Welcome to week 14!

A quicky review!

Emphasis on Final will be material listed here & in todays lecture.

Hypothesis Testing Framework (Ch. 4-6)

The general outline of the process:

English

1. Set the hypotheses.
   For a single proportion this will look like:
   H_0: p = null value
   H_A: p < or > or ≠ null value
2. Check assumptions and conditions
3. Calculate a test statistic and a p-value
4. Make a decision, and interpret it in context

Provides a rigorous way to determine the answer with a specific level of confidence.

Math

- If p-value < α, reject H_0,
  there is sufficient evidence for [H_A]

- If p-value > α, do not reject H_0,
  there is not sufficient for evidence for [H_A]

English

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(a) normal, large sample

(b) normal?, small sample

(c) observations & theory

<u>Test Statistics</u>

(a) Z-score -> P(Z)

(b) T-Score -> P(T)

(c) χ^2 -> P(χ^2)

Anatomy of a test statistic

The general form of a test statistic is

$$\frac{\text{point estimate} - \text{null value}}{\text{SE of point estimate}}$$

Only tricks are:

(1) picking what the point and null values are based on our hypotheses

(2) what the form of the standard errors is based on what our underlying distribution looks like (normal, t-distribution, χ^2)

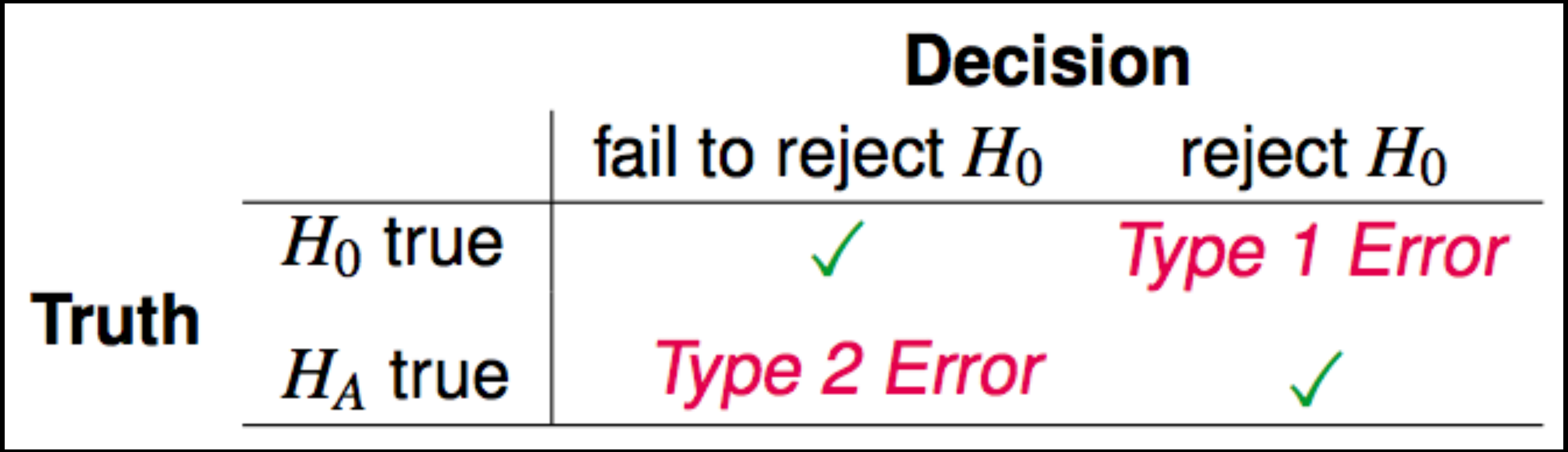
This construction is based on

- identifying the difference between a point estimate and an expected value if the null hypothesis was true, and
- standardizing that difference using the standard error of the point estimate.

These two ideas will help in the construction of an appropriate test statistic for count data.

Decision errors (cont.)

There are two competing hypotheses: the null and the alternative. In a hypothesis test, we make a decision about which might be true, but our choice might be incorrect.



		Decision	
		fail to reject H_0	reject H_0
Truth	H_0 true	✓	*Type 1 Error*
	H_A true	*Type 2 Error*	✓

A Type 1 Error is rejecting the null hypothesis when H_0 is true.
A Type 2 Error is failing to reject the null hypothesis when H_A is true.

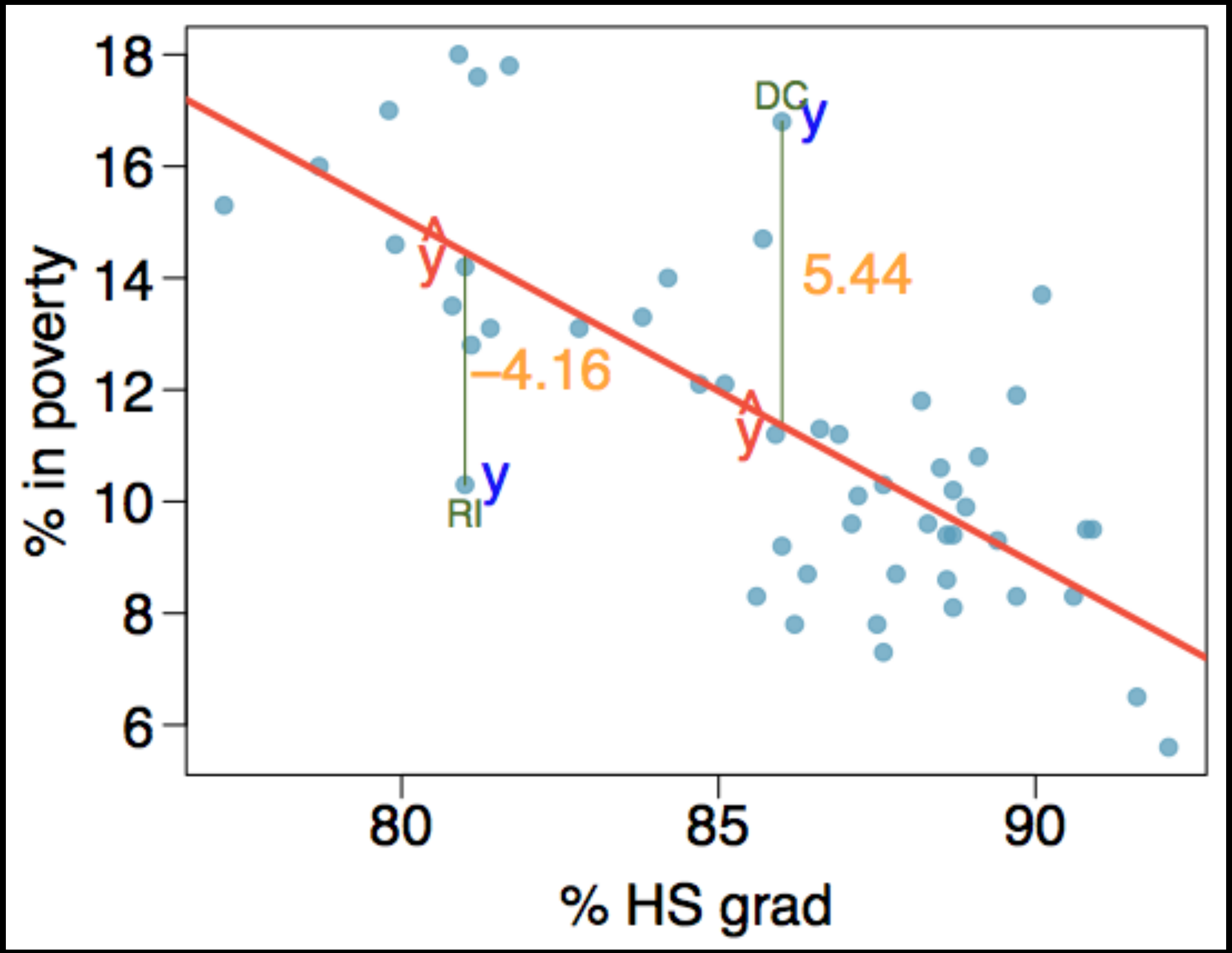
We (almost) never know if H_0 or H_A is true, but we need to consider all possibilities.

Residuals

Residuals are the leftovers from the model fit: Data = Fit + Residual

Aka a residual is the difference between the observed (y_i) and predicted $\hat{y}_i$.

$e_i = y_i - \hat{y}_i$

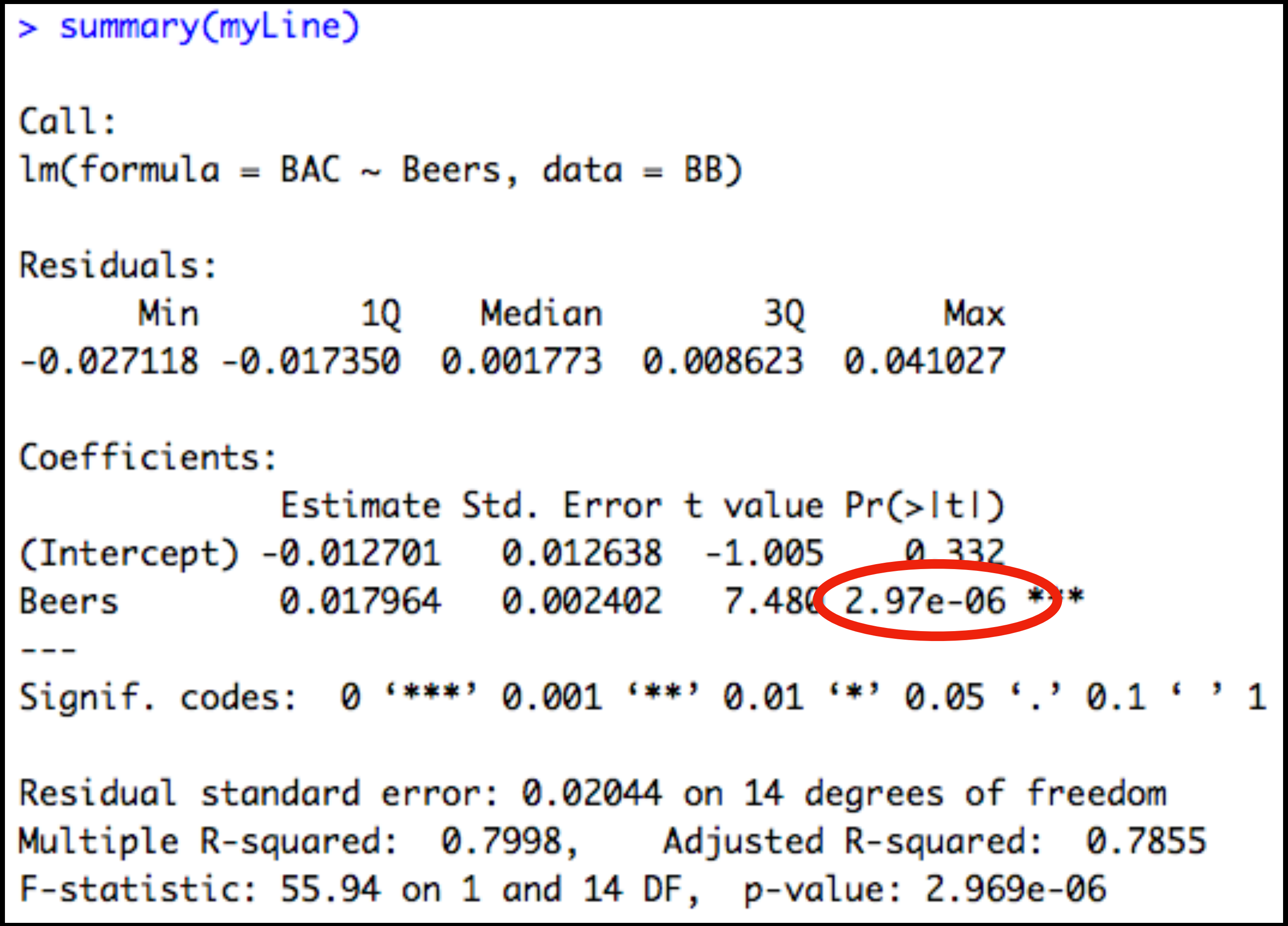


Here is a depiction of the residuals - how far each point is from our fitted line.

p-values for Linear Regression

What's really going on here? Just the same calculations we've been doing the past few weeks!

p-value > 0.05 so we fail to reject H_0



```
> summary(myLine)

Call:
lm(formula = BAC ~ Beers, data = BB)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.027118 -0.017350  0.001773  0.008623  0.041027 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.012701   0.012638  -1.005    0.332    
Beers        0.017964   0.002402   7.480 2.97e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.02044 on 14 degrees of freedom
Multiple R-squared:  0.7998,	Adjusted R-squared:  0.7855 
F-statistic: 55.94 on 1 and 14 DF,  p-value: 2.969e-06
```

H_0: There is no relation between Beers and BAC - slope = 0
H_A: There is a relationship between Beers and BAC - slope != 0

Conditions to use MLR

1. Independence of observations of responses

2. Linearity of *all* variables - linear relationship between response variable and each of the explanatory variables

3. Multicollinearity checked for - does not mean we cannot use MLR, but we should be aware of how predictor/explanatory variables are related when quoting our results

4. Constant variance

5. Normality of Residuals

6. No influential points (outliers with strong leverage)

Logistic Regression: A Morbid Example

Logistic regression is a GLM used to model a binary categorical variable using numerical and categorical predictors.

We assume a binomial distribution produced the outcome variable and we therefore want to model *p* the probability of success for a given set of predictors.

