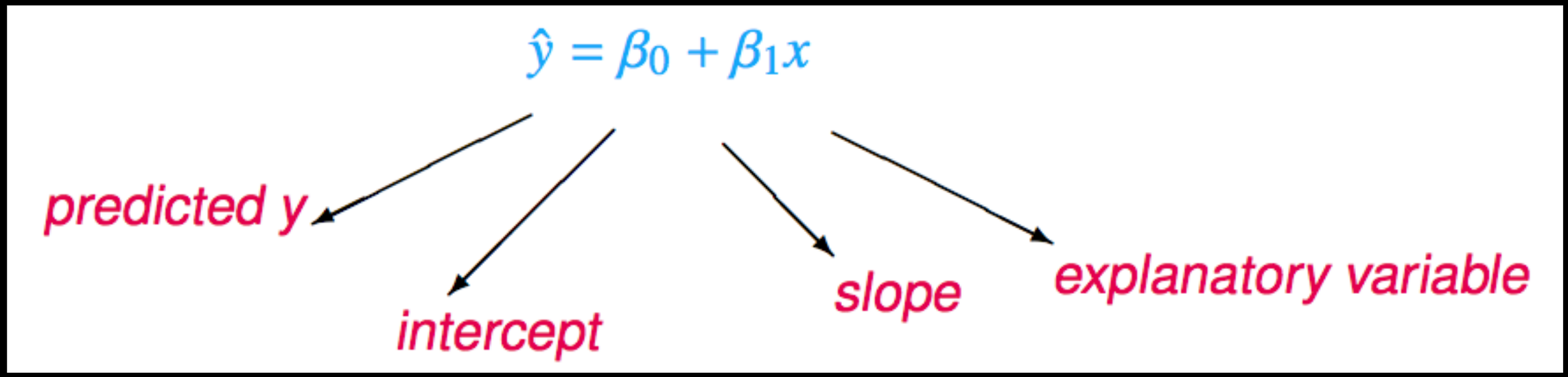
Welcome to Week #12!

Linear Regression → Multiple Linear Regression & Logistic Regression

SLR → MLR & LR

SLR: The least squares line: What are we actually fitting?



Intercept Notation

- Parameter: β_0
- Point estimate: b_0

Slope Notation

- Parameter: β_1
- Point estimate: b_1

SLR: Quantifying the relationship

Correlation (R) describes the strength of the linear association between two variables.

It takes values between -1 (perfect negative) and +1 (perfect positive).

A value of 0 indicates no *linear* association.

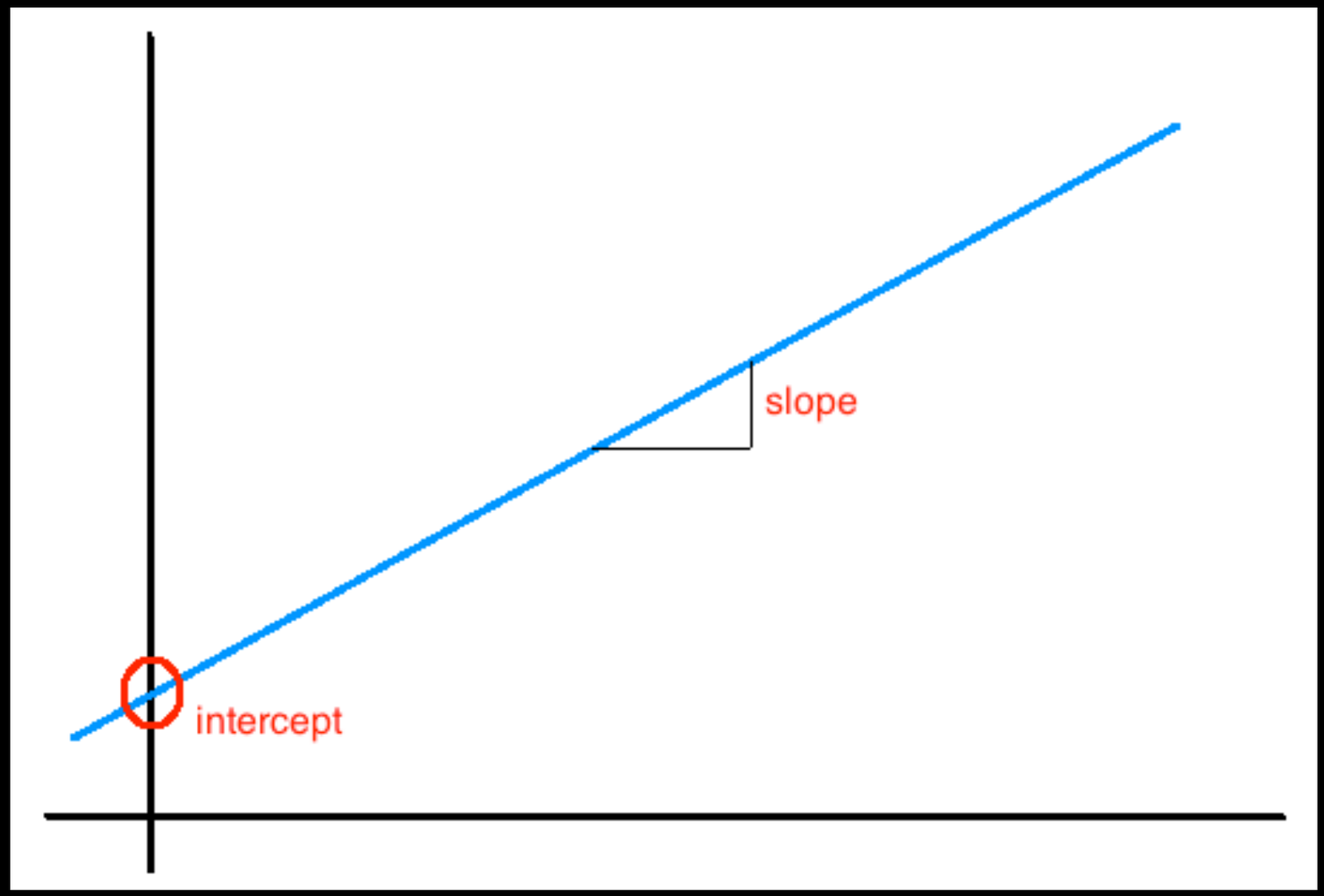
SLR: Interpretation of slope and intercept

Slope

For each unit in x, y is expected to increase / decrease on average by the slope.

Intercept

When x = 0, y is expected to equal the intercept.



