Homework Set 7 EE 1473

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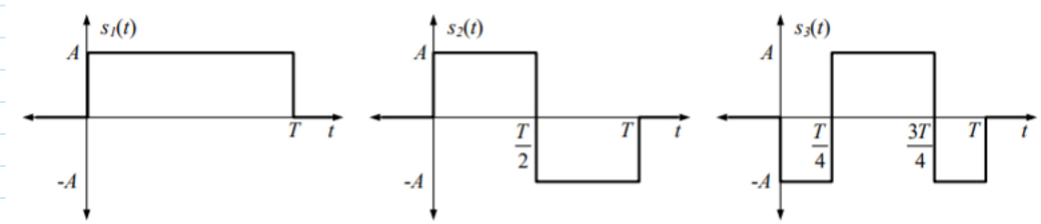
1. In this problem, we consider a ternary communication system in which one of 3 symbols is transmitted every T seconds using the signals shown in the figure below. When symbol $i \in \{1, 2, 3\}$ is transmitted, the received signal is

$$H_i : \quad r(t) = s_i(t) + n(t)$$

where $n(t)$ is a white Gaussian noise process with mean 0 and power spectral density $P_n(f) = N_0/2$. Each symbol is equally likely to be transmitted, so the priors are $P_i = 1/3$.



(a) Show that the signals $s_i(t)$ are mutually orthogonal.

Orthogonal if $\rho = 0$

$$\Rightarrow \int s_1(t)s_2(t)\,dt = \int s_1(t)s_3(t)\,dt = \int s_2(t)s_3(t)\,dt$$

2) $$\int s_1 s_2 = \frac{A^2T}{2} - \frac{A^2T}{2} = 0$$

$$\int s_1 s_3 = -\frac{A^2T}{4} + \frac{A^2T}{2} - \frac{A^2T}{4} = 0$$

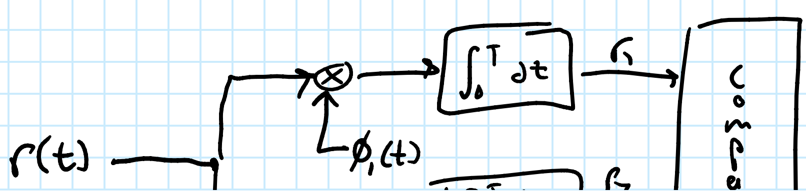
$$\int s_2 s_3 = \cancel{-\frac{A^2T}{4}} \cancel{+\frac{A^2T}{4}} \cancel{-\frac{A^2T}{4}} \cancel{+\frac{A^2T}{4}} = 0$$

(b) Determine signals $\phi_i(t)$ that are versions of the $s_i(t)$ that have been normalized to unit energy.

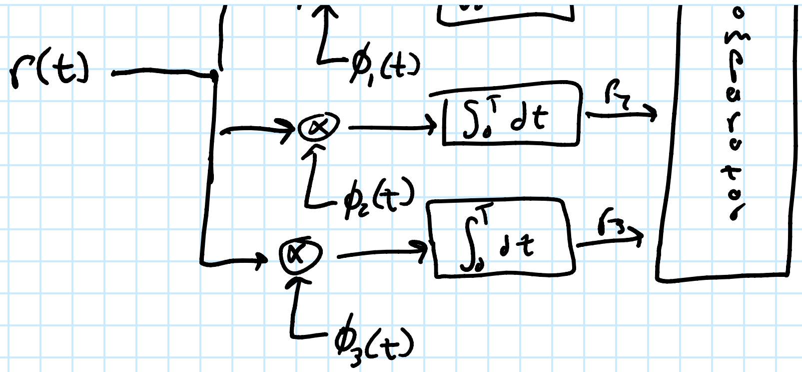
Unit energy $\rightarrow \int_0^T s_i^2\,dt = 1$

$$\phi_i: \quad E_{1,2,3} = A^2T \Rightarrow \boxed{\phi_i(t) = \sqrt{\frac{1}{A^2T}}\, s_i(t)}$$

(c) Sketch a block diagram of the optimal receiver for this problem, as a bank of 3 correlation receivers, followed by a decision rule. Each channel should correlate with one of the energy-normalized signals $\phi_i(t)$ from part (b). Let ri be the output of channel i in your receiver. Specify the decision rule that takes {r1, r2, r3} as inputs and determines which signal is most likely to have been transmitted. Then simplify the decision rule to comparing pairwise differences of the ri to thresholds.



