Homework Set 9 EE 1473

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2. For an antipodal signaling system such as BPSK in AWGN, the probability of a bit error is well known. To remind ourselves of the essentials, under H0 the receiver output is a random variable r with mean − √ E, where E is the energy in the transmitted signal,

E{r|H0} = − √ E.

Under H1, the mean of the receiver output is

E{r|H1} = √ E

The variance of the receiver output is the same under either hypothesis,

Var{r|H0} = Var{r|H1} = N0

It is also assumed that the prior probabilities of the hypotheses are equal. Using these results, the bit error rate is

Pr(ε) = Q (√ 2E/ N0) .

Now suppose that we implement a 4-level ASK system, where two bits are transmitted in each signaling interval using one of 4 amplitudes of a pulse, which is then modulated onto the carrier. The receiver demodulates the signal using a filter matched to the baseband pulse shape. Let the expected value of the receiver output under each hypothesis be

E{r|H00} = −3 √ E E{r|H11} = √ E

E{r|H01} = − √ E E{r|H10} = 3√ E,

where the subscript for each hypothesis indicates the transmitted bit pair. The transmitted signal energy under each hypothesis is the square of the mean of r. The variance of the receiver output is the same under every hypothesis,

Var{r|H00} = Var{r|H01} = Var{r|H10} = Var{r|H11} = N0 2 ,

and all hypotheses are equally likely a priori.

(a) Show that the expression for the bit error probability for BPSK, as given above, is correct.

$$\left.\begin{array}{l}\text{Under } H_0 \Rightarrow r \sim N(-\sqrt{E}, N_0/2) \\ H_1: r \sim N(\sqrt{E}, N_0)\end{array}\right\} \gamma = 0$$

$$P_r(\varepsilon) = P_0\big(P(r>0|H_0)\big) + P_1\big(P(r<0|H_1)\big)$$

$$P(r>0|H_0) \Rightarrow Q\left(\frac{0+\sqrt{E}}{\sqrt{N_0/2}}\right) = Q\left(\sqrt{\frac{2E}{N_0}}\right)$$

$$P(r<0|H_1) \Rightarrow Q\left(\frac{\sqrt{E}-0}{\sqrt{N_0/2}}\right) = Q\left(\sqrt{2E/N_0}\right)$$

$$P_0 = P_1 = 1/2 \Rightarrow \boxed{P_r(\varepsilon) = Q\left(\sqrt{2E/N_0}\right)}$$

(b) Determine the average energy per bit Eb for the 4-level system, in terms of E. State the decoding rule for the 4-level system.

$$\langle E_s \rangle = 2E \qquad E_b = E_s/2 = \boxed{E}$$

$$0 < r < 2\sqrt{E} \rightarrow H_{11}$$

$$r < -2\sqrt{E} \rightarrow H$$

$$0 < r < 2\sqrt{E} \rightarrow H_{11}$$
$$r > 2\sqrt{E} \rightarrow H_{10}$$
$$-2\sqrt{E} < r < 0 \rightarrow H_{01}$$
$$r < -2\sqrt{E} \rightarrow H_{00}$$

(c) Determine the conditional probability of a bit error for the 4-level system, assuming that H00 is true. Note that different symbol errors will produce different numbers of bit errors.

Under $H_{00} \rightarrow r \sim N(-3\sqrt{E}, N_0/2)$

1 bit error for $H01$ or $H10 \rightarrow P(-2\sqrt{E} < r < 2\sqrt{E})$

2 bit errors for $H_{11} \rightarrow Pr(r > 2\sqrt{E})$

$$Q\left(\frac{-3\sqrt{E}-2\sqrt{E}}{\sqrt{N_0/2}}\right) - Q\left(\frac{-3\sqrt{E}+2\sqrt{E}}{\sqrt{N_0/2}}\right) + 2Q\left(\frac{2\sqrt{E}+3\sqrt{E}}{\sqrt{N_0/2}}\right)$$

$$\boxed{Q\left(5\sqrt{\frac{2E}{N_0}}\right) - Q\left(\sqrt{\frac{2E}{N_0}}\right)}$$

(d) Determine the conditional probability of a bit error for the 4-level system, assuming that H01 is true.

Under $H01 \rightarrow r \sim N(-\sqrt{E}, N_0/2)$

1 bit error for H_{00} and H_{11} $Pr(0 < r < 2\sqrt{E}) + P(r < -2\sqrt{E})$

2 bit errors for $H_{10} \rightarrow Pr(r > 2\sqrt{E})$

$$Q\left(\frac{-\sqrt{E}-2\sqrt{E}}{\sqrt{N_0/2}}\right) - Q\left(\frac{-\sqrt{E}-0}{\sqrt{N_0/2}}\right) + Q\left(\frac{-\sqrt{E}-2\sqrt{E}}{\sqrt{N_0/2}}\right) + 2Q\left(\frac{2\sqrt{E}+\sqrt{E}}{\sqrt{N_0/2}}\right)$$

$$(-3) \quad - \quad (-1) \quad + \quad (-3) \quad + 2 \quad (3)$$

$$1 \cancel{- Q\left(3\sqrt{\frac{2E}{N_0}}\right)} + \cancel{2Q\left(3\sqrt{\frac{2E}{N_0}}\right)} + 1 \cancel{- Q\left(3\sqrt{2E/N_0}\right)}$$