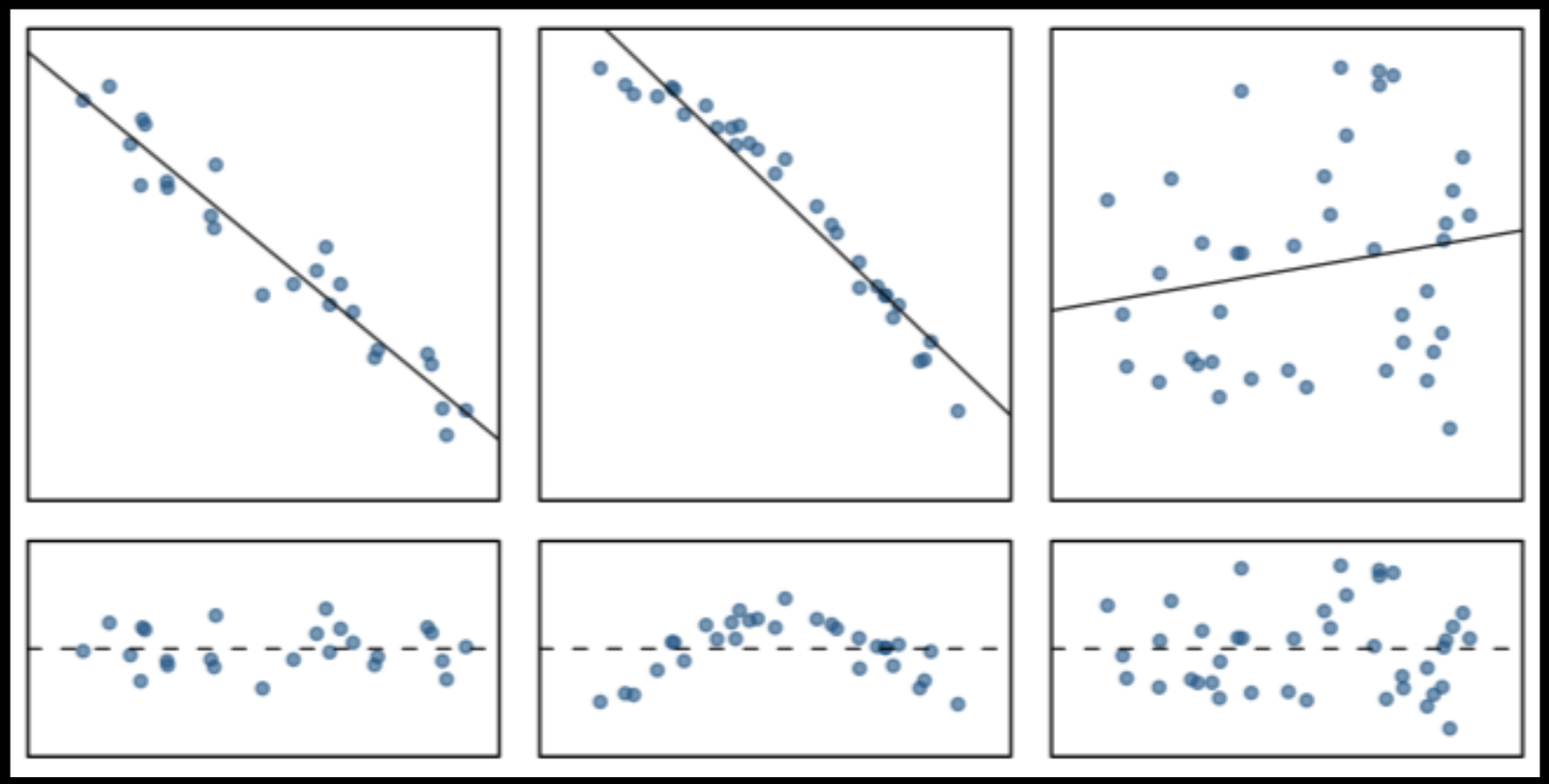
Note: These statements are not causal, unless the study is a randomized controlled experiment.

SLR: Conditions to using Linear Regression on Data

Conditions: (1) Linearity

The relationship between the explanatory and the response variable should be linear.
Methods for fitting a model to non-linear relationships exist, but we will not go into them in detail.

Check using a scatterplot of the data, or a residuals plot.

