Vehicle Suspension System Modelling

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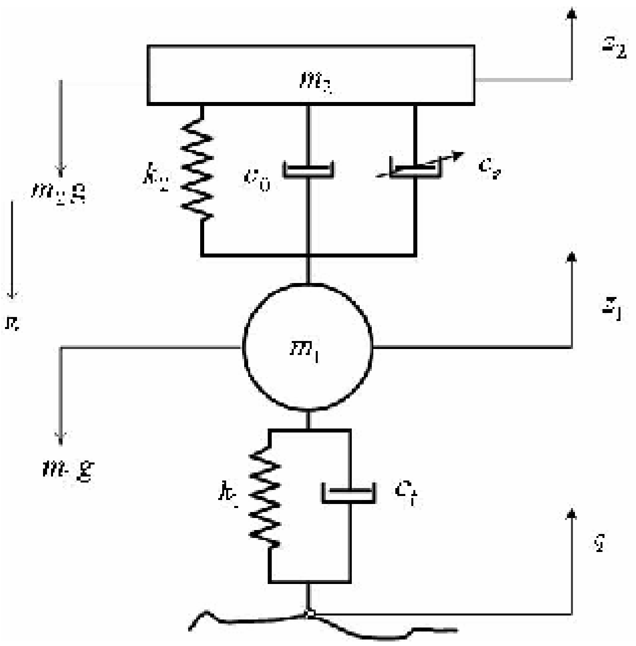
2-DOFs suspension system

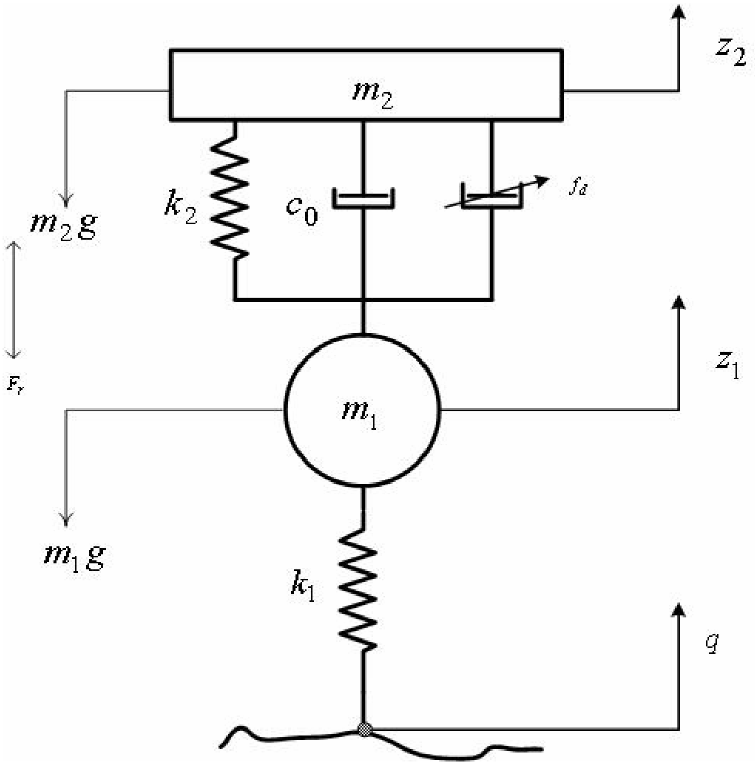


Dynamics formula:

$$\begin{cases} m_1 \ddot{z}_1 + (c_0 + c_e)\left(\dot{z}_1 - \dot{z}_2\right) + k_2(z_1 - z_2) + k_1(z_1 - q) + c_t\left(\dot{z}_1 - \dot{q}\right) - m_1 g - F_r = 0 \\ m_2 \ddot{z}_2 + (c_0 + c_e)\left(\dot{z}_2 - \dot{z}_1\right) + k_2(z_2 - z_1) - m_2 g = 0 \end{cases}$$

to Let $c_e\left(\dot{z}_2 - \dot{z}_1\right) = f_d$,

and to Ignore $c_t\left(\dot{z}_1 - \dot{q}\right) = F_t$, then:



$$\begin{cases} m_1 \ddot{z}_1 = -c_0\left(\dot{z}_1 - \dot{z}_2\right) - k_2\left(z_1 - z_2\right) - k_1\left(z_1 - q\right) + f_d - F_r + m_1 g \\ m_2 \ddot{z}_2 = -c_0\left(\dot{z}_2 - \dot{z}_1\right) - k_2\left(z_2 - z_1\right) - f_d + F_r + m_2 g = 0 \end{cases} \qquad (*)$$

Into matrix format:

$$[M]\left\{\ddot{Z}\right\} + [C_0]\left\{\dot{Z}\right\} + [K]\left\{Z\right\} + F_d = \{Q\}$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{z}_1 \\ \ddot{z}_2 \end{Bmatrix} + \begin{bmatrix} c_0 & -c_0 \\ -c_0 & c_0 \end{bmatrix}\begin{Bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{Bmatrix} z_1 \\ z_2 \end{Bmatrix} + \begin{Bmatrix} -f_d + F_r - m_1 g \\ f_d - F_r - m_2 g \end{Bmatrix} = \begin{Bmatrix} k_1 q \\ 0 \end{Bmatrix}$$

where:

$$M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \quad C_0 = \begin{bmatrix} c_0 & -c_0 \\ -c_0 & c_0 \end{bmatrix}$$

