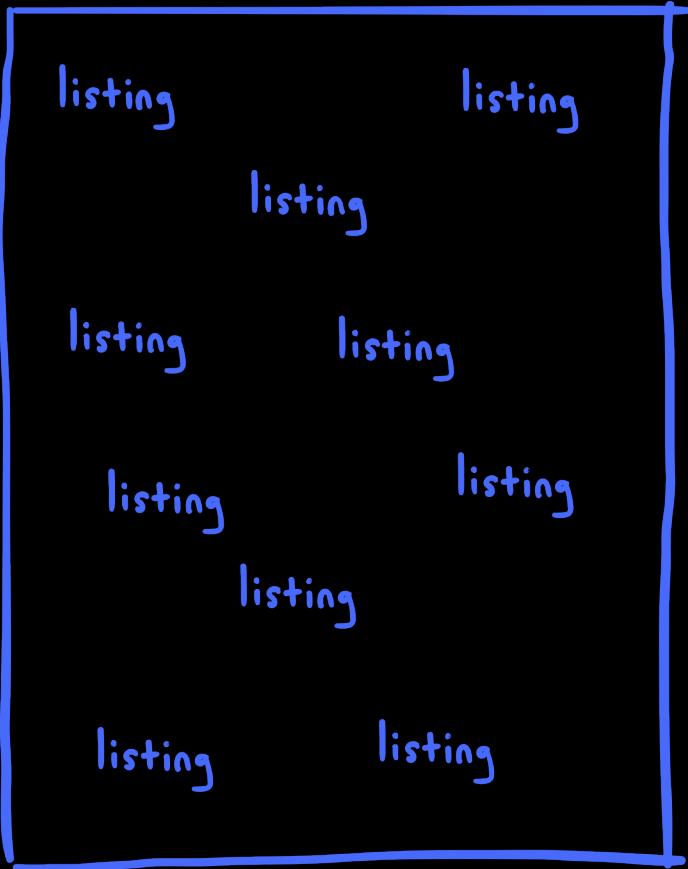


## 6. Connecting theory to practice: data and samples

## 6.1. Data and random samples

$S$



$\rightarrow R$

$X = \text{price}$

( listing 1, listing 2, listing 3, ..., listing n )

$x_1 \downarrow$

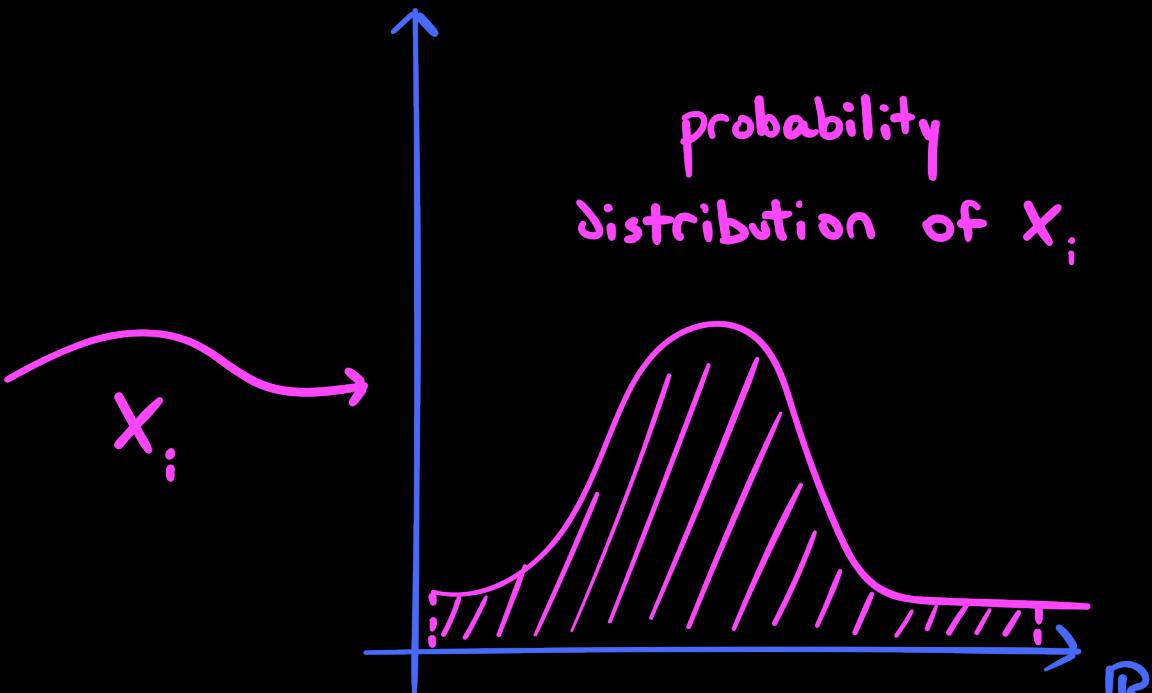
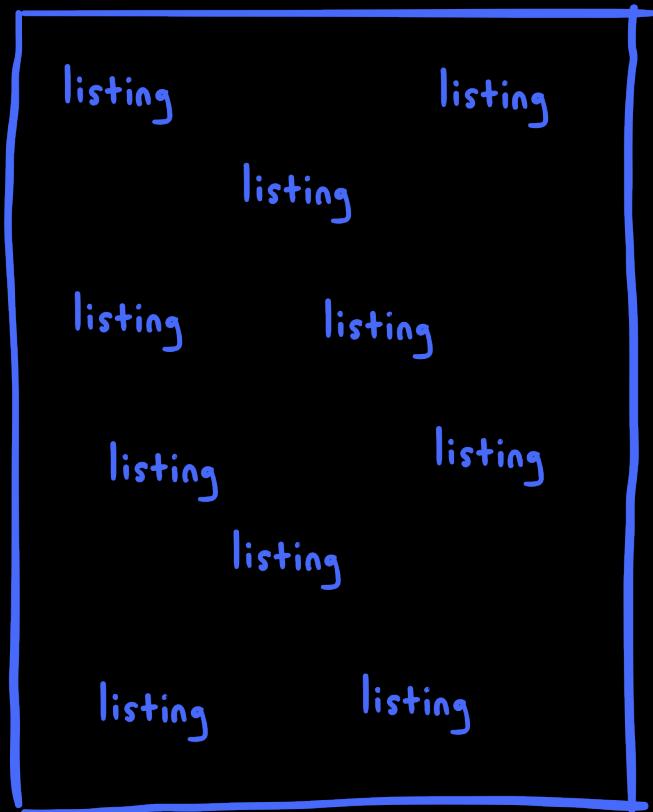
$x_2 \downarrow$

$x_3 \downarrow$

$x_n \downarrow$

( price 1, price 2, price 3, ..., price n )

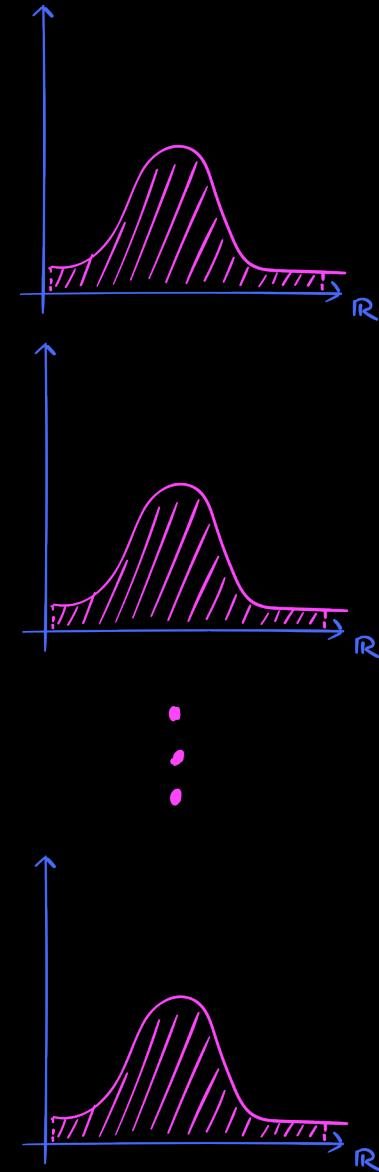
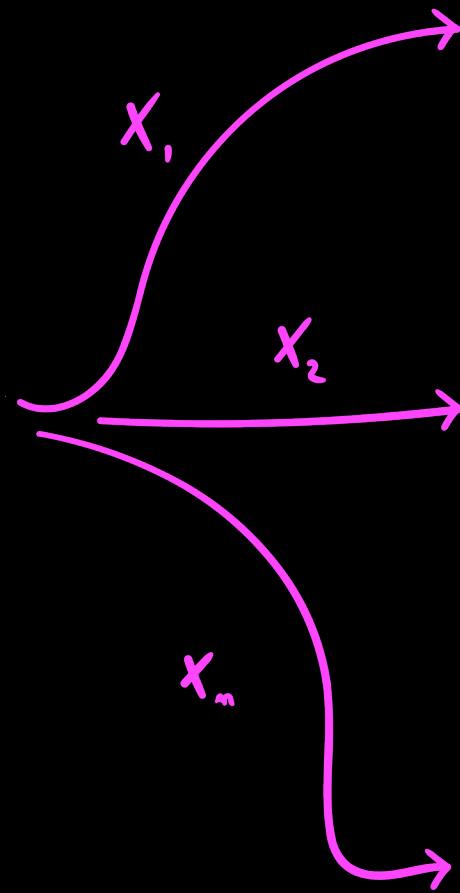
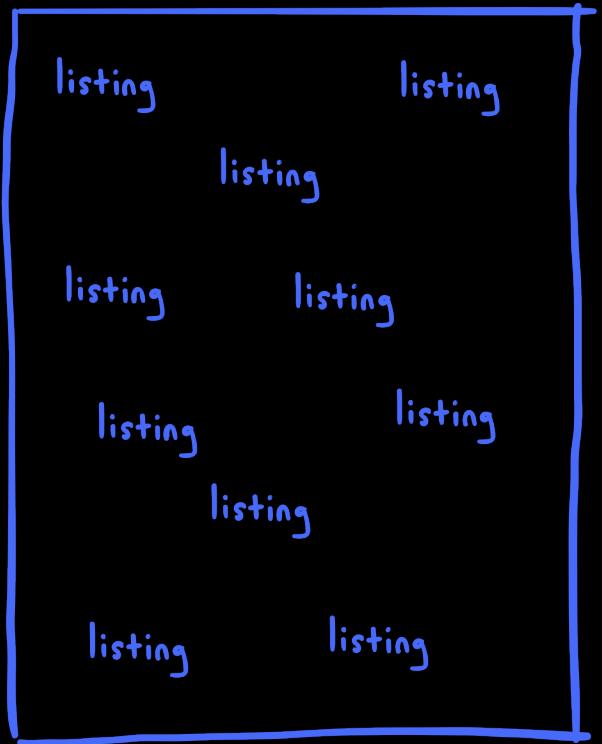
$S$



