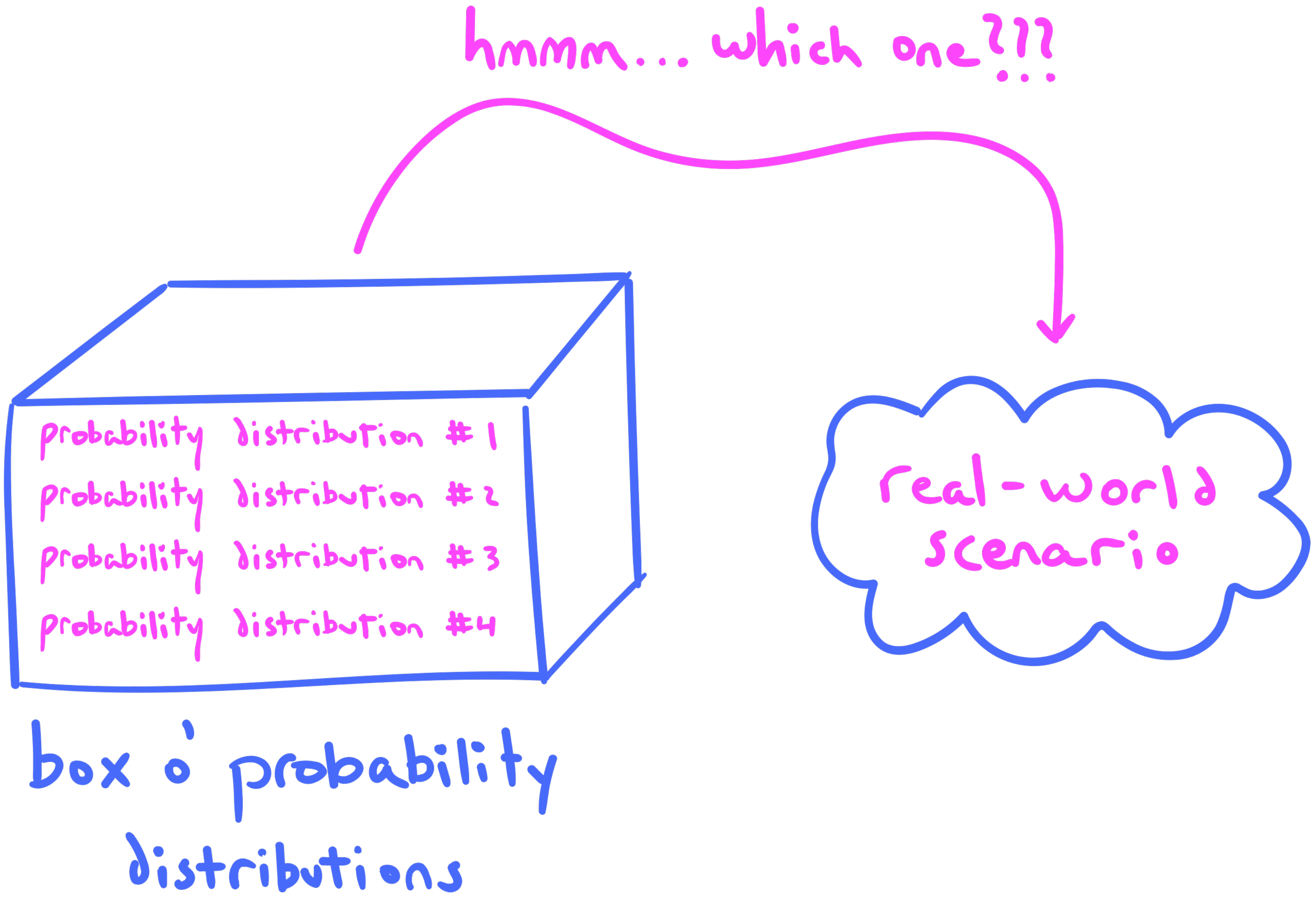
5. Examples of random variables


hmmm... which one???
probability distribution #1
probability distribution #2
probability distribution #3
probability distribution #4
box o' probability distributions
real-world scenario

