4.9. Linearity of expectation, part 1

Theorem 4.3 ("Weak" Linearity of Expectations)

Let X be a discrete or continuous random variable, let $y = g_1(x)$ and $y = g_2(x)$ be two real-valued functions, and let $c \in \mathbb{R}$ be a constant. Then:

$$E(g_1(X) + g_2(X)) = E(g_1(X)) + E(g_2(X)),$$

and

$$E(cX) = cE(X).$$

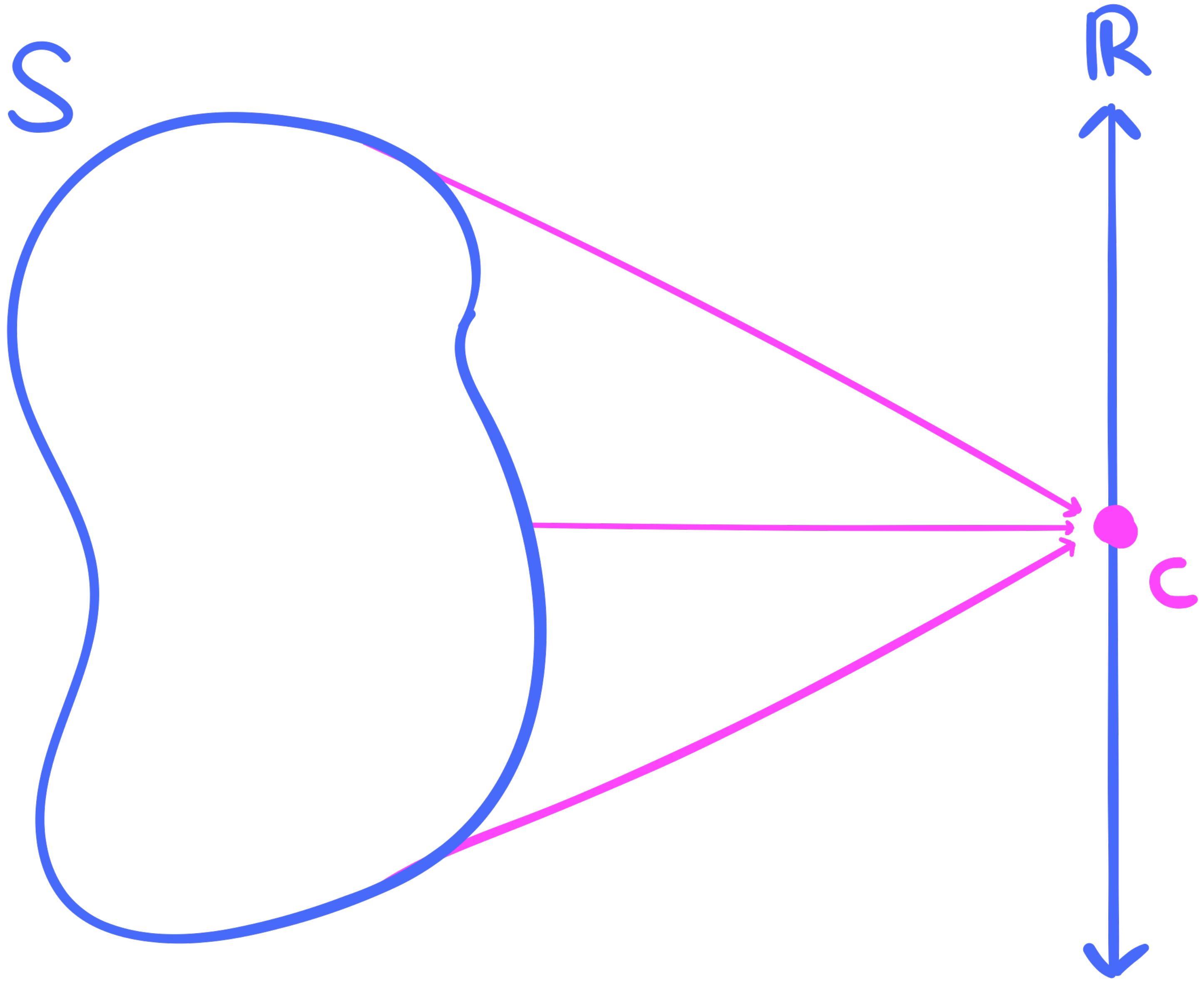
Theorem 4.2 (Linearity of Expectations)