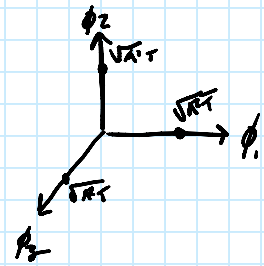
Choose largest



largest of r_1, r_2, r_3 to estimate H

$$\text{largest } r \iff \{r_i - r_j > 0\} \quad i \neq j$$

(d) Determine the signal constellation for this system, in terms of A and T.



basis $[\phi_1, \phi_2, \phi_3]$

$H_1 = [\sqrt{A^2T}, 0, 0]$

$H_2 = [0, \sqrt{A^2T}, 0]$

$H_3 = [0, 0, \sqrt{A^2T}]$

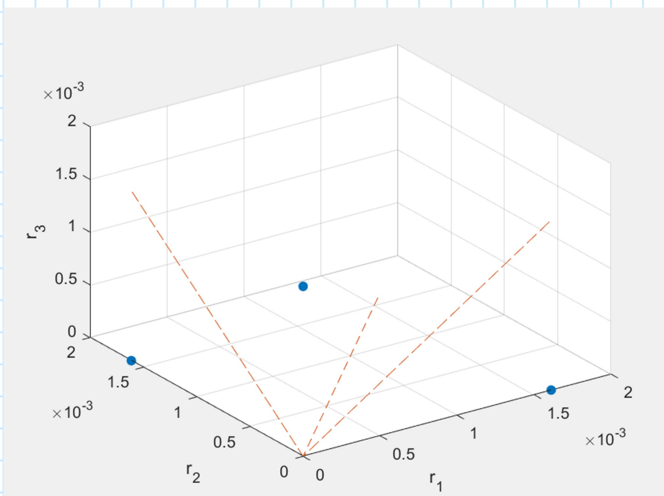
(e) Express the probability that a symbol will be decoded in error, in terms of differences between the receiver outputs, as defined in part (c). Then explain why it is difficult to express the symbol error probability in terms of the Gaussian cdf.

$$Pr(\varepsilon) = Pr(\varepsilon | H_i) \quad \text{for any } H_i$$

$$\Rightarrow 1 - Pr(\varepsilon | H_i) = Pr\left(\bigcap_{i \neq j} \{r_i - r_j > 0\} \,\middle|\, H_i\right)$$

This is difficult to evaluate because the events $r_i - r_j$ are not independent.

(f) Let the symbol rate be D = 1.544 Mbaud and A = 2 Volts. Use MATLAB to plot the signal constellation; what are the units for the axes? Since the decision space is three-dimensional, use the plot3 command in MATLAB to do this



$$T = 1/D \Rightarrow \sqrt{A^2T} = 0.0016$$

2. In this problem, we consider a modification of the system from Problem 1, in which one of 8 symbols is transmitted every T seconds, using the signals

y1(t), y2(t), . . . , y8(t).

Each of the 8 signals is equally likely to be transmitted. The system must conform to the following specifications:

• The receiver structure must be identical to that from Problem 1, i.e. a bank of three correlation receivers, using the same signals ϕi(t) from part (b). Obviously the decision rule will have to be modified to decode one of eight possible symbols from the three receiver outputs.

• The signals yi(t) must have the same duration (T) and peak amplitude (±A) as the signals si(t) from Problem 1. (To be clear, some of the yi(t) will have peak positive amplitude A, and others will have peak negative amplitude −A, but it is not necessary that they achieve both ±A.)

• The signals yi(t) must be chosen so as to optimize the performance of the system, i.e. minimize the symbol error rate.

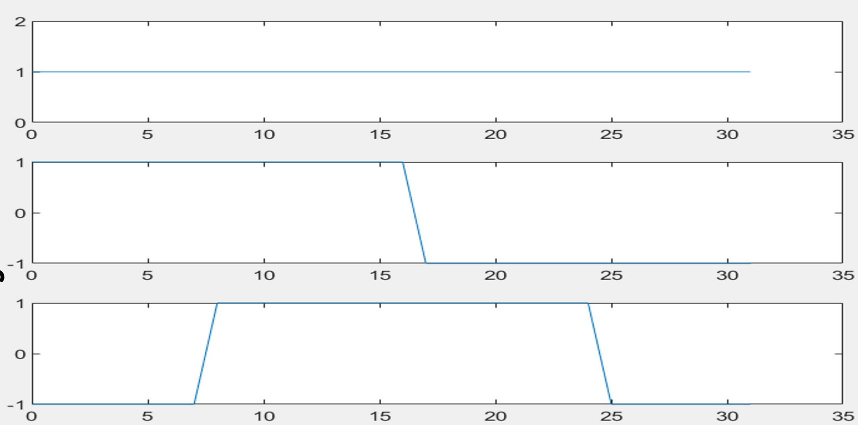
(a) Determine the set of signals yi(t) that should be used for this problem, and sketch them vs. t.

