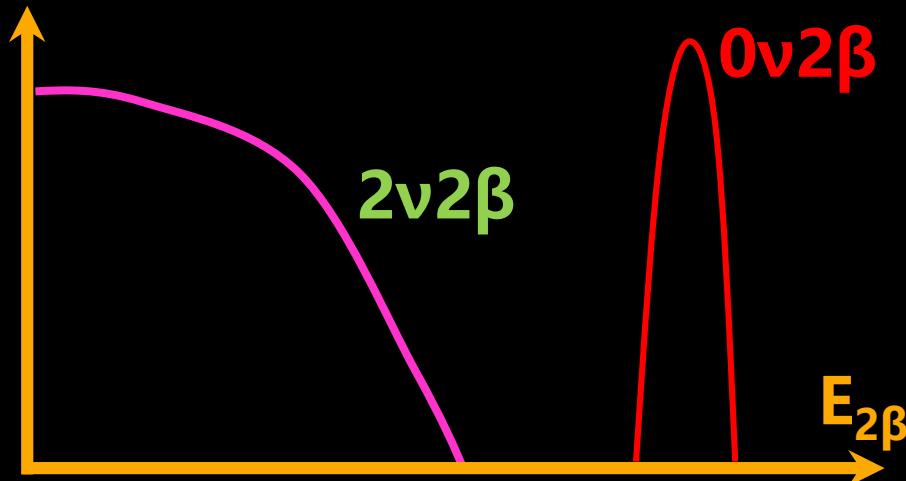
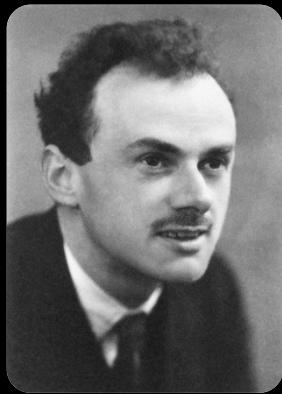


# Neutrinoless Double- $\beta$ Decays & $\nu$ Masses

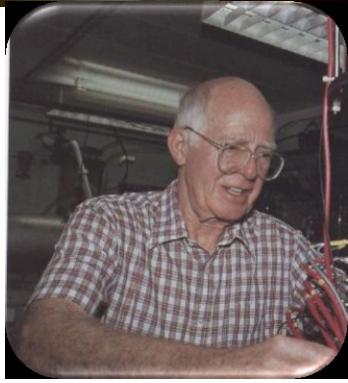
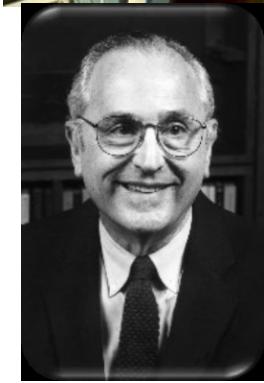


© E. Recami  
M. Majorana

Shun Zhou (IHEP & UCAS)

Workshop on Neutrinoless Double-Beta Decays, Sun Yat-Sen University,  
Zhuhai, 2021/5/21

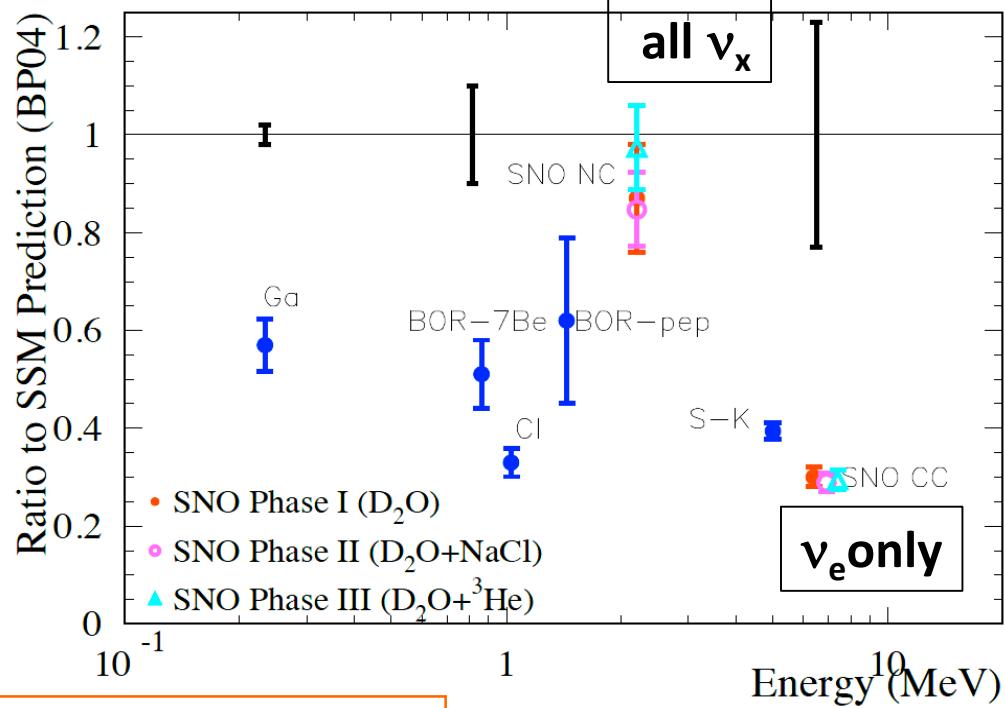
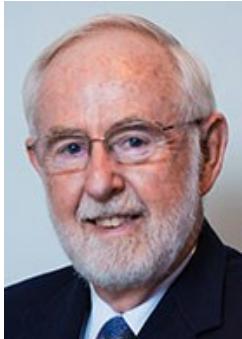
# Solar Neutrino Oscillations



**Discovery of solar neutrino oscillations**

**supported by  
KamLAND**

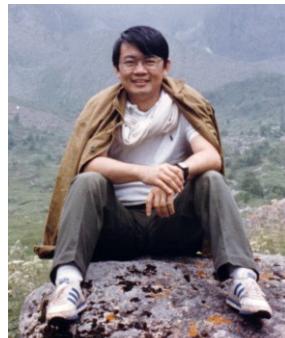
**A. B. McDonald**



CC:  $\nu_e + d \rightarrow p + p + e^-$

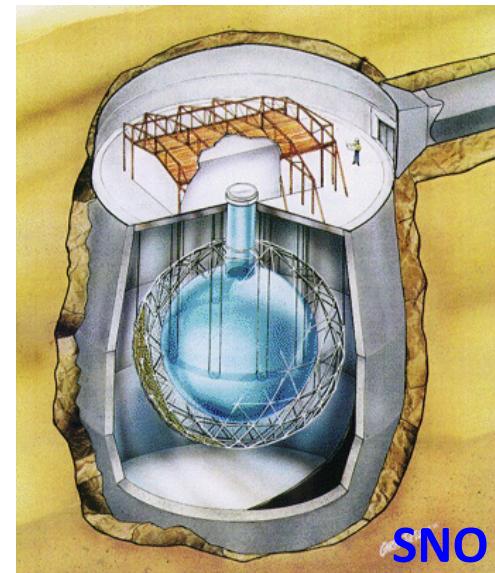
NC:  $\nu_\alpha + d \rightarrow p + n + \nu_\alpha$

ES:  $\nu_\alpha + e^- \rightarrow \nu_\alpha + e^-$



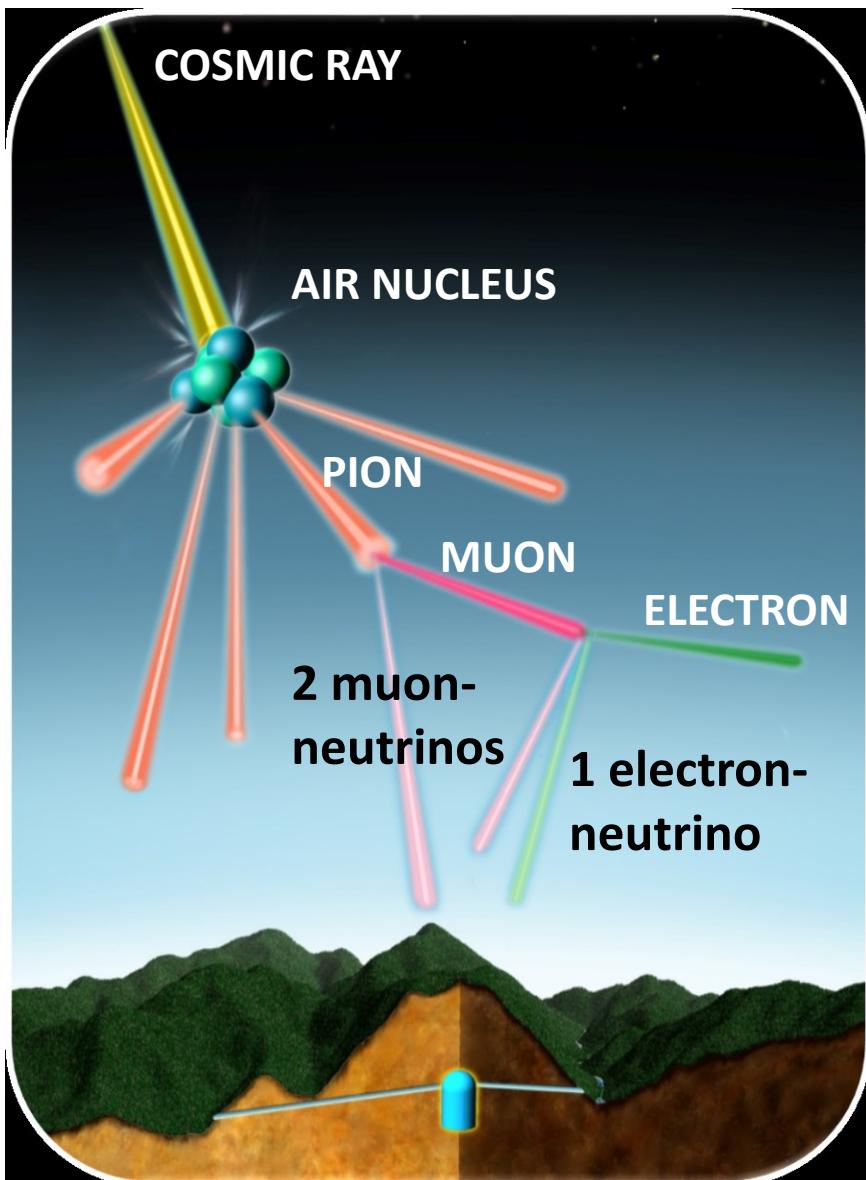
**Phys. Rev. Lett.  
55 (1985) 1534**

**Herbert H. Chen  
陈华森  
(1942-1987)**



# Atmospheric Neutrino Oscillations

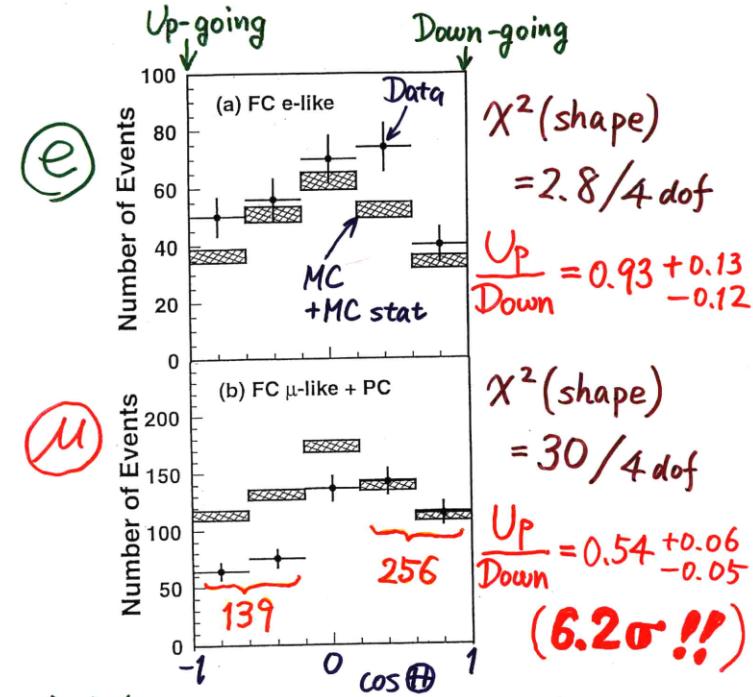
2



From Kajita, ICHEP 16

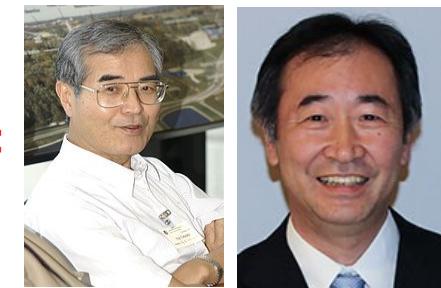
Super-Kamiokande @ Neutrino 98

Zenith angle dependence  
(Multi-GeV)



Discovery of atmospheric  
neutrino oscillations

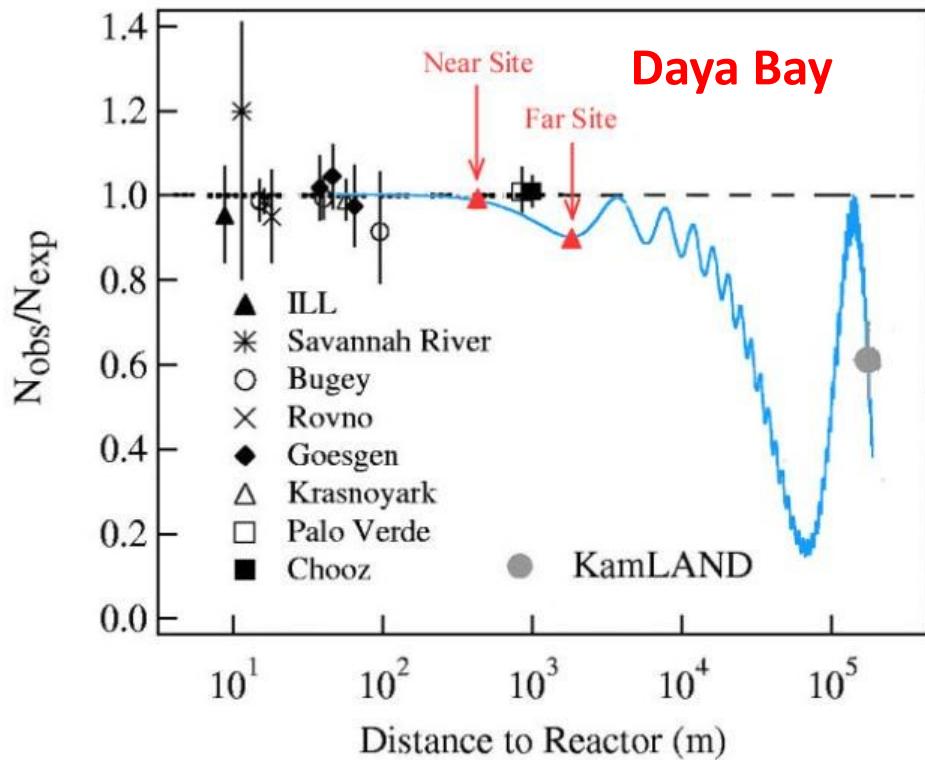
supported by  
K2K, MINOS, T2K, NOvA



Yoji Totsuka T. Kajita  
(1942-2008)