$$2 - 1 + Q\left(\sqrt{\frac{2E}{N_0}}\right)$$

$$P_r(\varepsilon | H_{01}) = \boxed{1 - Q\left(\sqrt{\frac{2E}{N_0}}\right)}$$

$$P_r(\varepsilon|H_{01}) = \boxed{1 - Q\left(\sqrt{\frac{2E}{N_0}}\right)}$$

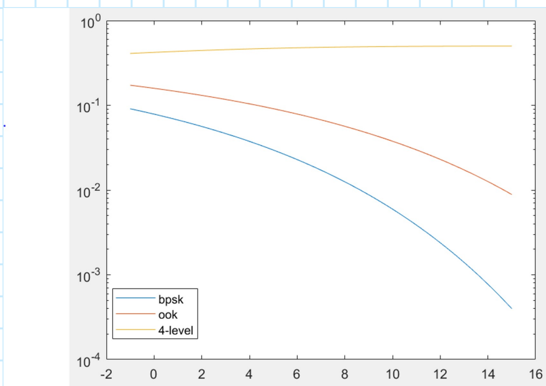
(e) Determine the overall bit error probability for the 4-level system, and express the result in terms of Eb/N0. (f) Use MATLAB to plot the bit error rates for the two systems, as well as the bit error rate for OOK,

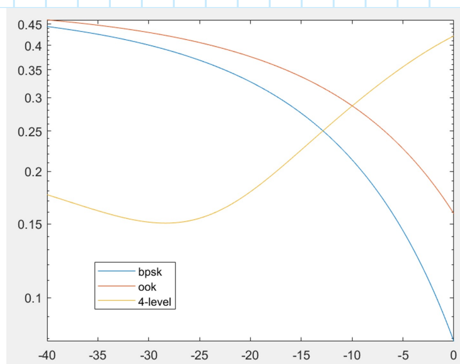
Pr(ε) = Q (√Eb/N0)

Make one plot with values of Eb/N0 between -1 and 15 dB, and a second plot with values between -40 and 0 dB. Both plots should have logarithmic scaling for the y-axis. Comment on the results, and quantify the degradation in performance if the 4-level system is used.

$$\frac{1}{4} Pr(\varepsilon|H_{00}) + \frac{1}{4} Pr(\varepsilon|H_{10}) + \frac{1}{4} Pr(\varepsilon|H_{11}) + \frac{1}{4} Pr(\varepsilon|H_{01})$$

$$\Rightarrow \boxed{\frac{1}{2} Pr(\varepsilon|H_{00}) + \frac{1}{2} Pr(\varepsilon|H_{01})}$$





3. Couch 5-67.

Assume that a QPSK signal is used to send data at a rate of 30 Mbits/s over a satellite transponder. The transponder has a bandwidth of 24 MHz.

(a) If the satellite signal is equalized to have an equivalent raised cosine filter characteristic, what is the rolloff factor r required?

$$B = (1+r)\,R/l \quad l=2 \Rightarrow \boxed{r = 0.6}$$

(b) Could a rolloff factor r be found so that a 50-Mbit/s data rate could be supported?

no, not for QPSK

$2B \;\; < 1 \Rightarrow r < ($

$$\frac{2B}{R} < 1 \Rightarrow \beta < 0$$

4. Suppose that the system described in Problem 3 was implemented, using the original bit rate of R = 30 Mbps, and the roll-off factor that was the solution to part (a). Referring to Couch, Figure 8-10, suppose that this signal was transmitted using downlink channel C13 of the Galaxy system transponder.

(a) Assume that, as in the previous problem, QPSK modulation is used. What range of frequencies will be occupied by the spectrum of the downlink signal? Will this spectrum fit within the bandwidth specified for channel C13? If not, indicate any adjacent channels that will experience interference from the QPSK spectrum. (Note: Due to the polarization diversity available with satellite communications, only the channels shown in part (a) of Figure 8-10 could experience interference.)

C13 down $\rightarrow$ 3960 MHz

with $r = 0.6$ and $R = 30$ Mbps

$B_T = 24$ MHz $\rightarrow f \in [3948, 3972]$ MHz

36 MHz allocated, so no interference, and yes it fits in the band

(b) Repeat part (a) if BPSK modulation is used.

$B_T = 48$ MHz $\quad f \in [3936, 3984]$ MHz

Does not fit in allocated spectrum, but will not interfere as the leakage is equal to the guardband.

(c) Repeat part (a) if 8-PSK modulation is used.

$l = 3 \quad B_T = \left(\frac{1.6}{3}\right)R = 16$ MHz

$f \in [3952, 3968]$ MHz no interference

(d) Repeat part (a) if 16-QAM modulation is used

12 MHz $\quad f \in [3954, 3966]$

no interference

5. Couch 5-68. In part (a), also calculate the absolute bandwidths of the complex envelope and the QPSK signal. In part (b), use MATLAB to produce the plot.
A QPSK signal is generated with nonrectangular data pulses on the I and Q channels. The data

pulses have spectra corresponding to the transfer function of a square-root raised cosine-rolloff filter. (a) Find the formula for the PSD of the QPSK signal. (b) Plot the PSD for the complex envelope of the QPSK signal, where the rolloff factor is r = 0.35 and the data rate is normalized to R = 1 bit/s. Plot your result in dB versus frequency normalized to the bit rate, similar to that shown in Fig. 5–33.

$P_g(f) = A_c^2 T_s H_e(f)$ $\quad H_e(f)$ is RCRO spectrum

$P_s(f) = \frac{1}{4} P_g(f \pm f_c)$

$\Rightarrow \frac{A_c^2 T_s}{4}\left(H_e(f+f_c) + H_e(f-f_c)\right)$

$B_T = \left(\frac{1.35}{2}\right)R = 0.675R$ $\quad$ total $-f_c + B_T$