# PA4-REPORT

## Score Sheet

| | |
|---|---|
| Name 1 | |
| Email | |
| Other contact information (optional) | |
| Name 2 | |
| Email | |
| Other contact information (optional) | |
| Signature (required) | I (we) have followed the rules in completing this assignment<br>Jiaming Zhang<br>ChongZun Yang. |

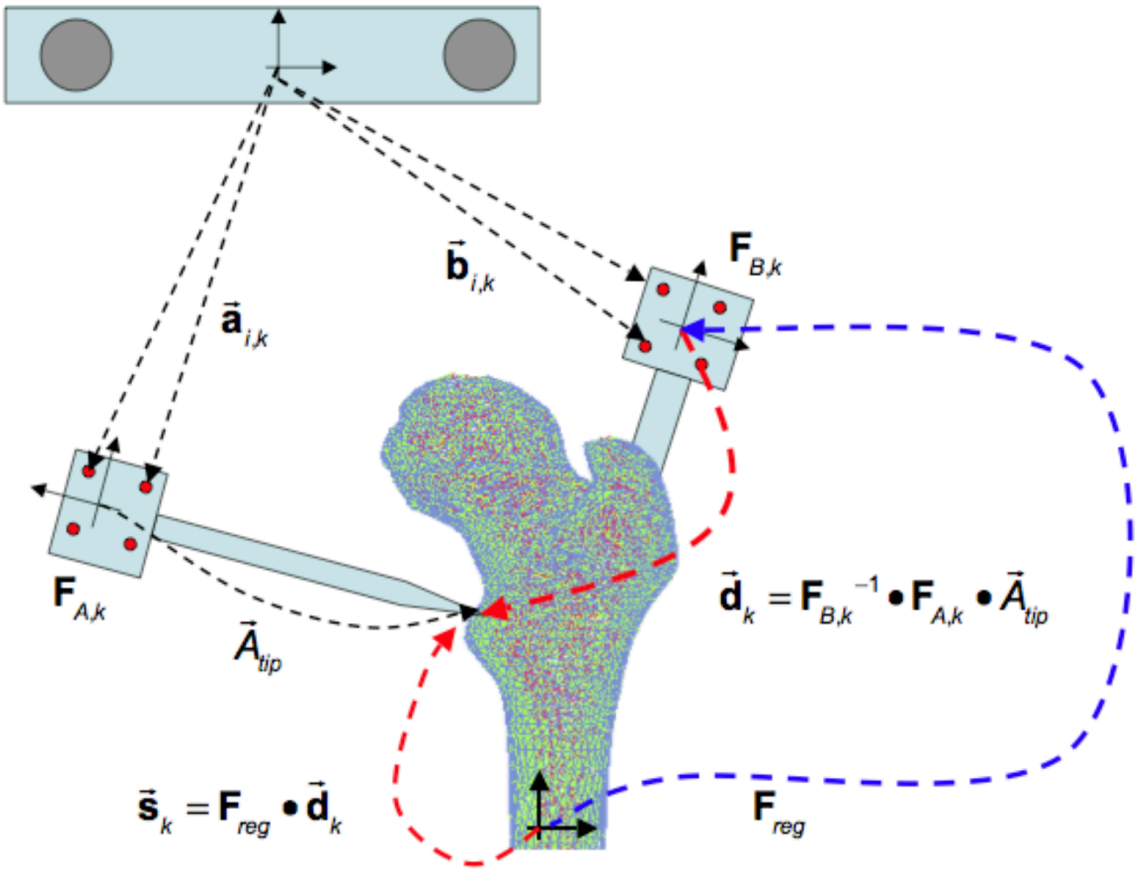
| **Grade Factor** | | |
|---|---|---|
| Program (40) | | |
| Design and overall program structure | 20 | |
| Reusability and modularity | 10 | |
| Clarity of documentation and programming | 10 | |
| Results (20) | | |
| Correctness and completeness | 20 | |
| Report (40) | | |
| Description of formulation and algorithmic approach | 15 | |
| Overview of program | 10 | |
| Discussion of validation approach | 5 | |
| Discussion of results | 10 | |
| TOTAL | 100 | |

# I. Mathematics & Algorithms Implementation

This section introduces the mathematical principles and implemented algorithm for locating the closest point to a given mesh file described in Programming Assignment 4.