5.1. Bernoulli distributions

Definition 5.1

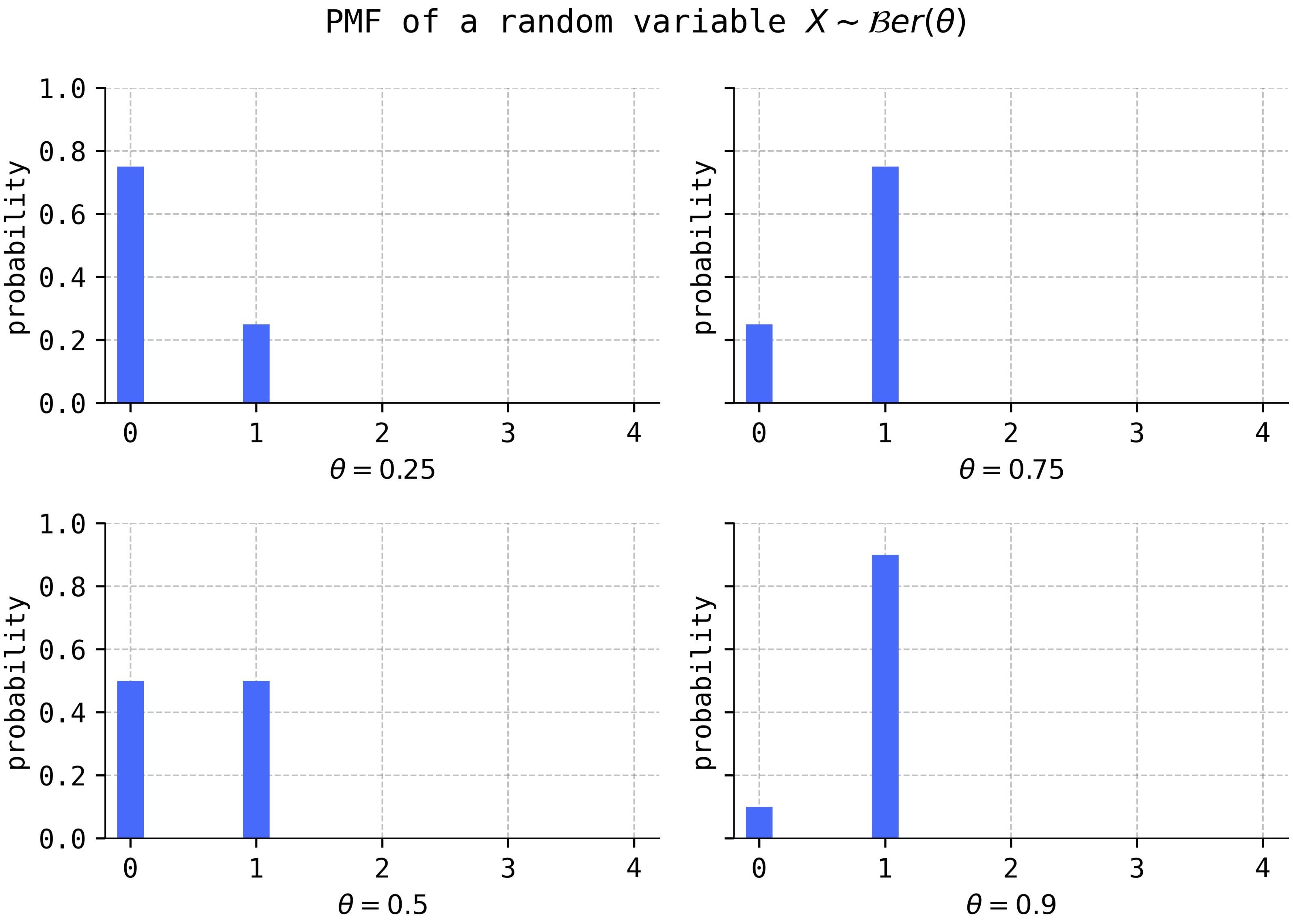
Let θ be a real number with $0 \leq \theta \leq 1$. A discrete random variable X is said to have a *Bernoulli distribution* with parameter θ, denoted

$$X \sim \mathcal{B}er(\theta), \tag{5.1}$$

if its probability mass function is given by

$$p(x; \theta) = \theta^x (1 - \theta)^{1-x}. \tag{5.2}$$

with support $\{0, 1\}$.


PMF of a random variable $X \sim \mathcal{B}er(\theta)$
probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
$\theta = 0.25$
$\theta = 0.75$
$\theta = 0.5$
$\theta = 0.9$

An archetypical Bernoulli scenario

Q: Suppose that a coin is flipped once and X is the number of heads obtained. Suppose further that we allow for the possibility that the coin is *loaded*, so that it lands heads with probability θ (which may not be 0.5!). What is the distribution of the random variable X?

A: $X \sim \mathcal{B}er(\theta)$.

Theorem 5.1 (Expectations and variances of Bernoulli variables)

