2.10.58

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Question

Let **P**, **Q**, **R** and **S** be the points on the plane with position vectors $-2\hat{i} - \hat{j}, 4\hat{i}, 3\hat{i} + 3\hat{j}$ and $-3\hat{i} + 2\hat{j}$ respectively. The quadrilateral **PQRS** must be a

1. parallelogram, which is neither a rhombus nor a rectangle
2. square
3. rectangle, but not a square
4. rhombus, but not a square

Equation

Given details:

$$\mathbf{P} = \begin{pmatrix} -2 \\ -1 \\ 0 \end{pmatrix} \ \mathbf{Q} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} \ \mathbf{R} = \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix} \ \mathbf{S} = \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix} \tag{1}$$

Theoretical Solution

Finding the sides:

$$\mathbf{Q}-\mathbf{P}=\begin{pmatrix}6\\1\\0\end{pmatrix}\ \mathbf{R}-\mathbf{Q}=\begin{pmatrix}-1\\3\\0\end{pmatrix} \tag{2}$$

$$\mathbf{S}-\mathbf{R}=\begin{pmatrix}-6\\-1\\0\end{pmatrix}\ \mathbf{P}-\mathbf{S}=\begin{pmatrix}1\\-3\\0\end{pmatrix} \tag{3}$$

Theoretical Solution

First let's check wheter the given opposite sides of the quadrilateral are parallel to each other
For the sides to be parallel

$$\mathbf{Q} - \mathbf{P} = \mathbf{S} - \mathbf{R} \tag{4}$$
$$\mathbf{R} - \mathbf{Q} = \mathbf{P} - \mathbf{S} \tag{5}$$
$$\tag{6}$$

Since:

$$\mathbf{Q} - \mathbf{P} = \mathbf{R} - \mathbf{S} = \begin{pmatrix} 6 \\ 1 \\ 0 \end{pmatrix} \tag{7}$$
$$\mathbf{R} - \mathbf{Q} = \mathbf{S} - \mathbf{P} = \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} \tag{8}$$

Theoretical Solution

Therefore the opposite sides are parallel to each other and Thus the given quadrilateral can be classified as a **Parallelogram**.
Now, checking for right angle, we check for inner product.

$$(\mathbf{Q}-\mathbf{P})^\top(\mathbf{R}-\mathbf{Q}) = \begin{pmatrix} 6 & 1 & 0 \end{pmatrix}\begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} = -3 \tag{9}$$

This implies that the parallelogram is neither a rectangle nor a square

Theoretical Solution

Checking for a rhombus:
The given quadrilateral is a rhombus if its diagonals are orthogonal,

$$(\mathbf{R}-\mathbf{P})^\top(\mathbf{S}-\mathbf{Q}) = \begin{pmatrix}5 & 4 & 0\end{pmatrix}\begin{pmatrix}-7\\2\\0\end{pmatrix} = -27 \tag{10}$$

Theoretical Solution

We can see that $(\mathbf{R}-\mathbf{P})^{\top}(\mathbf{S}-\mathbf{Q})$ is not equal to 0, that is, the diagonals are not orthogonal and therefore the quadrilateral **PQRS** is a parallelogram which is neither a rhombus nor a rectangle.

C Code (1) - Function to store the points

```c

#include <stdio.h>

void get_points(double *points) {
    double coords[8] = {-2,-1, 4,0, 3,3, -3,2};

    for (int i = 0; i < 8; i++) {
        points[i] = coords[i];


}
}
```

Python Code - Using Shared Object

```python
import ctypes
import numpy as np
import matplotlib.pyplot as plt

parallelogram_lib = ctypes.CDLL("./points.so")

parallelogram_lib.get_points.argtypes = [np.ctypeslib.ndpointer(
    dtype=np.double, ndim=1, flags="C")]

points = np.zeros(8, dtype=np.double)

parallelogram_lib.get_points(points)
```

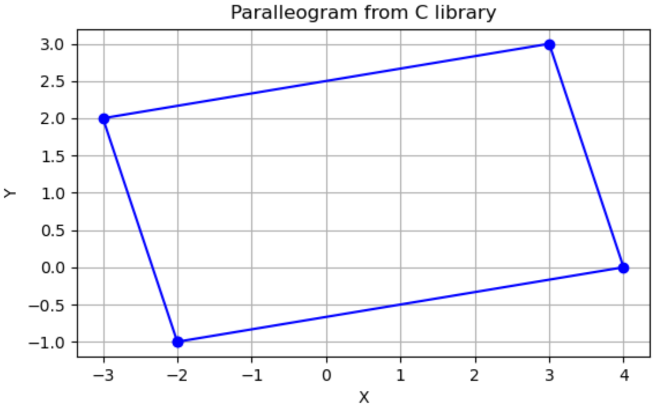
Python Code - Using Shared Object

```python
points = points.reshape((4,2))

points = np.vstack([points, points[0]])

plt.plot(points[:,0], points[:,1], "bo-")
plt.title("Paralleogram from C library")
plt.xlabel("X")
plt.ylabel("Y")
plt.gca().set_aspect("equal")
plt.grid(True)
plt.savefig('figs/parallelogram.png')
subprocess.run(shlex.split('termux-open ../figs/parallelogram.png
    '))
plt.show()
```

Plot-Using Both C and Python



Python Code

```python
import numpy as np
import matplotlib.pyplot as plt

points=np.array([[-2,-1],[4,0],[3,3],[-3,2]])
points=np.vstack([points,points[0]])
plt.plot(points[:,0],points[:,1],"bo-",linewidth=2)
plt.title("Parallelogram of 4 Points")
plt.xlabel("X-axis")
plt.ylabel("Y-axis")
plt.gca().set_aspect("equal")
plt.grid(True)
plt.savefig('figs/parallelogram2.png')
subprocess.run(shlex.split('termux-open ../figs/parallelogram.png
    '))
plt.show()
```

Plot-Using only Python