Simulation & Modeling Techniques

HW3: Modeling & Quadratic Growth

Libraries

In [1]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import scipy
import sympy
import pint
UNITS = pint.UnitRegistry()
import modsim
from modsim import*
```

0. Preliminary work

Here, we are going to load the data and prepare it for tasks we will do in this assignment.

The code here are the same used in the last assignment where we modeled population growth.

In [2]:
```
# Read data
tables = pd.read_html('World_population_estimates.html', header=0, index_col=0
, decimal="M")
len(tables)

# Get the 3rd element of the tables
table2 = tables[2]

# Replace new columns names
table2.columns = ['census', 'prb', 'un', 'maddison',
                  'hyde', 'tanton', 'biraben', 'mj',
                  'thomlinson', 'durand', 'clark']

# Write data in billion

census = table2.census
un = table2.un

census = census / 1e9
un = un / 1e9
```

In [3]:
```
# Calculate elapsed time

t_0 = get_first_label(census)
t_end = get_last_label(census)
elapsed_time = t_end - t_0

# Calculate growth

p_0 = get_first_value(census)
p_end = get_last_value(census)
total_growth = p_end - p_0

# Annual growth

annual_growth = total_growth / elapsed_time
```

1. Proportional growth (Chap. 6)

Define the system and growth rate

In [4]:
```
# Set different birth and death rate

## Before 1980:
birth_rate1 = 0.0310
death_rate1 = 0.0120

## After 1980:
birth_rate2 = 0.0370
death_rate2 = 0.0223

# Define growth rates: aplha1, before 1980 and alpha2, after 1980

alpha1 = birth_rate1 - death_rate1
alpha2 = birth_rate2 - death_rate2


# System with alpha1 and alpha2

system = System(t_0 = t_0,
                t_end = t_end,
                p_0 = p_0,
                alpha1 = alpha1,
                alpha2 = alpha2)
```

Write the updated function

In [5]:
```
def update_func(pop, t, system):
    """ Compute the population next year.
    pop: current population
    t: current year
    system: system object containing parameters of the model
    returns: population next year depending on which year t falls
    """
    t_cutoff = 1980 # cutoff t

    # Set condition to pick a rate depending to where t falls

    if t < t_cutoff:
        net_growth = system.alpha1 * pop

    else:
        net_growth = system.alpha2 * pop

    return pop + net_growth
```

run_simulation function

In [6]:
```
def run_simulation(system, update_func):
    """Simulate the system using any updtate function

    system: System object
    update_func: function that computes the population next year

    returns: TimeSeries
    """

    results = TimeSeries()
    results[system.t_0] = system.p_0

    for t in linrange(system.t_0, system.t_end):
        results[t+1] = update_func(results[t], t, system)

    return results
```

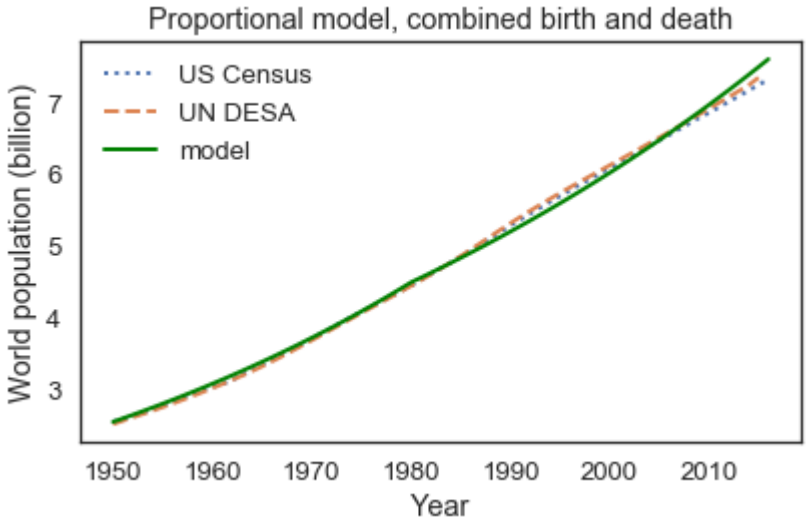
Test the results

Plot the results

In [7]:
```
def plot_results(census, un, timeseries, title):
    """Plot the estimates and the model.