## 0. Scenario

In this programming assignment, we are required to compute the registration transformation $F_{reg}$ using Iterative Closest Point (ICP). We use LED trackers to detect 2 rigid bodies on the bone and obtain the point cloud from intraoperative reality. The CT system could obtain information of the 3D surface model. By finding the transformation matrix between the preoperative surface mesh and intraoperative point cloud we can derive the frame transformation between the CT frame and Bone frame. While finding the closest triangle in the ICP algorithm, we implemented Brute-Force Search and Octree Search to determine the closest points. We also compared their performance in terms of searching efficiency.




1. Find Correspondence

For every $F_{reg}$ \vec q{*tip*}, *where**vec* *q*{tip} is the point in the point cloud, use *FindClosestPoint*() to find the matching triangles and the closest distance. Note that we assume the rotation and translation part of $Freg$ are $I$ and $[000]^T$ , respectively, in the first iteration. Also, we conduct robust pose estimation in this step to filter out the outliers.

2. Transformation update

   After getting the point pairs in the last step, the 3D to 3D point cloud registration package developed previously to update the $F_{reg}$. Meanwhile, we compute three residual-error-related parameters, which are used to evaluate whether terminate this loop in the future step.

3. Adjustment

   Update threshold $\eta_n$. This parameter is used in matching step (robust pose estimation) to restrict the influence of clearly wrong matches on the computation of $F_{reg}$.

4. Iteration

   If the current $F_{reg}$ satisfies the termination condition, this $F_{reg}$ is our result. If not, go to next iteration.

# 1. Iterative Closest Point Method

## 1) Mathematical Method

1. Find Corresponding Point Pairs

   Since we are finding the transformation between a point cloud and a surface mesh, the algorithm needs to identify the closest pair between the points cloud and surface mesh in the first place. As introduced in PA3, we implemented Brute Force Searching and Octree Searching methods to solve this problem. Here we use $F_n$ to denote the $F_{reg}$ updated in the previous iteration, $\vec{q}_k$ is the 4th point in the point cloud and $\vec{c}_k$ is the corresponding closest needlepoint on the surface model in the previous corresponding process.

2. Find Best Transformation

   Then we use the search method to search for the closest point on the surface mesh for each element in the point cloud. In this assignment, as a default option, we adopt the Octree searching method taught in the class. In addition to the closest point coordinates, the algorithm also saves the distance of each point pair.

3. Adjusting the Boundary for Outliers

   Another difference with PA3 is the robust estimation part. With the iteration proceeding, we can leverage the distance values of each point pairs and their residual errors serving as thresholds to filter out some clearly wrong matches.

## 2) Code Implementation

The code implemented in **"/PA4/pa4_compute_dk_test.py"**. The basic steps are:

| **Algorithm 1** | **Compute dk** |
|---|---|
| **Input**: | Point Clouds: LED readings $A_k$ and $B_k$ , rigid body description file $A$ and $B$ |
| **Output**: | $A_{tip}$ position w.r.t. $B$ body frame, i.e. $d_k$ |
| | Load $A_k$ , $B_k$ , $A$ and $B$ |
| | **FOR** each frame in $\{ A_k, B_k\}$ |
| | **CALL** $F_{A,k}$ = Registration($A_k$ , $A$) $F_{B,k}$ = Registration($B_k$ , $B$) |
| | **COMPUTE** $\vec{d}_k = F_{B,k}^{-1} F_{A,k} \vec{A}_{tip}$ |
| | **ENDFOR** |

Then we use the search method to search for the closest point on the surface mesh for each element in the point cloud. In this assignment, as a default option, we adopt the Octree searching method taught in the class. In addition to the closest point coordinates, the algorithm also saves the distance of each point pair.

Another difference with PA3 is the robust estimation part. With the iteration proceeding, we can leverage the distance values of each point pairs and their residual errors serving as thresholds to filter out some clearly wrong matches.

