5.5. Poisson distributions

Definition 5.5

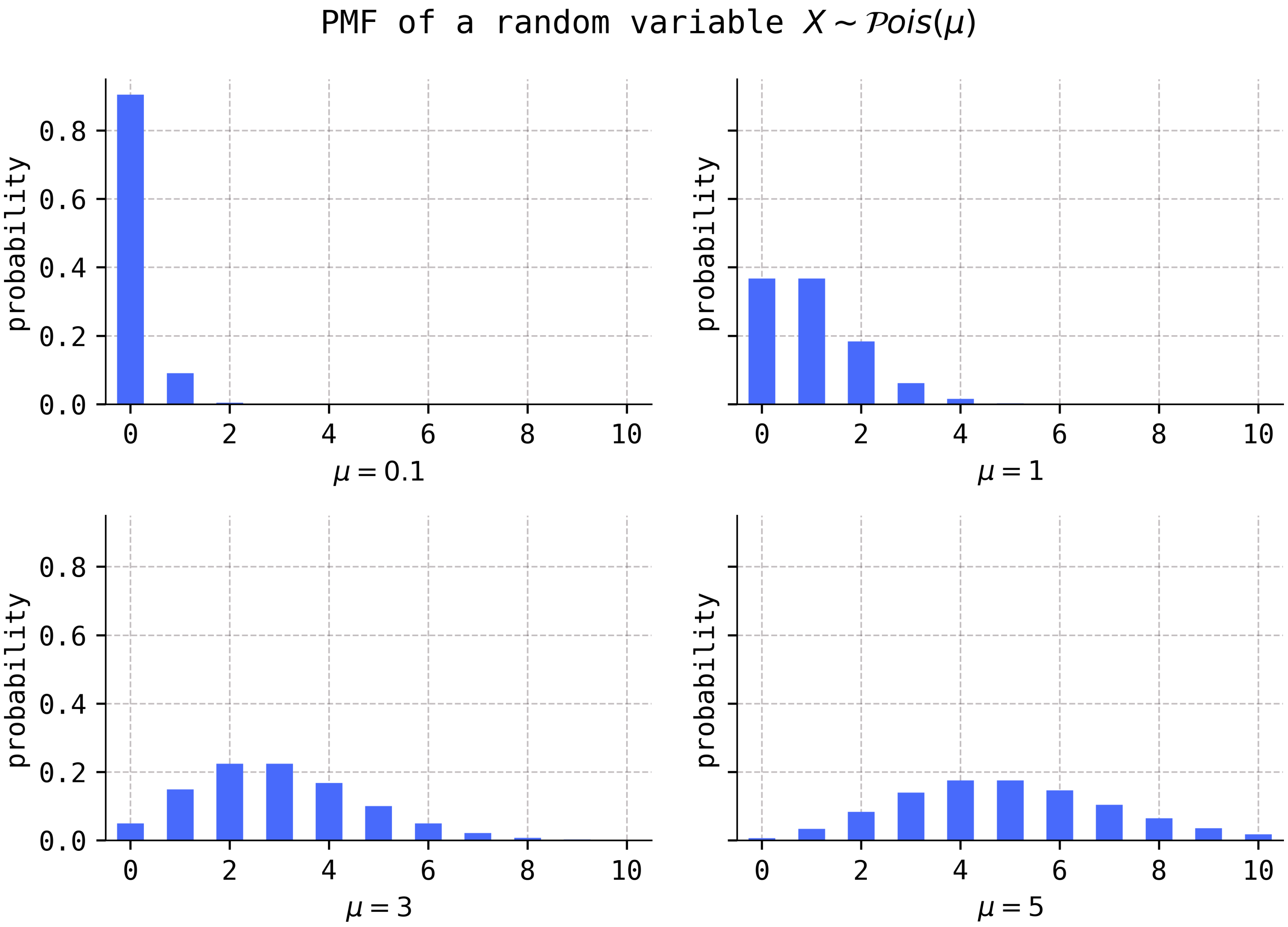
Let $\mu > 0$ be a real number. A discrete random variable X is said to have a *Poisson distribution* with parameter μ, denoted

$$X \sim \mathcal{P}ois(\mu),$$

if its probability mass function is given by

$$p(x; \mu) = \frac{\mu^x}{x!} e^{-\mu}$$

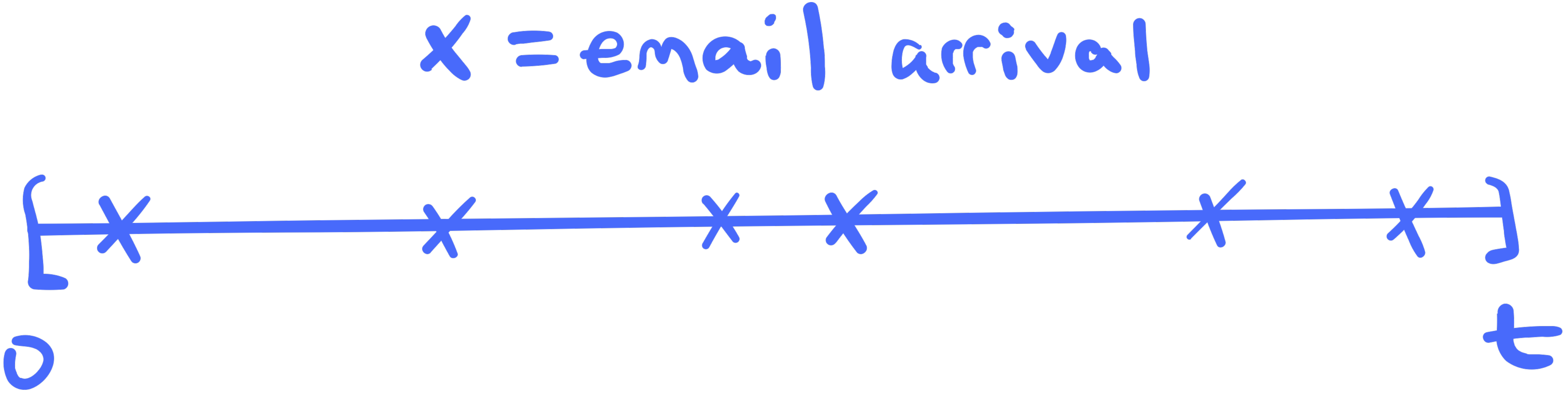
with support $\{0, 1, 2, \ldots\}$.

