Johns Hopkins Engineering

Power Electronics 525.725

Module 7 Lecture 7

Converter Transfer Functions

Some elements of design-oriented analysis, discussed in this chapter

- Writing transfer functions in normalized form, to directly expose salient features
- Obtaining simple analytical expressions for asymptotes, corner frequencies, and other salient features, allows element values to be selected such that a given desired behavior is obtained
- Use of inverted poles and zeroes, to refer transfer function gains to the most important asymptote
- Analytical approximation of roots of high-order polynomials
- Graphical construction of Bode plots of transfer functions and polynomials, to

  avoid algebra mistakes

  approximate transfer functions

  obtain insight into origins of salient features

8.1. Review of Bode plots

Decibels

$$\| G \|_{\mathrm{dB}} = 20 \log_{10}\left(\| G \| \right)$$

Decibels of quantities having units (impedance example): normalize before taking log

$$\| Z \|_{\mathrm{dB}} = 20 \log_{10}\left(\frac{\| Z \|}{R_{base}} \right)$$

Table 8.1. Expressing magnitudes in decibels

Actual magnitude	*Magnitude in dB*
1/2	– 6dB
1	0 dB
2	6 dB
5 = 10/2	20 dB – 6 dB = 14 dB
10	20dB
$1000 = 10^3$	$3 \cdot 20$dB = 60 dB

5Ω is equivalent to 14dB with respect to a base impedance of R_{base} = 1Ω, also known as 14dBΩ.

60dBμA is a current 60dB greater than a base current of 1μA, or 1mA.

Bode plot of f^n

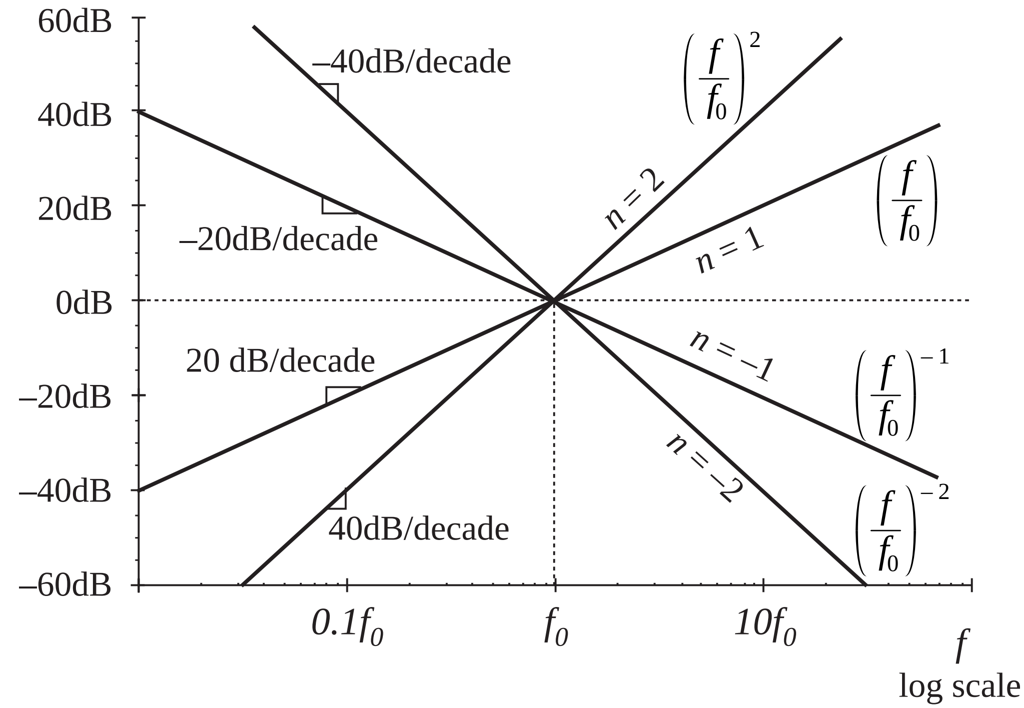
Bode plots are effectively log-log plots, which cause functions which vary as f^n to become linear plots. Given:

$$\| G \| = \left(\frac{f}{f_0} \right)^n$$

Magnitude in dB is

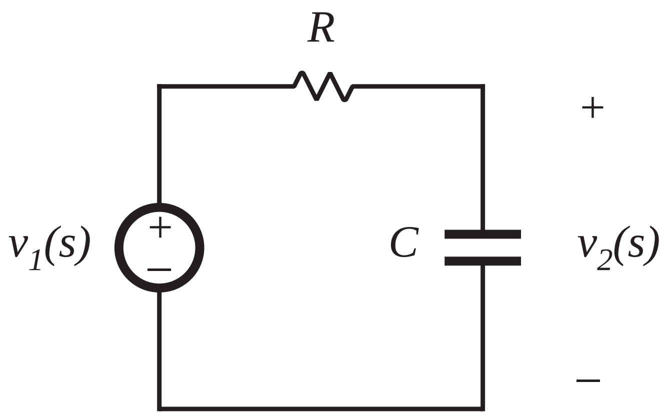
$$\| G \|_{dB} = 20 \log_{10} \left(\frac{f}{f_0} \right)^n = 20n \log_{10} \left(\frac{f}{f_0} \right)$$

- Slope is $20n$ dB/decade
- Magnitude is 1, or 0dB, at frequency $f = f_0$



8.1.1. Single pole response

Simple R-C example



Transfer function is

$$G(s) = \frac{v_2(s)}{v_1(s)} = \frac{\frac{1}{sC}}{\frac{1}{sC} + R}$$

Express as rational fraction:

$$G(s) = \frac{1}{1 + sRC}$$

This coincides with the normalized form

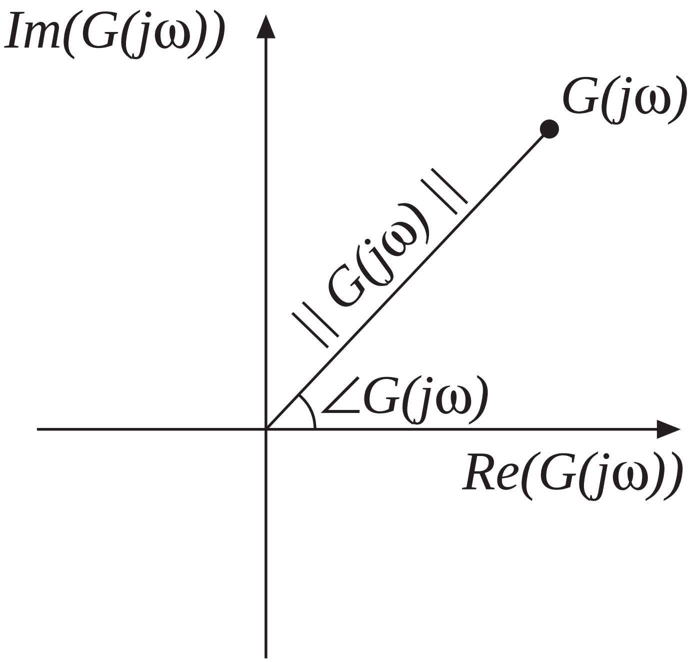
$$G(s) = \frac{1}{\left(1 + \frac{s}{\omega_0}\right)}$$

with $\omega_0 = \frac{1}{RC}$

$G(j\omega)$ and $\| G(j\omega) \|$

Let $s = j\omega$:

$$G(j\omega) = \frac{1}{\left(1 + j\frac{\omega}{\omega_0}\right)} = \frac{1 - j\frac{\omega}{\omega_0}}{1 + \left(\frac{\omega}{\omega_0}\right)^2}$$



Magnitude is

$$\left\| G(j\omega) \right\| = \sqrt{\left[\mathrm{Re}\left(G(j\omega)\right)\right]^2 + \left[\mathrm{Im}\left(G(j\omega)\right)\right]^2}$$
$$= \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