# Reactor Neutrino Oscillations

3



**Double Chooz (far detector):**

$$\sin^2 \theta_{13} = 0.022 \pm 0.013 \quad 1.7\sigma$$

**Daya Bay (near + far detectors):**

$$\sin^2 \theta_{13} = 0.024 \pm 0.004 \quad 5.2\sigma$$

**RENO (near + far detectors):**

$$\sin^2 \theta_{13} = 0.029 \pm 0.006 \quad 4.9\sigma$$

Dec. 2011

Mar. 2012

Apr. 2012



**Discovery of reactor neutrino oscillations**

**A complete picture of three-flavor neutrino oscillations!**

# Current Status of Neutrino Oscillations

4

## Basic neutrino parameters

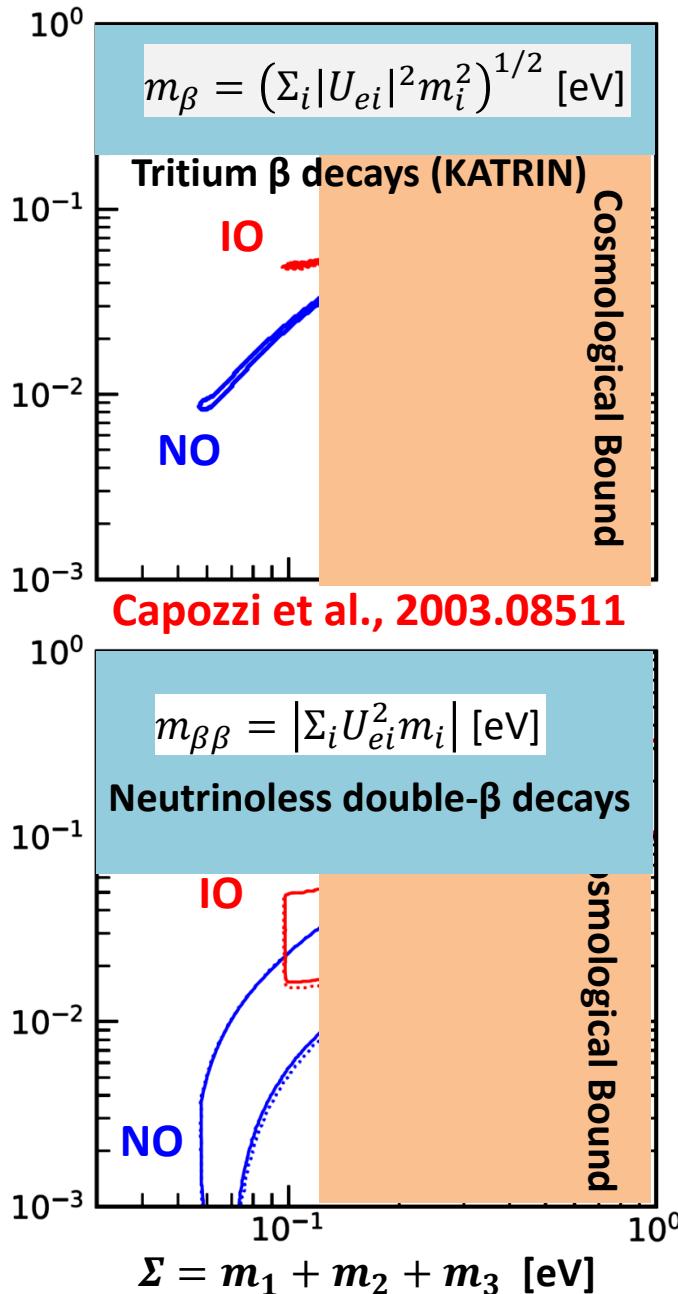
Esteban *et al.*, 2007.14792, NuFIT 5.0 (2020)

|                             | Normal Ordering (best fit)                        |                                 | Inverted Ordering ( $\Delta\chi^2 = 2.7$ ) |                                 |                               |
|-----------------------------|---|---------------------------------|--|---------------------------------|-------------------------------|
|                             | bfp $\pm 1\sigma$                                 | $3\sigma$ range                 | bfp $\pm 1\sigma$                          | $3\sigma$ range                 |                               |
| without SK atmospheric data | $\sin^2 \theta_{12}$                              | $0.304^{+0.013}_{-0.012}$       | $0.269 \rightarrow 0.343$                  | $0.304^{+0.013}_{-0.012}$       | $0.269 \rightarrow 0.343$     |
|                             | $\theta_{12}/^\circ$                              | $33.44^{+0.78}_{-0.75}$         | $31.27 \rightarrow 35.86$                  | $33.45^{+0.78}_{-0.75}$         | $31.27 \rightarrow 35.87$     |
|                             | $\sin^2 \theta_{23}$                              | $0.570^{+0.018}_{-0.024}$       | $0.407 \rightarrow 0.618$                  | $0.575^{+0.017}_{-0.021}$       | $0.411 \rightarrow 0.621$     |
|                             | $\theta_{23}/^\circ$                              | $49.0^{+1.1}_{-1.4}$            | $39.6 \rightarrow 51.8$                    | $49.3^{+1.0}_{-1.2}$            | $39.9 \rightarrow 52.0$       |
|                             | $\sin^2 \theta_{13}$                              | $0.02221^{+0.00068}_{-0.00062}$ | $0.02034 \rightarrow 0.02430$              | $0.02240^{+0.00062}_{-0.00062}$ | $0.02053 \rightarrow 0.02436$ |
|                             | $\theta_{13}/^\circ$                              | $8.57^{+0.13}_{-0.12}$          | $8.20 \rightarrow 8.97$                    | $8.61^{+0.12}_{-0.12}$          | $8.24 \rightarrow 8.98$       |
|                             | $\delta_{\text{CP}}/^\circ$                       | $195^{+51}_{-25}$               | $107 \rightarrow 403$                      | $286^{+27}_{-32}$               | $192 \rightarrow 360$         |
|                             | $\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$    | $7.42^{+0.21}_{-0.20}$          | $6.82 \rightarrow 8.04$                    | $7.42^{+0.21}_{-0.20}$          | $6.82 \rightarrow 8.04$       |
|                             | $\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$ | $+2.514^{+0.028}_{-0.027}$      | $+2.431 \rightarrow +2.598$                | $-2.497^{+0.028}_{-0.028}$      | $-2.583 \rightarrow -2.412$   |

- Future neutrino oscillation experiments will measure the octant of  $\theta_{23}$ , the CP-violating phase  $\delta$ , and the neutrino mass ordering
- The most restrictive bound on absolute neutrino masses is coming from cosmological observations:  $m_1 + m_2 + m_3 < 0.12 \text{ eV}$  (Planck)

# Current Status of Absolute Neutrino Masses

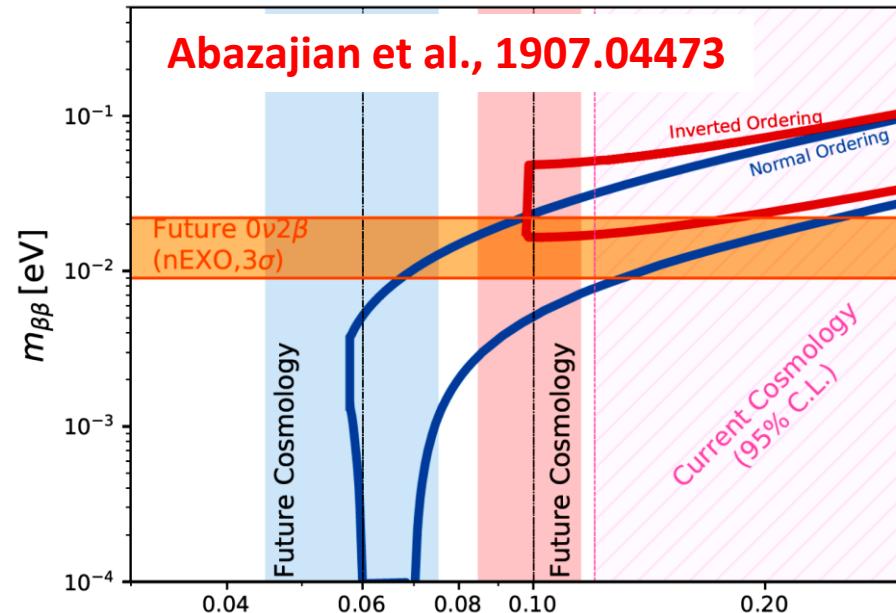
5



$m_1 < m_2 < m_3$  (NO) or  $m_3 < m_1 < m_2$  (IO)

Constraints on absolute neutrino masses

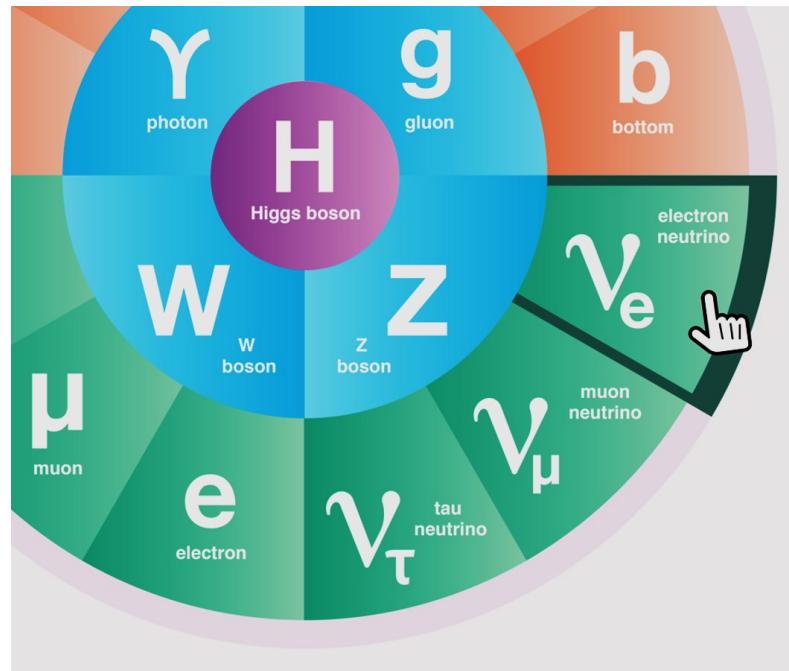
- Tritium β decays (95% C.L.)  
 $m_\beta < 0.8$  eV (KATRIN 2021)
- Neutrinoless double-β decays (90% C.L.)  
 $m_{\beta\beta} < (0.06 \sim 0.16)$  eV (KamLAND-Zen)  
 $(0.15 \sim 0.40)$  eV (EXO-200)  
 $(0.08 \sim 0.18)$  eV (GERDA-II)  
 $(0.08 \sim 0.35)$  eV (CUORE)
- Cosmological observations (95% probability)  
 $\Sigma < 0.12$  eV (Planck)



# Open Questions in Neutrino Physics

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- Normal or Inverted (sign of  $\Delta m_{31}^2$ ?)
- Leptonic CP Violation ( $\delta = ?$ )
- Octant of  $\theta_{23}$  ( $>$  or  $< 45^\circ$ ?)
- Absolute Neutrino Masses ( $m_{\text{lightest}} = 0?$ )
- Majorana or Dirac Nature ( $\nu = \nu^c ?$ )
- Majorana CP-Violating Phases (how?)



- Extra Neutrino Species
- Exotic Neutrino Interactions
- Various LNV & LFV Processes
- Leptonic Unitarity Violation

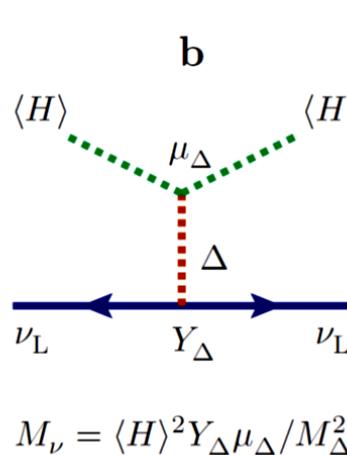
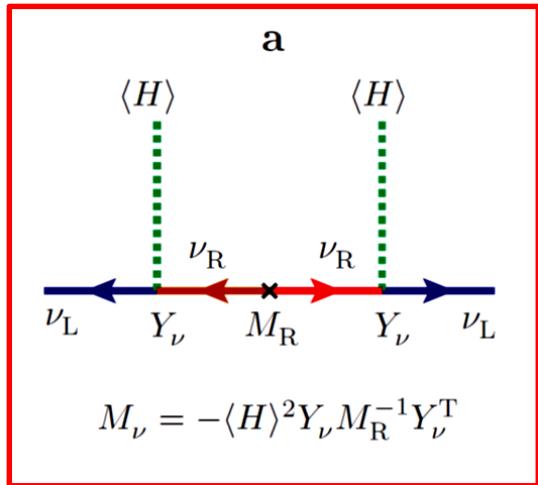
- Origin of Neutrino Masses
- Flavor Structure (Symmetry?)
- Quark-Lepton Connection
- Relations to DM, BAU, or NP