To finish specifying the Logistic model we just need to establish a reasonable link function that connects η to *p*. There are a variety of options but the most commonly used is the logit function.

$$logit(p) = \log\left(\frac{p}{1-p}\right), \text{ for } 0 \le p \le 1$$

Logistic Regression: A Morbid Example

Ok, so what does the totality of our model look like?

$$y_i \sim \text{Binom}(p_i)$$

$$\eta = \beta_0 + \beta_1 x_1 + \cdots + \beta_n x_n$$

$$\text{logit}(p) = \eta$$

From which we back out the probability of survival based on parameters 1-n, for the *i*th observation:

$$p_i = \frac{\exp(\beta_0 + \beta_1 x_{1,i} + \cdots + \beta_n x_{n,i})}{1 + \exp(\beta_0 + \beta_1 x_{1,i} + \cdots + \beta_n x_{n,i})}$$

So far…

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ldots + \beta_n x_n$$

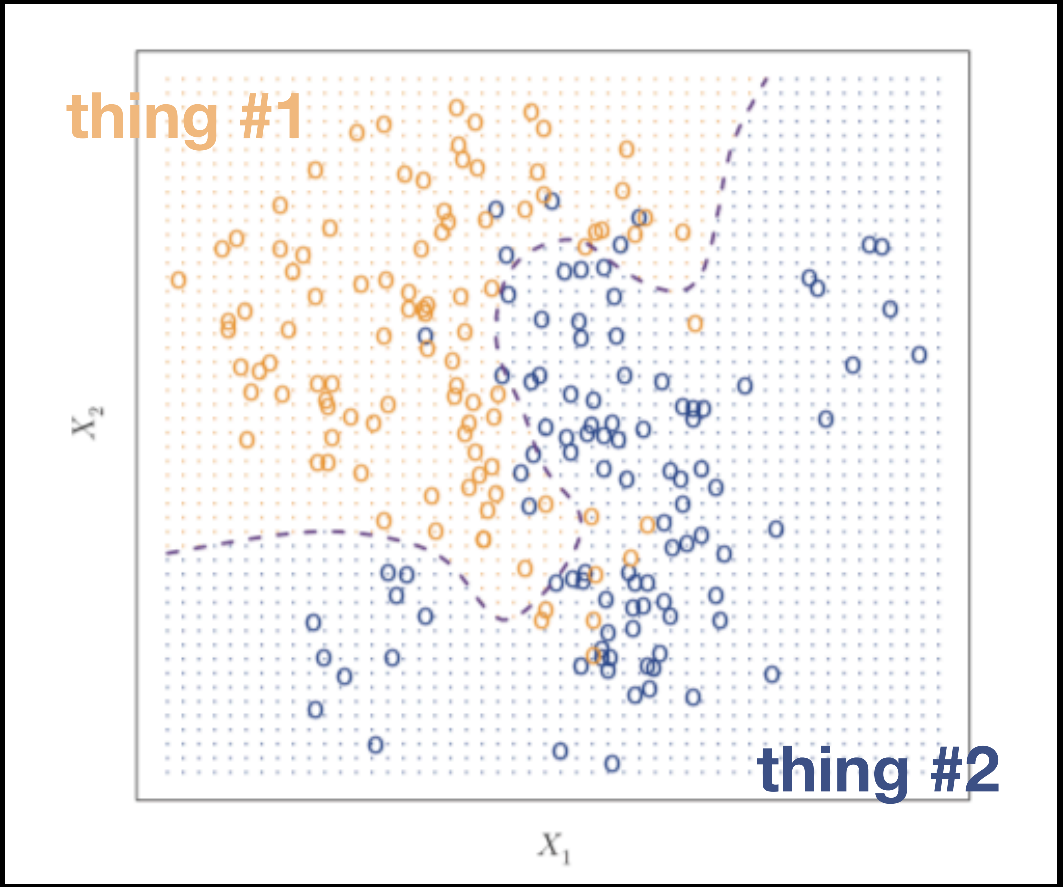
predicted y

intercept

So far we've been saying:
"I have a basic idea of what the functional form of y looks like (a line or a logit) - computer go find the parameters of that functional form.

This is nice because we have some hope of gaining intuition from our models.

Now we classify…



"I want to know where thing #1 and thing #2 live in some 2D space - computer, go figure out the boundary between these two things and let me know"

This is nice because we don't have to assume some model beforehand.

Bias-Variance Trade-Off (First Glance)

$$E\left(y_0 - \hat{f}(x_0)\right)^2 = \mathrm{Var}(\hat{f}(x_0)) + [\mathrm{Bias}(\hat{f}(x_0))]^2 + \mathrm{Var}(\epsilon)$$

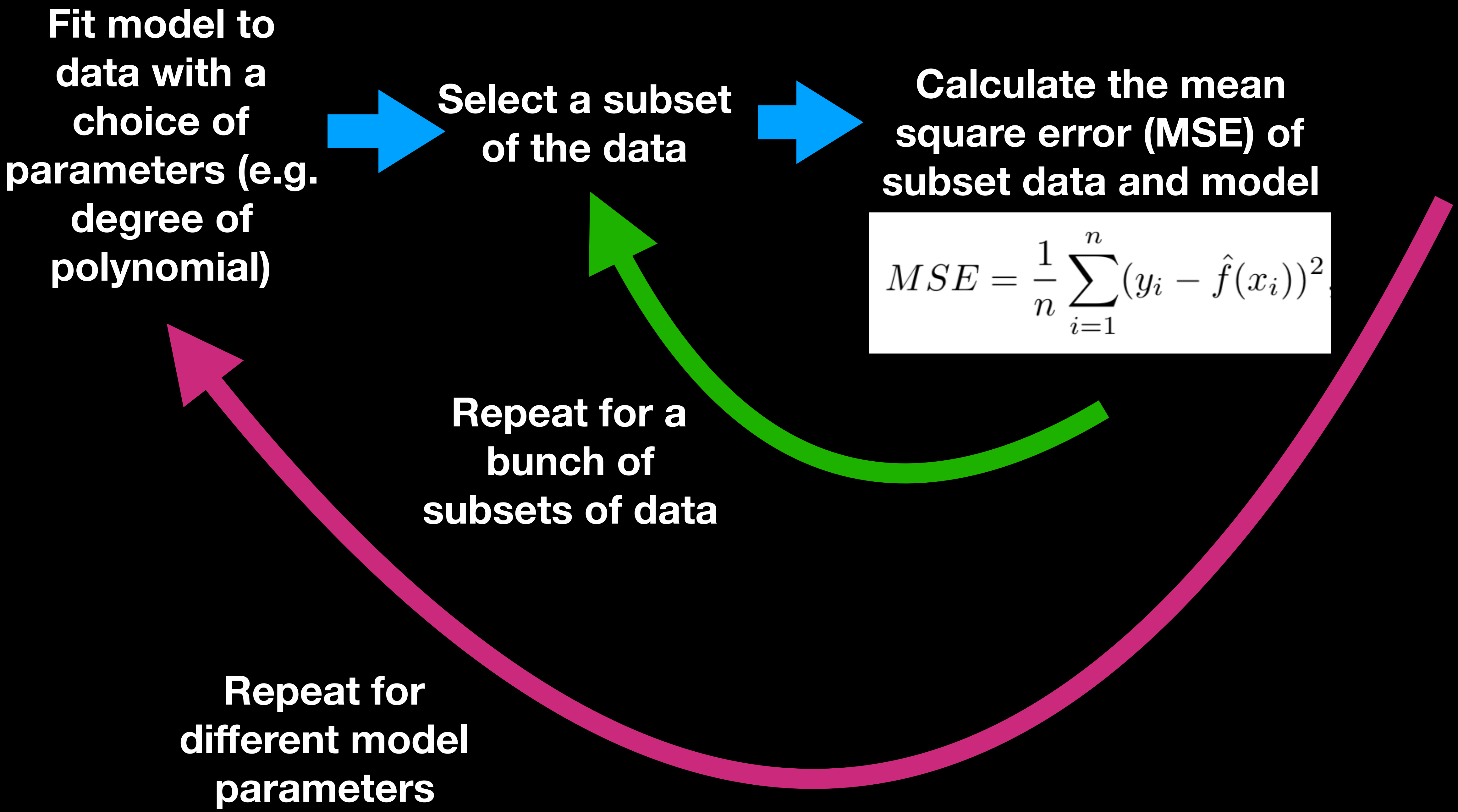
mean square error if we kept estimating response variable y by our fitted function of our explanatory variables, f(x) with different sample datasets at point x_0

how much our function, f, changes if we use a different random sample (variance)

Inherent error (bias) in the fact that any model is only an approximation to reality

Inherent error in our measurements

Cross-Validation Methods



Fit model to data with a choice of parameters (e.g. degree of polynomial)

Select a subset of the data

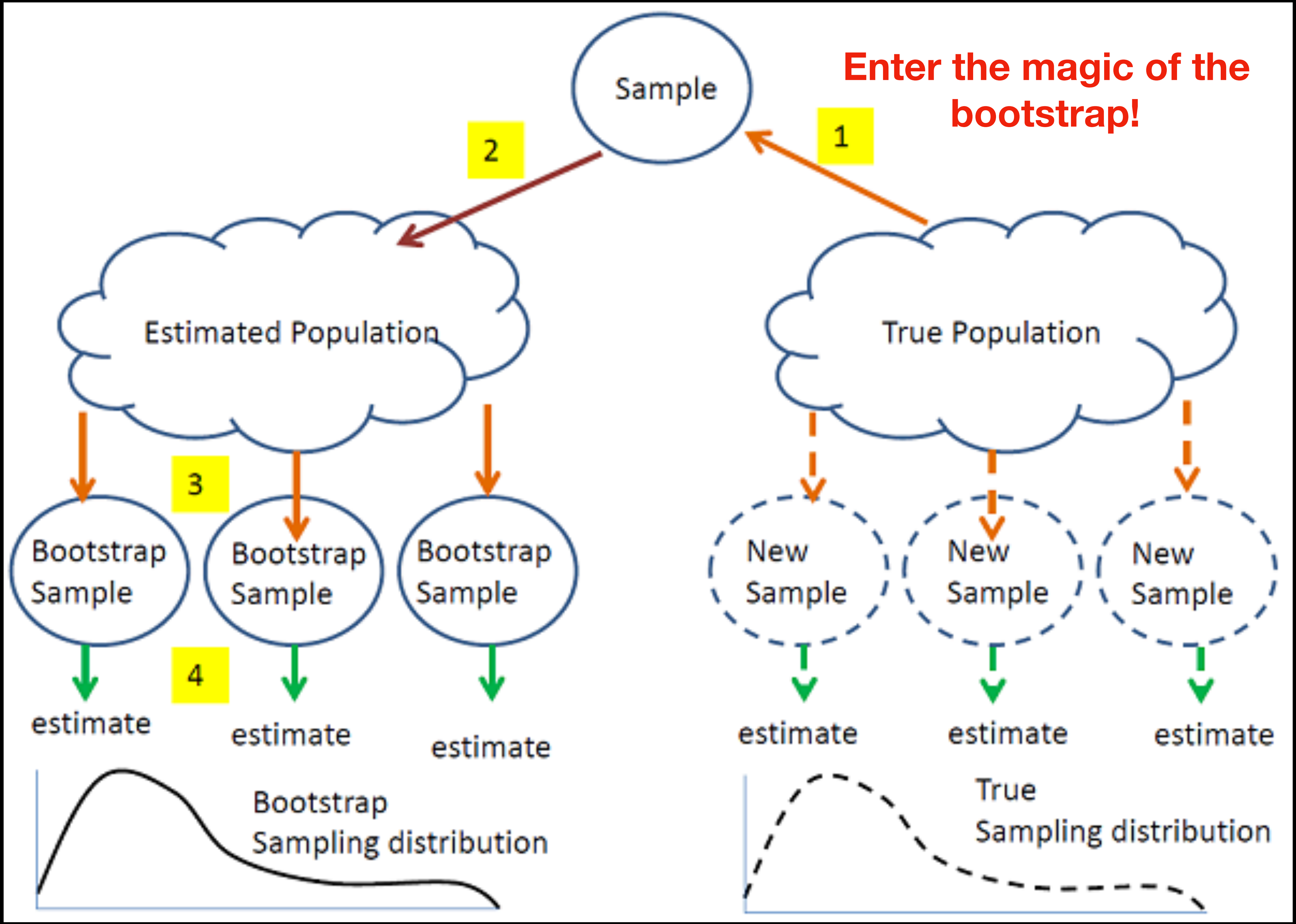
Calculate the mean square error (MSE) of subset data and model

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{f}(x_i))^2$$

Repeat for a bunch of subsets of data

Repeat for different model parameters

Bootstrapping



Distribution of means, proportions, etc

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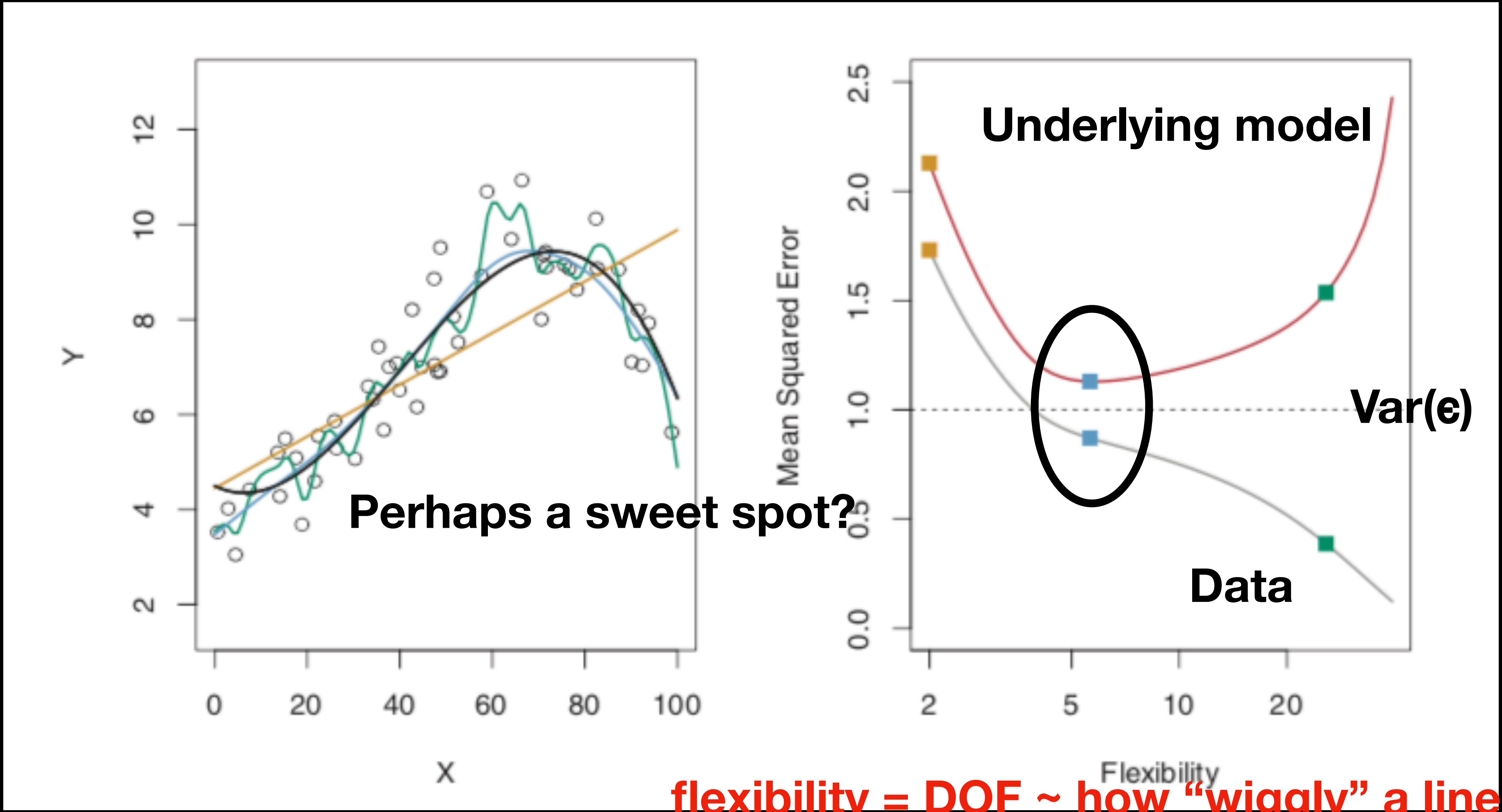