these prices  
are common...

...these ones  
less so

$S$



identical  
distributions!!!

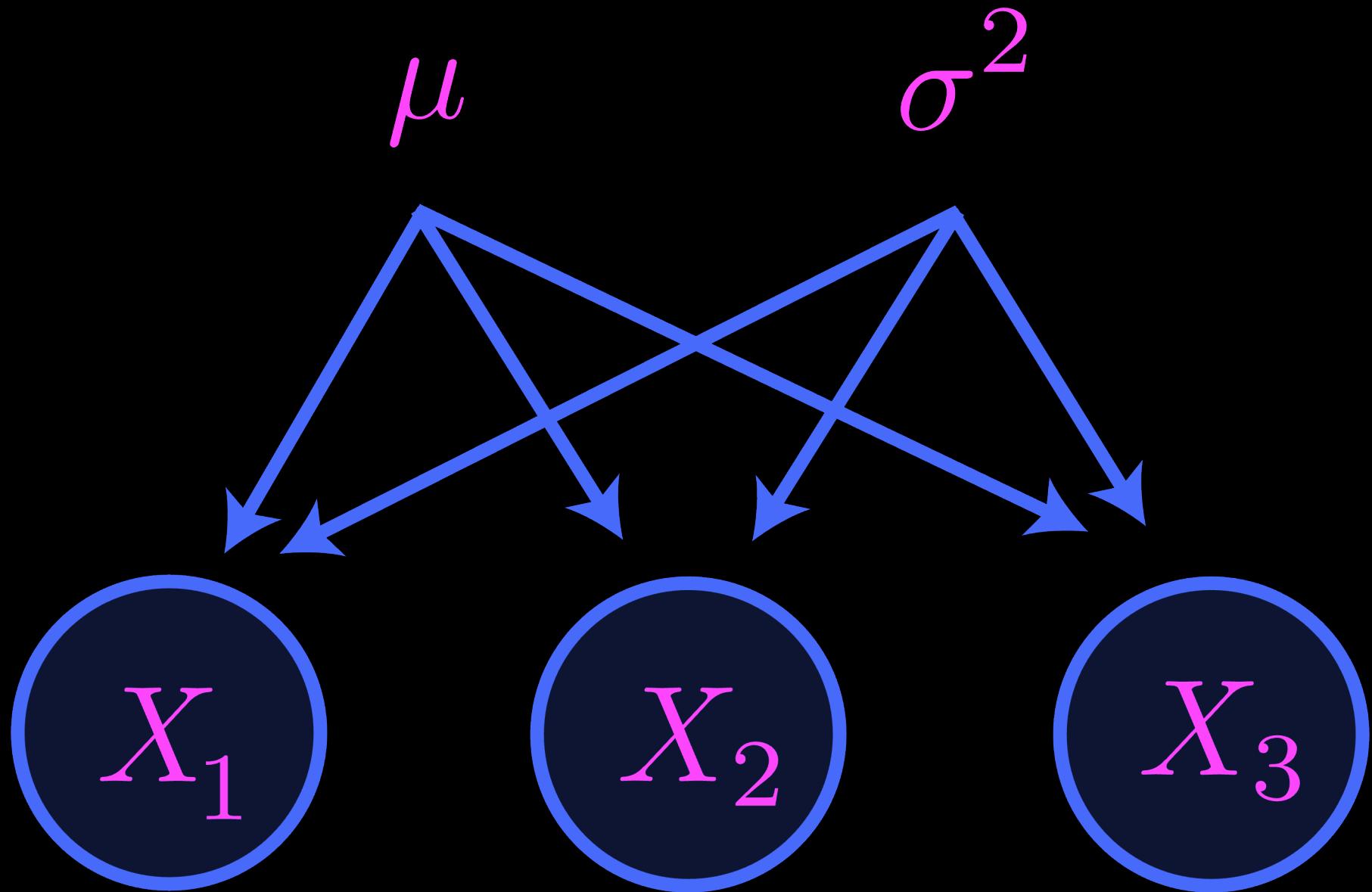
 **Definition 6.1**

Let  $X_1, X_2, \dots, X_m$  be a sequence of random variables, all defined on the same probability space.

- The random variables are called a *random sample* if they are *independent* and *identically distributed* (IID).

Provided that the sequence is a random sample, an *observed random sample*, or a *dataset*, is a sequence of real numbers  $x_1, x_2, \dots, x_m$  where  $x_i$  is an observed value of  $X_i$ .

