4.6. The algebra of random variables

s	$X(s)$	$Y(s)$
1	-1	0
2	1	2
3	3	-1
4	0	3

s	$X(s)$	$Y(s)$	$(X+Y)(s)$
1	-1	0	$-1 + 0 = -1$
2	1	2	$1 + 2 = 3$
3	3	-1	$3 - 1 = 2$
4	0	3	$0 + 3 = 3$

🔔 Problem Prompt

Do problem 16 on the worksheet.

s	$Y(s)$	$(4Y)(s)$
1	0	$4 \cdot 0 = 0$
2	2	$4 \cdot 2 = 8$
3	-1	$4 \cdot (-1) = -4$
4	3	$4 \cdot 3 = 12$

- $(X \pm Y)(s) = X(s) \pm Y(s),$
- $(XY)(s) = X(s)Y(s),$
- $(X/Y)(s) = X(s)/Y(s),$ when $Y(s) \neq 0,$
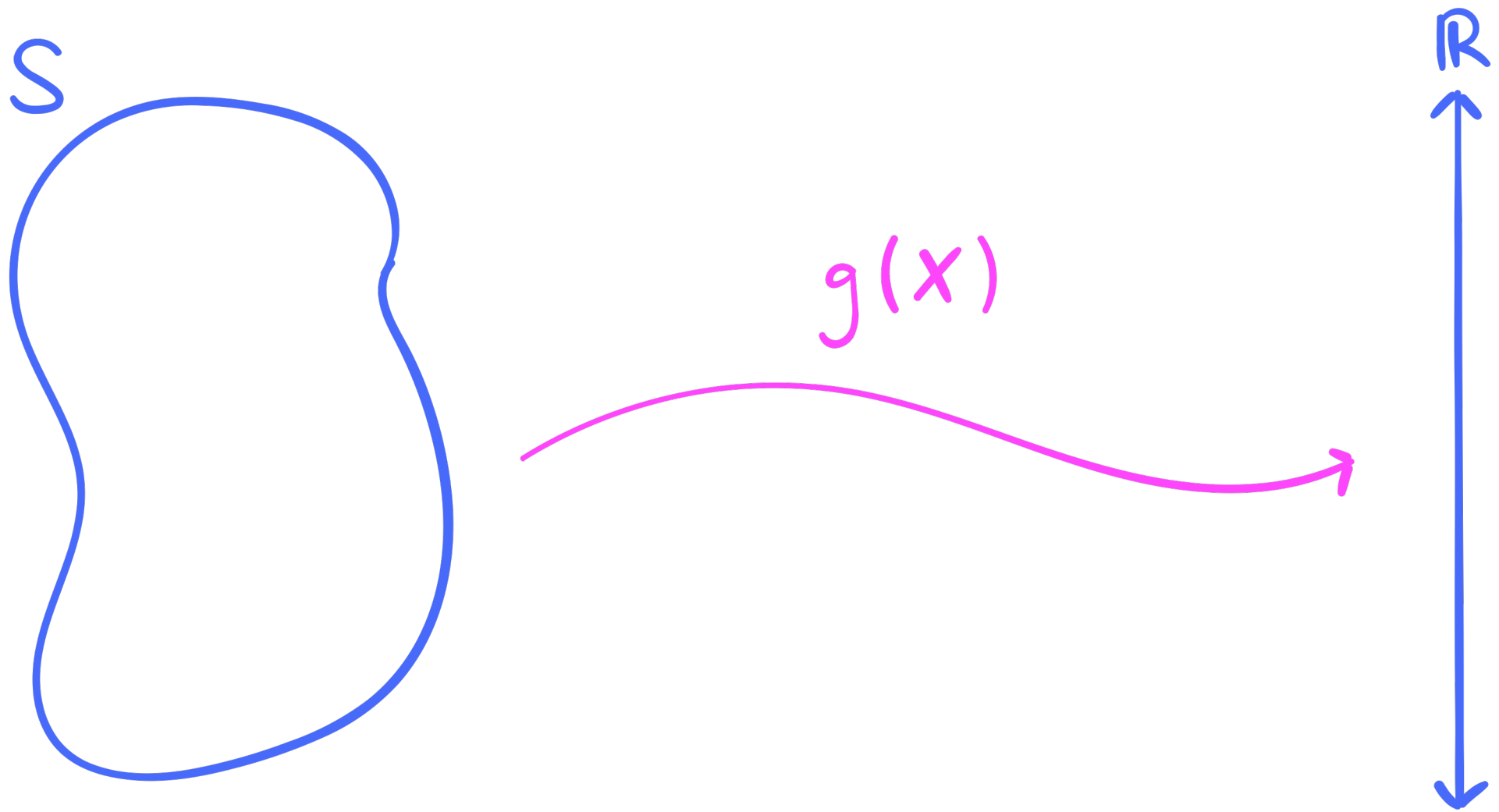
4.7. Functions of random variables