flexibility = DOF ~ how "wiggly" a line is

— Actual underlying function - y
o Simulated data with added error (ϵ)
— Linear fit
— Low "flexibility" smooth spline
— High "flexibility" smooth spline

fits data well, but underlying model badly

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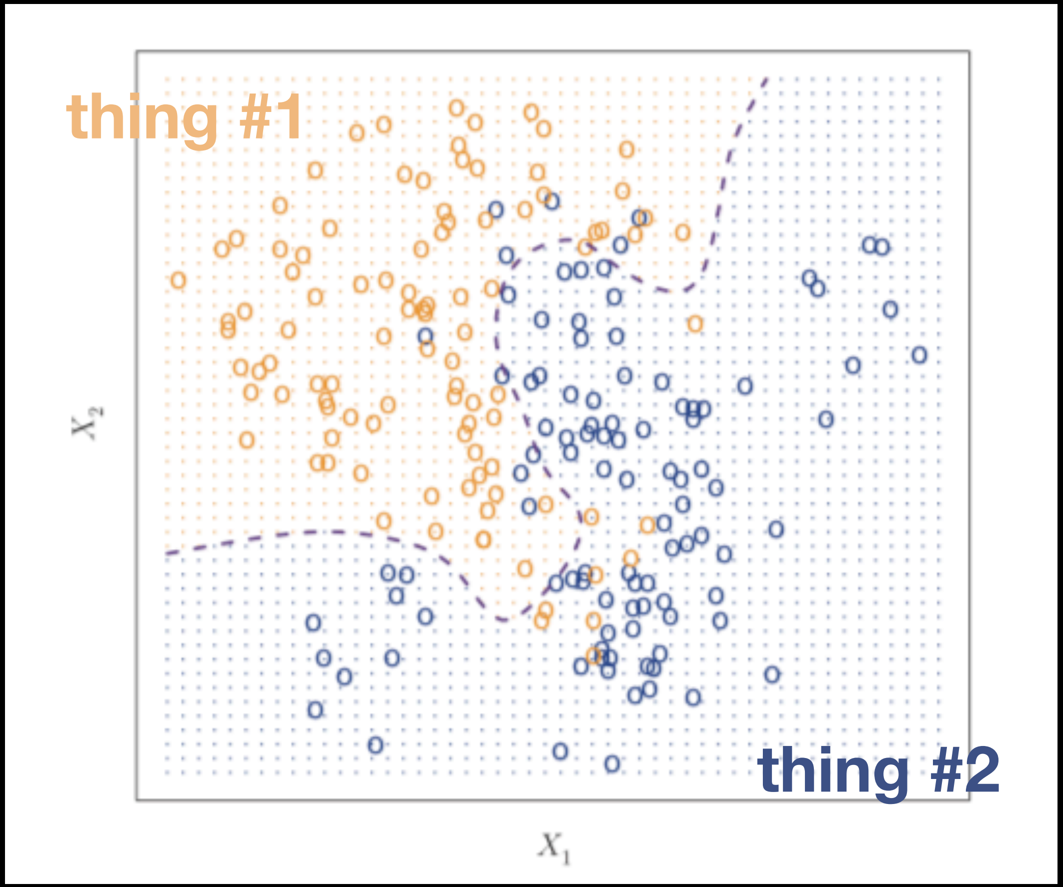

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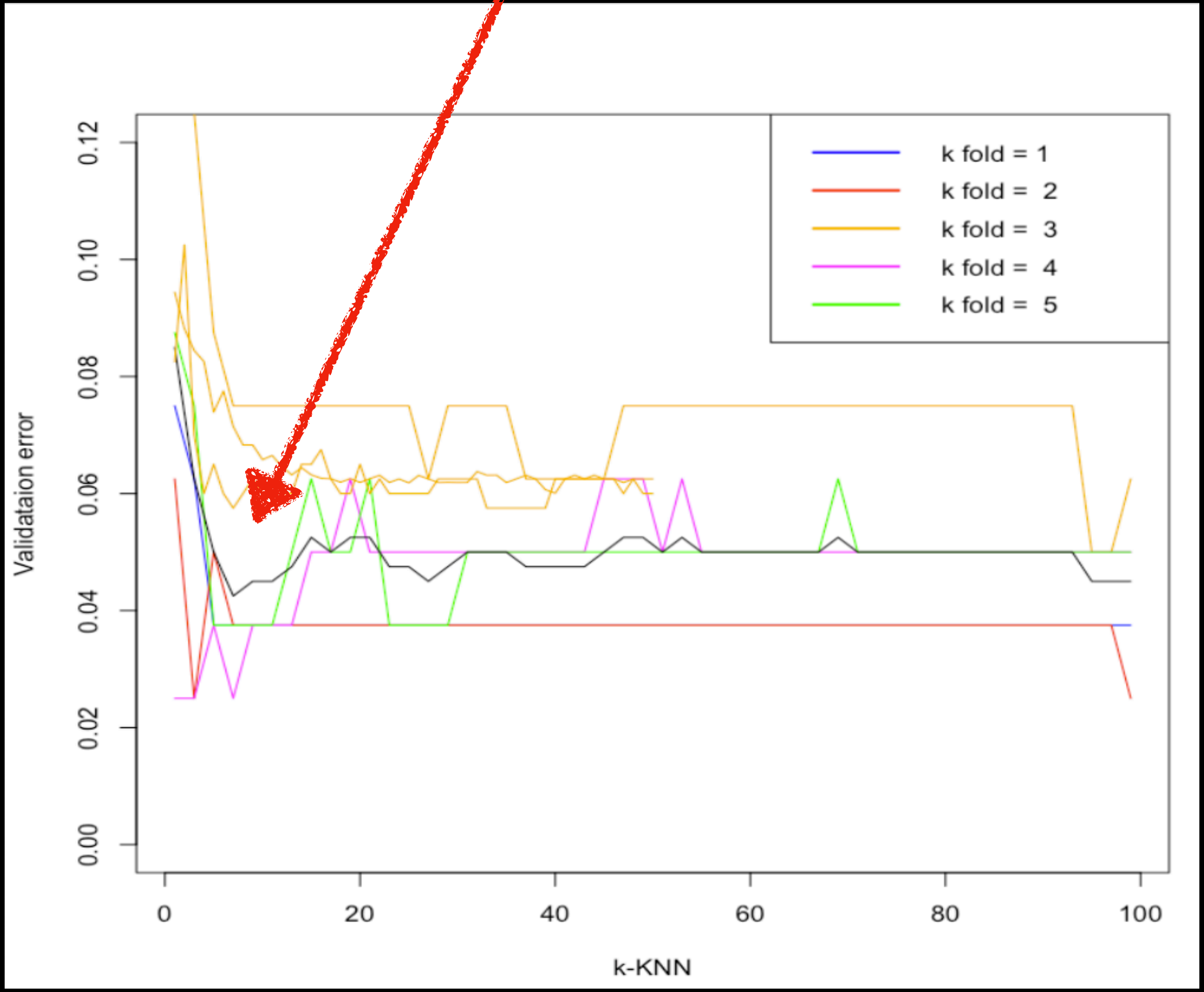
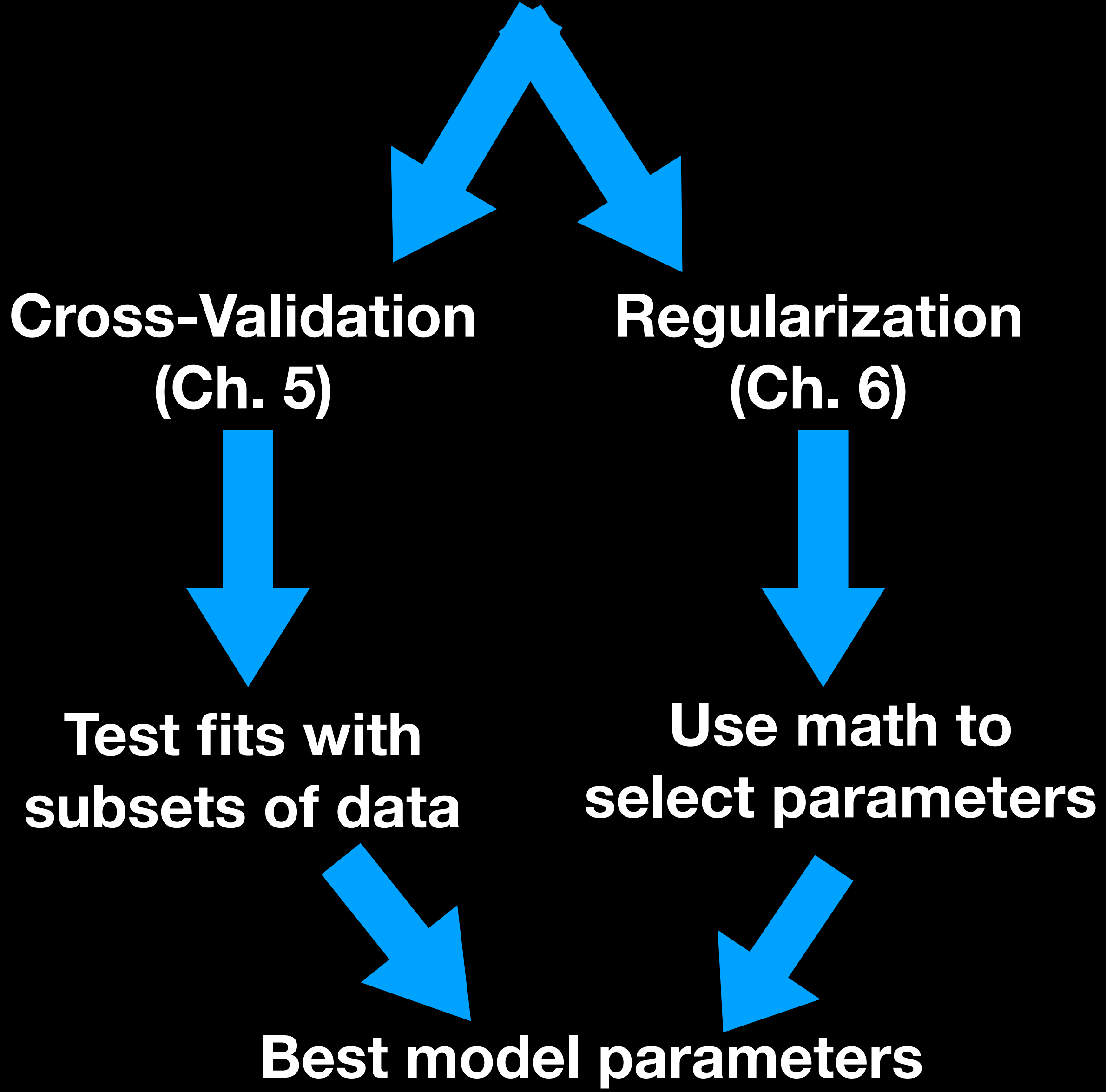


But we are only able to calculate the test error because we know the background distribution.



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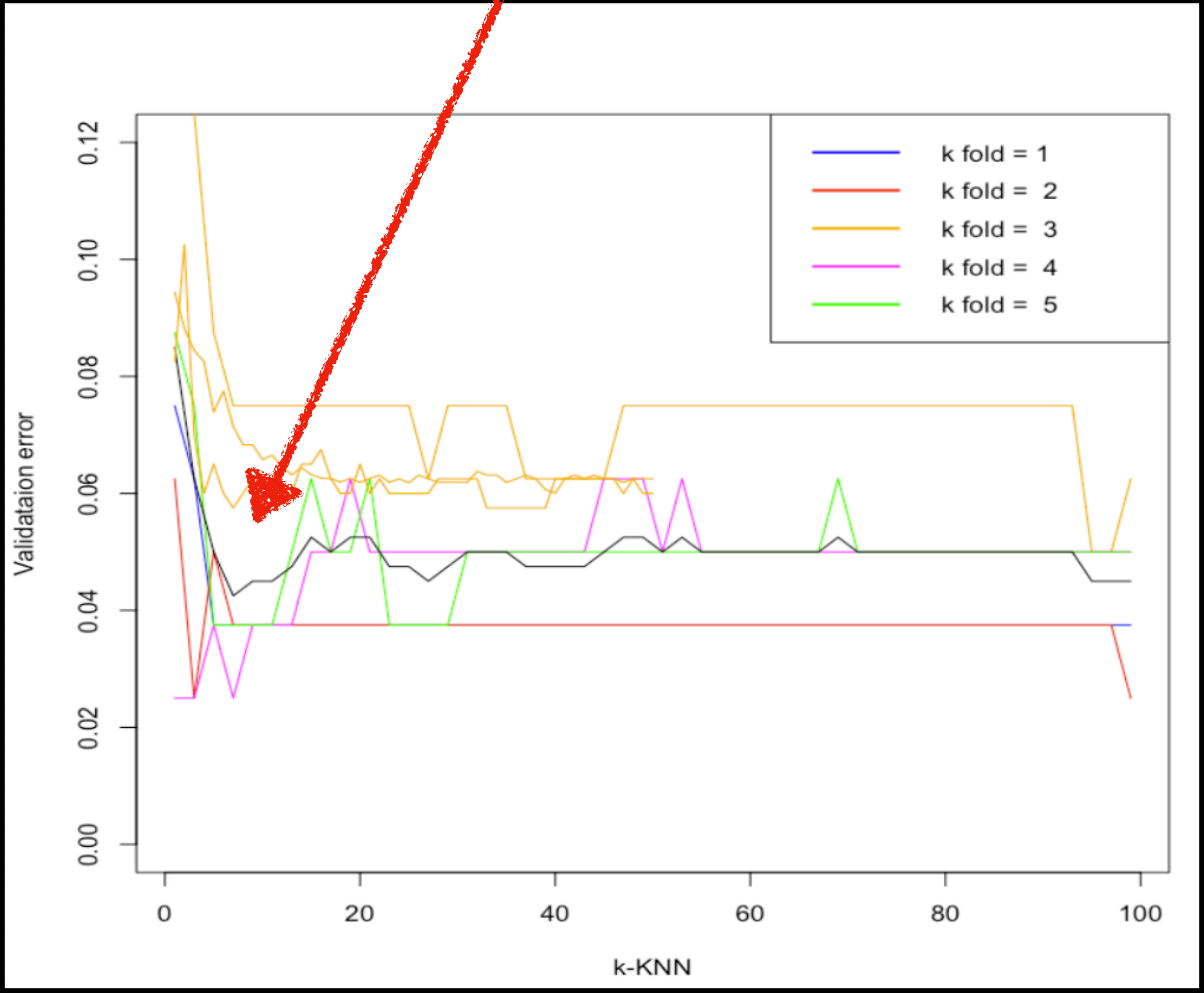