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \; F_d = \begin{Bmatrix} -f_d + F_r - m_1 g \\ f_d - F_r - m_2 g \end{Bmatrix} \quad Q = \begin{Bmatrix} k_1 q \\ 0 \end{Bmatrix}$$

F_r is a constant friction

State space:

$$\begin{cases} \dot{X} = AX + BQ + EU \\ Y = CX + DQ + FU \end{cases}$$

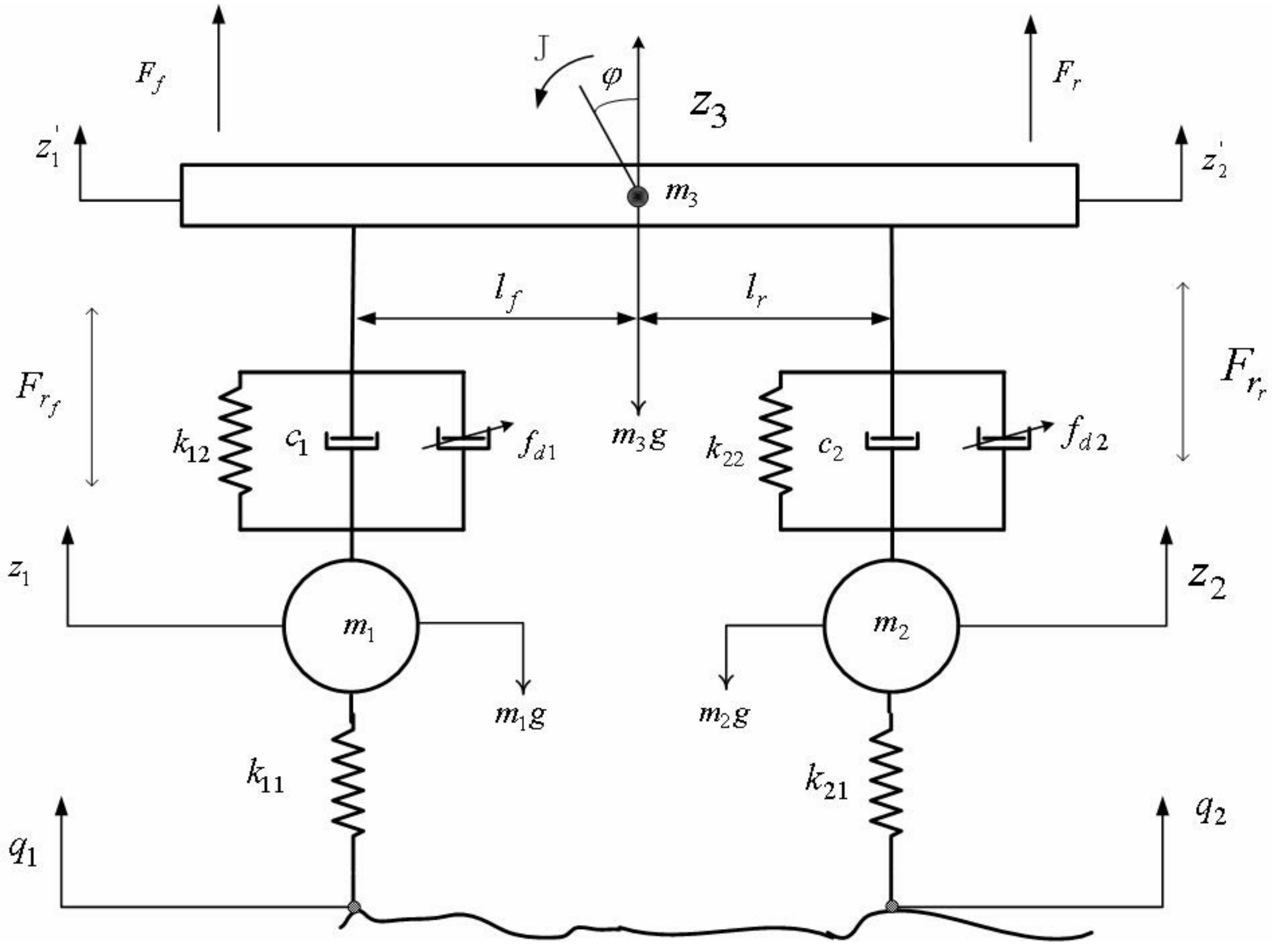
where:

$$X = \begin{Bmatrix} z_1 - q \\ z_2 - z_1 \\ \dot{z}_1 \\ \dot{z}_2 \end{Bmatrix} \quad Y = \begin{Bmatrix} \ddot{z}_2 \\ z_1 - q \\ z_2 - z_1 \end{Bmatrix} \quad Q = \begin{Bmatrix} \dot{q} \\ g \\ F_r \end{Bmatrix} \quad U = \{f_d\}$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ \frac{-k_1}{m_1} & \frac{k_2}{m_1} & \frac{-c_0}{m_1} & \frac{c_0}{m_1} \\ 0 & \frac{-k_2}{m_2} & \frac{c_0}{m_2} & \frac{-c_0}{m_2} \end{bmatrix} \quad B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & \frac{-1}{m_1} \\ 0 & 1 & \frac{1}{m_2} \end{bmatrix} \quad C = \begin{bmatrix} 0 & \frac{-k_2}{m_2} & \frac{c_0}{m_2} & \frac{-c_0}{m_2} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_1} \\ \frac{-1}{m_2} \end{bmatrix} \quad F = \begin{bmatrix} \frac{-1}{m_2} \\ 0 \\ 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 1 & \frac{1}{m_2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

4–DOFs suspension system -- Pitch



$$
\begin{cases}
m_1 \ddot{z}_1 = k_{11}(z_1 - q_1) + k_{12}\left(z_1^{'} - z_1\right) + c_1\left(\dot{z}_1^{'} - \dot{z}_1\right) + F_{r_f} + f_{d1} + m_1 g \\
m_2 \ddot{z}_2 = k_{21}(z_2 - q_2) + k_{22}\left(z_2^{'} - z_2\right) + c_2\left(\dot{z}_2^{'} - \dot{z}_2\right) + F_{r_r} + f_{d2} + m_2 g \\
m_3 \ddot{z}_3 = k_{12}\left(z_1 - z_1^{'}\right) + k_{22}\left(z_2 - z_2^{'}\right) + c_1\left(\dot{z}_1 - \dot{z}_1^{'}\right) + c_2\left(\dot{z}_2 - \dot{z}_2^{'}\right) - f_{d1} - f_{d2} - F_{r_f} - F_{r_r} + m_3 g \\
J\ddot{j} = -\left[k_{12}\left(z_1 - z_1^{'}\right) + c_1\left(\dot{z}_1 - \dot{z}_1^{'}\right)\right] l_f + \left[k_{22}\left(z_2 - z_2^{'}\right) + c_2\left(\dot{z}_2 - \dot{z}_2^{'}\right)\right] l_r - l_f f_{d1} + l_f F_{r_f} + l_r f_{d2} - l_r F_{r_r}
\end{cases}
$$

where:

$$z_1^{'} = z_3 - j\, l_f$$

$$z_2^{'} = z_3 + j\, l_r$$

State space:

$$\begin{cases} \dot{X} = AX + BQ + EU \\ Y = CX + DQ + FU \end{cases}$$

where:

$$X = \begin{Bmatrix} z_1 - z_1^{'} \\ z_2 - z_2^{'} \\ q_1 - z_1 \\ q_2 - z_2 \\ \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{j} \end{Bmatrix} \quad Y = \begin{Bmatrix} \ddot{z}_3 \\ z_1 - z_1^{'} \\ z_2 - z_2^{'} \\ q_1 - z_1 \\ q_2 - z_2 \end{Bmatrix} \quad U = \begin{Bmatrix} f_{d1} \\ f_{d2} \end{Bmatrix} \quad Q = \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ g \\ F_{r_f} \\ F_{r_r} \end{Bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & -1 & l_f \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & -l_r \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ \frac{-k_{12}}{m_1} & 0 & \frac{-k_{11}}{m_1} & 0 & \frac{-c_1}{m_1} & 0 & \frac{c_1}{m_1} & \frac{-c_1 l_f}{m_1} \\ 0 & \frac{-k_{22}}{m_2} & 0 & \frac{-k_{21}}{m_2} & 0 & \frac{-c_2}{m_2} & \frac{c_2}{m_2} & \frac{c_2}{m_2} \\ \frac{k_{12}}{m_3} & \frac{k_{22}}{m_3} & 0 & 0 & \frac{c_1}{m_3} & \frac{c_2}{m_3} & \frac{-(c_1 + c_2)}{m_3} & \frac{l_f c_1 - l_r c_2}{m_3} \\ -\frac{k_{12} l_f}{J} & \frac{k_{22} l_r}{J} & 0 & 0 & \frac{-c_1 l_f}{J} & \frac{c_2 l_r}{J} & \frac{c_1 l_f - c_2 l_r}{J} & \frac{-\left(c_1 l_f^2 + c_2 l_r^2\right)}{J} \end{bmatrix}$$