# 2. Transformation Update

## 1) Mathematical Method

With the matched point pairs, we adopted the 3D point-cloud-to-point-cloud registration method introduced in PA1 and PA2 to compute the best transformation. After updating the registration transformation, we calculate residuals for each transformed point with the corresponding closest needle point. Then we can compute the residual error based values shown as following:

$$\sigma = \sqrt{(\sum_k \vec{e} \cdot \vec{e}}/k \tag{1}$$

$$\varepsilon_{max} = max_k \sqrt{\vec{e}_k \cdot \vec{e}_k} \tag{2}$$

$$\varepsilon = \sum_k \sqrt{\vec{e}_k \cdot \vec{e}_k}/k \tag{3}$$

where $\vec{e}_k$ denote residual vector for the k-th point pair.

## 2) Code Implementation

The code implemented in **"/cispa/FindClosestPoint2Triangle.py"** and the test script is developped in **"/PA4/pa4_find_closest_on_triangle.py"**. The steps are as following:

| **Algorithm 2** | **Find Closest Point to Triangle** |
|---|---|
| **Input**: | Point $a$ and Triangle $T = \{p, q, r\}$ |
| **Output**: | Closest Point $h$ |
| | **CALL** $[\lambda \quad \mu]^T = \text{LeastSquare}([\vec{q} - \vec{p} \quad \vec{r} - \vec{p}], \vec{a} - \vec{p})$ |
| | **COMPUTE**: $h = p + \lambda(q - p) + \mu(r - p)$ |
| | **IF** $\lambda < 0$: |
| | $h = \text{ProjectOnSegment}(h, r, p)$ |
| | **ELIF** $\mu < 0$: |
| | $h = \text{ProjectOnSegment}(h, p, q)$ |
| | **ELIF** $\lambda + \mu > 1$: |
| | $h = \text{ProjectOnSegment}(h, q, r)$ |
| | **ENDIF** |
| **RETURN**: | $h$ is the closest point on the triangle |

# 3.Adjustment

## 1) Mathematical Method

In order to realize robust pose estimation in the matching step, we need a threshold $\eta_n$ to filter out the wrong pairs. Here we adopt $3\varepsilon_n$ as this threshold. In practice, the setting of $\eta_n$ is very tricky. On the on one hand, too tight threshold might delete some matching pairs that are actual correct. On the other hand, too loose threshold might not filter out the wrong pairs.

## 2) Code Implementation

The code implemented in**"cispa/FindClosestPoint2Mesh.py"** and the test script is **"/PA4/pa4_linear_search_test.py"**.

| **Algorithm3** | **Linear Search by Brute Force** |
|---|---|
| **INPUT** | Vertices Coordinates and Triangle Indices |
| **OUTPUT** | A tuple that contains closest point and its corresponding distance |
| | minimum distance = $\infty$ |
| | **FOR** $i$ = index of the triangles |
| | **CALL**: $\vec{C}_{k,i}^{nearest}$ = FindClosestPoint2Triangle($\vec{d}_k$, $triangle_i$) |
| | $\vec{d}_k = \|\|\vec{d}_k - \vec{C}_{k,i}^{nearest}\|\|$ |
| | **IF** $\vec{d}_k \leq$minimum distance **then** |
| | minimum distance =$\vec{d}_k$ |
| | closest point = $\vec{C}_{k,i}^{nearest}$ |
| | **ENDIF** |
| | **ENDFOR** |
| **RETURN** | [minimum distance, closest point] |

# 4 Iteration

## 1) Mathematical Method

The criterion of termination we adopted is to stop the iteration when $\sigma_n$ , $\varepsilon_n$, $(\varepsilon_max)_n$ are less than desired thresholds (all take 0.01) and $\gamma \leq \varepsilon_n/\varepsilon_{n-1} \leq 1$. Here we assume that $\gamma = 0.95$.

