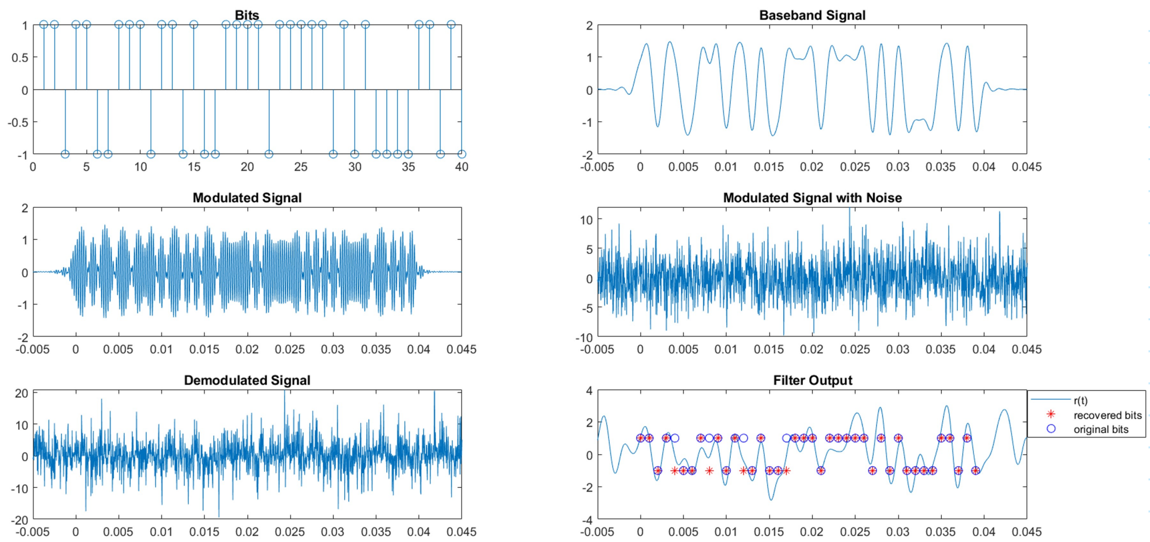
Josh Eaton

# Homework Set 8 EE 1473

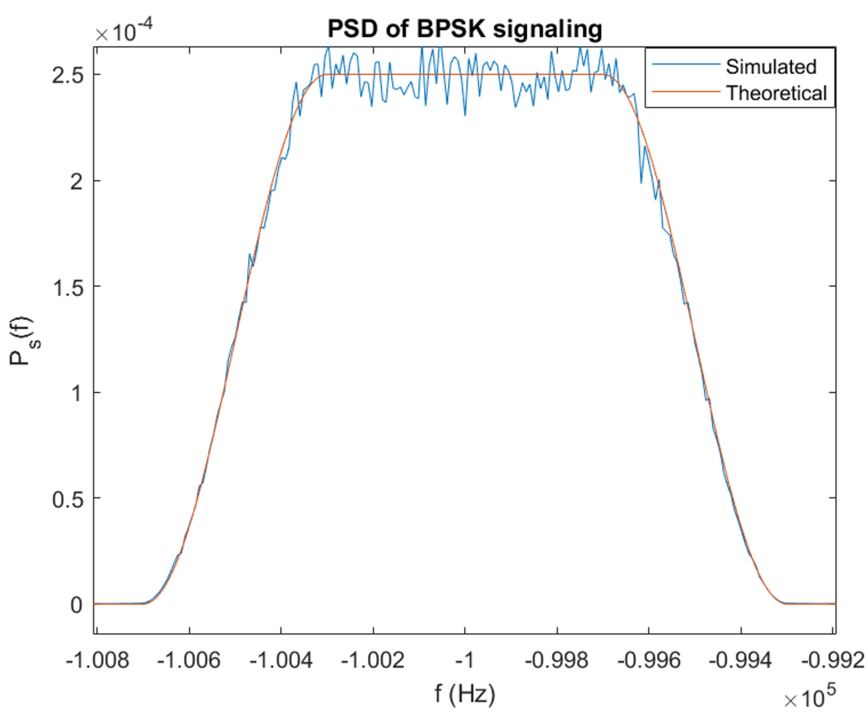
Monday, March 25, 2019 3:00 PM

**Problem 2**

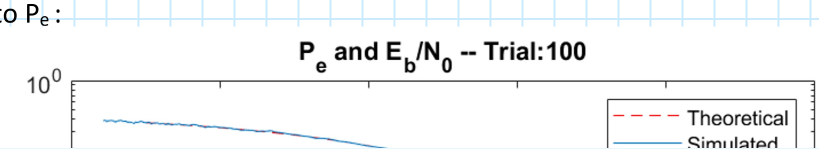
BPSK Signaling:

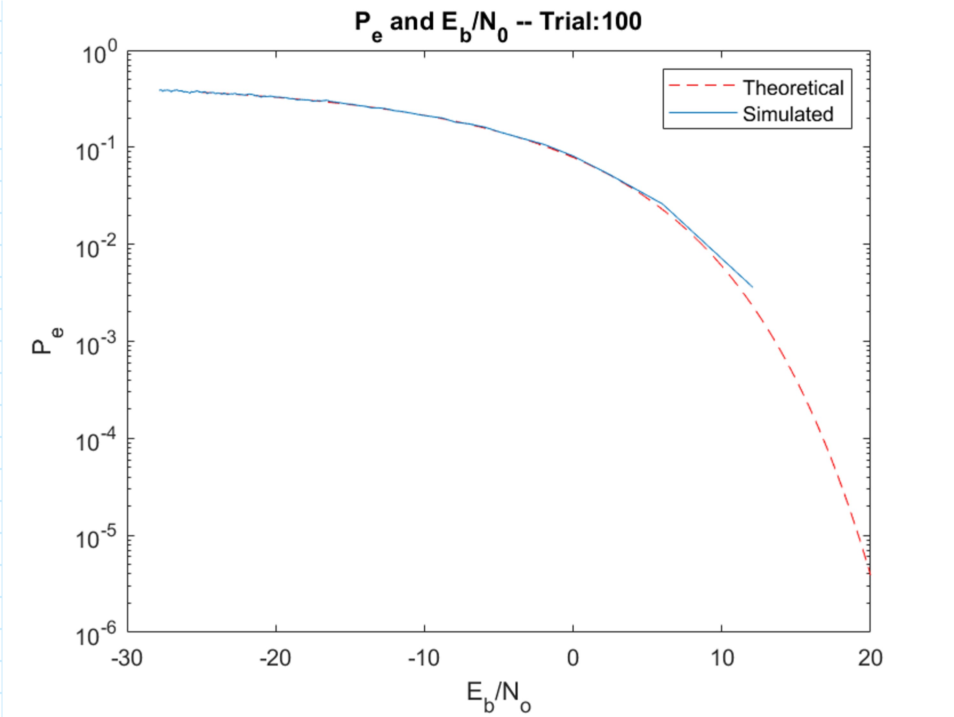



PSD Comparison:




BER comparison to $P_e$ :






3. Consider the design of a binary bandpass communication system using either OOK or BPSK. The baseband signal is formed using root-RCRO pulses with rolloff factor r = 0.5, and with Unipolar NRZ signaling in the case of OOK and Polar NRZ signaling for BPSK. The root-RCRO pulse is defined in Homework Assignment 6, Problem 1, and in that definition Tb is the time per bit in seconds, and R = 1/Tb is the bit rate.

The bandpass signal is formed by DSB-SC modulation, as described by Couch equation (5-13), where m(t) is the baseband data signal, and Ac is the carrier amplitude that you will choose as part of the design. Assume that the data symbols are ak ∈ {0, 1} for the unipolar baseband signal, and ak ∈ {−1, +1} for the polar signal, respectively.

The channel specifications are:

- The theoretical PSD of the bandpass signal must have an absolute bandwidth of no more than 15 MHz, centered at 1.92 GHz.
- The average power of the bandpass signal, when connected to a load of 50Ω, must not exceed 0.5 W.

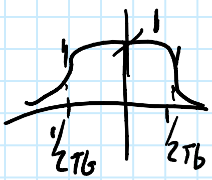
(a) Determine the PSD for the Polar NRZ signal. Show that the normalized average power in this signal is 1 W.

$$P_m(f) = \frac{1}{T_b}|F(f)|^2 \sum R_a[k]\, e^{-j\omega k T_b} \qquad F(f) = H_{\sqrt{rcro}}(f)$$

$\omega = 2\pi f$

$$R_a[k] = \begin{cases} A^2 & k=0 \\ 0 & \end{cases} \qquad A = 1$$

$$\boxed{P_m(f) = T_b |F(f)|^2} \qquad \boxed{P_m(f) = T_b H_{RCRO}(f)}$$




$$\Rightarrow (T_b)\left(\tfrac{1}{2}T_b + \tfrac{1}{2}T_b\right)$$

$$\boxed{P_m = 1W}$$

$$\boxed{P_m = 1W}$$

(b) Determine the PSD for the Unipolar NRZ signal, and the normalized average power in this signal.