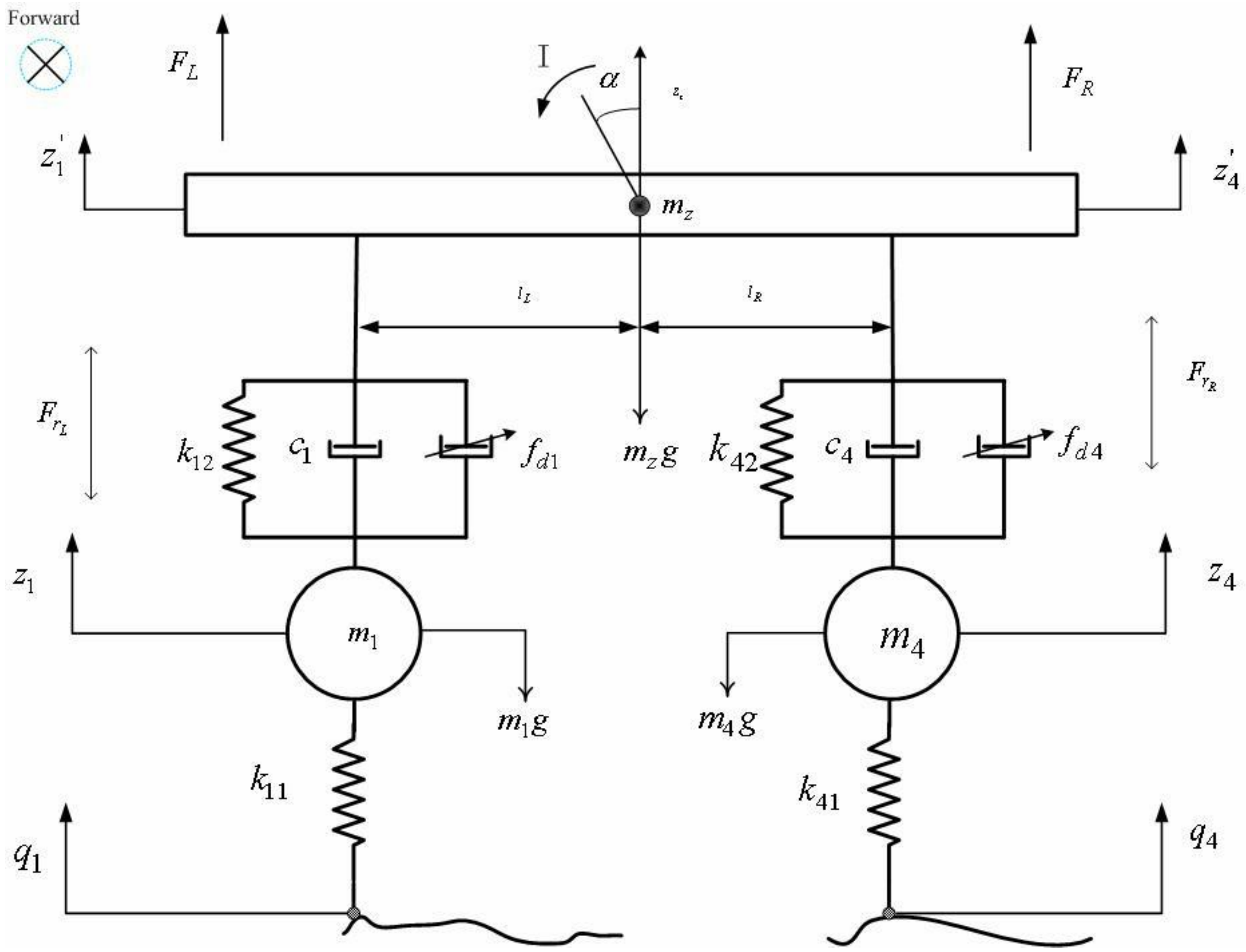
F_{r_f} is a constant friction for front half suspension

F_{r_r} is a constant friction for rear half suspension

$$B=\begin{bmatrix}0&0&0&0&0\\0&0&0&0&0\\1&0&0&0&0\\0&1&0&0&0\\0&0&1&1&0\\0&0&1&0&1\\0&0&1&-1&-1\\0&0&0&l_f&-l_r\end{bmatrix}\quad D=\begin{bmatrix}0&0&1&-1&-1\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}\quad E=\begin{bmatrix}0&0\\0&0\\0&0\\0&0\\\frac{1}{m_1}&0\\0&\frac{1}{m_2}\\-\frac{1}{m_3}&-\frac{1}{m_3}\\\frac{-l_f}{J}&\frac{l_r}{J}\end{bmatrix}$$

$$C=\begin{bmatrix}\frac{k_{12}}{m_3}&\frac{k_{22}}{m_3}&0&0&\frac{c_1}{m_3}&\frac{c_2}{m_3}&\frac{-(c_1+c_2)}{m_3}&\frac{l_fc_1-l_rc_2}{m_3}\\1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\end{bmatrix}\quad F=\begin{bmatrix}\frac{1}{m_3}&\frac{1}{m_3}\\0&0\\0&0\\0&0\\0&0\end{bmatrix}$$