## 2) Code Implementation

The code implemented in **"/cispa/FindBoundingSphere.py"**. We use spheres to represent the triangles and did the same brute force search process.

| **Algorithm4** | **Linear Search by Bounding Spheres** |
|---|---|
| **Step**. 1 | bound $\leftarrow\infty$ |
| **Step**. 2 | **for** $i = 1$ to number of triangles do |
| **Step**. 3 | **if** $\|\|q_i - a\|\| - r_i \leq bound$ **then** |
| **Step**. 4 | $c_i = \|\|\vec{d}_k - \vec{C}_{k,i}^{nearest}\|\|$ |
| **Step**. 5 | **if** $d_i \leq bound$ **then** |
| **Step**. 6 | bound$\leftarrow d_i$ |
| **Step**. 7 | closest point$\leftarrow c_i$ |
| **Step**. 8 | **end if** |
| **Step**. 9 | **end if** |
| **Step** 10 | **end for** |

####

# II. Overall Structure

The overall structure for the ../PROGRAMS folder is described as follows:

```
└── PROGRAMS
    ├── PA3                          # Test scripts are contained in this directory
    |   ├── Data                     # This DIR contains all the provided data
    |   ├── output                   # This DIR contains all the result of our program
    |   ├── pa3_main.py              # This is the main process that output the result
    |   ├── pa3_compute_dk_test.py   # compute the tip point
    |   ├── pa3_find_closest_on_triangle.py
    |   ├── pa3_octree_search_test.py  # Octree search to find closest point on mesh
    |   └── pa3_linear_search_test.py  # Linear search to find closest point on mesh
    └── cispa                        # Utility Functions are in this directory
        ├── CarteFrame.py
        ├── ComputeExpectValue.py
        ├── CorrectDistortion.py
        ├── DataProcess.py           # Import and Export Data
        ├── FindBoundingSphere.py
        ├── FindClosestPoint2Mesh.py     # Find the closest point on the mesh to point
        ├── FindClosestPoint2Triangle.py # Find the closest point on the triangle
        ├── HomoRepresentation.py
        ├── Octree.py                # Generate the octree based on given mesh
        ├── PivotCalibration.py
        └── Registration.py
```

# III. Unit Test and Debug

**NOTE**: Please change your directory to the ${PROGRAMS} directory before you start the unit test. In our case, the command is:

```
$ cd ~/*PARENT DIR*/PROGRAMS
```

And list all files to check whether you are in the correct directory.

```
$ ls
PA3   cispa
```

**Dear Graders**, Before starting to run our code, please remember to install **matplotlib** in your virtual environment.

# 1. Compute dk Unit Test

This script is implement to compute the tip positions w.r.t. B body frame for all LED data frames. In the ../PROGRAMS directory, run test script "/PA3/pa3_compute_dk_test.py". In the terminal, run the following command:

```
$ python3 /PA3/pa3_compute_dk_test.py
INFO     dk for PA3-A-Debug data:
                    [[  5.75770718  19.75264402  -3.01776797]
                     [ 17.90665894  25.32838102 -18.84286624]
                     [-39.30008812 -22.32569893 -30.75256506]
                     [ -9.82756727 -13.44235012  -1.22125489]
                     [-24.99293076 -28.7577944  -38.54036667]
                     [-41.37003445 -14.70559823 -29.90157133]
                     [ 33.83443216  16.2533508    7.01085956]
                     [ -2.48501569  -7.41132429  29.48201166]
                     [ 26.65360204  23.29655741 -10.08941296]
                     [  3.48440717   8.81466607 -14.49922852]
                     [-22.84699817 -12.85719931 -48.78797397]
                     [ 23.77236338  -6.19173927  13.76863119]
                     [ -6.24773552  10.3658506    2.06087393]
                     [-39.00797053 -17.63316542 -14.57114666]
                     [-24.84466034 -11.01359415  -6.08798191]]
```

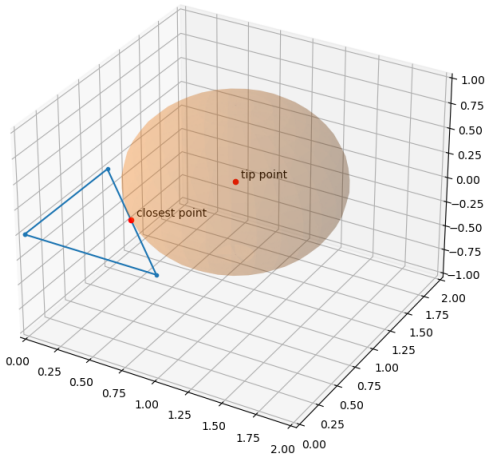
Compare the result to given answer, we can see our code is correct.

