## 1 $p(u)$

$$
\begin{aligned}
p\left(\mathbf{u}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{u}}, \mathbf{\Sigma}_{\mathbf{u}}\right) \\
\mathbf{m}_{\mathbf{u}} &= \mathbf{0} \\
\mathbf{\Sigma}_{\mathbf{u}} &= \mathbf{K}_{\mathbf{u},\mathbf{u}}
\end{aligned}
$$

## 2 $q_{FITC}(f|u)$

$$
\begin{aligned}
q_{FITC}\left(\mathbf{f}|\mathbf{u}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{f}|\mathbf{u}}, \mathbf{\Sigma}_{\mathbf{f}|\mathbf{u}}\right) \\
\mathbf{m}_{\mathbf{f}|\mathbf{u}} &= \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{u} \\
\mathbf{\Sigma}_{\mathbf{f}|\mathbf{u}} &= \mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}]
\end{aligned}
$$

## 3 $q_{FITC}(f_*|u)$

$$
\begin{aligned}
q_{FITC}\left(\mathbf{f}_*|\mathbf{u}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{f}_*|\mathbf{u}}, \mathbf{\Sigma}_{\mathbf{f}_*|\mathbf{u}}\right) \\
\mathbf{m}_{\mathbf{f}_*|\mathbf{u}} &= \mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{u} \\
\mathbf{\Sigma}_{\mathbf{f}_*|\mathbf{u}} &= \mathbf{K}_{\mathbf{f}_*,\mathbf{f}_*} - \mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}_*}
\end{aligned}
$$

## 4 $q_{FITC}(f, f_*|u)$

$$
\begin{aligned}
q_{FITC}\left(\mathbf{f}, \mathbf{f}_*|\mathbf{u}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{f},\mathbf{f}_*|\mathbf{u}}, \mathbf{\Sigma}_{\mathbf{f},\mathbf{f}_*|\mathbf{u}}\right) \\
\mathbf{m}_{\mathbf{f},\mathbf{f}_*|\mathbf{u}} &= \begin{bmatrix} \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{u} \\ \mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{u} \end{bmatrix} \\
\mathbf{\Sigma}_{\mathbf{f},\mathbf{f}_*|\mathbf{u}} &= \begin{bmatrix} \mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}] & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{\mathbf{f}_*,\mathbf{f}_*} - \mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}_*} \end{bmatrix}
\end{aligned}
$$

## 5 $q_{FITC}(f, f_*, u)$

$$
\begin{aligned}
q_{FITC}\left(\mathbf{f}, \mathbf{f}_*, \mathbf{u}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{f},\mathbf{f}_*,\mathbf{u}}, \mathbf{\Sigma}_{\mathbf{f},\mathbf{f}_*,\mathbf{u}}\right) \\
\mathbf{m}_{\mathbf{f},\mathbf{f}_*,\mathbf{u}} &= \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \\
\mathbf{\Sigma}_{\mathbf{f},\mathbf{f}_*,\mathbf{u}} &= \begin{bmatrix} \mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}] + \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} & \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}_*} & \mathbf{K}_{\mathbf{f},\mathbf{u}} \\ \mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} & \mathbf{K}_{\mathbf{f}_*,\mathbf{f}_*} & \mathbf{K}_{\mathbf{f}_*,\mathbf{u}} \\ \mathbf{K}_{\mathbf{u},\mathbf{f}} & \mathbf{K}_{\mathbf{u},\mathbf{f}_*} & \mathbf{K}_{\mathbf{u},\mathbf{u}} \end{bmatrix}
\end{aligned}
$$

## 6 $q_{FITC}(f, f_*)$

$$
\begin{aligned}
q_{FITC}\left(\mathbf{f}, \mathbf{f}_*\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{f},\mathbf{f}_*}, \mathbf{\Sigma}_{\mathbf{f},\mathbf{f}_*}\right) \\
\mathbf{m}_{\mathbf{f},\mathbf{f}_*} &= \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} \\
\mathbf{\Sigma}_{\mathbf{f},\mathbf{f}_*} &= \begin{bmatrix} \mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}] + \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} & \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}_*} \\ \mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} & \mathbf{K}_{\mathbf{f}_*,\mathbf{f}_*} \end{bmatrix}
\end{aligned}
$$