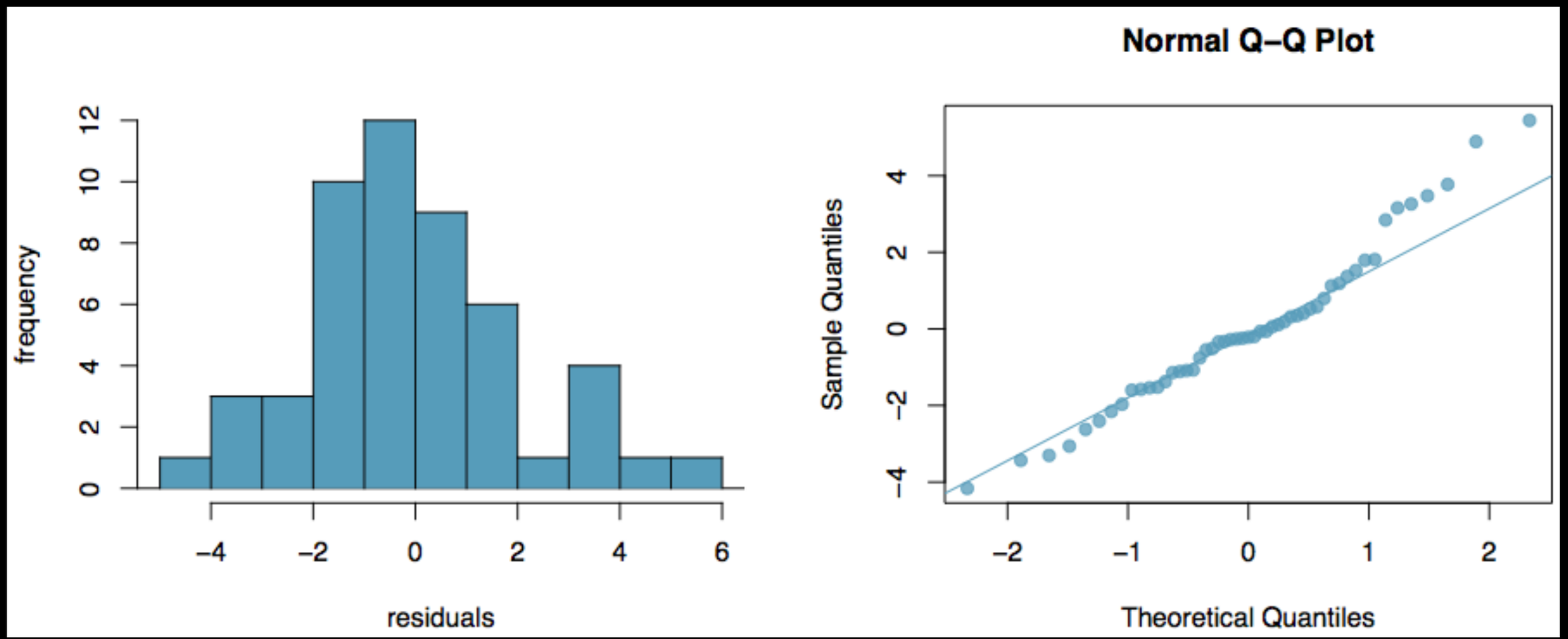


Conditions:
(2) Nearly normal residuals

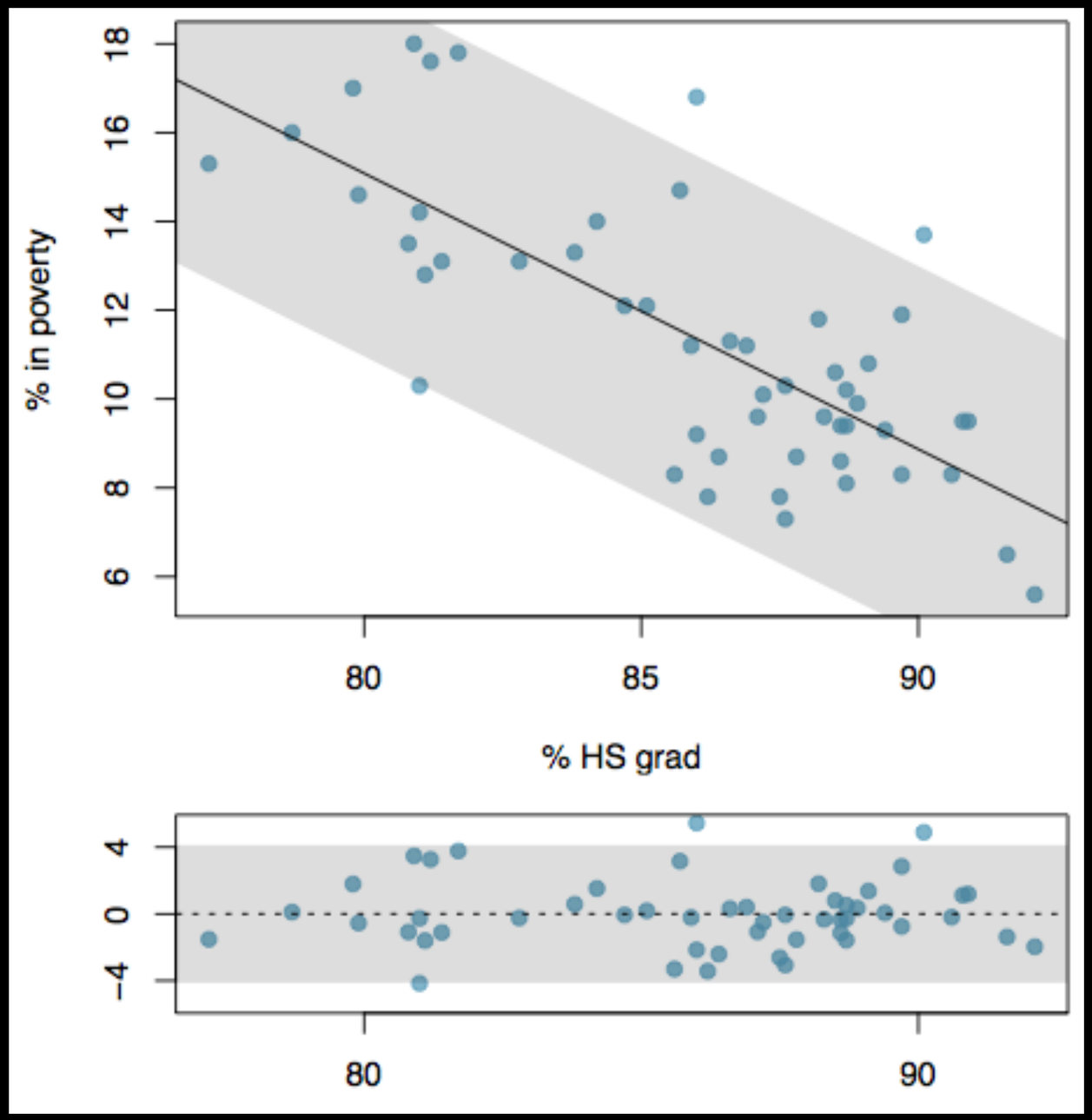
The residuals should be nearly normal.

This condition may not be satisfied when there are unusual observations that don't follow the trend of the rest of the data.

Check using a histogram or normal probability plot of residuals.