# Origin of Neutrino Masses

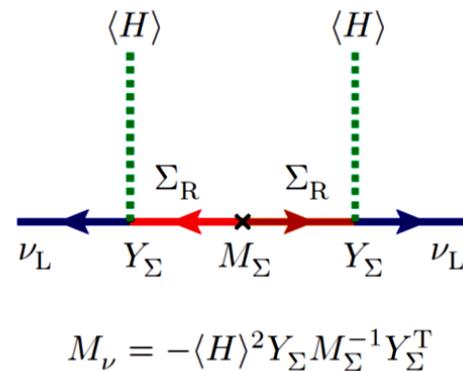
7

- Extend the SM with new particles but keep its gauge symmetries intact

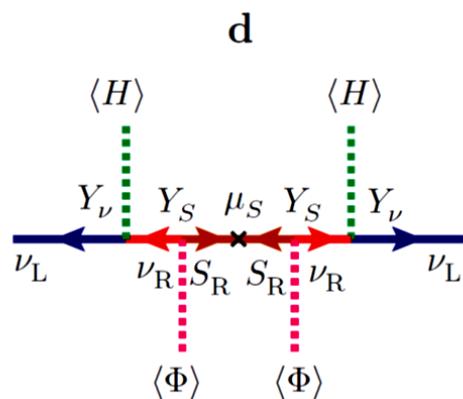
Canonical seesaw models



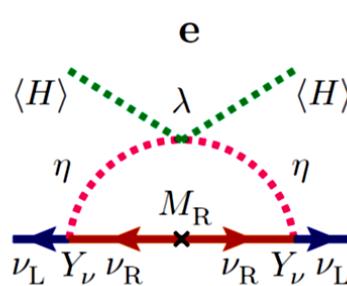
Ohlsson, SZ, Nat. Commun., 2014



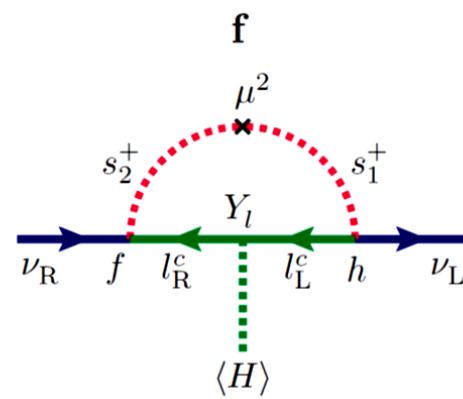
Inverse seesaw model



The scotogenic model



Radiative Dirac model



# History of $0\nu 2\beta$ Decays

SEPTEMBER 15, 1935

PHYSICAL REVIEW

VOLUME 48



## Double Beta-Disintegration

M. GOEPPERT-MAYER, *The Johns Hopkins University*

(Received May 20, 1935)



From the Fermi theory of  $\beta$ -disintegration the probability of simultaneous emission of two electrons (and two neutrinos) has been calculated. The result is that this process occurs sufficiently rarely to allow a half-life of over  $10^{17}$  years for a nucleus, even if its isobar of atomic number different by 2 were more stable by 20 times the electron mass.

In 1935, Maria Goeppert-Mayer published the first calculation of  $2\nu 2\beta$  decay rates based on the Fermi theory (proposed in 1934). It was acknowledged in her paper that Eugene Wigner suggested this problem.

DECEMBER 15, 1939

PHYSICAL REVIEW

VOLUME 56

## On Transition Probabilities in Double Beta-Disintegration

W. H. FURRY

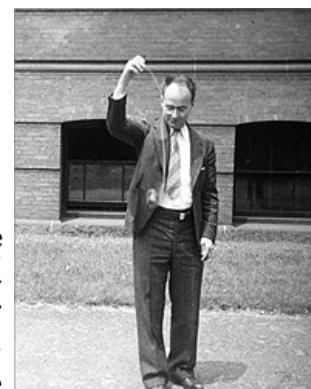
*Physics Research Laboratory, Harvard University, Cambridge, Massachusetts*

(Received October 16, 1939)



Ettore Majorana  
( $v=v^c$ ? 1937)

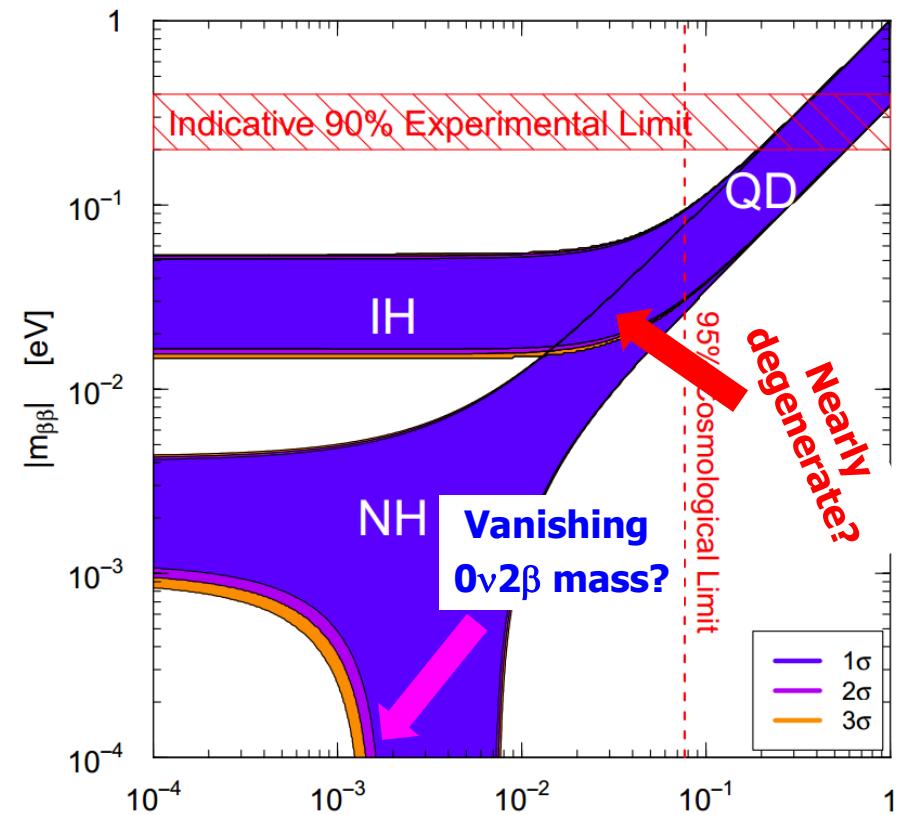
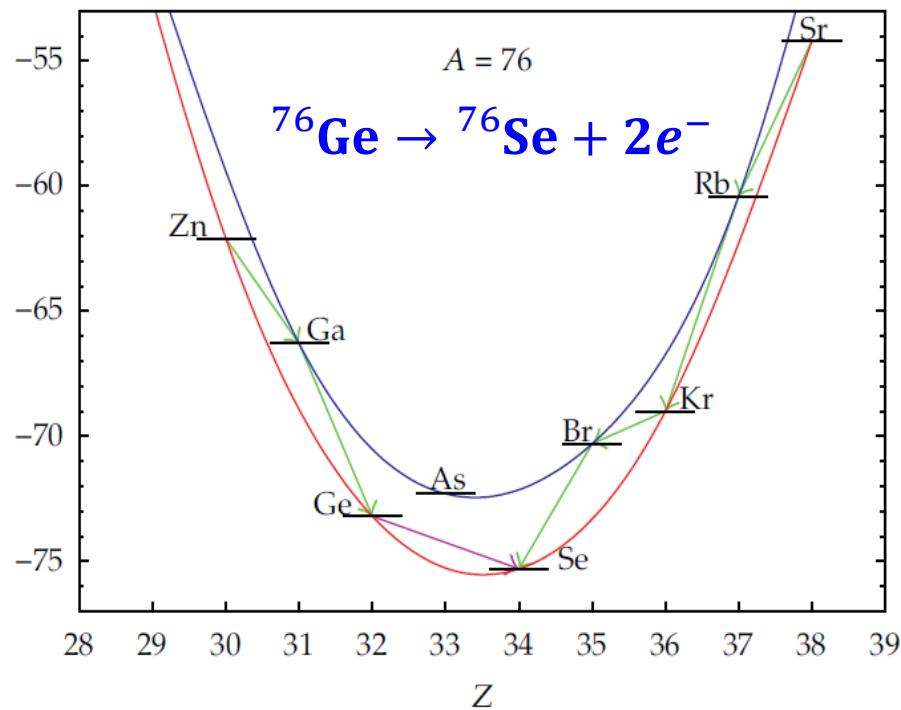
The phenomenon of double  $\beta$ -disintegration is one for which there is a marked difference between the results of Majorana's symmetrical theory of the neutrino and those of the original Dirac-Fermi theory. In the older theory double  $\beta$ -disintegration involves the emission of four particles, two electrons (or positrons) and two antineutrinos (or neutrinos), and the probability of disintegration is extremely small. In the Majorana theory only two particles—the electrons or positrons—have to be emitted, and the transition probability is much larger. Approximate values of this probability are calculated on the Majorana theory for the various Fermi and Konopinski-Uhlenbeck expressions for the interaction energy. The selection rules



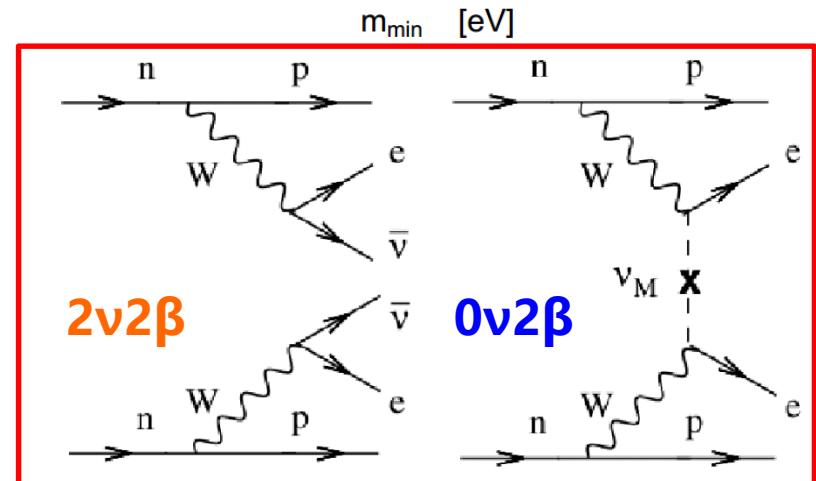
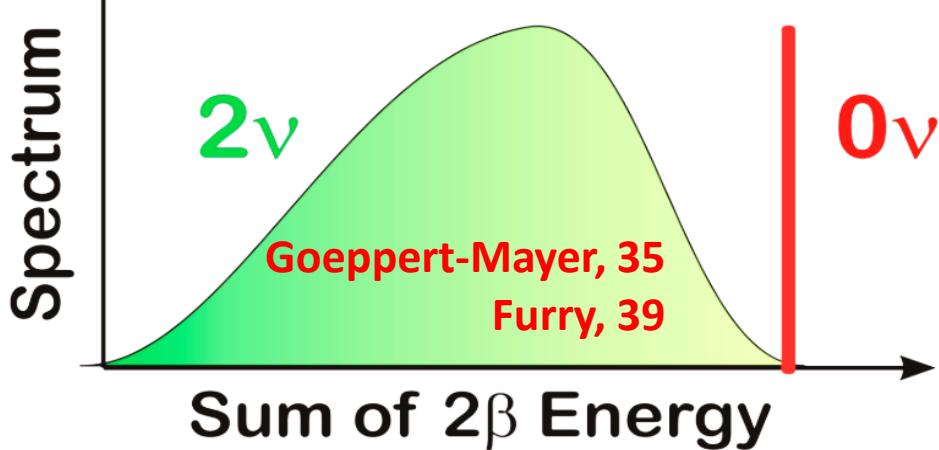
Wendell Furry  
(1907 – 1984)