## 6.2. Probabilistic models and empirical distributions



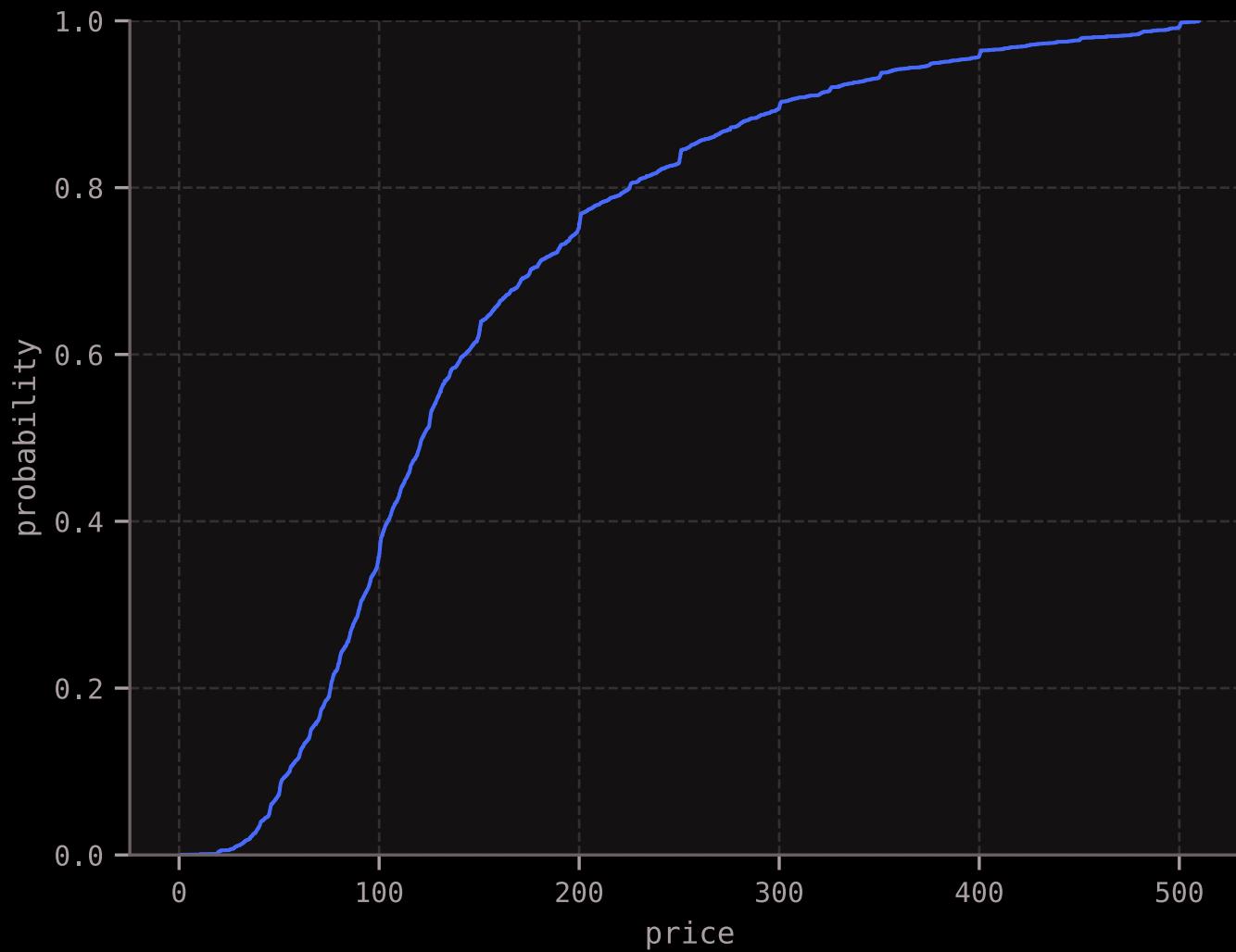
## Definition 6.2

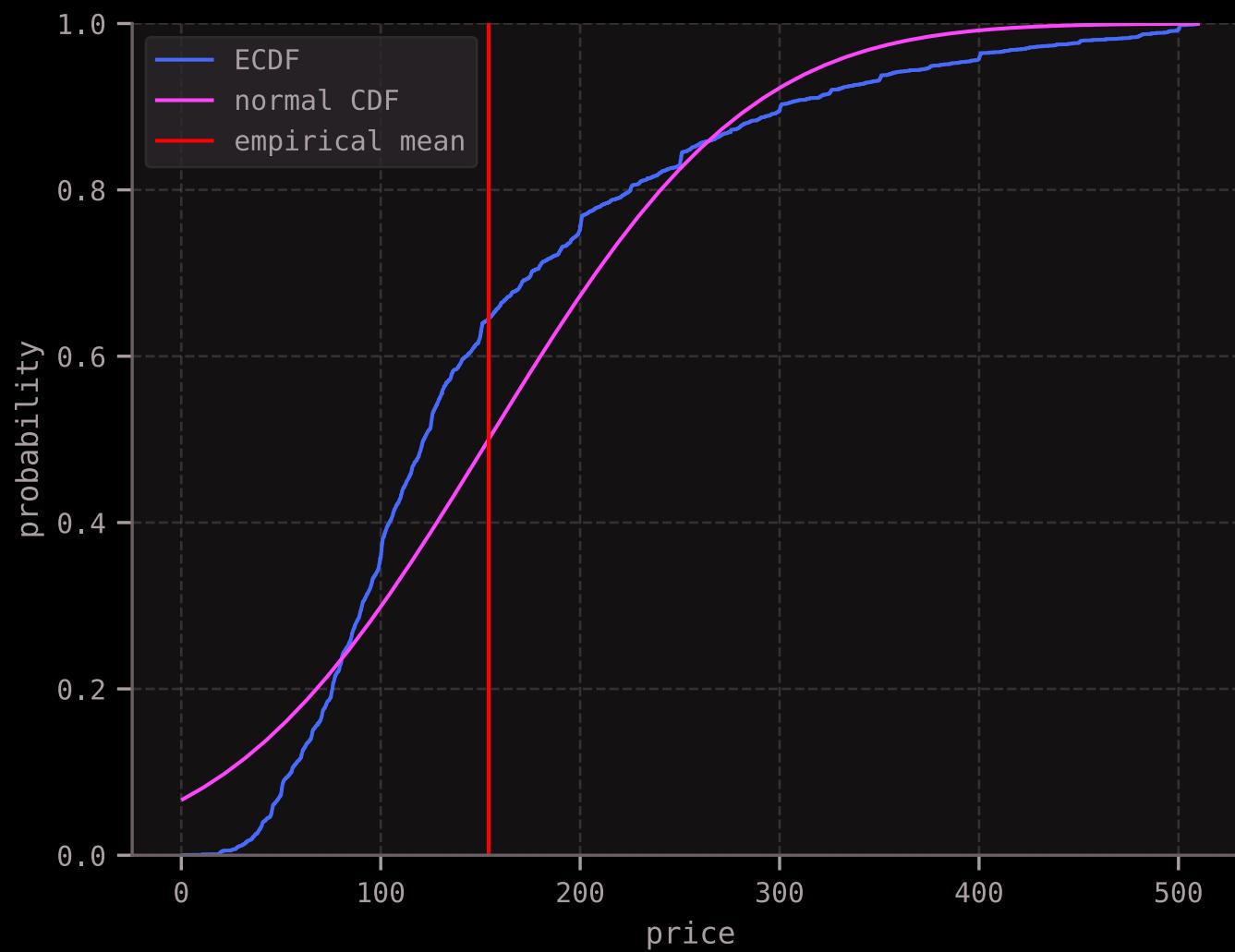
Let  $X_1, X_2, \dots, X_m \sim F$  be a random sample from an unknown distribution function  $F$ , and let  $x_1, x_2, \dots, x_m$  be an *observed* random sample. Then the *empirical distribution* of the dataset is the discrete probability measure on  $\mathbb{R}$  with probability mass function

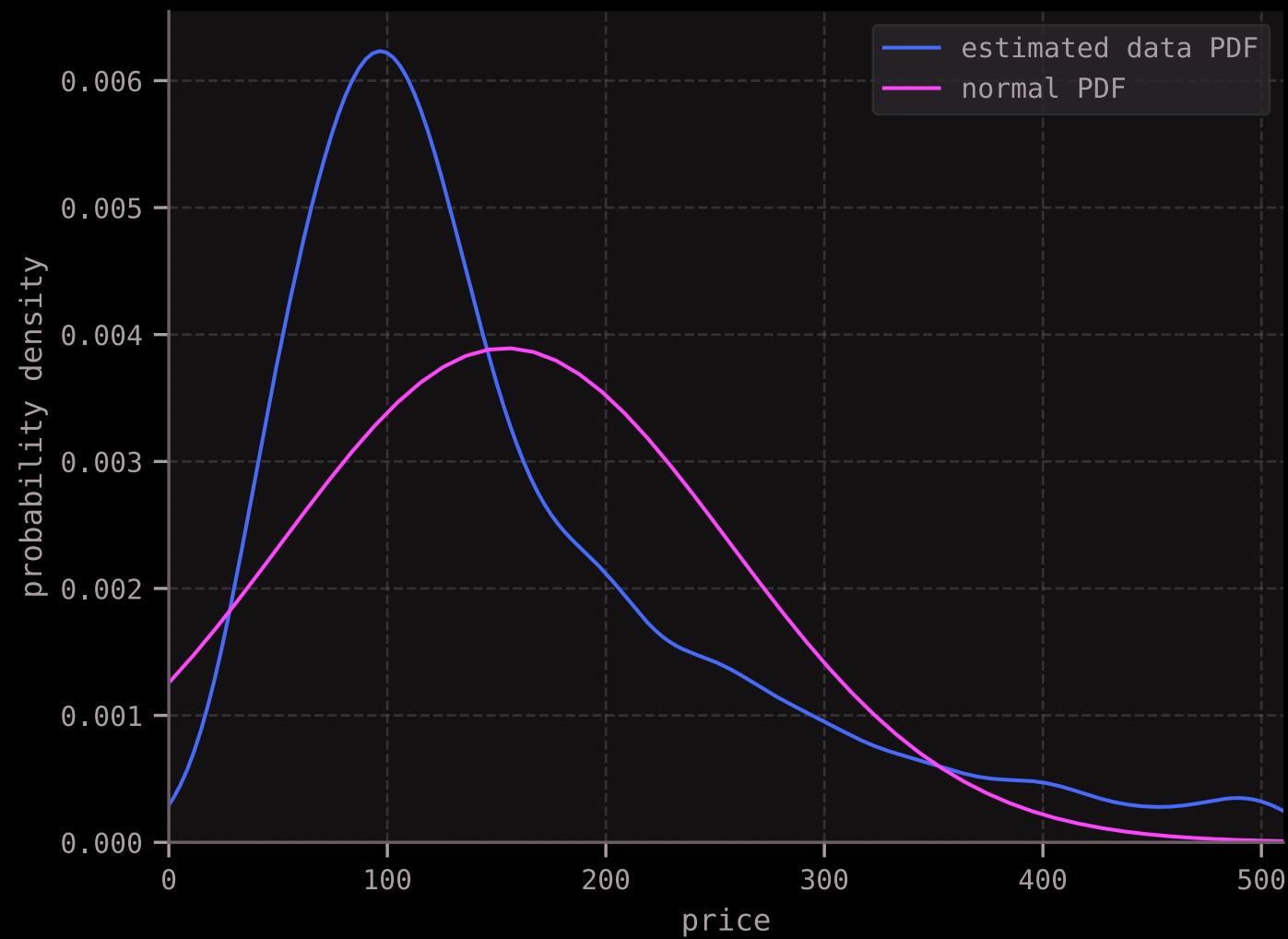
$$p(x) = \frac{\text{number of data points } x_i \text{ that match } x}{m}. \quad (6.1)$$