If $X \sim \mathcal{B}er(\theta)$, then

$$E(X) = \theta \quad \text{and} \quad V(X) = \theta(1-\theta).$$

5.2. Binomial distributions

Definition 5.2

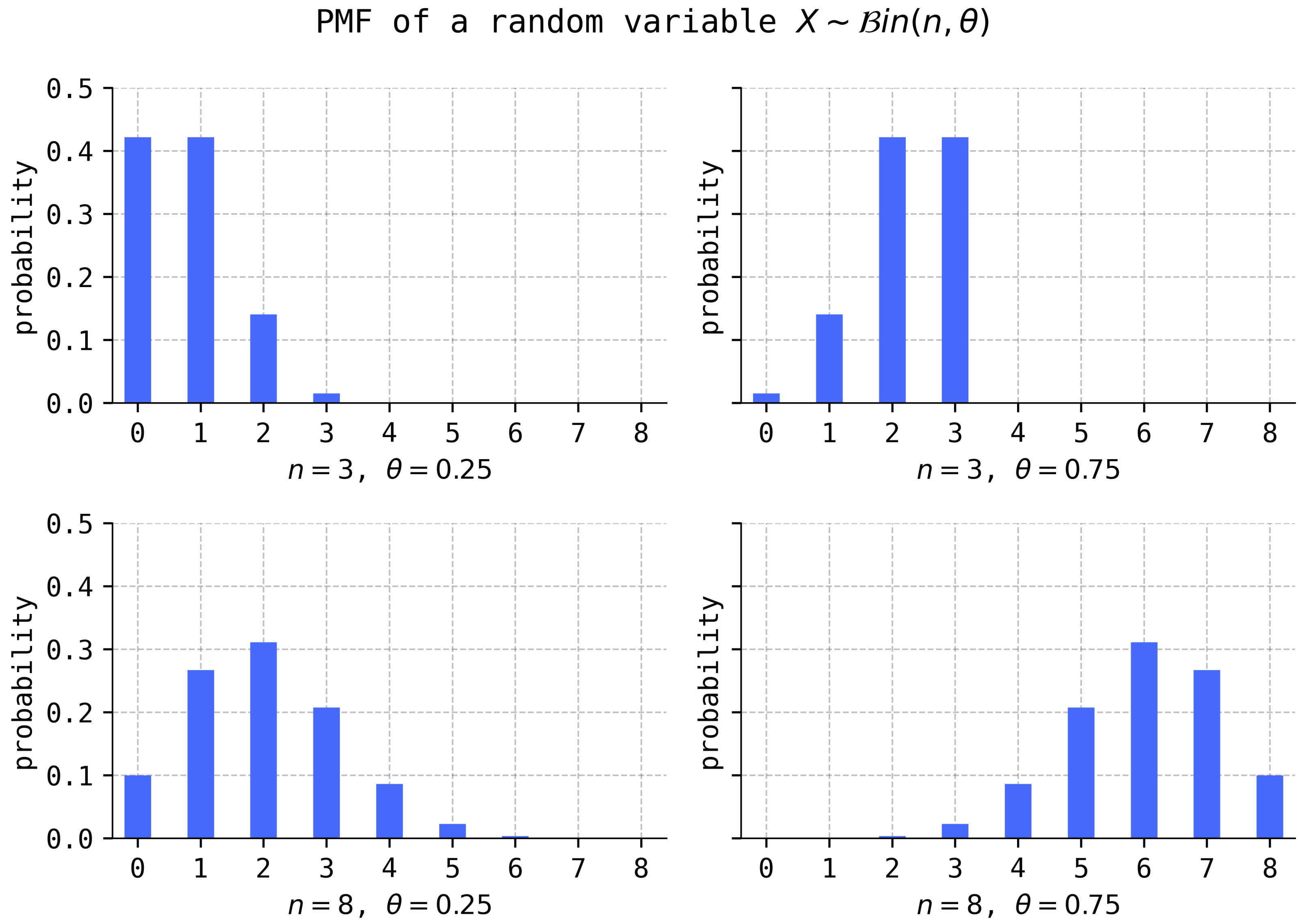
Let $n \geq 0$ be an integer and let θ be a real number with $0 \leq \theta \leq 1$. A discrete random variable X is said to have a *binomial distribution* with parameters n and θ, denoted

$$Y \sim \mathcal{B}in(n, \theta),$$

if its probability mass function is given by

$$p(x; n, \theta) = \binom{n}{x} \theta^x (1-\theta)^{n-x}$$

with support $\{0, 1, \dots, n\}$.


PMF of a random variable $X \sim \mathcal{B}in(n, \theta)$
probability
0.0
0.1
0.2
0.3
0.4
0.5
0 1 2 3 4 5 6 7 8
$n = 3, \ \theta = 0.25$
$n = 3, \ \theta = 0.75$
$n = 8, \ \theta = 0.25$
$n = 8, \ \theta = 0.75$

An archetypical binomial scenario

Q: Suppose that a coin is flipped n times and that each flip is independent of the others. Suppose further that we allow for the possibility that the coin is *loaded*, so that it lands heads with probability θ (which may not be 0.5!). If X is the number of heads obtained, what is the distribution of the random variable X?

A: $X \sim \mathcal{B}in(n, \theta)$.

Theorem 5.2 (Expectations and variances of binomial variables)

If $X \sim \mathcal{B}in(n, \theta)$, then

$$E(X) = n\theta \quad \text{and} \quad V(X) = n\theta(1-\theta). \tag{5.5}$$

Theorem 5.3 (Binomial variables as sums of Bernoulli variables)

Let $Y_1, Y_2, \ldots, Y_n$ be a sequence of independent Bernoulli variables, all with the same distribution $\mathcal{B}er(\theta)$. Then the random variable

$$X = Y_1 + Y_2 + \cdots + Y_n$$

is a $\mathcal{B}in(n, \theta)$ random variable.

Problem Prompt

Do problems 1 and 2 on the worksheet.