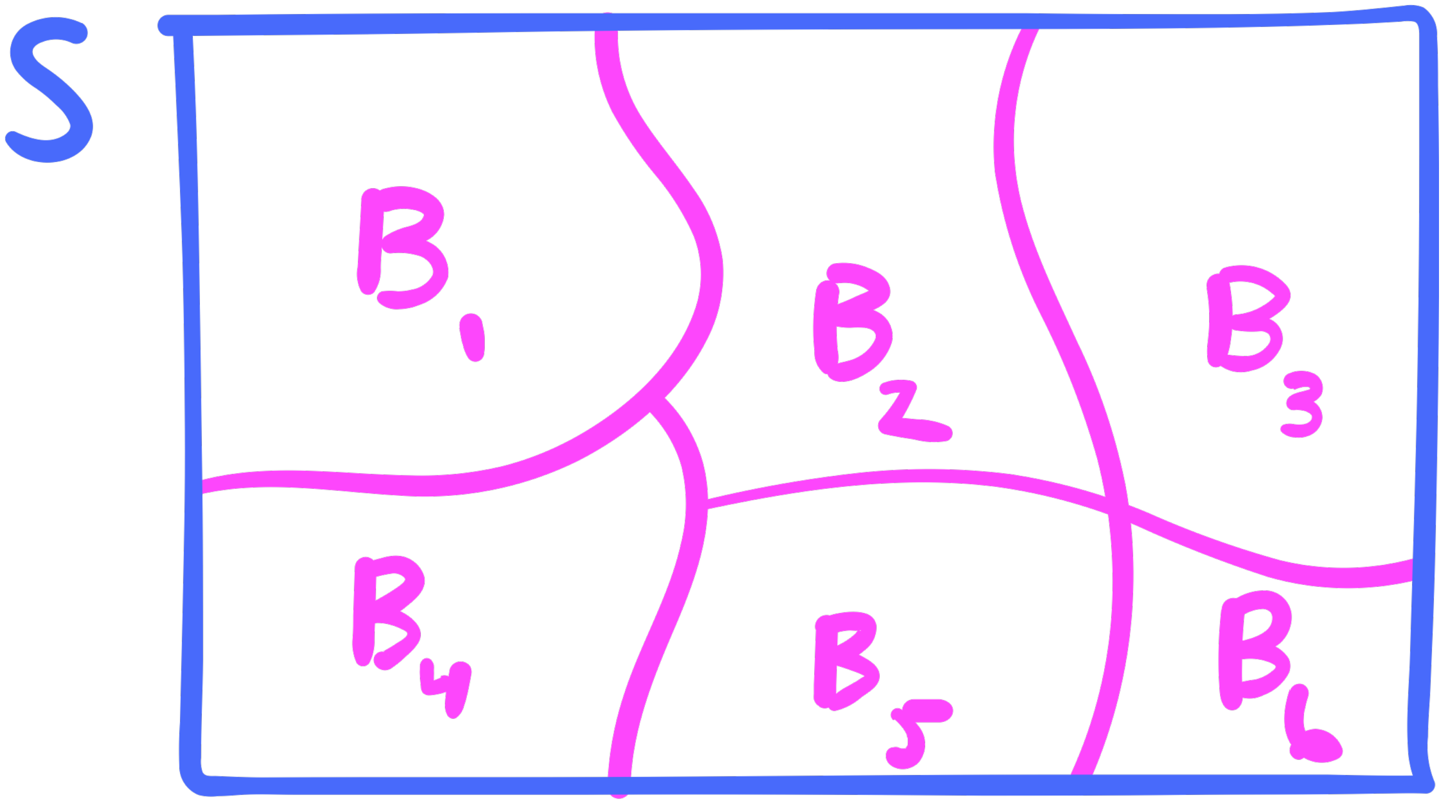
3.7. The Law of Total Probability and Bayes' Theorem

partitions



Theorem 3.8 (The Law of Total Probability)

Suppose that $\{B_1, B_2, \ldots, B_n\}$ is a partition of a sample space S, where each B_k is an event. Then for any event A, we have

$$P(A) = \sum_{k=1}^{n} P(A|B_k)P(B_k).$$

Theorem 3.9 (The Law of Total Probability (Two-Event Version))

Let A and B be two events in a sample space S. Then

$$P(A) = P(A|B)P(B) + P(A|B^c)P(B^c),$$

where I've written B^c for $S \setminus B$.