# Basics of $0\nu2\beta$ Decays

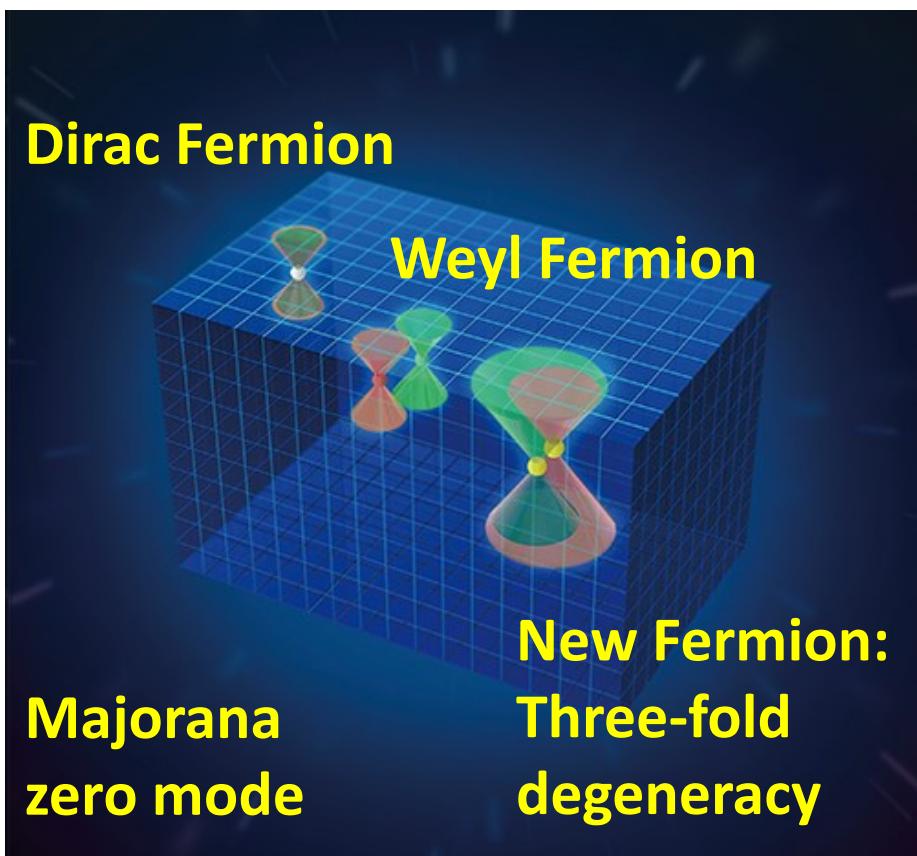
9



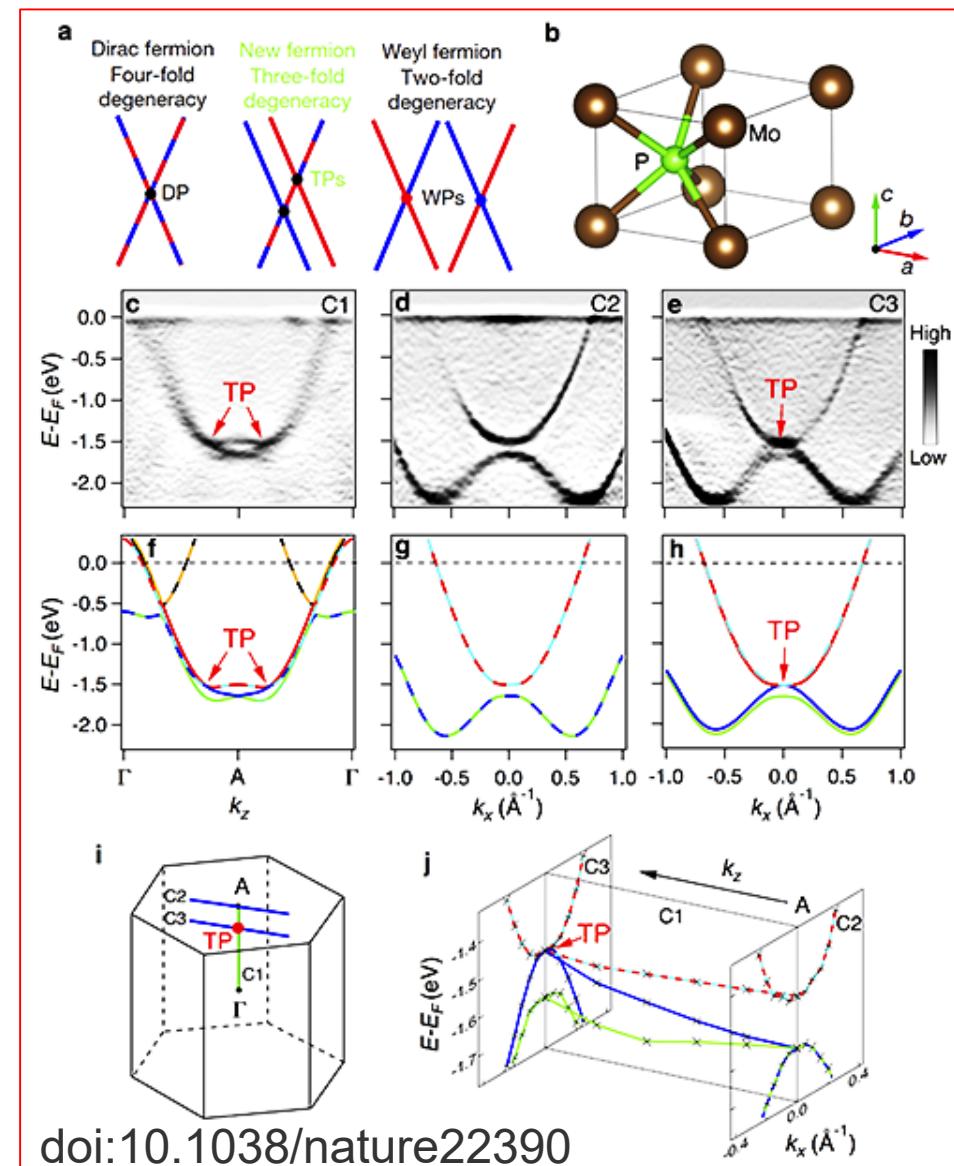
$$T_{1/2}^{0\nu} = G_{0\nu}^{-1} \cdot |\mathcal{M}_{0\nu}|^{-2} \cdot |m_{\beta\beta}|^{-2} \cdot m_e^2$$



## ★ A new feature of elementary particles: Majorana fermions



**HOT!!!** Weyl, Majorana, Dirac and new fermions in condensed matter physics,  
but **NOT** really elementary particles



## ★ Discovery of lepton number violation: baryon vs. lepton numbers

- Dirac Neutrinos

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \overline{\nu_R} i \not{d} \nu_R - \left[ \overline{\ell_L} Y_\nu \tilde{H} \nu_R + \text{h.c.} \right]$$

Generate Dirac  $\nu$  masses in a similar way to that for quarks and charged leptons, after the spontaneous gauge symmetry breaking

$$O(0.1 \text{ eV}) \xrightarrow{M_\nu = Y_\nu v} O(10^{-12}) \approx 174 \text{ GeV}$$

- Majorana Neutrinos

$$- \left[ \frac{1}{2} \overline{\nu_R^C} M_R \nu_R + \text{h.c.} \right]$$

Generate tiny Majorana  $\nu$  masses via the so-called seesaw mechanism

$$O(0.1 \text{ eV}) \xrightarrow{M_\nu = v^2 Y_\nu M_R^{-1} Y_\nu^T} O(10^{14} \text{ GeV})$$

- Need to introduce additional symmetries to the SM to forbid a Majorana mass for right-handed neutrino singlets
- Need to explain a strong hierarchy for the Yukawa couplings of the SM fermions

- Retain the SM symmetries
- GUT or TeV energy scale?

**Guide the theorists to build a model for tiny  $\nu$  masses**

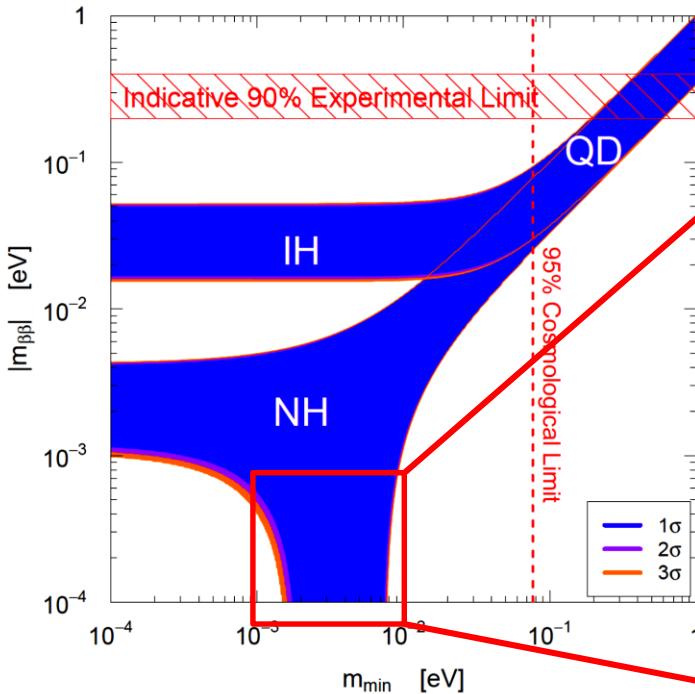
# Importance of $0\nu 2\beta$ Decays

12

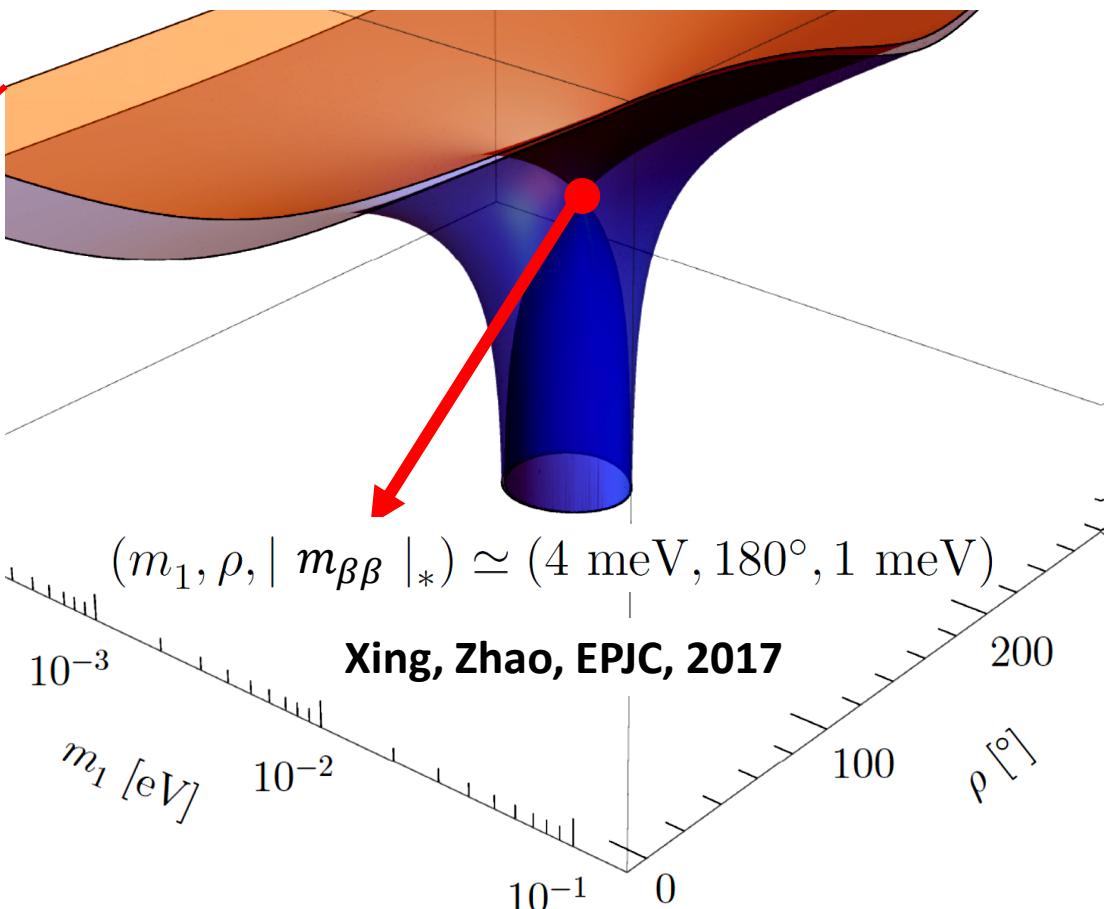
★ Constrain  $\nu$  masses and mixing parameters: Majorana CP phases?

$$T_{1/2}^{0\nu} = G_{0\nu}^{-1} \cdot |\mathcal{M}_{0\nu}|^{-2} \cdot |m_{\beta\beta}|^{-2} \cdot m_e^2$$

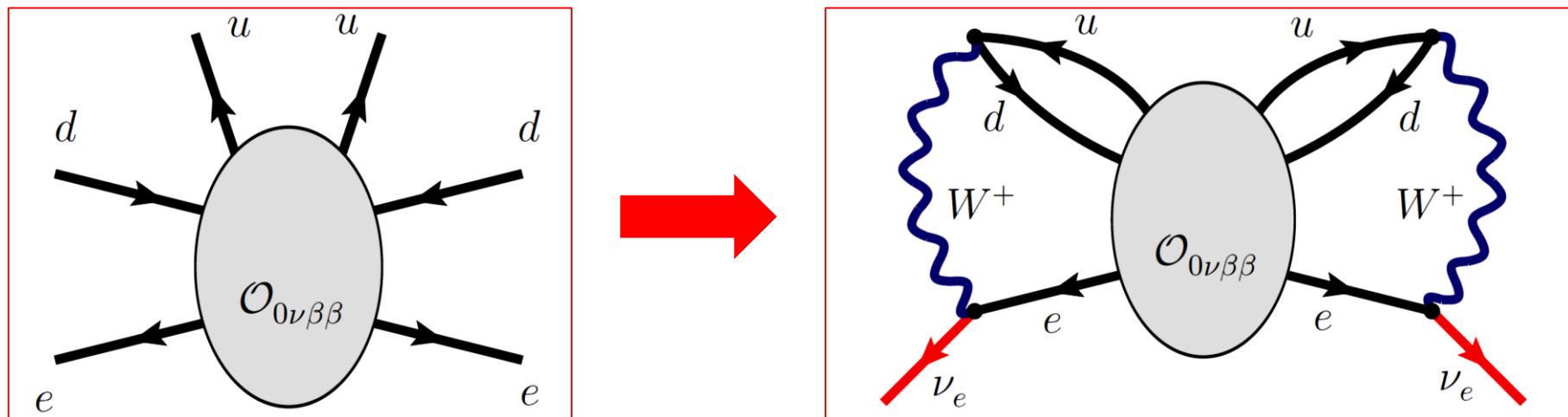
Rodejohann, IJMPE, 2011; JPG, 2012; Bilenky, Giunti, MPLA, 2012; IJMPA, 2015



$$m_{\beta\beta} = m_1 |U_{e1}|^2 e^{i\rho} + m_2 |U_{e2}|^2 + m_3 |U_{e3}|^2 e^{i\sigma}$$



- **$\nu$  mass ordering, lightest  $\nu$  mass, Majorana CP phases**
- **A unique way to constrain Majorana CP phases?**



**Schechter-Valle Theorem (82):** If the  $0\nu2\beta$  decay happens, there must exist an effective Majorana neutrino mass term.

