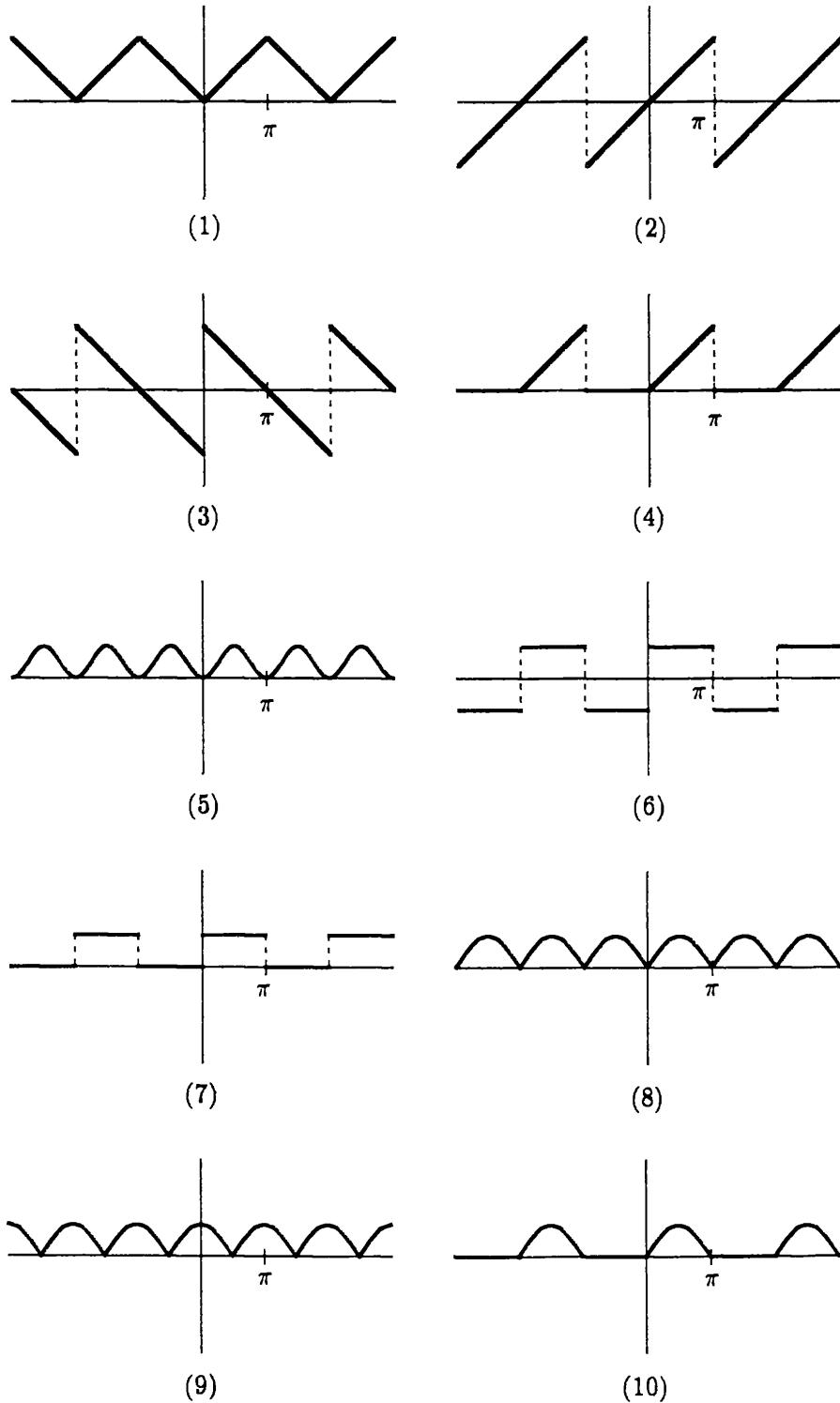


26 Chapter 2. Fourier Series

TABLE 1. FOURIER SERIES

The functions f in this table are all understood to be 2π -periodic. The formula for $f(\theta)$ on either $(-\pi, \pi)$ or $(0, 2\pi)$ (except perhaps at its points of discontinuity) is given in the left column; the Fourier series of f is given in the right column; and the graph of f is sketched on the facing page.