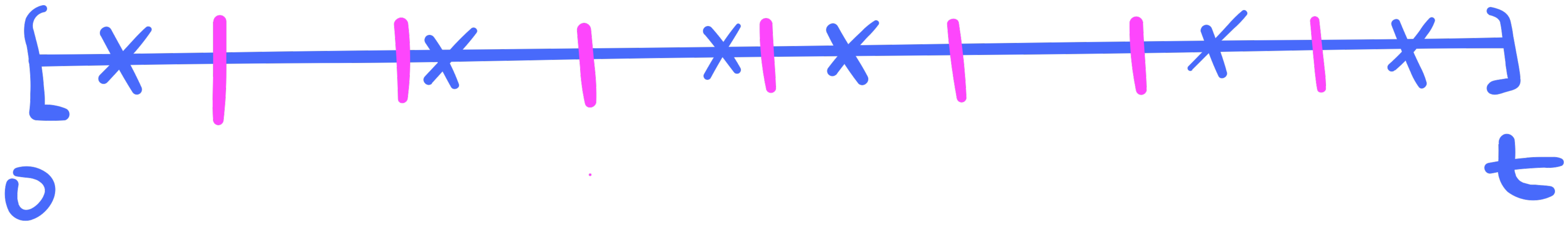

PMF of a random variable $X \sim \mathcal{P}ois(\mu)$
probability
0.0
0.2
0.4
0.6
0.8
0
2
4
6
8
10
$\mu = 0.1$
$\mu = 1$
$\mu = 3$
$\mu = 5$

An archetypical Poisson scenario

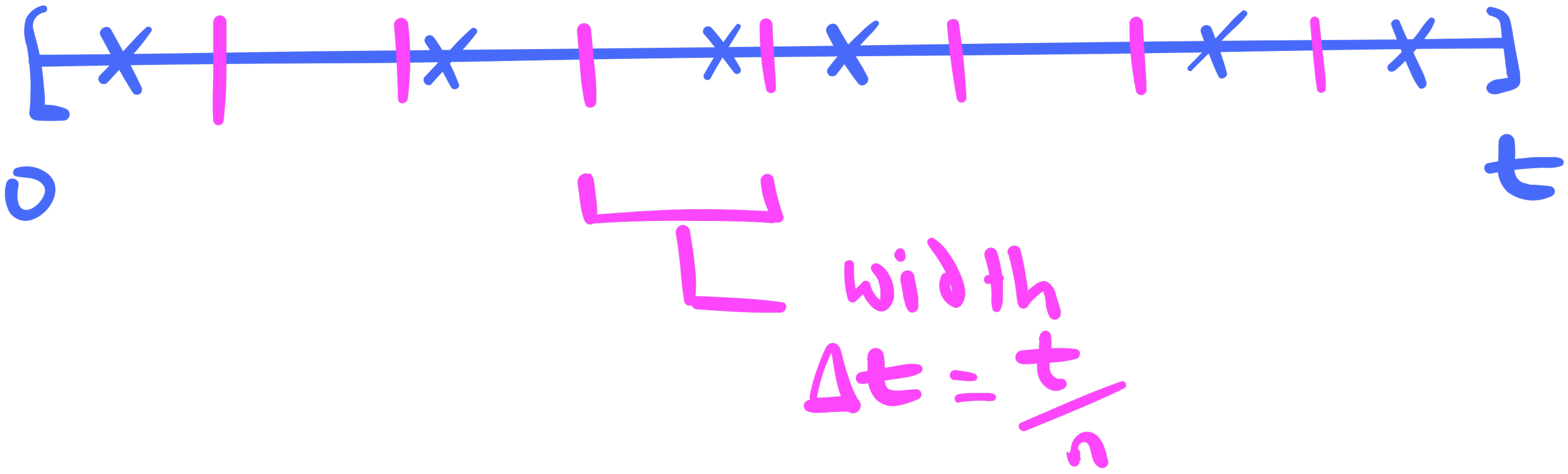
Q: Over a span of t hours, let X denote the number of emails that arrive in your inbox. Assuming that the emails arrive at random and independently of each other, and that the mean rate at which they arrive is λ (measured in units of reciprocal hours, or hour^{-1}), what is the distribution of the random variable X?

A: $X \sim \mathcal{P}ois(\lambda t)$.


x = email arrival
0
t



0
t


0
t
width
Δt = t/n

Theorem 5.6 (Expectations and variances of Poisson variables)

If $X \sim \mathcal{P}ois(\mu)$, then

$$E(X) = \mu \quad \text{and} \quad V(X) = \mu.$$

Problem Prompt

Do problems 7-9 on the worksheet.