4–DOFs suspension system – Roll



$$
\begin{cases}
m_1 \ddot{z}_1 = k_{11}(z_1 - q_1) + k_{12}(z_1' - z_1) + c_1\left(\dot{z}_1' - \dot{z}_1\right) + F_{r_L} + f_{d1} + m_1 g \\
m_4 \ddot{z}_4 = k_{41}(z_4 - q_4) + k_{42}(z_4' - z_4) + c_4\left(\dot{z}_4' - \dot{z}_4\right) + F_{r_R} + f_{d4} + m_4 g \\
m_z \ddot{z}_z = k_{12}(z_1 - z_1') + k_{42}(z_4 - z_4') + c_1\left(\dot{z}_1 - \dot{z}_1'\right) + c_4\left(\dot{z}_4 - \dot{z}_4'\right) - f_{d1} - f_{d4} - F_{r_L} - F_{r_R} + m_z g \\
I\ddot{a} = -\left[k_{12}(z_1 - z_1') + c_1\left(\dot{z}_1 - \dot{z}_1'\right)\right] l_L + \left[k_{42}(z_4 - z_4') + c_4\left(\dot{z}_4 - \dot{z}_4'\right)\right] l_R - l_L f_{d1} + l_L F_{r_L} + l_R f_{d4} - l_R F_{r_R}
\end{cases}
$$

where:

$$z_1' = z_z - a l_L$$

$$z_4' = z_z + a l_R$$

State space:

$$\begin{cases} \dot{X} = AX + BQ + EU \\ Y = CX + DQ + FU \end{cases}$$

and:

$$X = \begin{Bmatrix} z_1 - z_1' \\ z_4 - z_4' \\ q_1 - z_1 \\ q_4 - z_4 \\ \dot{z}_1 \\ \dot{z}_4 \\ \dot{z}_z \\ \dot{a} \end{Bmatrix} \qquad Y = \begin{Bmatrix} \ddot{z}_z \\ z_1 - z_1' \\ z_4 - z_4' \\ q_1 - z_1 \\ q_4 - z_4 \end{Bmatrix} \qquad U = \begin{Bmatrix} f_{d1} \\ f_{d4} \end{Bmatrix} \qquad Q = \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_4 \\ g \\ F_{r_L} \\ F_{r_R} \end{Bmatrix}$$

$$C = \begin{bmatrix} \frac{k_{12}}{m_z} & \frac{k_{42}}{m_z} & 0 & 0 & \frac{c_1}{m_z} & \frac{c_4}{m_z} & \frac{-(c_1+c_4)}{m_z} & \frac{l_L c_1 - l_R c_4}{m_z} \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & l_L \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & -l_R \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & -1 & l_f \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & -l_r \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ \frac{-k_{12}}{m_1} & 0 & \frac{-k_{11}}{m_1} & 0 & \frac{-c_1}{m_1} & 0 & \frac{c_1}{m_1} & \frac{-c_1 l_f}{m_1} \\ 0 & \frac{-k_{42}}{m_4} & 0 & \frac{-k_{41}}{m_4} & 0 & \frac{-c_4}{m_4} & \frac{c_4}{m_4} & \frac{c_4}{m_4} \\ \frac{k_{12}}{m_z} & \frac{k_{42}}{m_z} & 0 & 0 & \frac{c_1}{m_z} & \frac{c_4}{m_z} & \frac{-(c_1+c_4)}{m_z} & \frac{l_L c_1 - l_R c_4}{m_z} \\ -\frac{k_{12} l_L}{I} & \frac{k_{42} l_R}{I} & 0 & 0 & \frac{-c_1 l_L}{I} & \frac{c_4 l_R}{I} & \frac{c_1 l_L - c_4 l_R}{I} & \frac{-\left(c_1 l_L^2 + c_4 l_R^2\right)}{I} \end{bmatrix}$$