s	$X(s)$
1	0
2	2
3	-1
4	3

s	$X(s)$	$X^2(s)$
1	0	$0^2 = 0$
2	2	$2^2 = 4$
3	-1	$(-1)^2 = 1$
4	3	$3^2 = 9$

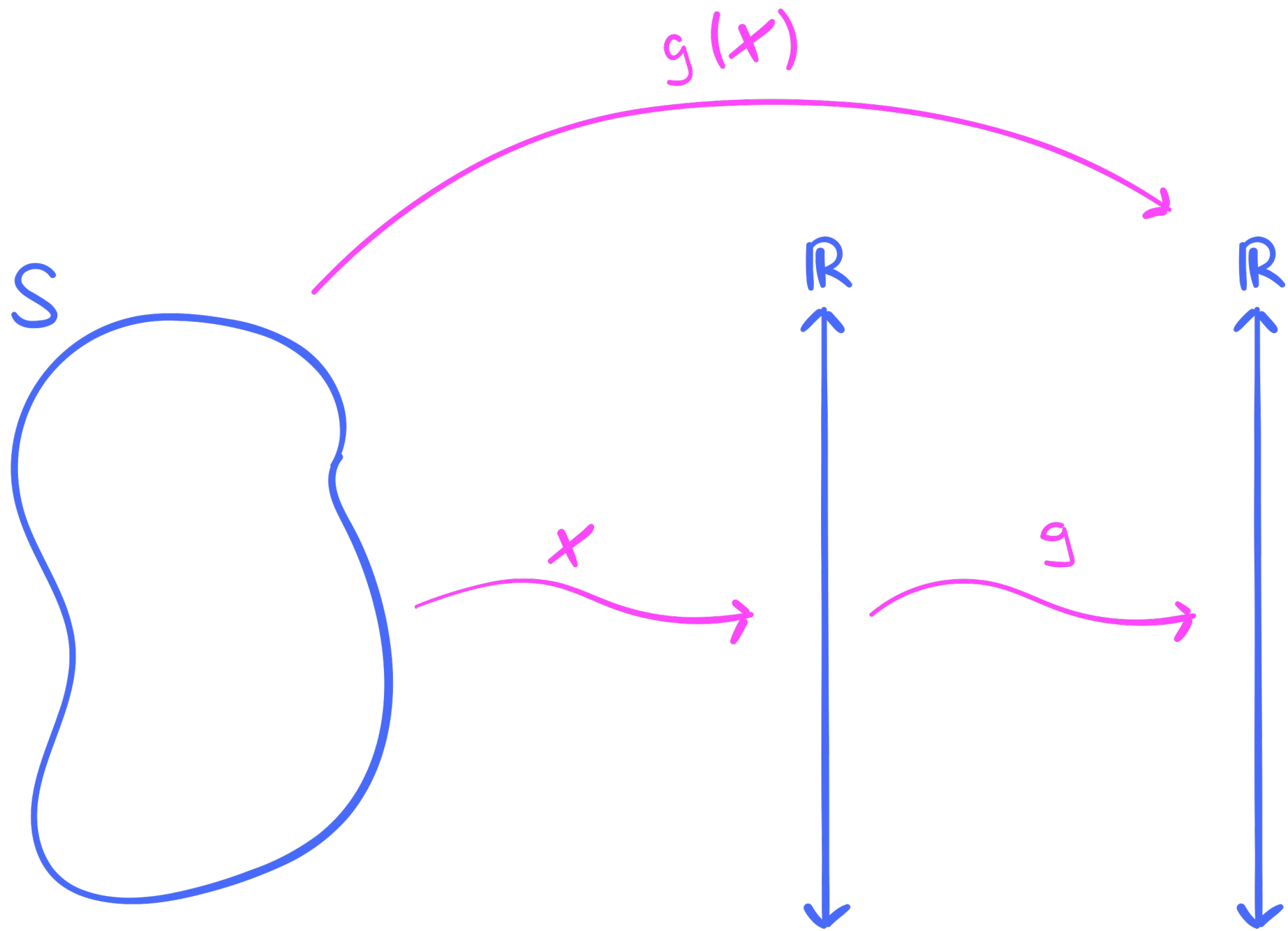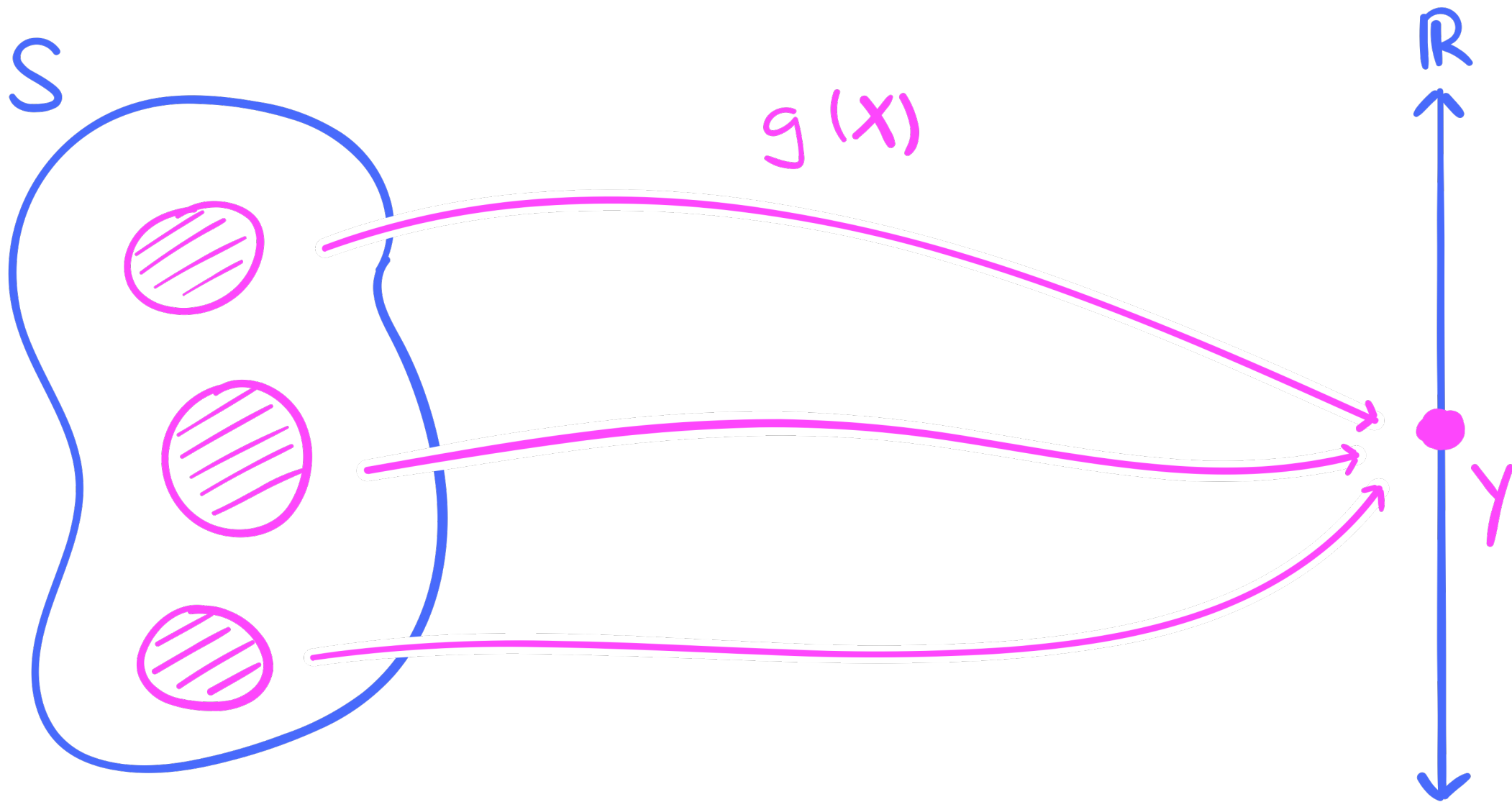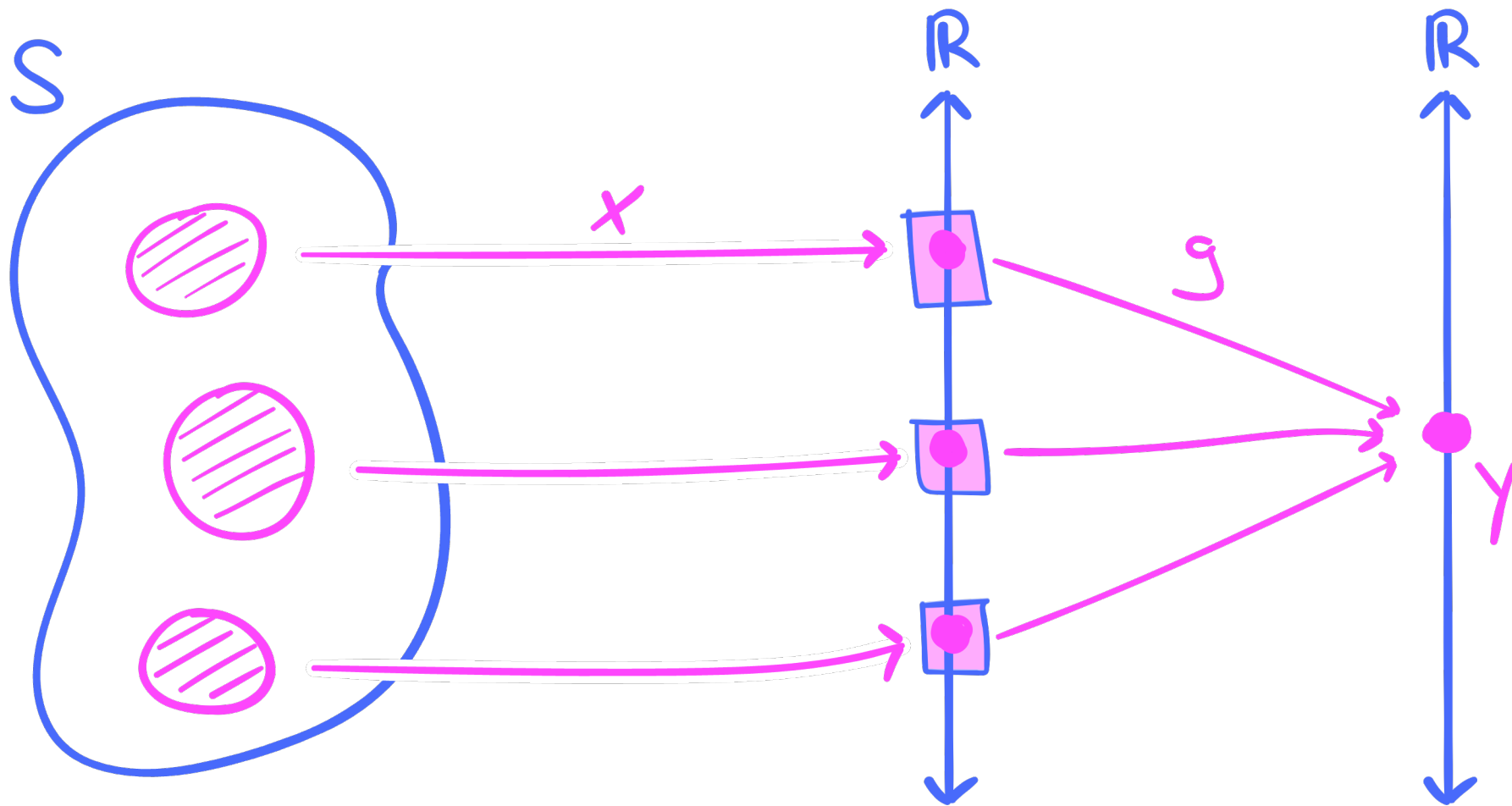## 🔔 Problem Prompt

Have a go at problem 17 on the worksheet.

4.8. Expectations of functions of random variables and the LotUS

S

$g(x)$

$\mathbb{R}$

$g(x)$

S

$\mathbb{R}$

$\mathbb{R}$

x

g

$$\square = g^{-1}(y)$$

$\{ s \in S : X(s) = x_1 \}$

S

$\{ s \in S : X(s) = x_2 \}$

X

$\mathbb{R}$

g

x_1

x_2

x_3

$\{ s \in S : X(s) = x_3 \}$

$\mathbb{R}$

y

$\blacksquare = g^{-1}(y)$

> 🔔 **Theorem 4.1 (Law of the Unconscious Statistician (LotUS))**
>
> Let X be a random variable and let $y = g(z)$ be a real-valued function on the real line $\mathbb{R}$.
>
> - If X is discrete with probability mass function $p(x)$, then
>
> $$E(g(X)) = \sum_{x \in \mathbb{R}} g(x) \cdot p(x).$$
>
> - If X is continuous with probability density function $f(x)$, then
>
> $$E(g(X)) = \int_{\mathbb{R}} g(x) \cdot f(x) \, dx.$$

🔔 Problem Prompt

Do problems 18 and 19 on the worksheet.