2.10.58

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Question Let $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ and $\mathbf{S}$ be the points on the plane with position vectors $-2\hat{i} - \hat{j}, 4\hat{i}, 3\hat{i} + 3\hat{j}$ and $-3\hat{i} + 2\hat{j}$ respectively. The quadrilateral **PQRS** must be a

1) parallelogram, which is neither a rhombus nor a rectangle
2) square
3) rectangle, but not a square
4) rhombus, but not a square

Solution Given details

$$\mathbf{P} = \begin{pmatrix} -2 \\ -1 \\ 0 \end{pmatrix} \mathbf{Q} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} \mathbf{R} = \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix} \mathbf{S} = \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix} \tag{1}$$

Finding the sides:

$$\mathbf{Q} - \mathbf{P} = \begin{pmatrix} 6 \\ 1 \\ 0 \end{pmatrix} \mathbf{R} - \mathbf{Q} = \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} \tag{2}$$

$$\mathbf{S} - \mathbf{R} = \begin{pmatrix} -6 \\ -1 \\ 0 \end{pmatrix} \mathbf{P} - \mathbf{S} = \begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix} \tag{3}$$

First let's check wheter the given opposite sides of the quadrilateral are parallel to each other
For the sides to be parallel

$$\mathbf{Q} - \mathbf{P} = \mathbf{S} - \mathbf{R} \tag{4}$$

$$\mathbf{R} - \mathbf{Q} = \mathbf{P} - \mathbf{S} \tag{5}$$

$$\tag{6}$$

Since:

$$\mathbf{Q} - \mathbf{P} = \mathbf{R} - \mathbf{S} = \begin{pmatrix} 6 \\ 1 \\ 0 \end{pmatrix} \tag{7}$$

$$\mathbf{R} - \mathbf{Q} = \mathbf{S} - \mathbf{P} = \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} \tag{8}$$

Therefore the opposite sides are parallel to each other and Thus the given quadrilateral can be classified as a **Parallelogram**.
Now, checking for right angle, we check for inner product.

$$(\mathbf{Q} - \mathbf{P})^\top (\mathbf{R} - \mathbf{Q}) = \begin{pmatrix} 6 & 1 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} = -3 \tag{9}$$

This implies that the parallelogram is neither a rectangle nor a square
Checking for a rhombus:
The given quadrilateral is a rhombus if its diagonals are orthogonal,

$$(\mathbf{R}-\mathbf{P})^{\top}(\mathbf{S}-\mathbf{Q}) = \begin{pmatrix}5 & 4 & 0\end{pmatrix}\begin{pmatrix}-7\\2\\0\end{pmatrix} = -27 \tag{10}$$

We can see that $(\mathbf{R}-\mathbf{P})^{\top}(\mathbf{S}-\mathbf{Q})$ is not equal to 0, that is, the diagonals are not orthogonal and therefore the quadrilateral **PQRS** is a parallelogram which is neither a rhombus nor a rectangle.

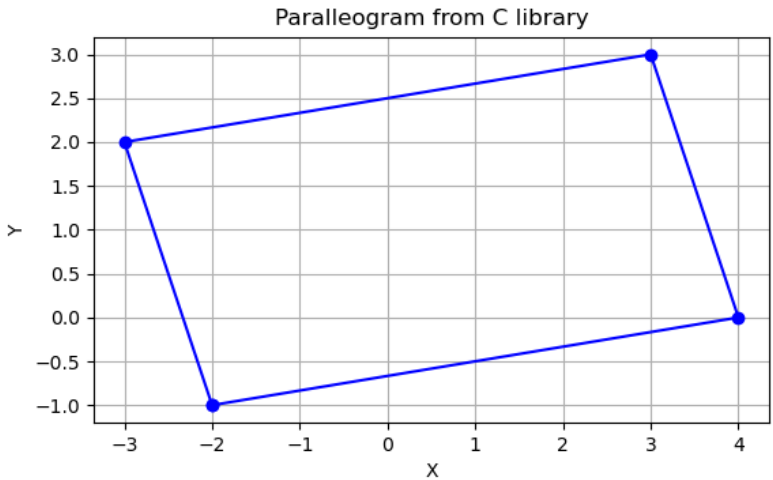

Fig. 4. paralleogram