**Question: How large is such a mass term?**

**Assumptions:** (1) no tree-level neutrino masses; (2)  $0\nu2\beta$  decays are mediated by heavy particles, i.e., short-range interactions:

Pas et al.,  
PLB, 2001

$$\mathcal{L}_{0\nu\beta\beta} = \frac{G_F^2}{2m_p} (\epsilon_1 J J j + \epsilon_2 J^{\mu\nu} J_{\mu\nu} j + \epsilon_3 J^\mu J_\mu j + \epsilon_4 J^\mu J_{\mu\nu} j^\nu + \epsilon_5 J^\mu J j_\mu)$$

Hadronic  
currents

$$J \equiv \bar{u}(1 \pm \gamma_5)d, \quad J^\mu \equiv \bar{u}\gamma^\mu(1 \pm \gamma_5)d, \quad J^{\mu\nu} \equiv \bar{u}\frac{i}{2}[\gamma^\mu, \gamma^\nu](1 \pm \gamma_5)d$$

leptonic  
currents

$$j = \bar{e}(1 \pm \gamma_5)e^c, \quad j^\mu = \bar{e}\gamma^\mu(1 \pm \gamma_5)e^c, \quad j^{\mu\nu} = \bar{e}\frac{i}{2}[\gamma^\mu, \gamma^\nu](1 \pm \gamma_5)e^c$$

- Current experimental upper bounds on the  $0\nu 2\beta$  decay width can be translated into **upper limits** on the coefficients  $\epsilon_i$ 's
- Nonzero quark and electron masses are crucial for us to draw the butterfly diagram, since we don't know the chirality of (u, d, e)

**Take one operator  
for example:**

$$\epsilon_3 J_R^\mu J_{\mu R} j_L$$

$$\epsilon_3 < 1.5 \times 10^{-8}$$

**Upper bound**

$$\delta m_\nu^{ee} = \frac{128g^4 G_F^2 \epsilon_3 m_u^2 m_d^2 m_e^2}{m_p} \mathcal{I}_{0\nu\beta\beta}$$

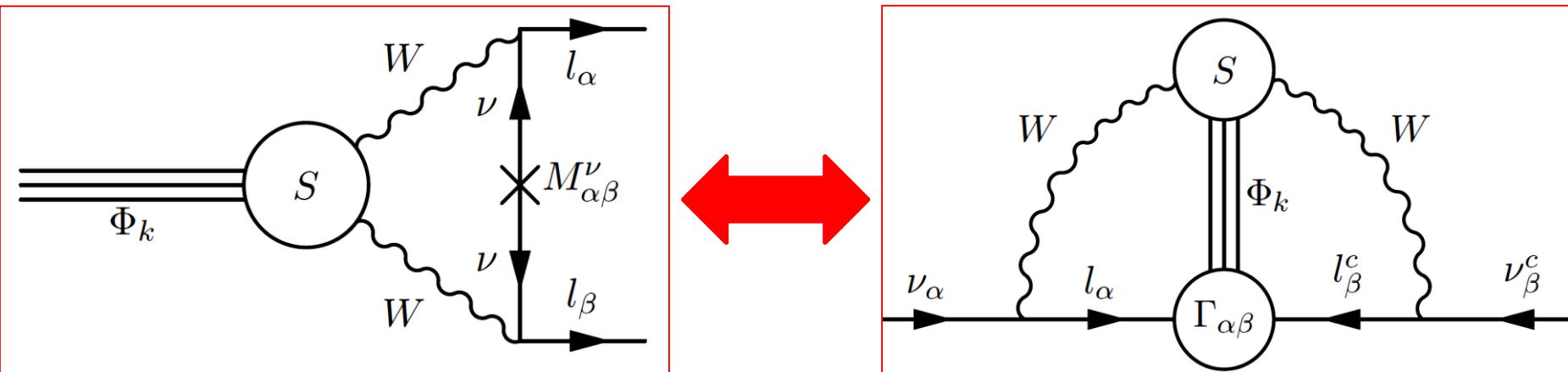
Duerr, Lindner,  
Merle, JHEP, 2011

Quantitatively, the 4-loop Majorana mass from the butterfly diagram is **EXTREMELY** small:

$$\delta m_\nu \lesssim O(10^{-28} \text{ eV})$$

Duerr, Lindner, Merle, JHEP, 2011;  
Liu, Zhang, Zhou, PLB, 2016

- Assume  $0\nu 2\beta$  decays are governed by short-distance operators
- The Schechter-Valle (Black Box) theorem is qualitatively correct, but the induced Majorana masses are **too small to be relevant** for neutrino oscillations
- Other mechanisms are needed to generate neutrino masses



**Hirsch, Kovalenko, Schmidt (06): One-to-one correspondence relation between LNV processes and corresponding elements of the Majorana neutrino mass matrix**

### Conclusion:

current neutrino oscillation data indicate a finite  
0v2β rate

$$A_1 : \begin{pmatrix} 0 & 0 & a \\ 0 & b & c \\ a & c & d \end{pmatrix}$$

$$\mathcal{M}_\nu^{(1)} = \begin{pmatrix} 0 & x & y \\ x & 0 & 0 \\ y & 0 & 0 \end{pmatrix} \quad \mathcal{M}_\nu^{(2)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & x & y \\ 0 & y & z \end{pmatrix} \quad \mathcal{M}_\nu^{(3)} = \begin{pmatrix} 0 & x & 0 \\ x & 0 & y \\ 0 & y & 0 \end{pmatrix}$$

All possible mass matrices with  $(M_\nu)_{ee} = 0$

$$\mathcal{M}_\nu^{(4)} = \begin{pmatrix} 0 & 0 & y \\ 0 & x & 0 \\ y & 0 & 0 \end{pmatrix} \quad \mathcal{M}_\nu^{(5)} = \begin{pmatrix} 0 & x & 0 \\ x & 0 & 0 \\ 0 & 0 & y \end{pmatrix}$$

The Majorana neutrino mass matrix  $A_1$  is allowed by oscillation data, leading to a nonzero 0v2β rate!

# Other Lepton-Number-Violating Decays

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**Meson Decays:**

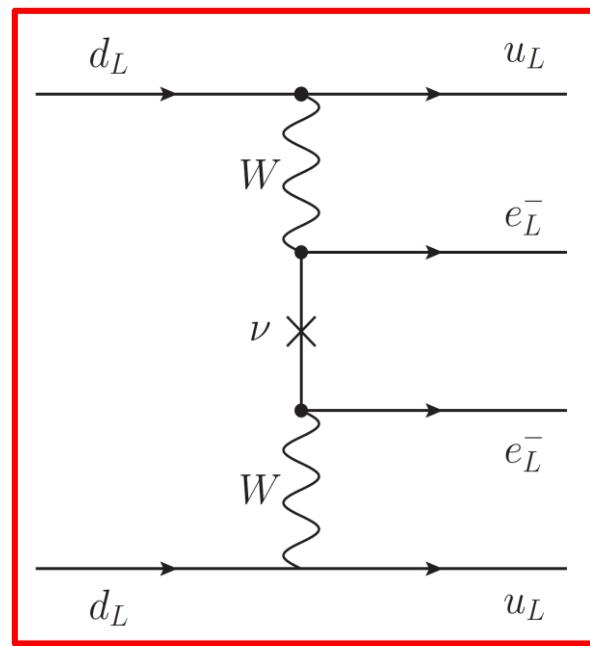
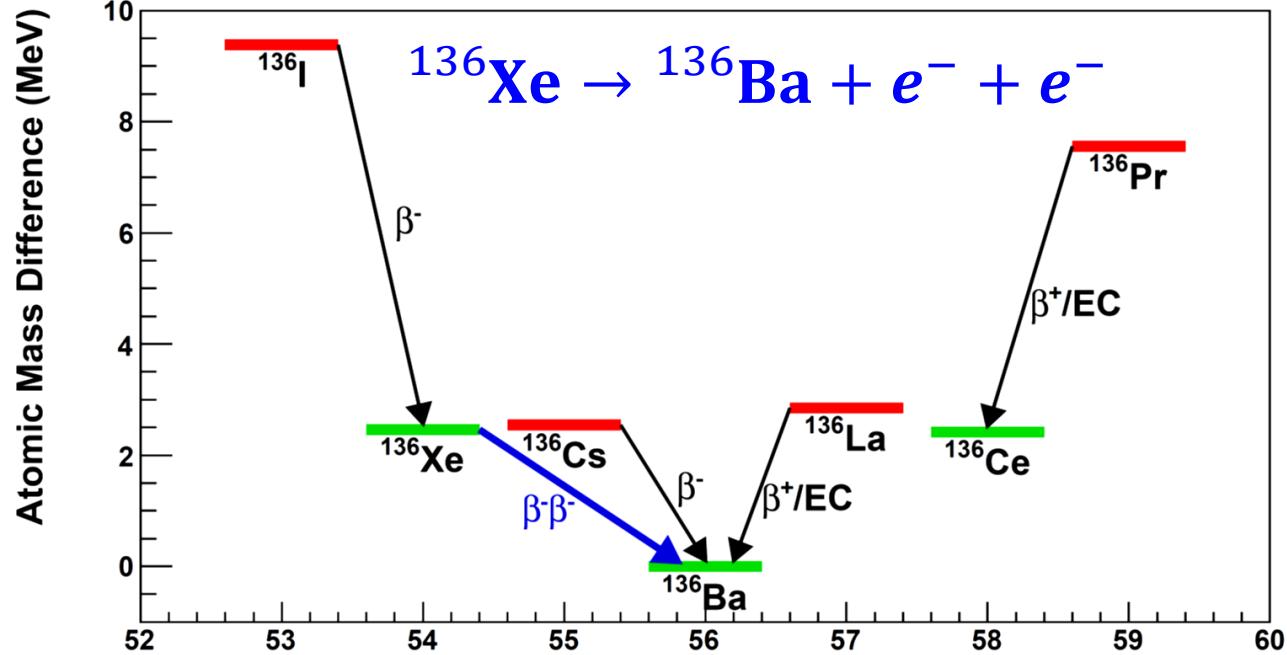
$$\mathcal{L}_{\text{MD}} = \varepsilon_{\alpha\beta} \frac{G_F^2}{2m_p} J_R^\mu J'_{\mu R} j_L$$

**Liu, Zhang, Zhou, PLB, 2016**

| Decay modes                            | Branching ratios        | Upper bounds on $\varepsilon_{\alpha\beta}$ | Upper bounds on $ \delta m_\nu^{\alpha\beta} $ (eV) |
|--|-------------------------|---|---|
| $K^- \rightarrow \pi^+ e^- e^-$        | $< 6.4 \times 10^{-10}$ | $9.0 \times 10^2$                           | $9.7 \times 10^{-18}$                               |
| $K^- \rightarrow \pi^+ \mu^- \mu^-$    | $< 1.1 \times 10^{-9}$  | $2.2 \times 10^3$                           | $1.0 \times 10^{-12}$                               |
| $K^- \rightarrow \pi^+ e^- \mu^-$      | $< 5.0 \times 10^{-10}$ | $7.3 \times 10^2$                           | $1.6 \times 10^{-15}$                               |
| $D^- \rightarrow \pi^+ e^- e^-$        | $< 1.1 \times 10^{-6}$  | $2.4 \times 10^4$                           | $7.3 \times 10^{-15}$                               |
| $D^- \rightarrow \pi^+ \mu^- \mu^-$    | $< 2.2 \times 10^{-8}$  | $3.5 \times 10^3$                           | $4.6 \times 10^{-11}$                               |
| $D^- \rightarrow \pi^+ e^- \mu^-$      | $< 2.0 \times 10^{-6}$  | $2.4 \times 10^4$                           | $1.5 \times 10^{-12}$                               |
| $D^- \rightarrow \rho^+ \mu^- \mu^-$   | $< 5.6 \times 10^{-4}$  | $1.0 \times 10^6$                           | $1.3 \times 10^{-8}$                                |
| $D^- \rightarrow K^+ e^- e^-$          | $< 9 \times 10^{-7}$    | $2.1 \times 10^4$                           | $2.5 \times 10^{-13}$                               |
| $D^- \rightarrow K^+ \mu^- \mu^-$      | $< 1.0 \times 10^{-5}$  | $7.2 \times 10^4$                           | $3.7 \times 10^{-8}$                                |
| $D^- \rightarrow K^+ e^- \mu^-$        | $< 1.9 \times 10^{-6}$  | $2.2 \times 10^4$                           | $5.5 \times 10^{-11}$                               |
| $D^- \rightarrow K^{*+} \mu^- \mu^-$   | $< 8.5 \times 10^{-4}$  | $1.7 \times 10^6$                           | $8.7 \times 10^{-7}$                                |
| $D_s^- \rightarrow \pi^+ e^- e^-$      | $< 4.1 \times 10^{-6}$  | $4.5 \times 10^4$                           | $5.5 \times 10^{-13}$                               |
| $D_s^- \rightarrow \pi^+ \mu^- \mu^-$  | $< 1.2 \times 10^{-7}$  | $7.9 \times 10^3$                           | $4.1 \times 10^{-9}$                                |
| $D_s^- \rightarrow \pi^+ e^- \mu^-$    | $< 8.4 \times 10^{-6}$  | $4.6 \times 10^4$                           | $1.2 \times 10^{-10}$                               |
| $D_s^- \rightarrow K^+ e^- e^-$        | $< 5.2 \times 10^{-6}$  | $4.7 \times 10^4$                           | $5.6 \times 10^{-12}$                               |
| $D_s^- \rightarrow K^+ \mu^- \mu^-$    | $< 1.3 \times 10^{-5}$  | $7.7 \times 10^4$                           | $3.9 \times 10^{-7}$                                |
| $D_s^- \rightarrow K^+ e^- \mu^-$      | $< 6.1 \times 10^{-6}$  | $3.7 \times 10^4$                           | $8.9 \times 10^{-10}$                               |
| $D_s^- \rightarrow K^{*+} \mu^- \mu^-$ | $< 1.4 \times 10^{-3}$  | $1.8 \times 10^6$                           | $9.1 \times 10^{-6}$                                |

# Experimental Sensitivities

17

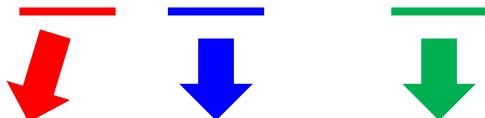


Gomez-Cadenas et al., 1109.5155

Atomic Number Z      Light  $\nu$  mediated  $0\nu\beta\beta$

The  $0\nu\beta\beta$ -decay half-life

$$T_{1/2}^{0\nu} = G_{0\nu}^{-1} \cdot |\mathcal{M}_{0\nu}|^{-2} \cdot |m_{\beta\beta}|^{-2} \cdot m_e^2$$



Phase space

Nuclear Matrix Element

Effective  $\nu$  mass

➤ Experimentalists: fix the half-life

➤ Nuclear theorists: calculate NME

➤ Particle theorists: effective mass  
 $\nu$  masses & Majorana CP phases

$$|m_{\beta\beta}| \equiv |m_1 \cos^2 \theta_{13} \cos^2 \theta_{12} e^{i\phi} + m_2 \cos^2 \theta_{13} \sin^2 \theta_{12} + m_3 \sin^2 \theta_{13} e^{i\sigma}|$$

## How to extract the $0\nu\beta\beta$ -decay half-life:

Number  
of events

$$N = \frac{\ln 2}{T_{1/2}^{0\nu}} \cdot t \cdot \frac{M_{\text{iso}} \cdot N_A}{m_{\text{iso}}} \cdot \epsilon$$

Detection  
efficiency

Toy model:

➤ **Background-free setup:**  $N = S$

Decay rate

Number of nuclei

$$T_{1/2}^{0\nu} = G_{0\nu}^{-1} \cdot |\mathcal{M}_{0\nu}|^{-2} \cdot |m_{\beta\beta}|^{-2} \cdot m_e^2$$

➤ **Require  $N < 1$**  →

$$T_{1/2}^{0\nu} > \ln 2 \cdot t \cdot \frac{M_{\text{iso}} \cdot N_A}{m_{\text{iso}}} \cdot \epsilon$$

