1
$$p(u)$$

$$\begin{aligned} p\left(\mathbf{u}\right) &= \mathcal{N}\left(\mathbf{m_u}, \boldsymbol{\Sigma_u}\right) \\ \mathbf{m_u} &= \mathbf{0} \\ \boldsymbol{\Sigma_u} &= \mathbf{K_{u,u}} \end{aligned}$$

$\mathbf{2} \quad q_{FITC}(f|u)$

$$\begin{aligned} q_{FITC}\left(\mathbf{f}|\mathbf{u}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{f}|\mathbf{u}}, \boldsymbol{\Sigma}_{\mathbf{f}|\mathbf{u}}\right) \\ \mathbf{m}_{\mathbf{f}|\mathbf{u}} &= \mathbf{K}_{\mathbf{f},\mathbf{u}} \mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1} \mathbf{u} \\ \boldsymbol{\Sigma}_{\mathbf{f}|\mathbf{u}} &= \mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}} \mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1} \mathbf{K}_{\mathbf{u},\mathbf{f}}] \end{aligned}$$

$$\mathbf{3} \quad q_{FITC}(f_*|u)$$

$$\begin{split} q_{FITC}\left(\mathbf{f}_{*}|\mathbf{u}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{f}_{*}|\mathbf{u}}, \boldsymbol{\Sigma}_{\mathbf{f}_{*}|\mathbf{u}}\right) \\ \mathbf{m}_{\mathbf{f}_{*}|\mathbf{u}} &= \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}} \mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1} \mathbf{u} \\ \boldsymbol{\Sigma}_{\mathbf{f}_{*}|\mathbf{u}} &= \mathbf{K}_{\mathbf{f}_{*},\mathbf{f}_{*}} - \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}} \mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1} \mathbf{K}_{\mathbf{u},\mathbf{f}_{*}} \end{split}$$

 $4 \quad q_{FITC}(f, f_*|u)$

$$\begin{split} q_{FITC}\left(\mathbf{f},\mathbf{f}_*|\mathbf{u}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{f},\mathbf{f}_*|\mathbf{u}}, \boldsymbol{\Sigma}_{\mathbf{f},\mathbf{f}_*|\mathbf{u}}\right) \\ \mathbf{m}_{\mathbf{f},\mathbf{f}_*|\mathbf{u}} &= \begin{bmatrix} \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1} \mathbf{u} \\ \mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1} \mathbf{u} \end{bmatrix} \\ \boldsymbol{\Sigma}_{\mathbf{f},\mathbf{f}_*|\mathbf{u}} &= \begin{bmatrix} \operatorname{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}] & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{\mathbf{f}_*,\mathbf{f}_*} - \mathbf{K}_{\mathbf{f}_*,\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}_*} \end{bmatrix} \end{split}$$

 $\mathbf{5} \quad q_{FITC}(f, f_*, u)$

$$\begin{split} q_{FITC}\left(\mathbf{f},\mathbf{f_*},\mathbf{u}\right) &= \mathcal{N}\left(\mathbf{m_{f,f_*,u}}, \boldsymbol{\Sigma_{f,f_*,u}}\right) \\ \mathbf{m_{f,f_*,u}} &= \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \\ \boldsymbol{\Sigma_{f,f_*,u}} &= \begin{bmatrix} \text{diag}[\mathbf{K_{f,f}} - \mathbf{K_{f,u}} \mathbf{K_{u,u}}^{-1} \mathbf{K_{u,f}}] + \mathbf{K_{f,u}} \mathbf{K_{u,u}}^{-1} \mathbf{K_{u,f}} \mathbf{K_{f,u}} \mathbf{K_{u,u}}^{-1} \mathbf{K_{u,f_*}} \mathbf{K_{f_*,u}} \\ \mathbf{K_{f_*,u}} \mathbf{K_{u,f}} \mathbf{K_{u,f}} \mathbf{K_{u,f_*}} \mathbf{K_{f_*,f_*}} \mathbf{K_{f_*,u}} \\ \mathbf{K_{u,f}} \mathbf{K_{u,f_*}} \mathbf{K_{u,f_*}} \mathbf{K_{u,u}} \end{bmatrix} \end{split}$$