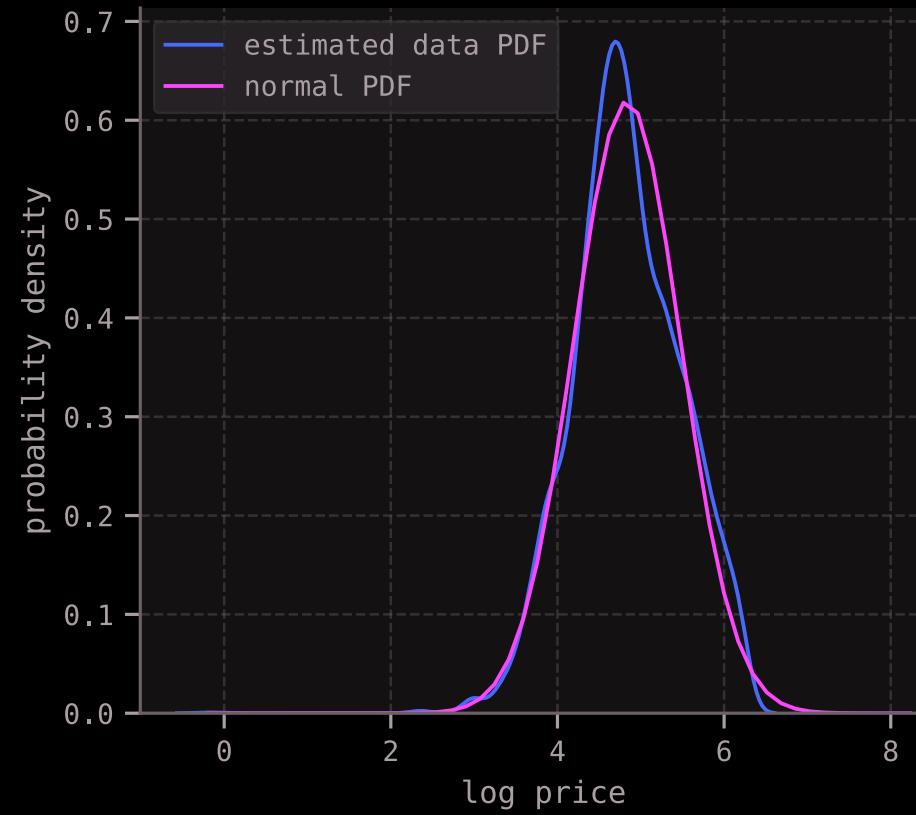
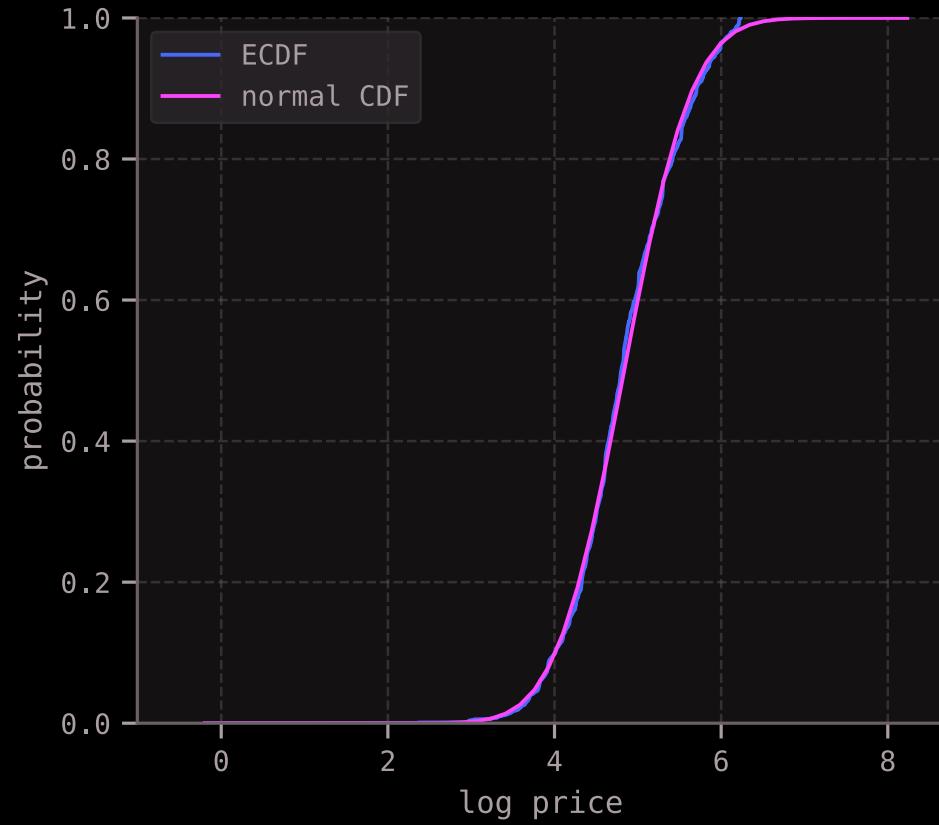
The *empirical cumulative distribution function* (ECDF) of the dataset is the CDF of the empirical distribution. It is often denoted  $\hat{F}(x)$ , and it is given by the usual formula

$$\hat{F}(x) = \sum_{y \leq x} p(y) = \frac{\text{number of data points } x_i \text{ with } x_i \leq x}{m}.$$







