4.7.15

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QuestionFind the vector equation of the plane which is at a distance of $\frac{6}{\sqrt{29}}$from the origin and its normal vector from the origin is $2\hat{i} - 3\hat{j} + 4\hat{k}$..

Solution Given details

$$\text{distance of plane from the origin} = d = \frac{6}{\sqrt{29}} \tag{1}$$

$$\text{normal vector} = \mathbf{n} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} \tag{2}$$

Generally a plane can be represented as:

$$\hat{\mathbf{n}}^\top (\mathbf{r} - \mathbf{r_o}) = 0 \tag{3}$$

For our convience we can choose $\mathbf{r_o}$ to be the point closest to origin.Therefore:

$$\mathbf{r_o} = d\hat{\mathbf{n}} \tag{4}$$

Substituting this in the plane equation:

$$\hat{\mathbf{n}}^\top (\mathbf{r} - d\hat{\mathbf{n}}) = 0 \tag{5}$$

$$\left(\hat{\mathbf{n}}^\top \mathbf{r}\right) - \left(d\hat{\mathbf{n}}^\top \hat{\mathbf{n}}\right) = 0 \tag{6}$$

$$\because \ \hat{\mathbf{n}}^\top \hat{\mathbf{n}} = 1 \tag{7}$$

$$\hat{\mathbf{n}}^\top \mathbf{r} = d \tag{8}$$

The unit normal vector is:

$$\hat{\mathbf{n}} = \frac{\mathbf{n}}{||\mathbf{n}||} \tag{9}$$

$$||\mathbf{n}|| = \sqrt{\mathbf{n}^\top \mathbf{n}} = \sqrt{2^2 + (-3)^2 + 4^2} = \sqrt{29} \tag{10}$$

$$\therefore \ \hat{\mathbf{n}} = \frac{1}{\sqrt{29}} \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} \tag{11}$$

Substituting it in the plane equation:

$$\frac{1}{\sqrt{29}} \begin{pmatrix} 2 & -3 & 4 \end{pmatrix}^\top \mathbf{r} = \frac{6}{\sqrt{29}} \tag{12}$$

The final plane equation is:

$$\begin{pmatrix} 2 & -3 & 4 \end{pmatrix}^\top \mathbf{r} = 6 \tag{13}$$

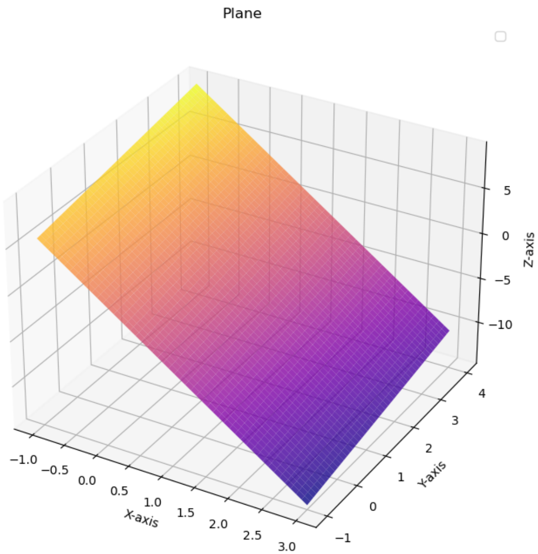

Fig. 0. plane