2.11. Bivariate continuous probability measures

Definition 2.7

Let P be a probability measure on a sample space S. We shall say P is *discrete* if every subset $A \subset S$ is an event and there is a function $p : S \to \mathbb{R}$ with discrete support such that

$$P(A) = \sum_{s \in A} p(s), \tag{2.2}$$

for all events A. In this case, the function p is called the *probability mass function* (PMF) of the probability measure P (or sometimes just the *probability function*), and S is called a *discrete probability space* (when equipped with P).

Definition 2.9

Let P be a probability measure on $\mathbb{R}$. We shall say P is *continuous* if there is a function $f : \mathbb{R} \to \mathbb{R}$ such that

$$P(A) = \int_A f(s)\,\mathrm{d}s \tag{2.4}$$

for all events $A \subset \mathbb{R}$. In this case, the function $f(s)$ is called the *probability density function* (PDF) of the probability measure P, and $\mathbb{R}$ is called a *continuous probability space* (when equipped with P).

Definition 2.12

Let P be a probability measure on $\mathbb{R}^2$. We shall say P is *continuous* if there is a function $f : \mathbb{R}^2 \to \mathbb{R}$ such that

$$P(C) = \iint_C f(s,t)\,\mathrm{d}s\mathrm{d}t$$

for all events $C \subset \mathbb{R}^2$. In this case, the function f is called the *probability density function* (PDF) of the probability measure P, and $\mathbb{R}^2$ is called a *continuous probability space* (when equipped with P).

Problem Prompt

Do problems 20 and 21 on the worksheet.

Theorem 2.9 (Properties of Probability Density Functions (bivariate version))

Let $f(s,t)$ be the probability density function of a continuous probability measure P on $\mathbb{R}^2$. Then:

1. $f(s,t) \geq 0$ for all $s, t \in \mathbb{R}$, and
2. $\iint_{\mathbb{R}^2} f(s,t)\,\mathrm{d}s\mathrm{d}t = 1$.

Theorem 2.10 (Continuous Probability Construction Lemma (bivariate version))

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function such that

1. $f(s,t) \geq 0$ for all $s, t \in \mathbb{R}$, and
2. $\iint_{\mathbb{R}^2} f(s,t)\,\mathrm{d}s\mathrm{d}t = 1$.

Then there is a unique continuous probability measure P on $\mathbb{R}^2$ such that

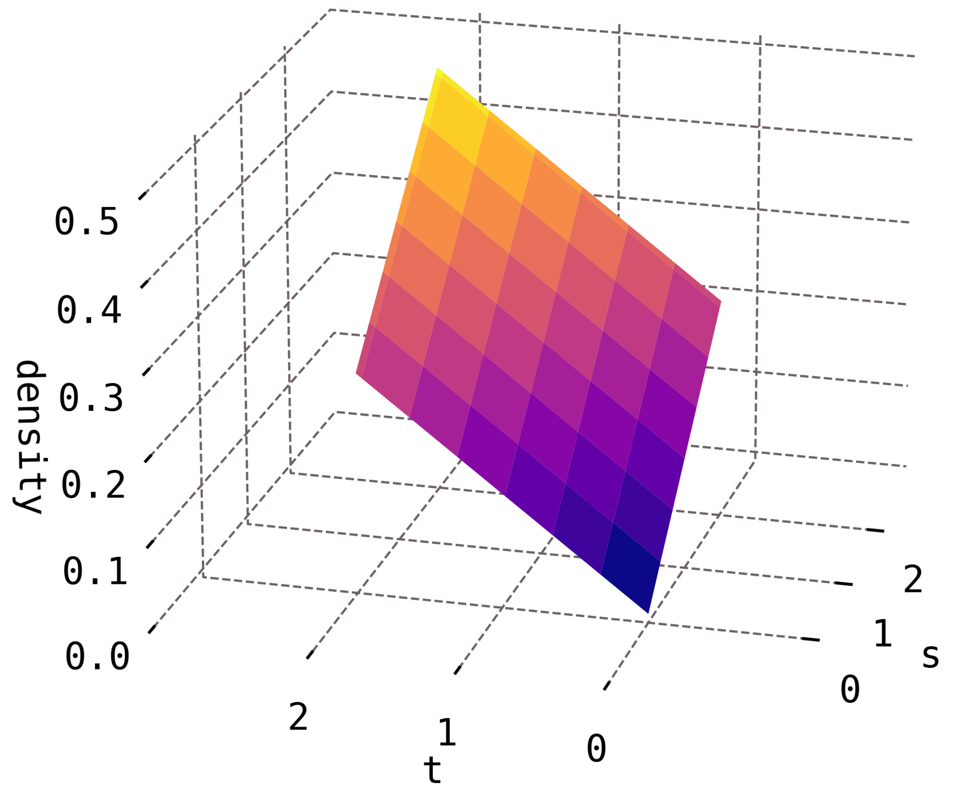
$$P(C) = \iint_C f(s,t)\,\mathrm{d}s\mathrm{d}t$$

