4.11.7

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QuestionThe equations to a pair of opposite sides of parallogram are $x^2 - 5x + 6 = 0$ and $y^2 - 6y + 5 = 0$, the equations to its diagonals are

1) $x + 4y = 13, y = 4x - 7$
2) $4x + y = 13, y = 4x - 7$
3) $4x + y = 13, 4y = x - 7$
4) $y - 4x = 13, y + 4x = 7$

Solution Given details
Equation 1:

$$x^2 - 5x + 6 = 0 \tag{1}$$

$$\text{This equation can be factored into:} \tag{2}$$

$$(x - 2)(x - 3) = 0 \tag{3}$$

$$\text{This gives us two vertical lines:} \tag{4}$$

$$x = 2 \tag{5}$$

$$x = 3 \tag{6}$$

Equation 2:

$$y^2 - 6y + 5 = 0 \tag{7}$$

$$\text{This equation can be factored into:} \tag{8}$$

$$(y - 1)(y - 5) = 0 \tag{9}$$

$$\text{This gives us two horizontal lines:} \tag{10}$$

$$y = 1 \tag{11}$$

$$y = 5 \tag{12}$$

Through the intersection of these 4 lines we can find the 4 vertices of the parallelogram

$$\text{Intersection of } x = 2 \text{ and } y = 1 \text{is the point} (2, 1) \text{.Let vector } \mathbf{A} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{13}$$

$$\text{Intersection of } x = 3 \text{ and } y = 1 \text{is the point} (3, 1) \text{.Let vector } \mathbf{B} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \tag{14}$$

$$\text{Intersection of } x = 3 \text{ and } y = 5 \text{is the point} (3, 5) \text{.Let vector } \mathbf{C} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \tag{15}$$

$$\text{Intersection of } x = 2 \text{ and } y = 5 \text{is the point} (2, 5) \text{.Let vector } \mathbf{D} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} \tag{16}$$

$$\tag{17}$$

The direction vector of the diagonal AC is:

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \tag{18}$$

The normal vector to vector $\begin{pmatrix} a \\ b \end{pmatrix}$ can be represented as $\begin{pmatrix} -b \\ a \end{pmatrix}$

Therefore the normal vector to the direction vector **AC** is:

$$\mathbf{n_{AC}}^\top = \begin{pmatrix} -4 & 1 \end{pmatrix} \tag{19}$$

Therefore the equation of diagonal can be represented as:

$$\mathbf{n_{AC}}^\top (\mathbf{r} - \mathbf{r_o}) = 0 \tag{20}$$

$$\begin{pmatrix} -4 & 1 \end{pmatrix} \left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right) = 0 \tag{21}$$

$$\begin{pmatrix} -4 & 1 \end{pmatrix} \left(\begin{pmatrix} x-2 \\ y-1 \end{pmatrix} \right) = 0 \tag{22}$$

$$-4(x-2) + (y-1) = 0 \tag{23}$$

$$y = 4x - 7 \tag{24}$$

The direction vector of the diagonal **BD** is:

$$\mathbf{D} - \mathbf{B} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} \tag{25}$$

$$\therefore \mathbf{n_{BD}}^\top = \begin{pmatrix} -4 & -1 \end{pmatrix} \tag{26}$$

The equation of diagonal can be represented as:

$$\mathbf{n_{BD}}^\top (\mathbf{r} - \mathbf{r_o}) = 0 \tag{27}$$

$$\begin{pmatrix} -4 & -1 \end{pmatrix} \left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \end{pmatrix} \right) = 0 \tag{28}$$

$$\begin{pmatrix} -4 & -1 \end{pmatrix} \left(\begin{pmatrix} x-3 \\ y-1 \end{pmatrix} \right) = 0 \tag{29}$$

$$-4(x-3) - 1(y-1) = 0 \tag{30}$$

$$4x + y = 13 \tag{31}$$

Therefore the equations of both the diagonals are:

$$y = 4x - 7 \tag{32}$$

$$4x + y = 13 \tag{33}$$

Hence the answer is option 2

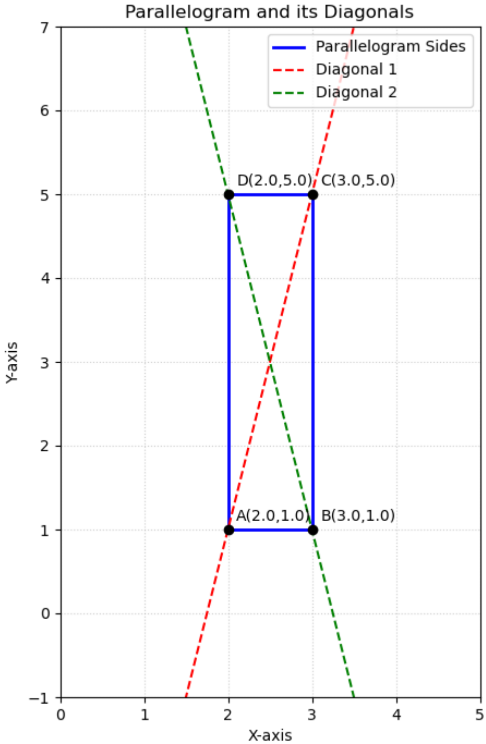

Fig. 4. diagonals