

# Nuclear ab initio methods

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## Introduction

# Introduction

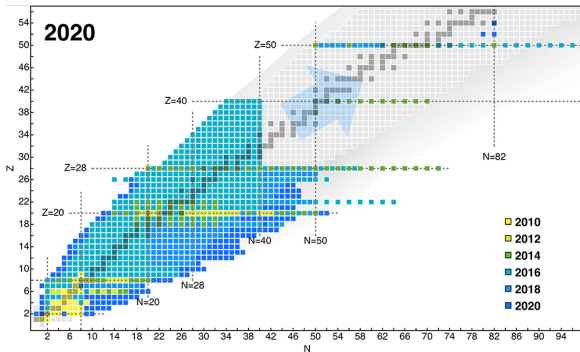
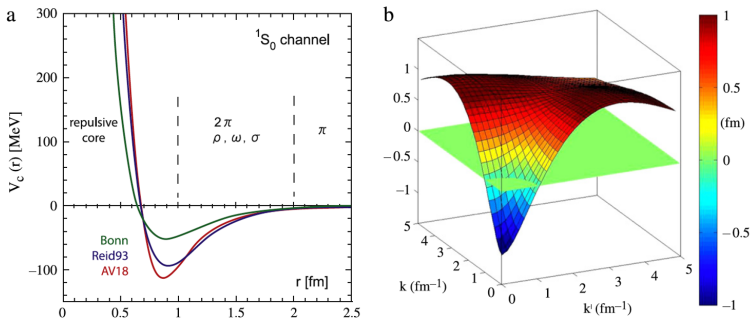


Figure: H. Hergert, Front. Phys. (2020)

Ab initio methods: Methods for solving the quantum mechanical many-body problem for all constituent nucleons, which

- describe the atomic nucleus from the bottom up with the bare nuclear forces compatible with the symmetries of QCD.
- This is done either exactly for very light nuclei or by employing certain well-controlled approximations for heavier nuclei.

# Introduction



**Figure:** Several phenomenological NN potentials in the  $^1S_0$  channel and momentum-space matrix elements of the Argonne v18 (AV18)  $^1S_0$  potential after Fourier (Bessel) transformation. S.K. Bogner et al. *Prog. Part. Nucl. Phys.* 65 (2010) 94.

- Nuclear structure calculations are complicated due to the coupling of low to high momenta by these potentials.

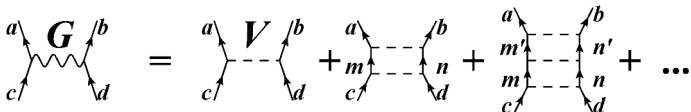
## The Brueckner-Hartree-Fock (BHF) theory



# The Brueckner-Hartree-Fock (BHF) theory

The G matrix is given by:

$$\langle ab|G(W)|cd\rangle = \langle ab|V|cd\rangle + \sum_{mn} \langle ab|V|mn\rangle \frac{Q(m,n)}{W - \epsilon_m - \epsilon_n} \langle mn|G(W)|cd\rangle$$



- $W$  is the starting energy, its value depends on the position of G-matrix in the diagram.
- $\epsilon_m, \epsilon_n$  are the HF single-particle energies.
- $Q$  is the Pauli operator which forbids the states being scattered below Fermi surface.

K. Brueckner, C. Levinson, H. Mahmoud, Phys. Rev. 95, 217 (1954)

# The Brueckner-Hartree-Fock (BHF) theory



- The HF equation is given by:

$$\sum_j (T + U)_{ij} D_{ja} = \epsilon_a D_{ia},$$

where the one-body mean-field potential is

$$U_{ij} = \sum_{c=1}^A \langle ic | \bar{G}(W) | jc \rangle.$$

Rel. BHF: [S.H. Shen et al., Prog. Part. Nucl. Phys. 109, 103713 \(2019\).](#)





# The Brueckner-Hartree-Fock (BHF) theory

## Procedure of the (R)BHF calculation

1. Initial single-particle basis  $\{|i\rangle\}$  trial for RBHF final solution  $\{|a\rangle\}$ .
2. Bethe-Goldstone equation 
$$\bar{G}_{aba'b'}(W) = \bar{V}_{aba'b'} + \frac{1}{2} \sum_{cd} \frac{\bar{V}_{abcd} Q(c, d)}{W - \varepsilon_c - \varepsilon_d} \bar{G}_{cda'b'}(W),$$
  
Solving with matrix inversion method  
*M. Haftel and F. Tabakin, NPA 158, 1 (1970)*
3. Single-particle potential 
$$U_{ab} = \sum_{c=1}^A \langle ac | \bar{G}(W) | bc \rangle.$$
4. RHF iteration 
$$\sum_j (T_{ij} + U_{ij}) D_{ji'} = \varepsilon_{i'} D_{ii'},$$
  
  
If converged  $\{|i'\rangle\} = \{|i\rangle\} = \{|a\rangle\}$  , RBHF iteration finishes.
5. Basis transformation 
$$\bar{V}_{k'l'm'n'} = \sum_{klmn} D_{kk'}^* D_{ll'}^* D_{mm'} D_{nn'} \bar{V}_{klmn}.$$
  
Go back to step 2.

## The no-core shell model

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## The Monte-Carlo methods

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## The many-body perturbation theory

# Spontaneous symmetry breaking



■ The

## The coupled-cluster theory



# Spontaneous symmetry breaking



■ The

## The in-medium similarity renormalization group

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