Magnitude in dB:

$$\left\| G(j\omega) \right\|_{\mathrm{dB}} = -20 \log_{10}\left(\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}\right) \text{ dB}$$

Asymptotic behavior: low frequency

For small frequency,
$\omega \ll \omega_0$ and $f \ll f_0$:

$$\left(\frac{\omega}{\omega_0}\right) \ll 1$$

Then $\| G(j\omega) \|$ becomes

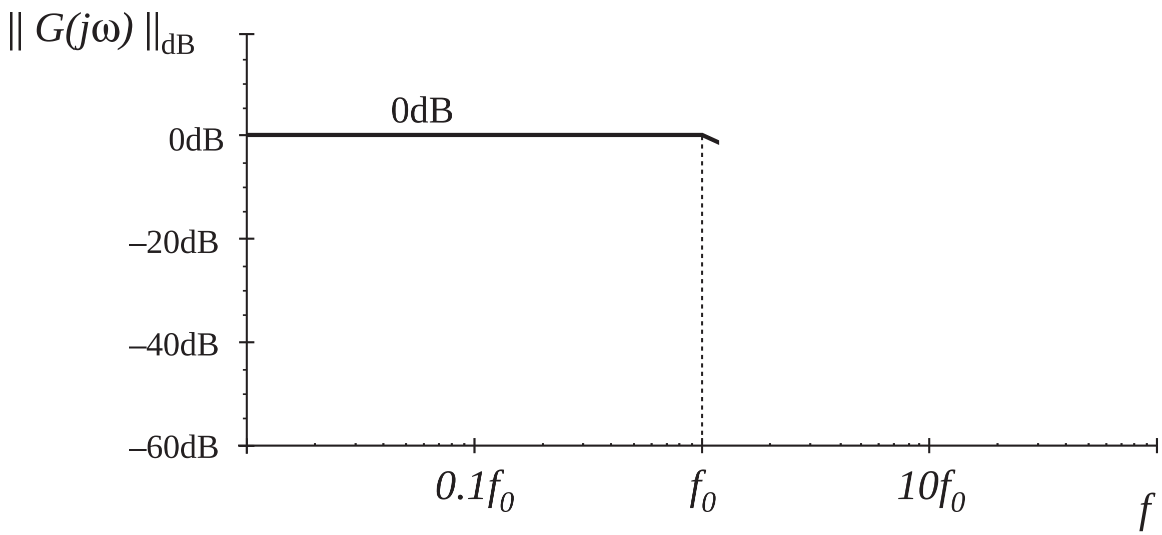
$$\left\| G(j\omega) \right\| \approx \frac{1}{\sqrt{1}} = 1$$

Or, in dB,

$$\left\| G(j\omega) \right\|_{\text{dB}} \approx 0\text{dB}$$

This is the low-frequency asymptote of $\| G(j\omega) \|$

$$\left\| G(j\omega) \right\| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$



Asymptotic behavior: high frequency

For high frequency,
$\omega >> \omega_0$ and $f >> f_0$:

