4.6. The algebra of random variables

s	$X(s)$	$Y(s)$
1	-1	0
2	1	2
3	3	-1
4	0	3

s	$X(s)$	$Y(s)$	$(X+Y)(s)$
1	-1	0	$-1+0=-1$
2	1	2	$1+2=3$
3	3	-1	$3-1=2$
4	0	3	$0+3=3$

Problem Prompt

Do problem 16 on the worksheet.

s	$Y(s)$	$(4Y)(s)$
1	0	$4 \cdot 0 = 0$
2	2	$4 \cdot 2 = 8$
3	-1	$4 \cdot (-1) = -4$
4	3	$4 \cdot 3 = 12$

- $(X \pm Y)(s) = X(s) \pm Y(s)$,
- $(XY)(s) = X(s)Y(s)$,
- $(X/Y)(s) = X(s)/Y(s)$, when $Y(s) \neq 0$,

4.7. Functions of random variables

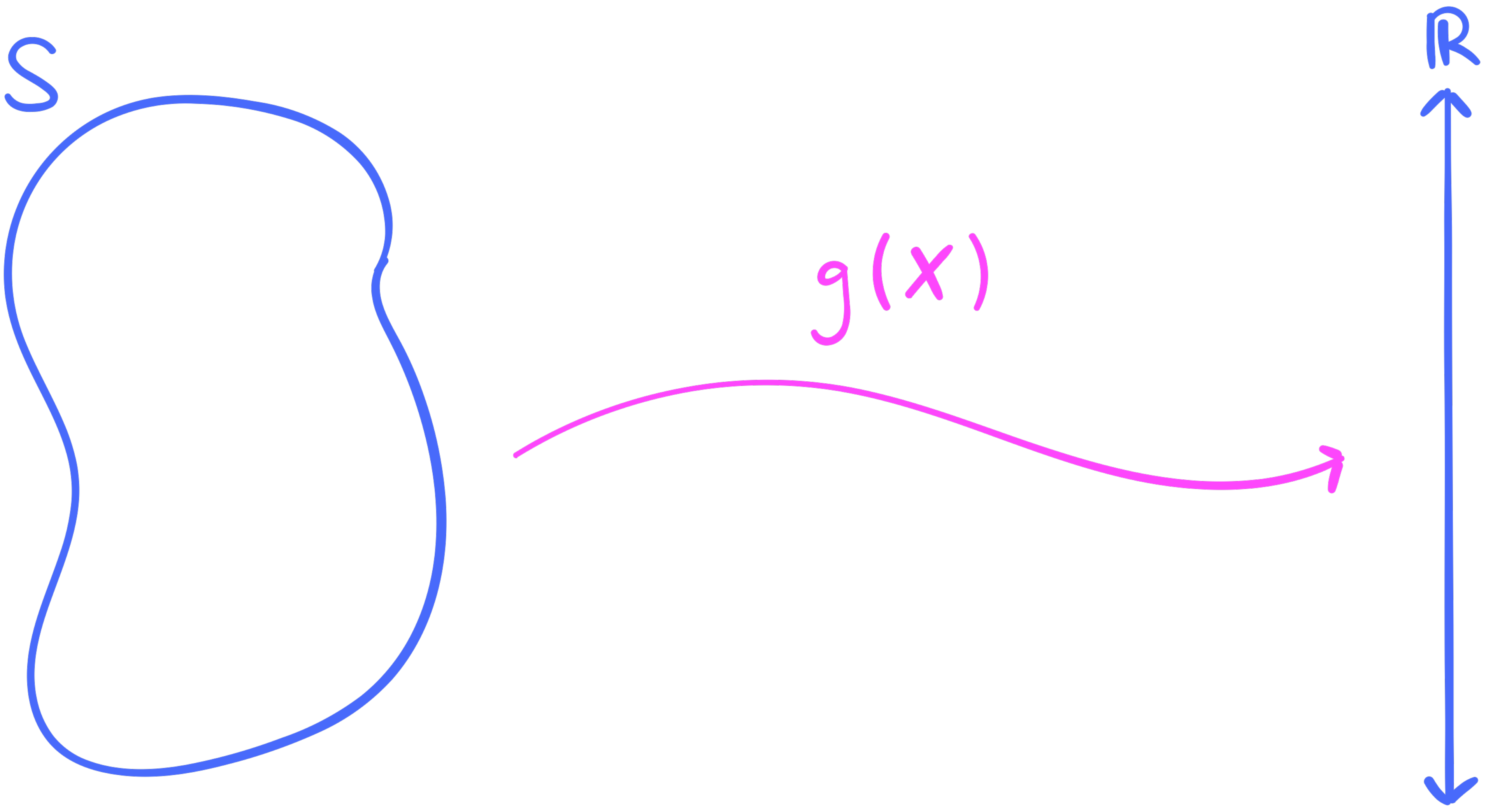
s	$X(s)$
1	0
2	2
3	-1
4	3

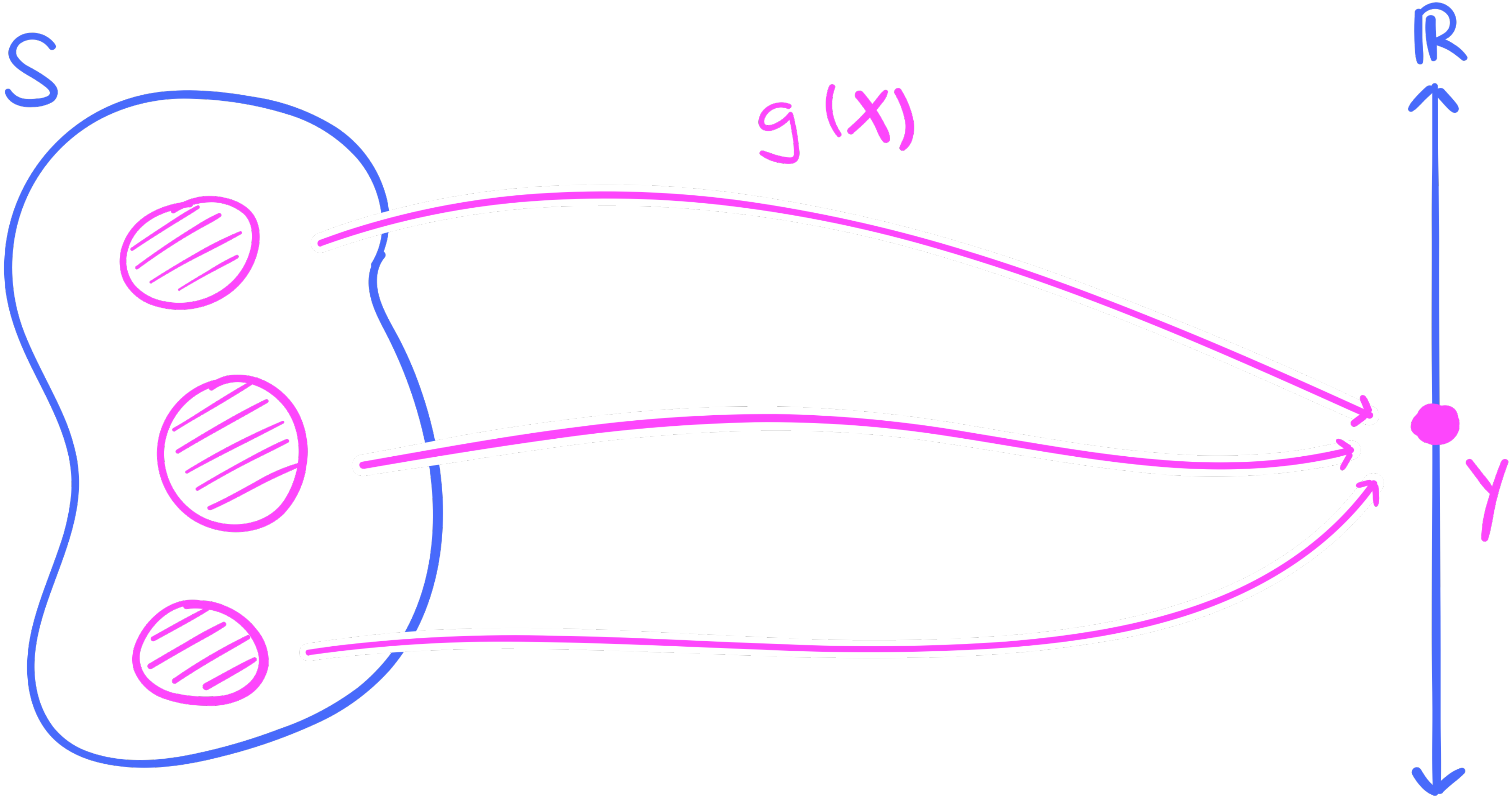
s	$X(s)$	$X^2(s)$
1	0	$0^2 = 0$
2	2	$2^2 = 4$
3	-1	$(-1)^2 = 1$
4	3	$3^2 = 9$