# 2. Closest Point on Triangle Unit Test

This script is to find closest point for a given point and a given triangle. In the ../PROGRAMS directory, run test script "/PA3/pa3_find_closest_on_triangle.py". In the terminal, run the following command:

```
$ python3 PA3/pa3_find_closest_on_triangle.py
```

And we use matplotlib to demonstrate our result:



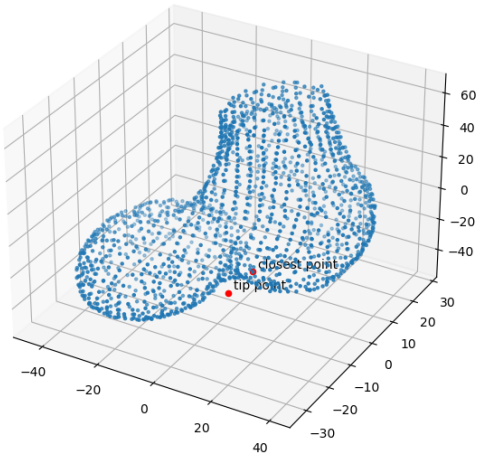

The sphere is a little bit distorted due to the unequal axis range. But as we can see, the result closest point is correct.

# 3. Linear Search Unit Test

This script is used to find closest point on mesh routine that works by linear search through all the triangles. In the ../PROGRAMS directory, run test script "/PA3/pa3_linear_search_test.py". In the terminal, run the following command:

```
$ python3 /PA3/pa3_linear_search_test.py
```

To demonstrate the result, we plotted the triangle and the point together with the corresponding closest point.




# 4. Compute Bounding Sphere Unit Test

This script is designed to check whether the Bounding Sphere for one given triangle is correctly generated.

```
$ python3 PA3/pa3_single_sphere_test
[0.5 0.5 0. ]
0.7071067811865476
```

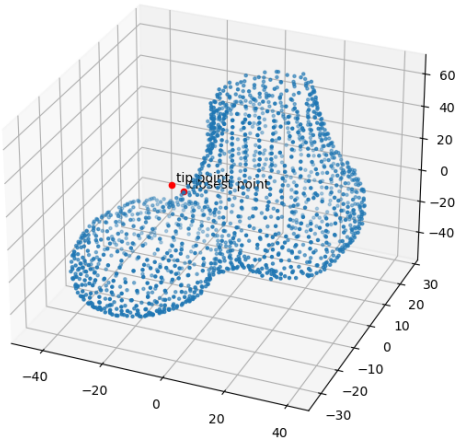
Here we use the triangle with three vertices, which are [[0, 0, 0], [1, 0, 0], [0, 1, 0]]. Through simple inspection, we can see that our result is correct.

# 5 Octree Search Unit Test

We generate a octree based on the given triangular mesh. And then we perform octree search. In the ../PROGRAMS directory, run test script "/PA3/pa3_octree_search_test.py". In the terminal, run the following command:

```
$ python3 PA3/pa3_octree_search_test.py
```

To demonstrate the result, we plotted the triangle and the point together with the corresponding closest point.




By simple inspection, we can see that our result is correct.

# 6. Discussion for the results of finding point pairs in different cases

## 1) Debug case

Table 1 shows us  a quantitative evaluation of various proposed methods to find the closest points. For both the Debug and Unknown cases, we compared the running time of the four methods to demonstrate the efficiency improvement.

In Table 2 , compared with the Brutal Search, the Octree Search has largely reduced running time. Beside the octree generation time comsuption, the search process only takes 10% of time for the brute force search. This could be caused by the performance in the implementation of python. During the process of the tree construction, there would be a series of calling and assignment operations for List type limiting the efficiency of Octree.