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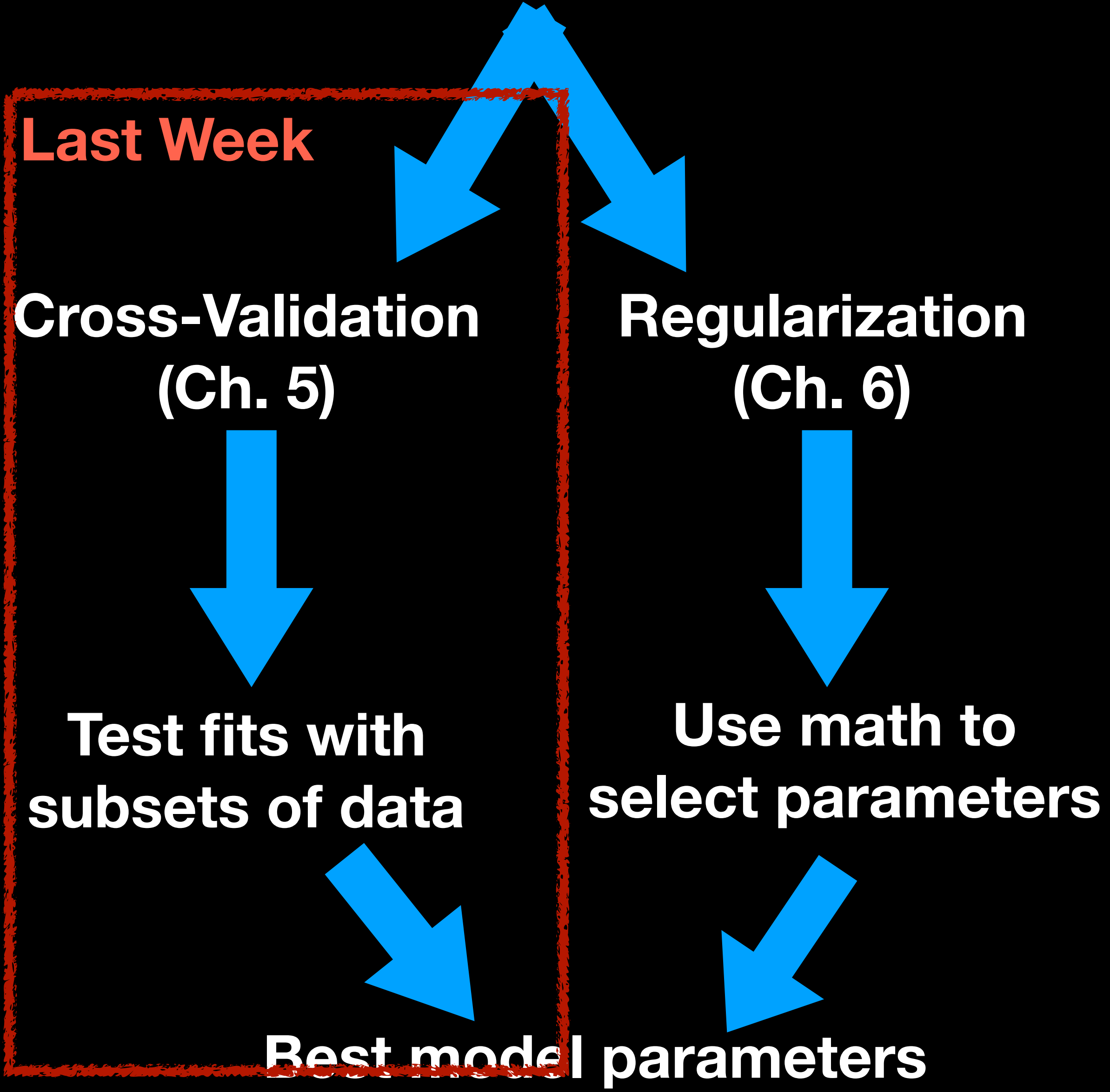

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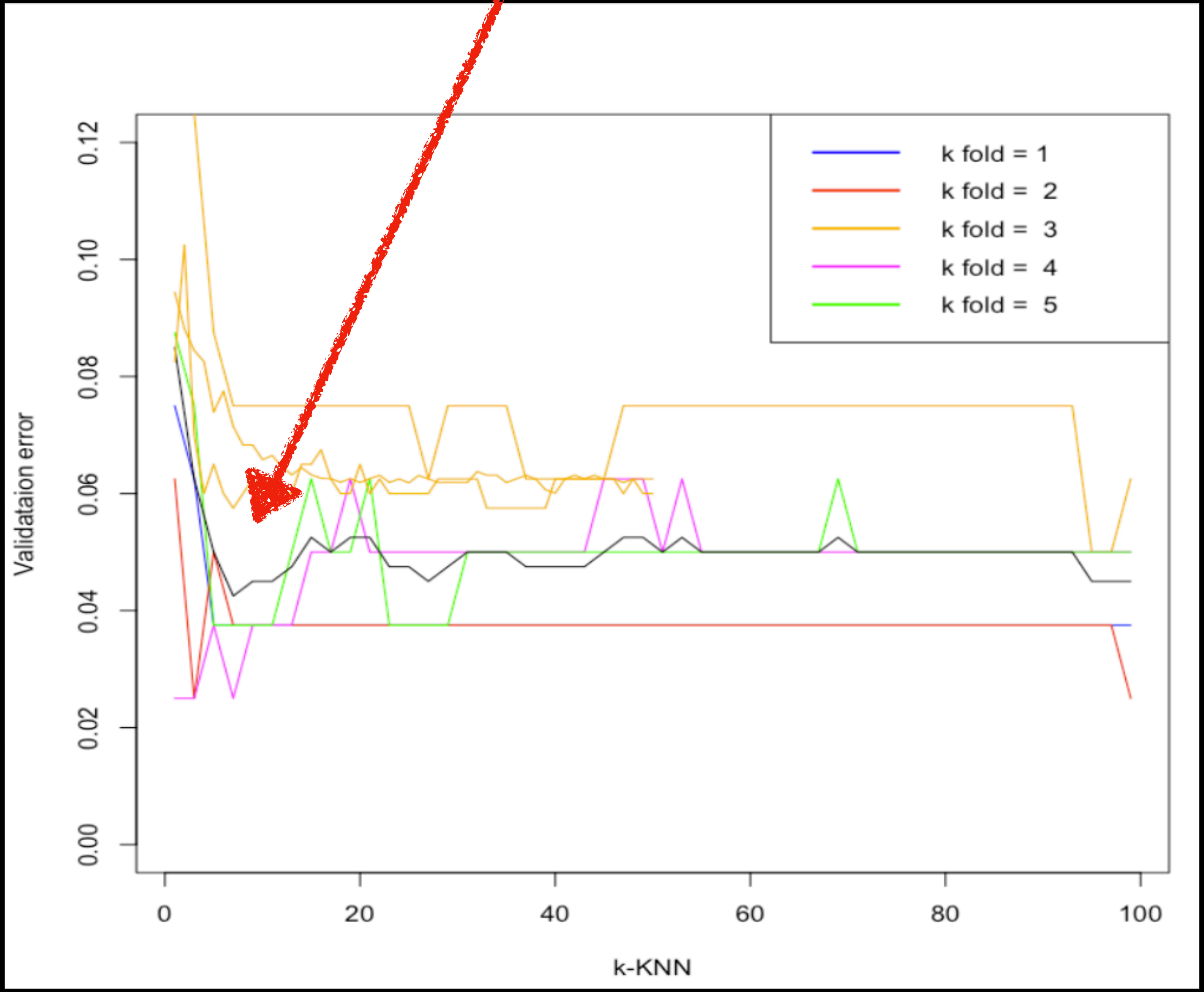

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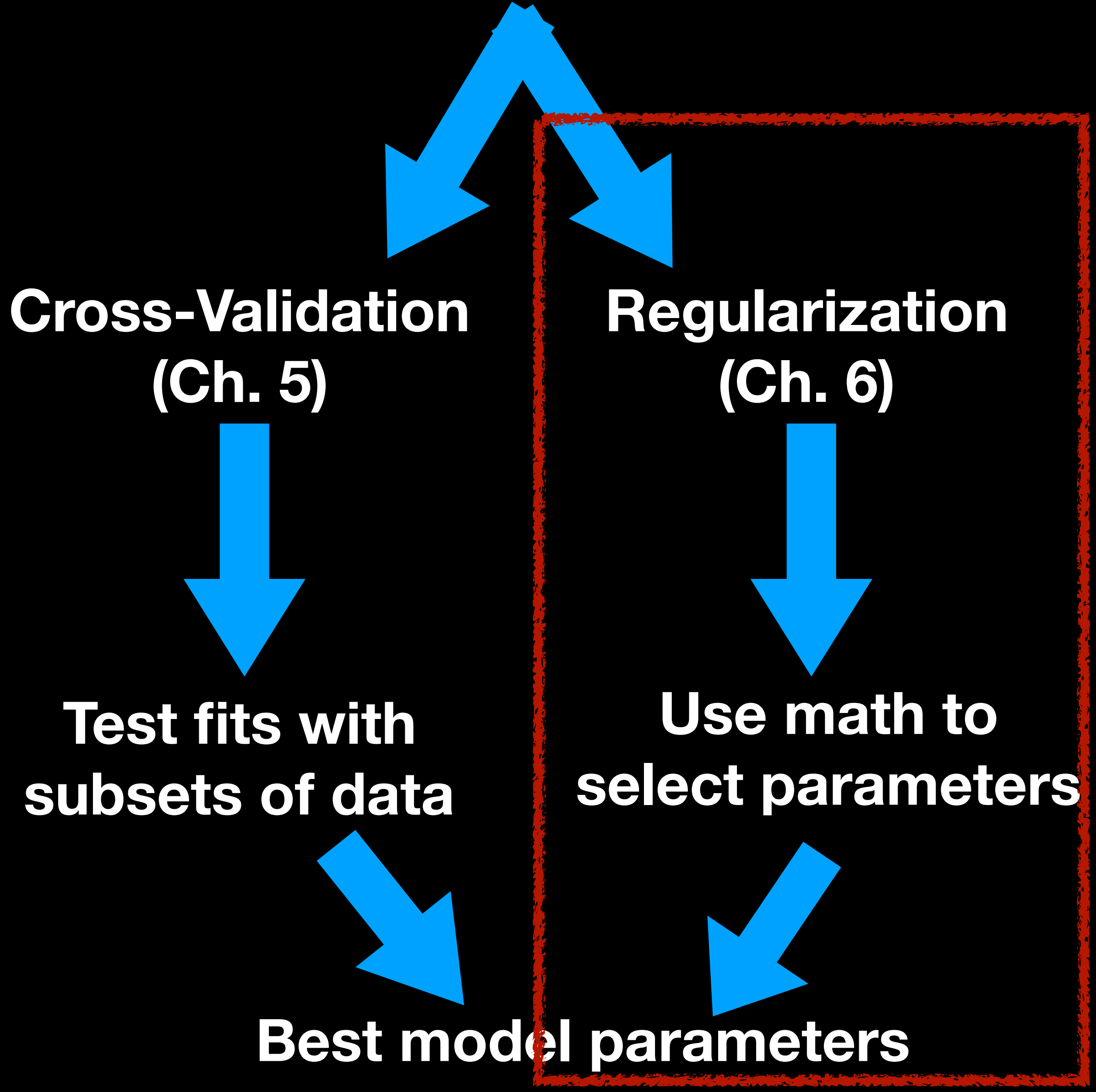

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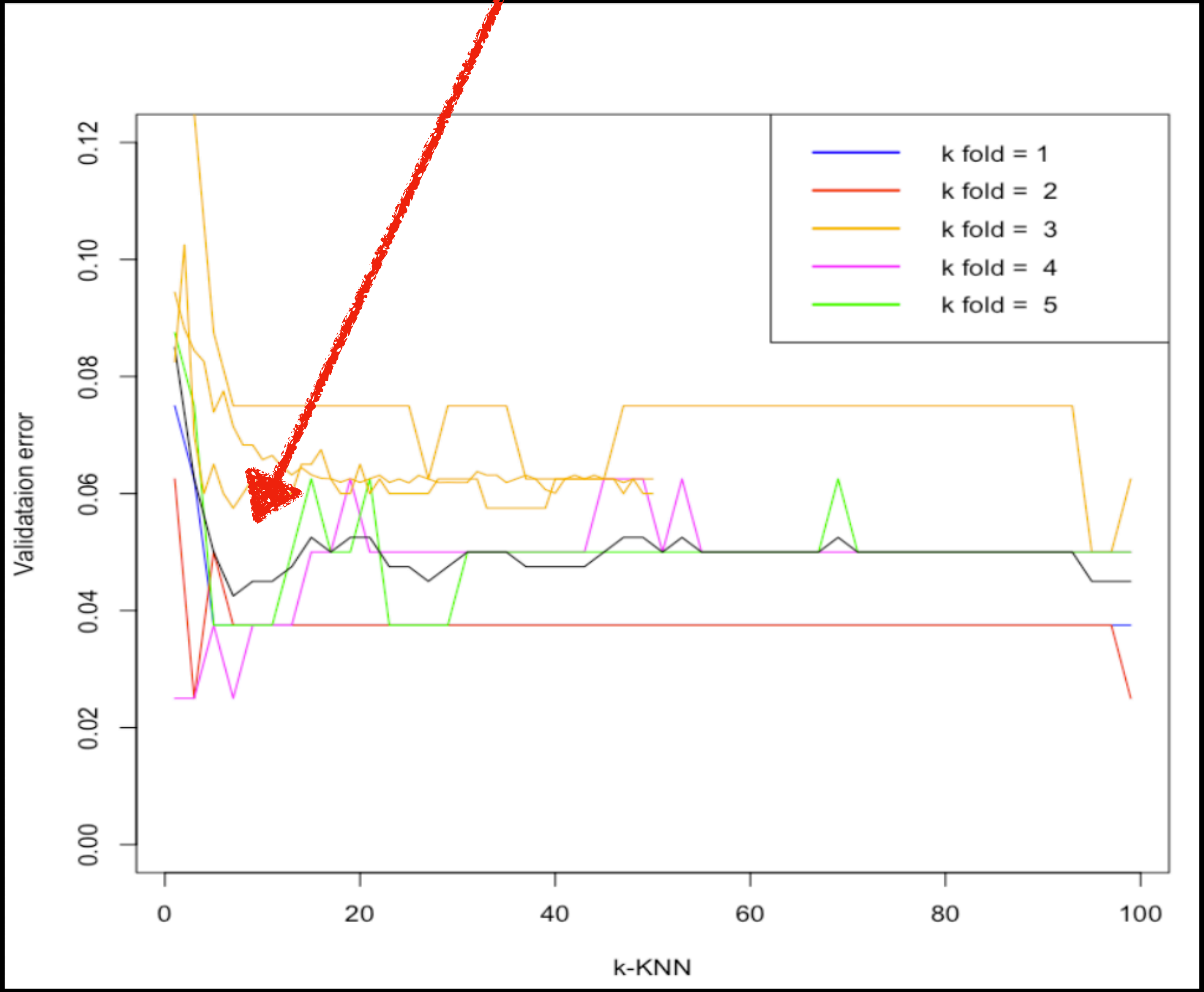

