Problem 1: Let A and B be two subsets of a set S. Let C be the subset of S consisting of elements that are in A or B, but not both. Express C in terms of A and B using only the basic set operations.

Problem 2: Draw Venn diagrams to illustrate DeMorgan's law, which states that, given two subsets A and B of a set S, that

$$(A \cup B)^c = A^c \cap B^c.$$

Problem 3: Identify appropriate sample spaces for each of the following scenarios.

(a) Rolling a die.

(b) Flipping a coin.

(c) Flipping a coin *three* times.

(d) Flipping a coin until it lands heads.

(e) Forecasting whether it will rain tomorrow.

(f) Conducting an experiment to measure the speed of light.

Problem 4: Describe some events in the following scenarios. Use the sample spaces that you identified in the previous problem.

(a) Rolling a die.

(b) Flipping a coin.

(c) Flipping a coin *three* times.

(d) Flipping a coin until it lands heads.

(e) Forecasting whether it will rain tomorrow.

(f) Conducting an experiment to measure the speed of light.

Problem 5: Suppose that you choose at random one of the twelve months of the year.

(a) Identify an appropriate sample space.

(b) Identify the event that you choose a month beginning with a "J".

(c) Identify the event that you choose a month whose name is four letters long.

(d) Using one of the basic set operations and your answers to the two previous parts, identify the event that you choose a month beginning with a "J" and whose name is four letters long.

Problem 6: Suppose that A and B are two events. Write expressions involving only the basic set operations for each of the following:

(a) Both events occur.

(b) At least one occurs.

(c) Neither occurs.

(d) Exactly one occurs.

Problem 7: Let P be the discrete probability measure on the sample space $S = \mathbb{R}$ with probability mass function

$$p(s) = \begin{cases} 1/4 & : s = 0, \\ 1/4 & : s = 2, \\ 1/2 & : s = 4, \\ 0 & : \text{otherwise.} \end{cases}$$

Compute the probabilities $P(A)$ of the following events.

(a) $A = [-2, -1]$

(b) $A = (-1, 1)$

(c) $A = (-1, 1) \cup (3, 5]$

(d) $A = \mathbb{R}$

Problem 8: Let P be the discrete probability measure on the sample space $S = \mathbb{R}$ with probability mass function

$$p(s) = \begin{cases} (0.25)(0.75)^{s-1} & : s = 1, 2, 3, \ldots, \\ 0 & : \text{otherwise.} \end{cases}$$

Compute the probabilities $P(A)$ of the following events.

(a) $A = (-\infty, 1)$

(b) $A = (-10, 4]$

(c) $A = \mathbb{R}$

(d) $A = \{\text{all positive even integers}\}$

Problem 9: Describe in complete detail a probability space that models each of the following scenarios. Be sure to check that the two requirements in the Discrete Probability Construction Lemma are both satisfied.

(a) Rolling a fair six-sided die.

(b) Flipping an **unfair** coin, with probability 0.25 of obtaining heads, and probability 0.75 of obtaining tails.

(c) Flipping an **unfair** coin twice, with probability 0.25 of obtaining heads, and probability 0.75 of obtaining tails. What is the probability that you flip *exactly* one head?

(d) Flipping an **unfair** fair coin until you obtain a head, with probability 0.25 of obtaining heads, and probability 0.75 of obtaining tails. What is the probability that you flip an odd number of tails before you see a head?

Problem 10: A sample space consists of five simple events, E_1, E_2, E_3, E_4, and E_5.

(a) If $P(E_1) = P(E_2) = 0.15$, $P(E_3) = 0.4$, and $P(E_4) = 2P(E_5)$, find the probabilities of E_4 and E_5.

(b) If $P(E_1) = 3P(E_2) = 0.3$, find the probabilities of the remaining simple events if you know that the remaining simple events are equally probable.

Problem 11: A survey classified a large number of adults according to whether they were diagnosed as needing eyeglasses to correct their reading vision and whether they use eyeglasses when reading. The proportions falling into the four resulting categories are given in the following table:

	Uses Eyeglasses for Reading	
Needs glasses	Yes	No
Yes	.44	.14
No	.02	.40

If a single adult is selected from the large group, find the probabilities of the events defined below. The adult

(a) needs glasses.

(b) needs glasses but does not use them.