## 7 $q_{FITC}(y, f_*, f)$

$$
\begin{aligned}
q_{FITC}\left(\mathbf{y}, \mathbf{f}_*, \mathbf{f}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{y},\mathbf{f}_*,\mathbf{f}}, \mathbf{\Sigma}_{\mathbf{y},\mathbf{f}_*,\mathbf{f}}\right) \\
\mathbf{m}_{\mathbf{y},\mathbf{f}_*,\mathbf{f}} &= \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \\
\mathbf{\Sigma}_{\mathbf{y},\mathbf{f}_*,\mathbf{f}} &= \begin{bmatrix} \sigma_y^2\mathbf{I}+\mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}}-\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}]+\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} & \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}_*} & \mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}}-\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}]+\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} \\ \mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} & \mathbf{K}_{\mathbf{f}_*,\mathbf{f}_*} & \mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} \\ \mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}}-\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}]+\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} & \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}_*} & \mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}}-\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}]+\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} \end{bmatrix}
\end{aligned}
$$

## 8 $q_{FITC}(f_*, f|y)$

$$
\begin{aligned}
q_{FITC}\left(\mathbf{f}_*, \mathbf{f}|\mathbf{y}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{f}_*,\mathbf{f}|\mathbf{y}}, \mathbf{\Sigma}_{\mathbf{f}_*,\mathbf{f}|\mathbf{y}}\right) \\
\mathbf{m}_{\mathbf{f}_*,\mathbf{f}|\mathbf{y}} &= \begin{bmatrix} \mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\left(\sigma_y^2\mathbf{I}+\mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}}-\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}]+\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right)^{-1}\mathbf{y} \\ \left(\mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}}-\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}]+\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right)\left(\sigma_y^2\mathbf{I}+\mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}}-\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}]+\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right)^{-1}\mathbf{y} \end{bmatrix} \\
\mathbf{\Sigma}_{\mathbf{f}_*,\mathbf{f}|\mathbf{y}} &= \begin{bmatrix} \mathbf{K}_{\mathbf{f}_*,\mathbf{f}_*}-\mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\left(\sigma_y^2\mathbf{I}+\mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}}-\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}]+\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right)^{-1}\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}_*} \\ \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}_*}-\left(\mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}}-\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}]+\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right)\left(\sigma_y^2\mathbf{I}+\mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}}-\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}]+\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right)^{-1}\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}_*} & \mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}}-\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1} \end{bmatrix}
\end{aligned}
$$

## 9 $q_{FITC}(f_*|y)$

$$
\begin{aligned}
q_{FITC}\left(\mathbf{f}_*|\mathbf{y}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{f}_*|\mathbf{y}}, \mathbf{\Sigma}_{\mathbf{f}_*|\mathbf{y}}\right) \\
\mathbf{m}_{\mathbf{f}_*|\mathbf{y}} &= \mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\left(\sigma_y^2\mathbf{I}+\mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}}-\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}]+\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right)^{-1}\mathbf{y} \\
\mathbf{\Sigma}_{\mathbf{f}_*|\mathbf{y}} &= \mathbf{K}_{\mathbf{f}_*,\mathbf{f}_*}-\mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\left(\sigma_y^2\mathbf{I}+\mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}}-\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}]+\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right)^{-1}\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}_*}
\end{aligned}
$$

## 10 $q_{FITC}(f_*|y)$

$$
\begin{aligned}
q_{FITC}\left(\mathbf{f}_*|\mathbf{y}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{f}_*|\mathbf{y}}, \mathbf{\Sigma}_{\mathbf{f}_*|\mathbf{y}}\right) \\
\mathbf{m}_{\mathbf{f}_*|\mathbf{y}} &= \mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\left(\sigma_y^2\mathbf{I} + \mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}] + \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right)^{-1}\mathbf{y} \\
\mathbf{\Sigma}_{\mathbf{f}_*|\mathbf{y}} &= \mathbf{K}_{\mathbf{f}_*,\mathbf{f}_*} - \mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\left(\sigma_y^2\mathbf{I} + \mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{f}}] + \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right)^{-1}\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}_*}
\end{aligned}
$$

## 11 $p(u)$

$$
\begin{aligned}
p\left(\mathbf{u}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{u}}, \mathbf{\Sigma}_{\mathbf{u}}\right) \\
\mathbf{m}_{\mathbf{u}} &= \mathbf{0} \\
\mathbf{\Sigma}_{\mathbf{u}} &= \mathbf{K}_{\mathbf{u},\mathbf{u}}
\end{aligned}
$$

