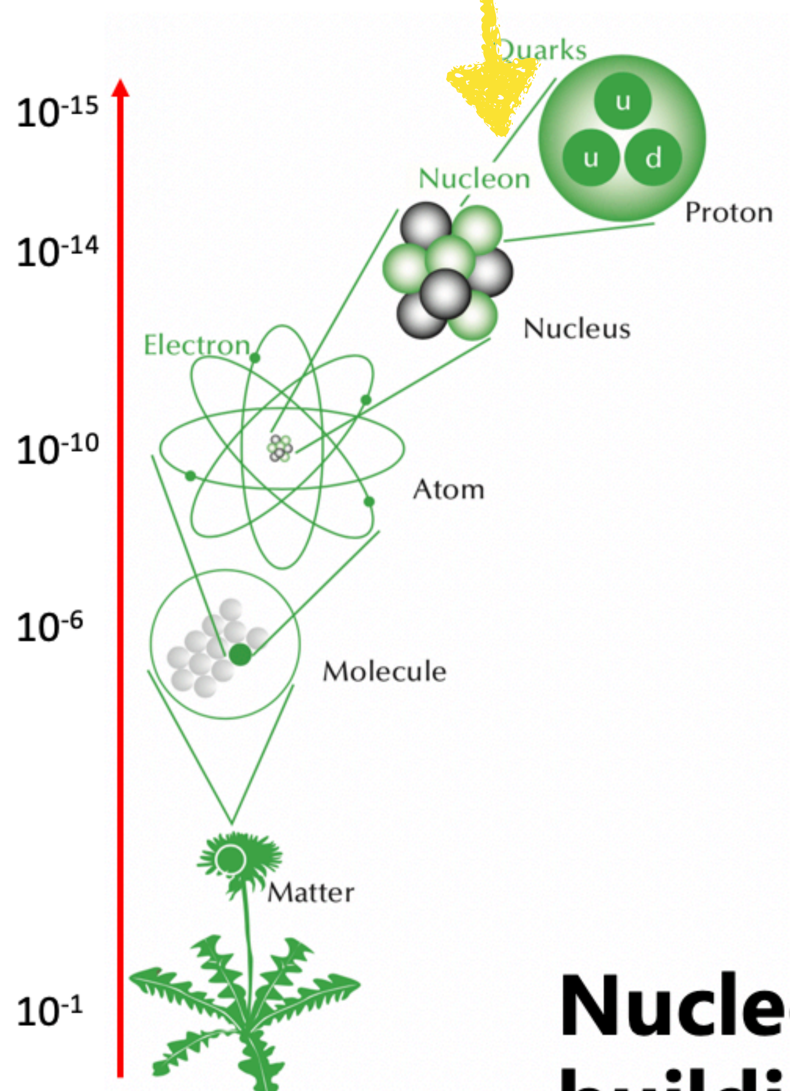
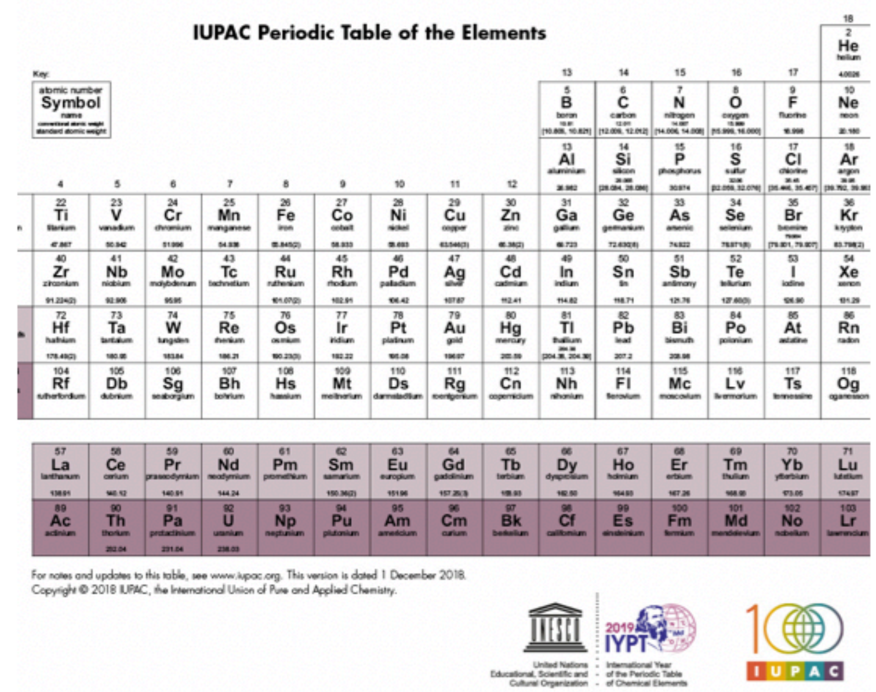
Chiral nuclear effective field theory

— Renormalization and power counting

龙炳蔚
四川大学

粒子物理与核物理结合，大有可为



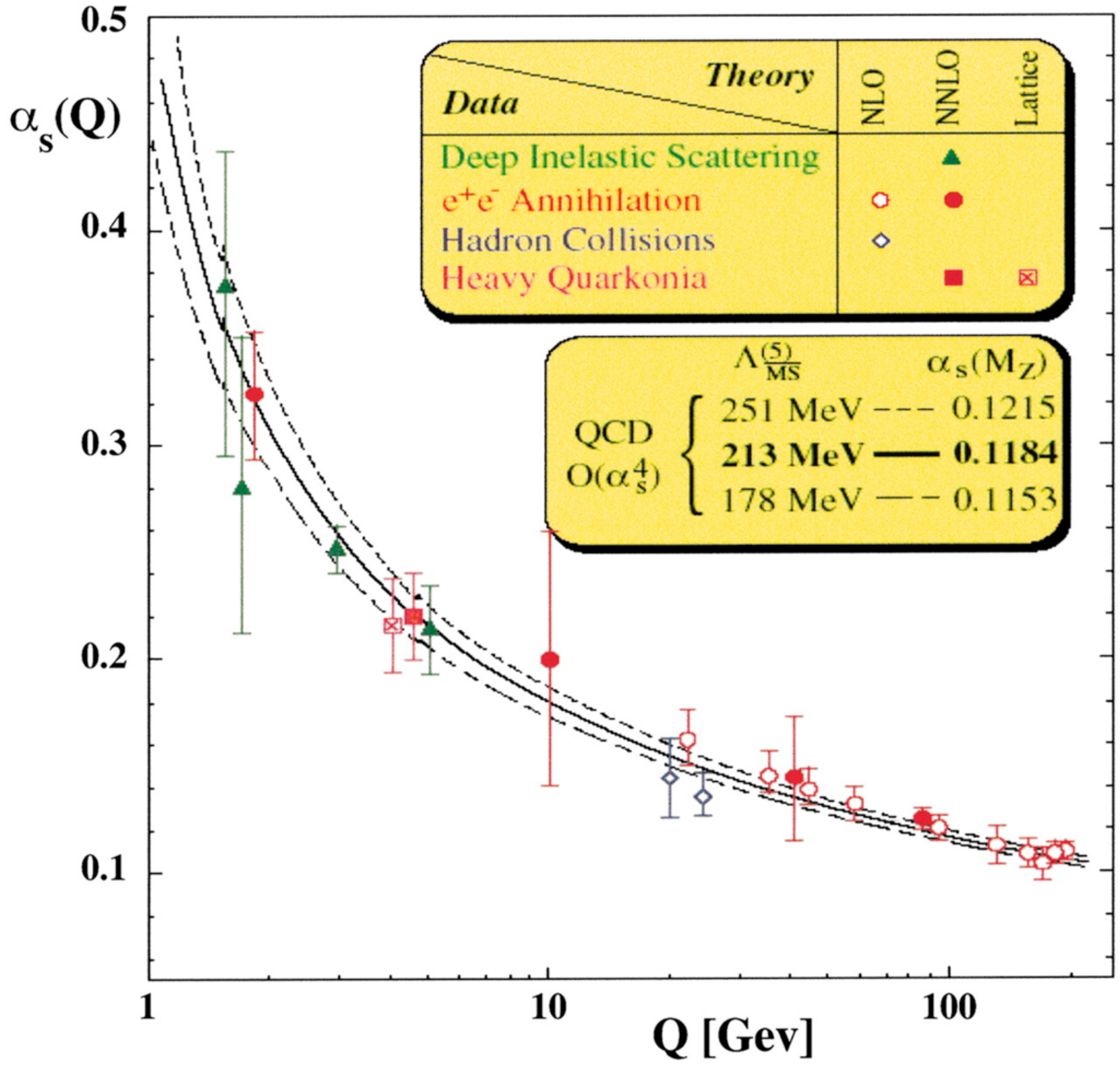


Nucleons are the essential building blocks of Matter!