(c) uses glasses whether the glasses are needed or not.

Problem 12: Let P be a continuous probability measure on the sample space $S = \mathbb{R}$ with probability density function

$$f(s) = \begin{cases} \frac{2}{9}s & : 0 \leq s \leq 3, \\ 0 & : \text{otherwise}. \end{cases}$$

Compute the probabilities $P(A)$ of the following events.

(a) $A = (-1, 1]$

(b) $A = (-1, 1) \cup [2, 4)$.

(c) $A = [0, 3]$.

(d) $A = \mathbb{R}$.

Problem 13: Let P be a continuous probability measure on the sample space $S = \mathbb{R}$ with probability density function

$$f(s) = \begin{cases} \dfrac{1}{s^2} & : s \geq 1, \\ 0 & : \text{otherwise.} \end{cases}$$

Compute the probabilities $P(A)$ of the following events.

(a) $A = (-10, 0)$

(b) $A = \mathbb{R}$.

Problem 14: A particle is located somewhere along the real line $\mathbb{R}$, but the only information that you have about its location is that it is at some point in the interval $[0, 4]$.

(a) Describe in complete detail a probability space that models this scenario.

(b) Suppose some new information comes to light that suggests the particle is closer to 4 than it is 0. Alter your probability model to reflect this new information.

Problem 15: Let P be the discrete probability measure on $\mathbb{R}$ with probability function

$$p(s) = \begin{cases} \frac{2!}{s!(2-s)!}\left(\frac{1}{4}\right)^s\left(\frac{3}{4}\right)^{2-s} & : s = 0, 1, 2, \\ 0 & : \text{otherwise.} \end{cases}$$

Compute the distribution function $F(s)$.

Problem 16: Let P be the continuous probability measure on $\mathbb{R}$ with density function

$$f(s) = \begin{cases} 3s^2 & : 0 \leq s \leq 1, \\ 0 & : \text{otherwise.} \end{cases}$$

Compute the distribution function $F(s)$.

Problem 17: Let P be a probability measure on $\mathbb{R}$ with distribution function

$$F(s) = \begin{cases} 0 & : s < 0, \\ s & : 0 \leq s \leq 1, \\ 1 & : s > 1. \end{cases}$$

Is P continuous? If so, compute its density function $f(s)$.

Problem 18: Let P be the discrete probability measure on $\mathbb{R}$ with probability function

$$p(s) = \begin{cases} 3/8 & : s = 1, \\ 7/16 & : s = 2, \\ 3/16 & : s = 3, \\ 0 & : \text{otherwise.} \end{cases}$$

Compute the following quantiles:

(a) $Q(0.25)$

(b) $Q(0.5)$ (the median of P)

(c) $Q(0.75)$

Problem 19: Let P be the continuous probability measure on $\mathbb{R}$ with density function

$$f(s) = \begin{cases} cs(1-s) & : 0 \leq s \leq 1, \\ 0 & : \text{otherwise.} \end{cases}$$

(a) Find the value of c that makes $f(s)$ a valid density function.

(b) Compute the quantile $Q(0.95)$.

Problem 20: Let P be a continuous probability measure on the sample space $S = \mathbb{R}^2$ with probability density function

$$f(s,t) = \begin{cases} \frac{1}{8}(s+t) & : 0 \leq s,t \leq 2, \\ 0 & : \text{otherwise.} \end{cases}$$

Compute the probabilities $P(C)$ of the following events.

(a) $C = [-1,1] \times (-1,1)$

(b) $C = \mathbb{R}^2$.

Problem 21: Let P be a continuous probability measure on the sample space $S = \mathbb{R}^2$ with probability density function

$$f(s,t) = \begin{cases} e^{-(s+t)} & : s,t > 0, \\ 0 & : \text{otherwise.} \end{cases}$$

Show that $P(\mathbb{R}^2) = 1$.

Problem 22: A particle is now located somewhere in the plane $\mathbb{R}^2$, but the only information that you have about its location is that it is at some point in the square

$$R = [0,3] \times [0,3].$$

(a) Describe in complete detail a probability space that models this scenario.

(b) Suppose some new information comes to light that suggests the particle is closer to the corner $(3,3)$ than to the origin $(0,0)$. Alter your probability model to reflect this new information.