On Cluster Synchronization for Linearly Coupled Complex Networks

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***Abstract*—In this paper, without assuming symmetry, we proved linearly coupled complex networks could achieve cluster synchronization under pinning control. Sufficient conditions guaranteeing cluster synchronization for any initial values are derived by using feedback schemes. Moreover, we propose an adaptive feedback algorithms to adjust the coupling strength. Several numerical examples are given to illustrate our theoretical results.**
***Keywords*-Cluster synchronization; pinning control; adaptive adjustment; asymmetry.**

I. Introduction

Recently, an increasing interest has been devoted to the study of complex networks. Among them, synchronization is the most interesting. In fact, synchronization of complex networks has been found to be a universal phenomenon in nature and has important potential applications in real-world dynamical systems. Great interests and attentions have been received for the synchronization of complex networks in many research and application fields including secure communication, seismology, parallel image processing, chemical reaction, and so on [1]-[6].

There are many widely studied synchronization patterns, such as complete synchronization [7], lag synchronization[8], cluster synchronization[9], phase synchronization[10], partial synchronization[11], etc. Among them, the studies on cluster synchronization have been received more and more attentions. The cluster synchronization requires that the coupled oscillators split into subgroups called clusters, such that the oscillators synchronize with one another in the same cluster, but there is no synchronization among different clusters, which could describe the behaviors of the complex network in the real world. For instance, the metabolic, neural, or software networks containing some different function communities. Thus, it is a natural idea to consider the cluster synchronization of such community networks.

The complex network we considered in this paper is the linearly coupled ordinary differential equations (LCODEs), which can be described as

$$\dot{x}_i(t) = f(x_i(t), t) + c\sum_{j=1}^{N} a_{ij}x_j(t), \quad i = 1, 2, \ldots, N, \quad (1)$$

where N is the networks size, $x_i(t) = (x_i^1(t), x_i^2(t), \ldots, x_i^n(t))^T \in R^n$ is the state vector of the ith oscillator, $f : R^n \times [0, \infty) \to R^n$ is a continues map, and $c > 0$ is the coupling strength. $A = (a_{ij}) \in R^{n\times n}$ is the coupling configuration matrix with zero-sum rows. It represents the topological structure of the network, in which $a_{ij} > 0$ if there is a connection from node j to node i ($i \neq j$) and is zero otherwise. Here, A need not be symmetric since asymmetric topological structures are most common in the real world. In fact, LCODEs are a large class of dynamical systems with continuous time and state, as well as discrete space, which are widely used to describe coupling oscillators. Nowadays, cluster synchronization of different kinds of LCODEs has been widely studied, and many results have already exist on the various properties of such problem. For instance, Z. Ma *et al.* [13] constructed a novel coupling scheme with cooperative and competitive weight couplings that guarantees the cluster synchronization of any connected networks with identical nodes. The authors also derived a sufficient condition for the global stability of cluster synchronization. Mentions should be made on the related paper [12]. In [12], the authors have discussed the problem of driving linearly coupled networks to an arbitrarily selected cluster synchronization pattern via pinning control. They introduced a single negative feedback controller for each cluster to pin the coupled system to the assigned cluster synchronization pattern for any initial values. Compared with their work, our results in this paper are more general, and specifically, the coupling matrix need not to be symmetric.

In this paper we investigate cluster synchronization of LCODEs under pinning control scheme. By utilizing the Lyapunov stability method, the global stability of cluster synchronization in networks is investigated, and several sufficient conditions for the global stability are given. Furthermore, we propose an adaptive feedback algorithms to adjust the coupling strength.

The paper is organized as follows. In section II, some necessary and useful definitions are given. In section III, we study the global cluster synchronization of LCODEs and give a sufficient condition for it. Then, the adaptive feedback algorithms on coupling strength are proposed to achieve cluster synchronization in the complex network. In section IV, numerical simulation are presented. We conclude the paper in section V.

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