Homework Set 10 EE 1473

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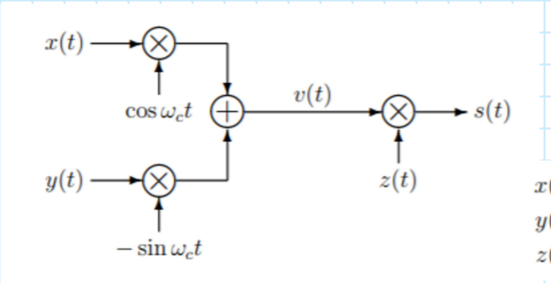
Problem 1.

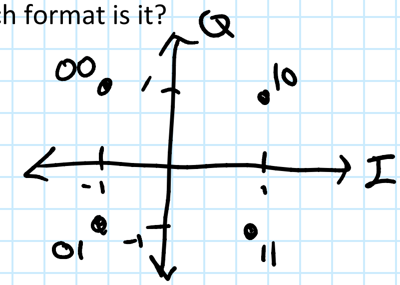


$$x(t) = 2a_k - 1$$
$$y(t) = 2b_k - 1$$
$$z(t) = c_k + 1.$$

(a) Sketch the signal constellation for v(t) on labeled axes. Label each constellation point with the corresponding bit pair ab. The signal v(t) is a bandpass signal in one of the standard digital modulation formats: ASK, OOK, BPSK, QPSK, 8-PSK, FSK, QAM. Which format is it?

00 ⇒ $x = -1$ $y = -1$
01 ⇒ $x = -1$ $y = 1$
11 ⇒ $x = 1$ $y = 1$
10 ⇒ $x = 1$ $y = -1$

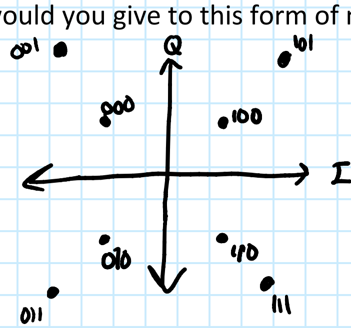
$V = x - y$



This is QPSK

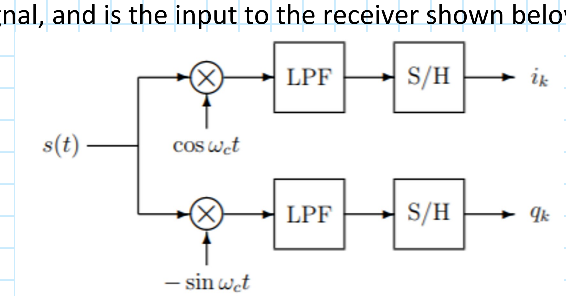
(b) Sketch the signal constellation for s(t) on labeled axes. Label each constellation point with the corresponding bit sequence abc. What name would you give to this form of modulation?

$c = 0 \Rightarrow s = v(t)$
$c = 1 \Rightarrow s = 2v(t)$



8 QAM

(c) The signal s(t) passes through a bandpass channel that adds white Gaussian noise, but otherwise does not distort the signal, and is the input to the receiver shown below.



Determine the impulse responses for the lowpass filters such that:

- the bit error rate is minimized, and
- the constellation for the receiver outputs {ik, qk} is identical to the signal constellation for s(t) from part (b).

What is the name for this type of receiver?

This is an IQ receiver

What is the name for this type of receiver?

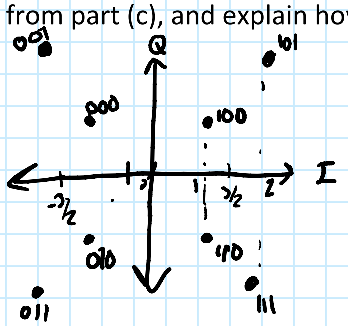
This is an IQ receiver

input to LPF $\Rightarrow$ $[\cos(w_c)][s(t) + n(t)]$

$[a_k \cos - b_k \sin + n][\cos] \Rightarrow a\cos^2 - b\sin\cos + n\cos$

$\boxed{\text{LPF}}$ = Rectangular pulse with 2x the amplitude

(d) Suppose that you must decode the bits {ak, bk, ck} by comparing the receiver outputs xk and yk to various thresholds. Determine appropriate thresholds, with specific reference to the signal constellation from part (c), and explain how the bit values are decoded from the threshold comparisons.