Let X and Y be two random variables and let $c \in \mathbb{R}$ be a constant. Then:

$$E(X+Y) = E(X) + E(Y), \tag{4.14}$$

and

$$E(cX) = cE(X). \tag{4.15}$$


S
R
c

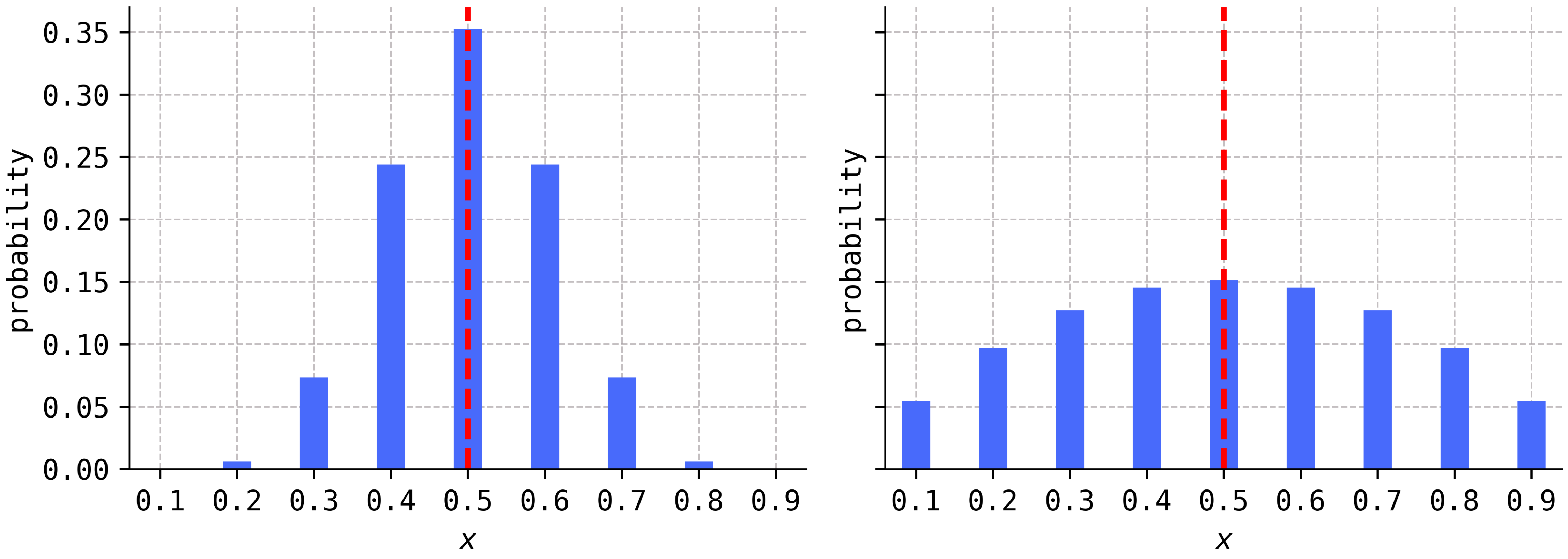
Theorem 4.4 (Expectations of Constants)

Let $c \in \mathbb{R}$ be a constant, viewed as a constant random variable. Then $E(c) = c$.

Problem Prompt

Do problem 20 on the worksheet.

4.10. Variances and standard deviations



0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
probability
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
x
probability
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
x

Definition 4.7

Let X be a random variable with expected value $\mu = E(X)$. The *variance* of X, denoted $V(X)$, is given by

$$V(X) = E\left((X - \mu)^2\right). \tag{4.17}$$

The variance of X is also denoted σ_X^2 or just σ^2.

Definition 4.8

Let X be a random variable. The *standard deviation* of X, denoted σ_X or just σ, is the positive square root of the variance:

$$\sigma_X = \sqrt{V(X)}.$$

Problem Prompt

Do problems 21-23 on the worksheet.

Theorem 4.5 (Shortcut Formula for Variance)

Let X be a random variable. Then

$$V(X) = E(X^2) - E(X)^2 = E(X^2) - \mu_X^2.$$

Theorem 4.6 (Variance of an Affine Transformation)

Let X be a random variable and a and b constants. Then

$$V(aX + b) = a^2 V(X). \tag{4.18}$$

Problem Prompt

Holy smokes. After a very, *very* long discussion of random variables, we've finally reached the end! Now finish off the worksheet and do problems 24 and 25.