**Table 1**: Quantitative evaluation of the proposed methods' accuracy.  All the  methods come to same accuracy. The error has been computed in  L2 Norm

| Case | $d$ error | $c$ error |
|---|---|---|
| A | 0.0053 | 0.0061 |
| B | 0.0054 | 0.0056 |
| C | 0.0061 | 0.0062 |
| D | 0.0087 | 0.0067 |
| E | 0.0081 | 0.0078 |
| F | 0.0082 | 0.0073 |

**Table 2**: Comparison of various methods in running time for Debug case A-F

| Method | Time(s) | | | | | |
|---|---|---|---|---|---|---|
| | A | B | C | D | E | F |
| Brutal Force Solver | 8.0982 | 405.4058 | 700.1229 | 1.4238 | 1.4501 | 1.4422 |
| Octree Solver | 0.135787 | 0.162758 | 0.161032 | 0.129104 | 0.1443400 | 0.099773 |

### 2) Unknown case

For Unknown cases, we save the results in ./output.

In the section, we also give time comparison. For the 3 cases, the Octree based Search method performs better than the Bounding Sphere Search method.

**Table 3**: Comparison of various methods in running time for Debug case G-K

| Method | Time(s) | | | |
|---|---|---|---|---|
| | G | H | I | J |
| Brute Force Solver | 1.4533 | 1.4526 | 1.4563 | 1.4601 |
| Octree Solver | 0.145598 | 0.139905 | 0.125763 | 0.173482 |

# IV. Result and Discussions

## 1. Discussion

### 1.1 Evaluation Metrics

The implementation of the ICP algorithm underlines the performance of both the accuracy and speed, which are respectively measured with Euclidean distance with ground-truth coordinates and the running time.

## 1.2 Unit test for robust pose estimation

We wrote a test program, *unit_ test.py*, for the most important subroutine *RobustPoseEstimation*(), which is used to filter out outliers (incorrect matching).
We artificially corrupt the given 'clean' surface points to simulate the noises in real applications. Specifically, we set noisy ratio as the ratio of correct points and noisy points which are corrupted by randomly shifting from the original coordinates. Based on these simulated datasets, we can investigate the effectiveness of the robust pose estimation module in alleviating the negative effects brought by mismatching.
With various noisy ratios, we compared the performance between algorithms with and without using robust pose estimation in terms of the average error with ground-truth. As shown in Fig. 1, the robust estimation demonstrates a strong capability of improving the robustness under various noises ratios.
Furthermore, we also adopted the robust pose estimation to facilitate the registration of the given Debug datasets. However, only Case E exhibits a reduction in error from 0.04 to 0.02 with robust pose estimation. Considering the discrepancy between the given output file and the answer file, we conjecture there could be some outliers leading to incorrect matches in this dataset.

## 1.3 Result of ICP

In this section, we first give a quantitative evaluation of the implemented ICP algorithm on various cases shown in Table 1. In addition, for both the Debug and Unknown cases, we compared the running time of the three made searching methods, which are Brute-Force Search, Simple Bounding Sphere Search, and Octree Search, to demonstrate the efficiency improvement.

# 1 A-Debug case

**TABLE 3 PA3-A-Debug-own-output**

| OUR RESULTS | GIVEN RESULTS |
|---|---|
| (17.90666 25.32838 -18.84287 17.90731 25.32410 -18.84304 0.00434) | (17.91 25.32 -18.84 17.91 25.32 -18.84 0.000) |
| ( -39.30009 -22.32570 -30.75257 -39.30331 -22.32773 -30.75337 0.00389) | (-39.30 -22.33 -30.75 -39.30 -22.33 -30.75 0.000) |
| (-9.82757 -13.44235 -1.22125 -9.82755 -13.44231 -1.22128 0.00005) | (-9.83 -13.44 -1.22 -9.83 -13.44 -1.22 0.000) |

# 2.B-Debug

The result is compared  with the "PA3-B-Debug-own-output.txt" and shown in the table below.

**TABLE 2 PA3-B-Debug-own-output**

| OUR RESULTS | GIVEN RESULTS |
|---|---|
| (1.15508 11.01661 -8.29899 1.18421 10.94645 -8.26507 0.08320) | (1.15 11.01 -8.30 1.18 10.95 -8.27 0.082) |
| (-31.61757 -5.26328 -12.85190 -32.46366 -3.96838 -9.86385 3.36468) | (-31.62 -5.26 -12.85 -32.46 -3.97 -9.86 3.366) |
| (-24.43960 -0.75184 -14.42152 -25.33479 0.98430 -12.44331 2.78008) | (-24.44 -0.76 -14.42 -25.34 0.98 -12.44 2.784) |

# 3.C-Debug

The result is compared  with the "PA3-C-Debug-own-output.txt" and shown in the table below.

