2.2.24

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QuestionShow that the points $(1, 7)$, $(4, 2)$, $(-1, -1)$ and $(-4, 4)$ are the vertices of a square.

Solution Given details:

$$\mathbf{A} = \begin{pmatrix} 1 \\ 7 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \mathbf{C} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \mathbf{D} = \begin{pmatrix} -4 \\ 4 \end{pmatrix} \tag{1}$$

For the points **ABCD** to represent a square:

$$||AB|| = ||BC|| = ||CD|| = ||DA|| \tag{2}$$

$$\angle BAD = \angle ABC = \angle DCA = \angle ADC = 90° \tag{3}$$

Find the sides

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 3 \\ -5 \end{pmatrix} \ \mathbf{C} - \mathbf{B} = \begin{pmatrix} -5 \\ -3 \end{pmatrix} \tag{4}$$

$$\mathbf{D} - \mathbf{C} = \begin{pmatrix} -3 \\ 5 \end{pmatrix} \ \mathbf{A} - \mathbf{D} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \tag{5}$$

Put these as columns of a 2×4 matrix V:

$$\mathbf{V} = \begin{pmatrix} B - A & C - B & D - C & A - B \end{pmatrix} \tag{6}$$

$$= \begin{pmatrix} 3 & -5 & -3 & 5 \\ -5 & -3 & 5 & 3 \end{pmatrix} \tag{7}$$

Compute the 4×4Gram matrix $\mathbf{G} = \mathbf{V^T V}$.Its entries are all possible inner products
Self inner-products (diagonal of G):

$$(\mathbf{A} - \mathbf{B})^T (\mathbf{A} - \mathbf{B}) = 3^2 + (-5)^2 = 34 \tag{8}$$

$$(\mathbf{B} - \mathbf{C})^T (\mathbf{B} - \mathbf{C}) = (-5)^2 + (-3)^2 = 34 \tag{9}$$

$$(\mathbf{C} - \mathbf{D})^T (\mathbf{C} - \mathbf{D}) = (-3)^2 + 5^2 = 34 \tag{10}$$

$$(\mathbf{D} - \mathbf{A})^T (\mathbf{D} - \mathbf{A}) = 5^2 + 3^2 = 34 \tag{11}$$

Adjacent inner products(off-diagonals for consecutive sides):

$$\left(\mathbf{B} - \mathbf{A}^T\right)(\mathbf{C} - \mathbf{B}) = 3(-5) + (-5)(-3) = -15 + 15 = 0 \tag{12}$$

$$\left(\mathbf{C} - \mathbf{B}^T\right)(\mathbf{D} - \mathbf{C}) = (-5)(-3) + (-3)(5) = 15 - 15 = 0, \tag{13}$$

$$\left(\mathbf{D} - \mathbf{C}^T\right)(\mathbf{A} - \mathbf{D}) = (-3)(5) + 5(3) = -15 + 15 = 0, \tag{14}$$

$$\left(\mathbf{A} - \mathbf{D}^T\right)(\mathbf{B} - \mathbf{A}) = 5(3) + 3(-5) = 15 - 15 = 0. \tag{15}$$

So the relevant part of the Gram matrix G (showing these entries) is:

$$\mathbf{G} = \mathbf{V^T V} = \begin{pmatrix} 34 & 0 & * & 0 \\ 0 & 34 & 0 & * \\ * & 0 & 34 & 0 \\ 0 & * & 0 & 34 \end{pmatrix} \tag{16}$$

Where * are the other inner products (not needed for the argument)
Interpretation:

1. Every diagonal element which is the self inner product has the same magnitude . It means all sides are of equal length.
2. Each adjacent pair has inner product zero. A zero inner product means the two vectors are perpendicular
Therefore the given 4 points represent a square

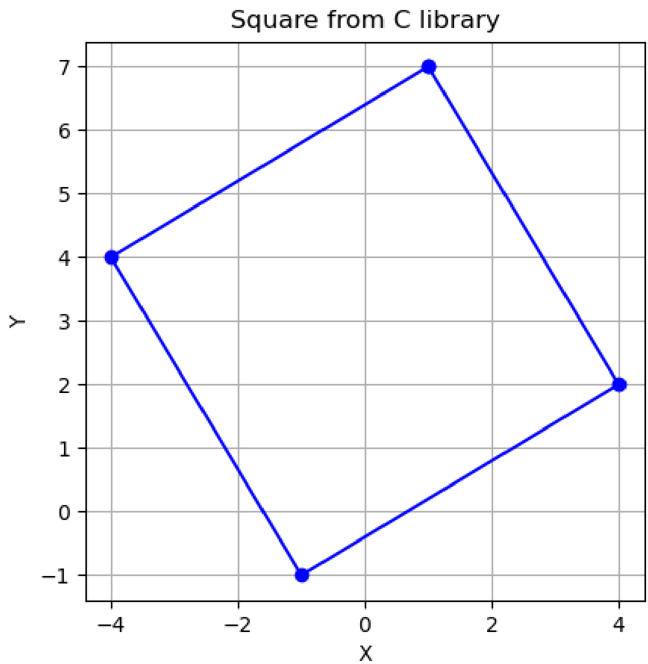

Fig. 0. Sqaure