Feb 15 2023 - RK Continued

This is due next Wednesday as well

RK4C Homework

$$\dot{y}=k(n_1-y/2)^2(n_2-(y/2))^2(n_3-(3y/4))^2$$

- $k=6.22e-19$
- $n_1=2e3$
- $n_2=2e3$
- $n_3=3e3$

We want to approximate y(0.2) using RK4

```
% Function RK4C,
% book answer is 2099, numerical approximation is w = 2703

a = 0;
b = 0;
ad = 0;
N = 10;

[T,W] = RK_4(a,b,ad,N);
w = round(W(N));
```

We have two answers that are vastly different so the idea is to double the number of steps for your system so you can get a much more accurate approximation of the system, **make step sizes smaller**, goal is to find optimal step size

Implementation

```
% Function RK4C,
% book answer is 2099, numerical approximation is w = 2703

a = 0;
b = 0;
ad = 0;
N = 10;

[T,W] = RK_4(a,b,ad,N);
w = round(W(N));

%Adjustments made here
N = 2 * N;
[T,W] = RK_4(a,b,ad, N);
wn = round(W(N));
```

```
%Do while loop here
while wn ~= w
    w = wn %reinitialize w with this approximation
    N = 2 * N;
    [T,W] = RK4(a,b,ad,N);
    wn = round(W(N));
end

N
```

Output shown should be:

```
w = 2703; %initial while loop
w = 2099; %first while loop
w = 2080; %second while loop
N = 160; %this is the final N you should get if done correctly
```

Utility Functions

```
function [T,W] = RK4(a,b,c,d)

    s1 = some_function(x, y)
    s2 = some_function(x + h/2, y + (h*s1)/2)
    s3 = some_function(x + h/2, y + (h*s2)/2)
    s4 = some_function(x + h, y[i] + h*s3)
    W = x
end


function fv = some_function(t,y)
    k = 6.22E-19;
    n1 = 2e3;
    n2 = 2e3;
    n3 = 3e3;
    fv = k(n_1-y/2)^2(n_2-(y/2))^2(n_3-(3y/4))^2; %the function you see at
the top page
end
```

RK4D Homework

Refer to exercise 2.8 from 5.4:

Water flows an inverted emial tank with a circular orifice at the rate $$\dot{y} = \frac{6\pi*r^2}{\sqrt{2g}}\frac{\sqrt{y}}{\pi y^2}$$

- $$y(0) = 8ft$$

- $$g = 32.2 ft/s$$
- $$r=0.1ft$$
- Part a:
    - Find the water level after 10 minutes using h = 205
- Part b:
    - When will the tank be empty within 20 secs?
- Time span $$0<t<b$$
- b is unknown so will be done via trial

Avoid y<0 otherwise we get a negative number and thus have a complex number from the square root of a negative value

Implementation

```
a = 0;
b = 1600; %seconds
alpha = 8; %initial guess
N = 80;

[T,W] = RK_4(a,b,al,N);

format short g;

%get last 6 indicies of T and W
T_W[T(75:80), W(75:80)]
```

Outputs should be as follows:

```
T_W
1500 + 0i   0.88464 + 0i
1520 + 0i   -1.3384 - 0.21151i
1540 + 0i
1560 + 0i
1580 + 0i
1600 + 0i   -0.61759 - 1.3302i
```

The tank will be emptied out between 1500s and 1520s

Homework 10

$$\dot{y}=\frac{2ty}{t^2-y^2}$$

- $$y(0)=3$$
- $$0<t<2$$
- What happens when at t=2?
- You will do 3 different approximations to see how reliable the approximations are with different step sizes, h

RK4E - Problem 10A

```
[T,W] = RK4(0,2,3, 20);  %h = 0.1
[T2,W2] = RK4(0,2,3,40); %h = 0.05
[T3,W3] = RK4(0,2,3,80); %h = 0.025
```

output the time and prediction values

```
2   1.1859 #W1
2   1.0842 #W2
2   5.9216 #W3 cant find a solution as t goes above 1.5
```