EE 1473 - Digital Communication Systems
Homework Set 6

Due: Thursday, March 7, 2019

1. The purpose of this problem is to simulate a baseband digital communications system. The simulation that you prepare could be a significant part of what you develop for the project that is due at the end of the term. Please submit your MATLAB code to the courseweb site.

   (a) Write a MATLAB function that will produce the Root Raised Cosine Rolloff pulse for $|t| \leq k_T T_b$, where T_b is the bit period and $2k_T$ represents the number of bit periods over which you will compute the pulse. The equation for the Root RCRO pulse is

   $$h(t) = \begin{cases} 1 - r + \dfrac{4r}{\pi} & t = 0 \\[2ex] \dfrac{r}{\sqrt{2}}\left[\left(1+\dfrac{2}{\pi}\right)\sin\left(\dfrac{\pi}{4r}\right) + \left(1-\dfrac{2}{\pi}\right)\cos\left(\dfrac{\pi}{4r}\right)\right] & t = \pm\dfrac{T_b}{4r} \\[2ex] \dfrac{\sin\left[\pi R t(1-r)\right] + 4Rrt\cos\left[\pi R t(1+r)\right]}{\pi R t\left[1-(4Rrt)^2\right]} & \text{otherwise,} \end{cases}$$

   where $R = 1/T_b$ is the bit rate. The inputs to your function should be k_T, T_b, the number of samples per bit, and the rolloff factor r.

   Call your function with $k_T = 5$, $R = 1$ kbps, at least 16 samples per bit, and rolloff factors $r = 0, 0.1, 0.2, \ldots 1$. Plot the resulting pulses versus time on a single pair of axes, and compare with Figure 3-26. Does this pulse satisfy the zero-ISI condition?

   (b) Write a MATLAB script that will generate random data with at least 20 bits and data values $a_n = \pm 1$. Then call the Root-RCRO pulse function from part (a) with $R = 28.8$ kbps and at least 16 samples per bit. You may choose any rolloff factor other than 0, i.e., $0 < r \leq 1$. State clearly the value you have chosen for r. Form a baseband signal from the data and pulse, and plot the resulting signal versus time.

   (c) Take the baseband signal from part (b) process it with a filter whose impulse response is the same Root-RCRO pulse you used to form the signal. Confirm that the filtered signal has zero ISI.

(d) Add Gaussian noise to your signal from part (b) (use the `randn` function in MATLAB), and filter the noisy signal as in part (c). Recover the data from the filtered noisy signal, compare the recovered bits to the original bits, and compute the bit error rate. Experiment with different noise variances to see the effects of noise. Show at least one case where the noise is strong enough to be noticeable, but does not cause any bit errors. Show another case where at least one bit error occurs.

(e) Perform your simulation with significantly more than 20 bits, and using a series of different noise variances, so that you achieve very small bit error rates, as well as bit error rates that are very close to 0.5. Your simulations from parts (b) - (d) plot the signals at various stages of the process, but you may want to turn that off for this part of the problem. Plot the bit error rate versus E_b/N_0, plot your results, and with Couch equation (7-26b). How will you compute E_b/N_0 for your simulation?

Repeat the simulation from Problem 1 many times (how many?) and compile the bit error rate in each case, and compute an average bit error rate for each value of E_n/N_0. Prepare a plot of the average results and compare to equation (7-26b). You should achieve good agreement between your simulated results and theoretical performance if you use enough bits in each transmitted signal and enough trials.

2. Consider a binary signaling system in which, when a binary 1 is to be sent, a pulse $\sqrt{E_1}\, s_1(t)$ is transmitted. Following the convention used in lecture, the signal energy is E_1, and the pulse $s_1(t)$ is assumed to have unit energy. When a binary 0 is to be sent, a negative version of the same pulse, is transmitted, i.e. $\sqrt{E_0}\, s_0(t) = -\sqrt{E_1}\, s_1(t)$. To simplify the analysis, we will restrict our attention to the transmission of a single bit, and the appropriate pulse is transmitted over the time interval $0 \le t \le T$.

The pulses are received in the presence of AWGN $n(t)$ with PSD

$$P_n(f) = \frac{N_0}{2}.$$

Therefore, the received signal model for this problem is

$$\begin{array}{llcll} \underline{H_1}: & r(t) & = & \sqrt{E_1}\, s_1(t) + n(t) & 0 \le t \le T \\ \underline{H_0}: & r(t) & = & -\sqrt{E_1}\, s_1(t) + n(t) & 0 \le t \le T \end{array}$$

(a) Sketch a block diagram of the MAP receiver in the form of a matched filter whose output is sampled. Specify the signal to which the filter is matched.

(b) Determine the optimal detection threshold, γ_0, for the MAP receiver in terms of the prior probabilities and the energy of the transmitted pulse.

(c) Evaluate γ_0 for the case when the priors are $P_1 = P_0 = 0.5$, and the noise PSD is

$$\frac{N_0}{2} = 5 \times 10^{-5} \text{ W/Hz} = 50 \text{ W/MHz}.$$

Furthermore, assume that the signal energy is proportional to the pulse duration, specifically

$$E_1 = E_0 = \frac{3\pi}{2} T,$$

and that bits are to be transmitted at a rate of $R = 10$ kbps. (This corresponds to the energy in a RCRO pulse with rolloff factor $r = 1$ under certain normalization constraints, but this information is not necessary to solve the problem.)

(d) Repeat part (c) for $P_1 = 0.7$.

(e) Repeat part (c) for $P_1 = 0.2$.

(f) Explain the effect of the prior probabilities on γ_0.

(g) Determine the bit error rates for the cases considered in parts (c), (d) and (e). In each case, compare to the bit error rate that would be achieved by a maximum-likelihood (ML) receiver.