Adaptive Synchronization for Nonlinearly-Coupled Complex Network with an Asymmetrical Coupling Matrix

Jingyi Wang, Jianwen Feng*, Yi Zhao
College of Mathematics and Computational Science
Shenzhen University
Shenzhen 518060, PR China
*Corresponding author. Email address: fengjw@szu.edu.cn

Abstract —**In this paper, the global adaptive synchronization for nonlinearly coupled identical nodes with an asymmetrical coupling matrix is investigated. Form the reason that the coupling strength can not be extraordinary large, we use a adaptive controller to give a sufficient condition for the global synchronization in practice. Numerical examples are also provided to demonstrate the effectiveness of the theory.**

Index Terms—Complex networks, Synchronization, Adaptive, Nonlinearly coupled, Asymmetrical coupling.

I. Introduction

In recent years, the studies of synchronization to complex networks have received great interest and attention in mathematical, physical, biological and physiological literature. Since the synchronization has been found to be a general phenomenon in nature and has important potential applications in real-world, it can be applied in fields including secure communication, seismology, parallel image processing, chemical reaction, and others [1]- [7]. Synchronization can be defined as a process wherein two (or many) dynamical systems adjust a given property of their motion to a common behavior as time goes to infinity, due to coupling or forcing [8].

In literature, there are many widely studied synchronization patterns, i.e., complete or identical synchronization, phase synchronization, cluster synchronization, and so on. In [9], the properties of invariant manifold were used to describe synchronization. The boundedness and synchronization problems for linearly coupled oscillators were considered via the semi-passivity property in [10]. The authors of [11] presented a master stability function based on the transverse Lyapunov exponents to study local synchronization. [12] proposed that a single controller could pin a coupled complex network to a homogenous solution and an effective approach to adapt the coupling strength. The authors of [13] constructed a novel coupling scheme with cooperative and competitive weight couplings to stabilize arbitrarily selected cluster synchronization patterns in connected networks with identical nodes, while, the cluster synchronization of dynamical networks with community structure and nonidentical nodes with or without time delay by feedback control schemes was studied in [14].

However, previous studies of synchronization mainly focussed on oscillators under linear coupling. Even in [12], [16], [17] and [18] which discussed nonlinear coupling, they assumed that the network should be bidirectional. However, unidirectional communication is more important in practical applications, i.e., via broadcasting and sensed information flow, which plays a central role in schooling and flocking. Both of them are typically not bidirectional [20].

In [20], the authors considered the complex network which consisted of N identical nonlinearly and diffusively coupled nodes, with each node being an n-dimensional dynamical system. The state equations of this network have the following form:

$$\dot{x}^i(t) = f(x^i(t)) - \sigma \sum_{i=1}^{N} l_{ij} h(x^j(t)), \quad i = 1, 2, \ldots, N, \quad (1)$$

where $\sigma > 0$ is the coupling strength. The diffusive coupling matrix $L = [l_{ij}] \in R^{N\times N}$ satisfies: $l_{ij} \leq 0$, $i \neq j$ and $l_{ii} = \sum_{j=1,i\neq j}^{N}, l_{ij}$, which can be regarded as the graph Laplacian if we assume the coupled complex networks as a weighted directed graph. If nodes i and j are connected, then $l_{ij} \neq 0$; otherwise $l_{ij} = 0$ and the diagonal elements of the coupling matrix L are $l_{ii} = -\sum_{i=1}^{N} l_{ij} = -k_i, i = 1, 2, ..., N$, where k_i denotes the degree of node i. The nonlinear coupling function $h(\cdot) : R^n \to R^n$ is continuous and has the form: $h(x^i(t)) = (h_1(x_1^i(t)), \ldots, h_n(x_n^i(t)))^T$, $i = 1, 2, \ldots, N$. And some criteria for the global synchronization are given by projecting the nonlinear coupling function onto a linear one and assuming the difference between them as a disturbing function. However, the coupling strength satisfying usch conditions is sufficiently large. It was pointed out in [19] that the theoretical value of the coupling strength is usually much larger than need. But it is not allowed that the coupling strength is extraordinary large in practice. In this paper, we will investigate the adaptive synchronization of nonlinearly coupled directional complex networks. Our works here is a nontrivial extension to the work in [20], where the coupling strength is constant. By constructing a adaptive coupling strength controller, we derive some criteria to ensure