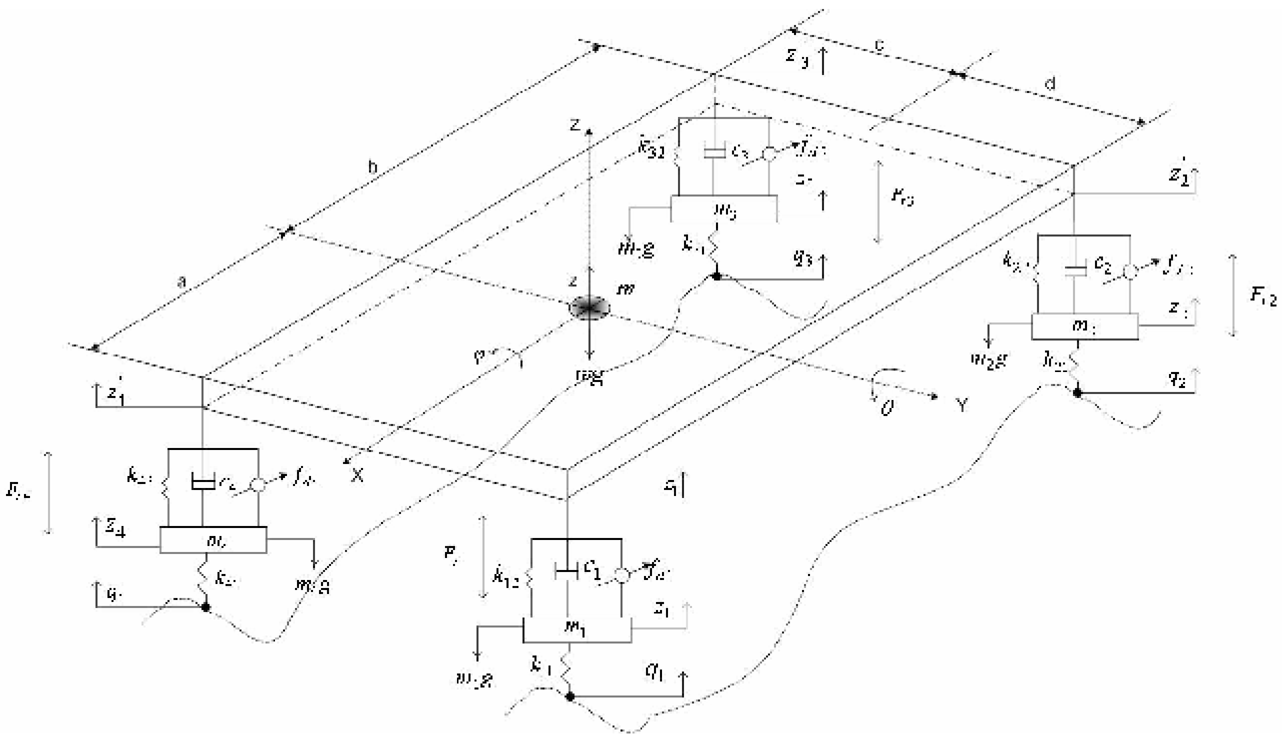
$$B=\begin{bmatrix}0&0&0&0&0\\0&0&0&0&0\\1&0&0&0&0\\0&1&0&0&0\\0&0&1&1&0\\0&0&1&0&1\\0&0&1&-1&-1\\0&0&0&l_L&-l_R\end{bmatrix}\quad D=\begin{bmatrix}0&0&1&-1&-1\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}\quad E=\begin{bmatrix}0&0\\0&0\\0&0\\0&0\\\frac{1}{m_1}&0\\0&\frac{1}{m_4}\\-\frac{1}{m_z}&-\frac{1}{m_z}\\\frac{-l_L}{I}&\frac{l_R}{I}\end{bmatrix}$$

$$C=\begin{bmatrix}\frac{k_{12}}{m_z}&\frac{k_{42}}{m_z}&0&0&\frac{c_1}{m_z}&\frac{c_4}{m_z}&\frac{-(c_1+c_4)}{m_z}&\frac{l_Lc_1-l_Rc_4}{m_z}\\1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\end{bmatrix}\quad F=\begin{bmatrix}\frac{1}{m_z}&\frac{1}{m_z}\\0&0\\0&0\\0&0\\0&0\end{bmatrix}$$

F_{r_L} is a constant friction for Left half suspension

F_{r_R} is a constant friction for Right half suspension

7 DOFs suspension system



$$
\begin{cases}
m_1 \ddot{z}_1 = k_{11}(q_1 - z_1) + k_{12}\left(z_1' - z_1\right) + c_1\left(\dot{z}_1' - \dot{z}_1\right) + f_{d1} + F_{r1} + m_1 g \\
m_2 \ddot{z}_2 = k_{21}(q_2 - z_2) + k_{22}\left(z_2' - z_2\right) + c_2\left(\dot{z}_2' - \dot{z}_2\right) + f_{d2} + F_{r2} + m_2 g \\
m_3 \ddot{z}_3 = k_{31}(q_3 - z_3) + k_{32}\left(z_3' - z_3\right) + c_3\left(\dot{z}_3' - \dot{z}_3\right) + f_{d3} + F_{r3} + m_3 g \\
m_4 \ddot{z}_4 = k_{41}(q_4 - z_4) + k_{42}\left(z_4' - z_4\right) + c_4\left(\dot{z}_4' - \dot{z}_4\right) + f_{d4} + F_{r4} + m_4 g \\
m \ddot{z} = k_{12}\left(z_1 - z_1'\right) + k_{22}\left(z_2 - z_2'\right) + k_{32}\left(z_3 - z_3'\right) + k_{42}\left(z_4 - z_4'\right) + c_1\left(\dot{z}_1 - \dot{z}_1'\right) + c_2\left(\dot{z}_2 - \dot{z}_2'\right) + c_3\left(\dot{z}_3 - \dot{z}_3'\right) + c_4\left(\dot{z}_4 - \dot{z}_4'\right) \\
- f_{d1} - f_{d2} - f_{d3} - f_{d4} + mg - F_{r1} - F_{r2} - F_{r3} - F_{r4} \\
J_x \ddot{j} = -\left[k_{32}\left(z_3 - z_3'\right) + c_3\left(\dot{z}_3 - \dot{z}_3'\right) + k_{42}\left(z_4 - z_4'\right) + c_4\left(\dot{z}_4 - \dot{z}_4'\right)\right]c + \left[k_{12}\left(z_1 - z_1'\right) + c_1\left(\dot{z}_1 - \dot{z}_1'\right) + k_{22}\left(z_2 - z_2'\right) + c_2\left(\dot{z}_2 - \dot{z}_2'\right)\right]d \\
-\left(f_{d3} + f_{d4}\right)c + \left(F_{r3} + F_{r4}\right)c + \left(f_{d1} + f_{d2}\right)d - \left(F_{r1} + F_{r2}\right)d \\
J_y \ddot{q} = -\left[k_{12}\left(z_1 - z_1'\right) + c_1\left(\dot{z}_1 - \dot{z}_1'\right) + k_{42}\left(z_4 - z_4'\right) + c_4\left(\dot{z}_4 - \dot{z}_4'\right)\right]a + \left[k_{22}\left(z_2 - z_2'\right) + c_2\left(\dot{z}_2 - \dot{z}_2'\right) + k_{32}\left(z_3 - z_3'\right) + c_3\left(\dot{z}_3 - \dot{z}_3'\right)\right]b \\
-\left(f_{d1} + f_{d4}\right)a + \left(F_{r1} + F_{r4}\right)a + \left(f_{d2} + f_{d3}\right)b - \left(F_{r2} + F_{r3}\right)b
\end{cases}
$$