$$R_a[k] = \begin{cases} A^2/2 & k=0 \\ A^2/4 & \text{———} \end{cases}$$

$$P_m(f) = \frac{1}{4}\left[\delta(f) + T_b (F(f))^2\right]$$

$$P_m = \frac{1}{4} + \frac{1}{4} = \boxed{\frac{1}{2} W}$$

(c) Determine the PSD for the OOK signal.

$$P_s(f) = \frac{A_c^2}{4}\left[P_m(f+f_c) + P_m(f-f_c)\right]$$

$$\Rightarrow \frac{A_c^2}{16}\left[\delta(f+f_c) + \delta(f-f_c) + T_b\left\{F(f-f_c)^2 + F(f+f_c)^2\right\}\right]$$

(d) Determine the PSD for the BPSK signal.

$$P_s(f) \; \frac{A_c^2 T_b}{4}\left[F(f-f_c)^2 + F(f+f_c)^2\right]$$

(e) Determine maximum values for R and Ac such that the channel specification will be met. Are the answers different for OOK and BPSK?

$$\text{BW of RCRO} \Rightarrow \frac{1}{2T_b}(1+r) = \frac{R}{2}(1+r)$$

$$\frac{R}{2}(1+r) < 15\,MHz \Rightarrow R < \frac{30M}{1+r}$$

$$\boxed{R_{max} = 20\,Mbps}$$

$$50\Omega \text{ power} = P/50$$

$$\text{OOK: } P = \frac{A_c^2}{16}(4)/50$$

$$\text{BPSK: } P = \frac{A_c^2}{4}(2)/50$$

$$\text{OOK} \quad A_c^2 < \sqrt{\frac{(25)(16)}{4}} \Rightarrow \boxed{A_c < 10}$$

$$\text{BPSK} \quad A_c^2 < \sqrt{\frac{25 \cdot 4}{2}} \Rightarrow \boxed{A_c < \sqrt{50} = 7.071}$$

(f) Assume OOK signaling, and that the optimal coherent receiver is used. Determine the value of Eb/N0 necessary to achieve bit error rates of Pe = 10−5 , 10−6 , and 10−7 . For each case, calculate the corresponding signal-to-noise ratio, and the value for N0 assuming that the transmitted signal exactly meets the channel constraints from part (e). Summarize your results in a table, showing all values in dB units.

OOK BER (orthogonal) = $Q\left(\sqrt{\frac{E_b}{N_0}}\right)$ $R = 20\,\text{Mbps}$ $A_c = 10$

$$\frac{S}{N} = \frac{E_b/T_b}{N_0 B} = \frac{E_b}{N_0}\frac{R}{B}$$

| $P_e$ | $Q^{-1}$ | $E_b/N_0$ | $N_0$ | $S/N$ |
|---|---|---|---|---|
| $10^{-5}$ | 4.265 | 18.18 | -4.2 | 19.42 |
| $10^{-6}$ | 4.7534 | 22.5998 | -8.61 | 23.84 |
| $10^{-7}$ | 5.1993 | 27.0331 | -13.13 | 28.27 |

$\frac{E_b}{N_0} = E_b - N_0$

$N_0 = E_b - \frac{E_b}{N_0}$ $\boxed{13.98\text{ dB}}$

$\left(\frac{E_b}{N_0}\right)\left(\frac{R}{B}\right) = \frac{S}{N} \Rightarrow \boxed{+1.24\text{dB}}$

$E_b = A_c^2/4$

(g) Repeat part (f) for BPSK signaling, and compare your answers to part those from (f).

$P_e = Q\left(\sqrt{2\frac{E_b}{N_0}}\right)$ (Antipodal) $\frac{E_b}{N_0}$ will be ½ of OOK

| $P_e$ | $E_b/N_0$ | $N_0$ | $S/N$ |
|---|---|---|---|
| $10^{-5}$ | 9.09 | 4.89 | 10.33 |
| $10^{-6}$ | 11.297 | 2.683 | 12.54 |
| $10^{-7}$ | 13.517 | 0.463 | 14.76 |

$E_b = A_c^2/2$

$N_0 \Rightarrow \boxed{13.98 - E_b/N_0}$

$\frac{S}{N} \Rightarrow \boxed{+1.24\text{ dB}}$

The BPSK requires a lower SNR and thereby can tolerate a higher noise floor to achieve the same BER in a given channel.