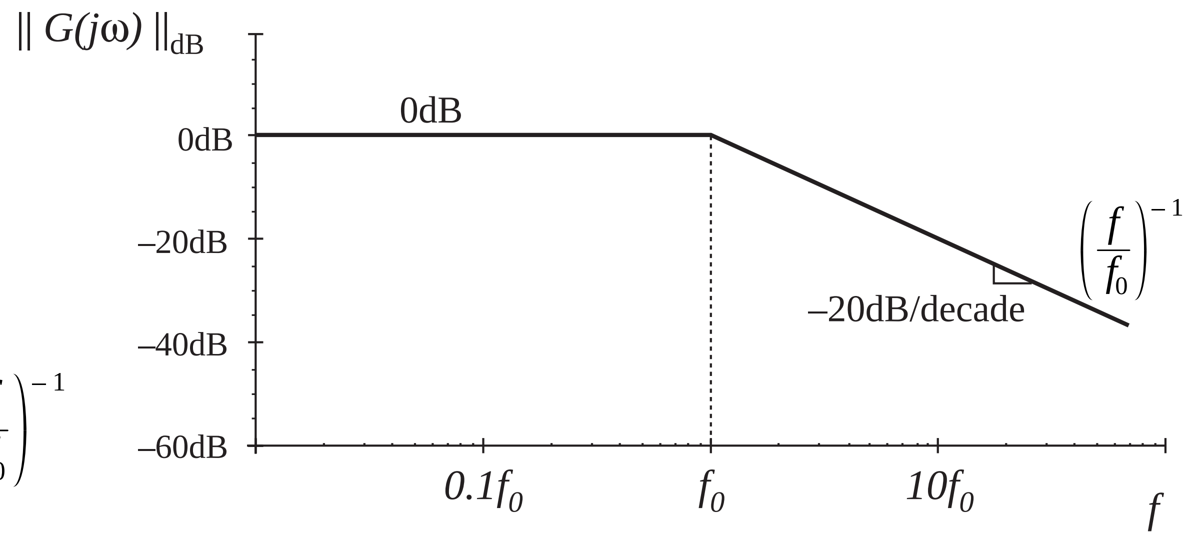
$$\left(\frac{\omega}{\omega_0}\right) >> 1$$

$$1 + \left(\frac{\omega}{\omega_0}\right)^2 \approx \left(\frac{\omega}{\omega_0}\right)^2$$

Then $\| G(j\omega) \|$ becomes

$$\left\| G(j\omega) \right\| \approx \frac{1}{\sqrt{\left(\frac{\omega}{\omega_0}\right)^2}} = \left(\frac{f}{f_0}\right)^{-1}$$

$$\left\| G(j\omega) \right\| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$



The high-frequency asymptote of $\| G(j\omega) \|$ varies as f^{-1}. Hence, $n = -1$, and a straight-line asymptote having a slope of -20dB/decade is obtained. The asymptote has a value of 1 at $f = f_0$.

Deviation of exact curve near $f = f_0$

Evaluate exact magnitude:

at $f = f_0$:

