2.2.24

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QuestionShow that the points $(1,7)$, $(4,2)$, $(-1,-1)$ and $(-4,4)$ are the vertices of a square.

Solution Given details:

$$\mathbf{A} = \begin{pmatrix}1\\7\end{pmatrix}\mathbf{B} = \begin{pmatrix}4\\2\end{pmatrix}\mathbf{C} = \begin{pmatrix}-1\\-1\end{pmatrix}\mathbf{D} = \begin{pmatrix}-4\\4\end{pmatrix} \tag{1}$$

Find the sides

$$\mathbf{B}-\mathbf{A} = \begin{pmatrix}3\\-5\end{pmatrix}\ \mathbf{C}-\mathbf{B} = \begin{pmatrix}-5\\-3\end{pmatrix} \tag{2}$$

$$\mathbf{D}-\mathbf{C} = \begin{pmatrix}-3\\5\end{pmatrix}\ \mathbf{A}-\mathbf{D} = \begin{pmatrix}5\\3\end{pmatrix} \tag{3}$$

First let's check wheter the given opposite sides of the polygon are parallel to each other For the sides to be parallel

$$\mathbf{B}-\mathbf{A} = \mathbf{D}-\mathbf{C} \tag{4}$$

$$\mathbf{C}-\mathbf{B} = \mathbf{A}-\mathbf{D} \tag{5}$$

$$\tag{6}$$

Since:

$$\mathbf{B}-\mathbf{A} = \mathbf{C}-\mathbf{D} = \begin{pmatrix}3\\-5\end{pmatrix} \tag{7}$$

$$\mathbf{C}-\mathbf{B} = \mathbf{D}-\mathbf{A} = \begin{pmatrix}-5\\-3\end{pmatrix} \tag{8}$$

Therefore the opposite sides are parallel to each other and Thus the given polygon can be classified as a **Parallelogram**.

Now put these sides as columns of a 2×4 matrix V:

$$\mathbf{V} = \begin{pmatrix}B-A & C-B & D-C & A-B\end{pmatrix} \tag{9}$$

$$= \begin{pmatrix}3 & -5 & -3 & 5\\ -5 & -3 & 5 & 3\end{pmatrix} \tag{10}$$

Compute the 4×4Gram matrix $\mathbf{G} = \mathbf{V^TV}$.Its entries are all possible inner products

Adjacent inner products of the Gram matrix would be(off-diagonals for consecutive sides):

$$\left(\mathbf{B}-\mathbf{A}^T\right)(\mathbf{C}-\mathbf{B}) = 3(-5) + (-5)(-3) = -15 + 15 = 0 \tag{11}$$

$$\left(\mathbf{C}-\mathbf{B}^T\right)(\mathbf{D}-\mathbf{C}) = (-5)(-3) + (-3)(5) = 15 - 15 = 0, \tag{12}$$

$$\left(\mathbf{D}-\mathbf{C}^T\right)(\mathbf{A}-\mathbf{D}) = (-3)(5) + 5(3) = -15 + 15 = 0, \tag{13}$$

$$\left(\mathbf{A}-\mathbf{D}^T\right)(\mathbf{B}-\mathbf{A}) = 5(3) + 3(-5) = 15 - 15 = 0. \tag{14}$$

Since all are zero ,it means all the sides are perpendicular to each other . Therefore the given parallelogram can be classified as a **Rectangle**.

In the matrix G self inner-products would be (diagonal of G):

$$(\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{B}) = 3^2 + (-5)^2 = 34 \tag{15}$$

$$(\mathbf{B}-\mathbf{C})^T(\mathbf{B}-\mathbf{C}) = (-5)^2 + (-3)^2 = 34 \tag{16}$$

$$(\mathbf{C}-\mathbf{D})^T(\mathbf{C}-\mathbf{D}) = (-3)^2 + 5^2 = 34 \tag{17}$$

$$(\mathbf{D}-\mathbf{A})^T(\mathbf{D}-\mathbf{A}) = 5^2 + 3^2 = 34 \tag{18}$$

since all are equal to 34 , it means all the side lengths are equal , therefore the given rectangle can be classified as a **Square**.

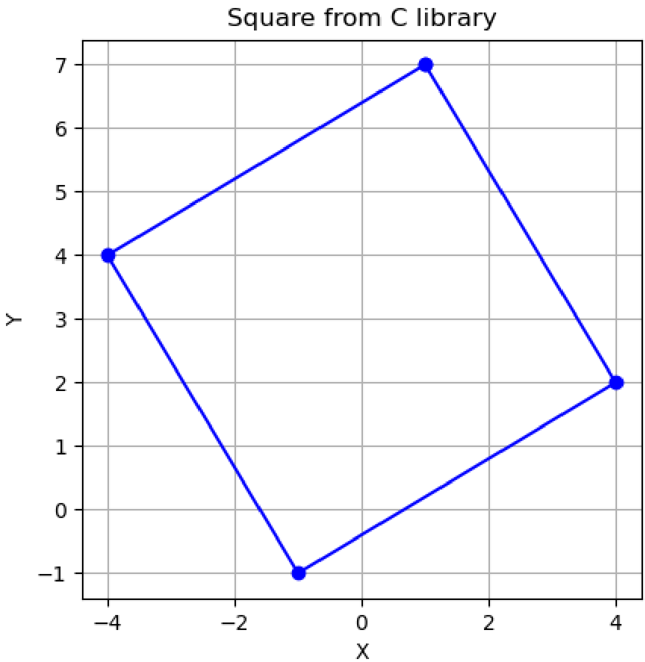

Fig. 0. Sqaure