2.8.10

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Question If with reference to the right handed system of mutually perpendicular unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}, \alpha = 3\hat{i} - \hat{j}, \beta = 2\hat{i} + \hat{j} - 3\hat{k}$, then express β in the form $\beta = \beta_1 + \beta_2$ where β_1 is parallel to α and β_2 is perpendicular to α .

Solution Given details:

$$\alpha = 3\hat{i} - \hat{j} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} \tag{1}$$

$$\beta = 2\hat{i} + \hat{j} - 3\hat{k} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} \tag{2}$$

β_1 is a projection of β on α
The projection formula for projection is:

$$\beta_1 = \frac{\beta^{\mathbf{T}}\alpha}{\left\|\alpha^{\mathbf{2}}\right\|}\alpha \tag{3}$$

$$= \frac{\begin{pmatrix} 2 & 1 & -3 \end{pmatrix}\begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}}{(3)^2 + (-1)^2 + (0)^2}\begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} \tag{4}$$

$$= \frac{5}{10}\begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} \tag{5}$$

$$= \begin{pmatrix} \frac{3}{2} \\ \frac{-1}{2} \\ 0 \end{pmatrix} \tag{6}$$

Now according to the given equation :

$$\beta = \beta_1 + \beta_2 \tag{7}$$

$$\beta_2 = \beta - \beta_1 \tag{8}$$

$$\beta_2 = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} - \begin{pmatrix} \frac{3}{2} \\ \frac{-1}{2} \\ 0 \end{pmatrix} \tag{9}$$

$$\beta_2 = \begin{pmatrix} \frac{1}{2} \\ \frac{3}{2} \\ -3 \end{pmatrix} \tag{10}$$

Lets verify wheter β_2 is perpendicular to α
For that:

$$\alpha^T.\beta_2 = 0 \tag{11}$$

$$\begin{pmatrix} 3 & -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ \frac{3}{2} \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \end{pmatrix} \tag{12}$$

Therefore β_2 is perpendicular to α
Therefore β is:

$$\beta = \beta_1 + \beta_2 \tag{13}$$

$$\beta = \begin{pmatrix} \frac{3}{2} \\ \frac{-1}{2} \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ \frac{3}{2} \\ -3 \end{pmatrix} \tag{14}$$

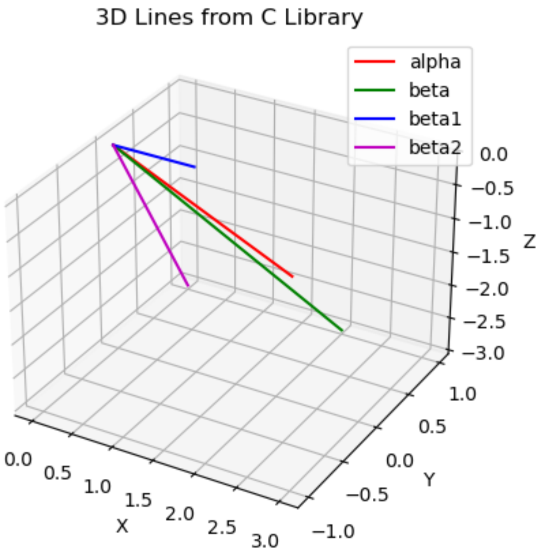

Fig. 0. vectors