 $6 \quad q_{FITC}(f, f_*)$

$$\begin{split} q_{FITC}\left(\mathbf{f},\mathbf{f}_{*}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{f},\mathbf{f}_{*}},\boldsymbol{\Sigma}_{\mathbf{f},\mathbf{f}_{*}}\right) \\ \mathbf{m}_{\mathbf{f},\mathbf{f}_{*}} &= \left[\begin{smallmatrix} \mathbf{0} \\ \mathbf{0} \end{smallmatrix} \right] \\ \boldsymbol{\Sigma}_{\mathbf{f},\mathbf{f}_{*}} &= \left[\begin{smallmatrix} \operatorname{diag}\left[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right] + \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} & \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} \\ \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} & \mathbf{K}_{\mathbf{f}_{*},\mathbf{f}_{*}} \end{smallmatrix} \right] \end{split}$$

7
$$q_{FITC}(y, f_*, f)$$

$$\begin{split} q_{FITC}\left(\mathbf{y},\mathbf{f}_{*},\mathbf{f}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{y},\mathbf{f}_{*},\mathbf{f}},\boldsymbol{\Sigma}_{\mathbf{y},\mathbf{f}_{*},\mathbf{f}}\right) \\ \mathbf{m}_{\mathbf{y},\mathbf{f}_{*},\mathbf{f}} &= \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \\ \boldsymbol{\Sigma}_{\mathbf{y},\mathbf{f}_{*},\mathbf{f}} &= \begin{bmatrix} \sigma_{y}^{2}\mathbf{I} + \operatorname{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}] + \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} & \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{d}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} + \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{d}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} \\ \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{d}} & \mathbf{K}_{\mathbf{f}_{*},\mathbf{f}_{*}} & \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} \\ \operatorname{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}] + \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} & \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{u}} + \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} \end{bmatrix} \end{split}$$

$q_{FITC}(f_*, f|y)$

$$\begin{split} q_{FITC}\left(\mathbf{f}_{*},\mathbf{f}|\mathbf{y}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{f}_{*},\mathbf{f}|\mathbf{y}},\boldsymbol{\Sigma}_{\mathbf{f}_{*},\mathbf{f}|\mathbf{y}}\right) \\ \mathbf{m}_{\mathbf{f}_{*},\mathbf{f}|\mathbf{y}} &= \begin{bmatrix} \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} \left(\sigma_{y}^{2}\mathbf{I} + \mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right)^{-1}\mathbf{y} \\ \left(\mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right) \left(\sigma_{y}^{2}\mathbf{I} + \mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right)^{-1}\mathbf{y} \end{bmatrix} \\ \boldsymbol{\Sigma}_{\mathbf{f}_{*},\mathbf{f}|\mathbf{y}} &= \begin{bmatrix} \mathbf{K}_{\mathbf{f}_{*},\mathbf{f}_{*}} - \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} \left(\sigma_{y}^{2}\mathbf{I} + \mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} \right)^{-1}\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{d}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} \\ \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{u}} + \left(\mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right) \left(\sigma_{y}^{2}\mathbf{I} + \mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{d}} \right)^{-1}\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{d}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{d}} \right) \\ \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{u}} + \left(\mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{d}}\right) \left(\sigma_{y}^{2}\mathbf{I} + \mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{d}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{d}}\right)^{-1}\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{d}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{d}} \right) \\ \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{d}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{d}} + \left(\mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{d}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{d}}\right) \left(\sigma_{y}^{2}\mathbf{I} + \mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{d}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{d}}\right)^{-1}\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{d}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{d}}\right) \\ \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{d}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{d}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{d}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{d}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{d}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{d}}\right) \\ \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{d}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{d}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{d}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{d}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{d}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{d}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{d}}^{-1}\mathbf{K}_{\mathbf$$

