4.11.7

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Question The equations to a pair of opposite sides of a parallelogram are $x^2-5x+6=0$ and $y^2-6y+5=0$. The equations to its diagonals are:

1) $x+4y=13, y=4x-7$
2) $4x+y=13, y=4x-7$
3) $4x+y=13, 4y=x-7$
4) $y-4x=13, y+4x=7$

Solution Given details Equation 1:

$$x^2-5x+6=0 \tag{1}$$

$$\text{This equation can be factored into:} \tag{2}$$

$$(x-2)(x-3)=0 \tag{3}$$

$$\text{This gives us two vertical lines:} \tag{4}$$

$$x=2 \tag{5}$$

$$x=3 \tag{6}$$

Equation 2:

$$y^2-6y+5=0 \tag{7}$$

$$\text{This equation can be factored into:} \tag{8}$$

$$(y-1)(y-5)=0 \tag{9}$$

$$\text{This gives us two horizontal lines:} \tag{10}$$

$$y=1 \tag{11}$$

$$y=5 \tag{12}$$

Through the intersection of these 4 lines we can find the 4 vertices of the parallelogram:

$$\text{Intersection of } x=2 \text{ and } y=1 \text{ is the point A}\,(2,1)\,. \tag{13}$$

$$\text{Intersection of } x=3 \text{ and } y=1 \text{ is the point B}\,(3,1)\,. \tag{14}$$

$$\text{Intersection of } x=3 \text{ and } y=5 \text{ is the point C}\,(3,5)\,. \tag{15}$$

$$\text{Intersection of } x=2 \text{ and } y=5 \text{ is the point D}\,(2,5)\,. \tag{16}$$

The equations of the diagonals can be found using the vector equation of a line, which is given by $\mathbf{r}(t)=\mathbf{p_0}+t\mathbf{d}$, where $\mathbf{p_0}$ is a starting point and $\mathbf{d}$ is the direction vector.

The equation of the diagonal **AC**, passing through **A** (2, 1) and **C** (3, 5), is:

$$\text{The starting point vector is } \mathbf{p_0} = \mathbf{A} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{17}$$

$$\text{The direction vector is } \mathbf{d} = \mathbf{C} - \mathbf{A} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \tag{18}$$

$$\text{The vector equation is } \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 4 \end{pmatrix} \tag{19}$$

$$\text{From this, we get } x = 2 + t \implies t = x - 2 \tag{20}$$

$$\text{And } y = 1 + 4t \tag{21}$$

$$\text{Substituting for t: } y = 1 + 4(x - 2) \tag{22}$$

$$y = 1 + 4x - 8 \tag{23}$$

$$y = 4x - 7 \tag{24}$$

The equation of the diagonal **BD**, passing through **B** (3, 1) and **D** (2, 5), is:

$$\text{The starting point vector is } \mathbf{p_0} = \mathbf{B} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \tag{25}$$

$$\text{The direction vector is } \mathbf{d} = \mathbf{D} - \mathbf{B} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} \tag{26}$$

$$\text{The vector equation is } \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} + t \begin{pmatrix} -1 \\ 4 \end{pmatrix} \tag{27}$$

$$\text{From this, we get } x = 3 - t \implies t = 3 - x \tag{28}$$

$$\text{And } y = 1 + 4t \tag{29}$$

$$\text{Substituting for t: } y = 1 + 4(3 - x) \tag{30}$$

$$y = 1 + 12 - 4x \tag{31}$$

$$4x + y = 13 \tag{32}$$

Therefore the equations of both the diagonals are:

$$y = 4x - 7 \tag{33}$$

$$4x + y = 13 \tag{34}$$

Hence the answer is option 2.

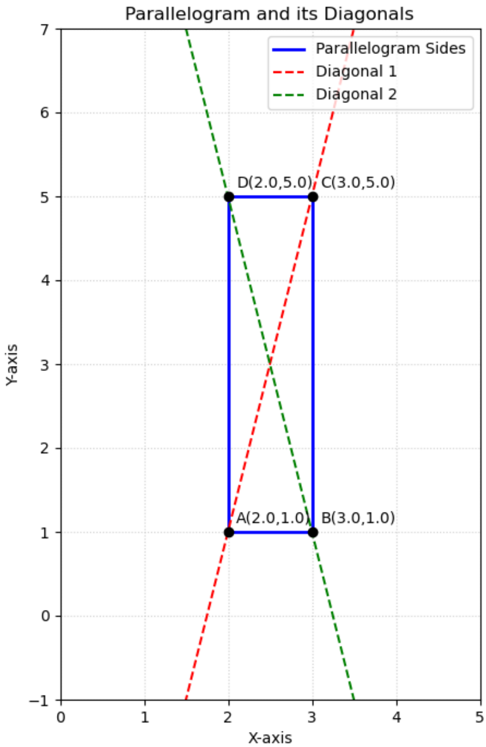

Fig. 4. diagonals