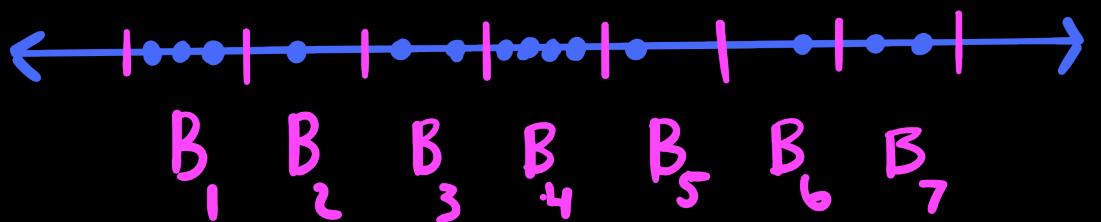


## 6.3. Histograms

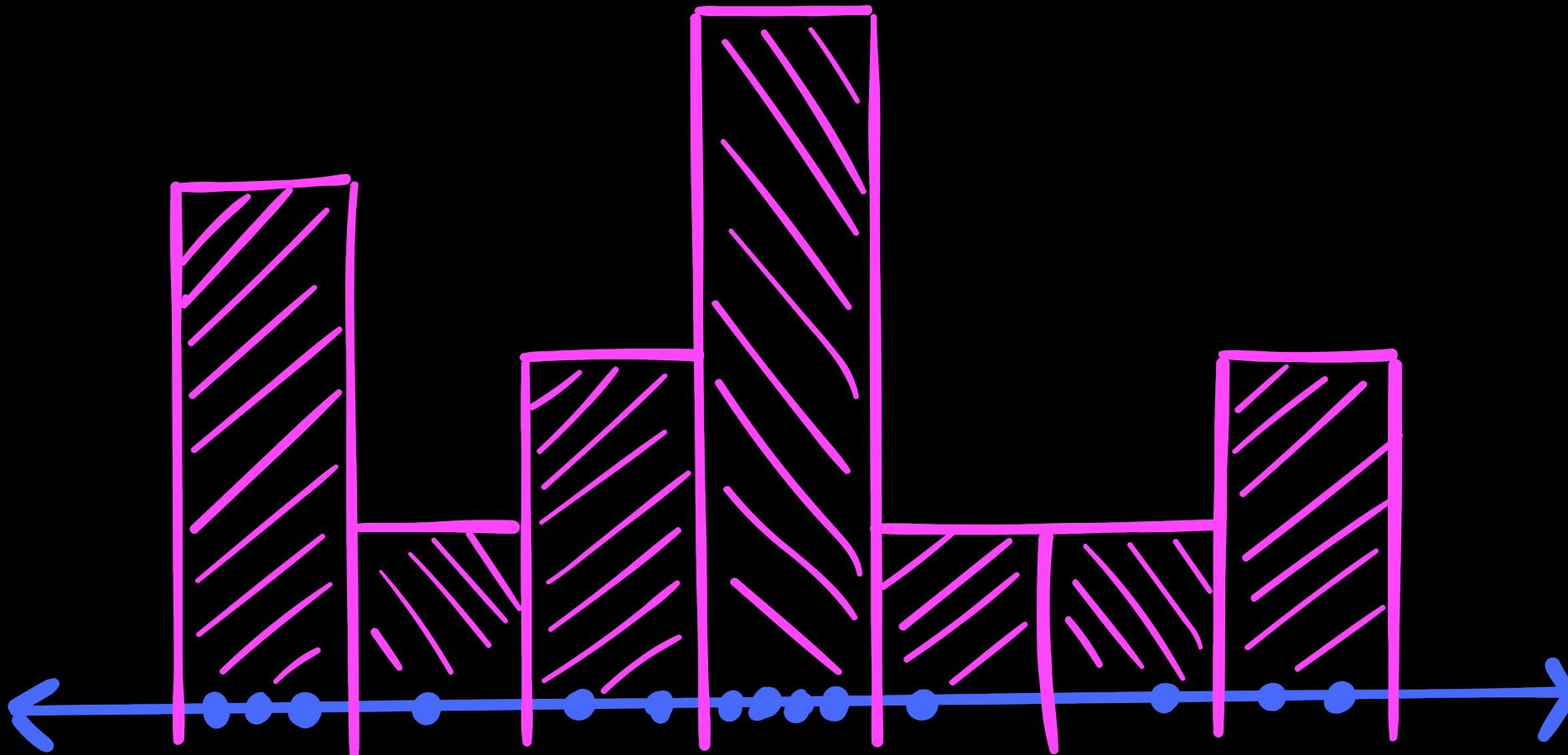
dataset

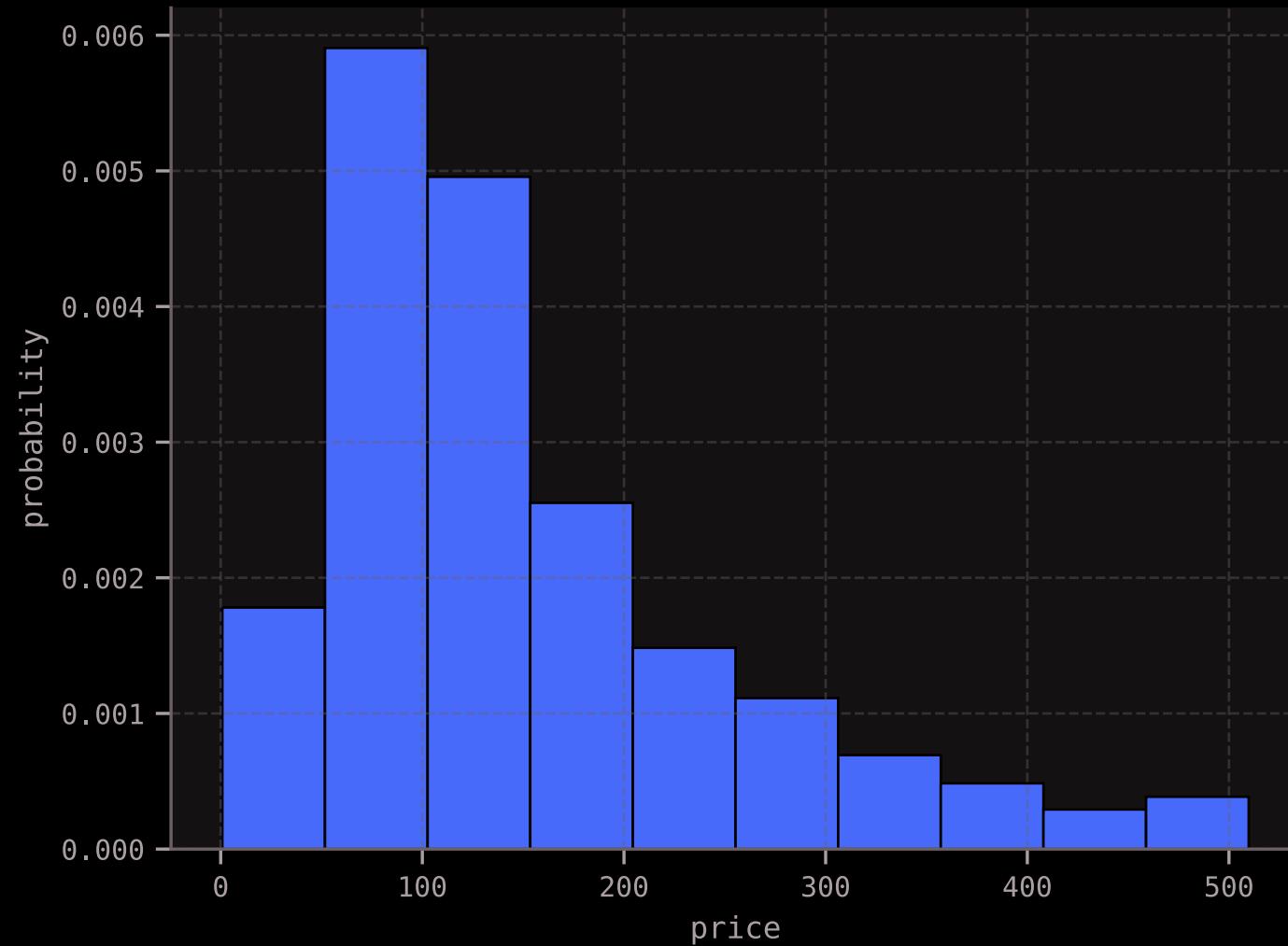


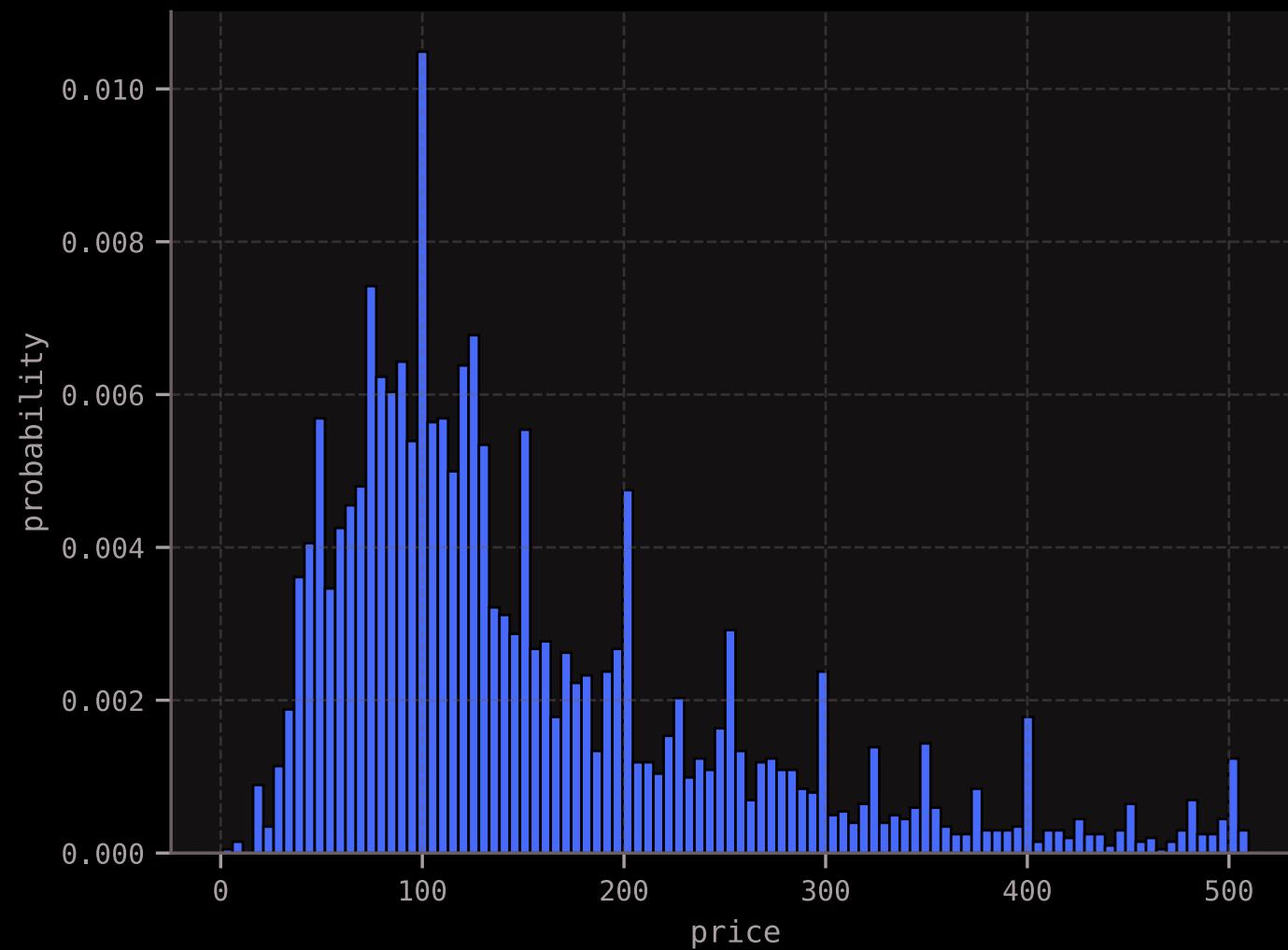
dataset in bins

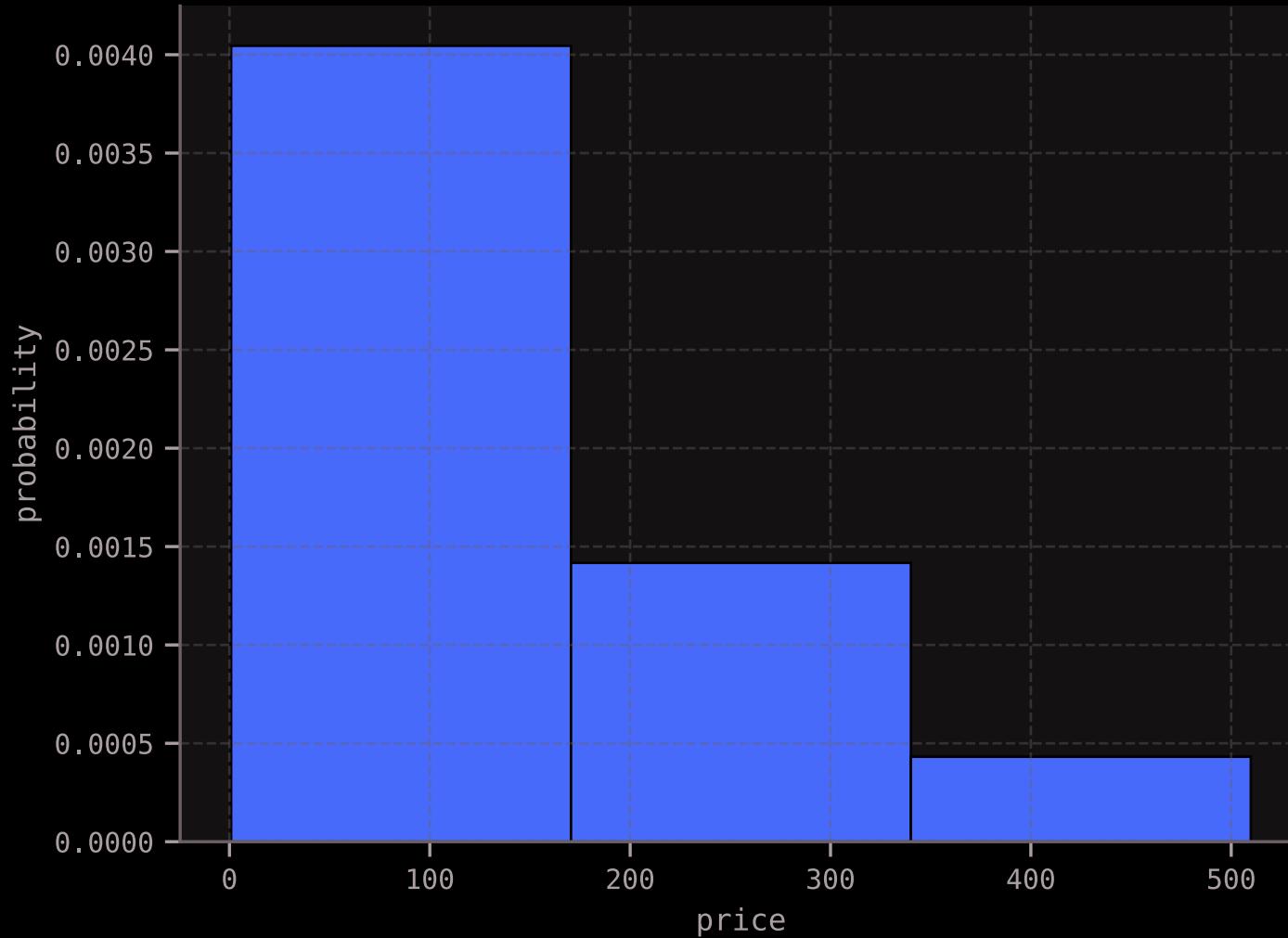


# histogram

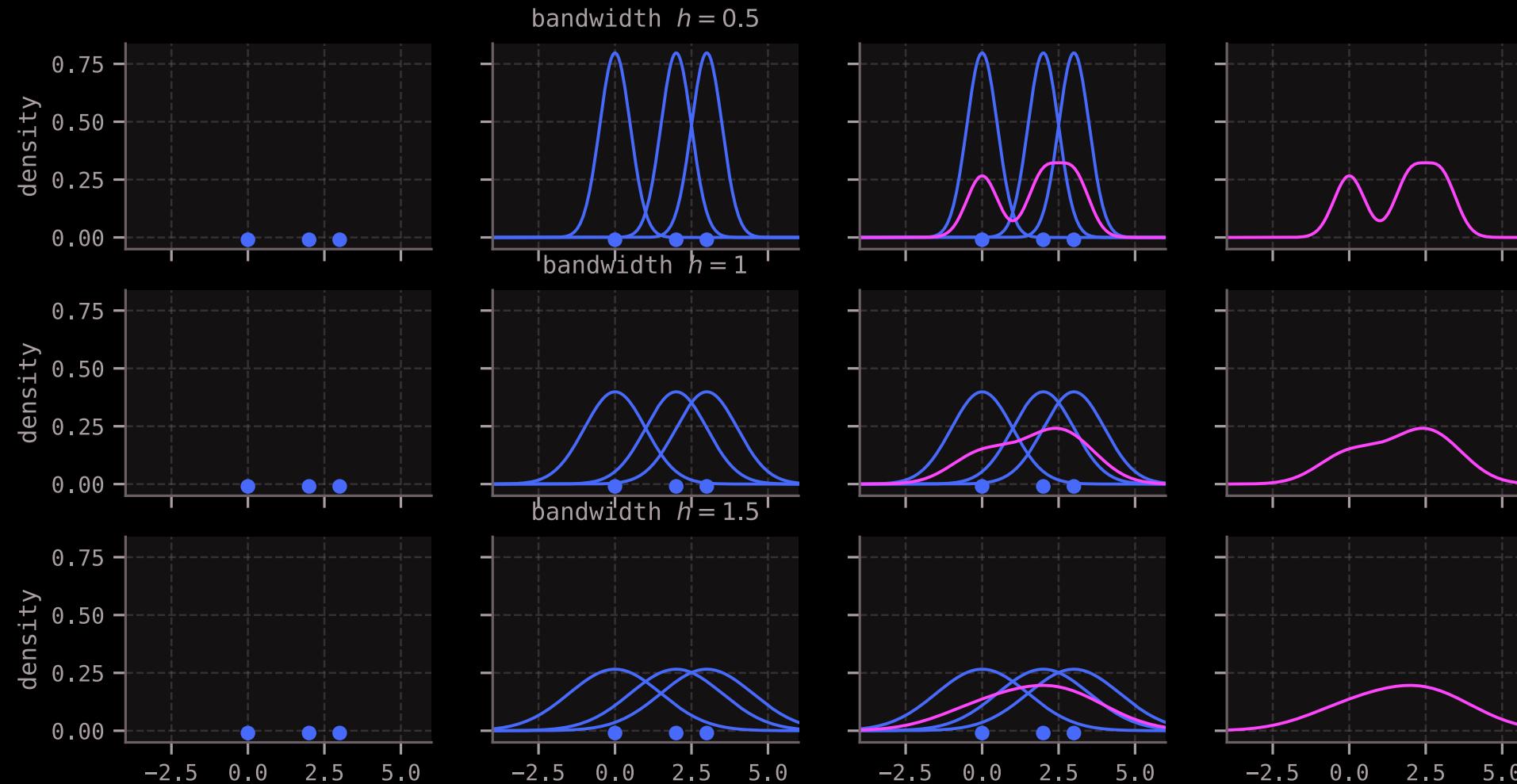


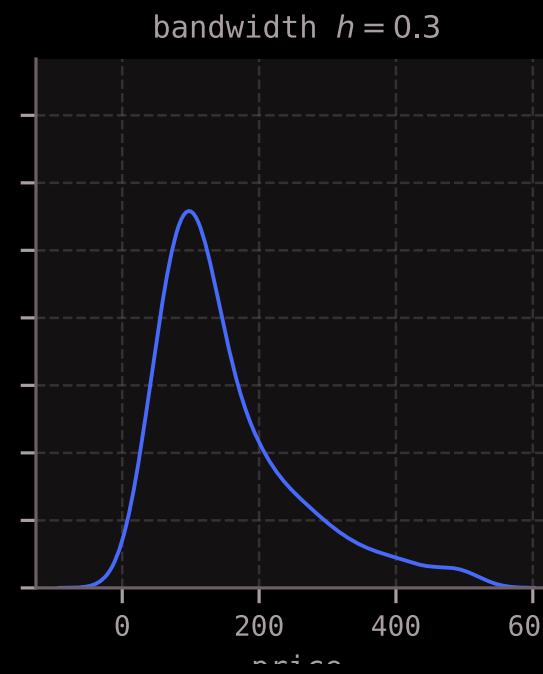
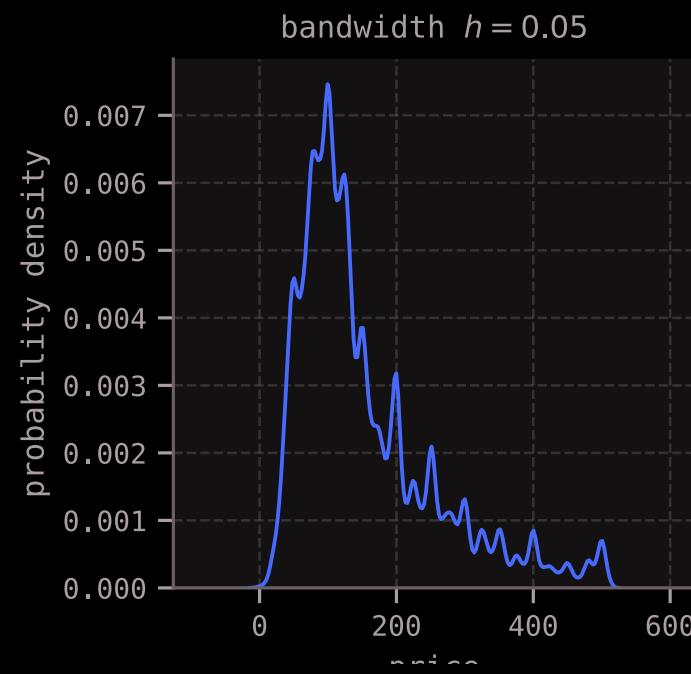






## 6.4. Kernel density estimation





## **6.5. Empirical statistics**



### Definition 6.3

Let  $x_1, x_2, \dots, x_m$  be an observed random sample (i.e., a dataset). The *empirical mean* is defined to be the number

$$\bar{x} = \frac{1}{m} \sum_{i=1}^m x_i,$$

while the *empirical variance* is defined to be the number

$$s^2 = \frac{1}{m-1} \sum_{i=1}^m (x_i - \bar{x})^2.$$

The *empirical standard deviation*  $s$  is defined, as usual, as the positive square root of the empirical variance,  $s = \sqrt{s^2}$ .