Problem Prompt

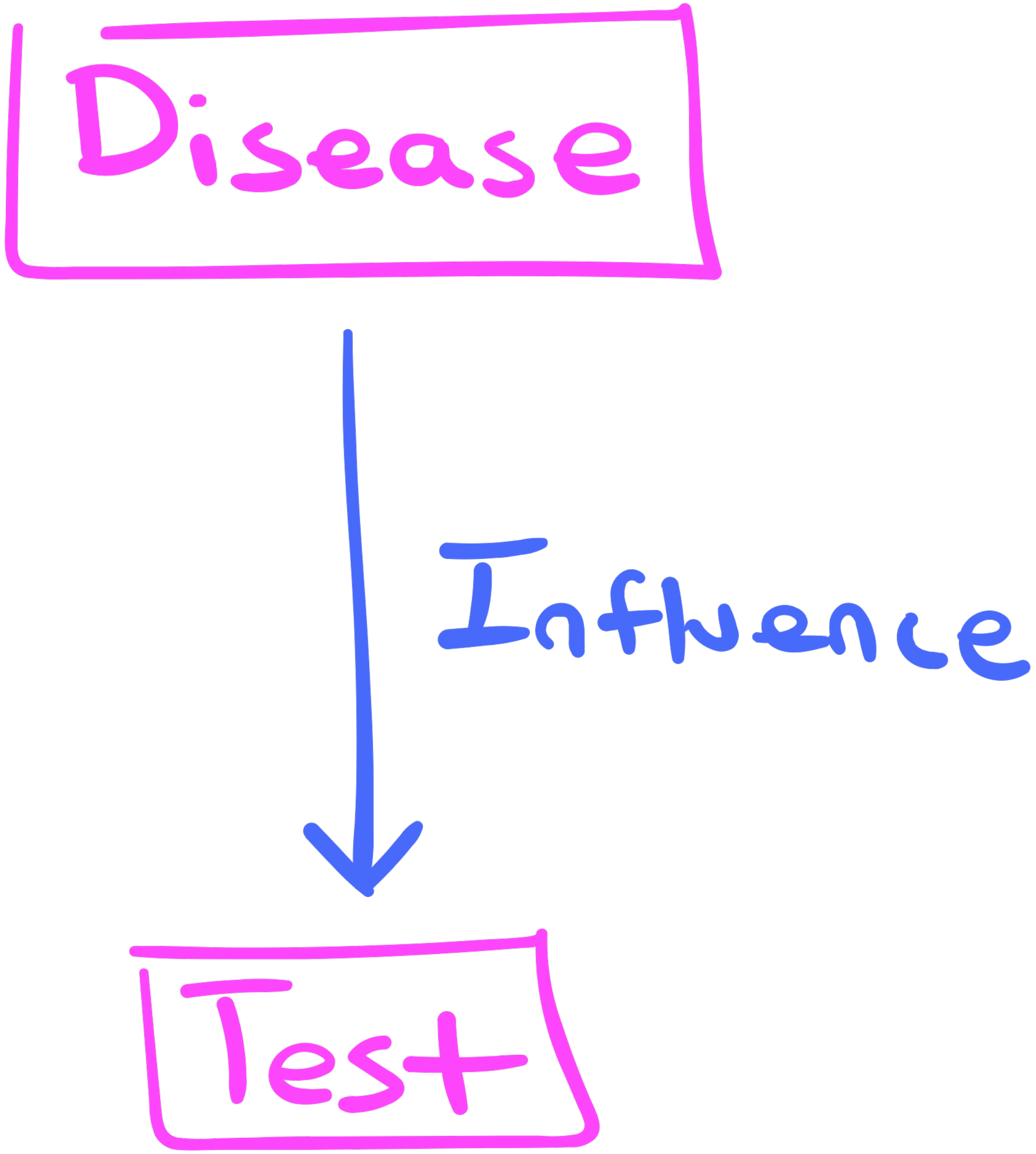
Do problem 10 on the worksheet.

The Canonical Example of Bayes' Theorem

Suppose that a test has been devised to detect a certain disease. Moreover, suppose that:

- The disease affects 0.1% of the population.
- The test does not produce any false negatives.
- The test produces false positives at a rate of 5%.