$q_{FITC}(f_*|y)$

$$\begin{aligned} q_{FITC}\left(\mathbf{f}_{*}|\mathbf{y}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{f}_{*}|\mathbf{y}}, \boldsymbol{\Sigma}_{\mathbf{f}_{*}|\mathbf{y}}\right) \\ \mathbf{m}_{\mathbf{f}_{*}|\mathbf{y}} &= \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\left(\sigma_{y}^{2}\mathbf{I} + \mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right] + \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right)^{-1}\mathbf{y} \\ \boldsymbol{\Sigma}_{\mathbf{f}_{*}|\mathbf{y}} &= \mathbf{K}_{\mathbf{f}_{*},\mathbf{f}_{*}} - \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\left(\sigma_{y}^{2}\mathbf{I} + \mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right) + \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} \\ \boldsymbol{\Sigma}_{\mathbf{f}_{*}|\mathbf{y}} &= \mathbf{K}_{\mathbf{f}_{*},\mathbf{f}_{*}} - \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\left(\sigma_{y}^{2}\mathbf{I} + \mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right)^{-1}\mathbf{Y} \\ \boldsymbol{\Sigma}_{\mathbf{f}_{*}|\mathbf{y}} &= \mathbf{K}_{\mathbf{f}_{*},\mathbf{f}_{*}} - \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} \\ \boldsymbol{\Sigma}_{\mathbf{f}_{*}|\mathbf{y}} &= \mathbf{K}_{\mathbf{f}_{*},\mathbf{f}_{*}} - \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{f}} \\ \boldsymbol{\Sigma}_{\mathbf{f}_{*}|\mathbf{y}} &= \mathbf{K}_{\mathbf{f}_{*},\mathbf{f}_{*}} - \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{f}} \\ \boldsymbol{\Sigma}_{\mathbf{f}_{*}|\mathbf{y}} &= \mathbf{K}_{\mathbf{f}_{*},\mathbf{f}_{*}} - \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{f}} \\ \boldsymbol{\Sigma}_{\mathbf{f}_{*}|\mathbf{y}} &= \mathbf{K}_{\mathbf{f}_{*},\mathbf{g}} - \mathbf{K}_{\mathbf{f}_{*},\mathbf{g}} \\ \boldsymbol{\Sigma}_{\mathbf{f}_{*}|\mathbf{y}} &= \mathbf{K}_{\mathbf{f}_{*},\mathbf{g}} - \mathbf{K}_{\mathbf{f}_{*},\mathbf{g}} \\ \boldsymbol{\Sigma}_{\mathbf{f}_{*}|\mathbf{g}} &= \mathbf{K}_{\mathbf{f}_{*},\mathbf{g}} - \mathbf{K}_{\mathbf{f}_{*},\mathbf{g}} \\ \boldsymbol{\Sigma}_{\mathbf{f}_{*}|\mathbf{g}} &= \mathbf{K}_{\mathbf{f}_{*},\mathbf{g}} - \mathbf{K}_{\mathbf{f}_{*},\mathbf{g}} \\ \boldsymbol{\Sigma}_{\mathbf{g}} &= \mathbf{K}_{\mathbf{f}_{*},\mathbf{g}} - \mathbf{K}_{\mathbf{g}_{*}|\mathbf{g}} \\ \boldsymbol{\Sigma}_{\mathbf{g}} &= \mathbf{K}_{\mathbf{g}_{*},\mathbf{g}} - \mathbf{K}_{\mathbf{g}_{*}|\mathbf{g}} \\ \boldsymbol{\Sigma}_{\mathbf{g}} &= \mathbf{K}_{\mathbf{g}_{*},\mathbf{g}} - \mathbf{K}_{\mathbf{g}_{*}|\mathbf{g}} \\ \boldsymbol{\Sigma}_{\mathbf{g}} &= \mathbf{K}_{\mathbf{g}_{*},\mathbf{g}} - \mathbf{K}_{\mathbf{g}_{*}|\mathbf{g}} \\ \boldsymbol{\Sigma}_{\mathbf{g}} &= \mathbf{K}_{\mathbf{g}_{*}|\mathbf{g}} - \mathbf{K}_{\mathbf{g}_{*}|\mathbf{g}} \\ \boldsymbol{\Sigma}_{\mathbf{g}} &= \mathbf{K}_{\mathbf{g}_{*}|\mathbf{g}} - \mathbf{K}_{\mathbf{g}_{*}|\mathbf{g}} - \mathbf{K}_{\mathbf{g}_{*}|\mathbf{g}} \\ \boldsymbol{\Sigma}_{\mathbf{g}} &= \mathbf{K}_{\mathbf{g}_{*}|\mathbf{g}} - \mathbf{K}_{\mathbf{g}_{*}|\mathbf{g}} \\ \boldsymbol{\Sigma}_{\mathbf{g}} &= \mathbf{K}_{\mathbf{g}_{*}|\mathbf{g}} - \mathbf{K}_{\mathbf{g}_{*}|\mathbf{g}} - \mathbf{K}_{\mathbf{g}_{*}|\mathbf{g}} \\ \boldsymbol{\Sigma}_{\mathbf{g}} - \mathbf{K}_{\mathbf{g}_{*$$

