# 2.2.24

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**Question**Show that the points $(1,7)$, $(4,2)$, $(-1,-1)$ and $(-4,4)$ are the vertices of a square.

**Solution** Given details:

$$\mathbf{A} = \begin{pmatrix} 1 \\ 7 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \mathbf{C} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \mathbf{D} = \begin{pmatrix} -4 \\ 4 \end{pmatrix} \tag{1}$$

Find the sides

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 3 \\ -5 \end{pmatrix} \ \mathbf{C} - \mathbf{B} = \begin{pmatrix} -5 \\ -3 \end{pmatrix} \tag{2}$$

$$\mathbf{D} - \mathbf{C} = \begin{pmatrix} -3 \\ 5 \end{pmatrix} \ \mathbf{A} - \mathbf{D} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \tag{3}$$

First let's check wheter the given opposite sides of the quadrilateral are parallel to each other
For the sides to be parallel

$$\mathbf{B} - \mathbf{A} = \mathbf{D} - \mathbf{C} \tag{4}$$

$$\mathbf{C} - \mathbf{B} = \mathbf{A} - \mathbf{D} \tag{5}$$

$$\tag{6}$$

Since:

$$\mathbf{B} - \mathbf{A} = \mathbf{C} - \mathbf{D} = \begin{pmatrix} 3 \\ -5 \end{pmatrix} \tag{7}$$

$$\mathbf{C} - \mathbf{B} = \mathbf{D} - \mathbf{A} = \begin{pmatrix} -5 \\ -3 \end{pmatrix} \tag{8}$$

Therefore the opposite sides are parallel to each other and Thus the given quadrilateral can be classified as a **Parallelogram**.
Now, checking for right angle, we check for inner product.

$$(\mathbf{B} - \mathbf{A})^\top (\mathbf{C} - \mathbf{B}) = \begin{pmatrix} 3 & -5 \end{pmatrix} \begin{pmatrix} -5 \\ -3 \end{pmatrix} = 0 \tag{9}$$

This implies that the angle at **B** is one right angle. A parallelogram with a right angle is a rectangle.
Checking for a square:
The given quadrilateral is a square if its diagonals are orthogonal,

$$(\mathbf{C} - \mathbf{A})^\top (\mathbf{D} - \mathbf{B}) = \begin{pmatrix} -2 & -8 \end{pmatrix} \begin{pmatrix} -8 \\ 2 \end{pmatrix} = 0 \tag{10}$$

We can see that $(\mathbf{C} - \mathbf{A})^\top (\mathbf{D} - \mathbf{B})$ is equal to 0, that is, the diagonals are orthogonal and therefore the quadrilateral **ABCD** is a square.
$\therefore$ The quadrilateral **ABCD** is a Square.

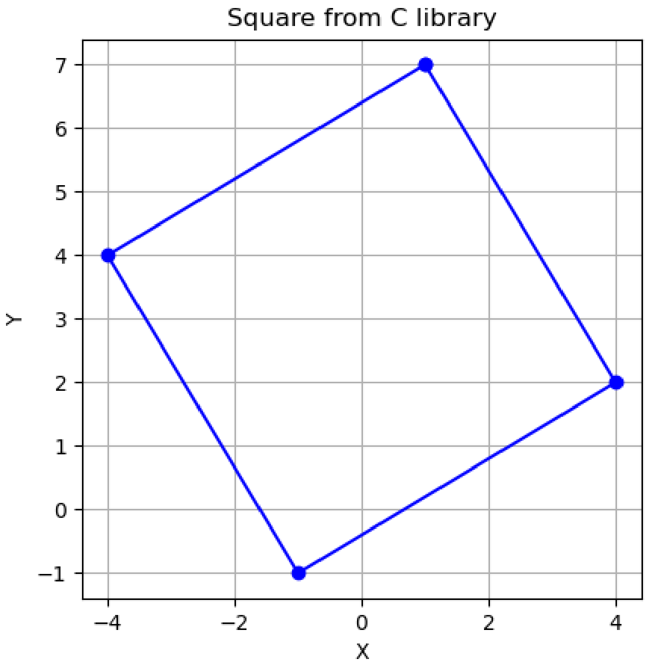


Fig. 0. Sqaure