Given that a randomly selected individual tests positive for the disease, what is the probability that they have it?



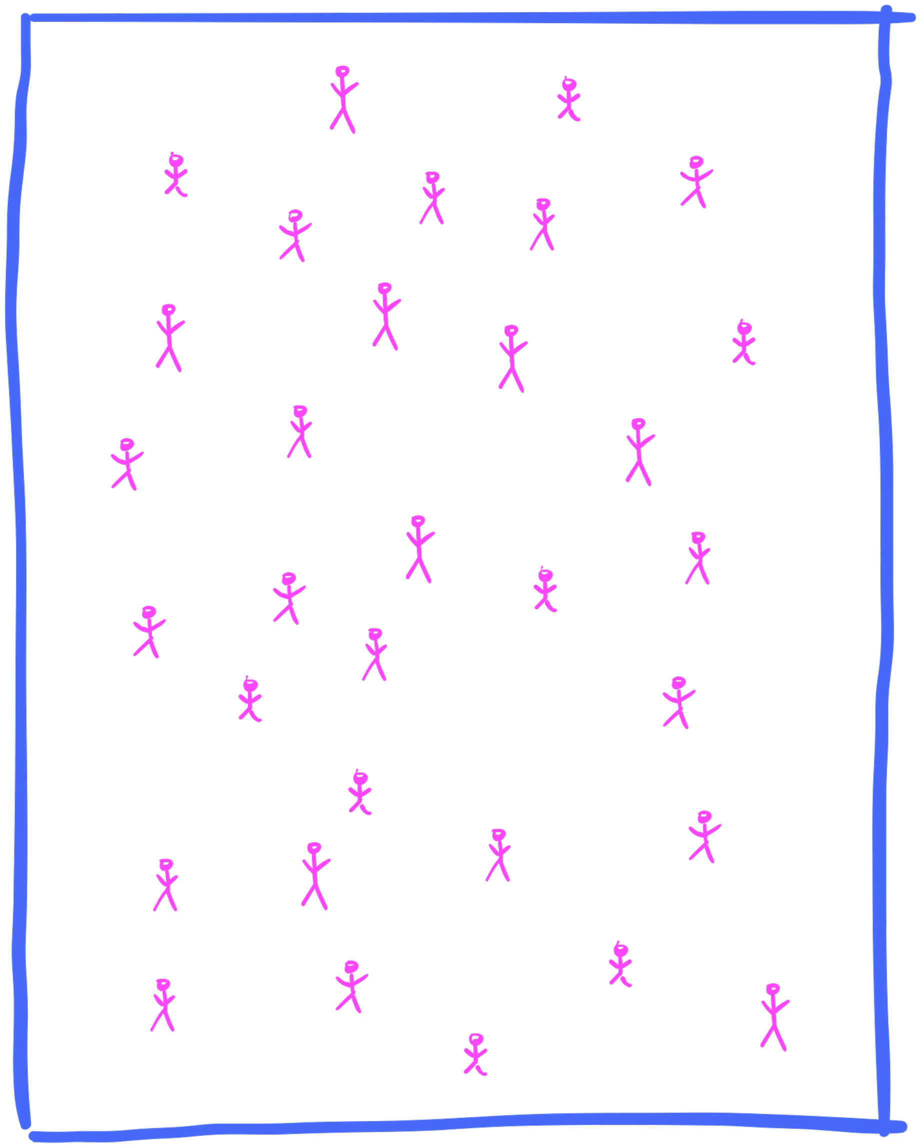
Disease
Influence
Test

Theorem 3.10 (Bayes' Theorem)