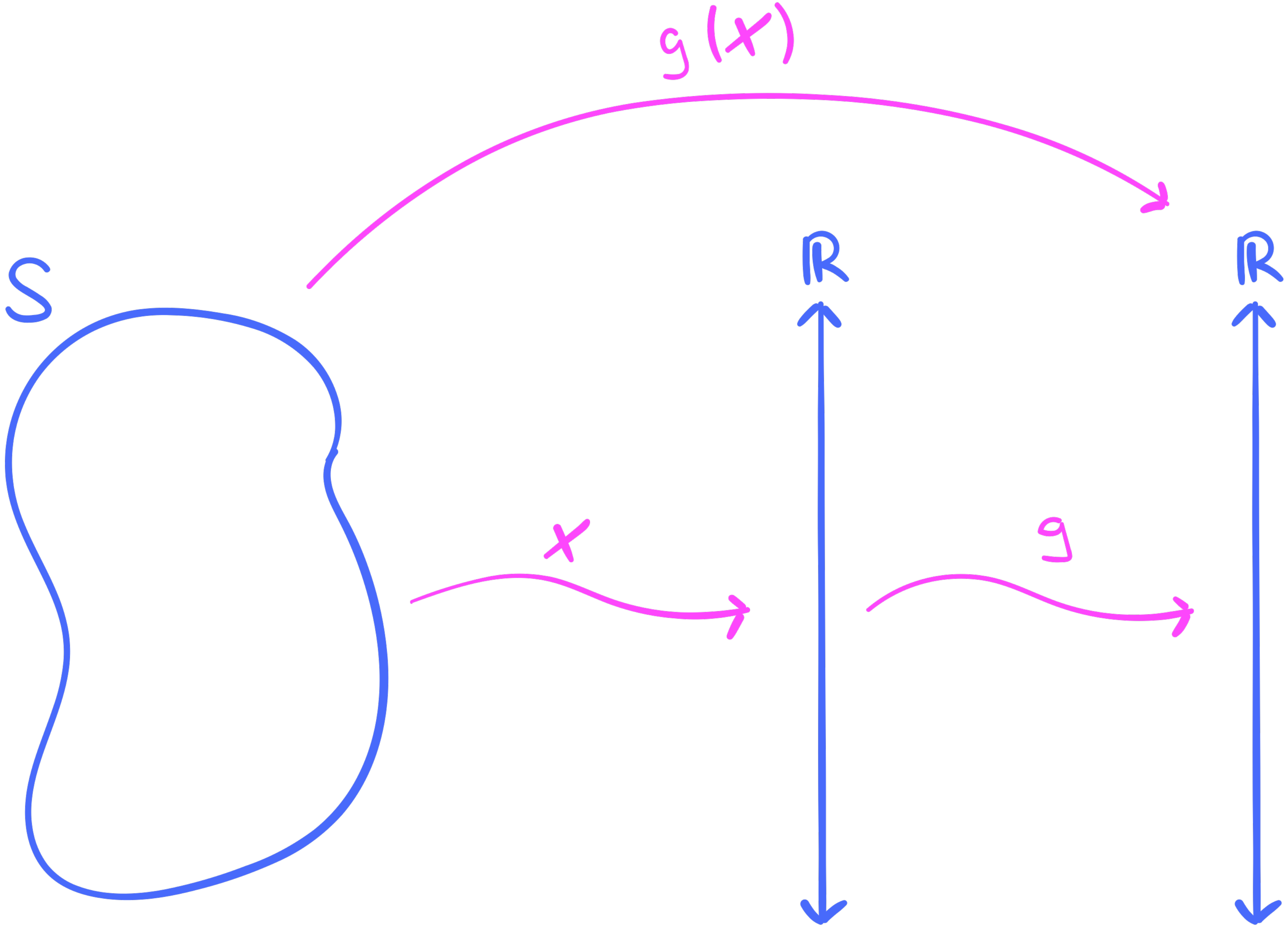
Problem Prompt

Have a go at problem 17 on the worksheet.

