

**Problem 1:** Suppose  $P$  is a probability measure defined on  $S = \{1, 2, 3, 4, 5\}$  with mass function

$s$	$p(s)$
1	0.1
2	0.3
3	0.2
4	0.3
5	0.1

Compute the information content  $I(s)$  of each sample point and the entropy  $H(P)$ .

**Problem 2:** Let  $X \sim \mathcal{Ber}(\theta)$  for  $\theta \in [0, 1]$ . Compute a formula for  $H(X)$  in terms of  $\theta$ .

**Problem 3:** Suppose  $P$  and  $Q$  are probability measures defined on  $S = \{1, 2, 3, 4, 5\}$  with mass functions

$s$	$p(s)$	$q(s)$
1	0.1	0.05
2	0.3	0.15
3	0.2	0.7
4	0.3	0.03
5	0.1	0.07

Compute the cross entropies  $H_P(Q)$  and  $H_Q(P)$ .

**Problem 4:** Suppose  $P$  and  $Q$  are probability measures defined on  $S = \{1, 2, 3, 4, 5\}$  with mass functions

$s$	$p(s)$	$q(s)$
1	0.1	0.05
2	0.3	0.15
3	0.2	0.7
4	0.3	0.03
5	0.1	0.07

Compute  $D(P \parallel Q)$  and  $D(Q \parallel P)$ .

**Problem 5:** While it is **not** true that  $D(P \parallel Q) = D(Q \parallel P)$  for **all**  $P$  and  $Q$ , there are examples of special distributions  $P$  and  $Q$  for which equality does hold. Find examples.

**Problem 6:** For each  $\phi \in [0, 1]$ , consider the communication channel with transition matrix

$$\mathbf{K} = \begin{bmatrix} 1 - \phi & \phi \\ \phi & 1 - \phi \end{bmatrix}$$

These are called *binary symmetric channels*. Suppose that  $X \sim \mathcal{Ber}(\alpha)$  for some  $\alpha \in [0, 1]$ , with the range of  $X$  enumerated as  $x_0 = 0$  and  $x_1 = 1$ . Show that  $X$  and the communication channel determine a random variable  $Y$  with range  $y_0 = 0$  and  $y_1 = 1$ . Determine its distribution.

**Problem 7:** Suppose  $X$  and  $Y$  are Bernoulli random variables with joint mass function given by

$p(x, y)$	$y = 0$	$y = 1$
$x = 0$	0.3	0.1
$x = 1$	0.36	0.24

(a) Compute the transition matrix of the communication channel induced from  $X$  and  $Y$ .

(b) Compute the mutual information  $I(X, Y)$ .