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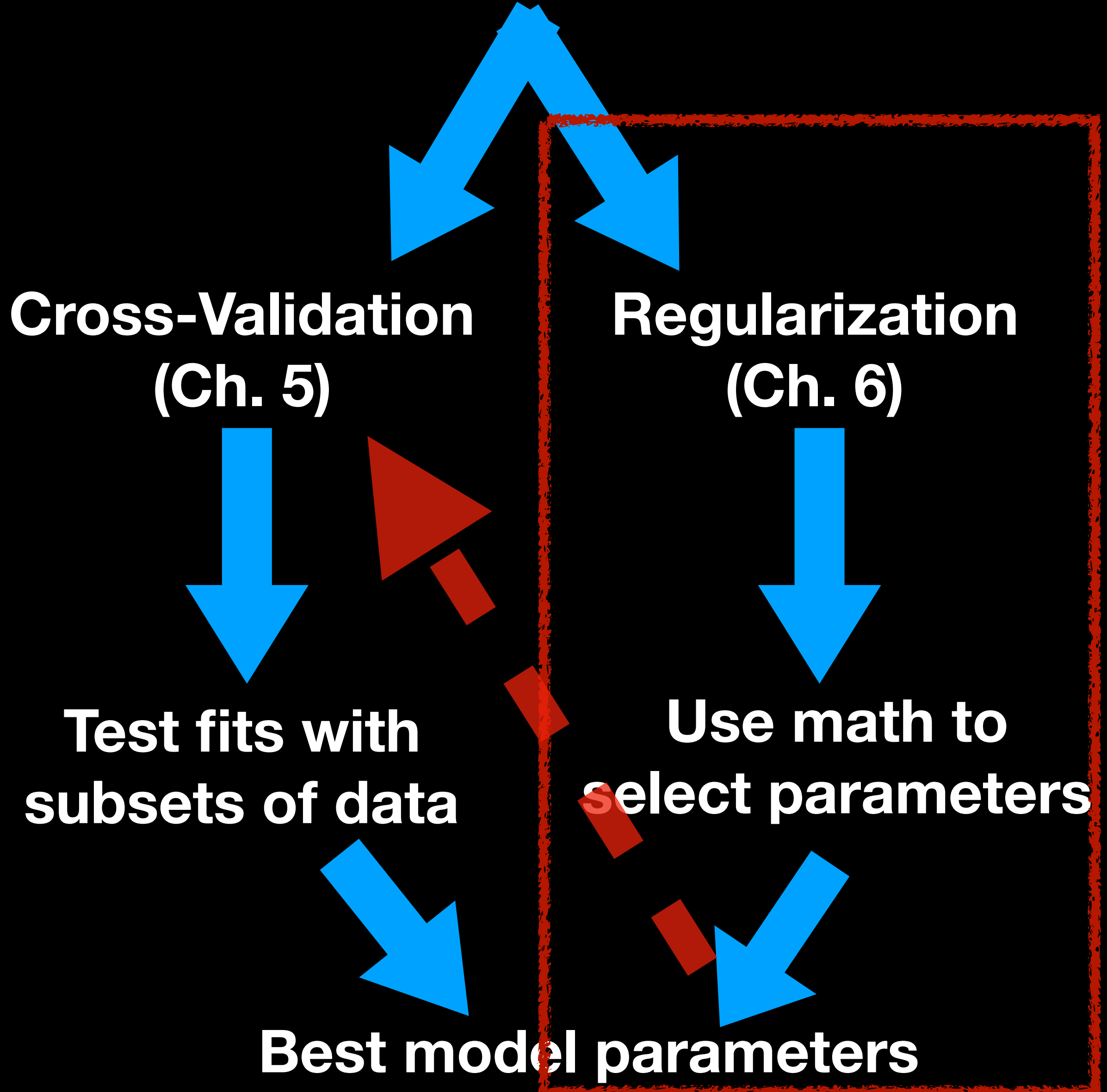

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Prediction Accuracy: especially when $p > n$, to control the variance.

Model Interpretability: By removing irrelevant features — that is, by setting the corresponding coefficient estimates to zero — we can obtain a model that is more easily interpreted. We will present some approaches for automatically performing feature selection.

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A few more details…

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   (a) Fit all $\binom{p}{k}$ models that contain exactly k predictors.

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this number can get big ~ 2^p
p=2: 4 (including null)
p=3: 8
...
p=10: 1024

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Great news: we already covered this!

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- Backward selection requires that the number of samples n is larger than the number of variables p (so that the full model can be fit). In contrast, forward stepwise can be used even when $n < p$, and so is the only viable subset method when p is very large.

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$$\mathrm{AIC} = \frac{1}{n\hat{\sigma}^2}\left(\mathrm{RSS} + 2d\hat{\sigma}^2\right)$$

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Or: large values as R_{adj}^2

Subset selection: Forward & Backward (Review)

Find models that minimize:

$$\mathrm{RSS} = \sum_{i=1}^{n}\left(y_i - \beta_0 - \sum_{j=1}^{p}\beta_j x_{ij}\right)^2$$

$$C_p = \frac{1}{n}\left(\mathrm{RSS} + 2d\hat{\sigma}^2\right)$$

d predictors, n data points

estimate from $SE^2/n = \sigma^2$ or RSE

$$\mathrm{AIC} = \frac{1}{n\hat{\sigma}^2}\left(\mathrm{RSS} + 2d\hat{\sigma}^2\right)$$

Idea here is to get a measurement of the mean square error that accounts for # of parameters in a model

$$\mathrm{BIC} = \frac{1}{n\hat{\sigma}^2}\left(\mathrm{RSS} + \log(n)d\hat{\sigma}^2\right)$$

With four parameters I can fit an elephant, and with five I can make him wiggle his trunk.
- von Neumann

Or: large values as R_{adj}^2

Optional R notes.

Shrinkage Methods

Shrinkage Methods

Lets recall how we find a linear model:

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Least squares minimization (i.e. minimize RSS)

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Don't panic, its just a bit more math (and we'll get R to do it for us)

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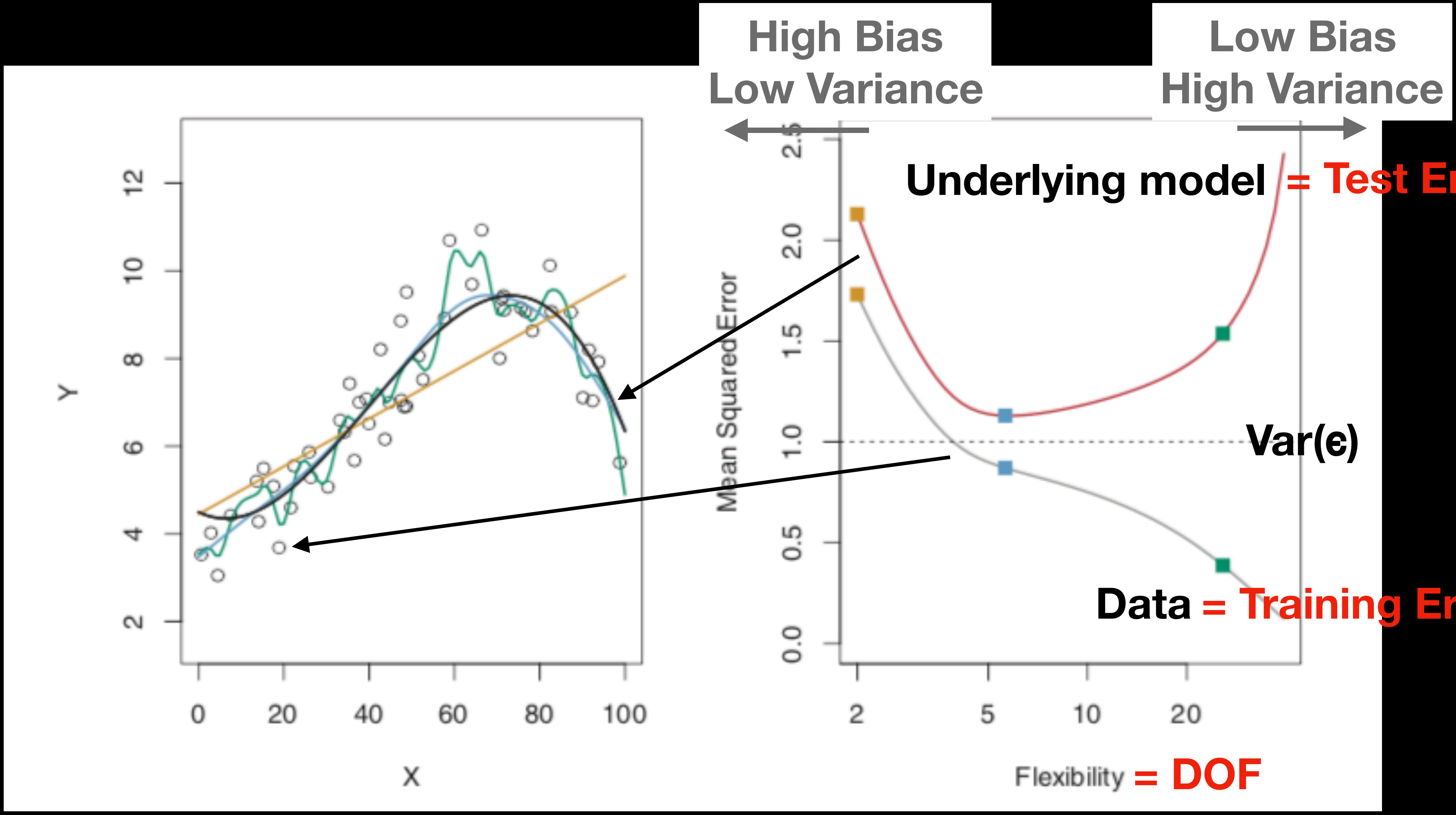

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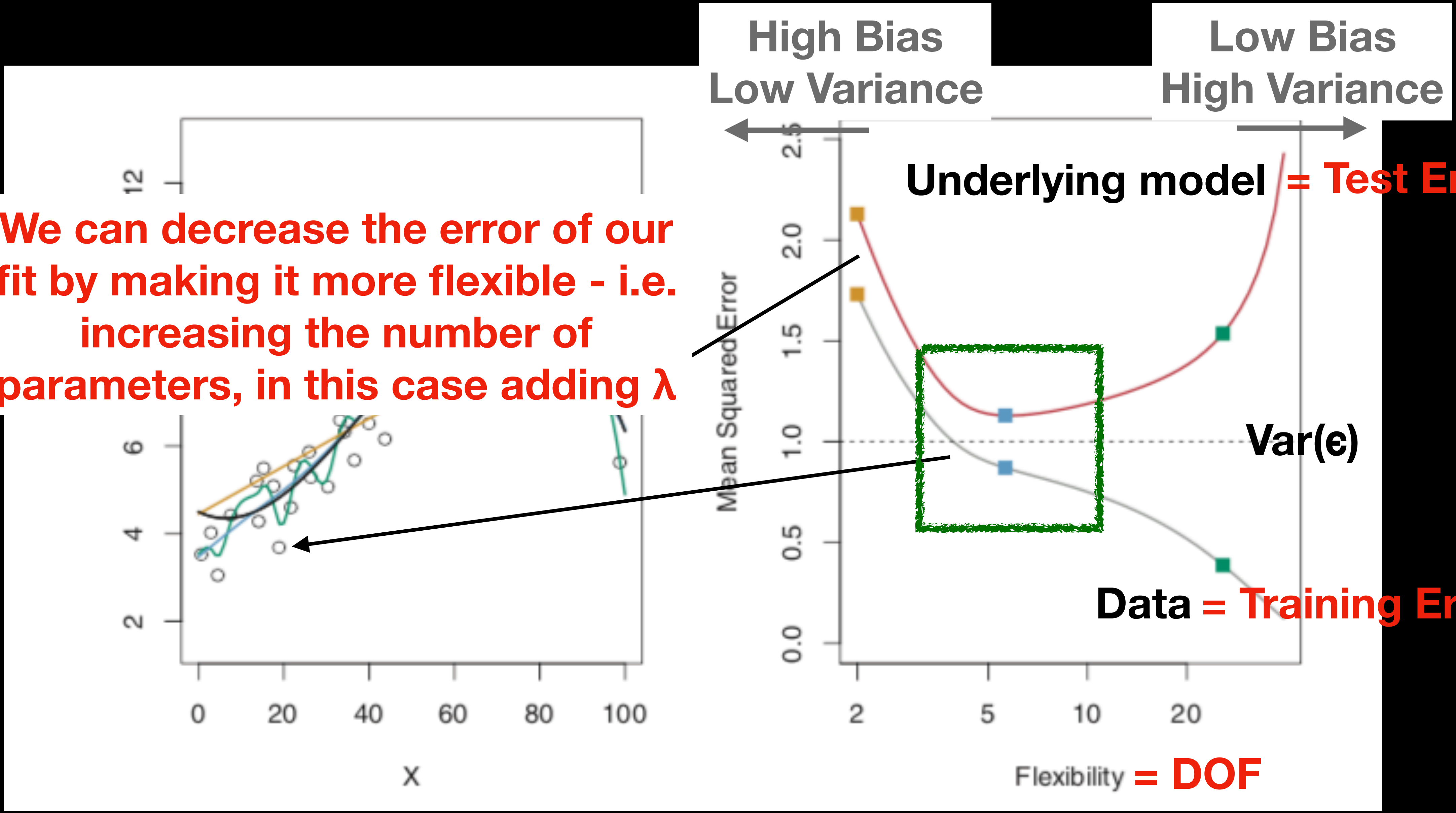