$|x_k| > 3/2 \quad c_k = 1$, else $c_k = 0$

$y_k < 0 \quad b_k = 1$, else $b_k = 0$

$x_k > 0 \quad a_k = 1$, else $a_k = 0$

2. The FCC has allocated the wireless spectrum from 960 MHz to 1215 MHz for aeronautical radio navigation. In this problem, you will consider communication of digital data through this channel. The baseband signal(s) will be formed using root-RCRO pulses with a rolloff factor of r = 0.25.

(a) Suppose that OOK or BPSK modulation will be used. Determine the maximum bit rate that can be communicated through the channel, such that the bandwidth of the modulated signal will fit within the specified frequencies, and intersymbol interference can be avoided in the receiver. Are the answers different for OOK and BPSK?

$B_T = (1+r)R \qquad B_T = 255\ \text{MHz} \Rightarrow \boxed{R = 204\ \text{Mbps}}$

same for BPSK and OOK

(b) One figure of merit for digital communication system is the data efficiency η, which is defined as the ratio of the bit rate to the channel bandwidth. Determine the data efficiency for the systems from part (a), in units of bits per second per Hertz.

$\eta = 1/(1+r) = \boxed{0.8}$

(c) Next, suppose that QPSK or 8-PSK modulation will be used. Repeat parts (a) and (b) for these cases.

$B_T = (1+r)D \Rightarrow D = R/2 \qquad R = \boxed{408\ \text{Mbps}}$ (QPSK)

$D = R/3 \qquad \boxed{\eta = 1.6}$

$\boxed{612\ \text{Mbps}}$ (8-PSK)

$\boxed{\eta = 2.4}$

(d) Now suppose that 16-QAM, 64-QAM or 256-QAM modulation will be used. Repeat parts (a) and (b) for these cases.

$16 \Rightarrow D = R/4 \qquad R = 816\ \text{Mbps} \quad \eta = 3.2$

$64 \Rightarrow D = R/6 \qquad R = 1224\ \text{Mbps} \quad \eta = 4.8$

$256 \Rightarrow D = R/7 \qquad R = 1428\ \text{Mbps} \quad \eta = 5.6$

256 ⇒ $D = R/7$ | $R = 1428$ Mbps $\eta = 5.6$

(e) For each of the modulation formats considered in this problem, determine the probability of a bit error, assuming that the SNR is 15 dB, and that the optimal receiver is used. If an exact expression for the bit error probability is not available, compute an upper bound using equations from Couch Chapter 7.

$OOK = Q\left(\sqrt{E_b/N_0}\right)$ QPSK/BPSK $= Q\left(\sqrt{2E_b/N_0}\right)$

Symbol Error Bounds

$M=8$ 8-PSK ⇒ $Q\left[\sqrt{\frac{2E_b}{N_0}(\log_2 M)\sin^2\left(\frac{\pi}{M}\right)}\right]$

MQAM ⇒ $4Q\left[\sqrt{2\frac{E_b}{N_0}\eta_M}\right]$ $\eta_{16} = -4,\ \eta_{64} = -8.5\ \ \eta_{256} = -13.3$

WER to BER

$P_e \le \frac{M/2}{M-1} P(E)$

$SNR = (E_b/N_0) + 20\log(R/B) \Rightarrow \frac{E_b}{N_0} = SNR - 20\log\frac{R}{B}$