BSM nuclear physics

BSM	Λ_{BSM}	$10^{?}$ TeV	
Standard M	M_{H}	100 GeV	
	Λ_{χ}	1 GeV	chiral sym. breaking
chiral EFT	Λ_{nuc}	$\sim$ 100 MeV	avg. nucleon momentum inside nuclei
pionless EFT	γ_D	45 MeV	very small nuclear scales
	$1/a_{1S0}$	8 MeV	

Perturbative QCD N/A

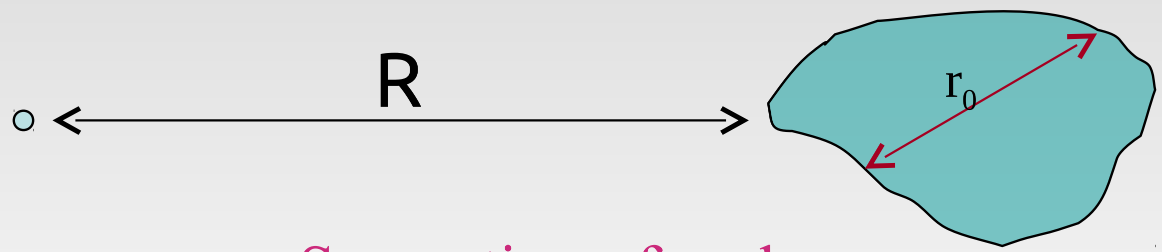


Nucl-ex/th is here!

What is EFT – a classical example

multipole expansions in electrodynamics

Charge distribution



Separation of scales

Symmetries of E&M
(Without knowing $\rho(\mathrm{R})$) $\Longrightarrow$ $R \gg r_0$

$$V = \frac{q}{R} + \frac{d_i R_i}{R^3} + \frac{Q_{ij} R_i R_j}{R^5} + \cdots$$

Low energy constants (LECs)

$$\left. \begin{array}{l} |d_i| \sim q r_0 \\ |Q_{ij}| \sim q r_0^2 \end{array} \right\}$$

Power counting

Based on dimensional analysis

Systematic low- energy approximation $\Longrightarrow$

$$V = \frac{q}{R}\left[1 + \mathcal{O}\left(\frac{r_0}{R}\right) + \mathcal{O}\left(\frac{r_0^2}{R^2}\right) + \cdots\right]$$

Much more nontrivial in quantum systems!

Chiral EFT

- Low-energy approximation of QCD, expansion in Q/M_hi
  Q: small external momenta
  M_{hi} : EFT breakdown scale (~500 MeV?)

$$\mathcal{M} = \sum_n \left(\frac{Q}{M_{hi}}\right)^n \mathcal{F}_n\left(\frac{Q}{M_{lo}}\right)$$

Q: generic external momenta,

$M_{hi} = \Lambda_{SB}, m_\rho, \cdots \sim 1\text{GeV}$

$M_{lo} = m_\pi, f_\pi \sim 100\text{MeV}$

Systematic approximation
$\rightarrow$ able to estimate theoretical errors

Chiral EFT

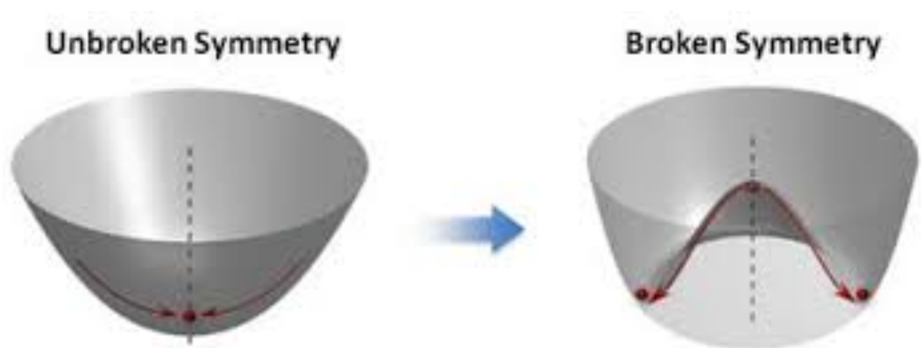
- Includes all symmetries of QCD, especially (approximate) chiral symmetry and its spontaneous breaking

$$\mathcal{L}_{\mathrm{QCD}} = \sum_{f=u,d,s,\ldots} \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4}\mathcal{G}_{a\mu\nu}\mathcal{G}_a^{\mu\nu}$$

$$q_L \equiv \begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix} \mapsto \Big(\mathbf{SU(3)}_L\Big)\begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix} \qquad q_R \equiv \begin{pmatrix} u_R \\ d_R \\ s_R \end{pmatrix} \mapsto \Big(\mathbf{SU(3)}_R\Big)\begin{pmatrix} u_R \\ d_R \\ s_R \end{pmatrix}$$

- Only two flavors used in our work

Lagrangian invariant when $m_f \to 0$, but broken by QCD ground state

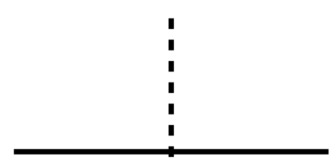


→ chiral symmetry nonlinearly realized by hadronic Dofs

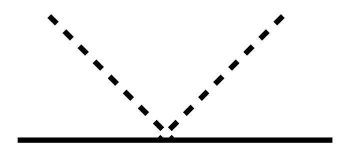
CCWZ; Weinberg; …

Chiral EFT

- The most general Lagrangian has infinitely many parameters



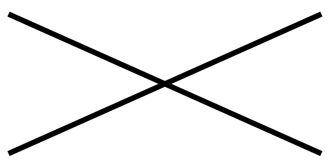
$$-\frac{g_A}{2f_\pi}N^\dagger\tau_a\vec{\sigma}\cdot\vec{\nabla}\pi_a N$$



$$-\frac{1}{4f_\pi^2}N^\dagger\epsilon_{abc}\tau_a\pi_b\dot{\pi}_c N$$

...