 $10 \quad q_{FITC}(f_*|y)$

$$\begin{split} q_{FITC}\left(\mathbf{f}_{*}|\mathbf{y}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{f}_{*}|\mathbf{y}}, \boldsymbol{\Sigma}_{\mathbf{f}_{*}|\mathbf{y}}\right) \\ \mathbf{m}_{\mathbf{f}_{*}|\mathbf{y}} &= \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\left(\sigma_{y}^{2}\mathbf{I} + \mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}] + \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right)^{-1}\mathbf{y} \\ \boldsymbol{\Sigma}_{\mathbf{f}_{*}|\mathbf{y}} &= \mathbf{K}_{\mathbf{f}_{*},\mathbf{f}_{*}} - \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\left(\sigma_{y}^{2}\mathbf{I} + \mathrm{diag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{f}}] + \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right)^{-1}\mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}_{*}} \end{split}$$

p(u)

$$\begin{split} \mathit{p}\left(\mathbf{u}\right) &= \mathcal{N}\left(\mathbf{m_{u}}, \boldsymbol{\Sigma_{u}}\right) \\ \mathbf{m_{u}} &= \mathbf{0} \\ \boldsymbol{\Sigma_{u}} &= \mathbf{K_{u,u}} \end{split}$$

 $q_{PITC}(f|u)$

$$\begin{split} q_{PITC}\left(\mathbf{f}|\mathbf{u}\right) &= \mathcal{N}\left(\mathbf{m_{f|u}}, \boldsymbol{\Sigma_{f|u}}\right) \\ \mathbf{m_{f|u}} &= \mathbf{K_{f,u}} \mathbf{K_{u,u}^{-1}} \mathbf{u} \\ \boldsymbol{\Sigma_{f|u}} &= \mathrm{blockdiag}[\mathbf{K_{f,f}} - \mathbf{K_{f,u}} \mathbf{K_{u,u}^{-1}} \mathbf{K_{u,f}}] \end{split}$$

13 $q_{PITC}(f_*|u)$

$$\begin{split} q_{PITC}\left(\mathbf{f}_{*}|\mathbf{u}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{f}_{*}|\mathbf{u}}, \boldsymbol{\Sigma}_{\mathbf{f}_{*}|\mathbf{u}}\right) \\ \mathbf{m}_{\mathbf{f}_{*}|\mathbf{u}} &= \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}} \mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1} \mathbf{u} \\ \boldsymbol{\Sigma}_{\mathbf{f}_{*}|\mathbf{u}} &= \mathbf{K}_{\mathbf{f}_{*},\mathbf{f}_{*}} - \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}} \mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1} \mathbf{K}_{\mathbf{u},\mathbf{f}_{*}} \end{split}$$

