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Ouestion:

The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class using matrices.

Solution: To find the total number of students, we first define our variables and set up a system of linear equations. Let x be the number of rows and y be the number of students per row. The total number of students is xy.

From the problem statement, we derive two equations:

$$(x-1)(y+3) = xy \implies 3x - y = 3$$

 $(x+2)(y-3) = xy \implies -3x + 2y = 6$

We can solve this system using an augmented matrix and Gaussian elimination. We begin by creating an augmented matrix $(A \mid B)$ for this system.

The augmented matrix for this system is:

$$(A \mid B) = \begin{pmatrix} 3 & -1 \mid 3 \\ -3 & 2 \mid 6 \end{pmatrix}$$
 (1) The goal is to use elementary row operations to transform the left side of the augmented

matrix into row-echelon form. We perform the operation $R_2 \rightarrow R_2 + R_1$:

$$\begin{pmatrix} 3 & -1 & 3 \\ -3+3 & 2+(-1) & 6+3 \end{pmatrix}$$
 (2)

After performing the operation, the matrix becomes:

$$\begin{pmatrix}
3 & -1 & | & 3 \\
0 & 1 & | & 9
\end{pmatrix}$$
(3)

From this row-echelon form, we can use back-substitution. The second row gives us the equation 0x + 1y = 9, which means y = 9.

Substituting y = 9 into the first row's equation, 3x - y = 3:

$$3x - 9 = 3$$
$$3x = 12$$

$$x = 4$$

We have found there are x = 4 rows and y = 9 students per row.

The total number of students is $x \times y = 4 \times 9 = 36$.