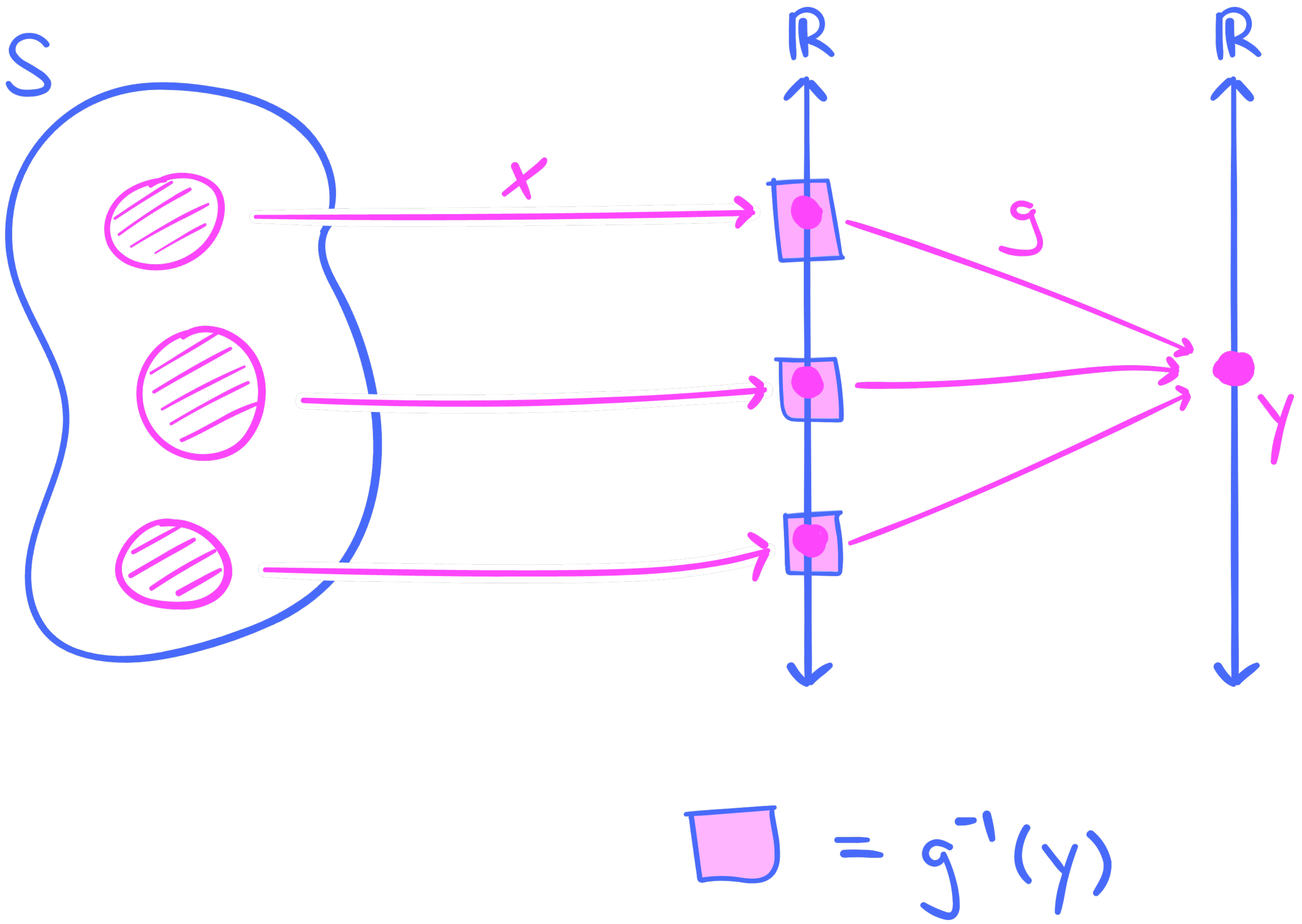
4.8. Expectations of functions of random variables and the LotUS