## 12 $q_{PITC}(f|u)$

$$
\begin{aligned}
q_{PITC}\left(\mathbf{f}|\mathbf{u}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{f}|\mathbf{u}}, \mathbf{\Sigma}_{\mathbf{f}|\mathbf{u}}\right) \\
\mathbf{m}_{\mathbf{f}|\mathbf{u}} &= \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{u} \\
\mathbf{\Sigma}_{\mathbf{f}|\mathbf{u}} &= \mathrm{blockdiag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}]
\end{aligned}
$$

## 13 $q_{PITC}(f_*|u)$

$$
\begin{aligned}
q_{PITC}\left(\mathbf{f}_*|\mathbf{u}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{f}_*|\mathbf{u}}, \mathbf{\Sigma}_{\mathbf{f}_*|\mathbf{u}}\right) \\
\mathbf{m}_{\mathbf{f}_*|\mathbf{u}} &= \mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{u} \\
\mathbf{\Sigma}_{\mathbf{f}_*|\mathbf{u}} &= \mathbf{K}_{\mathbf{f}_*,\mathbf{f}_*} - \mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}_*}
\end{aligned}
$$

## 14 $q_{PITC}(f, f_*|u)$

$$
\begin{aligned}
q_{PITC}\left(\mathbf{f}, \mathbf{f}_*|\mathbf{u}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{f},\mathbf{f}_*|\mathbf{u}}, \mathbf{\Sigma}_{\mathbf{f},\mathbf{f}_*|\mathbf{u}}\right) \\
\mathbf{m}_{\mathbf{f},\mathbf{f}_*|\mathbf{u}} &= \begin{bmatrix} \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{u} \\ \mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{u} \end{bmatrix} \\
\mathbf{\Sigma}_{\mathbf{f},\mathbf{f}_*|\mathbf{u}} &= \begin{bmatrix} \text{blockdiag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}] & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{\mathbf{f}_*,\mathbf{f}_*} - \mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}_*} \end{bmatrix}
\end{aligned}
$$

## 15 $q_{PITC}(f, f_*, u)$

$$
\begin{aligned}
q_{PITC}\left(\mathbf{f}, \mathbf{f}_*, \mathbf{u}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{f},\mathbf{f}_*,\mathbf{u}}, \mathbf{\Sigma}_{\mathbf{f},\mathbf{f}_*,\mathbf{u}}\right) \\
\mathbf{m}_{\mathbf{f},\mathbf{f}_*,\mathbf{u}} &= \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \\
\mathbf{\Sigma}_{\mathbf{f},\mathbf{f}_*,\mathbf{u}} &= \begin{bmatrix} \text{blockdiag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}] + \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} & \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}_*} & \mathbf{K}_{\mathbf{f},\mathbf{u}} \\ \mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} & \mathbf{K}_{\mathbf{f}_*,\mathbf{f}_*} & \mathbf{K}_{\mathbf{f}_*,\mathbf{u}} \\ \mathbf{K}_{\mathbf{u},\mathbf{f}} & \mathbf{K}_{\mathbf{u},\mathbf{f}_*} & \mathbf{K}_{\mathbf{u},\mathbf{u}} \end{bmatrix}
\end{aligned}
$$

## 16 $q_{PITC}(f, f_*)$

$$
\begin{aligned}
q_{PITC}\left(\mathbf{f}, \mathbf{f}_*\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{f},\mathbf{f}_*}, \boldsymbol{\Sigma}_{\mathbf{f},\mathbf{f}_*}\right) \\
\mathbf{m}_{\mathbf{f},\mathbf{f}_*} &= \left[\begin{smallmatrix}\mathbf{0}\\ \mathbf{0}\end{smallmatrix}\right] \\
\boldsymbol{\Sigma}_{\mathbf{f},\mathbf{f}_*} &= \begin{bmatrix} \text{blockdiag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}] + \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} & \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}_*} \\ \mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} & \mathbf{K}_{\mathbf{f}_*,\mathbf{f}_*} \end{bmatrix}
\end{aligned}
$$