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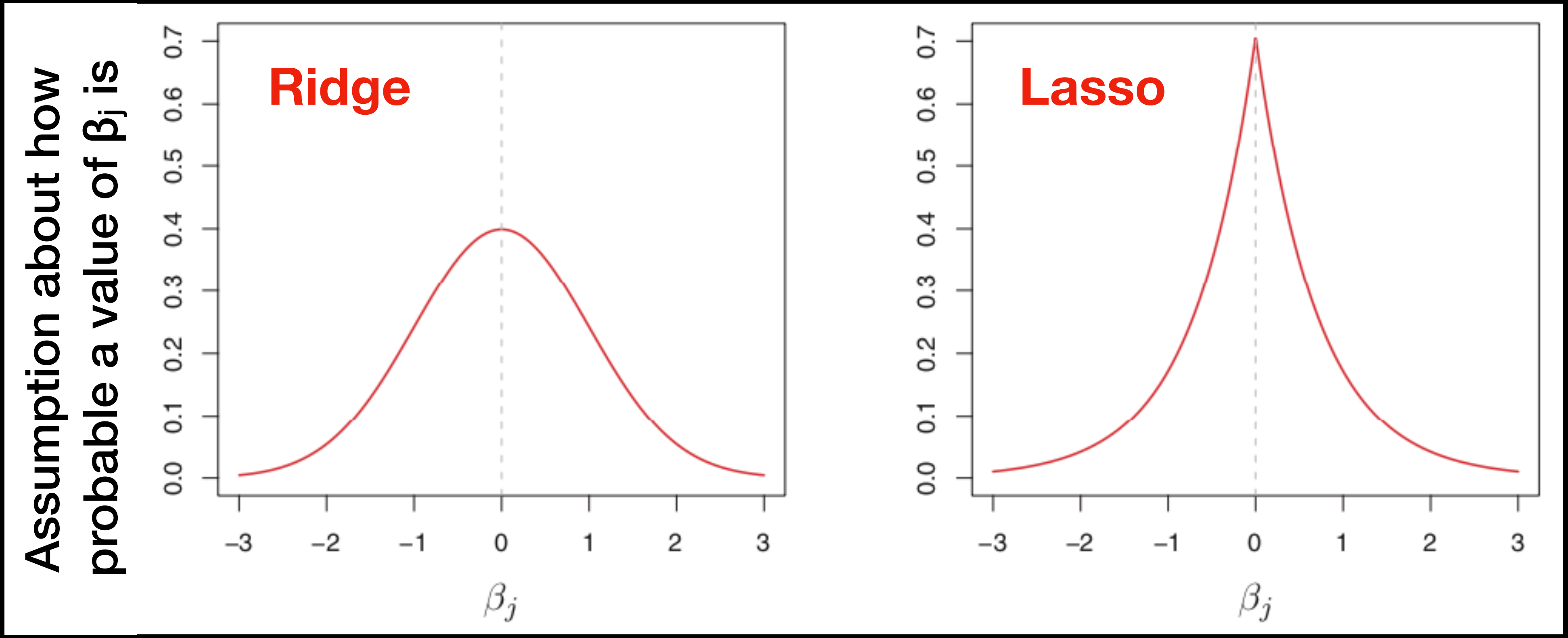

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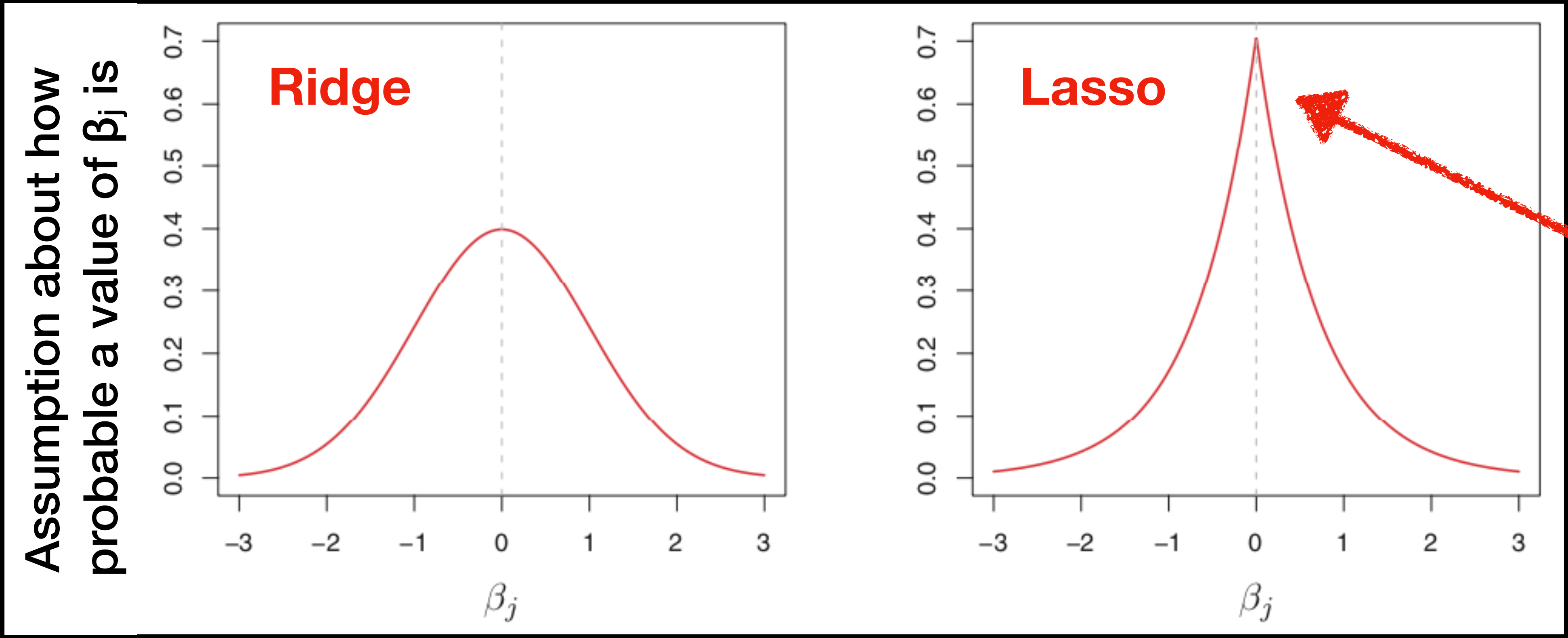

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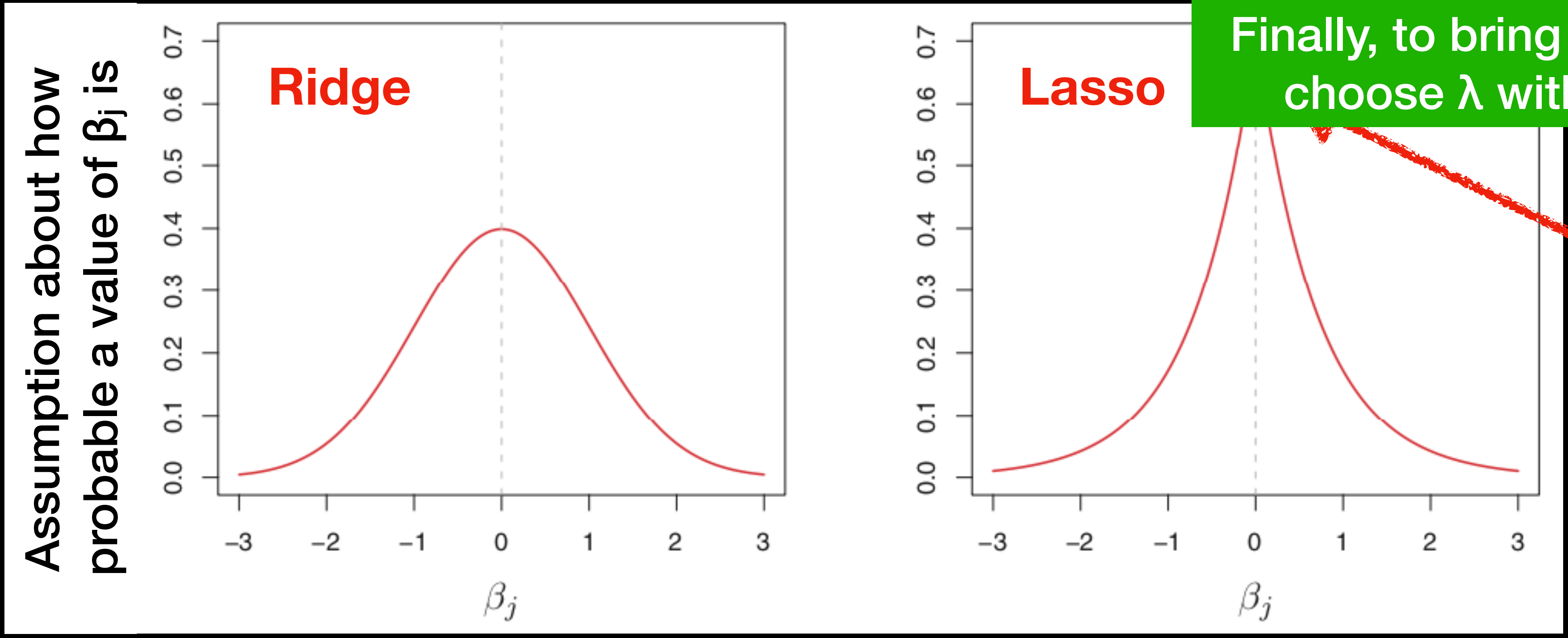
Assumption about how probable a value of β_j is



Finally, to bring it full circle - we can choose λ with Cross-Validation

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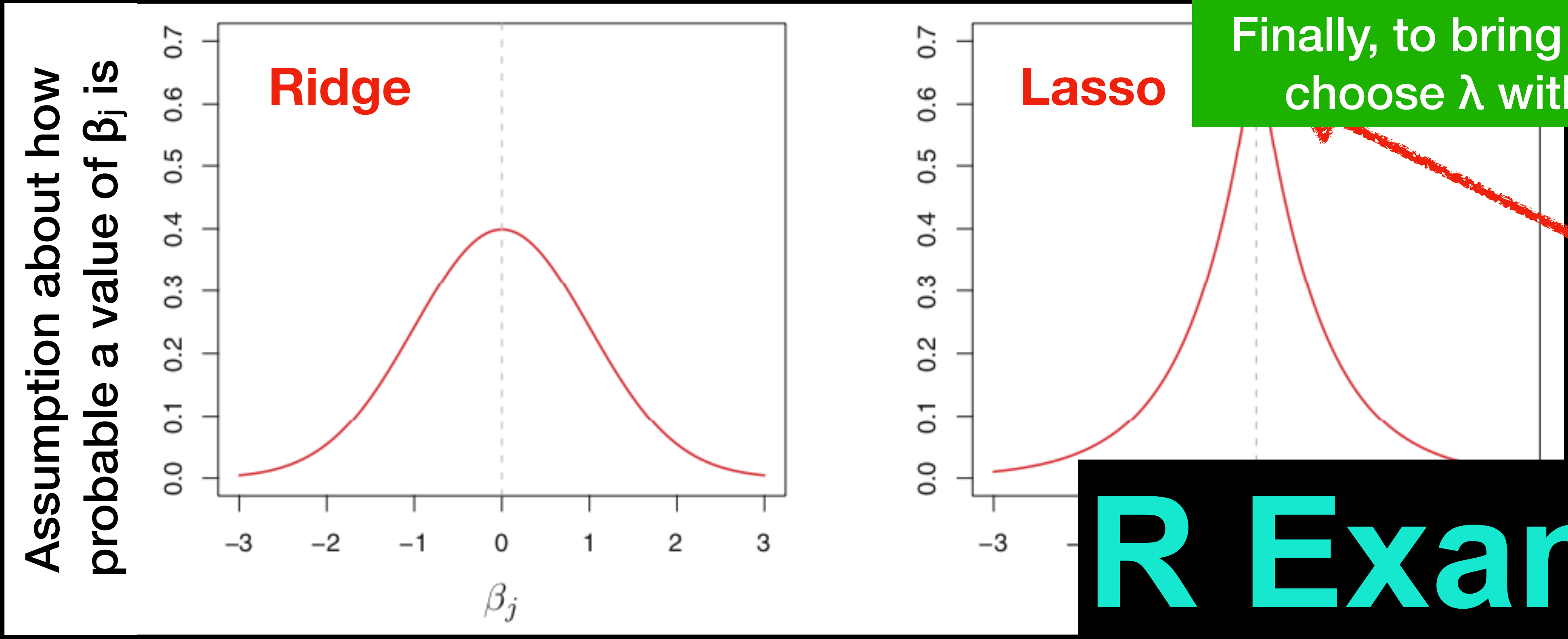

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R Examples!

Unsupervised Learning:
An intro to Principle Component Analysis

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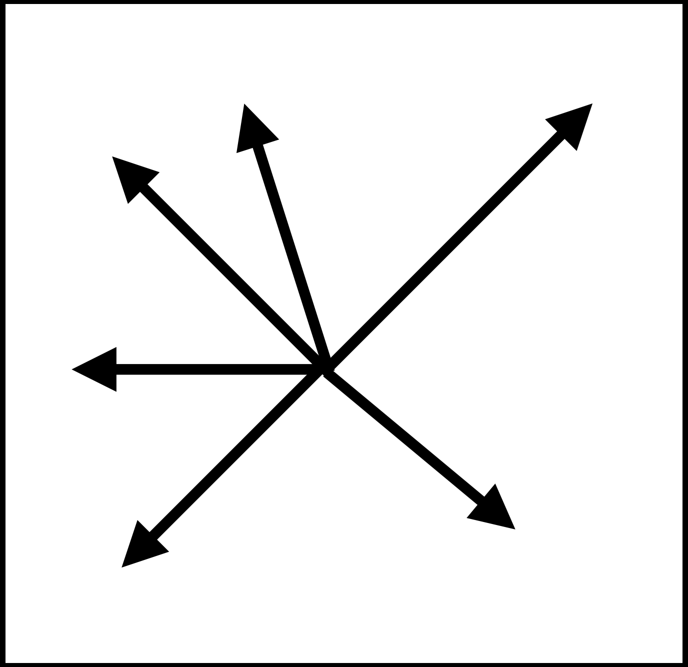
… what do we do when we don’t know anything

Principle Component Analysis: In Pictures



How many vectors do I need to define a 2D space?