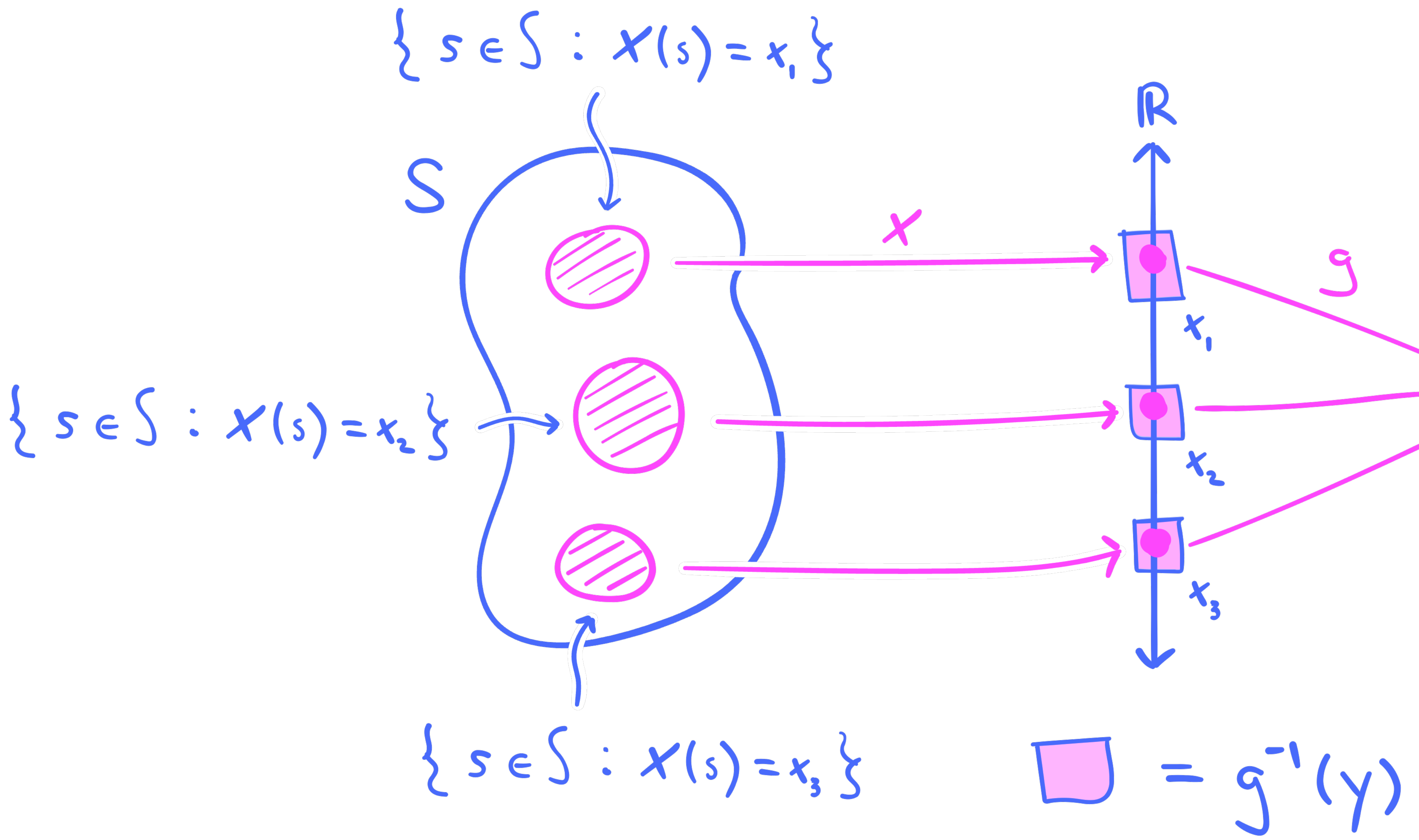

S
g(x)
ℝ


S
g(x)
ℝ
y



g(x)
S
x
g
R
R


S
ℝ
ℝ
x
g
y
= g⁻¹(y)


$\{s \in S : X(s) = x_1\}$
S
$\{s \in S : X(s) = x_2\}$
$\{s \in S : X(s) = x_3\}$
X
$\mathbb{R}$
$\mathbb{R}$
g
x_1
x_2
x_3
y
$= g^{-1}(y)$

Theorem 4.1 (Law of the Unconscious Statistician (LotUS))

Let X be a random variable and let $y = g(z)$ be a real-valued function on the real line $\mathbb{R}$.

- If X is discrete with probability mass function $p(x)$, then

$$E(g(X)) = \sum_{x \in \mathbb{R}} g(x) \cdot p(x).$$

- If X is continuous with probability density function $f(x)$, then

$$E(g(X)) = \int_{\mathbb{R}} g(x) \cdot f(x) \, \mathrm{d}x.$$

Problem Prompt

Do problems 18 and 19 on the worksheet.