

5.4.24

Jnanesh Sathisha Karmar- EE25BTECH11029

Question:

Find the inverse of the matrix $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$ using the Gauss-Jordan method.

Solution: To find the inverse of a matrix \mathbf{A} , we use the Gauss-Jordan elimination method. We begin by creating an augmented matrix by placing the identity matrix \mathbf{I} to the right of matrix \mathbf{A} , forming $(\mathbf{A}|\mathbf{I})$.

The augmented matrix for $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$ is:

$$(\mathbf{A} | \mathbf{I}) = \left(\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 4 & 2 & 0 & 1 \end{array} \right) \quad (1)$$

The goal is to use elementary row operations to transform the left side of the augmented matrix into the identity matrix. The right side will then become the inverse, \mathbf{A}^{-1} . We perform the operation $R_2 \rightarrow R_2 - 2R_1$:

$$\left(\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 4 - 2(2) & 2 - 2(1) & 0 - 2(1) & 1 - 2(0) \end{array} \right) \quad (2)$$

After performing the operation, the matrix becomes:

$$\left(\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 0 & 0 & -2 & 1 \end{array} \right) \quad (3)$$

Because a row of zeros has appeared on the left-hand side, it is impossible to continue the process to form the identity matrix. This indicates that the original matrix \mathbf{A} is singular (its determinant is zero). Therefore, the inverse of the matrix does not exist.