minimize P_e by maximizing distance between constellation points

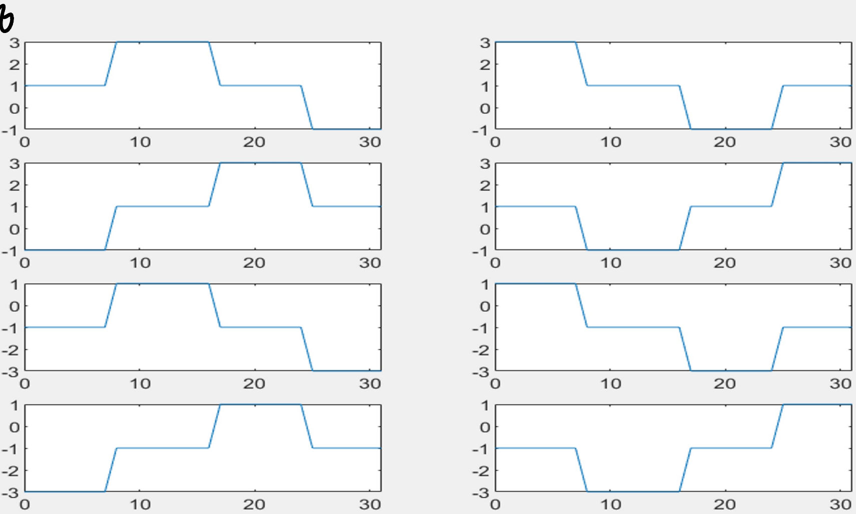
$\Rightarrow \sqrt{A^2 T}\,[\cdot]\,[\phi_1, \phi_2, \phi_3]$

$[1,1,1]$, $[1,1,-1]$, $[1,-1,1]$, $[1,-1,-1]$, $[-1,1,1]$,
$[-1,1,-1]$, $[-1,-1,1]$, $[-1,-1,-1]$

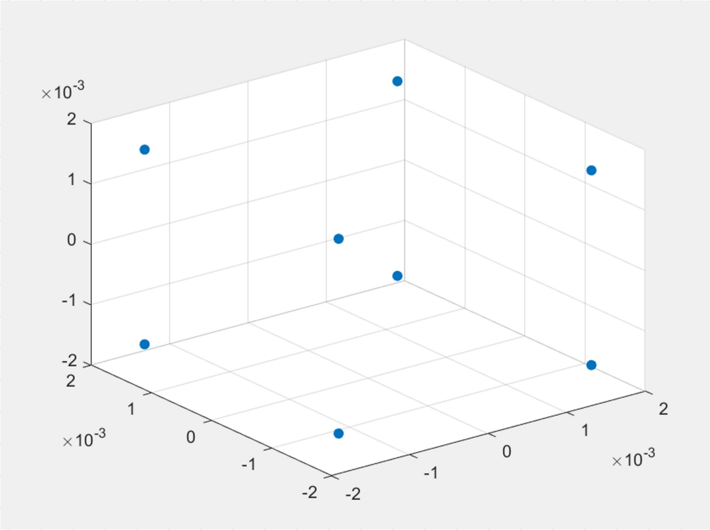
ϕ_1, ϕ_2, ϕ_3



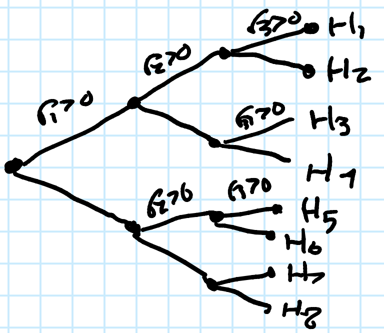
$y_1 \cdots y_8$



(b) Use MATLAB to plot the signal constellation, using the same values for R and A from Problem 1(f).



(c) Determine the decision rule to choose which signal was transmitted from the outputs {r1, r2, r3} from the correlation receivers, so that the symbol error rate is minimized.



Choose nearest constellation point

i.e. which quadrant in 3-D space.

