

# Cluster synchronization of nonlinearly-coupled complex networks with nonidentical nodes and asymmetrical coupling matrix

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**Abstract** In this paper, we investigate the cluster synchronization problem for networks with nonlinearly coupled nonidentical dynamical systems and asymmetrical coupling matrix by using pinning control. We derive sufficient conditions for cluster synchronization for any initial values through a feedback scheme and propose an adaptive feedback algorithm that adjusts the coupling strength. Some numerical examples are then given to illustrate the theoretical results.

**Keywords** Complex networks · Pinning control · Cluster synchronization · Adaptive · Nonlinearly-coupled networks · Asymmetrical

## 1 Introduction

Complex networks synchronization is an important mathematical problem in both the physical and biological sciences that has potential applications to diverse fields such as communications security, seismology, and parallel image processing [1–7]. Synchronization is the process in which two (or more) dynamical systems adjust a certain property of their motion to a common behavior by coupling or forcing in the

limit as time goes to infinity [8] and common synchronization patterns include complete synchronization [9], lag synchronization [10], cluster synchronization [11], phase synchronization [12], and partial synchronization [13]. This paper considers the cluster synchronization of nonlinearly-coupled complex networks with nonidentical nodes and an asymmetrical coupling matrix.

Roughly speaking, cluster synchronization is the phenomenon in which the nodes of every cluster of a network are synchronized and those in distinct clusters are not and many results already exist on the various properties of this phenomenon. Kaneko [14] showed that cluster synchronization is significant in biological science and communication engineering. Yoshiooka [15] analyzes a cluster state in periodically firing spiking neurons networks that are interconnected with chemical synapses, where an ensemble of neurons are subdivided into a few clusters, in each of which neurons exhibit perfect synchronization. Wu et al. [16], for instance, investigated the pinning control method for linearly-coupled complex networks with identical nodes and obtained sufficient conditions for cluster synchronization for any initial values and Ma et al. [17] constructed a novel coupling scheme with cooperative and competitive weight couplings that guarantees the cluster synchronization of any connected networks with identical nodes. The authors also derived a sufficient condition for the global stability of cluster synchronization. A number of sufficient conditions were similarly obtained by Lu [18] for the clus-

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