Conditions: (3) Constant variability

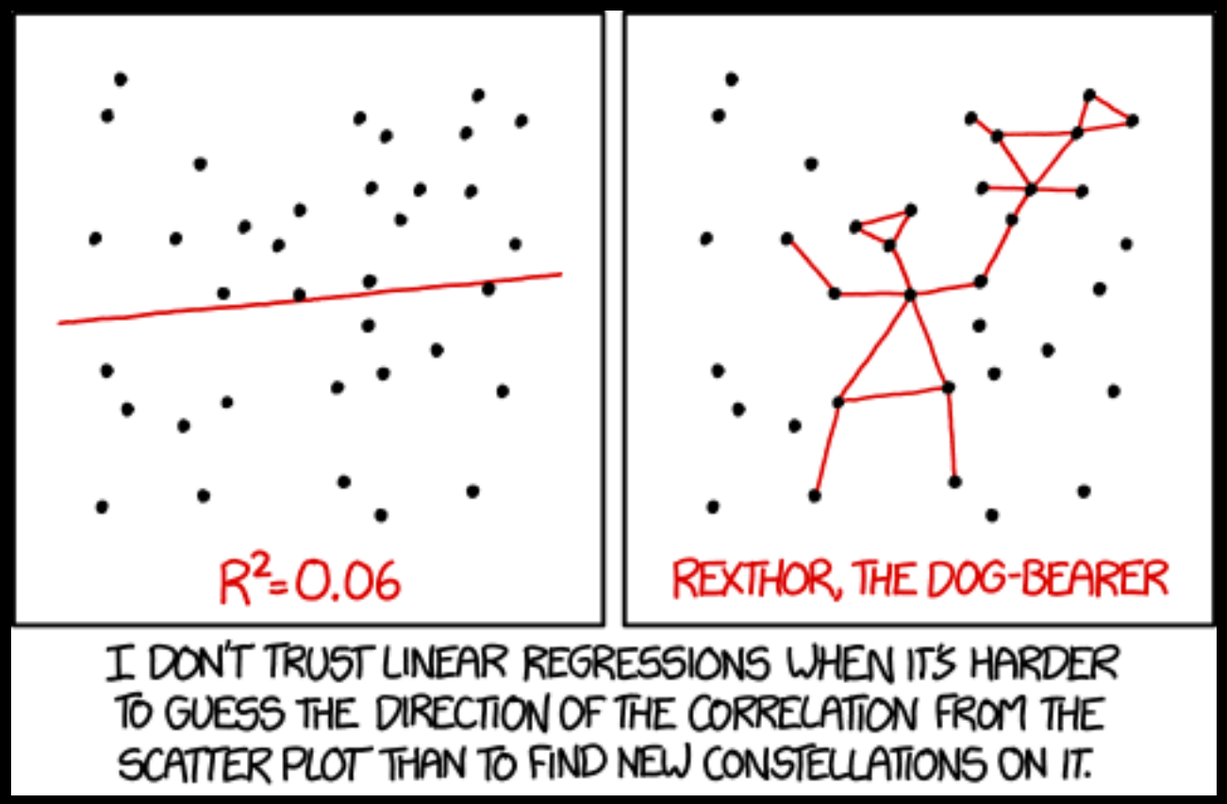


The variability of points around the least squares line should be roughly constant.

This implies that the variability of residuals around the 0 line should be roughly constant as well.

Also called homoscedasticity. Check using a histogram or normal probability plot of residuals.

Interpreting values for R^2 is a matter of practice



www.xkcd.com

SLR: p-values for Linear Regression

What's really going on here? Just the same calculations we've been doing the past few weeks!

p-value < 0.05
so we can reject H_0

```
> summary(myLine)

Call:
lm(formula = BAC ~ Beers, data = BB)

Residuals:
      Min        1Q    Median        3Q       Max
-0.027118 -0.017350  0.001773  0.008623  0.041027

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.012701   0.012638  -1.005    0.332
Beers        0.017964   0.002402   7.480 2.97e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.02044 on 14 degrees of freedom
Multiple R-squared:  0.7998,    Adjusted R-squared:  0.7855
F-statistic: 55.94 on 1 and 14 DF,  p-value: 2.969e-06
```

H_0: There is no relation between Beers and BAC - slope = 0
H_A: There is a relationship between Beers and BAC - slope != 0

p-values for Linear Regression

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$$t = \frac{\hat{\beta}_1 - 0}{\mathrm{SE}(\hat{\beta}_1)},$$

$$\mathrm{SE}(\hat{\beta}_1)^2 = \frac{\sigma^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

how different are the individual variances from the common variance?

H_0: There is no relation between Beers and BAC - slope = 0
H_A: There is a relationship between Beers and BAC - slope != 0

MLR: For multiple linear parameters

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ldots + \beta_n x_n$$

predicted y

intercept

"slopes along different parameters"

one response

many explanatory variables

Visualization in 3D for 2 explanatory variables: http://miabellaai.net/

SLR to MLR: R^2 to R_{adj}^2

The strength of the fit of a linear model is most commonly evaluated using R^2.

R^2 is calculated as the square of the correlation coefficient.

It tells us what percent of variability in the response variable is explained by the model.

$$R^2 = 1 - \frac{\text{variability in residuals}}{\text{variability in the outcome}} = 1 - \frac{Var(e_i)}{Var(y_i)}$$

