5.6. Normal distributions

Definition 5.6

Let μ and σ be real numbers with $\sigma>0$. A continuous random variable X is said to have a normal distribution (or Gaussian distribution) with parameters μ and σ , denoted

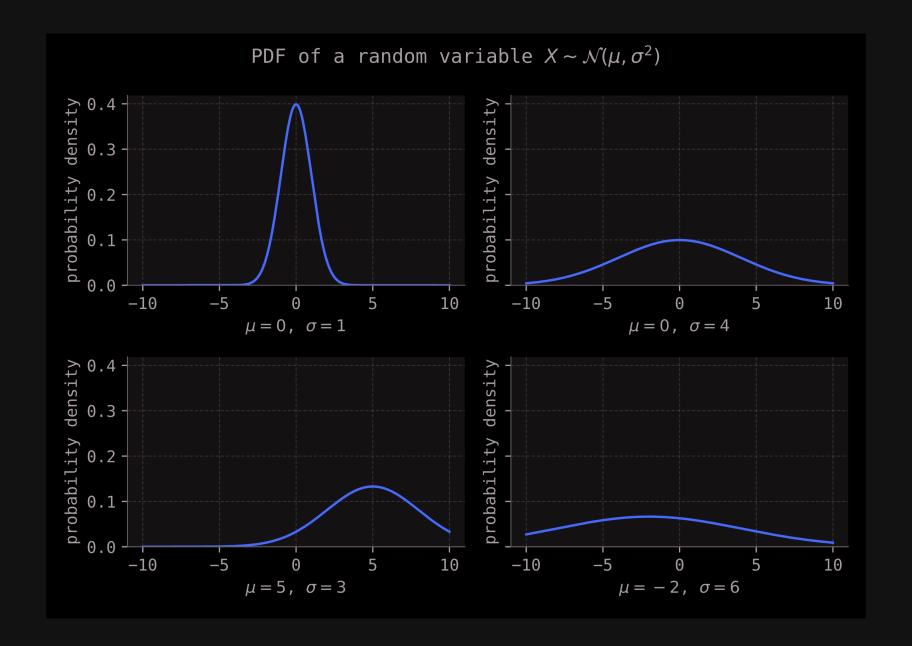
$$X \sim \mathcal{N}(\mu, \sigma^2),$$

if its probability density function is given by

$$f(x;\mu,\sigma) = rac{1}{\sigma\sqrt{2\pi}} \mathrm{exp}\left[-rac{1}{2}igg(rac{x-\mu}{\sigma}igg)^2
ight]$$

with support \mathbb{R} .

If $\mu=0$ and $\sigma=1$, then X is called a *standard normal variable* (or *standard Gaussian variable*) and is often represented as Z.



†

Theorem 5.7 (Affine transformations of normal variables)

Let $X\sim \mathcal{N}(\mu,\sigma^2)$ and a and b be two constants with $a\neq 0$. Then Y=aX+b is a normal random variable with $\mu_Y=a\mu+b$ and $\sigma_Y=|a|\sigma$.

Corollary 5.1 (Standardization of normal variables)

If $X \sim \mathcal{N}(\mu, \sigma^2)$, then $Z = (X - \mu)/\sigma$ has a standard normal distribution.

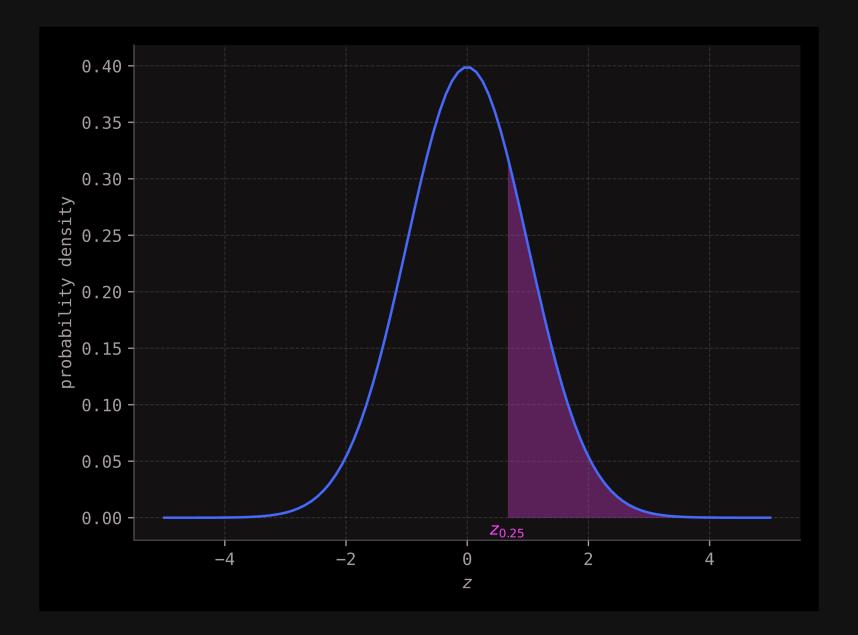
Definition 5.7

Let $Z \sim \mathcal{N}(0,1)$ be a standard normal random variable.

- The probability density function of Z is denoted $\phi(z)$.
- The cumulative distribution function of Z is denoted $\Phi(z)$.
- For each real number α , we define z_{α} to be the number for which the area under the density curve of Z to the *right* is exactly α . In other words, we have

$$z_lpha = \Phi^{-1}(1-lpha).$$

The number z_{α} is called a *critical value*.



Theorem 5.8 (Expectations and variances of normal variables)

If
$$X \sim \mathcal{N}(\mu, \sigma^2)$$
, then

$$E(X) = \mu \quad ext{and} \quad V(X) = \sigma^2.$$



Problem Prompt

Do problems 10-13 on the worksheet.