# 3.1. The Product and Sum Rules for Counting

**Theorem 3.1 (The Product Rule for Counting)**

Suppose that a procedure can be broken down into a sequence of two tasks. If there are $m$ ways to do the first task, and for each of these ways of doing the first task there are $n$ ways to do the second task, then there are $mn$ total ways to do the procedure.

**Theorem 3.2 (The Sum Rule for Counting)**

If a procedure can be done either in one of $m$ ways or in one of $n$ ways, where none of the set of $m$ ways is the same as any of the set of $n$ ways, then there are $m + n$ ways to do the procedure.

**Problem Prompt**

Do problem 1 on the worksheet.

## 3.2. Permutations and combinations

# permutations

box of n objects

choose objects

(object 1, object 2, ..., object k)

k-tuple = order matters

# combinations

box of $n$ objects

choose objects

$\{\text{object } 1, \text{object } 2, \ldots, \text{object } k\}$

set = order does not matter

**Definition 3.1**

An ordered collection of $k$ distinct objects is called a *permutation* of those objects. The number of permutations of $k$ objects selected from a collection of $n$ objects will be designated by the symbol $P_k^n$.

**Definition 3.2**

An unordered collection of $k$ distinct objects is called a *combination* of those objects. The number of combinations of $k$ objects selected from a collection of $n$ objects will be designated by the symbol $C_k^n$ or $\binom{n}{k}$.

**Theorem 3.3 (Formula for Counting Permutations)**

For $0 \leq k \leq n$, we have

$$P_k^n = \frac{n!}{(n-k)!}.$$

*Note*: Remember the convention that $0! = 1$.

**Problem Prompt**

Do problem 2 on the worksheet.

### Theorem 3.4 (Formula for Counting Combinations)

For $0 \leq k \leq n$, we have

$$\binom{n}{k} = C_k^n = \frac{n!}{(n-k)!k!}.$$

*Note*: Remember the convention that $0! = 1$.

**Problem Prompt**

Do problem 3 on the worksheet.