Let A and B be two events in a sample space S with $P(B) > 0$. Then

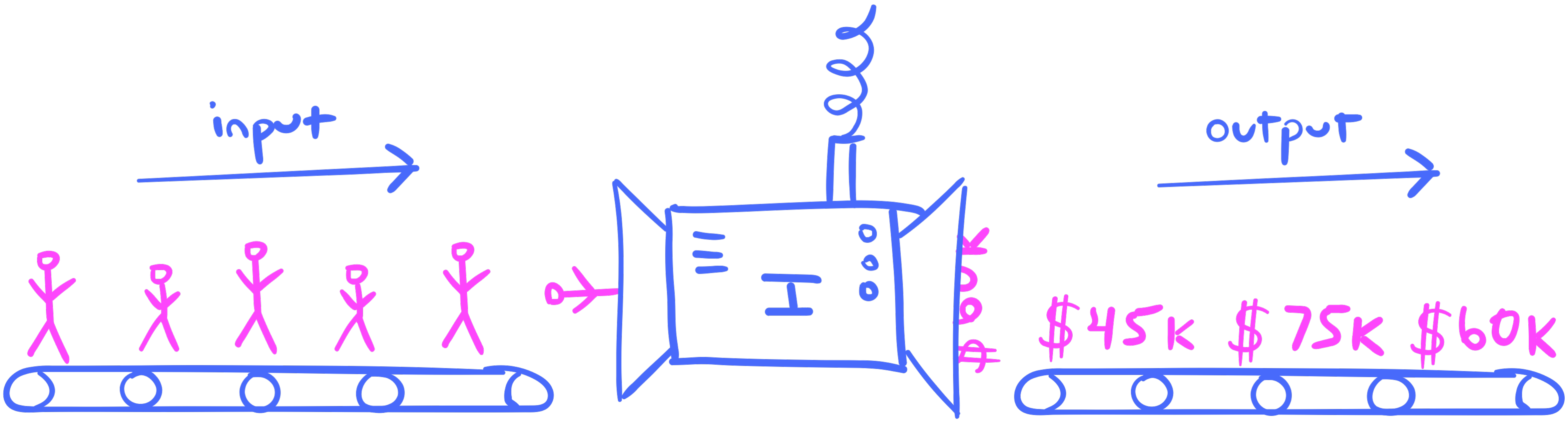
$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}.$$

4.1. Random variables

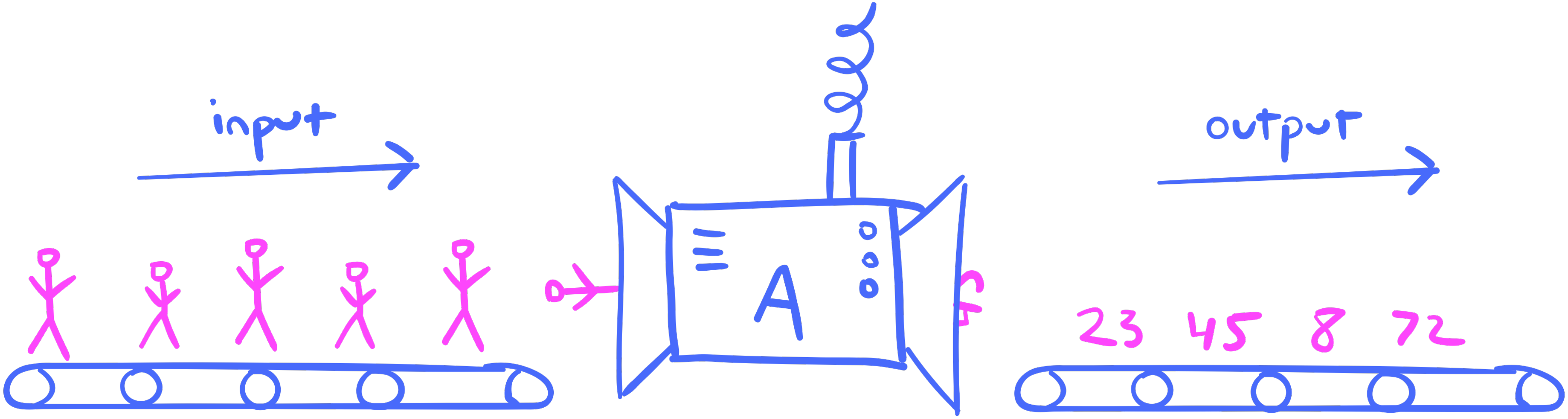


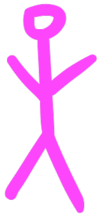
population

annual income machine



age machine



I() = \$70k

A() = 32

Definition 4.1

Let S be a probability space. A *random variable* on S is a function $X : S \to \mathbb{R}$.

Problem Prompt

Let’s get some practice with random variables! Do problems 1-4 on the worksheet.