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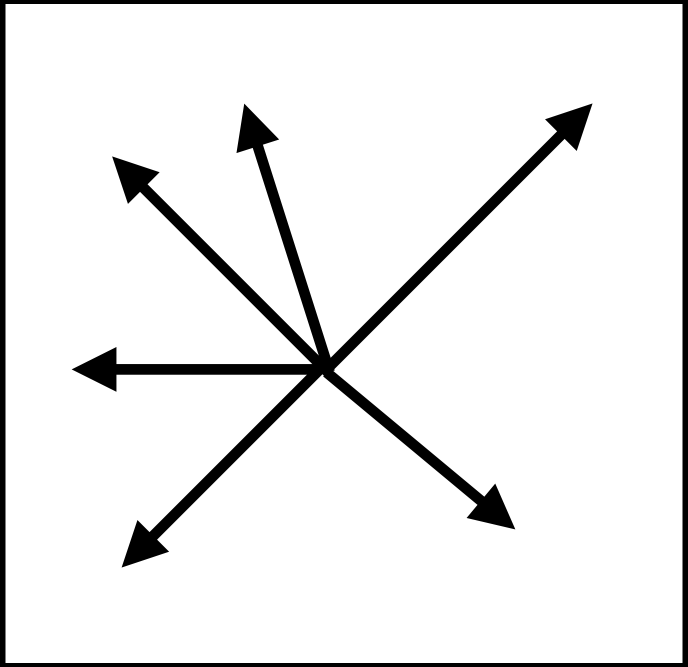

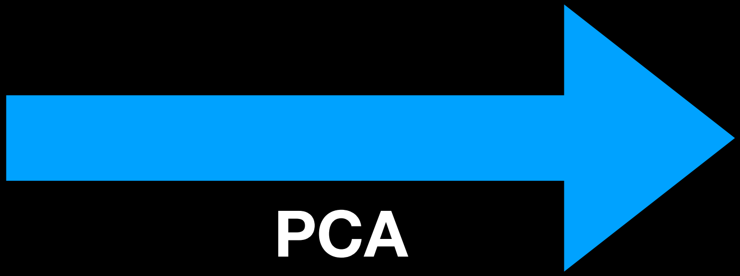

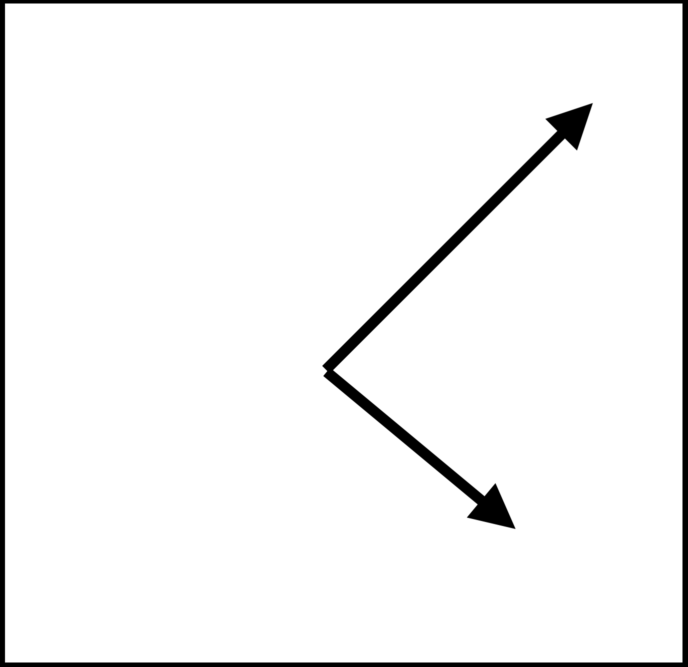

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Minimum number of vectors to define a space in a certain number of dimensions

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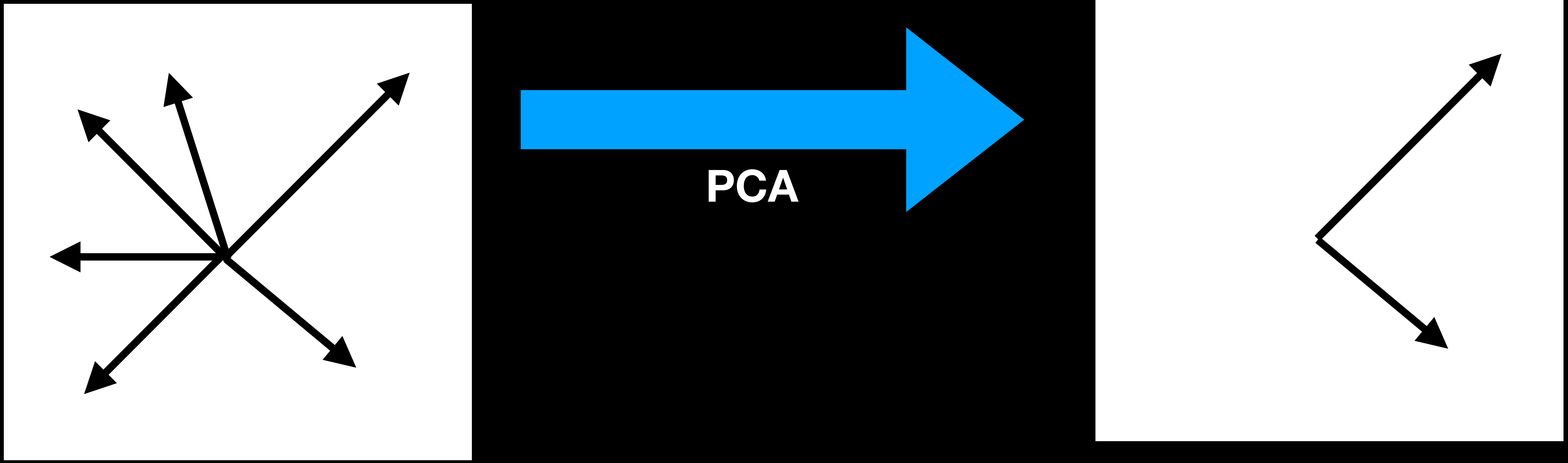

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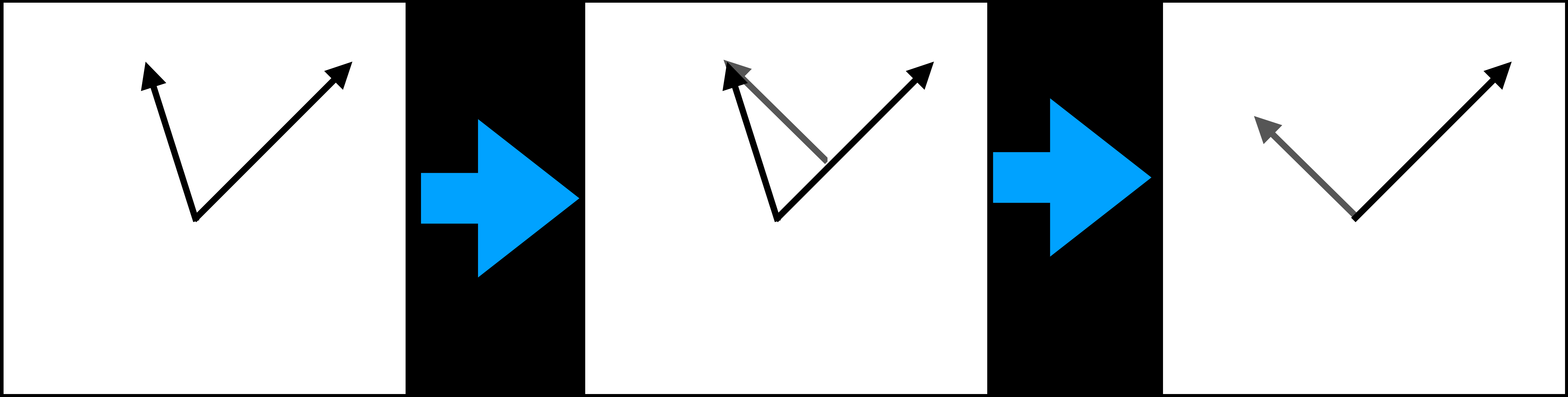

Constructing orthogonal vectors

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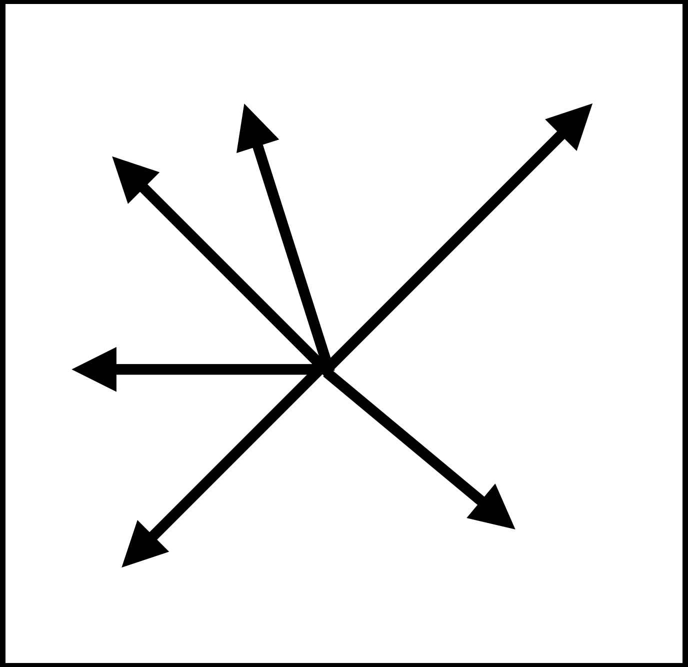

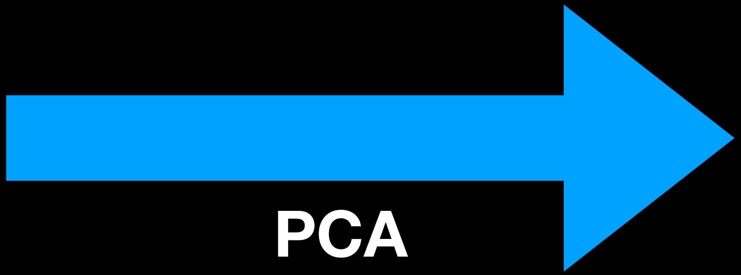

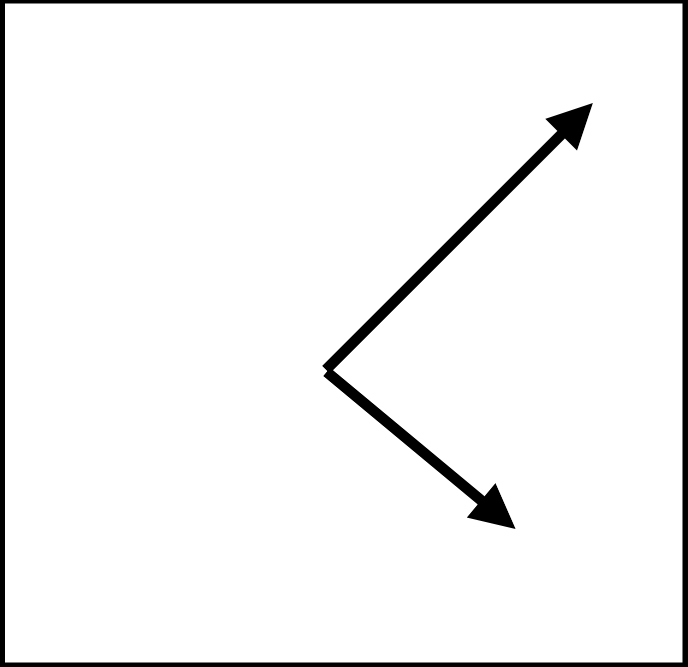

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The “dimensions” of our space is dictated by the number of parameters we have

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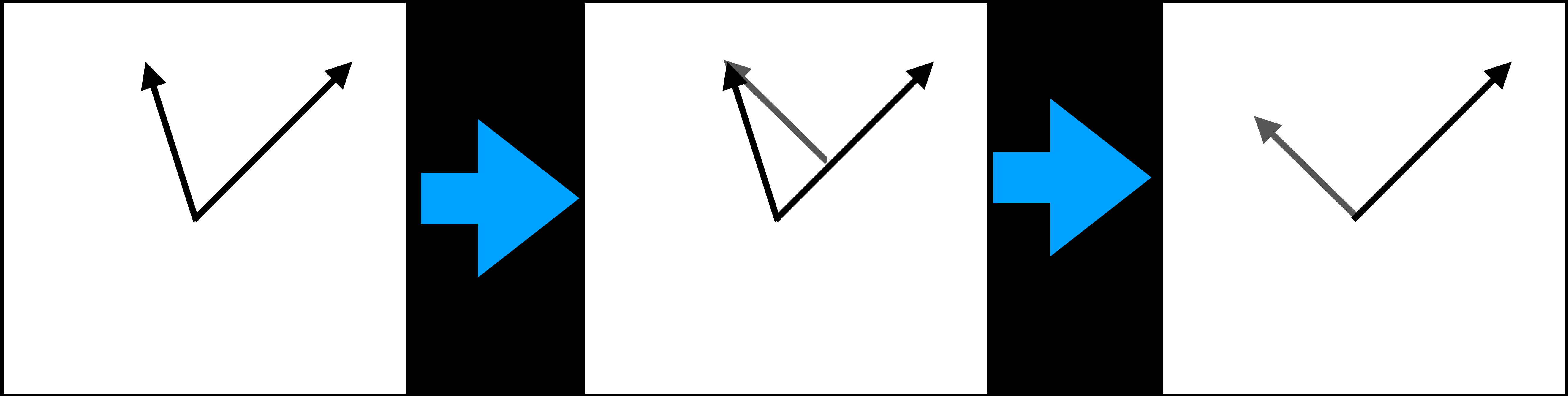