for all events $C \subset \mathbb{R}^2$.

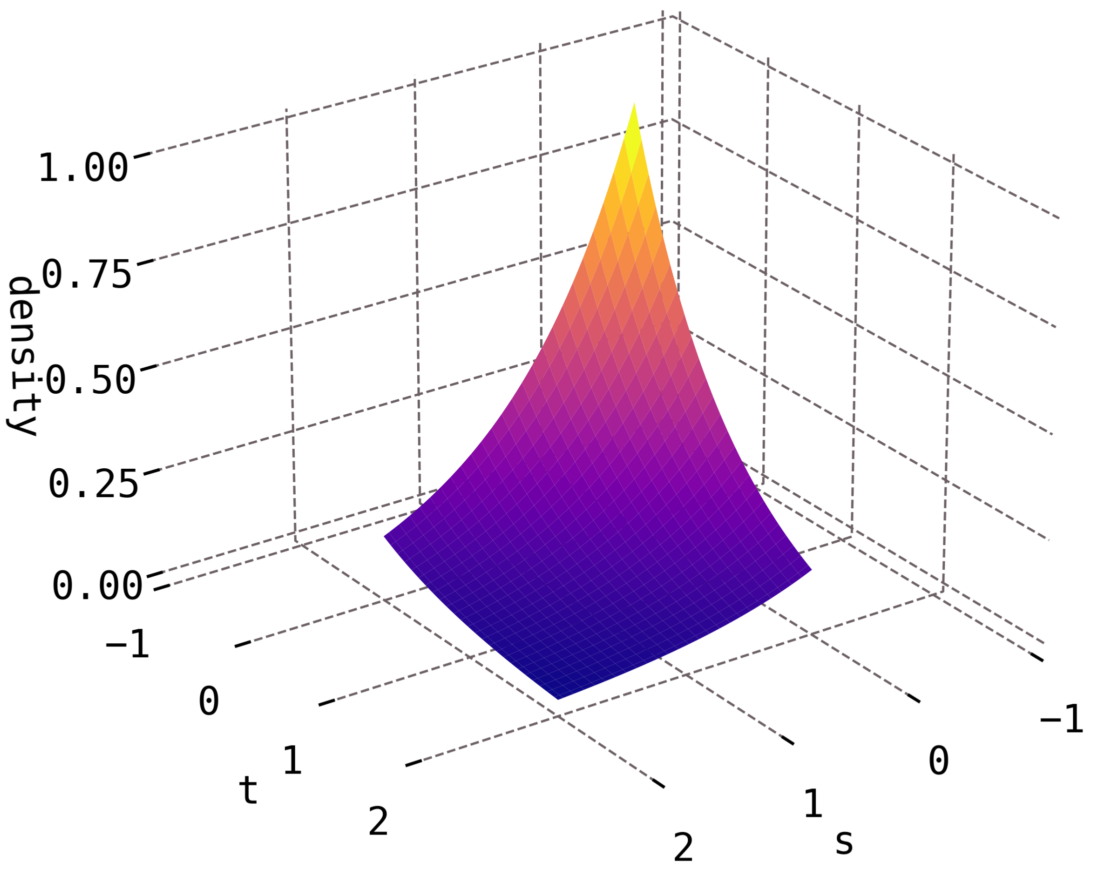
Problem Prompt

Do problem 22 on the worksheet.

2.12. Probability density graphs (bivariate versions)



$$f(s,t) = \begin{cases} \frac{1}{8}(s+t) & : 0 \leq s, t \leq 2, \\ 0 & : \text{otherwise.} \end{cases}$$



$$f(s,t) = \begin{cases} e^{-(s+t)} & : s,t > 0, \\ 0 & : \text{otherwise}, \end{cases}$$