

Teambook Sindicato de Transporte 2880

Universidad Mayor de San Simón



2880

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Chapter 1

Introduction

The following document represents the Teambook for the team Sindicato de Transporte 2880. This version was elaborated for the Latin American Regional phase of 2022's ICPC.

The template for the C++ code is presented:

```
#include <bits/stdc++.h>
// #include <ext/pb_ds/assoc_container.hpp>
// #include <ext/pb_ds/assoc_container.hpp>
// #include <ext/pb_ds/tree_policy.hpp>
// #include <ext/rope>
#define int ll
#define mp      make_pair
#define pb      push_back
#define all(a)   (a).begin(), (a).end()
#define sz(a)    (int)a.size()
#define eq(a, b) (fabs(a - b) < EPS)
#define md(a, b) ((a) % b + b) % b
#define mod(a)   md(a, MOD)
#define _max(a, b) ((a) > (b) ? (a) : (b))
#define srt(a)   sort(all(a))
#define mem(a, h) memset(a, (h), sizeof(a))
#define f        first
#define s        second
#define forn(i, n) for(int i = 0; i < n; i++)
#define fore(i, b, e) for(int i = b; i < e; i++)
#define forg(i, b, e, m) for(int i = b; i < e; i+=m)
#define index int mid = (b + e) / 2, l = node * 2 + 1, r = l + 1;
#define DBG(x) cerr<<#x<<"_="<<(x)<<endl
#define RAYA cout<<"===== "<<'\\n'
```

```
// int in(){int r=0,c;for(c=getchar();c<=32;c=getchar());if(c=='-')
return -in();for(;c>32;r=(r<<1)+(r<<3)+c-'0',c=getchar());
return r;}
```

```
using namespace std;
// using namespace __gnu_pbds;
// using namespace __gnu_cxx;

// #pragma GCC target ("avx2")
// #pragma GCC optimization ("O3")
// #pragma GCC optimization ("unroll-loops")

typedef long long ll;
typedef long double ld;
typedef unsigned long long ull;
typedef pair<int, int> ii;
typedef pair<pair<int, int>, int> iii;
typedef vector<int> vi;
typedef vector<ii> vii;
typedef vector<ll> vll;
// typedef tree<int,null_type,less<int>,rb_tree_tag,
// tree_order_statistics_node_update> ordered_set;
// find_by_order kth largest order_of_key <
// mt19937 rng(chrono::steady_clock::now().time_since_epoch().count
// ());
// rng
const int tam = 200010;
const int MOD = 1000000007;
```

```
const int MOD1 = 998244353;
const double DINF=1e100;
const double EPS = 1e-9;
const long double PI = acos(-1.0L);

signed main()
{
    ios::sync_with_stdio(0); cin.tie(0); cout.tie(0);
    // freopen("asd.txt", "r", stdin);
    // freopen("qwe.txt", "w", stdout);

    return 0;
}
```

../Template.cpp

Chapter 2

Mathematics

This chapter is about some useful mathematical tools needed in order to solve problems.

2.1 GCD and LCM

In order to find the greatest common divisor (GCD) of two numbers, the Euclidean algorithm can be used. The implementation is as follows:

```
11 gcd(11 a, 11 b){return b==0? a:gcd(b,a%b);}

int x, y, d;
void extendedEuclid(int a, int b)//ecuacion diofantica ax + by = d
{
    if(b==0) {x=1; y=0; d=a; return;}
    extendedEuclid(b,a%b);
    int x1=y;
    y = x-(a/b)*y;
    x=x1;
}
```

../Mathematics/Euclid.cpp

Another (and faster) way to find the GCD is by using the following code:

```
int gcd(int a, int b) {
    if (!a || !b)
        return a | b;
    unsigned shift = __builtin_ctz(a | b);
    a >>= __builtin_ctz(a);
```

```
do {
    b >>= __builtin_ctz(b);
    if (a > b)
        swap(a, b);
    b -= a;
} while (b);
return a << shift;
}
```