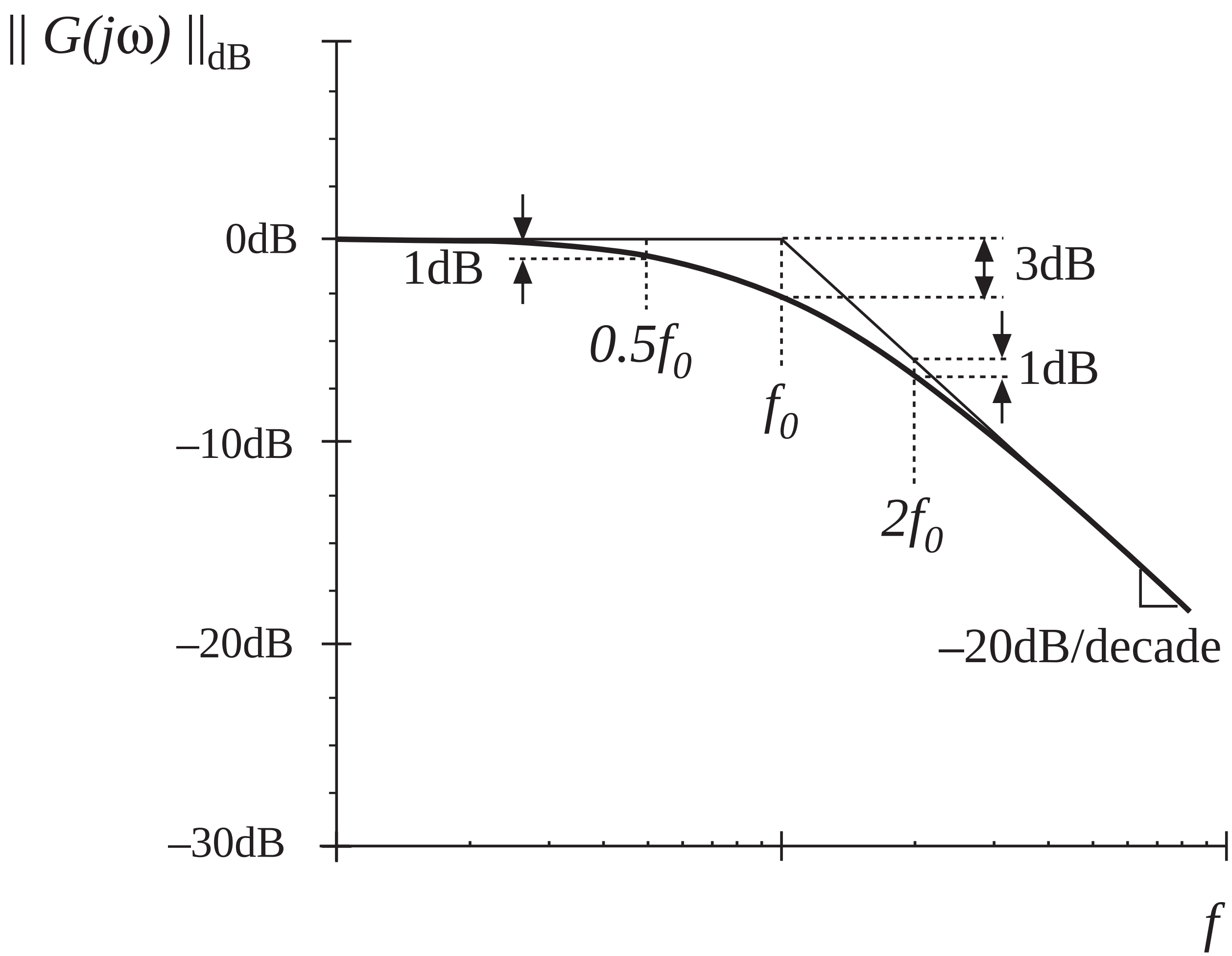
$$\left\| G(j\omega_0) \right\| = \frac{1}{\sqrt{1 + \left(\frac{\omega_0}{\omega_0} \right)^2}} = \frac{1}{\sqrt{2}}$$

$$\left\| G(j\omega_0) \right\|_{dB} = -20 \log_{10} \left(\sqrt{1 + \left(\frac{\omega_0}{\omega_0} \right)^2} \right) \approx -3 \text{ dB}$$

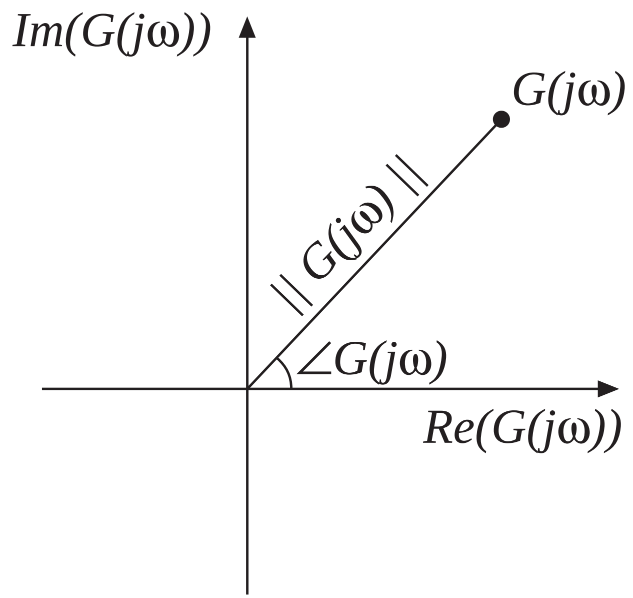
at $f = 0.5f_0$ *and* $2f_0$:

Similar arguments show that the exact curve lies 1dB below the asymptotes.

