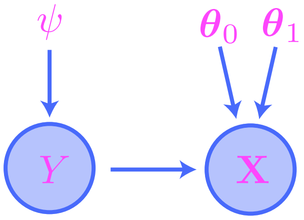
Problem 1: Consider a *Naive Bayes model* as described in the programming assignment for chapter 12. The underlying graph is of the form



where $\mathbf{X} \in \mathbb{R}^n$. The parameters are given by a number $\psi \in [0,1]$ which parametrizes the distribution of $Y \sim \mathcal{B}er(\psi)$, as well as two vectors $\boldsymbol{\theta}_0, \boldsymbol{\theta}_1 \in [0,1]^n$. The link function at $\mathbf{X}$ is given by

$$p(\mathbf{x} \mid y;\ \boldsymbol{\theta}_0, \boldsymbol{\theta}_1) = \prod_{j=1}^{n} \phi_j^{x_j}(1-\phi_j)^{1-x_j}$$

where

$$\boldsymbol{\phi} = (1-y)\boldsymbol{\theta}_0 + y\boldsymbol{\theta}_1$$

and $\boldsymbol{\phi}^\intercal = (\phi_1, \ldots, \phi_n)$.

(a) Assuming that Naive Bayes models are trained as **generative** models, write down a formula for the model likelihood function $\mathcal{L}_{\text{model}}(\psi, \boldsymbol{\theta}_0, \boldsymbol{\theta}_1)$. For simplicity, your formula should contain the ϕ_j's rather than the parameters $\boldsymbol{\theta}_0$ and $\boldsymbol{\theta}_0$ themselves.

(b) Using your answer from part (a), write down a formula for the model surprisal function $\mathcal{I}_{\text{model}}(\psi, \boldsymbol{\theta}_0, \boldsymbol{\theta}_1)$. For simplicity, your formula should contain the ϕ_j's rather than the parameters $\boldsymbol{\theta}_0$ and $\boldsymbol{\theta}_0$ themselves.

(c) Using your answer from part (b), write down an explicit formula for the cross entropy stochastic objective function $J(\psi, \boldsymbol{\theta}_0, \boldsymbol{\theta}_1)$ for a dataset of size m.

Problem 2: Consider the observed dataset

$$(0,0),(1,1),(2,3) \in \mathbb{R}^2.$$

Using this dataset, compute the exact MLEs for the parameters β_0 and β_1 of a simple linear regression model (with known variance).

Problem 3: For the neural network trained in Section 13.5, compute the following:

(a) The number of gradient steps per epoch.

(b) The *exact* number of gradient steps over all epochs.

(c) The number of trainable parameters in the network.