(d) Determine the probability of symbol error for this system

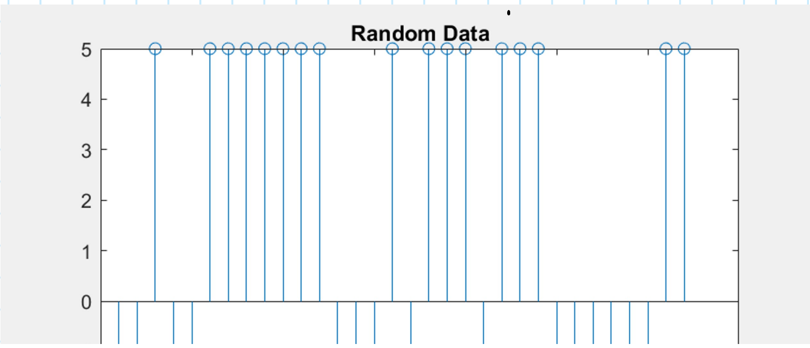
$$P(\varepsilon) = 1 - P(\varepsilon|H_i) \Rightarrow 1 - P(\cap\{r_i < 0\} | H_i\}$$

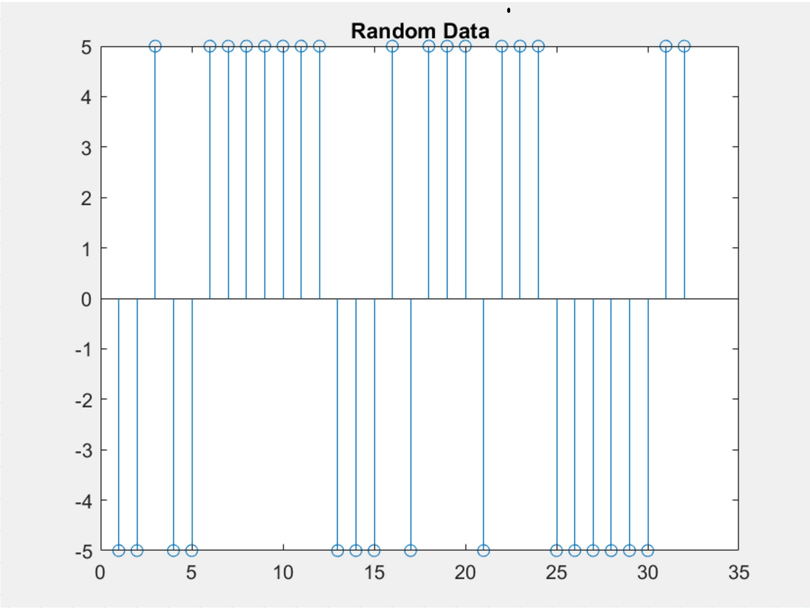
given H_i: $r \sim N(\sqrt{A^2T}, N_0/2)$

$$\Rightarrow 1 - \left[Q\left(\frac{-\sqrt{A^2T}}{\sqrt{N_0/2}}\right)\right]^3 \Rightarrow \boxed{1 - \left[Q\left(-\sqrt{\frac{2A^2T}{N_0}}\right)\right]^3}$$

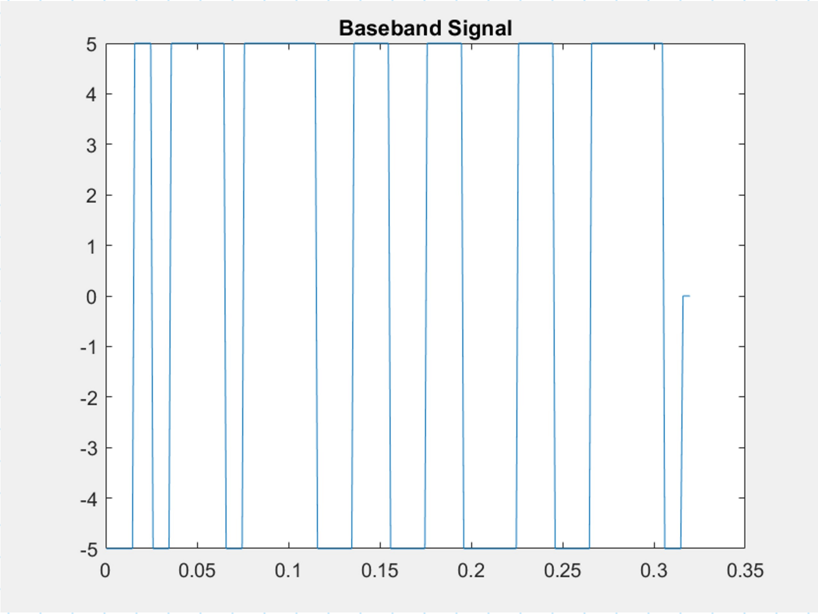
3. In this problem, you will approximate the PSD for random polar NRZ data through simulation. (Read Couch, Section 6-2, with particular emphasis on Subsection "Measurement of PSD, Numerical Computation of the PSD.")