- Short-range interactions: large numbers of 4N operators



$$-C^{(s)}(N^T P_i^{(s)} N)^\dagger (N^T P_i^{(s)} N) - C_2^{(s)} \left[(N^T P_i^{(s)} N)^\dagger (N^T P_i^{(s')} \overleftrightarrow{\nabla}^2 N) + h.c. \right] \quad \dots$$

$$-C^{(ss')}(N^T P_i^{(s)} N)^\dagger (N^T P_i^{(s')} N) \quad \dots$$

s, s' = ^{1}S$_0$, ^{3}S$_1$, ^{3}P$_0$, ...

$$P_i^{(^1S_0)} = \frac{(i\sigma_2)(i\tau_2\tau_i)}{2\sqrt{2}}$$

$$P_i^{(^3S_1)} = \frac{(i\sigma_2\sigma_i)(i\tau_2)}{2\sqrt{2}}$$

$$P_i^{(^3D_1)} = \left(\overleftrightarrow{\nabla}_i \overleftrightarrow{\nabla}_j - \frac{\delta_{ij}}{n} \overleftrightarrow{\nabla}^2 \right) P_j^{(^3S_1)}$$

NN contact pot. in mom. spapce

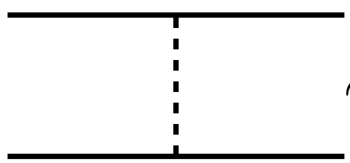
$$V_{1S0} = c_0^{1S0} + c_2^{1S0}(p^2 + p'^2) + \cdots$$

$$V_{3P0} = c_0^{3P0} p p' + \cdots$$

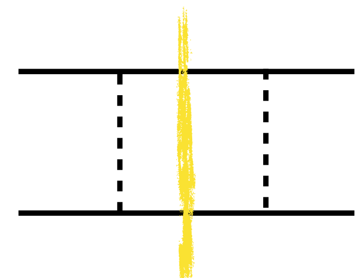
Naive dim. analysis (NDA): $\quad c_2^{1S0} \sim c_0^{3P0} \sim \dfrac{c_0^{1S0}}{M_{hi}^2}$

Power counting: long-range physics

Typical size of external momenta: $Q \sim m_\pi$



$$\sim \frac{1}{f_\pi^2}\frac{Q^2}{m_\pi^2+Q^2} \sim \frac{1}{f_\pi^2}$$

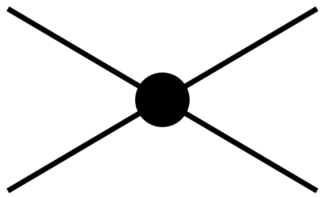


$$\sim \frac{1}{f_\pi^2}\frac{m_N}{4\pi f_\pi}\frac{Q}{a_l f_\pi}$$

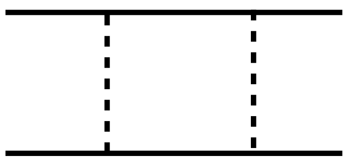
NN reducible

- Focus on loop momenta ~ external momenta Q
- Pion line or photon line ~ $1/Q^2$, nucleon line in irreducible diagrams ~ $1/Q$
- Nucleon line in reducible diagrams ~ m_N/Q^2
  $\Rightarrow$ Explain why we solve the Schrodinger eqn
  $\Rightarrow$ Explain why nuclei bound
- Strength of OPE ~ $a_l f_\pi$ (numerical factor $a_l \sim 1$ for small l, $a_l \gg 1$ for large l by centrifugal suppression)

Power counting: short-range physics



Renormalizing



$$\sim \frac{1}{f_\pi^2}\frac{m_N}{4\pi f_\pi}\frac{Q}{a_l f_\pi}$$

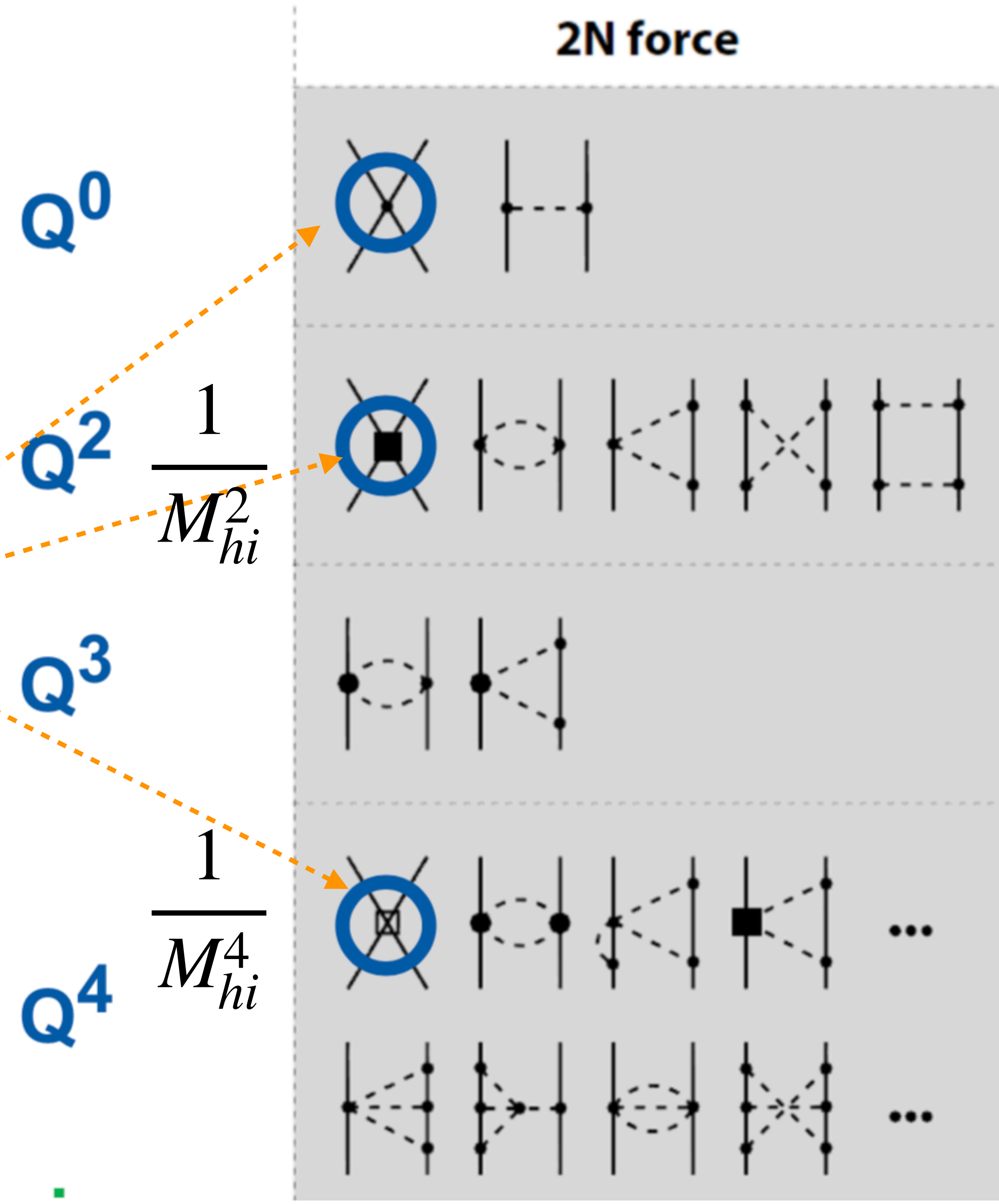
- Strength of OPE $a_l f_\pi$ may have impact on contacts through renormalization

- Coexistence of $a_l f_\pi$ and M_{hi} makes NDA no longer reliable

- Operators gaining large anomalous dimension through nuclear dynamics → "irrelevant" operators become relevant

Need to re-examine contact operators

24 contacts
in NDA up to Q^4



Counting short-range operators

- At any given order in a power counting scheme, there must be enough operators to satisfy renormalization group invariance