Upper limit on  
effective mass

Realistic case:

$$T_{1/2}^{0\nu} = \ln 2 \cdot \frac{N_A \cdot \xi \cdot \epsilon}{m_{\text{iso}} \cdot S(B)}$$

$\xi \equiv M_{\text{iso}} \cdot t$  exposure

$S(B)$  Number of signal events  
given background  $B$  in ROI

$$\text{PDF}_{\text{Poisson}}(n, \mu) = e^{-\mu} \mu^n / n!$$

$$\text{CDF}_{\text{Poisson}}(n, \mu) \equiv \sum_{k=0}^n \text{PDF}_{\text{Poisson}}(k, \mu)$$

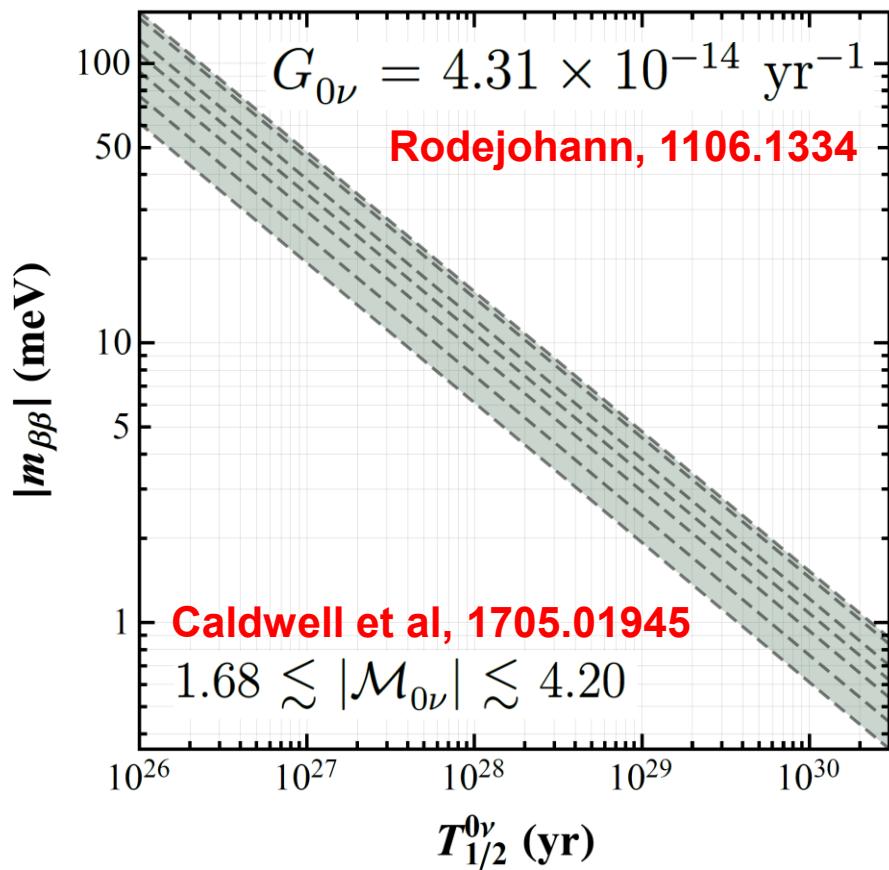
$$\text{CDF}_{\text{Poisson}}(n_p, B) \geq p$$

$$\overline{\text{CDF}}_{\text{Poisson}}(n_p, B + S) = q$$

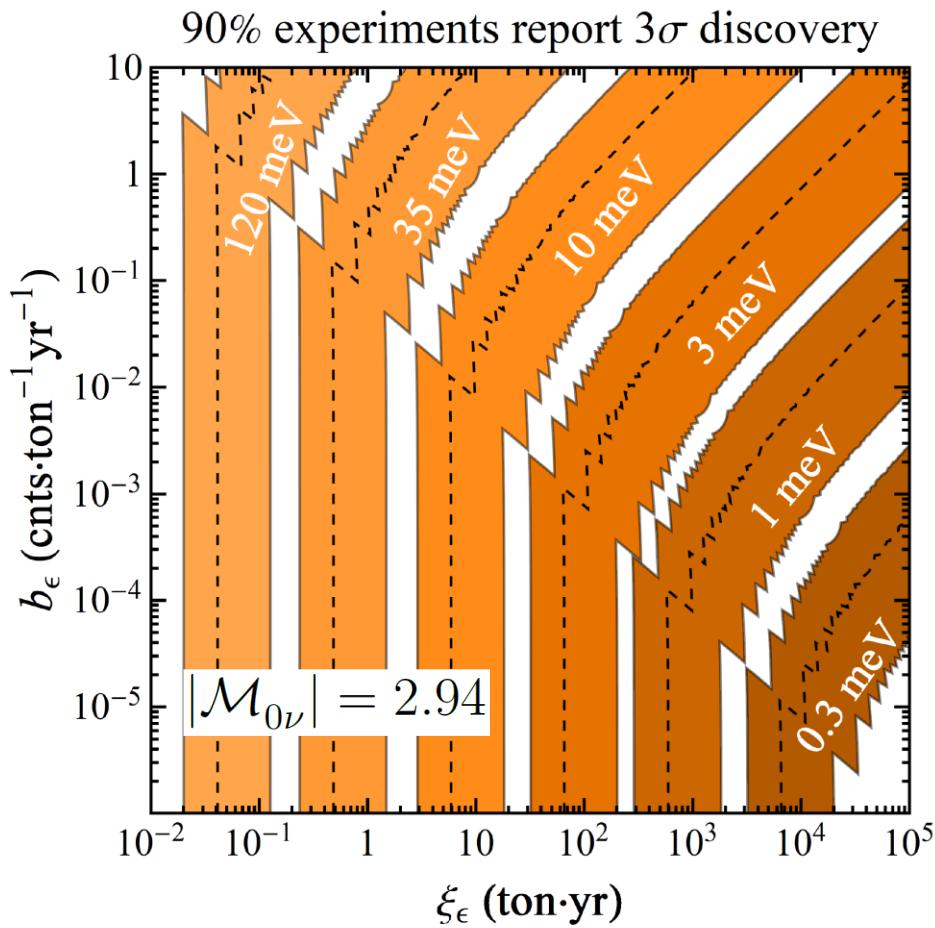
$$\overline{\text{CDF}}_{\text{Poisson}} = 1 - \text{CDF}_{\text{Poisson}}$$

# Experimental Sensitivities

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G.Y. Huang, SZ, 2010.16281



$$|m_{\beta\beta}| = 10 \text{ meV} \rightarrow T_{1/2}^{0\nu} \in (3.8 \times 10^{27} \dots 2.5 \times 10^{28}) \text{ yr}$$

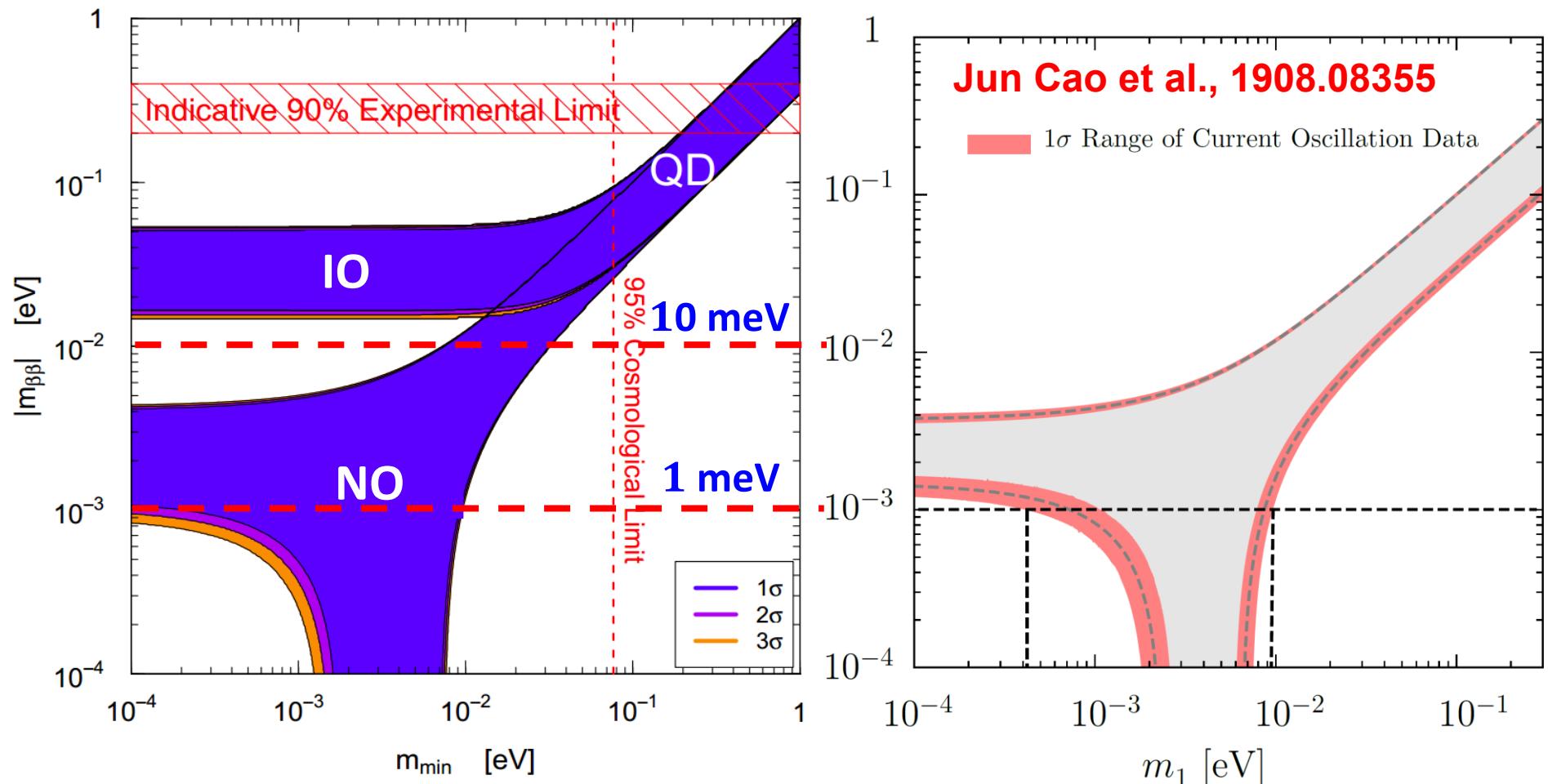
$$|m_{\beta\beta}| = 1 \text{ meV} \rightarrow T_{1/2}^{0\nu} \in (3.8 \times 10^{29} \dots 2.5 \times 10^{30}) \text{ yr}$$

**1 meV sensitivity**

$$\xi_\epsilon \simeq 300 \text{ ton} \cdot \text{yr}$$

# Probe v Masses & Majorana CP Phases

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**Effective v mass**

$$|m_{\beta\beta}| \equiv |m_1 \cos^2 \theta_{13} \cos^2 \theta_{12} e^{i\rho} + m_2 \cos^2 \theta_{13} \sin^2 \theta_{12} + m_3 \sin^2 \theta_{13} e^{i\sigma}|$$

- We assume Majorana neutrinos with NO, and observe no signal events
- If this is true even for future 1 meV sensitivity, then what can we learn?

**Bayes' Theorem**  $P(\mathcal{H}_i|\mathcal{D}) = \frac{P(\mathcal{D}|\mathcal{H}_i)P(\mathcal{H}_i)}{P(\mathcal{D})}$

Posterior probability      likelihood      → Prior probability  
↓                          → normalization

**Model selection:**  $\frac{P(\mathcal{H}_i|\mathcal{D})}{P(\mathcal{H}_j|\mathcal{D})} = \frac{\mathcal{Z}_i}{\mathcal{Z}_j} \frac{P(\mathcal{H}_i)}{P(\mathcal{H}_j)}$

**Bayes' factor**  
 $\mathcal{B} \equiv \mathcal{Z}_i / \mathcal{Z}_j$

**Parameter estimation:**  $P(\Theta|\mathcal{H}_i, \mathcal{D}) = \frac{P(\mathcal{D}|\mathcal{H}_i, \Theta)P(\Theta|\mathcal{H}_i)}{P(\mathcal{D}|\mathcal{H}_i)}$