../Mathematics/FastGCD.cpp

The way Halim suggests to find the GCD and the LCM is given by the following code:

```
int gcd(int a, int b) { return b == 0 ? a : gcd(b, a%b); }
int lcm(int a, int b) { return a / gcd(a, b) * b; }
```

../Mathematics/HalimGCD.cpp

2.2 Prime Numbers

The fastest way to check the primality of a number is by using Erathostenes' sieve. The typical implementation is as follows:

```
bitset<100000> bi;
vi primos; //primos
vector<ll> pric; //primos al cuadrado
void criba()
{
    bi.set();
```

```

for(int i=2;i<100000;i++)
    if(bi[i])
    {
        for(int j=i+i;j<100000;j+=i)
            bi[j]=0;
        primos.push_back(i);
        pric.push_back((ll)i*(ll)i);
    }
}
int euler(int n)
{
    int res=n;
    for(int i=0;pric[i]<=n;i++)
    {
        if(n%primos[i]==0)
        {
            res-= res/primos[i];
            while(n%primos[i]==0) n/=primos[i];
        }
    }
    if(n!=1) res-=res/n;
    return res;
}

```

../Mathematics/Erathostenes.cpp

Nevertheless, the following implementation is faster, since the statement `if (i % prime[j] == 0) break;` terminates the loop when `p` divides `i`. The inner loop is executed only once for each composite. Hence, the code performs in $O(n)$ complexity, resulting in the 'linear' sieve:

```

// This algorithm allows to find Eratosthenes sieve in  $O(n \log n)$ 
// time.

std::vector<int> prime;
bool is_composite[MAXN];

void sieve (int n) {
    std::fill (is_composite, is_composite + n, false);
    for (int i = 2; i < n; ++i) {
        if (!is_composite[i]) prime.push_back (i);

```

```

        for (int j = 0; j < prime.size () && i * prime[j] < n; ++j) {
            is_composite[i * prime[j]] = true;
            if (i % prime[j] == 0) break;
        }
    }

    // An application of this linear sieve is to find the Euler
    // totient function of a number in  $O(n \log n)$  time.
    std::vector<int> prime;
    bool is_composite[MAXN];
    int phi[MAXN];

    void sieve (int n) {
        std::fill (is_composite, is_composite + n, false);
        phi[1] = 1;
        for (int i = 2; i < n; ++i) {
            if (!is_composite[i]) {
                prime.push_back (i);
                phi[i] = i - 1; //i is prime
            }
            for (int j = 0; j < prime.size () && i * prime[j] < n; ++j) {
                is_composite[i * prime[j]] = true;
                if (i % prime[j] == 0) {
                    phi[i * prime[j]] = phi[i] * prime[j]; //prime[j]
                    divides i
                    break;
                } else {
                    phi[i * prime[j]] = phi[i] * phi[prime[j]]; //prime[j]
                    does not divide i
                }
            }
        }
    }
}

```

../Mathematics/LinearSieve.cpp

2.3 Modular Arithmetic

The modular inverse is defined by the following equation:

$$a \cdot a^{-1} \equiv 1 \pmod{m} \quad (2.1)$$

The following code shows how to find the modular inverse of a number:

```
int ModPow(int a, int b, int m) {
    int res = 1;
    while (b > 0) {
        if (b & 1) res = (res * a) % m;
        a = (a * a) % m;
        b >>= 1;
    }
    return res;
}

// Language: java
public static int modPow(int a, int b, int m) {
    int res = 1;
    while (b > 0) {
        if ((b & 1) == 1) res = (res * a) % m;
        a = (a * a) % m;
        b >>= 1;
    }
    return res;
}

int ModInverse(int a, int m) {
    return ModPow(a, m - 2, m);
}
```

../Mathematics/ModularInverse.cpp

Some other useful relationships in modular arithmetic are:

