

ECE 1473 - Digital Communication Systems

Homework Set 9

Due: Thursday, April 4, 2019

1. During the class meeting on Thursday, March 28, you were assigned a modulation format to implement for the simulation project. Conduct research on the assigned format, consider how you will implement the system in software, and be prepared to describe your design to the instructor when asked.
2. For an antipodal signaling system such as BPSK in AWGN, the probability of a bit error is well known. To remind ourselves of the essentials, under H_0 the receiver output is a random variable r with mean $-\sqrt{E}$, where E is the energy in the transmitted signal,

$$E\{r|H_0\} = -\sqrt{E}.$$

Under H_1 , the mean of the receiver output is

$$E\{r|H_1\} = \sqrt{E}.$$

The variance of the receiver output is the same under either hypothesis,

$$\text{Var}\{r|H_0\} = \text{Var}\{r|H_1\} = \frac{N_0}{2}.$$

It is also assumed that the prior probabilities of the hypotheses are equal. Using these results, the bit error rate is

$$\text{Pr}(\varepsilon) = Q\left(\sqrt{\frac{2E}{N_0}}\right).$$

Now suppose that we implement a 4-level ASK system, where two bits are transmitted in each signaling interval using one of 4 amplitudes of a pulse, which is then modulated onto the carrier. The receiver demodulates the signal using a filter matched to the baseband pulse shape. Let the expected value of the receiver output under each hypothesis be

$$\begin{aligned} E\{r|H_{00}\} &= -3\sqrt{E} & E\{r|H_{11}\} &= \sqrt{E} \\ E\{r|H_{01}\} &= -\sqrt{E} & E\{r|H_{10}\} &= 3\sqrt{E}, \end{aligned}$$

where the subscript for each hypothesis indicates the transmitted bit pair. The transmitted signal energy under each hypothesis is the square of the mean of r . The variance of the receiver output is the same under every hypothesis,

$$\text{Var}\{r|H_{00}\} = \text{Var}\{r|H_{01}\} = \text{Var}\{r|H_{10}\} = \text{Var}\{r|H_{11}\} = \frac{N_0}{2},$$

and all hypotheses are equally likely *a priori*.

- (a) Show that the expression for the bit error probability for BPSK, as given above, is correct.
- (b) Determine the average energy per bit E_b for the 4-level system, in terms of E . State the decoding rule for the 4-level system.
- (c) Determine the conditional probability of a bit error for the 4-level system, assuming that H_{00} is true. Note that different symbol errors will produce different numbers of bit errors.
- (d) Determine the conditional probability of a bit error for the 4-level system, assuming that H_{01} is true.
- (e) Determine the overall bit error probability for the 4-level system, and express the result in terms of E_b/N_0 .
- (f) Use MATLAB to plot the bit error rates for the two systems, as well as the bit error rate for OOK,

$$\Pr(\varepsilon) = Q \left(\sqrt{\frac{E_b}{N_0}} \right).$$

Make one plot with values of E_b/N_0 between -1 and 15 dB, and a second plot with values between -40 and 0 dB. Both plots should have logarithmic scaling for the y-axis. Comment on the results, and quantify the degradation in performance if the 4-level system is used.

- 3. Couch 5-67.
- 4. Suppose that the system described in Problem 3 was implemented, using the original bit rate of $R = 30$ Mbps, and the roll-off factor that was the solution to part (a). Referring to Couch, Figure 8-10, suppose that this signal was transmitted using downlink channel C13 of the Galaxy system transponder.
 - (a) Assume that, as in the previous problem, QPSK modulation is used. What range of frequencies will be occupied by the spectrum of the downlink signal? Will this spectrum fit within the bandwidth specified for channel C13? If not, indicate any adjacent channels that will experience interference from the QPSK spectrum. (*Note:* Due to the polarization diversity available with satellite communications, only the channels shown in part (a) of Figure 8-10 could experience interference.)
 - (b) Repeat part (a) if BPSK modulation is used.
 - (c) Repeat part (a) if 8-PSK modulation is used.
 - (d) Repeat part (a) if 16-QAM modulation is used.
- 5. Couch 5-68. In part (a), also calculate the absolute bandwidths of the complex envelope and the QPSK signal. In part (b), use MATLAB to produce the plot.