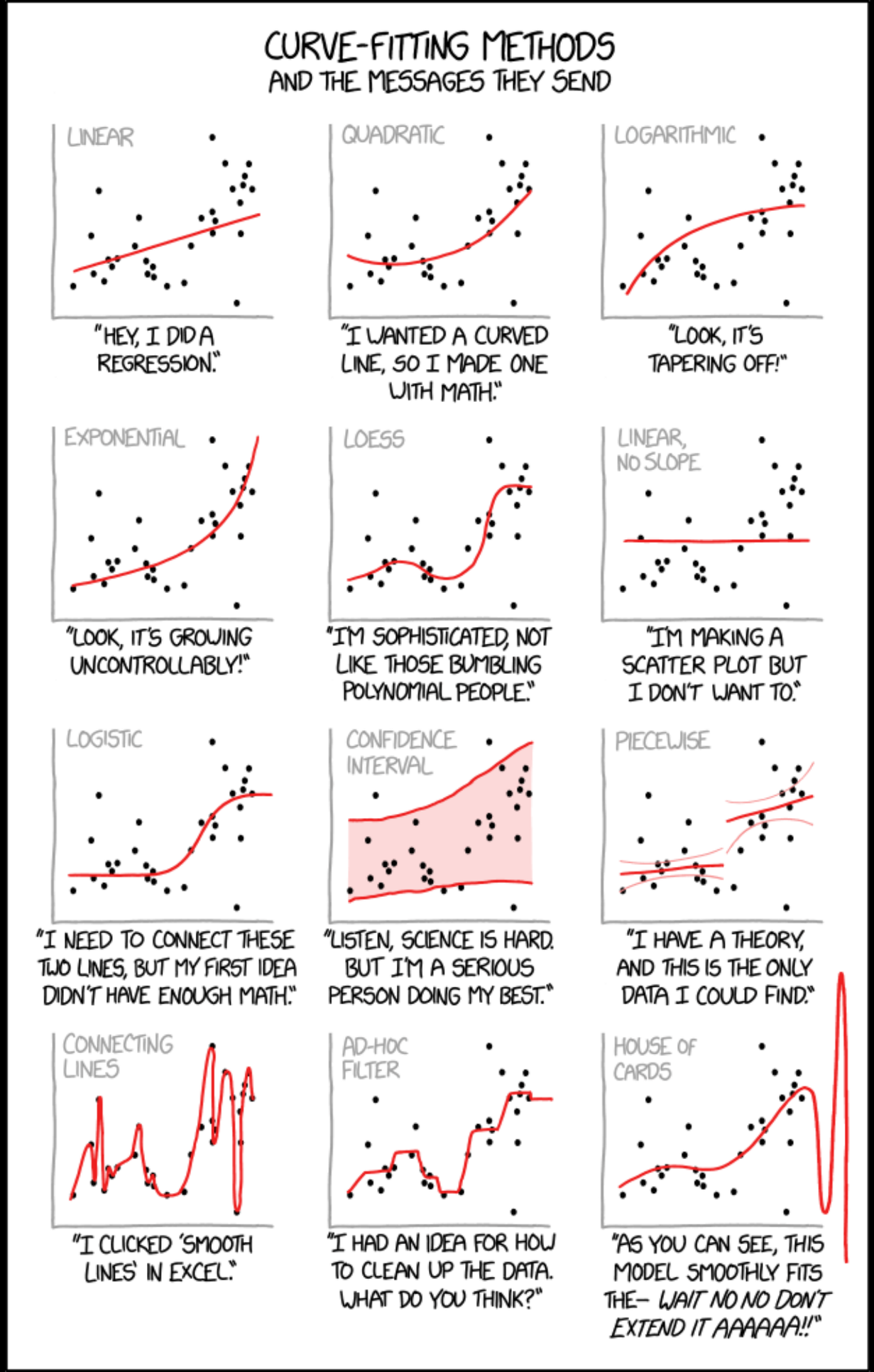
n = # of data points

k = # of parameters in our model

$$R^2_{adj} = 1 - \frac{Var(e_i)/(n-k-1)}{Var(y_i)/(n-1)} = 1 - \frac{Var(e_i)}{Var(y_i)} \times \frac{n-1}{n-k-1}$$

So like R^2, but takes into account the number of degrees of freedom

More parameters (higher k) means worse R_{adj}^2 - adjusts for the fact that we can fit anything if we have a large enough number of parameters!



And in general: we should avoid making things overly complicated

xkcd.com

Enough chit-chat: Lets do an R example!

Intercepts in MLR don't usually make a lot of sense

```
Call:
lm(formula = Snow.Depth ~ Elevation + Min.Temp + Max.Temp, data = snotel2)

Residuals:
    Min      1Q  Median      3Q     Max
-29.508  -7.679  -3.139   9.627  26.394

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -10.506529  99.616286  -0.105   0.9170
Elevation     0.012332   0.006536   1.887   0.0731 .
Min.Temp     -0.504970   2.042614  -0.247   0.8071
Max.Temp     -0.561892   0.673219  -0.835   0.4133
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 13.6 on 21 degrees of freedom
Multiple R-squared:  0.6485,    Adjusted R-squared:  0.5983
F-statistic: 12.91 on 3 and 21 DF,  p-value: 5.328e-05
```

“at 0 feet elevation, 0F minimum and maximum daily temperature the snowfall should be -10.5 inches”

Conditions to use MLR

1. Independence of observations of responses

2. Linearity of *all* variables - linear relationship between response variable and each of the explanatory variables

3. Multicollinearity checked for - does not mean we cannot use MLR, but we should be aware of how predictor/explanatory variables are related when quoting our results

4. Constant variance

5. Normality of Residuals

6. No influential points (outliers with strong leverage)

Lets check in R!

Few notes on Collinearity & Model Selection

- Two predictor variables are said to be collinear when they are correlated, and this collinearity complicates model estimation.
  Remember: Predictors are also called explanatory or independent variables. Ideally, they would be independent of each other.

- We don't like adding predictors that are associated with each other to the model, because often times the addition of such variable brings nothing to the table. Instead, we prefer the simplest best model, i.e. parsimonious model.
  It is OK to consider multiple reasons to select a model but it is dangerous to "shop" for a model across many possible models – a practice which is sometimes called ***data-dredging*** and leads to a high chance of spurious results from a single model that is usually reported based on this type of exploration.

- While it's impossible to avoid collinearity from arising in observational data, experiments are usually designed to prevent correlation among predictors.

vif(our model) =

$$\mathrm{VIF}(\hat{\beta}_j) = \frac{1}{1 - R^2_{X_j|X_{-j}}},$$

Few notes on Collinearity & Model Selection

- Two "general" methods for model selection:
  - ★ Forward: choose lowest p-value/highest R_{adj}^2 parameters first
  - ★ Backward: subtract parameters to minimize/maximize p-value/R_{adj}^2