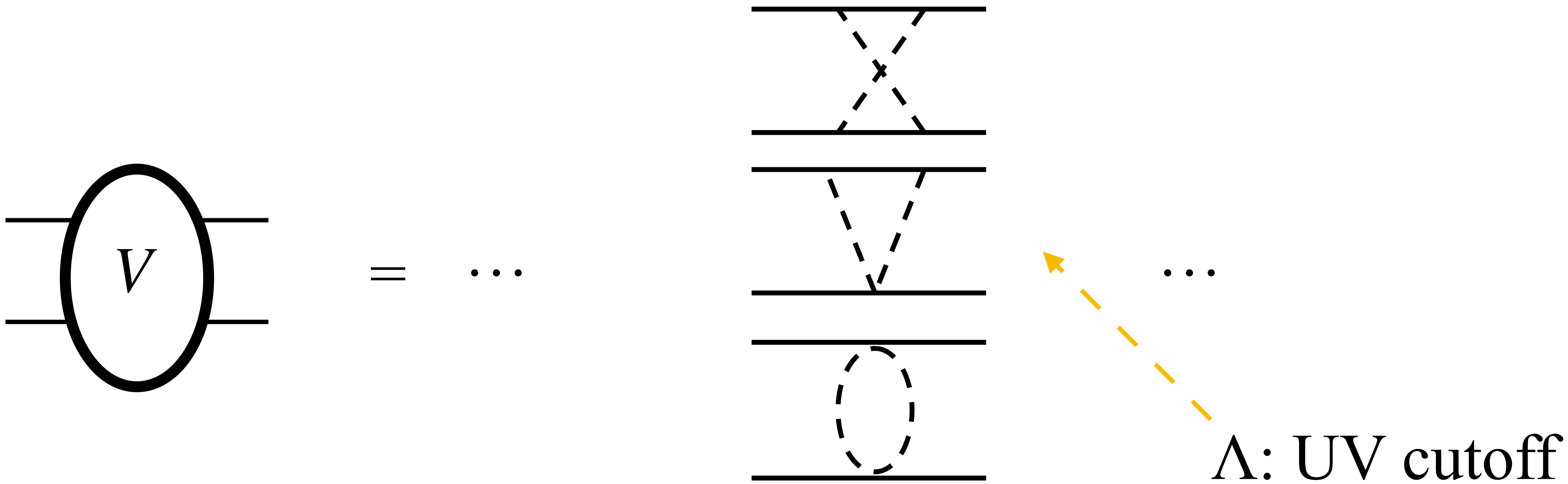
  $\Rightarrow$ explicit checking UV cutoff independence
  $\Rightarrow$ model independence

- In terms of Wilson's RG

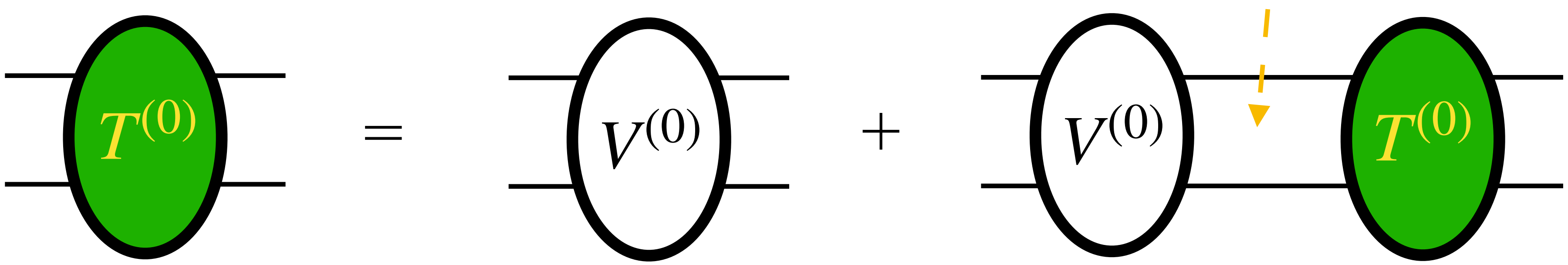
  1. Assume O be irrelevant (statement of PC)
  2. Run RGE
  3. Will O stay irrelevant in EFT?

What cutoff?

- Potentials: two-nucleon irreducible diagrams



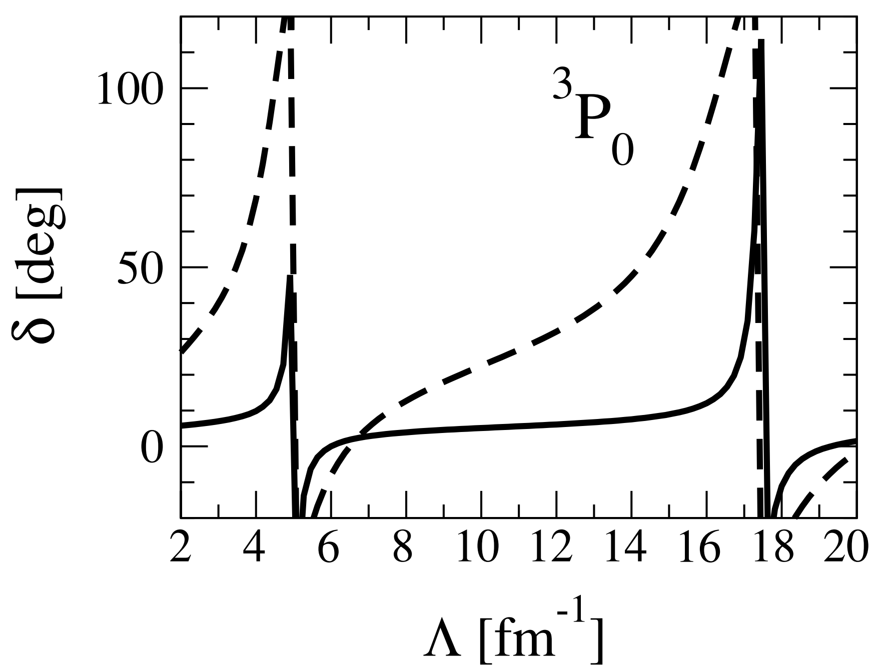
- Lippmann-Schwinger eqn (equivalent to Schrodinger eqn)



Renormalizing singular attraction

Nogga, Timmerman & van Kolck (2005)

Phase shifts vs. Λ



Solid: Tlab = 10 MeV, dashed: 50 MeV

$$C_{3P0}\vec{p}\cdot\vec{p}' \sim \frac{Q^2}{m_{hi}^2} \qquad C_{3P0}\vec{p}\cdot\vec{p}' \sim \frac{Q^2}{m_{lo}^2}$$