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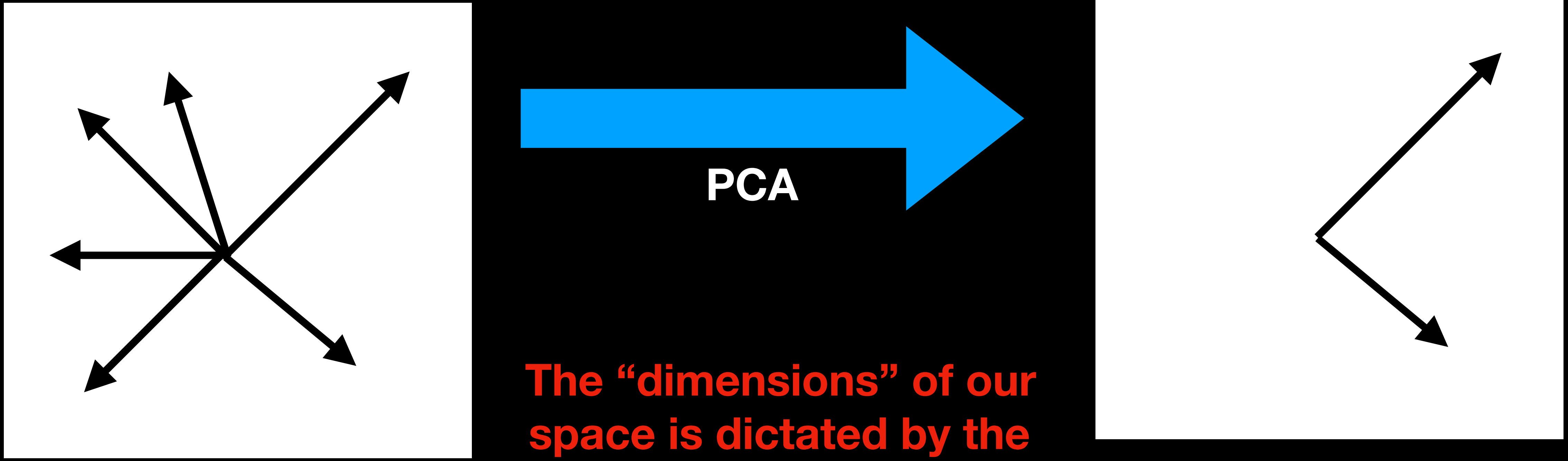

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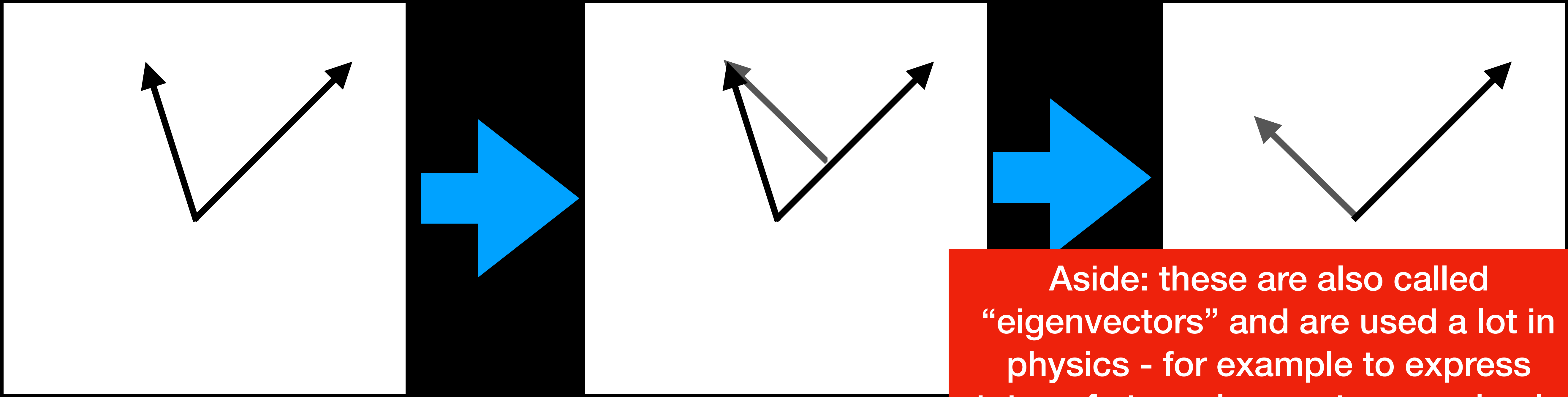

Aside: these are also called "eigenvectors" and are used a lot in physics - for example to express states of atoms in quantum mechanics

Constructing orthogonal vectors

Principle Component Analysis

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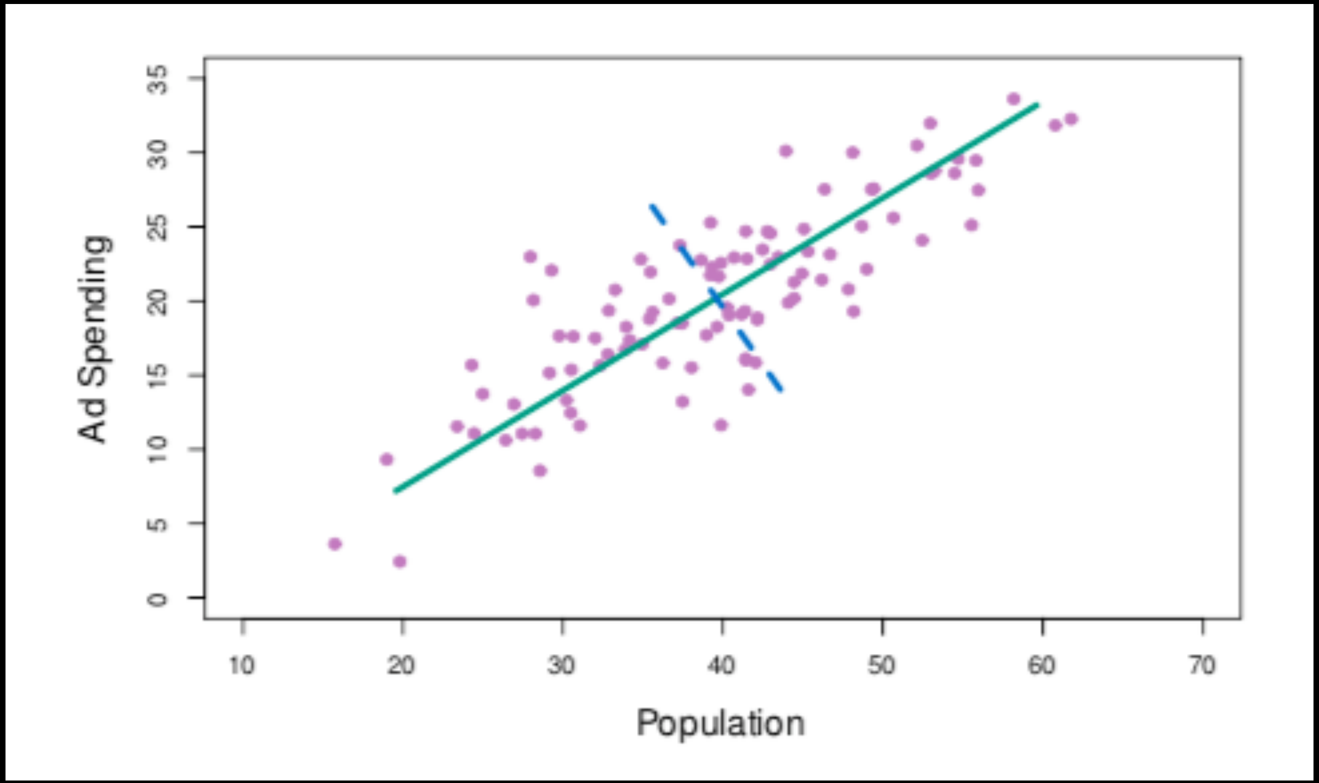

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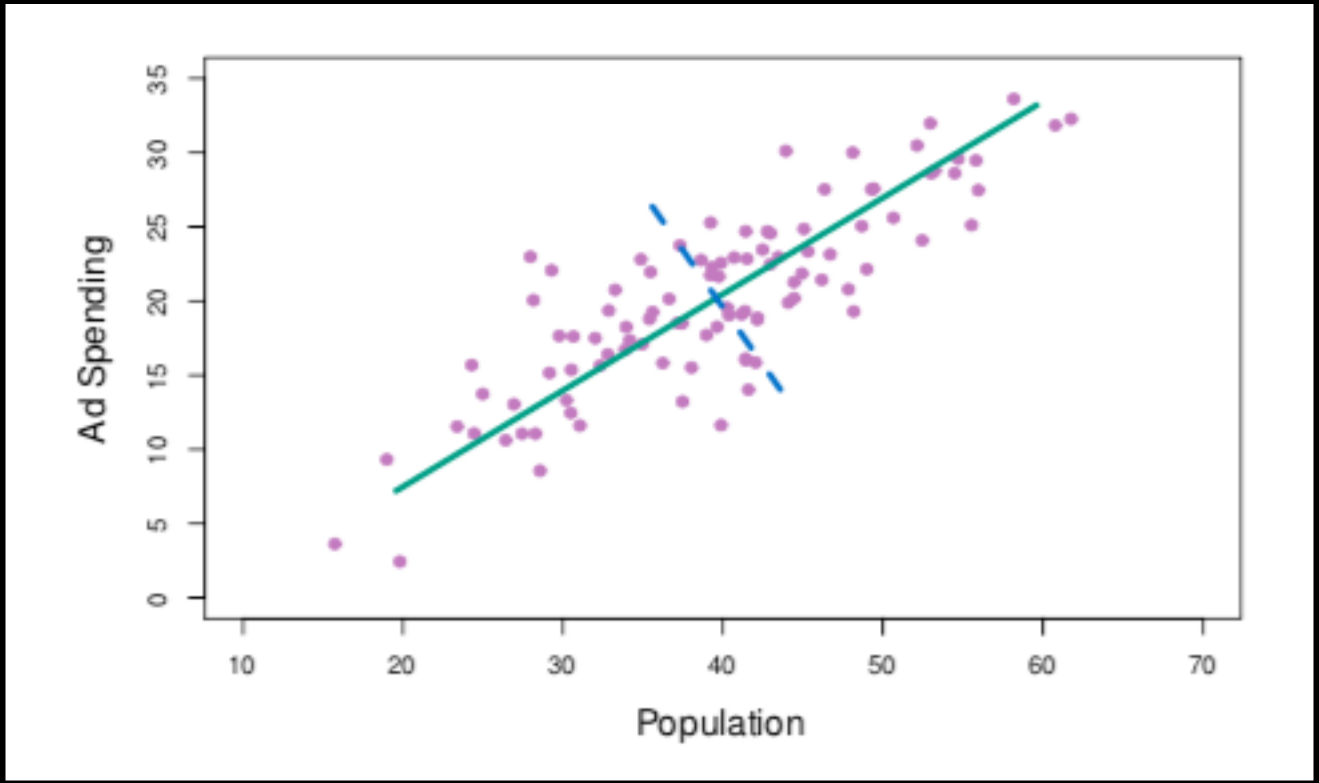

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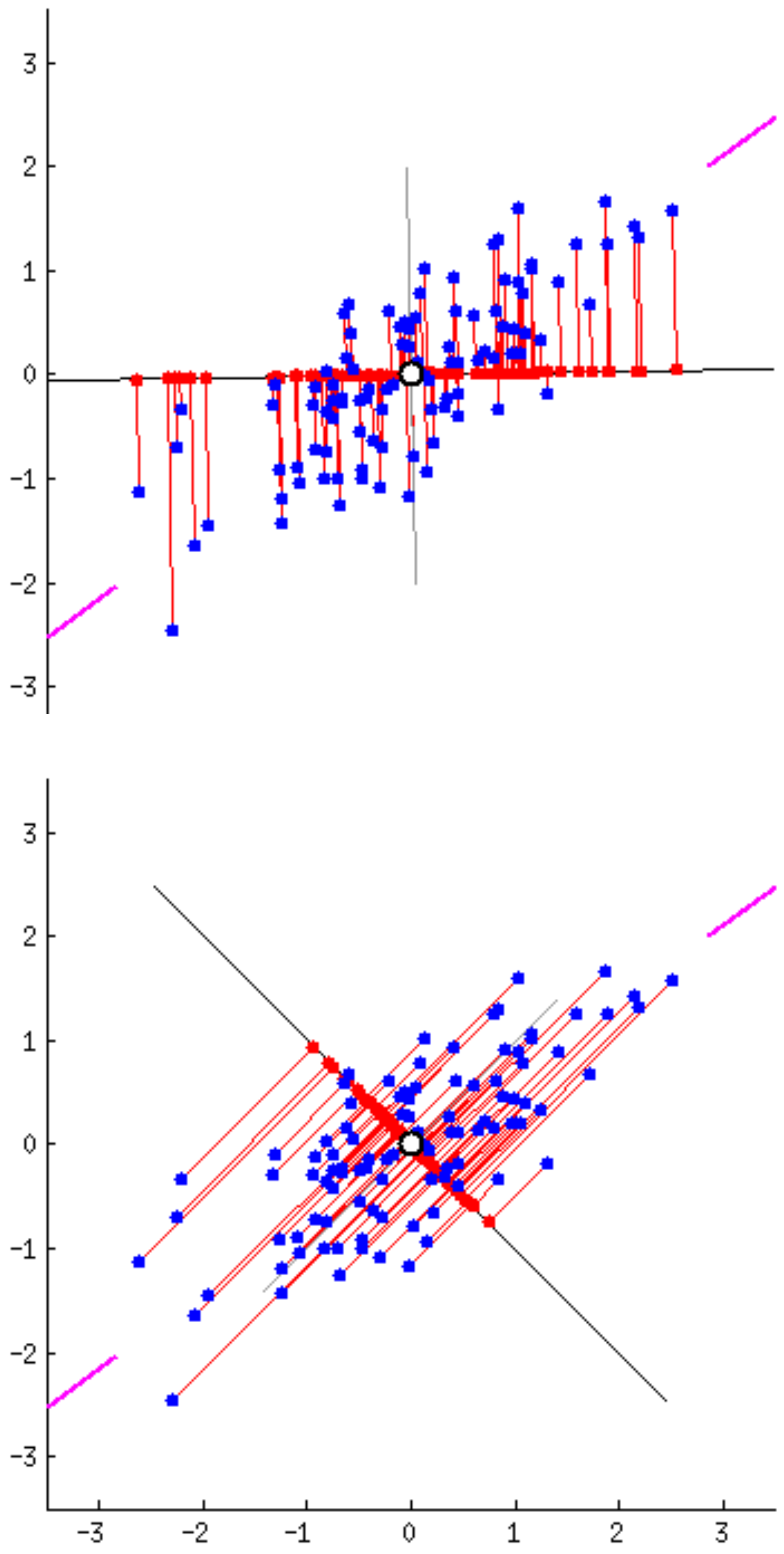
Quick R example!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