- $(a + b) \pmod{m} = (a \pmod{m} + b \pmod{m}) \pmod{m}$
- $(a - b) \pmod{m} = (a \pmod{m} - b \pmod{m}) \pmod{m}$
- $(a * b) \pmod{m} = (a \pmod{m} * b \pmod{m}) \pmod{m}$
- $(a/b) \pmod{m} = (a \pmod{m} * b^{-1} \pmod{m}) \pmod{m}$
- $(a^b) \pmod{m} = (a \pmod{m})^b \pmod{m}$
- $(a^b) \pmod{m} = (a \pmod{m})^{b \pmod{\phi(m)}} \pmod{m}$

- $(a^b) \pmod{m} = (a \pmod{m})^{b \pmod{m-1}} \pmod{m}$
- $\frac{a}{k} \equiv \frac{a}{k} \pmod{m} \iff a \equiv k \pmod{m}$
- $\frac{a}{k} \equiv \frac{a}{k} \pmod{\frac{n}{\gcd(n,k)}}$

2.4 Matrix Exponentiation

The following code shows how to find the nth power of a *mat*, noting that a data structure of type matrix is defined as follows:

```
typedef vector<vector<ll>> mat;
mat ans;
void mult(mat m1, mat m2)
{
    assert(m1[0].size() == m2.size());
    ans.clear();
    ll answer = 0;
    for(i, 0, m1.size())
    {
        vector<ll> fila;
        for(j, 0, m2[0].size())
        {
            answer = 0;
            for(k, 0, m2.size())
                answer = (answer + m1[i][k] * m2[k][j]) % MOD;
            fila.pb(answer);
        }
        ans.pb(fila);
    }
}

void pot(mat base, ll exp)
{
    mat res(base.size(), vector<ll>(base.size(), 0));
    for(i, 0, base.size())
        res[i][i] = 1;
    while(exp)
    {
        if(exp & 1)
        {
            mult(res, base);
        }
    }
}
```

```
        res = ans;
    }
    mult(base, base);
    base = ans;
    exp /= 2;
}
ans = res;
}
```

../Mathematics/MatrixPower.cpp

Chapter 3

Graphs

This chapter shows some of the basic algorithms and implementations required to solve problems that include graphs.

3.1 Depth First Search (DFS)

The DFS algorithm is a recursive algorithm that visits all the nodes of a graph. It is used to find connected components, topological sorting, and to find bridges and articulation points. The algorithm is as follows:

The implementation can be done as follows:

```
vector<vector<int>> g(tam);
vector<bool> vis(tam);

void dfs(int u){
    vis[u]=true;
    ans++;
    for(int v: g[u]){
        if(!vis[v]){
            dfs(v);
        }
    }
}

signed main()
{
    int n,m;
    cin>>n>>m; // n nodes, m edges
    g.assign(tam,vector<int>());
```

```
    vis.assign(tam, false);
    for(int i=0; i<m;i++){
        int u,v;
        cin>>u>>v;
        g[u].push_back(v);
        g[v].push_back(u);
    }
    ll res = 0;
    for(int i=1; i<=n;i++){
        if(!vis[i]){
            ans=0;
            dfs(i);
            res = max(res,ans);
        }
    }
    g.clear();
    vis.clear();
    return 0;
}
```

../Graphs/DFS.cpp

An application of this algorithm in order to find the shortest path between two nodes can be done as follows:

```
// The following code represents the implementation of a DFS
// algorithm
// to find the shortest path between two nodes in a graph.
// The graph is represented as an adjacency list.
// The algorithm is implemented using a stack.
```