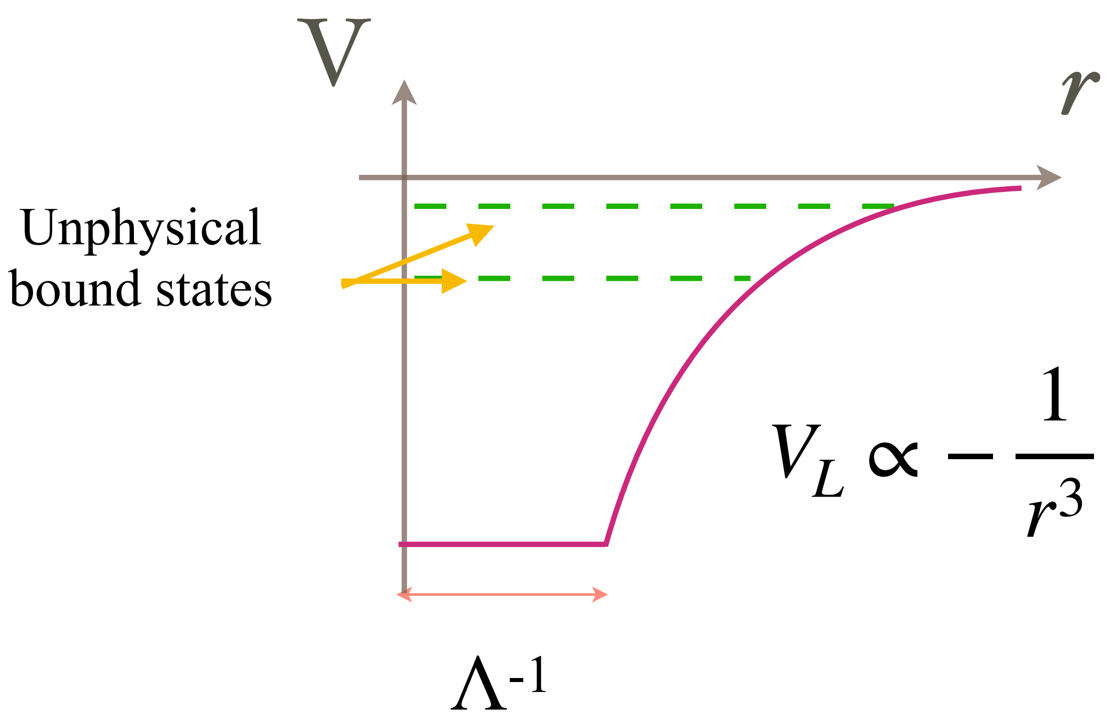
$O(Q^2)$ $\qquad$ $O(1)$

WPC $\qquad$ RG inv. counting

- Contacts needed at LO in attractive triplet channels: 3P2 - 3F2, 3D2, 3D3 …

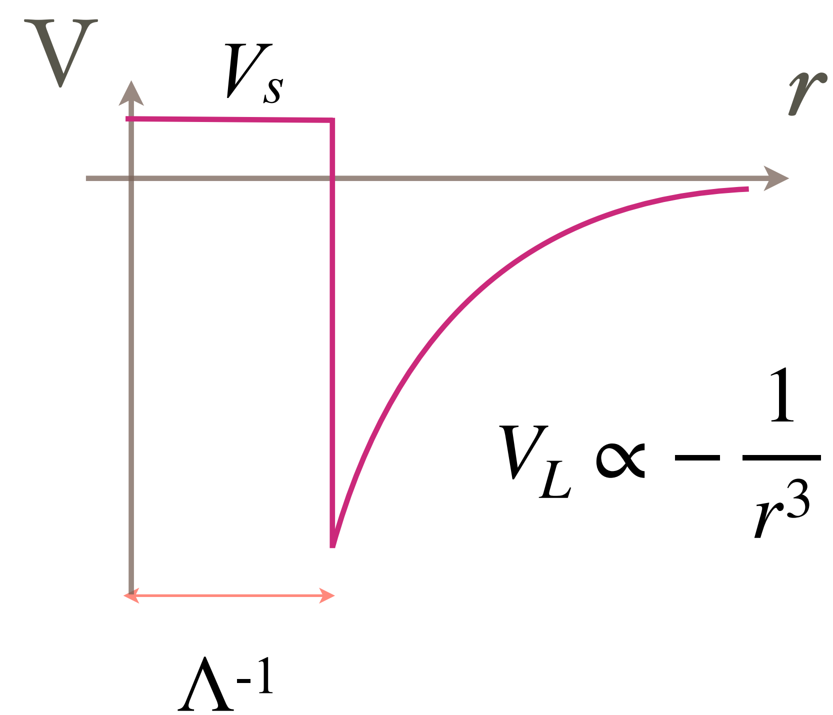
Renormalizing singular attraction

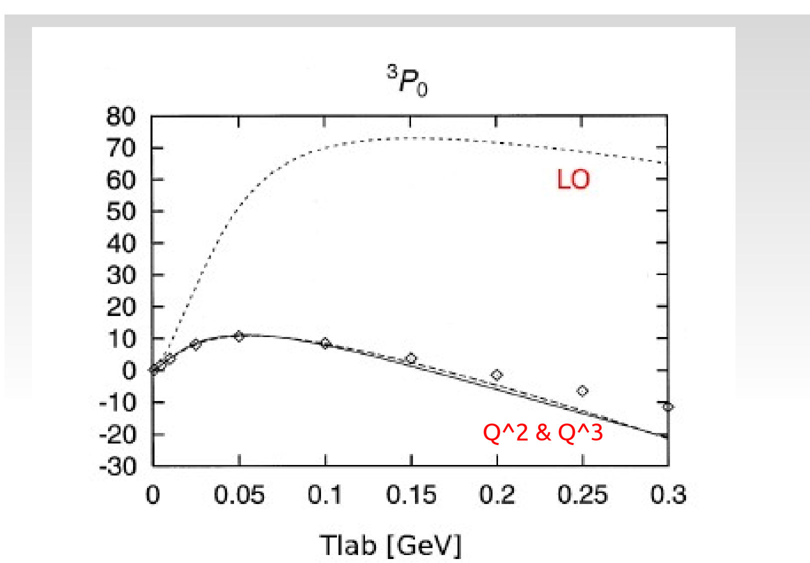
Beane et al ('01)
Pavon Valderrama & Ruiz Arriola ('05 ~ '07)
Nogga et al ('05)



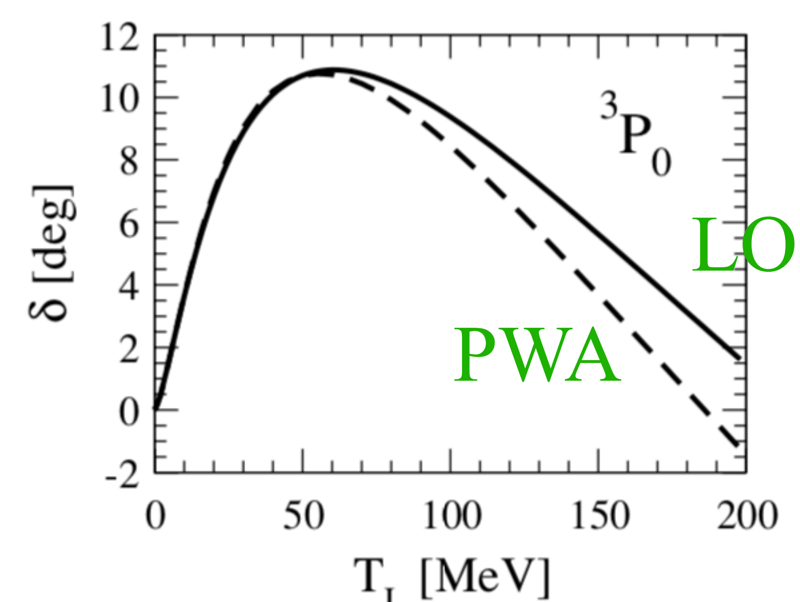
Modify PC

$\Longrightarrow$





Epelgaum et at, NPA 671, 295



Nogga et al ('05)

Modified power counting for chiral nuclear forces

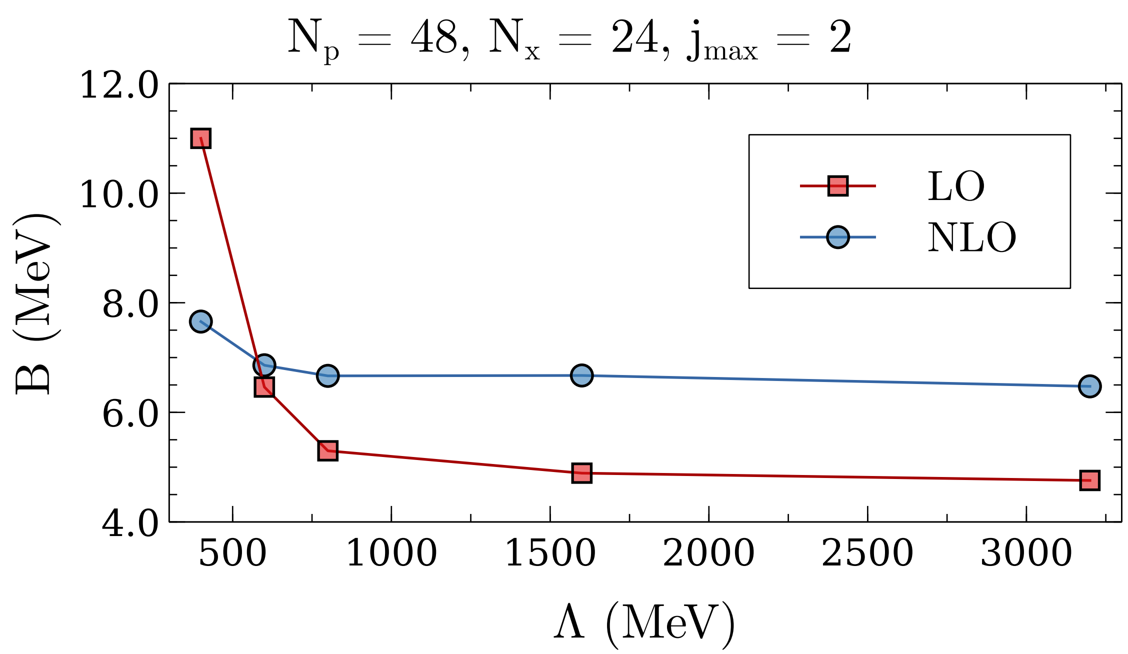
Nogga et al. '05
BwL & Yang '11, '12
Wu & BwL '19