1.	$f(\theta) = \theta \quad (-\pi < \theta < \pi)$	$2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin n\theta$
2.	$f(\theta) = \theta \quad (-\pi < \theta < \pi)$	$\frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2n-1)\theta}{(2n-1)^2}$
3.	$f(\theta) = \pi - \theta \quad (0 < \theta < 2\pi)$	$2 \sum_{n=1}^{\infty} \frac{\sin n\theta}{n}$
4.	$f(\theta) = \begin{cases} 0 & (-\pi < \theta < 0) \\ \theta & (0 < \theta < \pi) \end{cases}$	$\frac{\pi}{4} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2n-1)\theta}{(2n-1)^2} + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin n\theta$
5.	$f(\theta) = \sin^2 \theta$	$\frac{1}{2} - \frac{1}{2} \cos 2\theta$
6.	$f(\theta) = \begin{cases} -1 & (-\pi < \theta < 0) \\ 1 & (0 < \theta < \pi) \end{cases}$	$\frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2n-1)\theta}{2n-1}$
7.	$f(\theta) = \begin{cases} 0 & (-\pi < \theta < 0) \\ 1 & (0 < \theta < \pi) \end{cases}$	$\frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2n-1)\theta}{2n-1}$
8.	$f(\theta) = \sin \theta $	$\frac{2}{\pi} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos 2n\theta}{4n^2-1}$
9.	$f(\theta) = \cos \theta $	$\frac{2}{\pi} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n \cos 2n\theta}{4n^2-1}$
10.	$f(\theta) = \begin{cases} 0 & (-\pi < \theta < 0) \\ \sin \theta & (0 < \theta < \pi) \end{cases}$	$\frac{1}{\pi} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos 2n\theta}{4n^2-1} + \frac{1}{2} \sin \theta$



28 Chapter 2. Fourier Series

TABLE 1 (continued)

11.	$f(\theta) = \begin{cases} \theta & (-a < \theta < a) \\ a \frac{\pi-\theta}{\pi-a} & (a < \theta < \pi) \\ a \frac{\pi+\theta}{a-\pi} & (-\pi < \theta < -a) \end{cases}$	$\frac{2}{\pi-a} \sum_{n=1}^{\infty} \frac{\sin na}{n^2} \sin n\theta$
12.	$f(\theta) = \begin{cases} (2a)^{-1} & (\theta < a) \\ 0 & (a < \theta < \pi) \end{cases}$	$\frac{1}{2\pi} + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{\sin na}{na} \cos n\theta$
13.	$f(\theta) = \begin{cases} (2a)^{-1} & (\theta - \theta_0 < a) \\ 0 & (a < \theta - \theta_0 < \pi) \end{cases}$	$\frac{1}{2\pi} + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{\sin na}{na} (\cos n\theta_0 \cos n\theta + \sin n\theta_0 \sin n\theta)$
14.	$f(\theta) = \begin{cases} 1 & (-a < \theta < a) \\ -1 & (2a < \theta < 4a) \\ 0 & \text{elsewhere in } (-\pi, \pi) \end{cases}$	$\sum_{n=1}^{\infty} \frac{\sin na}{n} [(1 - \cos 3na) \cos n\theta - \sin 3na \sin n\theta]$
15.	$f(\theta) = \begin{cases} a^{-2}(a - \theta) & (\theta < a) \\ 0 & (a < \theta < \pi) \end{cases}$	$\frac{1}{2\pi} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1 - \cos na}{n^2 a^2} \cos n\theta$
16.	$f(\theta) = \theta^2 \quad (-\pi < \theta < \pi)$	$\frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos n\theta$
17.	$f(\theta) = \theta(\pi - \theta) \quad (-\pi < \theta < \pi)$	$\frac{8}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2n-1)\theta}{(2n-1)^3}$
18.	$f(\theta) = e^{b\theta} \quad (-\pi < \theta < \pi)$	$\frac{\sinh b\pi}{\pi} \sum_{n=-\infty}^{\infty} \frac{(-1)^n}{b - in} e^{in\theta}$
19.	$f(\theta) = e^{b\theta} \quad (0 < \theta < 2\pi)$	$\frac{e^{2\pi b} - 1}{2\pi} \sum_{n=-\infty}^{\infty} \frac{e^{in\theta}}{b - in}$
20.	$f(\theta) = \sinh \theta \quad (-\pi < \theta < \pi)$	$\frac{2 \sinh \pi}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{n^2 + 1} \sin n\theta$