14 $q_{PITC}(f, f_*|u)$

$$\begin{split} q_{PITC}\left(\mathbf{f},\mathbf{f}_{*}|\mathbf{u}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{f},\mathbf{f}_{*}|\mathbf{u}}, \boldsymbol{\Sigma}_{\mathbf{f},\mathbf{f}_{*}|\mathbf{u}}\right) \\ \mathbf{m}_{\mathbf{f},\mathbf{f}_{*}|\mathbf{u}} &= \begin{bmatrix} \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{u} \\ \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{u} \end{bmatrix} \\ \boldsymbol{\Sigma}_{\mathbf{f},\mathbf{f}_{*}|\mathbf{u}} &= \begin{bmatrix} \text{blockdiag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}] & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{\mathbf{f}_{*},\mathbf{f}_{*}} - \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} \end{bmatrix} \end{split}$$

15 $q_{PITC}(f, f_*, u)$

$$\begin{split} q_{PITC}\left(\mathbf{f},\mathbf{f_*},\mathbf{u}\right) &= \mathcal{N}\left(\mathbf{m_{f,f_*,u}}, \boldsymbol{\Sigma_{f,f_*,u}}\right) \\ \mathbf{m_{f,f_*,u}} &= \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \\ \boldsymbol{\Sigma_{f,f_*,u}} &= \begin{bmatrix} \text{blockdiag}[\mathbf{K_{f,f}} - \mathbf{K_{f,u}} \mathbf{K_{u,u}}^{-1} \mathbf{K_{u,f}}] + \mathbf{K_{f,u}} \mathbf{K_{u,u}}^{-1} \mathbf{K_{u,f}} \mathbf{K_{f,u}} \mathbf{K_{u,f_*}} \mathbf{K_{f,u}} \mathbf{K_{u,f_*}} \mathbf{K_{f,u}} \mathbf{K_{u,f_*}} \mathbf{K_{f,u}} \mathbf{K_{u,u}} \mathbf{$$

16 $q_{PITC}(f, f_*)$

$$\begin{split} q_{PITC}\left(\mathbf{f},\mathbf{f_*}\right) &= \mathcal{N}\left(\mathbf{m_{f,f_*}}, \boldsymbol{\Sigma_{f,f_*}}\right) \\ \mathbf{m_{f,f_*}} &= \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} \\ \boldsymbol{\Sigma_{f,f_*}} &= \begin{bmatrix} \text{blockdiag}[\mathbf{K_{f,f}} - \mathbf{K_{f,u}K_{u,u}^{-1}K_{u,f}}] + \mathbf{K_{f,u}K_{u,u}^{-1}K_{u,f}} & \mathbf{K_{f,u}K_{u,u}^{-1}K_{u,f_*}} \\ \mathbf{K_{f_*,u}K_{u,u}^{-1}K_{u,u}K_{u,f}} & \mathbf{K_{f_*,f_*}} \end{bmatrix} \end{split}$$

17 p(y|f)

$$p(\mathbf{y}|\mathbf{f}) = \mathcal{N}\left(\mathbf{m}_{\mathbf{y}|\mathbf{f}}, \mathbf{\Sigma}_{\mathbf{y}|\mathbf{f}}\right)$$
$$\mathbf{m}_{\mathbf{y}|\mathbf{f}} = \mathbf{f}$$
$$\mathbf{\Sigma}_{\mathbf{y}|\mathbf{f}} = \sigma_y^2 \mathbf{I}$$