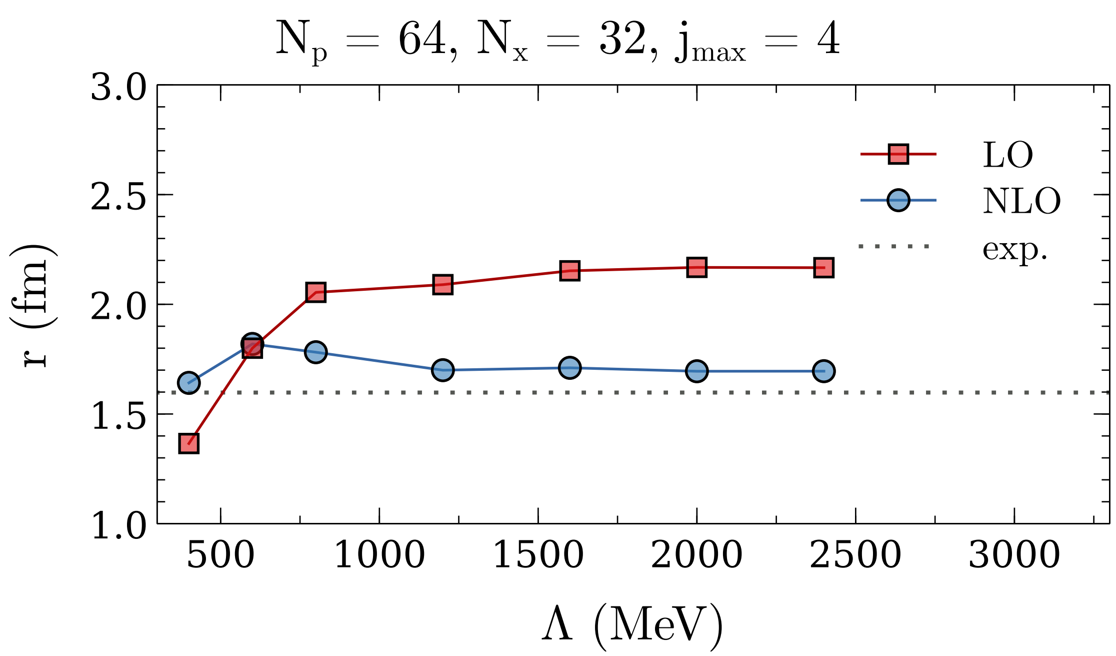
- LO : (C + OPE) for 1S0, 3S1, 3P0

(perturbative OPE for most waves)

- NLO : Q^2 C.T. for 1S0; OPE for 1P1, 3P1, 3P2…
- N2LO: (Q^4 C.T. + TPE) for 1S0; (Q^2 C.T. + TPE) for 3S1-3D1 and 3P0
- N3LO: ….

Triton BE and charge radius Preliminary





nd elastic scattering

Preliminary

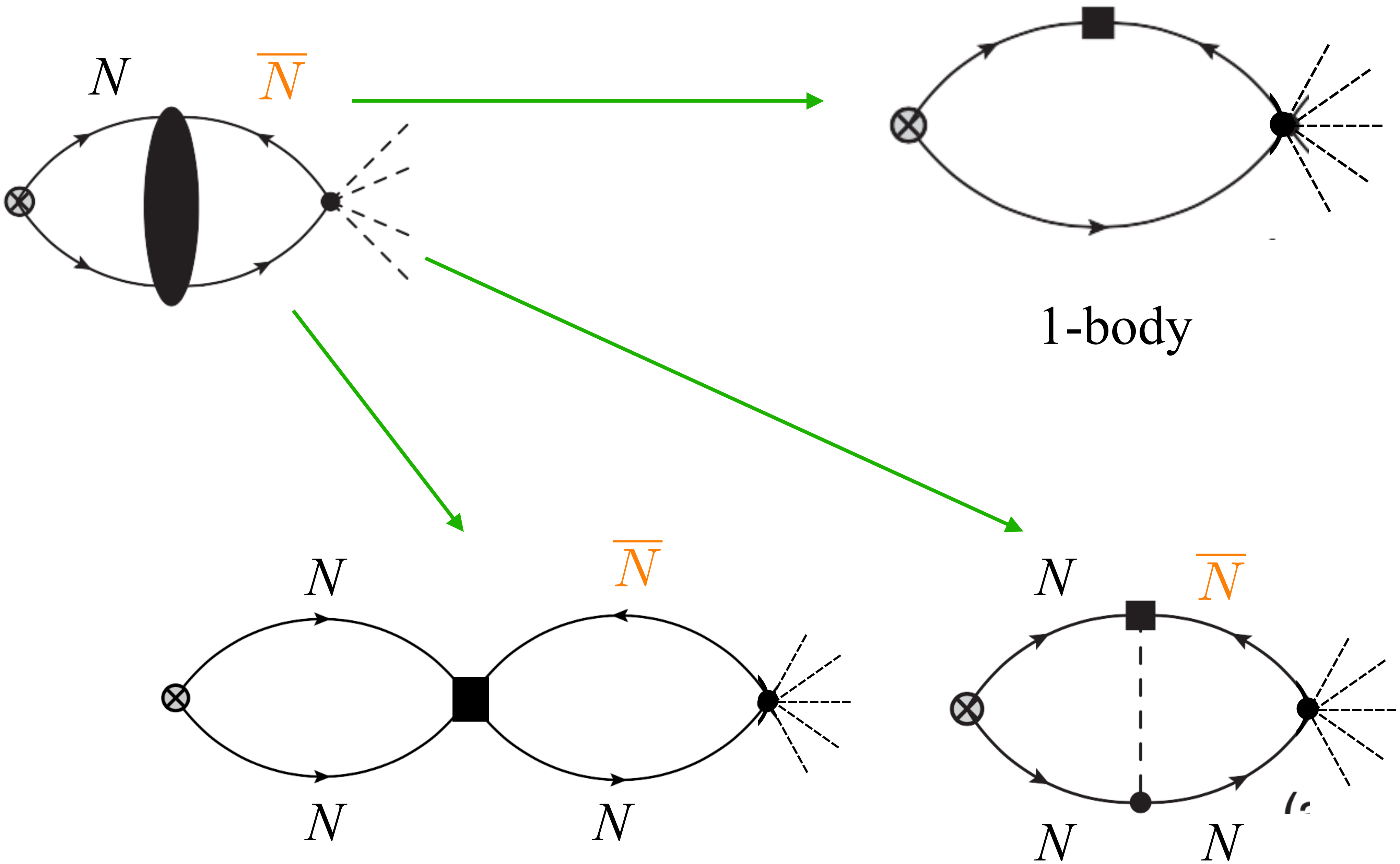
Phase shift in degrees ($j^{\pi} = 1/2^{+}$, S wave)

Tlab	1 MeV	2 MeV	3 MeV
LO	-20.2	-28.3	-37
NLO	-22.2	-33.1	-40.4
AV14	-17.8	-28	-34.9