**TABLE 3 PA3-C-Debug-own-output**

| OUR RESULTS | GIVEN RESULTS |
|---|---|
| (27.51716 -8.17727 6.19635 27.51501 -8.14967 6.18664 0.02933) | (27.52 -8.18 6.20 27.51 -8.15 6.19 0.030) |
| (-8.77465 6.25957 46.45507 -6.95442 6.25081 46.40286 1.82100) | (-8.78 6.25 46.45 -6.95 6.25 46.40 1.823) |
| (27.63058 -11.79476 -12.06240 27.82258 -12.01527 -11.99464 0.30014) | (27.63 -11.80 -12.06 27.82 -12.02 -12.00 0.298) |

# 4.D-Debug

The result is compared  with the "PA3-D-Debug-own-output.txt" and shown in the table below.

**TABLE 4 PA3-D-Debug-own-output**

| OUR RESULTS | GIVEN RESULTS |
|---|---|
| (15.42274 19.93007 35.51362 15.44614 21.76373 36.08316 1.92021) | ( 15.42 19.93 35.51 15.45 21.76 36.08 1.922) |
| (-5.15204 -18.60613 -8.49794 -5.19629 -18.17795 -8.68783 0.47048) | (-5.15 -18.60 -8.50 -5.19 -18.18 -8.69 0.470) |
| (3.33449 22.29966 27.79139 3.45898 23.17946 27.91469 0.89708) | (3.34 22.29 27.79 3.46 23.18 27.92 0.902) |

# 5.E-Debug

The result is compared  with the "PA3-E-Debug-own-output.txt" and shown in the table below.

**TABLE 5 PA3-E-Debug-own-output**

| OUR RESULTS | GIVEN RESULTS |
|---|---|
| ( -2.28196 -5.06556 -27.61830 -0.78391 -4.32855 -28.66912 1.97271) | (-2.28 -5.07 -27.62 -0.78 -4.34 -28.67 1.976) |
| (-41.51150 -13.61948 -42.49447 -38.68369 -13.27770 -40.83609 3.29599) | (-41.51 -13.61 -42.49 -38.68 -13.28 -40.84 3.296) |
| (30.09727 15.19825 -6.44416 33.02990 17.64569 -6.29841 3.82251) | (30.09 15.21 -6.45 33.02 17.65 -6.30 3.816) |

# 6.F-Debug

The result is compared  with the "PA3-F-Debug-own-output.txt" and shown in the table below.

**TABLE 6 PA3-F-Debug-own-output**

| OUR RESULTS | GIVEN RESULTS |
|---|---|
| ( -9.27443 -29.37243 -38.63571 -10.81595 -27.29781 -37.41410 2.85878) | (-9.27 -29.37 -38.63 -10.81 -27.29 -37.41 2.856) |
| (-38.37045 0.91475 -25.07241 -39.95337 1.99688 -25.06008 1.91750) | (-38.36 0.91 -25.07 -39.95 2.00 -25.06 1.924) |
| (-5.78249 -29.91310 -19.62803 -7.35776 -27.20945 -20.30206 3.20085) | (-5.79 -29.91 -19.63 -7.36 -27.21 -20.30 3.195) |

# 7.G-Debug

The result is compared  with the "PA3-G-Unknown-own-output.txt" and shown in the table below.

**TABLE 7 PA3-G-Unknown-own-output.txt**

| OUR RESULTS |
|---|
| (-13.66093 12.38037 30.02801 -13.96138 11.96082 30.24026 0.55799) |
| (16.00902 24.41337 8.53591 16.55037 26.03651 9.01081 1.77572) |
| (9.75001 15.57415 -10.16029 9.92203 15.53096 -9.94482 0.27908) |

# 8.H-Debug

The result is compared  with the "PA3-H-Unknown-own-output.txt" and shown in the table below.