Algorithm 1 Depth First Search (DFS)

```

1: procedure DFS( $G$ )
2:    $visited \leftarrow \emptyset$ 
3:    $time \leftarrow 0$ 
4:    $parent \leftarrow \emptyset$ 
5:    $low \leftarrow \emptyset$ 
6:    $disc \leftarrow \emptyset$ 
7:    $AP \leftarrow \emptyset$ 
8:    $bridge \leftarrow \emptyset$ 
9:   for all  $v \in V$  do
10:     $visited[v] \leftarrow false$ 
11:     $parent[v] \leftarrow -1$ 
12:     $low[v] \leftarrow \infty$ 
13:     $disc[v] \leftarrow \infty$ 
14:   end for
15:   for all  $v \in V$  do
16:     if  $visited[v] = false$  then
17:       DFSUtil( $G, v, visited, time, parent, low, disc, AP, bridge$ )
18:     end if
19:   end for
20: end procedure
21: procedure DFSUTIL( $G, v, visited, time, parent, low, disc, AP, bridge$ )
22:    $visited[v] \leftarrow true$ 
23:    $disc[v] \leftarrow time$ 
24:    $low[v] \leftarrow time$ 
25:    $time \leftarrow time + 1$ 
26:    $children \leftarrow 0$ 
27:   for all  $u \in Adj(v)$  do
28:     if  $visited[u] = false$  then
29:        $parent[u] \leftarrow v$ 
30:        $children \leftarrow children + 1$ 
31:       DFSUtil( $G, u, visited, time, parent, low, disc, AP, bridge$ )
32:        $low[v] \leftarrow \min(low[v], low[u])$ 
33:       if  $parent[v] = -1$  and  $children > 1$  then
34:          $AP[v] \leftarrow true$ 
35:       end if
36:       if  $parent[v] \neq -1$  and  $low[u] \geq disc[v]$  then
37:          $AP[v] \leftarrow true$ 
38:       end if
39:       if  $low[u] > disc[v]$  then
40:          $bridge[v][u] \leftarrow true$ 
41:       end if
42:     else
43:        $low[v] \leftarrow \min(low[v], disc[u])$ 
44:     end if
45:   end for

```

```

#include <bits/stdc++.h>

using namespace std;

vector<int> DFS(vector<vector<int>> &adj, int s, int t) {
    stack<vector<int>> path_stack;
    vector<int> path;
    vector<int> visited(adj.size(), 0);
    path_stack.push({s});
    while (!path_stack.empty()) {
        path = path_stack.top();
        path_stack.pop();
        int last = path[path.size() - 1];
        if (last == t) {
            return path;
        }
        if (visited[last] == 0) {
            visited[last] = 1;
            for (int i = 0; i < adj[last].size(); i++) {
                if (visited[adj[last][i]] == 0) {
                    vector<int> new_path(path);
                    new_path.push_back(adj[last][i]);
                    path_stack.push(new_path);
                }
            }
        }
    }
    return {};
}

int main() {
    int n, m;
    cin >> n >> m;
    vector<vector<int>> adj(n, vector<int>());
    for (int i = 0; i < m; i++) {
        int x, y;
        cin >> x >> y;
        adj[x - 1].push_back(y - 1);
        adj[y - 1].push_back(x - 1);
    }
}

```

```

}
int x, y;
cin >> x >> y;
x--, y--;
vector<int> path = DFS(adj, x, y);
for (int i = 0; i < path.size(); i++) {
    cout << path[i] + 1 << " ";
}
}

```

../Graphs/DFS-application.cpp

3.2 Breadth First Search (BFS)

The BFS algorithm is a non-recursive algorithm that visits all the nodes of a graph. It is used to find connected components, topological sorting, and to find bridges and articulation points, to better understand it, a propagating fire can be imagined. The algorithm is as follows:

The implementation can be done as follows:

```

#include <bits/stdc++.h>

using namespace std;
signed main()
{
    vector<vector<int>> adj; // adjacency list representation
    int n; // number of nodes
    int s; // source vertex

    queue<int> q;
    vector<bool> used(n);
    vector<int> d(n), p(n);

    q.push(s);
    used[s] = true;
    p[s] = -1;
    while (!q.empty()) {
        int v = q.front();
        q.pop();
        for (int u : adj[v]) {
            if (!used[u]) {