Deuteron decay by *NN* → *NNbar*

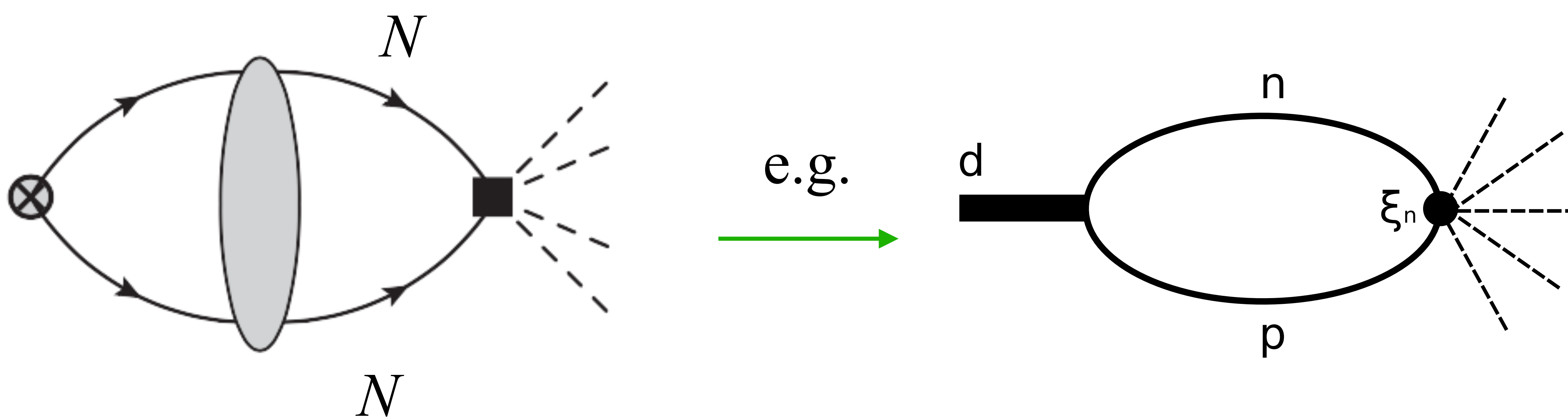
Oosterhof, BWL, de Vries, Timmermans & van Kolck
PRL 122 (2019) 17, 172501

neutron-antineutron osc.



2-body direct transitions

Direct *NN* annihilation



- To what extent can we disentangle these mechanisms?

Finally…

Oosterhof, BwL, de Vries, Timmermans & van Kolck
PRL 122 (2019) 17, 172501

$$R_d \equiv \Gamma_d^{-1}/\tau_{n\bar{n}}^2$$

Re($a_{\text{nbar-p}}$)

NN range

pion

$NN \leftrightarrow N\bar{N}$
w/ unknown B_0

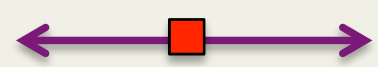
$$R_d = -\left[\frac{m_N}{\kappa}\mathrm{Im}\,a_{\bar{n}p}(1 + 0.40 + 0.20 - 0.13 \pm 0.4)\right]^{-1}$$
$$= (1.1 \pm 0.3) \times 10^{22}\ \mathrm{s}^{-1}.$$

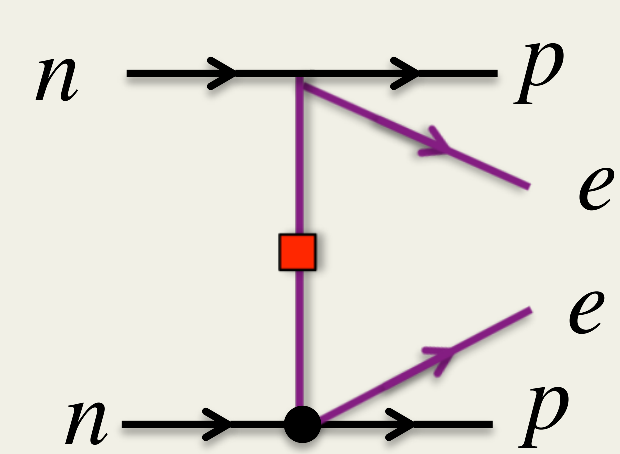
- Perturbative pion allows for analytic expression
- Loosely bound neutron helps sensitivity (nuclei with neutron halo?)
- B_0 gives largest uncertainty
- W/ nonperturbative pion EFT, unknown LECs may have smaller impact

Chiral effective field theory

~ GeV $L = L_{QCD} + L_{Fermi} - m_{\beta\beta}\nu_L^T C \nu_L$ light quarks and gluons + electrons + neutrinos

~100 MeV **Neutrinos are still degrees of freedom in the low-energy EFT**

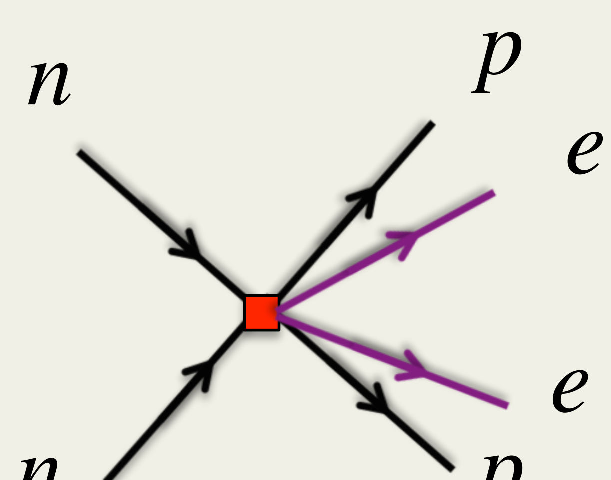
LO interaction : ν_L  ν_L $\sim m_{\beta\beta}$



Leads to long-range nn→ pp + ee $\sim \dfrac{m_{\beta\beta}}{q^2}$

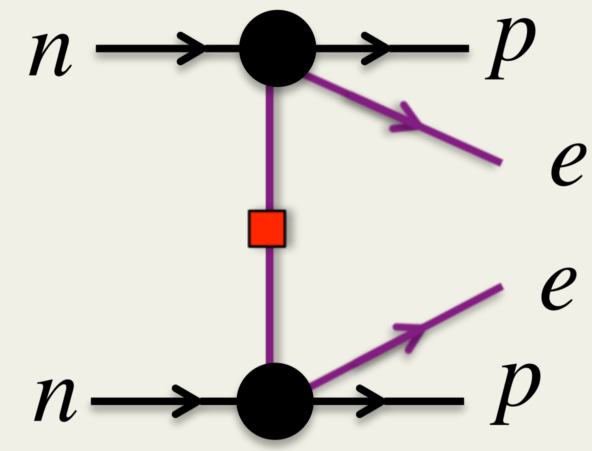
$q \sim k_F \sim m_\pi$

\`Hard' neutrino exchange $(E, |\vec{p}| > \Lambda_\chi)$ → short-range operators



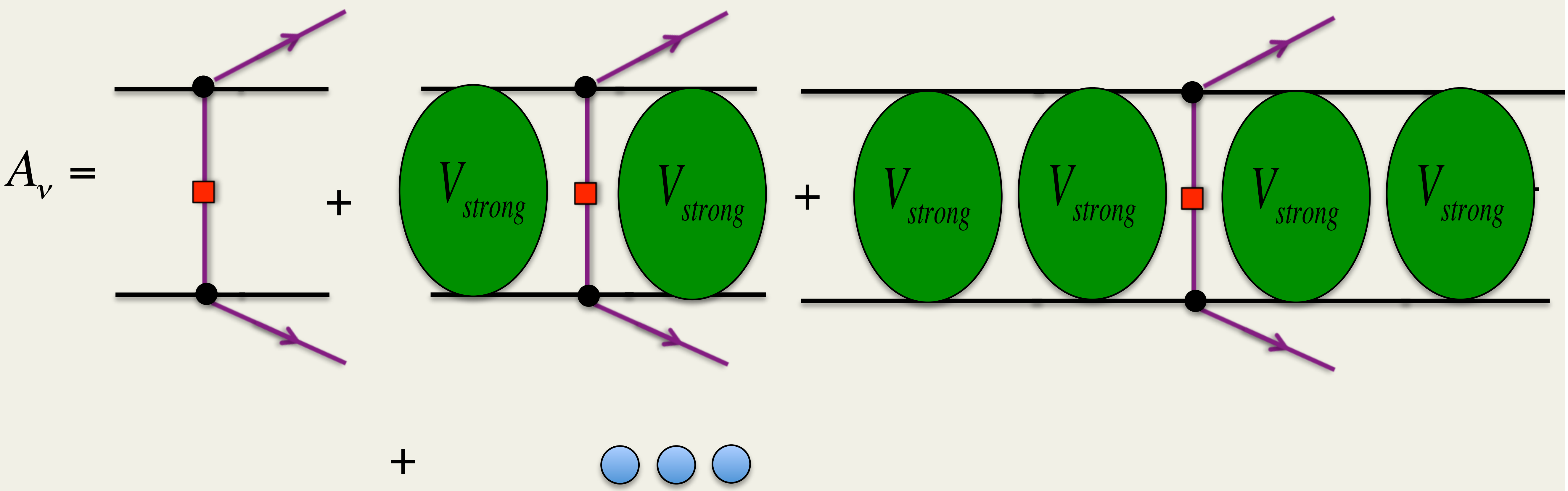
Expected at N²LO $\sim \dfrac{m_{\beta\beta}}{\Lambda_\chi^2}$