## 17 $p(y|f)$

$$
\begin{aligned}
p\left(\mathbf{y}|\mathbf{f}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{y}|\mathbf{f}}, \boldsymbol{\Sigma}_{\mathbf{y}|\mathbf{f}}\right) \\
\mathbf{m}_{\mathbf{y}|\mathbf{f}} &= \mathbf{f} \\
\boldsymbol{\Sigma}_{\mathbf{y}|\mathbf{f}} &= \sigma_y^2\mathbf{I}
\end{aligned}
$$

## 18 $q_{PITC}(y, f_*, f)$

$$
\begin{aligned}
q_{PITC}\left(\mathbf{y}, \mathbf{f}_*, \mathbf{f}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{y},\mathbf{f}_*,\mathbf{f}}, \boldsymbol{\Sigma}_{\mathbf{y},\mathbf{f}_*,\mathbf{f}}\right) \\
\mathbf{m}_{\mathbf{y},\mathbf{f}_*,\mathbf{f}} &= \begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix} \\
\boldsymbol{\Sigma}_{\mathbf{y},\mathbf{f}_*,\mathbf{f}} &= \begin{bmatrix} \sigma_y^2\mathbf{I} + \text{blockdiag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}] + \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} & \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}_*} & \text{blockdiag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}] + \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} \\ \mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} & \mathbf{K}_{\mathbf{f}_*,\mathbf{f}_*} & \mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} \\ \text{blockdiag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}] + \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} & \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}_*} & \text{blockdiag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}] + \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} \end{bmatrix}
\end{aligned}
$$

## 19 $q_{PITC}(f_*, f|y)$

$$
\begin{aligned}
q_{PITC}\left(\mathbf{f}_*, \mathbf{f}|\mathbf{y}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{f}_*,\mathbf{f}|\mathbf{y}}, \mathbf{\Sigma}_{\mathbf{f}_*,\mathbf{f}|\mathbf{y}}\right) \\
\mathbf{m}_{\mathbf{f}_*,\mathbf{f}|\mathbf{y}} &= \begin{bmatrix} \mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\left(\sigma_y^2\mathbf{I}+\text{blockdiag}[\mathbf{K}_{\mathbf{f},\mathbf{f}}-\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}]+\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right)^{-1}\mathbf{y} \\ \left(\text{blockdiag}[\mathbf{K}_{\mathbf{f},\mathbf{f}}-\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}]+\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right)\left(\sigma_y^2\mathbf{I}+\text{blockdiag}[\mathbf{K}_{\mathbf{f},\mathbf{f}}-\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}]+\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right)^{-1}\mathbf{y} \end{bmatrix} \\
\mathbf{\Sigma}_{\mathbf{f}_*,\mathbf{f}|\mathbf{y}} &= \begin{bmatrix} \mathbf{K}_{\mathbf{f}_*,\mathbf{f}_*}-\mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\left(\sigma_y^2\mathbf{I}+\text{blockdiag}[\mathbf{K}_{\mathbf{f},\mathbf{f}}-\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}]+\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right)^{-1}\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}_*} \\ \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}_*}-\left(\text{blockdiag}[\mathbf{K}_{\mathbf{f},\mathbf{f}}-\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}]+\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right)\left(\sigma_y^2\mathbf{I}+\text{blockdiag}[\mathbf{K}_{\mathbf{f},\mathbf{f}}-\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}]+\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right)^{-1}\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}_*} & \text{blockdiag}[ \end{bmatrix}
\end{aligned}
$$

## 20 $q_{PITC}(f_*|y)$

$$
\begin{aligned}
q_{PITC}\left(\mathbf{f}_*|\mathbf{y}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{f}_*|\mathbf{y}}, \mathbf{\Sigma}_{\mathbf{f}_*|\mathbf{y}}\right) \\
\mathbf{m}_{\mathbf{f}_*|\mathbf{y}} &= \mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\left(\sigma_y^2\mathbf{I}+\text{blockdiag}[\mathbf{K}_{\mathbf{f},\mathbf{f}}-\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}]+\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right)^{-1}\mathbf{y} \\
\mathbf{\Sigma}_{\mathbf{f}_*|\mathbf{y}} &= \mathbf{K}_{\mathbf{f}_*,\mathbf{f}_*}-\mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\left(\sigma_y^2\mathbf{I}+\text{blockdiag}[\mathbf{K}_{\mathbf{f},\mathbf{f}}-\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}]+\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right)^{-1}\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}_*}
\end{aligned}
$$