a)

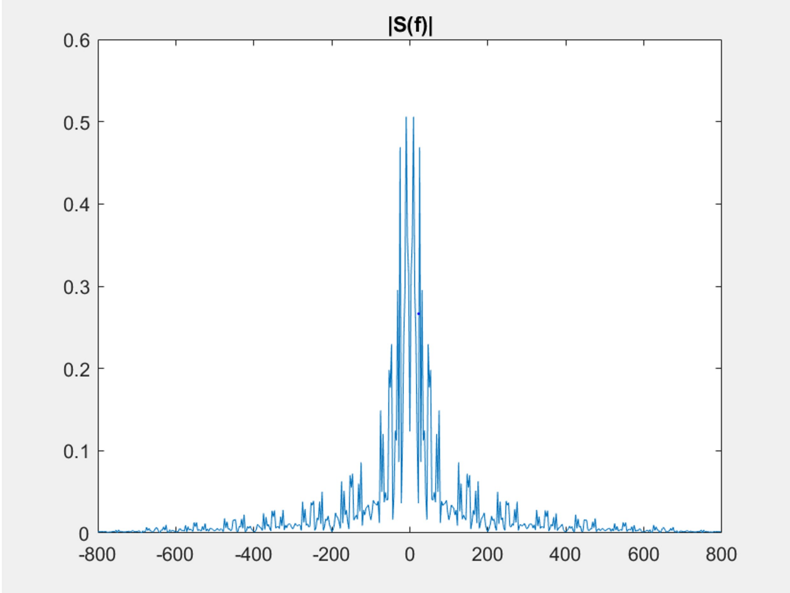



Random Data
5
4
3
2
1
0
-1
-2
-3
-4
-5
0
5
10
15
20
25
30
35

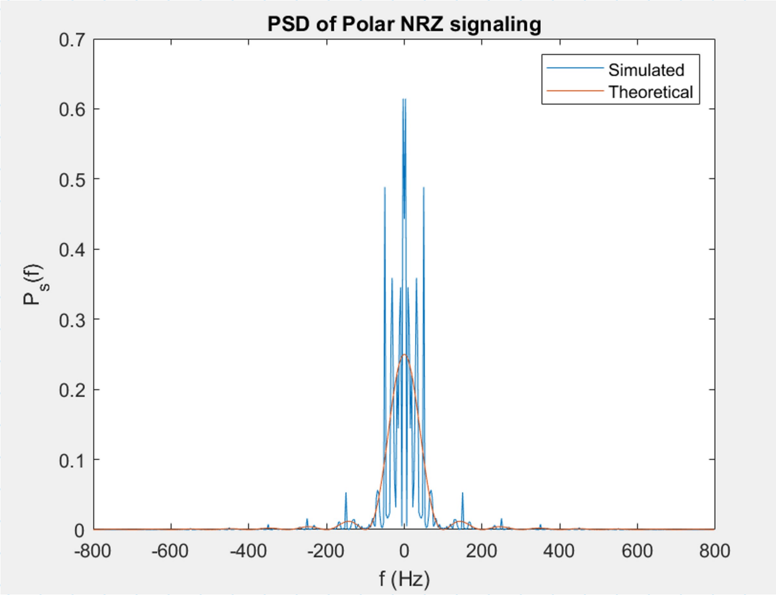
b)


Baseband Signal
5
4
3
2
1
0
-1
-2
-3
-4
-5
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35

