5 Project 5: 3DVar with Lorenz'63

5.1 Model Description

The 3-variable Lorenz'63 system:

$$\frac{dx}{dt} = \sigma(y - x), \tag{15}$$

$$\frac{dy}{dt} = x(\rho - z) - y, \tag{16}$$

$$\frac{dz}{dt} = xy - \beta z. \tag{17}$$

with $\sigma = 10$, $\rho = 28$, $\beta = 8/3$. Integrate with RK4, $\Delta t = 0.01$.

5.2 Objective

Test 3DVar and ML-based correction in a low-dimensional chaotic model.

5.3 Nature Run and Observations

Simulate for $T = 200$, record at $\Delta t_{obs} = 0.02$, with noise $\sim \mathcal{N}(0, 1^2)$.

5.4 3DVar and ML Correction

Analogous to Project 1: train an MLP on $(\mathbf{x}^b, \mathbf{y})$ to predict the increment $\mathbf{x}^t - \mathbf{x}^b$.

5.5 Tasks

1. Compute RMSE between multiple Lorenz'63 runs to assess model uncertainty.
2. Construct and visualize the B matrix.
3. Implement the 3DVar twin experiment with the ML-based analysis increment correction.
4. Analyze innovations and compare to theoretical expectations.
5. Vary the observation noise $R \in \{1, 2, 5\}$ and assess the impact on precision.
6. Test partial observations (e.g., observing only x) and their effect on y and z.

5.6 Evaluation Metrics

RMSE, phase-space portraits comparing analyses, and innovation statistics.