    census: TimeSeries of population estimates
    un: TimeSeries of population estimates
    timeseries: TimeSeries of simulation results
    title: string
    """

    plot(census, ':', label="US Census")
    plot(un, '--', label='UN DESA')
    plot(timeseries, color='green', label='model')

    decorate(xlabel='Year',
             ylabel='World population (billion)',
             title=title)
```

Run the simulation

In [8]:
```
results = run_simulation(system, update_func)
plot_results(census, un, results, 'Proportional model, combined birth and deat
h')
```



2. Quadratic growth (Chap. 7)

We are going to use same functions as defined previously with some modifications adapted to quadratic model.

Define parameters

In [9]:
```
alpha = 0.025
beta = -0.0018

# Max growth rate
r = alpha

# carrying capacity
K = -alpha / beta
```

First parameterization method: Book example

In [10]:
```
sys1 = System(t_0 = t_0,
              t_end = t_end,
              p_0 = p_0,
              p_end = p_end,
              alpha = alpha,
              beta = beta)
```

In [11]:
```
def update_func1(pop, t, sys1):

    net_growth = sys1.alpha * pop + sys1.beta * pop**2

    return pop + net_growth
```

Second system: New parameterization with carrying capacity

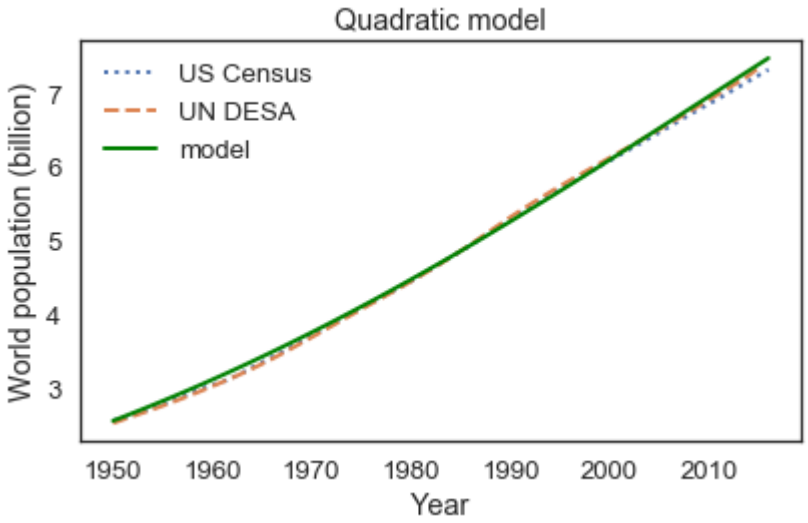
In [12]:
```
sys2 = System(t_0 = t_0,
              t_end = t_end,
              p_0 = p_0,
              p_end = p_end,
              r = r,
              K = K)
```

Updated function

In [13]:
```
def update_func2(pop, t, sys2):

    p = pop

    delta_p = sys2.r*p*(1 - p/sys2.K) # population net growth

    return p + delta_p
```

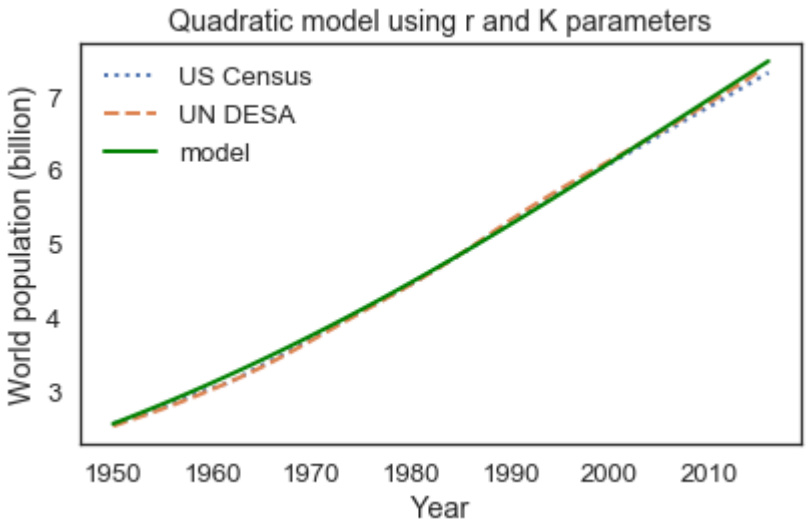
run_simulation & plot: first parameterization

In [14]:
```
results = run_simulation(sys1, update_func1)
plot_results(census, un, results, 'Quadratic model')
```



run_simulation & plot: second parameterization with carrying capacity

In [15]:
```
results = run_simulation(sys2, update_func2)
plot_results(census, un, results, 'Quadratic model using r and K parameters')
```



We can confirm that we get the same results from both parameterization methods