Example: For a forward model

```
Call:
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Play with this in R!

Some “non-linear” linear models

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ldots + \beta_n x_n$$

predicted y

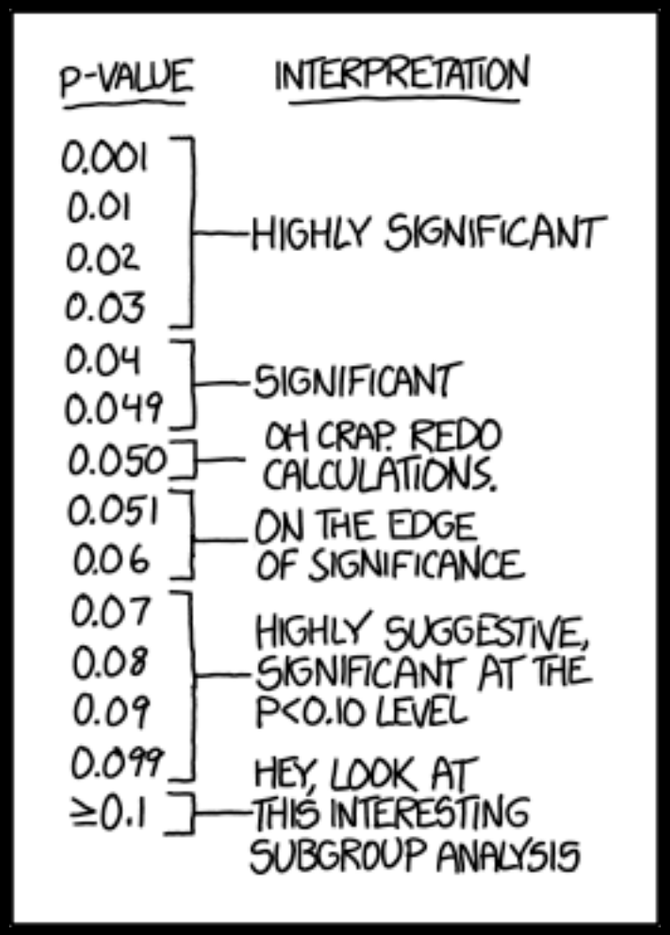
intercept

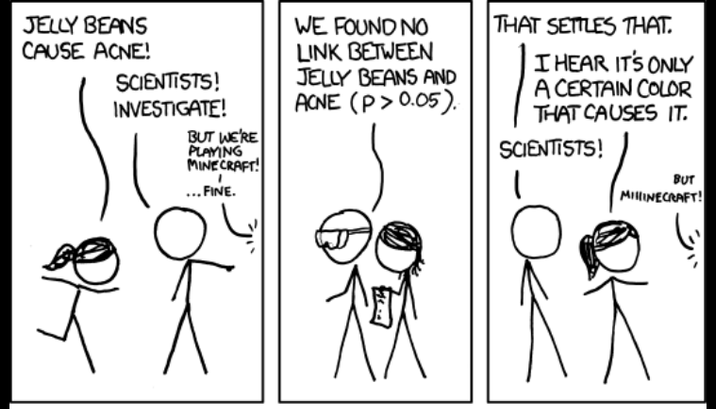
“slopes along different parameters”

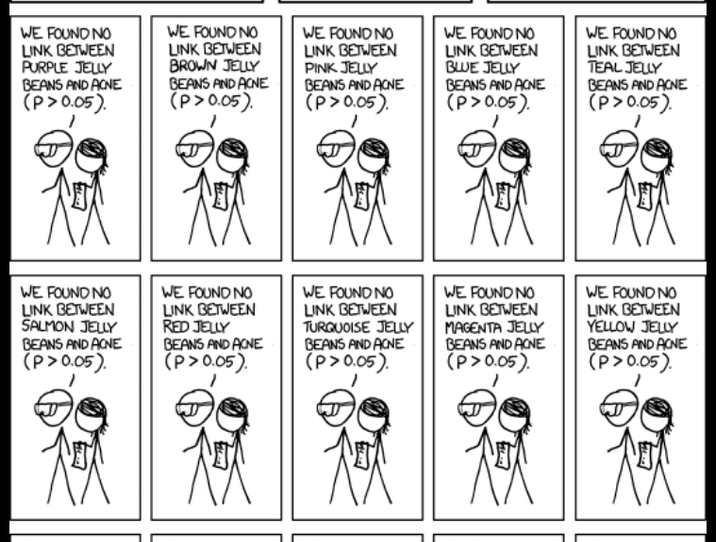
what can x_n be here? For example, I can say $x_4 = x_3^2$?

Issues with p-values

Issues with p-values




JELLY BEANS CAUSE ACNE!
SCIENTISTS! INVESTIGATE!
BUT WE'RE PLAYING MINECRAFT!
... FINE.
WE FOUND NO LINK BETWEEN JELLY BEANS AND ACNE (P > 0.05).
THAT SETTLES THAT.
I HEAR IT'S ONLY A CERTAIN COLOR THAT CAUSES IT.
SCIENTISTS!
BUT MIIIINECRAFT!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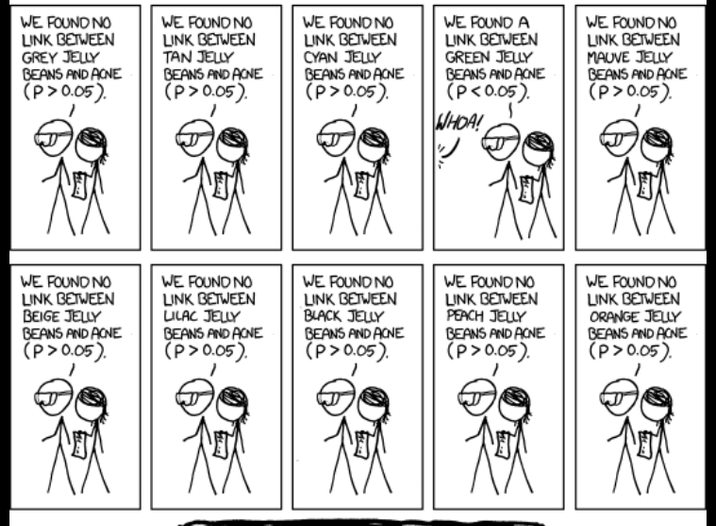
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WHOA!
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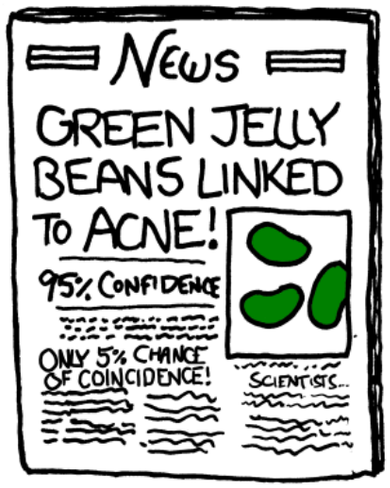
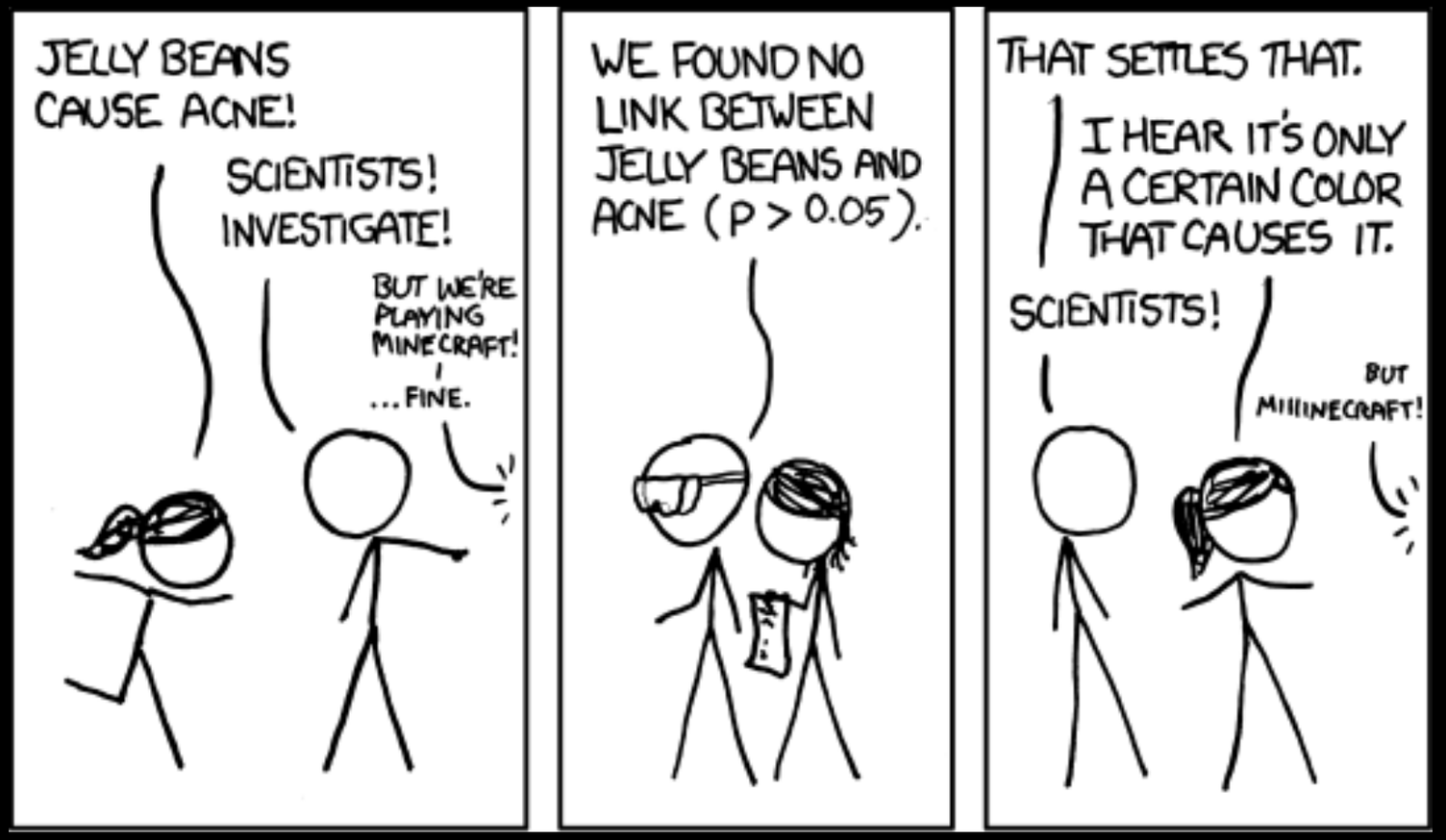
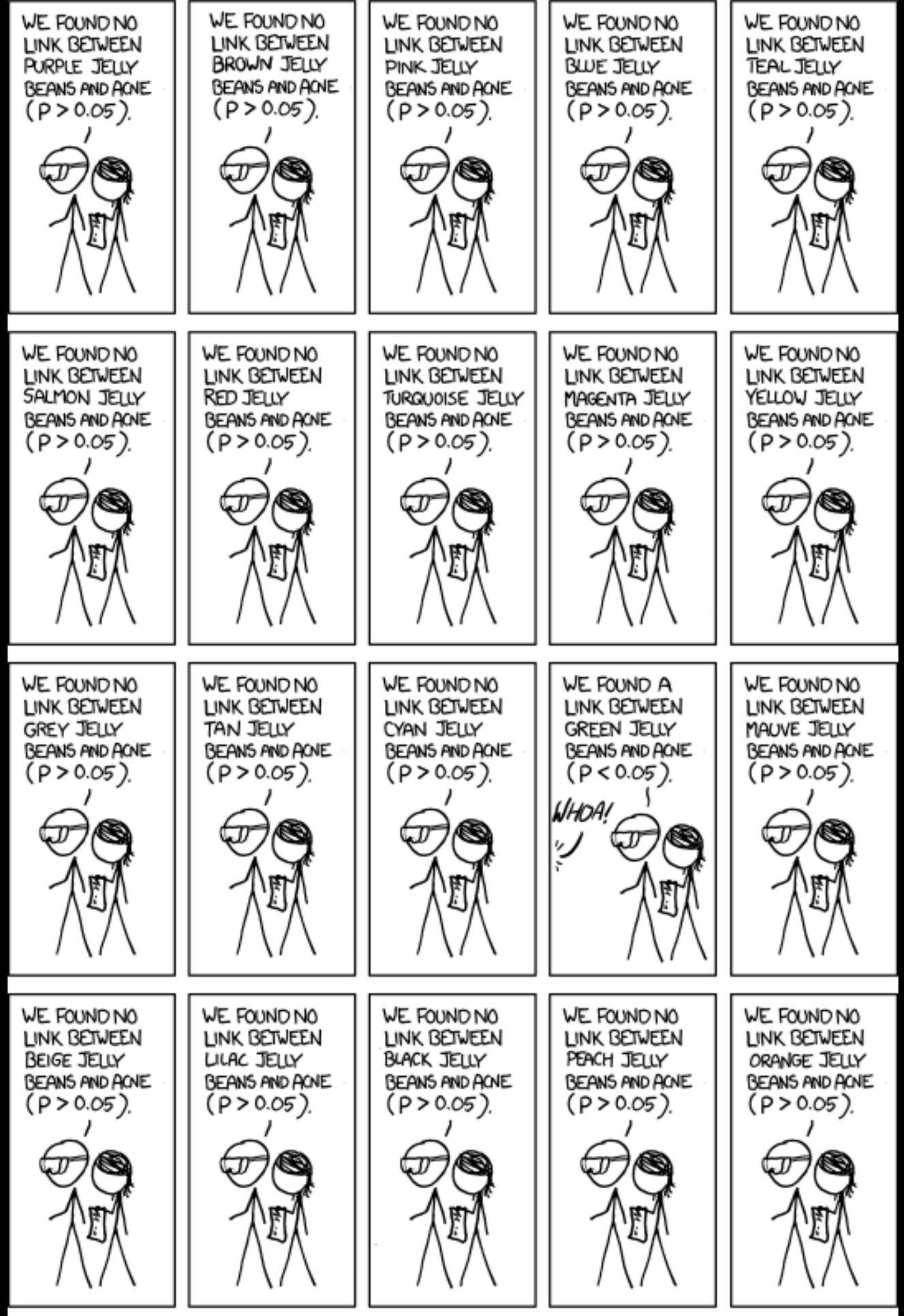

News
GREEN JELLY BEANS LINKED TO ACNE!
95% CONFIDENCE
ONLY 5% CHANCE OF COINCIDENCE!
SCIENTISTS...

Issues with p-values

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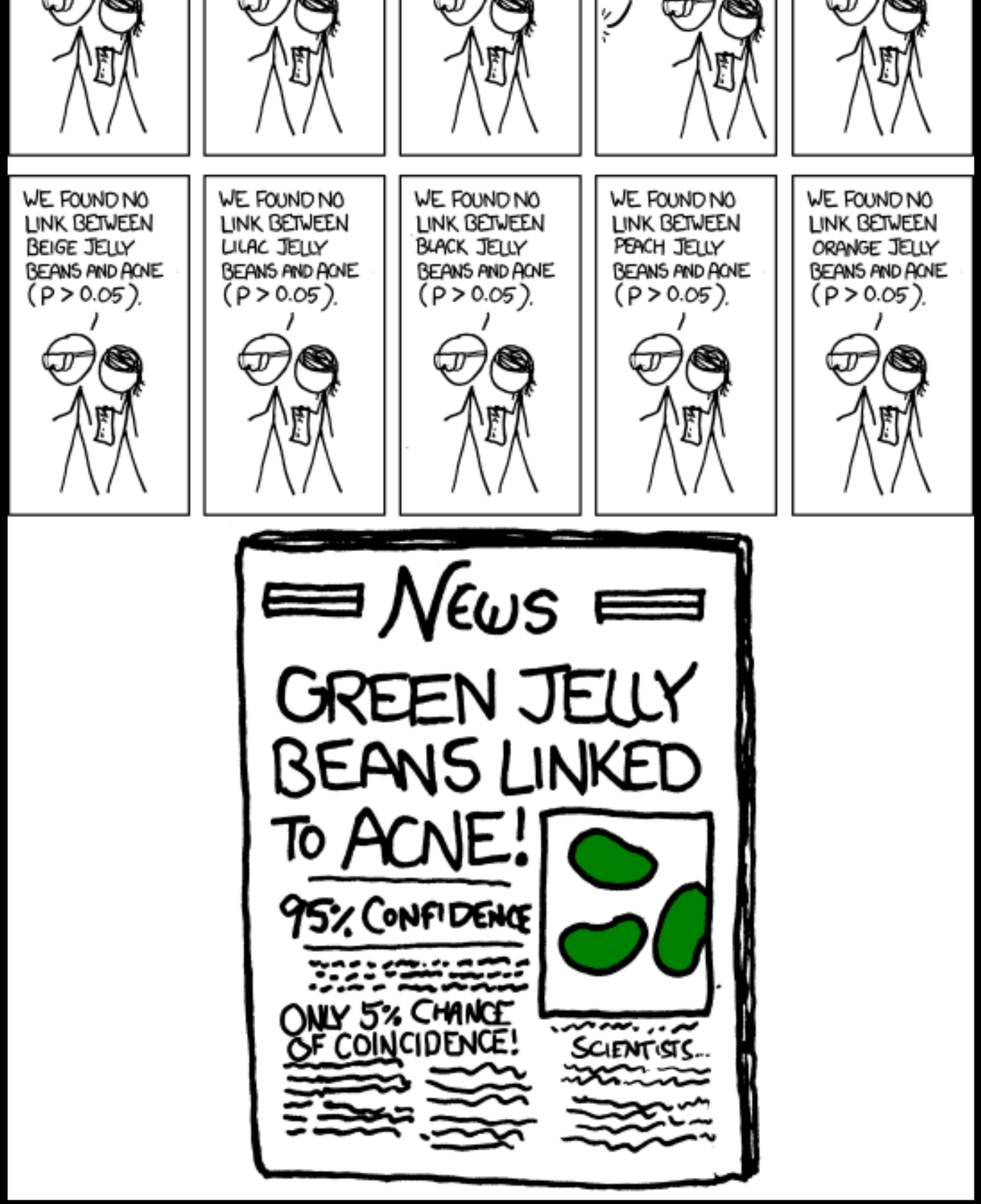
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Issues with p-values

https://www.amstat.org//asa/files/pdfs/P-ValueStatement.pdf

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AMERICAN STATISTICAL ASSOCIATION RELEASES STATEMENT ON STATISTICAL SIGNIFICANCE AND *P*-VALUES

