4.11.7

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QuestionFind the equation of the planes passing through the intersection of the planes $\mathbf{r}.\left(3\hat{i}+6\hat{j}\right)+12=0$ and $\mathbf{r}.\left(3\hat{i}-\hat{j}+4\hat{k}\right)=0$ are at a unit distance from the origin.

Solution Given details
Plane 1:

$$\mathbf{r}.\left(3\hat{i}+6\hat{j}\right)+12=0 \tag{1}$$

$$\begin{pmatrix}3 & 6 & 0\end{pmatrix}\mathbf{r}+12=0 \tag{2}$$

$$\text{dividing the equation with 3:} \tag{3}$$

$$\begin{pmatrix}1 & 2 & 0\end{pmatrix}\mathbf{r}+4=0 \tag{4}$$

$$\Rightarrow \mathbf{n}_1^\top\mathbf{r}+d_1=0 \tag{5}$$

$$\text{Therefore :} \tag{6}$$

$$\mathbf{n_1}^\top=\begin{pmatrix}1 & 2 & 0\end{pmatrix}\text{and } d_1=4 \tag{7}$$

Plane 2:

$$\mathbf{r}.\left(3\hat{i}-\hat{j}+4\hat{k}\right)=0 \tag{8}$$

$$\begin{pmatrix}3 & -1 & 4\end{pmatrix}\mathbf{r}+0=0 \tag{9}$$

$$\text{similarly:} \tag{10}$$

$$\mathbf{n_2}^\top=\begin{pmatrix}3 & -1 & 4\end{pmatrix}\text{and } d_2=0 \tag{11}$$

plane passing through the intersection of the two given planes can be represented as a linear combination of their equations.

$$\left(\mathbf{n_1}^\top+d_1\right)+\lambda\left(\mathbf{n_2}^\top+d_2\right)=0 \tag{12}$$

Rearranging them:

$$\left(\mathbf{n_1}^\top+\lambda\mathbf{n_2}^\top\right)+(d_1+\lambda d_2)=0 \tag{13}$$

Substituting the values we get:

$$\left(\begin{pmatrix}1 & 2 & 0\end{pmatrix}+\lambda\begin{pmatrix}3 & -1 & 4\end{pmatrix}\right)+(4+\lambda.0)=0 \tag{14}$$

$$\begin{pmatrix}1+3\lambda & 2-\lambda & 4\lambda\end{pmatrix}\mathbf{r}+4=0 \tag{15}$$

This is the general equation for any plane passing through the line of intersection. Let's call the new normal vector $\mathbf{n}^\top=\begin{pmatrix}1+3\lambda & 2-\lambda & 4\lambda\end{pmatrix}$ and the constant $d=4$
Since the plane is at a unit distance from the origin

$$1=\frac{|d|}{||\mathbf{n}||} \tag{16}$$

$$1=\frac{4}{\sqrt{(1+3\lambda)^2+(2-\lambda)^2+(4\lambda)^2}} \tag{17}$$

$$26\lambda^2+2\lambda-11=0 \tag{18}$$

On solving the equation we get:

$$\lambda = \frac{-2 \pm 2\sqrt{287}}{52} = \frac{-1 \pm \sqrt{287}}{26} \tag{19}$$

This gives us two possible values for λ, which means there are two planes that satisfy the given conditions.

The final plane equations are:

$$\mathbf{r}\left(\left(23 + 3\sqrt{287}\right)\hat{i} + \left(53 - \sqrt{287}\right)\hat{j} + \left(4\sqrt{287} - 4\right)\hat{k}\right) + 104 = 0 \tag{20}$$

$$\mathbf{r}\left(\left(23 - 3\sqrt{287}\right)\hat{i} + \left(53 + \sqrt{287}\right)\hat{j} - \left(4 + 4\sqrt{287}\right)\hat{k}\right) + 104 = 0 \tag{21}$$

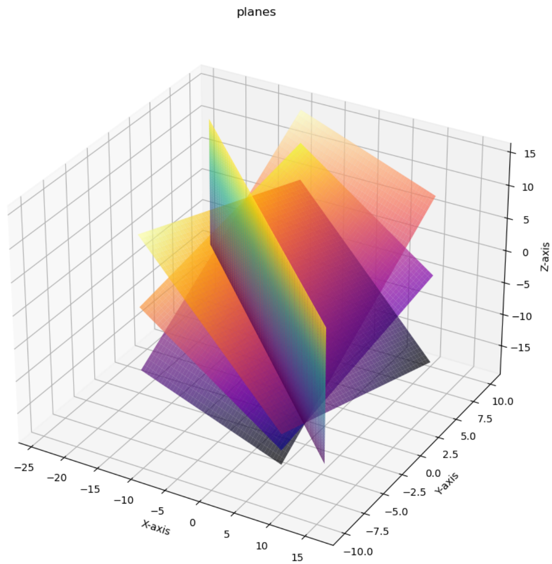

Fig. 0. plane