5. Examples of random variables

hmmm... which one??? probability distribution #1 real-world probability distribution #2 Scenario probability distribution #3 probability distribution #4

box o probability
distributions

5.1. Bernoulli distributions

Definition 5.1

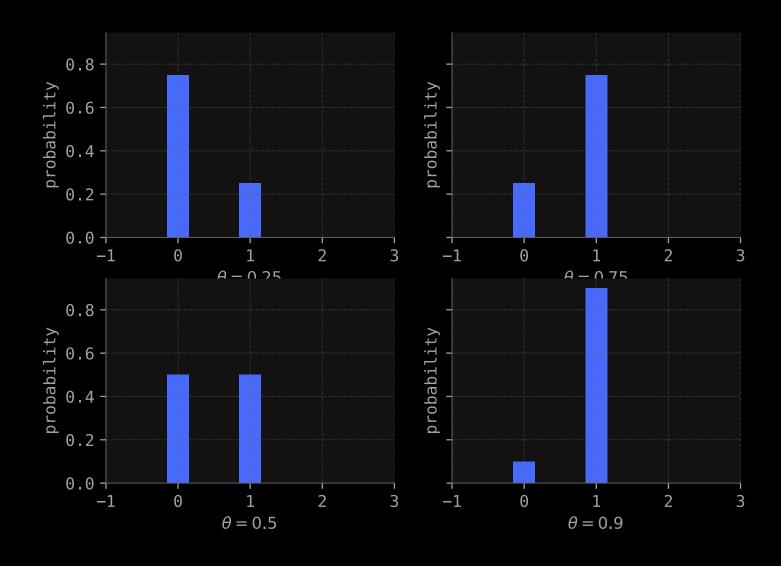
Let θ be a real number with $0 \le \theta \le 1$. A discrete random variable X is said to have a Bernoulli distribution with parameter θ , denoted

$$X \sim \mathcal{B}er(\theta),$$
 (5.1)

if its probability mass function is given by

$$p(x;\theta) = \theta^x (1-\theta)^{1-x}. \tag{5.2}$$

with support $\{0,1\}$.





An archetypical Bernoulli scenario

Q: Suppose that a coin is flipped once and X is the number of heads obtained. Suppose further that we allow for the possibility that the coin is *loaded*, so that it lands heads with probability θ (which may not be 0.5!). What is the distribution of the random variable X?

A: $X \sim \mathcal{B}er(\theta)$.

Theorem 5.1 (Expectations and variances of Bernoulli variables)

If $X \sim \mathcal{B}er(heta)$, then

$$E(X) = \theta \quad ext{and} \quad V(X) = \theta(1 - \theta).$$

5.2. Binomial distributions

Definition 5.2

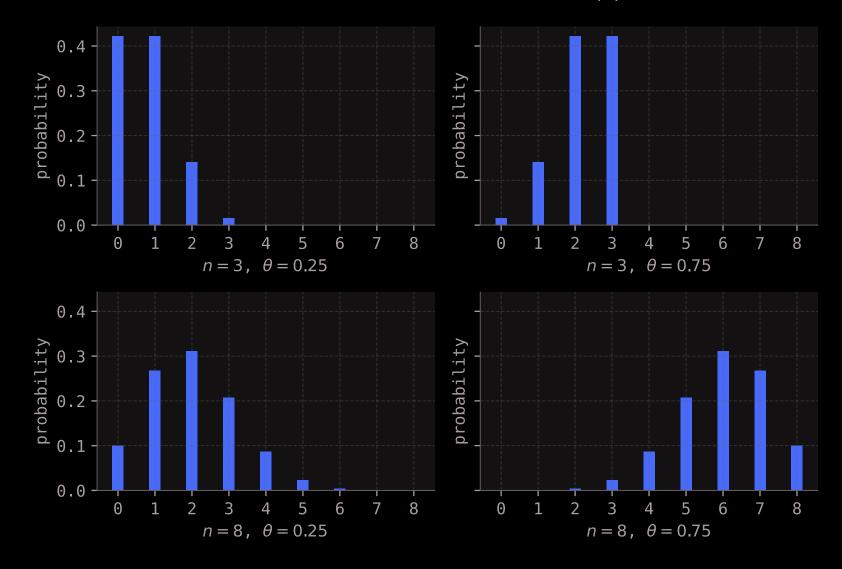
Let $n \geq 0$ be an integer and let θ be a real number with $0 \leq \theta \leq 1$. A discrete random variable X is said to have a binomial distribution with parameters n and θ , denoted

$$X \sim \mathcal{B}in(n, heta),$$

if its probability mass function is given by

$$p(x;n, heta)=inom{n}{x} heta^x(1- heta)^{n-x}$$

with support $\{0, 1, \dots, n\}$.





An archetypical binomial scenario

Q: Suppose that a coin is flipped n times and that each flip is independent of the others. Suppose further that we allow for the possibility that the coin is *loaded*, so that it lands heads with probability θ (which may not be 0.5!). If X is the number of heads obtained, what is the distribution of the random variable X?

A: $X \sim \mathcal{B}in(n, \theta)$.

†

Theorem 5.2 (Expectations and variances of binomial variables)

If $X \sim \mathcal{B}in(n, heta)$, then

$$E(X) = n\theta$$
 and $V(X) = n\theta(1-\theta)$. (5.5)

À

Theorem 5.3 (Binomial variables as sums of Bernoulli variables)

Let Y_1, Y_2, \ldots, Y_n be a sequence of independent Bernoulli variables, all with the same distribution $\mathcal{B}er(\theta)$. Then the random variable

$$X = Y_1 + Y_2 + \dots + Y_n$$

is a $\mathcal{B}in(n,\theta)$ random variable.



Problem Prompt

Do problems 1 and 2 on the worksheet.