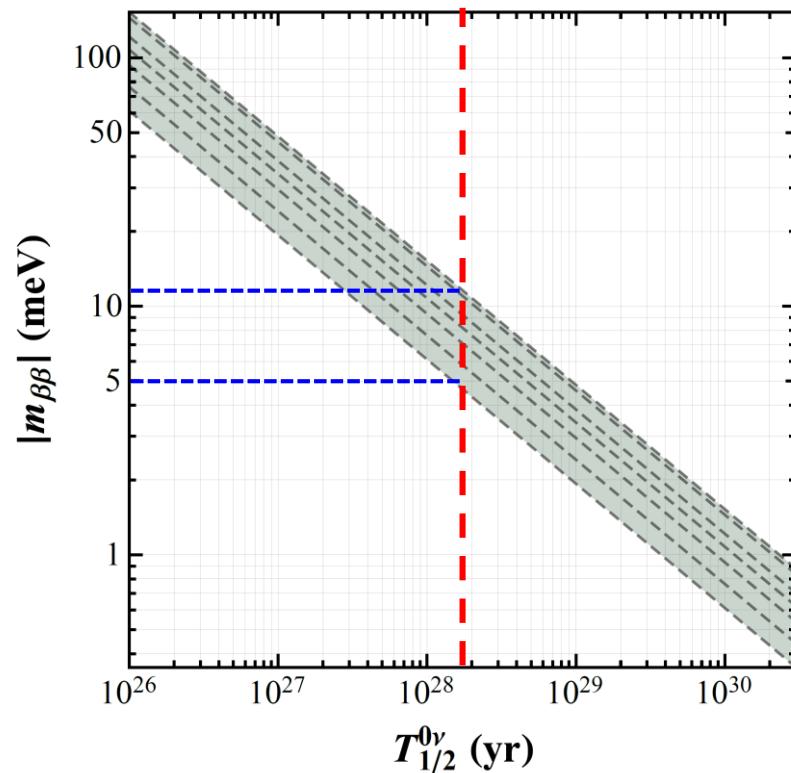
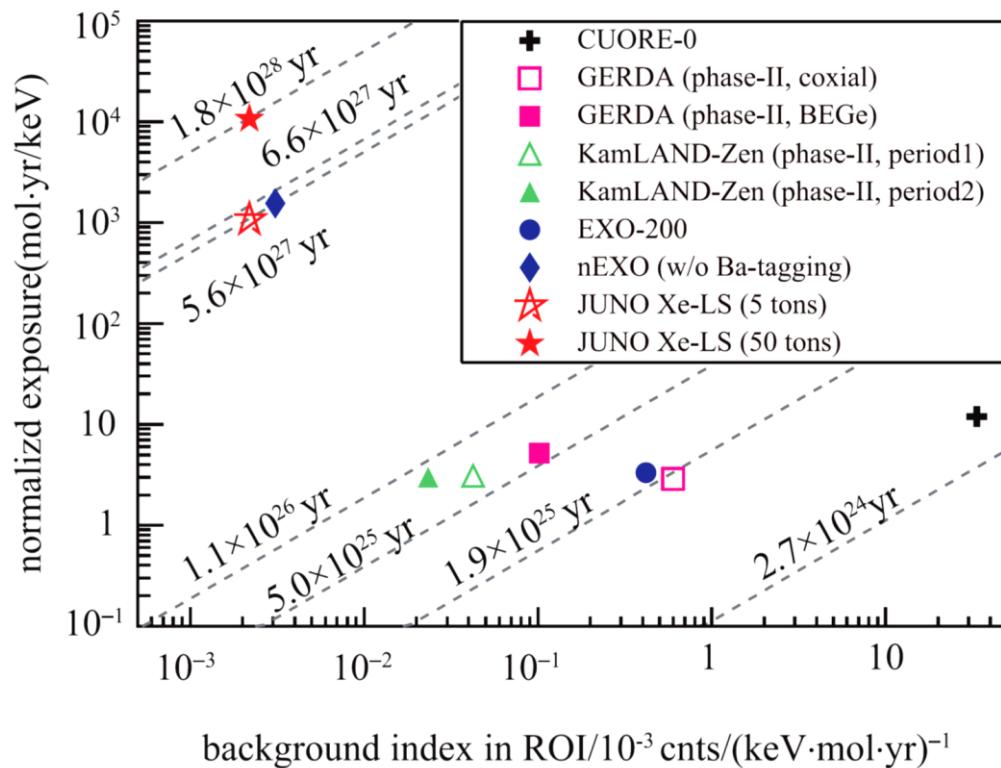
**Calculate the evidence**  $\mathcal{Z}_i = \int P(\mathcal{D}|\mathcal{H}_i, \Theta)P(\Theta|\mathcal{H}_i)d\Theta$

**Central idea:** Sivia & Skilling, *Data Analysis: A Bayesian Tutorial*, 2006

- Before including the new experimental data, we have to specify prior info.
- Update our knowledge by using the posterior of previous expts. as priors

## Motivation for two setups

J. Zhao, L.J. Wen, Y.F. Wang, J. Cao, 1610.07143



**Setup I: JUNO Xe-LS**     $\xi = 50 \text{ ton}\cdot 5 \text{ yr}$      $\epsilon = 0.634$      $b = 1.35 \text{ ton}^{-1} \cdot \text{yr}^{-1}$

$$T_{1/2}^{0\nu} = 6.24 \times 10^{27} \text{ yr} \quad |m_{\beta\beta}| = (7.9 \dots 19.7) \text{ meV} \quad @3\sigma \text{ C.L.}$$

**Setup II: Ideal scenario**     $\xi = 400 \text{ ton}\cdot 5 \text{ yr}$      $\epsilon = 1$      $b = 0 \text{ ton}^{-1} \cdot \text{yr}^{-1}$

$$T_{1/2}^{0\nu} = 2.67 \times 10^{30} \text{ yr} \quad |m_{\beta\beta}| = (0.38 \dots 0.95) \text{ meV} \quad @3\sigma \text{ C.L.}$$

- First, we specify the model (i.e., Majorana neutrinos & NO) and the model parameters are the lightest neutrino mass  $m_1$ , two neutrino mass-squared differences  $\{\Delta m_{21}^2, \Delta m_{31}^2\}$ , two mixing angles  $\{\theta_{12}, \theta_{13}\}$  and two Majorana CP phases  $\{\rho, \sigma\}$
- Second, we state our current knowledge on these parameters, i.e., priors.  
 Neutrino oscillations:  $\{\Delta m_{21}^2, \Delta m_{31}^2\}$  and  $\{\theta_{12}, \theta_{13}\}$ ; Uniform distribution:  $\{\rho, \sigma\}$  in the whole range  $[0, 2\pi]$ ; Uniform distribution for NME in whole range  $[1.68, 4.20]$ ; Gaussian distribution for the phase factor; Uniform distribution:  $m_1$

**Flat:**  $m_1/\text{eV} \in [10^{-7}, 10]$

**Log:**  $\log_{10}(m_1/\text{eV}) \in [-7, 1]$

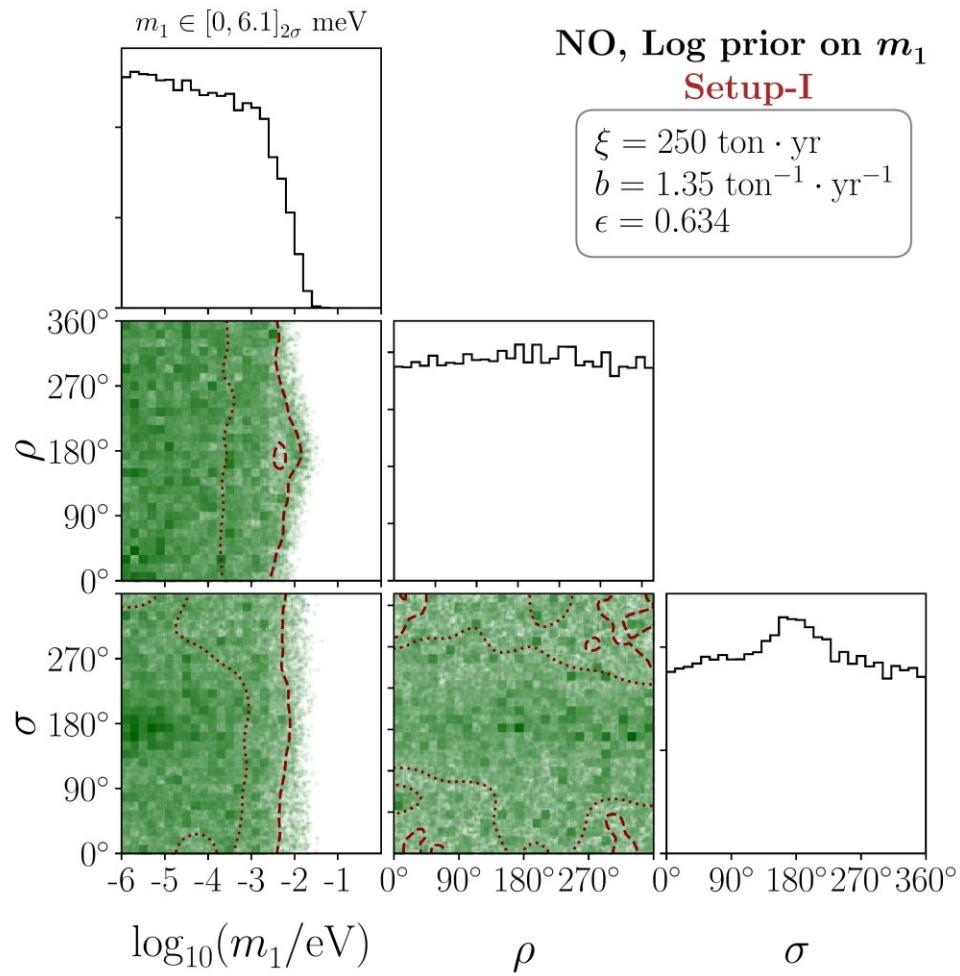
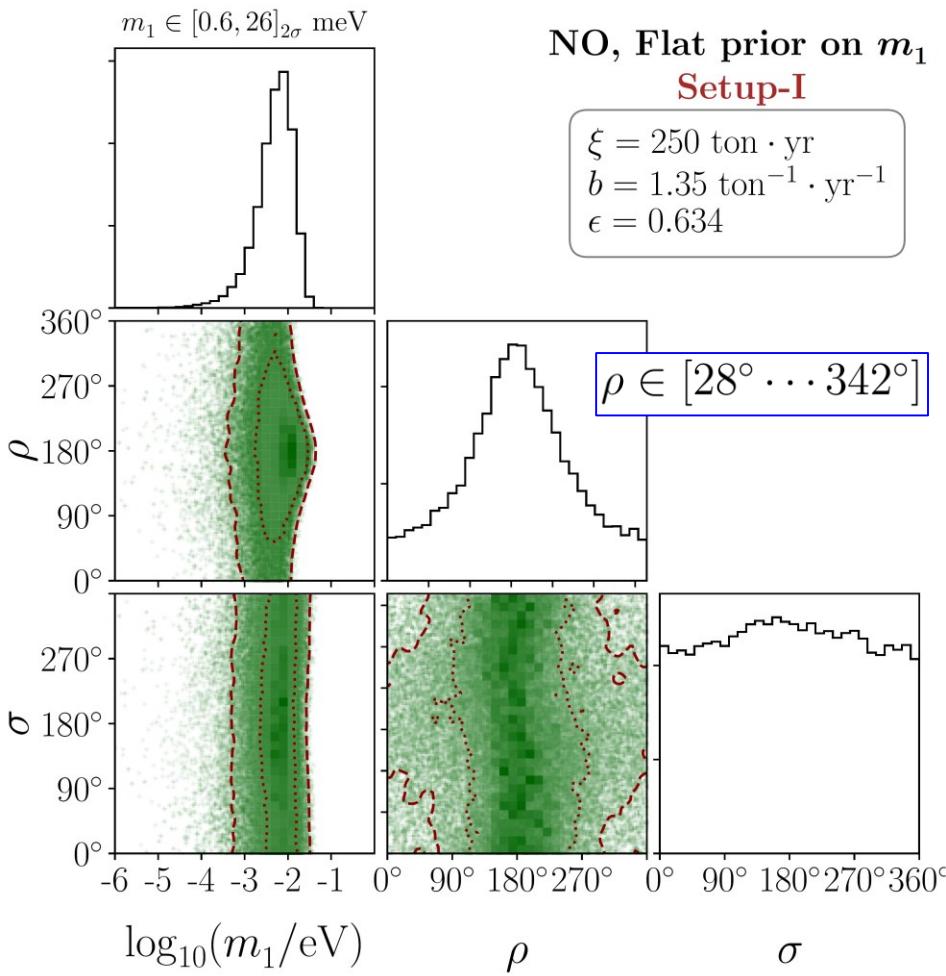
- Third, given the parameters, we calculate the expected number  $N^{0\nu}$  of the signal events; the data  $n_{\text{tot}}$  can be simulated and we simply take  $n_{\text{tot}} = B$ ; then the likelihood is

$$\mathcal{L}_{0\nu\beta\beta}^{\text{meV}}(N^{0\nu}) = \frac{(N^{0\nu} + B)^{n_{\text{tot}}}}{n_{\text{tot}}!} \cdot e^{-(N^{0\nu} + B)}$$

- Fourth, calculate the posterior probabilities for the model parameters and project them onto one or two-dimensional parameter space

## Final Results for Setup-I

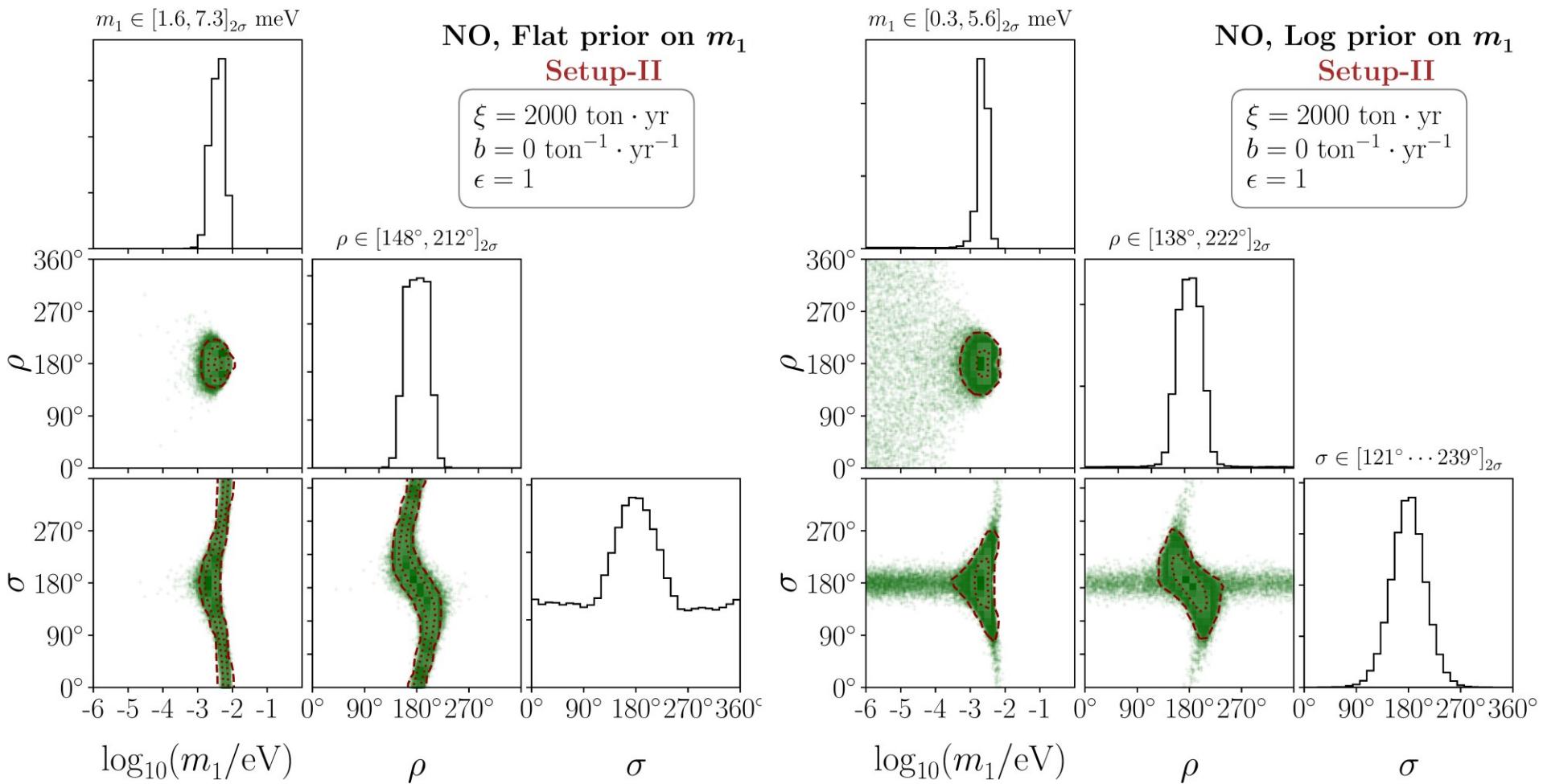
G.Y. Huang, SZ, 2010.16281



$$\begin{cases} m_1 \in [0.6 \cdots 26] \text{ meV}, \\ m_1 \in [0 \cdots 6.1] \text{ meV}, \end{cases} \quad \begin{cases} \Sigma \equiv m_1 + m_2 + m_3 < 0.11 \text{ eV}, \\ \Sigma \equiv m_1 + m_2 + m_3 < 0.067 \text{ eV}, \end{cases} \quad \begin{array}{l} \text{for flat prior on } m_1 \\ \text{for log prior on } m_1 \end{array}$$