BPSK ⇒ 6.86×10^{-5}

OOK ⇒ 0.004

QPSK ⇒ 0.004

8PSK ⇒ $P_e \le 0.043$

16QAM $P_e \le 0.145$

64 QAM $P_e \le 0.354$

256 QAM $P_e \le 0.512$

3. In this problem, we consider using OFDM to transmit data over the channel from Problem 2. You will calculate the maximum bit rate that can be achieved over this channel for different forms of OFDM. The baseband signal will be formed using rootRCRO pulses with r = 0.25. As a result, the complex envelope as described by Couch, equation (5-117a) must be modified to

$$g(t) = A_c f(t) \sum_{n=0}^{N-1} w_n \varphi_n(t),$$

where f(t) is the root-RCRO pulse. This equation only represents the complex envelope of the OFDM signal for transmitting the first N symbols {w0, w1, . . . , wN−1}, but it can be used to determine the PSD Pg(f). Note that the expression in Couch equation (5-118) must also be modified, because the sinc2 term that appears there corresponds to rectangular pulses in the baseband signal.

(a) Suppose that OFDM is used, with either 2, 4, 8 or 16 carriers, and BPSK signaling on each carrier. Determine the maximum bit rate that can be achieved in each case, while not exceeding the channel bandwidth. Compare your results to the bit rate from Problem 2 with BPSK signaling.

$B_T = (N+1)(1+r)R$

$R = \frac{B_T}{(N+1)(1+r)} N$

2 carriers ⇒ $R = 136$ Mbps

4 carriers ⇒ $R = 163.2$ Mbps

8 carriers ⇒ $R = 181.33$ Mbps

$$R = \frac{B_T}{(N+1)(1+r)} N$$

8 carriers $\Rightarrow$ $R = 181.33$ Mbps

$R_{OFDM} < R_{BPSK}$

16 carriers $\Rightarrow$ $R = 192$ Mbps

(b) Now suppose that QPSK signaling is used on each OFDM carrier, with 2, 4, 8 or 16 carriers. Determine the maximum bit rates for these cases, and compare to the bit rate from Problem 2 for QPSK signaling.

Double R for each case, still less than QPSK alone

2 carriers $\Rightarrow$ 272 Mbps

8 carriers $\Rightarrow$ 362.66 Mbps

4 carriers $\Rightarrow$ 326.4 Mbps

16 carriers $\Rightarrow$ 384 Mbps

4. For this problem, you will consider FSK to transmit data over the channel from Problem 2. As in Problems 2 and 3, the baseband signal will be formed using root-RCRO pulses with r = 0.25.

(a) Suppose that binary FSK will be used, with a modulation index of h = 0.5, 1, or 2. For each case, determine the maximum bit rate such that the FSK bandwidth as predicted by Carson's rule does not exceed the channel bandwidth. Compare your results to the bit rate from Problem 2 for BPSK signaling.

$$h = \frac{2\Delta F}{R} \qquad B_T = 2\Delta F + (1+r)R \qquad B_T = 255 \text{ MHz}$$

$$hR = 2\Delta F \Rightarrow B_T = (h+1+r)R$$

$h = \frac{1}{2} \Rightarrow R = 145$ Mbps $\quad h = 1 \Rightarrow R = 113$ Mbps $\quad h = 2 \Rightarrow R = 78$ Mbps

(b) Now suppose 4-ary FSK is used, i.e. two-bit symbols are transmitted using one of four frequencies, with a modulation index of h = 0.5, 1 or 2. For each case, determine the maximum bit rate such that the FSK bandwidth as predicted by Carson's rule does not exceed the channel bandwidth. Is there an advantage to using 4-ary FSK?

$$B_T = (M-1)\Delta F + (1+r)R \Rightarrow M = 2$$

$$h = \frac{2\Delta F}{D} \qquad D = \frac{R}{2} \Rightarrow \frac{hR}{4} = \Delta F$$

$$\Rightarrow B_T = \left(3\left(\frac{h}{4}\right) + 1 + r\right) R$$

@ $\frac{1}{2}$ $R = 156$ Mbps

@ 1 $R = 127$ Mbps

@ 2 $R = 92$ Mbps