where:

$$
\begin{aligned}
z_1' &= z - \left(aq + dj\right) \\
z_2' &= z + \left(bq - dj\right) \\
z_3' &= z + \left(bq + cj\right) \\
z_4' &= z + \left(aq - cj\right)
\end{aligned}
$$

state space: $\begin{cases} \dot{X} = AX + BQ + EU \\ Y = CX + DQ + FU \end{cases}$

where:

$$X=\begin{Bmatrix} z_1-z_1^{'} \\ z_2-z_2^{'} \\ z_3-z_3^{'} \\ z_4-z_4^{'} \\ q_1-z_1 \\ q_2-z_2 \\ q_3-z_3 \\ q_4-z_4 \\ \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \\ \dot{z} \\ \dot{j} \\ \dot{q} \end{Bmatrix} \qquad E=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{m_1} & 0 & 0 & 0 \\ 0 & \frac{1}{m_2} & 0 & 0 \\ 0 & 0 & \frac{1}{m_3} & 0 \\ 0 & 0 & 0 & \frac{1}{m_4} \\ -\frac{1}{m} & -\frac{1}{m} & -\frac{1}{m} & -\frac{1}{m} \\ \frac{d}{J_x} & \frac{d}{J_x} & \frac{-c}{J_x} & \frac{-c}{J_x} \\ \frac{-a}{J_y} & \frac{b}{J_y} & \frac{b}{J_y} & \frac{-a}{J_y} \end{bmatrix}$$

$$Y=\begin{Bmatrix} \ddot{z} \\ \ddot{j} \\ \ddot{q} \\ z_1-z_1^{'} \\ z_2-z_2^{'} \\ z_3-z_3^{'} \\ z_4-z_4^{'} \\ q_1-z_1 \\ q_2-z_2 \\ q_3-z_3 \\ q_4-z_4 \end{Bmatrix} \qquad U=\begin{Bmatrix} f_{d1} \\ f_{d2} \\ f_{d3} \\ f_{d4} \end{Bmatrix} \qquad Q=\begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ g \\ F_{r1} \\ F_{r2} \\ F_{r3} \\ F_{r4} \end{Bmatrix}$$

$$A=\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & d & a \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & d & -b \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & -c & -b \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & c & -a \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
\frac{-k_{12}}{m_1} & 0 & 0 & 0 & \frac{k_{11}}{m_1} & 0 & 0 & 0 & \frac{-c_1}{m_1} & 0 & 0 & 0 & \frac{c_1}{m_1} & \frac{-c_1 d}{m_1} & \frac{-c_1 a}{m_1} \\
0 & \frac{-k_{22}}{m_2} & 0 & 0 & 0 & \frac{k_{21}}{m_2} & 0 & 0 & 0 & \frac{-c_2}{m_2} & 0 & 0 & \frac{c_2}{m_2} & \frac{-c_2 d}{m_2} & \frac{c_2 b}{m_2} \\
0 & 0 & \frac{-k_{32}}{m_3} & 0 & 0 & 0 & \frac{k_{31}}{m_3} & 0 & 0 & 0 & \frac{-c_3}{m_3} & 0 & \frac{c_3}{m_3} & \frac{c_3 c}{m_3} & \frac{c_3 b}{m_3} \\
0 & 0 & 0 & \frac{-k_{42}}{m_4} & 0 & 0 & 0 & \frac{k_{41}}{m_4} & 0 & 0 & 0 & \frac{-c_4}{m_4} & \frac{c_4}{m_4} & \frac{-c_4 c}{m_4} & \frac{c_4 a}{m_4} \\
\frac{k_{12}}{m} & \frac{k_{22}}{m} & \frac{k_{32}}{m} & \frac{k_{42}}{m} & 0 & 0 & 0 & 0 & \frac{c_1}{m} & \frac{c_2}{m} & \frac{c_3}{m} & \frac{c_4}{m} & \frac{-(c_1+c_2+c_3+c_4)}{m} & \frac{(c_1+c_2)d-(c_3-c_4)c}{m} & \frac{(c_1-c_4)a-(c_2+c_3)b}{m} \\
\frac{k_{12}d}{J_x} & \frac{k_{22}d}{J_x} & \frac{-k_{32}c}{J_x} & \frac{-k_{42}c}{J_x} & 0 & 0 & 0 & 0 & \frac{c_1 d}{J_x} & \frac{c_2 d}{J_x} & \frac{-c_3 c}{J_x} & \frac{c_4 c}{J_x} & \frac{(c_3-c_4)c-(c_1+c_2)d}{J_x} & \frac{(c_3+c_4)c^2+(c_1+c_2)d^2}{J_x} & \frac{(c_3 b-c_4 a)c+(c_1 a-c_2 b)d}{J_x} \\
\frac{-k_{12}a}{J_y} & \frac{k_{22}b}{J_y} & \frac{k_{32}b}{J_y} & \frac{k_{42}a}{J_y} & 0 & 0 & 0 & 0 & \frac{c_1 a}{J_y} & \frac{c_2 b}{J_y} & \frac{c_3 b}{J_y} & \frac{c_4 a}{J_y} & -\frac{(c_1+c_4)a+(c_2+c_3)b}{J_y} & \frac{(c_1 a+c_2 b)d+(c_4 a-c_3 b)c}{J_y} & \frac{(c_1-c_4)a^2-(c_2+c_3)b^2}{J_y}
\end{bmatrix}$$