## Definition 6.4

Let  $x_1, x_2, \dots, x_m$  be an observed random sample, written in non-decreasing order:

$$x_1 \leq x_2 \leq \dots \leq x_m. \quad (6.2)$$

For each  $i = 1, 2, \dots, m$ , the datapoint  $x_i$  is called the *empirical q-quantile* where

$$q = \frac{i - 1}{m - 1}. \quad (6.3)$$



## Definition 6.5

Let  $x_1, x_2, \dots, x_m$  be an observed random sample. The *empirical interquartile range* (*empirical IQR*) is the difference

(empirical 0.75-quantile) – (empirical 0.25-quantile).



## Definition 6.6

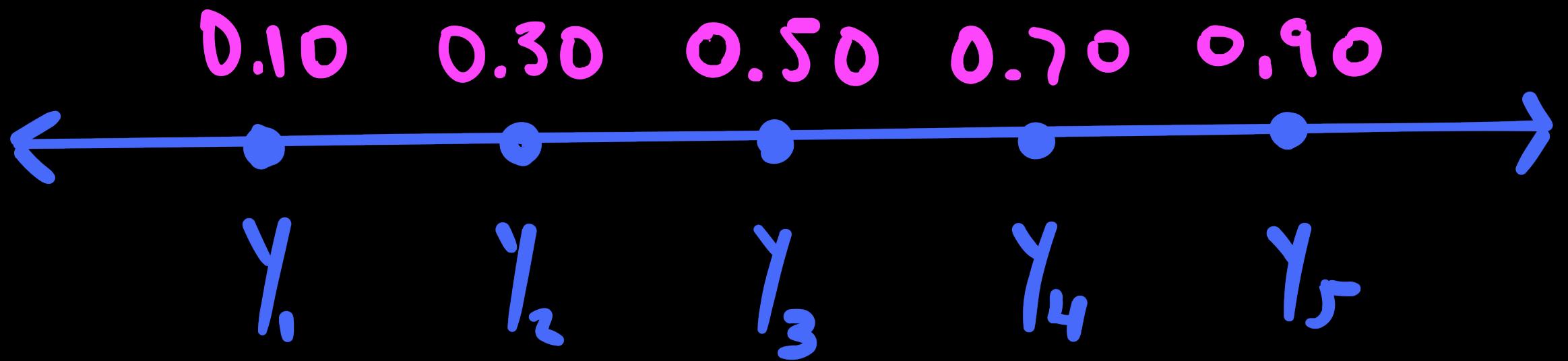
Let  $x_1, x_2, \dots, x_m$  be an observed random sample. Then a data point  $x_i$  is called an *outlier* if it is above an upper threshold value

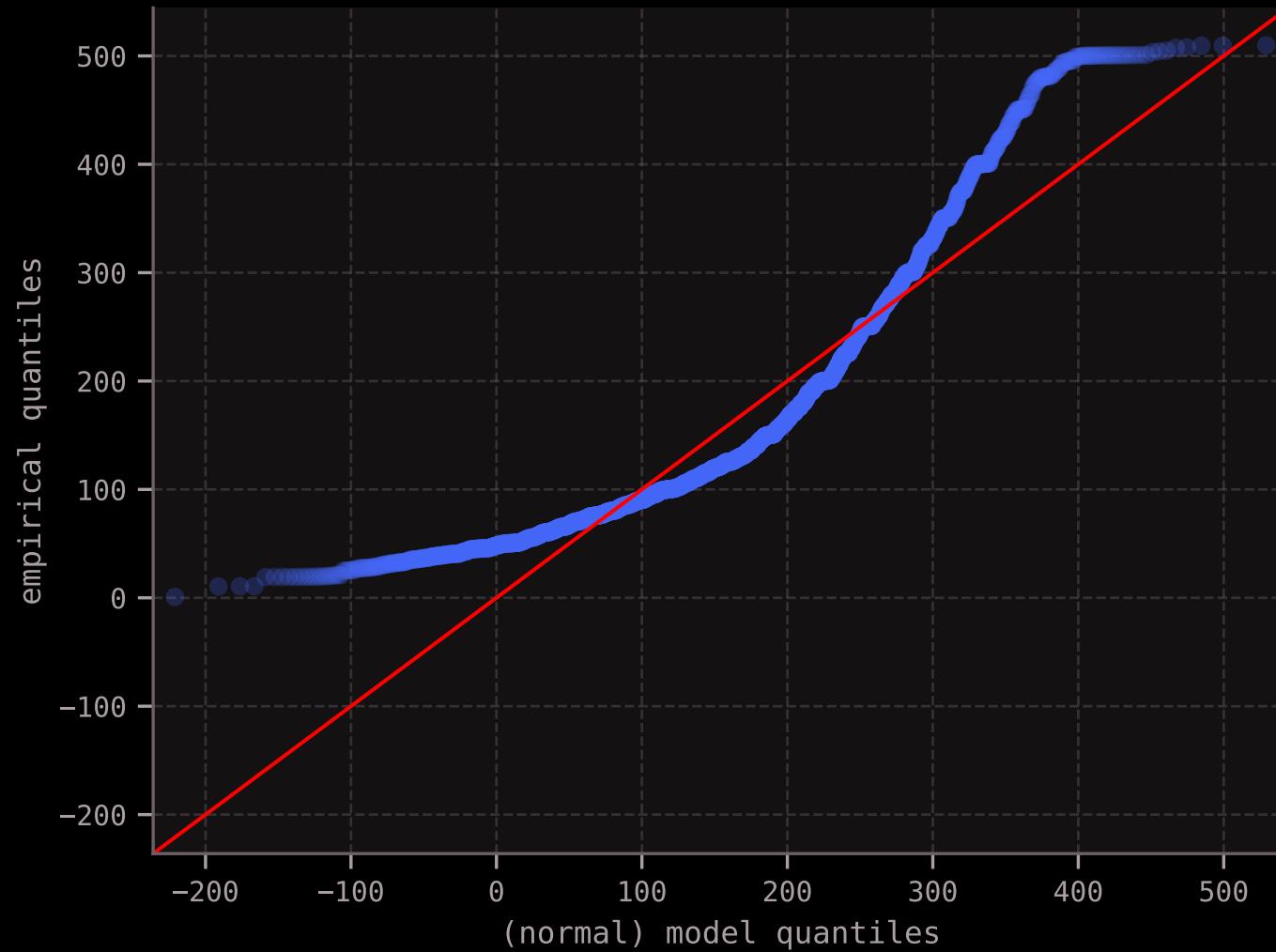
$$x_i > (\text{empirical 0.75-quantile}) + 1.5 \times (\text{empirical IQR}),$$