```

Algorithm 2 Breadth First Search (BFS)

```

1: procedure BFS( $G$ )
2:    $visited \leftarrow \emptyset$ 
3:    $time \leftarrow 0$ 
4:    $parent \leftarrow \emptyset$ 
5:    $low \leftarrow \emptyset$ 
6:    $disc \leftarrow \emptyset$ 
7:    $AP \leftarrow \emptyset$ 
8:    $bridge \leftarrow \emptyset$ 
9:   for all  $v \in V$  do
10:     $visited[v] \leftarrow false$ 
11:     $parent[v] \leftarrow -1$ 
12:     $low[v] \leftarrow \infty$ 
13:     $disc[v] \leftarrow \infty$ 
14:   end for
15:   for all  $v \in V$  do
16:     if  $visited[v] = false$  then
17:       BFSUtil( $G, v, visited, time, parent, low, disc, AP, bridge$ )
18:     end if
19:   end for
20: end procedure
21: procedure BFSUtil( $G, v, visited, time, parent, low, disc, AP, bridge$ )
22:    $visited[v] \leftarrow true$ 
23:    $disc[v] \leftarrow time$ 
24:    $low[v] \leftarrow time$ 
25:    $time \leftarrow time + 1$ 
26:    $children \leftarrow 0$ 
27:   for all  $u \in Adj(v)$  do
28:     if  $visited[u] = false$  then
29:        $parent[u] \leftarrow v$ 
30:        $children \leftarrow children + 1$ 
31:       BFSUtil( $G, u, visited, time, parent, low, disc, AP, bridge$ )
32:        $low[v] \leftarrow \min(low[v], low[u])$ 
33:       if  $parent[v] = -1$  and  $children > 1$  then
34:          $AP[v] \leftarrow true$ 
35:       end if
36:       if  $parent[v] \neq -1$  and  $low[u] \geq disc[v]$  then
37:          $AP[v] \leftarrow true$ 
38:       end if
39:       if  $low[u] > disc[v]$  then
40:          $bridge[v][u] \leftarrow true$ 
41:       end if
42:     else
43:        $low[v] \leftarrow \min(low[v], disc[u])$ 
44:     end if
45:   end for

```

```

        used[u] = true;
        q.push(u);
        d[u] = d[v] + 1;
        p[u] = v;
    }
}
return 0;
}

```

../Graphs/BFS.cpp

3.3 Flows

One of the useful flows algorithms is Dinic, which allows to find the maximum flow that could go through the system of nodes. This code is initially done for a directed graph with many nodes. The implementation of this algorithm is:

```

struct flowEdge
{
    int to, rev, f, cap;
};

vector<vector<flowEdge>> G;

void addEdge(int st, int en, int cap) {
    // Anade arista (st --> en) con su capacidad
    flowEdge A = {en, (int)G[en].size(), 0, cap};
    flowEdge B = {st, (int)G[st].size(), 0, 0};
    G[st].pb(A);
    G[en].pb(B);
}

int nodes, S, T; // asignar estos valores al armar el grafo G
                // nodes = nodos en red de flujo. Hacer G.clear();
    G.resize(nodes);
vi work, lvl;
int Q[200010];

bool bfs() {

```

```

    int qt = 0;
    Q[qt++] = S;
    lvl.assign(nodes, -1);
    lvl[S] = 0;
    for (int qh = 0; qh < qt; qh++) {
        int v = Q[qh];
        for (flowEdge &e : G[v]) {
            int u = e.to;
            if (e.cap <= e.f || lvl[u] != -1) continue;
            lvl[u] = lvl[v] + 1;
            Q[qt++] = u;
        }
    }
    return lvl[T] != -1;
}

int dfs(int v, int f) {
    if (v == T || f == 0) return f;
    for (int &i = work[v]; i < G[v].size(); i++) {
        flowEdge &e = G[v][i];
        int u = e.to;
        if (e.cap <= e.f || lvl[u] != lvl[v] + 1) continue;
        int df = dfs(u, min(f, e.cap - e.f));
        if (df) {
            e.f += df;
            G[u][e.rev].f -= df;
            return df;
        }
    }
    return 0;
}

int maxFlow() {
    int flow = 0;
    while (bfs()) {
        work.assign(nodes, 0);
        while (true) {
            int df = dfs(S, INF);
            if (df == 0) break;
            flow += df;
        }
    }
}

```

```
    }  
    return flow;  
}
```

../Graphs/Dinic.cpp