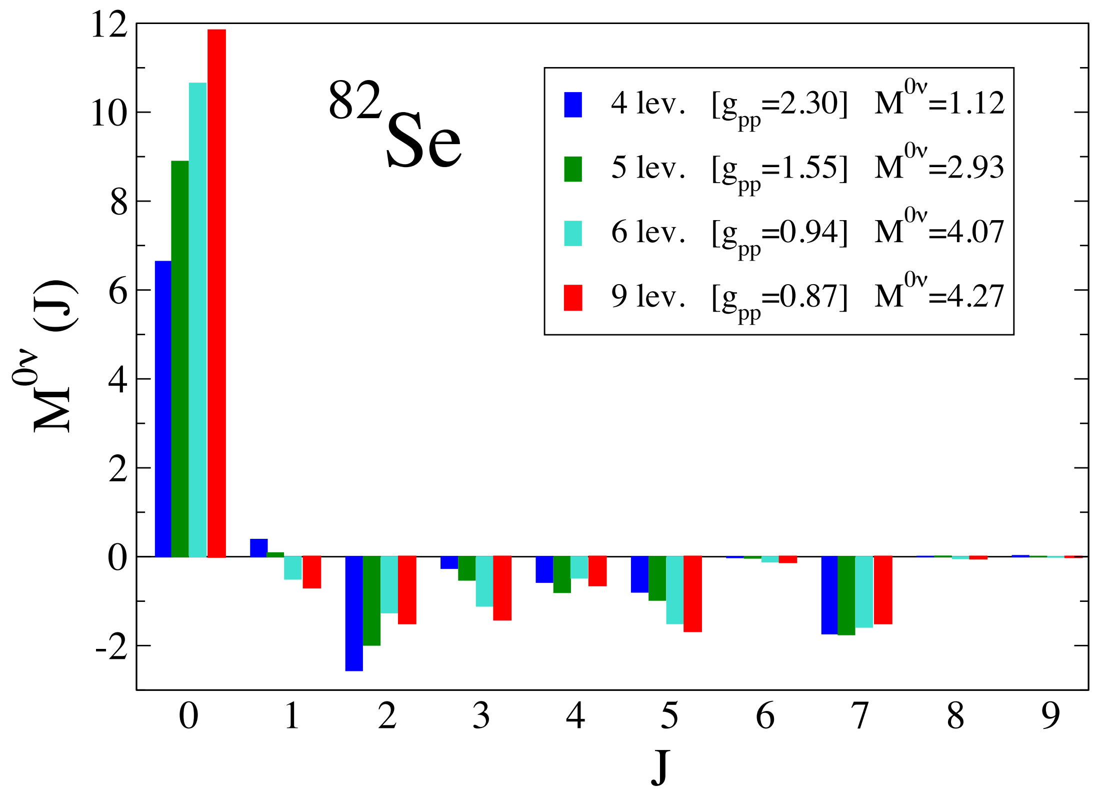


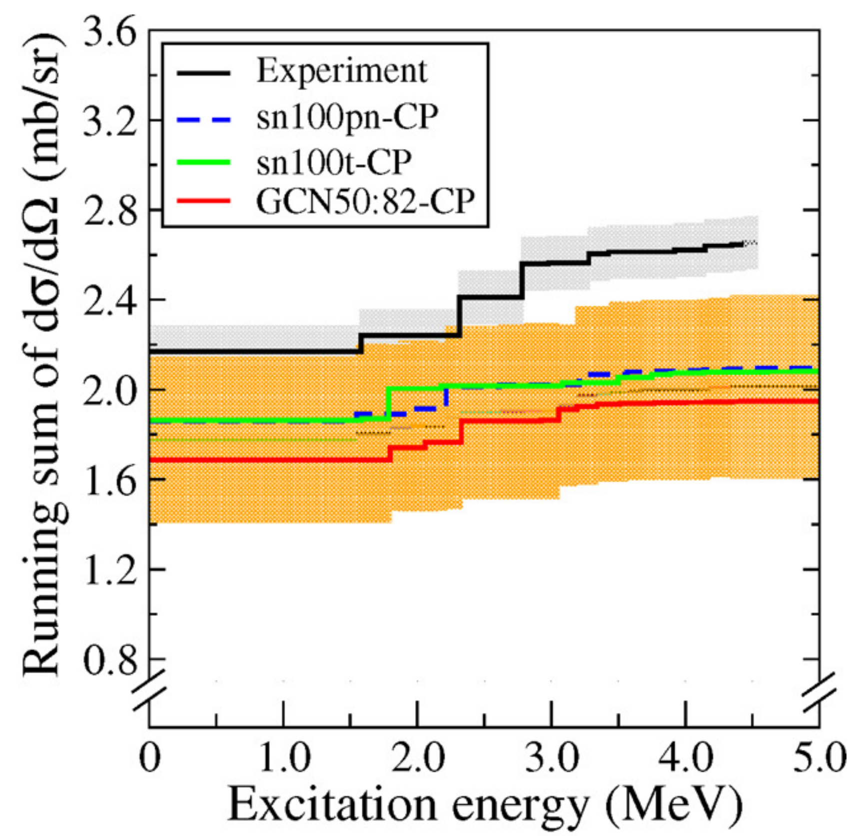


NME from experiments

- Above results has a similar nucleon pair structure as double beta decay

Rebeiro 20'





- Two nucleon removal amplitude constrained with charge changing (p,t) reactions

Conclusion

- New formalism of double beta decay based on SMEFT frame has been developed
- The requirements of NME calculations are urgent for new physics survey
- Deviations among traditional many-body approaches are large and we are trying to understand the reason
- There are also efforts of constraining the NMEs from experiment side

Thanks for your attention