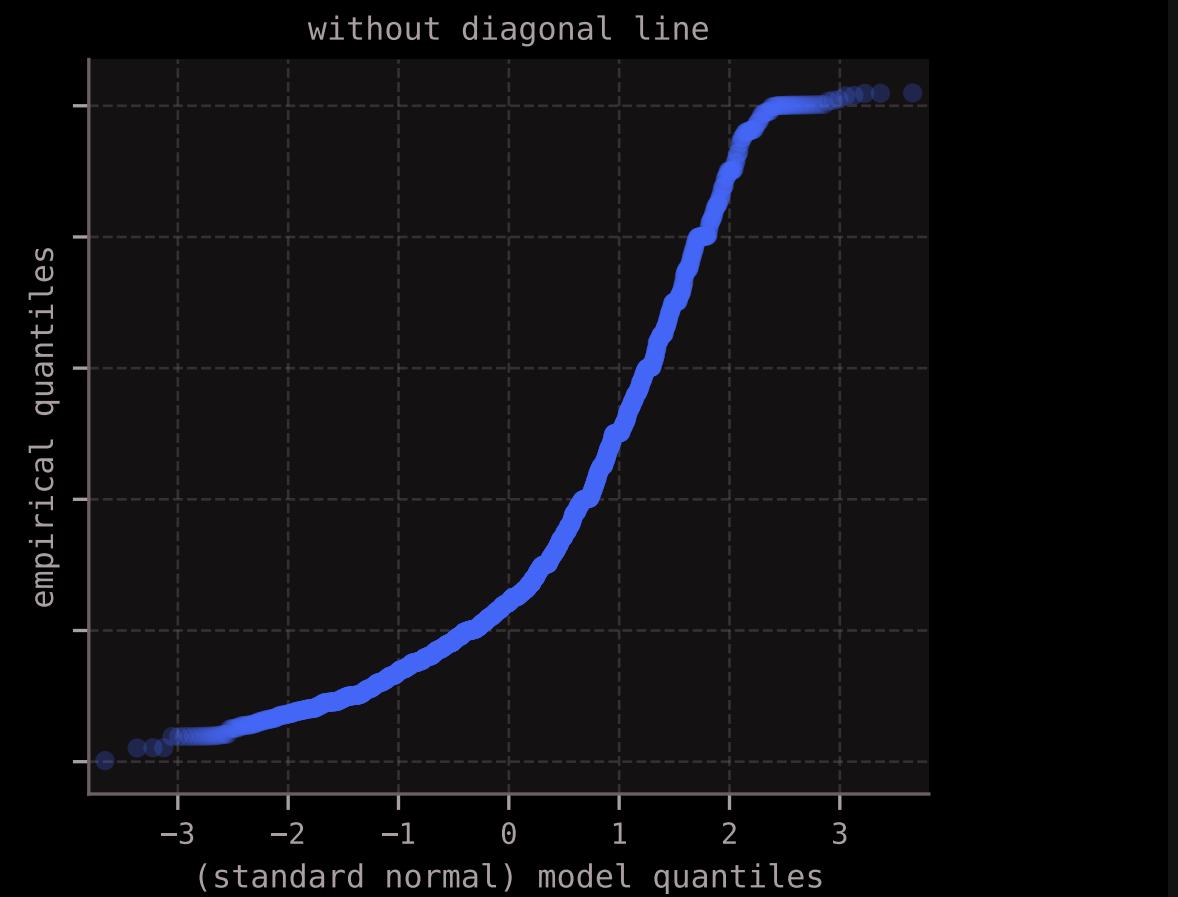
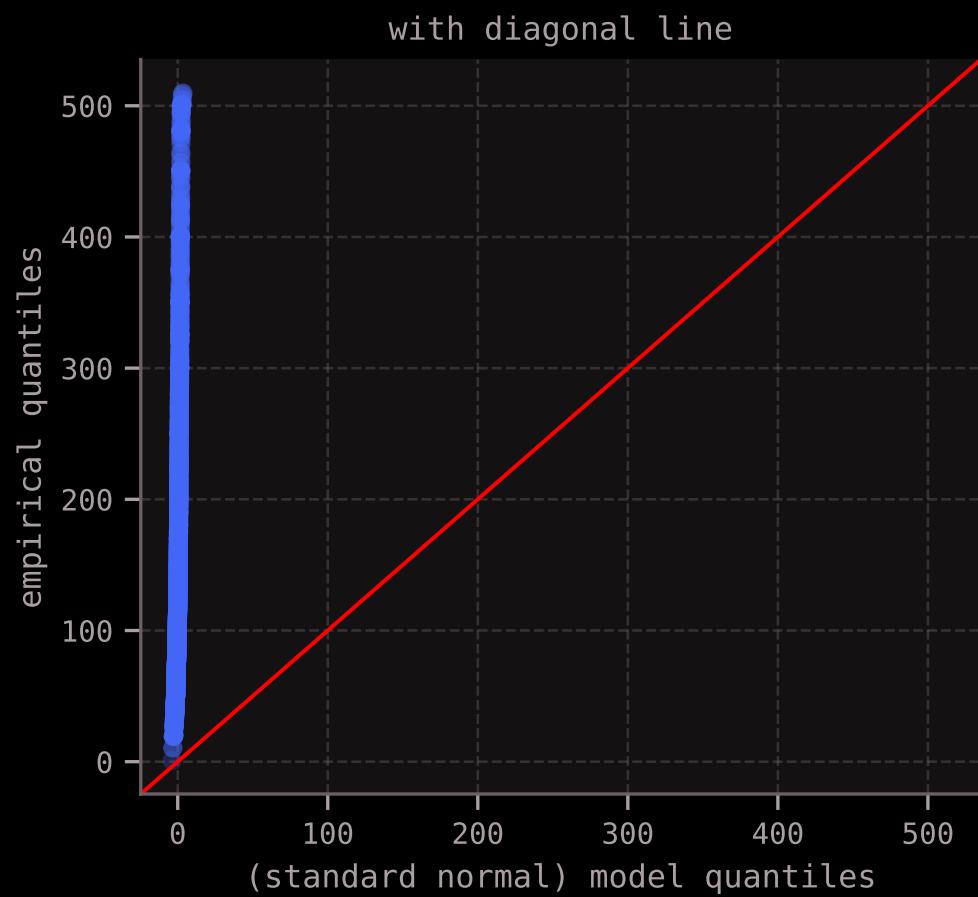
or if it is below a lower threshold value

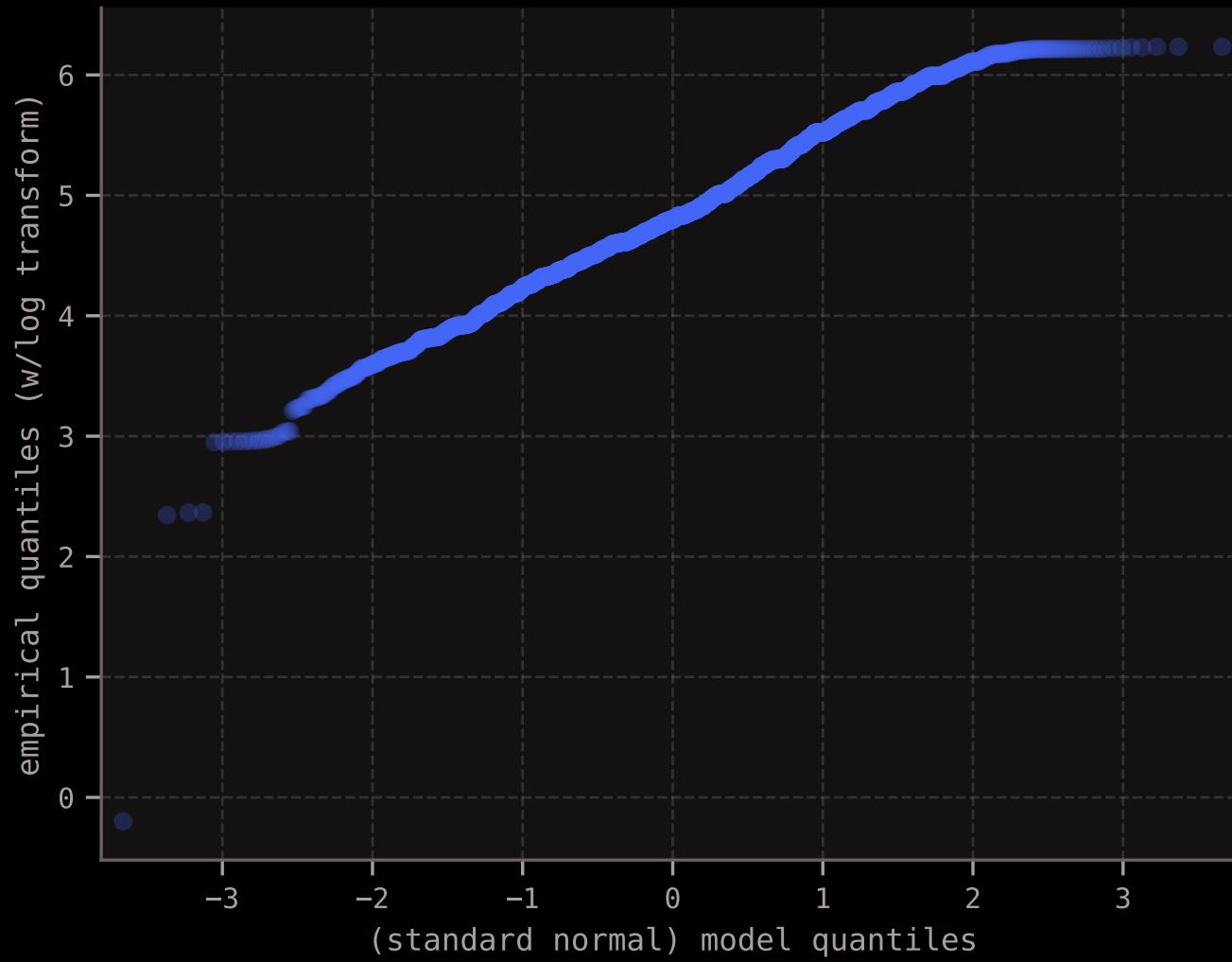
$$x_i < (\text{empirical 0.25-quantile}) - 1.5 \times (\text{empirical IQR}).$$

## 6.6. QQ-plots









## 6.7. Box plots and violin plots

