5.2.32

Jnanesh Sathisha Karmar- EE25BTECH11029

Question Solve the following system of linear equations
$x + 2y - 4 = 0$
$2x + 4y - 12 = 0$
Solution Given details

$$x + 2y - 4 = 0 \tag{1}$$

$$2x + 4y - 12 = 0 \tag{2}$$

$$\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 12 \end{pmatrix} \tag{3}$$

$$\mathbf{Ax} = \mathbf{B} \tag{4}$$

To determine if a unique solution exists, we calculate the determinant of the coefficient matrix

$$\det(\mathbf{A}) = 4 - 4 = 0 \tag{5}$$

Since the determinant is zero, the matrix $\mathbf{A}$ is singular (it has no inverse). This means that the system does not have a unique solution. It will either have no solution or infinitely many solutions.
To find out which case it is, we use an augmented matrix $\left(\mathbf{A} \mid \mathbf{B}\right)$and apply row reduction.

$$\left(\begin{array}{cc|c} 1 & 2 & 4 \\ 2 & 4 & 12 \end{array}\right) \xrightarrow{R_2 \to R_2 - 2R_1} \left(\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & 0 & 4 \end{array}\right)$$

Since the second row of the reduced matrix corresponds to the equation $0x + 0y = 4$, which is a contradiction, the system is inconsistent and has no solution.

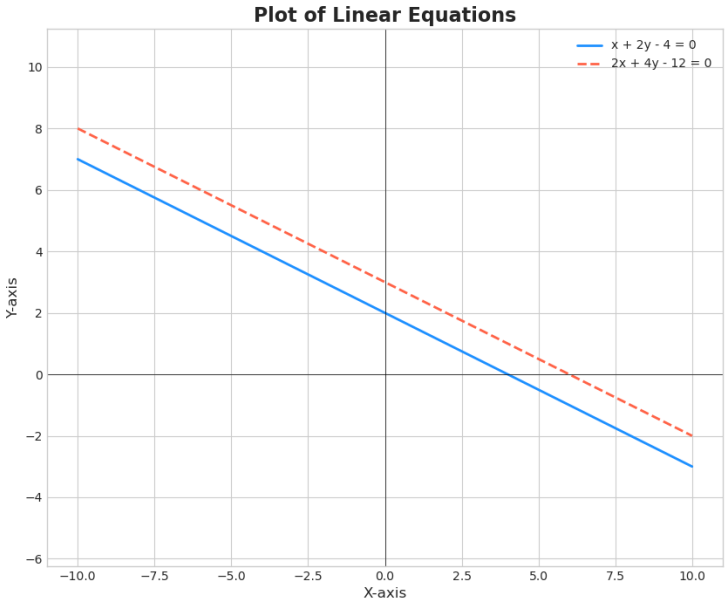

Fig. 0. diagonals