On Cluster Synchronization for Nonlinearly-Coupled Complex Network with Nonidentical Nodes

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***Abstract*—In this paper, cluster synchronization in networks of nonlinearly coupled nonidentical dynamical systems under pinning control is investigated. Sufficient conditions guaranteeing cluster synchronization for any initial values are derived by using feedback schemes. Moreover, we propose an adaptive feedback algorithms to adjust the coupling strength. Several numerical examples are given to illustrate our theoretical results.**
***Keywords*—Cluster synchronization, pinning control, adaptive adjustment, nonlinear coupling.**

I. INTRODUCTION

Synchronization can be defined as a process wherein two (or many) dynamical systems adjust a given property of their motion to a common behavior as time goes to infinity, due to coupling or forcing[12]. There are many widely studied synchronization patterns, such as complete synchronization [7], lag synchronization[8], cluster synchronization[9], phase synchronization[10], partial synchronization[11], etc. Great interests and attentions have been received for the synchronization of complex networks in many research and application fields including secure communication, seismology, parallel image processing, chemical reaction, and so on [1]-[6].

In the real world, many networks contain some different function communities and the local dynamics between two function communities are different, e.g., metabolic, neural, or software networks. Thus, it is a natural idea to consider the cluster synchronization of community networks with nonidentical nodes [10], which requires that the coupled oscillators split into subgroups called clusters, such that the oscillators synchronize with one another in the same cluster, but there is no synchronization among different clusters. In current literature, most of those works have considered cluster synchronization of linearly coupled complex network with nonidentical nodes, i.e., the coupled function is linear. For example, in [14], the authors studied the cluster synchronization of dynamical networks with community structure and nonidentical nodes with or without time delay by feedback control schemes, obtained several sufficient conditions for achieving cluster synchronization. The authors of [15] presented conditions guaranteeing cluster synchronization and investigated the relation between cluster synchronization and the unweighted graph topology.

However, nonlinearly coupling exists in real world rather than artificial world. For instances, in neural and metabolic network, the coupling configuration is oscillated up and down by external disturbance. Thus, in this paper we investigate cluster synchronization of nonlinearly coupled complex network with nonidentical nodes under pinning control scheme. By utilizing the Lyapunov stability method, the global stability of cluster synchronization in networks is investigated, and several sufficient conditions for the global stability are given. Furthermore, we propose an adaptive feedback algorithms to adjust the coupling strength.

The paper is organized as follows. In section II, a model of nonlinear coupled complex network is proposed and some necessary and useful definitions, lemmas and assumptions are given. In section III, we study the global cluster synchronization in the network with nonidentical nodes and give a sufficient condition for it. Then, the adaptive feedback algorithms on coupling strength are proposed to achieve cluster synchronization in the complex network. In section IV, numerical simulation are presented. We conclude the paper in section V.

II. PRELIMINARIES

For simplicity, we now make some assumptions for a network with community structure as follows. Suppose that these networks have N nodes and m communities with $N > m \geq 2$. If node i belongs to the jth community, then we let $\mu_i = j$. We denote by U_i the set of all nodes in the ith community and $\tilde{U}_i$, which is the subset of U_i, be the index set of all nodes in the ith community having direct connections to other communities. The state equations of this network have the following form

$$\dot{x}_i(t) = f_{\mu_i}(x_i(t), t) + \sum_{j=1}^{N} a_{ij}\Gamma g(x_j(t)), \quad i = 1, 2, \ldots, N, \tag{1}$$

where $x_i = (x_{1i}, x_{2i}, ..., x_{ni})^T \in R^n$ are the state variables of node i. The function $f_{\mu_i}(\cdot)$ describes the local dynamics of nodes in the μ_ith community, which is differentiable and capable of performing abundant dynamical behaviors. For any pair of indices i and j, if $\mu_i \neq \mu_j$, which means node i and node j belong to different communities, then