Summary: magnitude



Phase of $G(j\omega)$

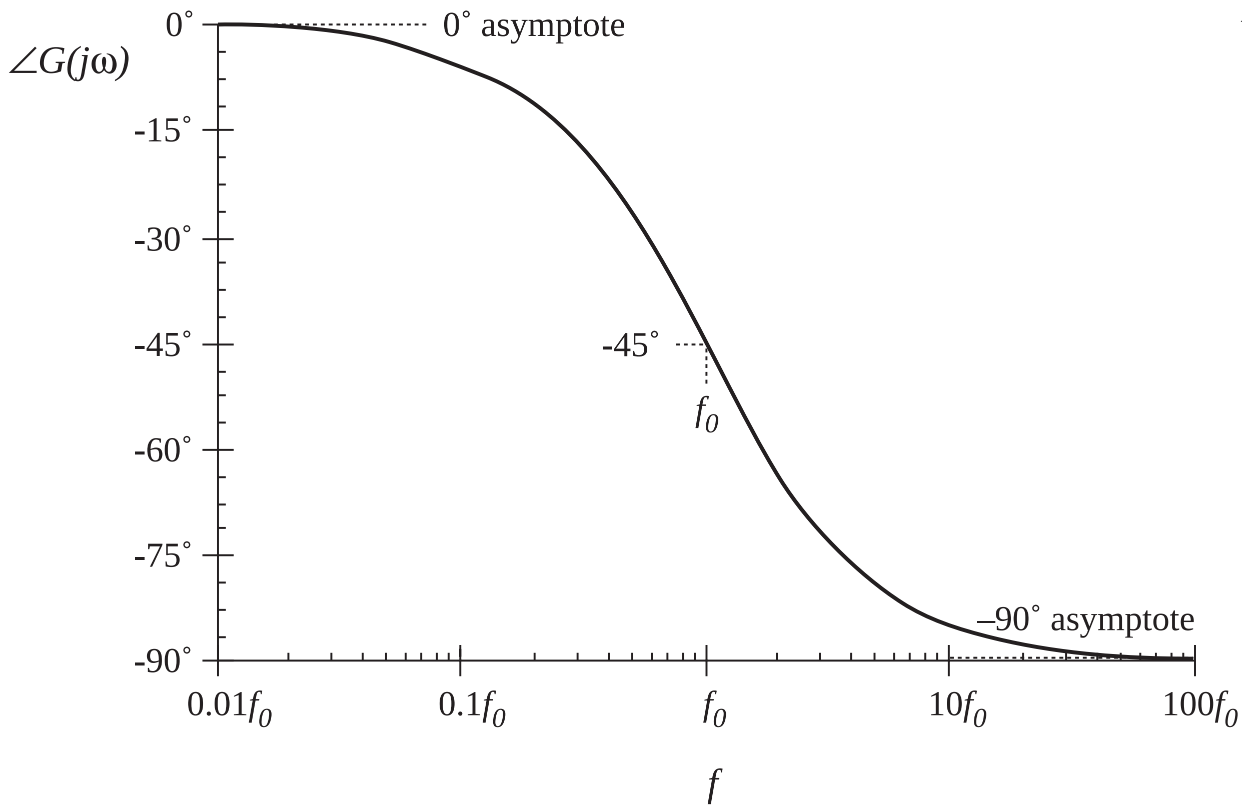


$$\angle G(j\omega) = \tan^{-1}\left(\frac{\mathrm{Im}\left(G(j\omega)\right)}{\mathrm{Re}\left(G(j\omega)\right)}\right)$$

$$G(j\omega) = \frac{1}{\left(1 + j\dfrac{\omega}{\omega_0}\right)} = \frac{1 - j\dfrac{\omega}{\omega_0}}{1 + \left(\dfrac{\omega}{\omega_0}\right)^2}$$