Provides Principles to Improve the Conduct and Interpretation of Quantitative Science

March 7, 2016

The American Statistical Association (ASA) has released a "Statement on Statistical Significance and *P*-Values" with six principles underlying the proper use and interpretation of the *p*-value [http://amstat.tandfonline.com/doi/abs/10.1080/00031305.2016.1154108#.Vt2XlOaE2MN]. The ASA releases this guidance on *p*-values to improve the conduct and interpretation of quantitative science and inform the growing emphasis on reproducibility of science research. The statement also notes that the increased quantification of scientific research and a proliferation of large, complex data sets has expanded the scope for statistics and the importance of appropriately chosen techniques, properly conducted analyses, and correct interpretation.

Good statistical practice is an essential component of good scientific practice, the statement observes, and such practice "emphasizes principles of good study design and conduct, a variety of numerical and graphical summaries of data, understanding of the phenomenon under study, interpretation of results in context, complete reporting and proper logical and quantitative understanding of what data summaries mean."

"The *p*-value was never intended to be a substitute for scientific reasoning," said Ron Wasserstein, the ASA's executive director. "Well-reasoned statistical arguments contain much

Issues with p-values

https://www.amstat.org//asa/files/pdfs/P-ValueStatement.pdf

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"The *p*-value was never intended to be a substitute for scientific reasoning," said Ron Wasserstein, the ASA's executive director. "Well-reasoned statistical arguments contain much more than the value of a single number and whether that number exceeds an arbitrary threshold. The ASA statement is intended to steer research into a 'post $p<0.05$ era.'"

"Over time it appears the *p*-value has become a gatekeeper for whether work is publishable, at least in some fields," said Jessica Utts, ASA president. "This apparent editorial bias leads to the 'file-drawer effect,' in which research with statistically significant outcomes are much more likely to get published, while other work that might well be just as important scientifically is never seen in print. It also leads to practices called by such names as '*p*-hacking' and 'data dredging' that emphasize the search for small *p*-values over other statistical and scientific reasoning."

Issues with p-values

https://www.amstat.org//asa/files/pdfs/P-ValueStatement.pdf

1. *P-values can indicate how incompatible the data are with a specified statistical model.*
2. *P-values do not measure the probability that the studied hypothesis is true, or the probability that the data were produced by random chance alone.*
3. *Scientific conclusions and business or policy decisions should not be based only on whether a p-value passes a specific threshold.*
4. *Proper inference requires full reporting and transparency.*
5. *A p-value, or statistical significance, does not measure the size of an effect or the importance of a result.*
6. *By itself, a p-value does not provide a good measure of evidence regarding a model or hypothesis.* **SCIENCE!**