$$B=\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & -1 & -1 & -1 & -1 \\
0 & 0 & 0 & 0 & 0 & \frac{-d}{J_x} & \frac{-d}{J_x} & \frac{c}{J_x} & \frac{c}{J_x} \\
0 & 0 & 0 & 0 & 0 & \frac{-a}{J_y} & \frac{b}{J_y} & \frac{b}{J_y} & \frac{-a}{J_y}
\end{bmatrix}$$

F_{r_1} is a constant friction for 1 - front left half suspension

F_{r_2} is a constant friction for 2 - rear left half suspension

F_{r_3} is a constant friction for 3 - rear right half suspension

F_{r_4} is a constant friction for 4 - front right half suspension

$$C=\begin{bmatrix}
\frac{k_{12}}{m} & \frac{k_{22}}{m} & \frac{k_{32}}{m} & \frac{k_{42}}{m} & 0 & 0 & 0 & 0 & \frac{c_1}{m} & \frac{c_2}{m} & \frac{c_3}{m} & \frac{c_4}{m} & \frac{-(c_1+c_2+c_3+c_4)}{m} & \frac{(c_1+c_2)d-(c_3-c_4)c}{m} & \frac{(c_1-c_4)a-(c_2+c_3)b}{m} \\
\frac{k_{12}d}{J_x} & \frac{k_{22}d}{J_x} & \frac{-k_{32}c}{J_x} & \frac{-k_{42}c}{J_x} & 0 & 0 & 0 & 0 & \frac{c_1 d}{J_x} & \frac{c_2 d}{J_x} & \frac{-c_3 c}{J_x} & \frac{c_4 c}{J_x} & \frac{(c_3-c_4)c-(c_1+c_2)d}{J_x} & \frac{(c_3+c_4)c^2+(c_1+c_2)d^2}{J_x} & \frac{(c_3 b-c_4 a)c+(c_1 a-c_2 b)d}{J_x} \\
\frac{-k_{12}a}{J_y} & \frac{k_{22}b}{J_y} & \frac{k_{32}b}{J_y} & \frac{k_{42}a}{J_y} & 0 & 0 & 0 & 0 & \frac{c_1 a}{J_y} & \frac{c_2 b}{J_y} & \frac{c_3 b}{J_y} & \frac{c_4 a}{J_y} & -\frac{(c_1+c_4)a+(c_2+c_3)b}{J_y} & \frac{(c_1 a+c_2 b)d+(c_4 a-c_3 b)c}{J_y} & \frac{(c_1-c_4)a^2-(c_2+c_3)b^2}{J_y} \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$D=\begin{bmatrix}
0 & 0 & 0 & 0 & 1 & -1 & -1 & -1 & -1 \\
0 & 0 & 0 & 0 & 0 & \frac{-d}{J_x} & \frac{-d}{J_x} & \frac{c}{J_x} & \frac{c}{J_x} \\
0 & 0 & 0 & 0 & 0 & \frac{-a}{J_x} & \frac{b}{J_x} & \frac{b}{J_x} & \frac{-a}{J_x} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$F=\begin{bmatrix}
-\frac{1}{m} & -\frac{1}{m} & -\frac{1}{m} & -\frac{1}{m} \\
\frac{d}{J_x} & \frac{d}{J_x} & \frac{-c}{J_x} & \frac{-c}{J_x} \\
\frac{-a}{J_y} & \frac{b}{J_y} & \frac{b}{J_y} & \frac{-a}{J_y} \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}$$