$$\angle G(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

Phase of $G(j\omega)$



$$\angle G(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

ω	$\angle G(j\omega)$
0	0˚
ω_0	–45˚
∞	–90˚

Phase asymptotes

Low frequency: 0˚

High frequency: –90˚

Low- and high-frequency asymptotes do not intersect

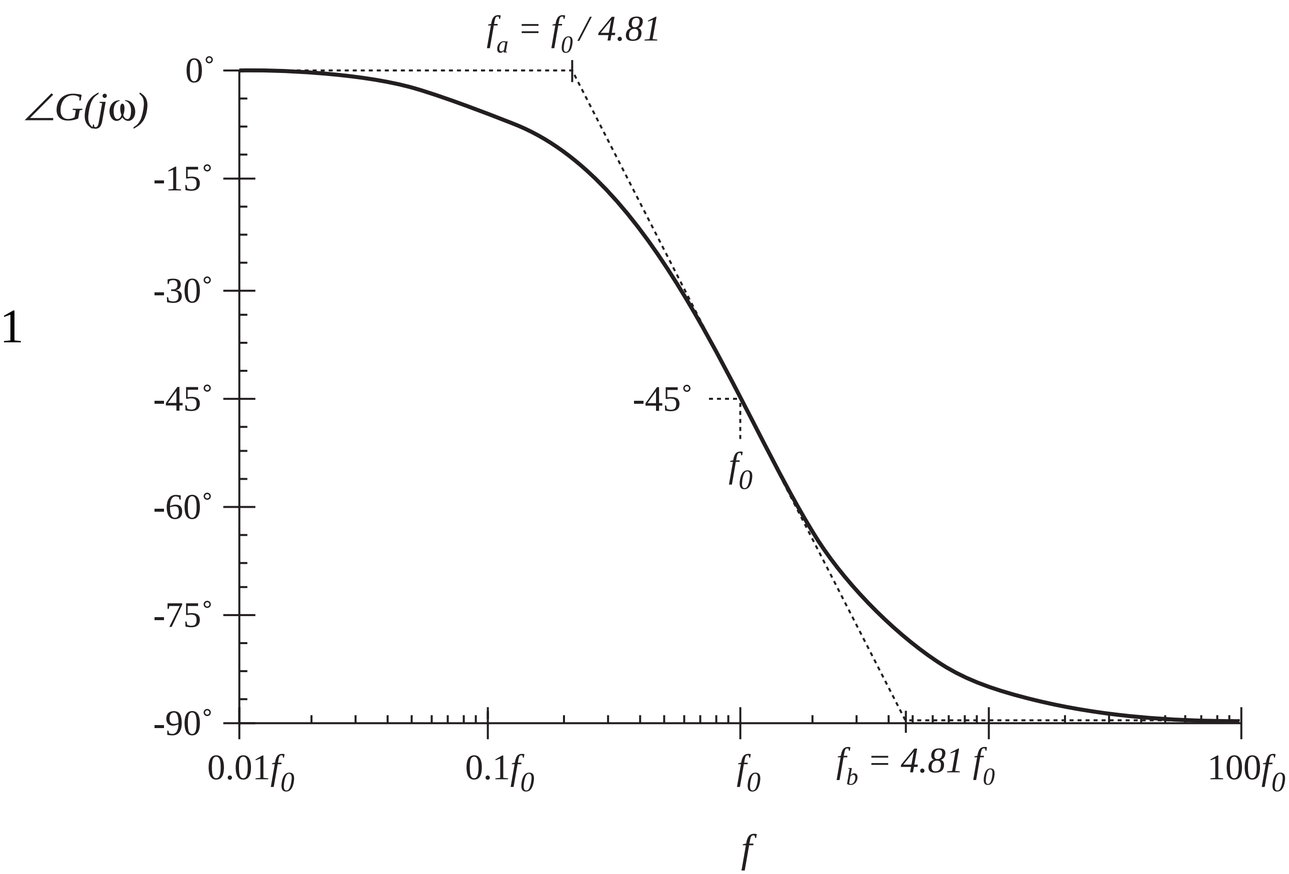
Hence, need a midfrequency asymptote

Try a midfrequency asymptote having slope identical to actual slope at the corner frequency f_0. One can show that the asymptotes then intersect at the break frequencies

$$f_a = f_0\, e^{-\pi/2} \approx f_0 / 4.81$$
$$f_b = f_0\, e^{\pi/2} \approx 4.81\, f_0$$

