4.11.7

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Question The equations to a pair of opposite sides of a parallelogram are $x^2-5x+6=0$ and $y^2-6y+5=0$. The equations to its diagonals are:

1) $x+4y=13, y=4x-7$
2) $4x+y=13, y=4x-7$
3) $4x+y=13, 4y=x-7$
4) $y-4x=13, y+4x=7$

Solution Given details

Equation 1:

$$x^2-5x+6=0 \tag{1}$$

This equation can be factored into: (2)

$$(x-2)(x-3)=0 \tag{3}$$

This gives us two vertical lines: (4)

$$x=2 \tag{5}$$

$$x=3 \tag{6}$$

Equation 2:

$$y^2-6y+5=0 \tag{7}$$

This equation can be factored into: (8)

$$(y-1)(y-5)=0 \tag{9}$$

This gives us two horizontal lines: (10)

$$y=1 \tag{11}$$

$$y=5 \tag{12}$$

Through the intersection of these 4 lines we can find the 4 vertices of the parallelogram:

Intersection of $x=2$ and $y=1$ is the point $\mathbf{A}(2,1)$. (13)

Intersection of $x=3$ and $y=1$ is the point $\mathbf{B}(3,1)$. (14)

Intersection of $x=3$ and $y=5$ is the point $\mathbf{C}(3,5)$. (15)

Intersection of $x=2$ and $y=5$ is the point $\mathbf{D}(2,5)$. (16)

The equations of the diagonals can be found using the matrix method. The equation of a line through (x_1,y_1) and (x_2,y_2) is given by setting the determinant of the matrix of coordinates to zero, as three collinear points form a triangle with zero area.

$$\det\begin{pmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{pmatrix} = 0$$

The equation of the diagonal **AC**, passing through **A**$(2,1)$ and **C**$(3,5)$, is:

$$\det\begin{pmatrix} x & y & 1 \\ 2 & 1 & 1 \\ 3 & 5 & 1 \end{pmatrix} = 0 \tag{17}$$

$$x(1\cdot 1 - 5\cdot 1) - y(2\cdot 1 - 3\cdot 1) + 1(2\cdot 5 - 3\cdot 1) = 0 \tag{18}$$

$$x(1-5) - y(2-3) + 1(10-3) = 0 \tag{19}$$

$$-4x - y(-1) + 7 = 0 \tag{20}$$

$$-4x + y + 7 = 0 \tag{21}$$

$$y = 4x - 7 \tag{22}$$

The equation of the diagonal **BD**, passing through **B**$(3,1)$ and **D**$(2,5)$, is:

$$\det\begin{pmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 2 & 5 & 1 \end{pmatrix} = 0 \tag{23}$$

$$x(1\cdot 1 - 5\cdot 1) - y(3\cdot 1 - 2\cdot 1) + 1(3\cdot 5 - 2\cdot 1) = 0 \tag{24}$$

$$x(1-5) - y(3-2) + 1(15-2) = 0 \tag{25}$$

$$-4x - y(1) + 13 = 0 \tag{26}$$

$$-4x - y + 13 = 0 \tag{27}$$

$$4x + y = 13 \tag{28}$$

Therefore the equations of both the diagonals are:

$$y = 4x - 7 \tag{29}$$

$$4x + y = 13 \tag{30}$$

Hence the answer is option 2.

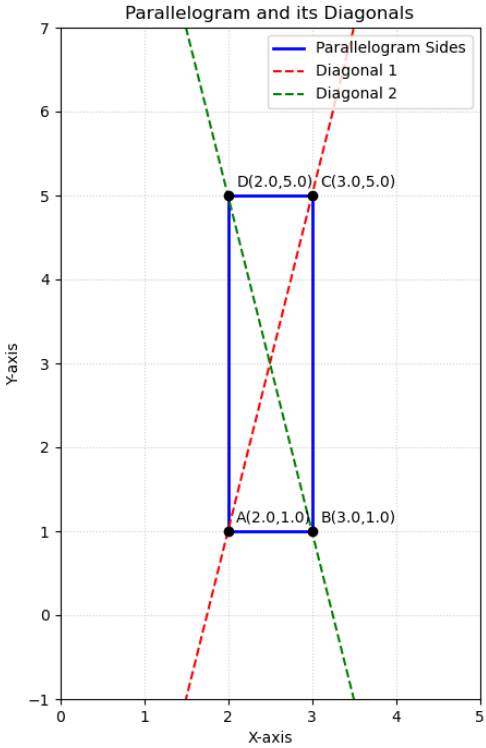

Fig. 4. diagonals