## Final Results for Setup-II

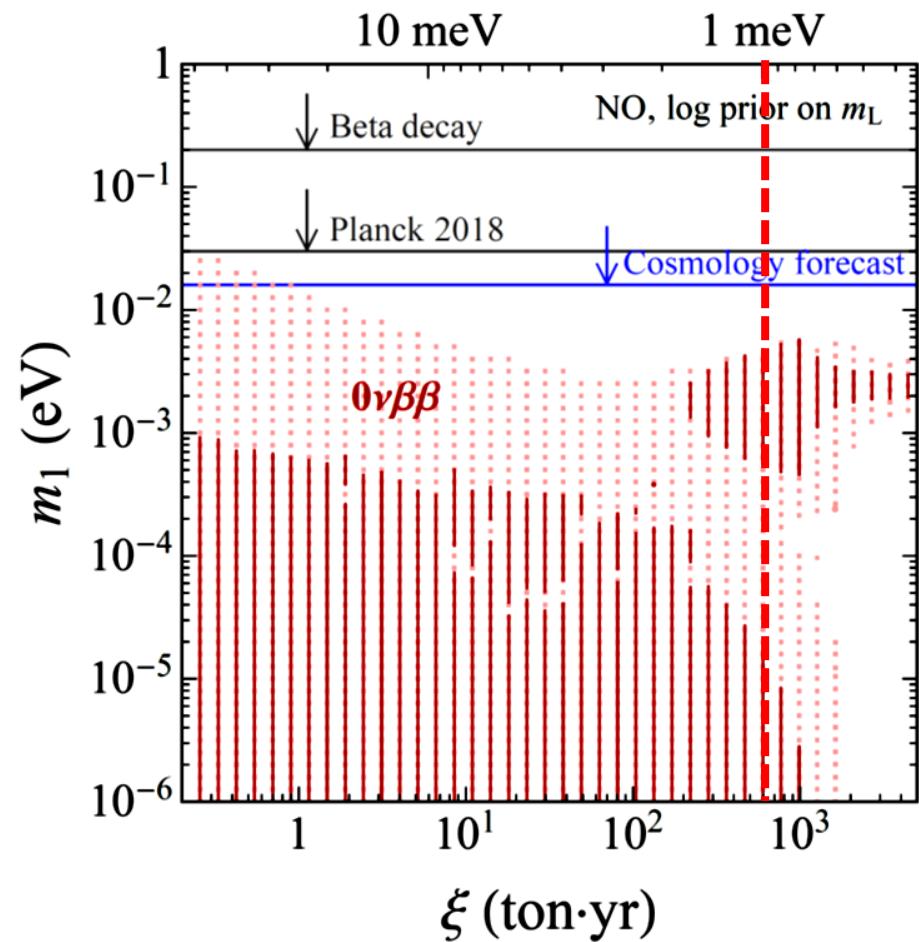
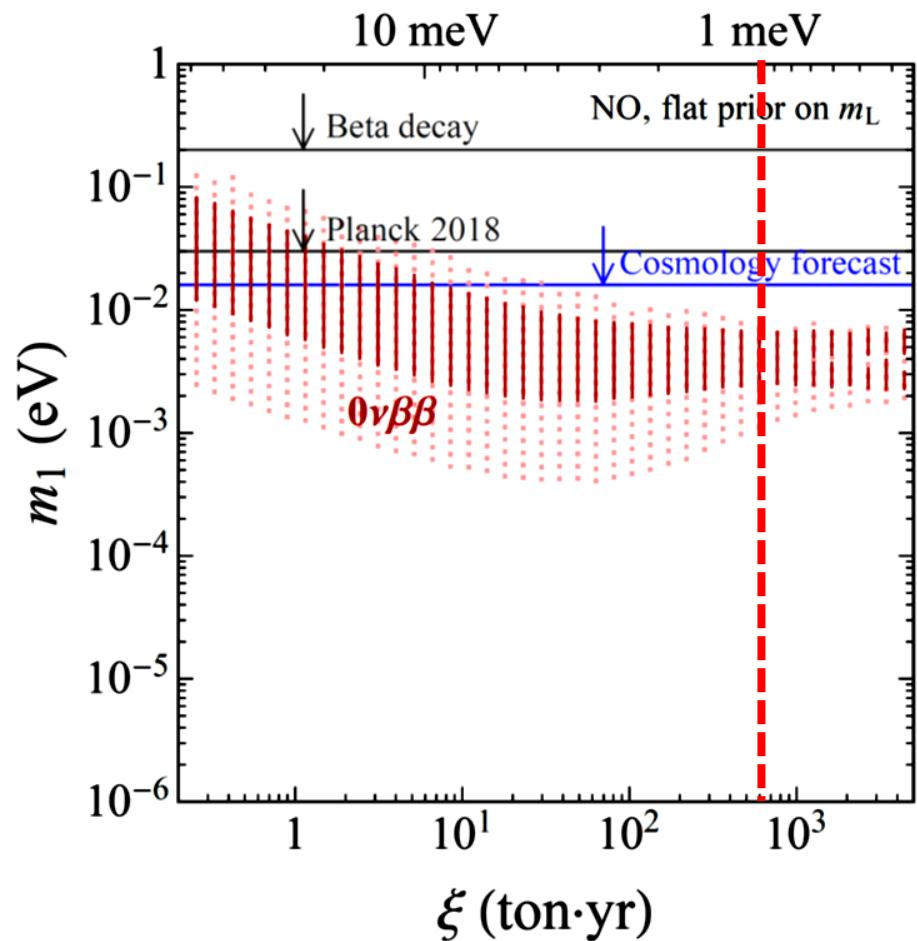
G.Y. Huang, SZ, 2010.16281



$$\left\{ \begin{array}{l} m_1 \in [1.6 \dots 7.3] \text{ meV} , \\ m_1 \in [0.3 \dots 5.6] \text{ meV} , \end{array} \right. \quad \left\{ \begin{array}{l} \rho \in [148^\circ \dots 212^\circ] , \quad \text{for flat prior on } m_1 \\ \rho \in [138^\circ \dots 222^\circ] , \quad \text{for log prior on } m_1 \end{array} \right.$$

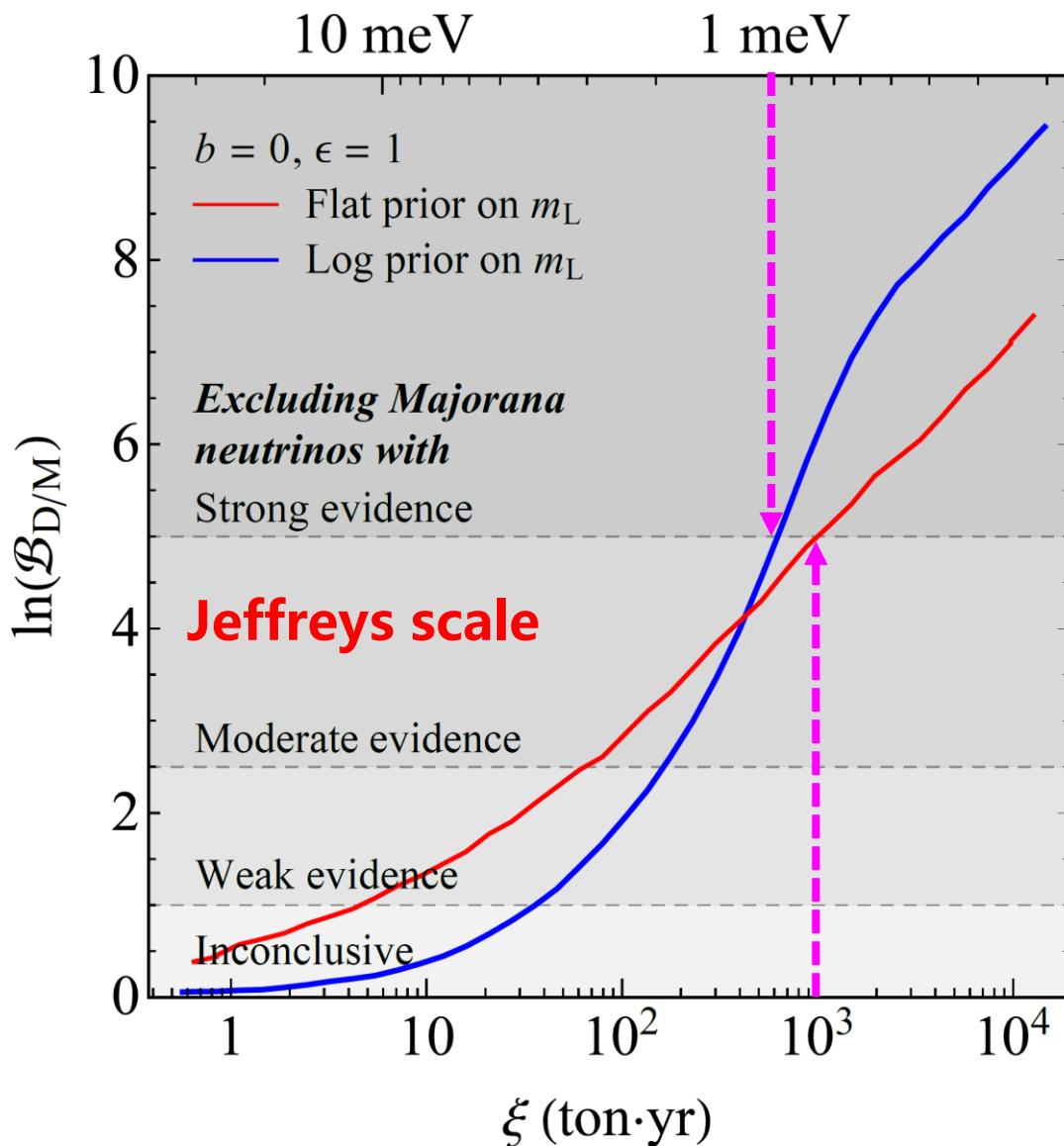
## Future sensitivity from cosmology

$$\sigma(\Sigma) \sim 14 \text{ meV}$$



- Background-free environment, average NME, and varying total exposure
- Better sensitivity to lightest ν mass than future cosmological observations

If no signals observed, is it possible to **exclude** Majorana neutrinos?



**Setup I: JUNO Xe-LS**

**NO**

$$\ln(\mathcal{B}_{D/M}) = 0.16 \quad \text{Log}$$

$$\ln(\mathcal{B}_{D/M}) = 1.0 \quad \text{Flat}$$

**Inconclusive**

**IO**

$$\ln(\mathcal{B}_{D/M}) = 11.7 \quad \text{Log}$$

$$\ln(\mathcal{B}_{D/M}) = 12.3 \quad \text{Flat}$$

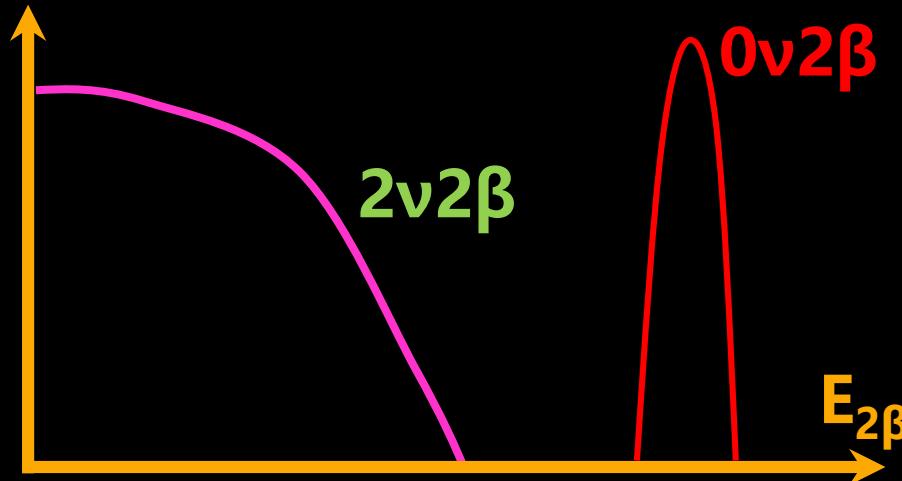
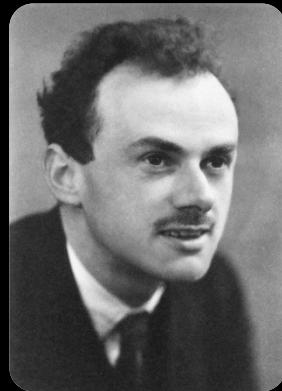
**Very strong evidence**

**Setup II: NO case**

**Strong evidence  $\ln(B_{D/M})=5$  requires the total exposure:**

$$\xi = 1000 \text{ ton} \cdot \text{yr}$$

# Summary



- The  $0\nu 2\beta$  decays offer a feasible and promising way to establish the Majorana nature of massive neutrinos
  - ★ Discovery of a new feature of elementary fermions
  - ★ Discovery of LNV: a guide for theorists
  - ★ Majorana CP Phases
- The implications of the Schechter-Valle theorem are discussed and extended to other LNV processes: a Majorana mass
- More efforts on experimental & theoretical sides hopefully help us explore the intrinsic properties of neutrinos