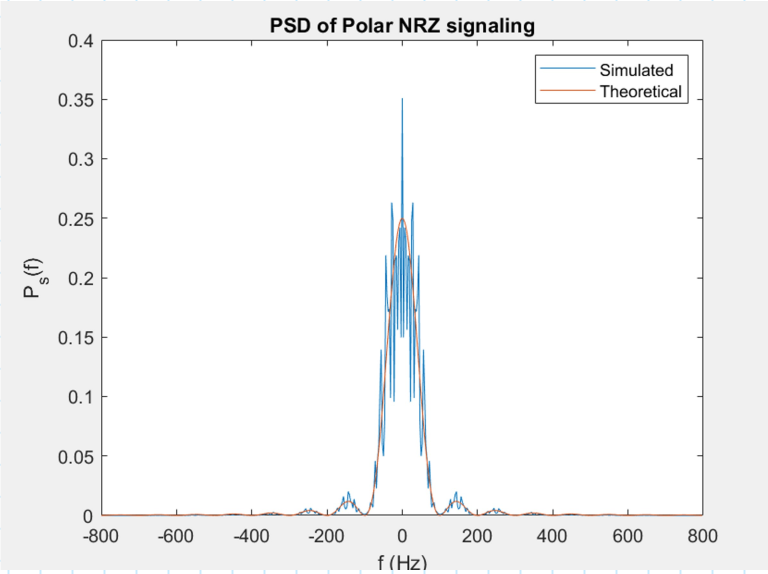
c)


|S(f)|
0.6
0.5
0.4
0.3
0.2
0.1
0
-800
-600
-400
-200
0
200
400
600
800

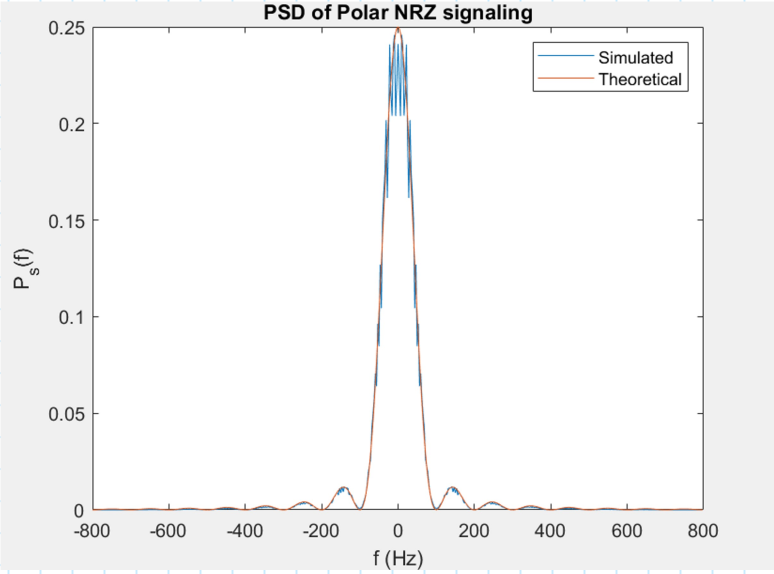
d)


PSD of Polar NRZ signaling
Simulated
Theoretical
P_s(f)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-800
-600
-400
-200
0
200
400
600
800
f (Hz)

e)

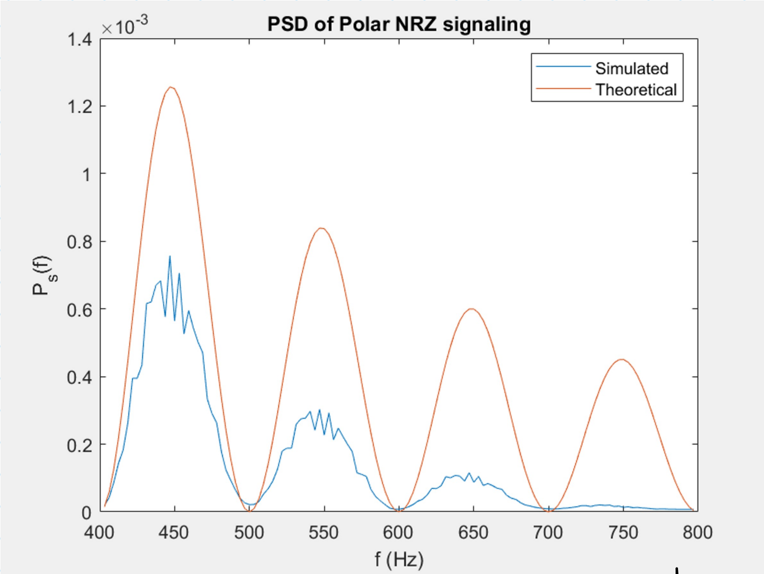

PSD of Polar NRZ signaling
Simulated
Theoretical
P_s(f)
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-800
-600
-400
-200
0
200
400
600
800
f (Hz)

f)



g) Agreement increases with more signals, more so at lower frequencies, due to the finite window T, which effectively convolves the spectra with a sinc "spectral leakage"

h)



The finite window prevents accuracy in this range, as explained above.