4.3. Discrete and continuous random variables

Definition 4.3

Let $X : S \to \mathbb{R}$ be a random variable.

1. We shall say X is *discrete* if there exists a function $p : \mathbb{R} \to \mathbb{R}$ such that

$$P(X \in A) = \sum_{x \in A} p(x)$$

   for all events $A \subset \mathbb{R}$. In this case, $p(x)$ is called the *probability mass function of* X.

2. We shall say X is *continuous* if there exists a function $f : \mathbb{R} \to \mathbb{R}$ such that

$$P(A) = \int_A f(x)\, \mathrm{d}x$$

   for all events $A \subset \mathbb{R}$. In this case, $f(x)$ is called the *probability density function of* X.

Recognizing Discrete and Continuous Random Variables

- If the range of a random variable is *finite* or *countably infinite*, then it is discrete.
- If the range of a random variable is a *continuum* of values, then it is continuous.

Problem Prompt

Let’s get some practice recognizing discrete and continuous random variables, and computing some of their probability measures. Do problems 8 and 9 on the worksheet.

4.4. Distribution and quantile functions

Definition 4.4

Let X be a random variable. The *distribution function of* X is the function $F : \mathbb{R} \to \mathbb{R}$ defined by

$$F(x) = P(X \leq x).$$

In particular:

1. If X is discrete with probability mass function $p(x)$, then

$$F(x) = \sum_{y \leq x} p(y),$$

   where the sum ranges over all $y \in \mathbb{R}$ with $y \leq x$.
2. If X is continuous with density function $f(x)$, then

$$F(x) = \int_{-\infty}^{x} f(y)\, \mathrm{d}y.$$

Definition 4.5

Let X be a random variable with distribution function $F : \mathbb{R} \to [0, 1]$. The *quantile function of* X is the function $Q : [0, 1] \to \mathbb{R}$ defined so that

$$Q(p) = \min\{x \in \mathbb{R} : p \leq F(x)\}.$$

In other words, the value $x = Q(p)$ is the smallest $x \in \mathbb{R}$ such that $p \leq F(x)$.

1. The value $Q(p)$ is called the *p-th quantile of* X.
2. The quantile $Q(0.5)$ is called the *median of* X.

Problem Prompt

Do problem 10 on the worksheet.