18 $q_{PITC}(y, f_*, f)$

$$\begin{split} q_{PITC}\left(\mathbf{y},\mathbf{f}_{*},\mathbf{f}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{y},\mathbf{f}_{*},\mathbf{f}},\boldsymbol{\Sigma}_{\mathbf{y},\mathbf{f}_{*},\mathbf{f}}\right) \\ \mathbf{m}_{\mathbf{y},\mathbf{f}_{*},\mathbf{f}} &= \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} \\ \boldsymbol{\Sigma}_{\mathbf{y},\mathbf{f}_{*},\mathbf{f}} &= \begin{bmatrix} \sigma_{y}^{2}\mathbf{I} + \mathrm{blockdiag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}] + \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} & \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} \\ \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{u}} & \mathbf{K}_{\mathbf{f}_{*},\mathbf{f}} & \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} \\ \mathrm{blockdiag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}] + \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} & \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{u}} + \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}} \end{bmatrix} \\ \end{split}$$

19 $q_{PITC}(f_*, f|y)$

$$\begin{split} q_{PITC}\left(\mathbf{f}_{*},\mathbf{f}|\mathbf{y}\right) &= \mathcal{N}\left(\mathbf{m}_{\mathbf{f}_{*},\mathbf{f}|\mathbf{y}},\boldsymbol{\Sigma}_{\mathbf{f}_{*},\mathbf{f}|\mathbf{y}}\right) \\ \mathbf{m}_{\mathbf{f}_{*},\mathbf{f}|\mathbf{y}} &= \begin{bmatrix} \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\left(\sigma_{y}^{2}\mathbf{I} + \mathrm{blockdiag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right)^{-1}\mathbf{y} \\ \left(\mathrm{blockdiag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right) + \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\right)\left(\sigma_{y}^{2}\mathbf{I} + \mathrm{blockdiag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{u}}\right)^{-1}\mathbf{y} \end{bmatrix} \\ \boldsymbol{\Sigma}_{\mathbf{f}_{*},\mathbf{f}|\mathbf{y}} &= \begin{bmatrix} \mathbf{K}_{\mathbf{f}_{*},\mathbf{f}_{*}} - \mathbf{K}_{\mathbf{f}_{*},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{f}}\left(\sigma_{y}^{2}\mathbf{I} + \mathrm{blockdiag}[\mathbf{K}_{\mathbf{f},\mathbf{f}} - \mathbf{K}_{\mathbf{f},\mathbf{u}}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{-1}\mathbf{K}_{\mathbf{u},\mathbf{u}}^{$$

20 $q_{PITC}(f_*|y)$

$$\begin{split} q_{PITC}\left(\mathbf{f_*}|\mathbf{y}\right) &= \mathcal{N}\left(\mathbf{m_{f_*|y}}, \boldsymbol{\Sigma_{f_*|y}}\right) \\ \mathbf{m_{f_*|y}} &= \mathbf{K_{f_*,u}}\mathbf{K_{u,u}^{-1}}\mathbf{K_{u,f}}\left(\sigma_y^2\mathbf{I} + \mathrm{blockdiag}[\mathbf{K_{f,f}} - \mathbf{K_{f,u}}\mathbf{K_{u,u}^{-1}}\mathbf{K_{u,f}}] + \mathbf{K_{f,u}}\mathbf{K_{u,u}^{-1}}\mathbf{K_{u,f}}\right)^{-1}\mathbf{y} \\ \boldsymbol{\Sigma_{f_*|y}} &= \mathbf{K_{f_*,f_*}} - \mathbf{K_{f_*,u}}\mathbf{K_{u,u}^{-1}}\mathbf{K_{u,f}}\left(\sigma_y^2\mathbf{I} + \mathrm{blockdiag}[\mathbf{K_{f,f}} - \mathbf{K_{f,u}}\mathbf{K_{u,u}^{-1}}\mathbf{K_{u,f}}] + \mathbf{K_{f,u}}\mathbf{K_{u,u}^{-1}}\mathbf{K_{u,f}}\right)^{-1}\mathbf{K_{f,u}}\mathbf{K_{u,u}^{-1}}\mathbf{K_{u,f_*}} \end{split}$$