**TABLE 8 PA3-H-Unknown-own-output.txt**

| OUR RESULTS |
|---|
| ( 2.51675 -13.64304 8.64420 2.83203 -11.96537 8.47300 1.71560) |
| (-4.31288 -12.33908 -37.45976 -2.14691 -12.27012 -38.16811 2.27991) |
| (-35.97575 -7.62909 -42.13914 -36.66262 -7.35385 -42.91870 1.07483) |

# 9.J-Debug

The result is compared  with the "PA3-J-Unknown-own-output.txt" and shown in the table below.

**TABLE 9 PA3-J-Unknown-own-output.txt**

| OUR RESULTS |
|---|
| (21.48054 -0.92945 49.72730 20.59832 -0.49203 49.58690 0.99467) |
| (25.36021 6.03934 25.74393 27.53223 5.82745 26.43426 2.28892) |
| (8.03087 10.08615 -11.70728 7.97719 10.55353 -11.99916 0.55365) |

## 10.1 Debug case

According to Table 1, the error less than 0.01 means that the predicted number is exactly the same as the given reference ground-truth, which verifies the correctness of our implemented module in finding the best rigid body transformation from point cloud to surface model by utilizing ICP algorithm. For cases D, E, and F, the errors are more than 0.01 regardless of whether implementing robust pose estimation.

**TABLE 1 Quantitative evaluation of the implemented methods' accuracy. Due to the same closest point on mesh w.r.t. point, all the four methods achieve the same accuracy. The error is computed in terms of L2 Norm**

| DataSet | $c^{expected}$ error | pivot error | nav error |
|---|---|---|---|
| debug-a | 0.0048 | 0.0037 | 0.0052 |
| debug-b | 0.8524 | 0.1824 | 0.0249 |
| debug-c | 0.8349 | 0.4538 | 1.2514 |
| debug-d | 0.0190 | 0.0068 | 0.0058 |
| debug-e | 3.5046 | 1.6100 | 3.1573 |
| debug-f | 2.8998 | 1.5632 | 2.9556 |

We guess that this phenomenon is because the current robust pose estimation method is not effective enough to filter out the outliers in the dataset. Another explanation may be that the ICP algorithm suffers from local optimal solution. Its performance critically relies on the quality of initialization, and only local optimality is guaranteed.

**TABLE 2 Comparison of various methods in terms of running time for Debug case A-F.**

| OUR RESULTS | GIVEN RESULTS |
|---|---|
| (17.90666 25.32838 -18.84287 17.90731 25.32410 -18.84304 0.00434) | (17.91 25.32 -18.84 17.91 25.32 -18.84 0.000) |
| ( -39.30009 -22.32570 -30.75257 -39.30331 -22.32773 -30.75337 0.00389) | (-39.30 -22.33 -30.75 -39.30 -22.33 -30.75 0.000) |
| (-9.82757 -13.44235 -1.22125 -9.82755 -13.44231 -1.22128 0.00005) | (-9.83 -13.44 -1.22 -9.83 -13.44 -1.22 0.000) |

As shown in Table 2, 'Octree' denoting the Octree Search demonstrates the highest speed in implementing ICP in every case. Compared to naive Brutal Search, all the two smarter methods significantly improved the computation efficiency, especially Octree Search. Table 2 demonstrates that the clever search algorithm saves much time.

### 10.2 Unknown case

For Unknown cases, we save the results in ./OUTPUT. As shown in Table 3, we simply give the time comparison in this section. For these three cases, the Octree based Search method significantly outperforms the Bounding Sphere Search method. Both of these two clever algorithms are much faster than Brute-Force Search method.

| **Method** | **Time(s)** | | | |
|---|---|---|---|---|
| | G | H | I | J |
| Brute Force Solver | 1.4533 | 1.4526 | 1.4563 | 1.4601 |
| Octree Solver | 0.145598 | 0.139905 | 0.125763 | 0.173482 |

# Contributions

Jiaming Zhang developed Octree Based ICP algorithm; Chongjun Yang developed other search methods based ICP algorithm of this PA. Both team member complete a part of this report.

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