

Problem 1: Suppose P and Q are probability measures defined on $S = \{1, 2, 3, 4, 5\}$ with mass functions

s	$p(s)$	$q(s)$
1	0.1	0.05
2	0.3	0.15
3	0.2	0.7
4	0.3	0.03
5	0.1	0.07

Compute $D(P \parallel Q)$ and $D(Q \parallel P)$.

Using technology, we compute:

$$D(P \parallel Q) = \sum_{s=1}^5 p(s) \log_2 \left(\frac{p(s)}{q(s)} \right) \approx 1.087 \quad \text{and} \quad D(Q \parallel P) = \sum_{s=1}^5 q(s) \log_2 \left(\frac{q(s)}{p(s)} \right) \approx 0.929.$$

Problem 2: Compute the entropies $H(P \parallel Q)$ and $H(P)$ for the distributions in the previous problem.

Using technology, we compute:

$$H(P \parallel Q) = - \sum_{s=1}^5 p(s) \log_2(q(s)) \approx 3.258 \quad \text{and} \quad H(P) = - \sum_{s=1}^5 p(s) \log_2(p(s)) \approx 2.171$$