The neutrino amplitude



- At LO the 'standard' mechanism is long-range

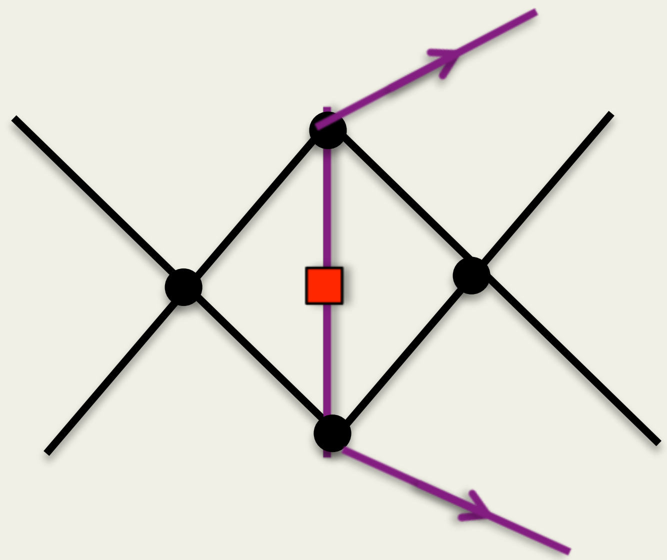
$$V_\nu = (2G_F^2 m_{\beta\beta})\tau_1^+\tau_2^+ \frac{1}{\vec{q}^2}\left[1 - g_A^2\left(\vec{\sigma}_1\cdot\vec{\sigma}_2 - \vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}\,\frac{2m_\pi^2+\vec{q}^2}{(m_\pi^2+\vec{q}^2)^2}\right)\right]\otimes \bar{e}_L e_L^c$$



Non-perturbative renormalization

Kaplan et al ’98

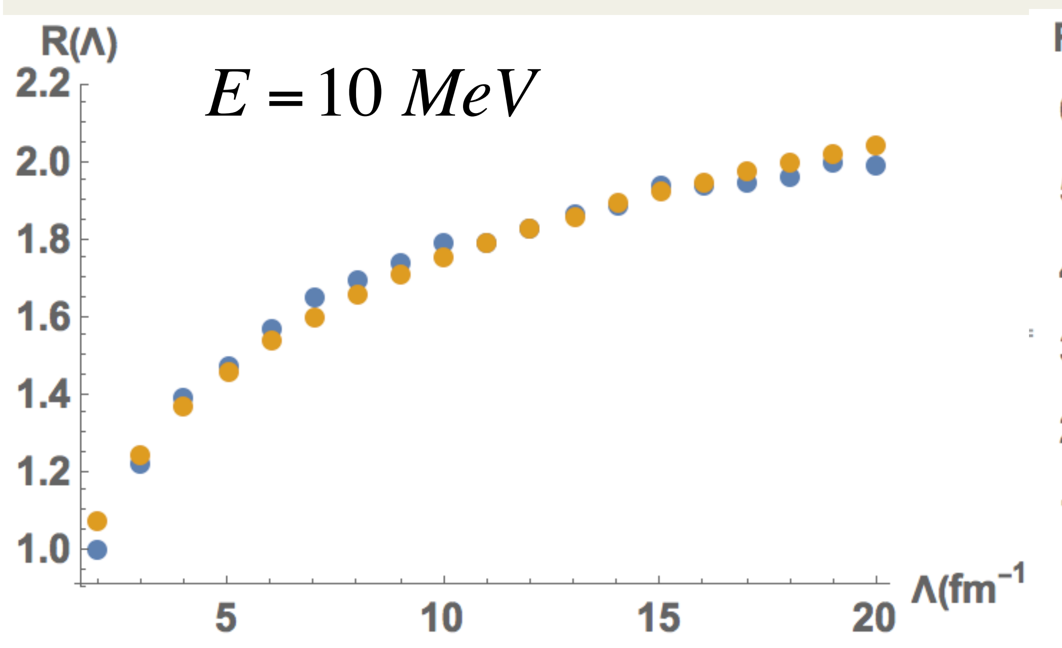
Can show analytically (dim-reg) two-loop diagram with two C_0 is UV divergent

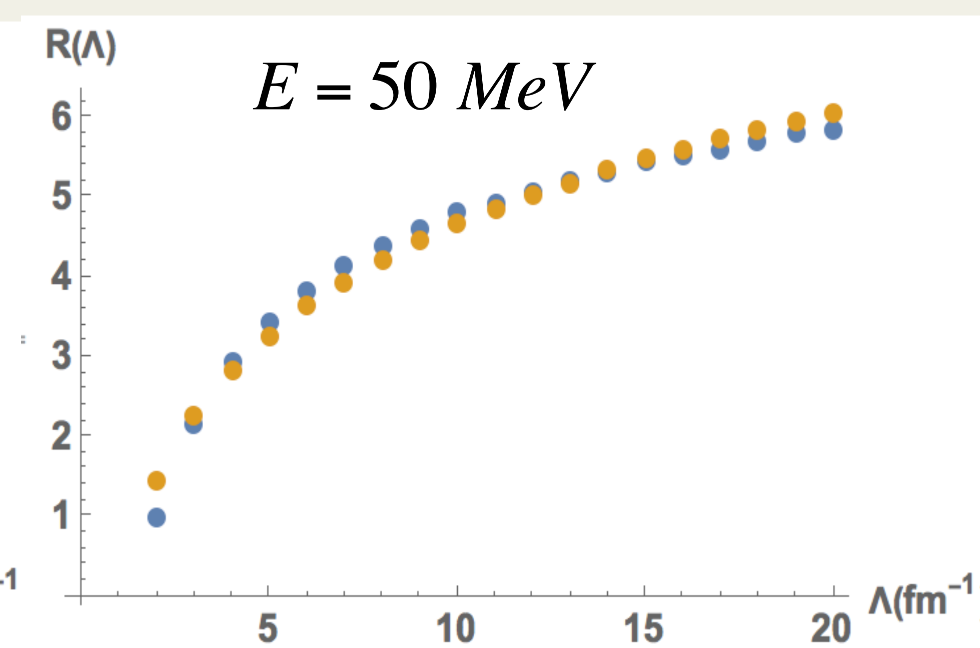


$$\sim (1+2g_A^2)\left(\frac{m_N C_0}{4\pi}\right)^2\left(\frac{1}{\varepsilon}+\log\frac{\mu^2}{p^2}\right)$$

Confirmed numerically for nn → pp +ee

$$R(\Lambda,E)=\frac{A_\nu(\Lambda,E)}{A_\nu(\Lambda=2\,fm^{-1},E)}$$





● numerical result

● fit to:

$$R(\Lambda,E)=$$
$$a+b\log(\Lambda\ fm)$$

Summary

- Chiral EFT has infinite number of LECs
  → power counting is crucial

- NDA good for counting long-range physics, but unreliable for short-range interactions

- RG analysis (UV cutoff independence) can be used as guideline to test PC for short-range physics