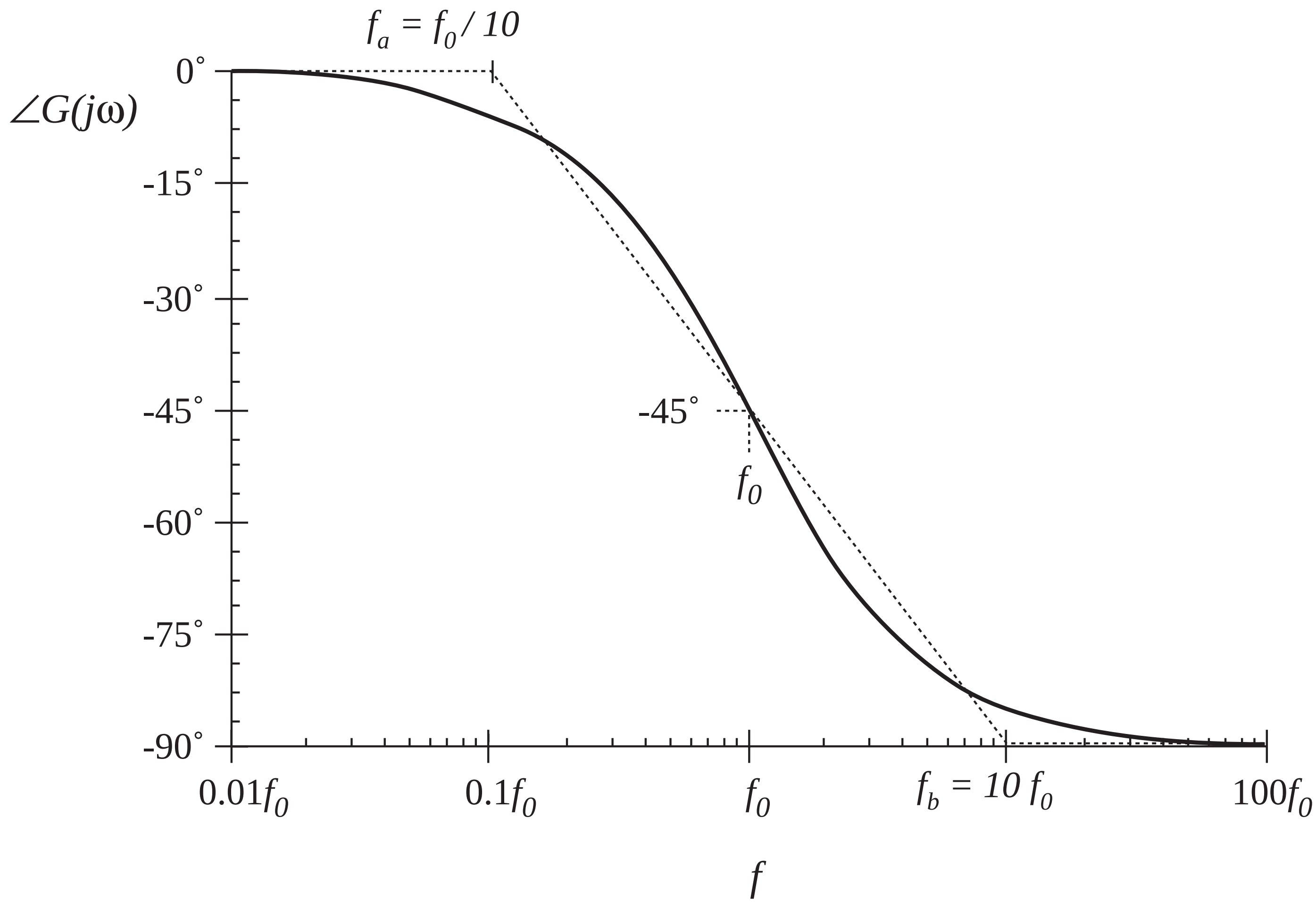
Phase asymptotes

$$f_a = f_0\, e^{-\pi/2} \approx f_0 / 4.81$$
$$f_b = f_0\, e^{\pi/2} \approx 4.81\, f_0$$



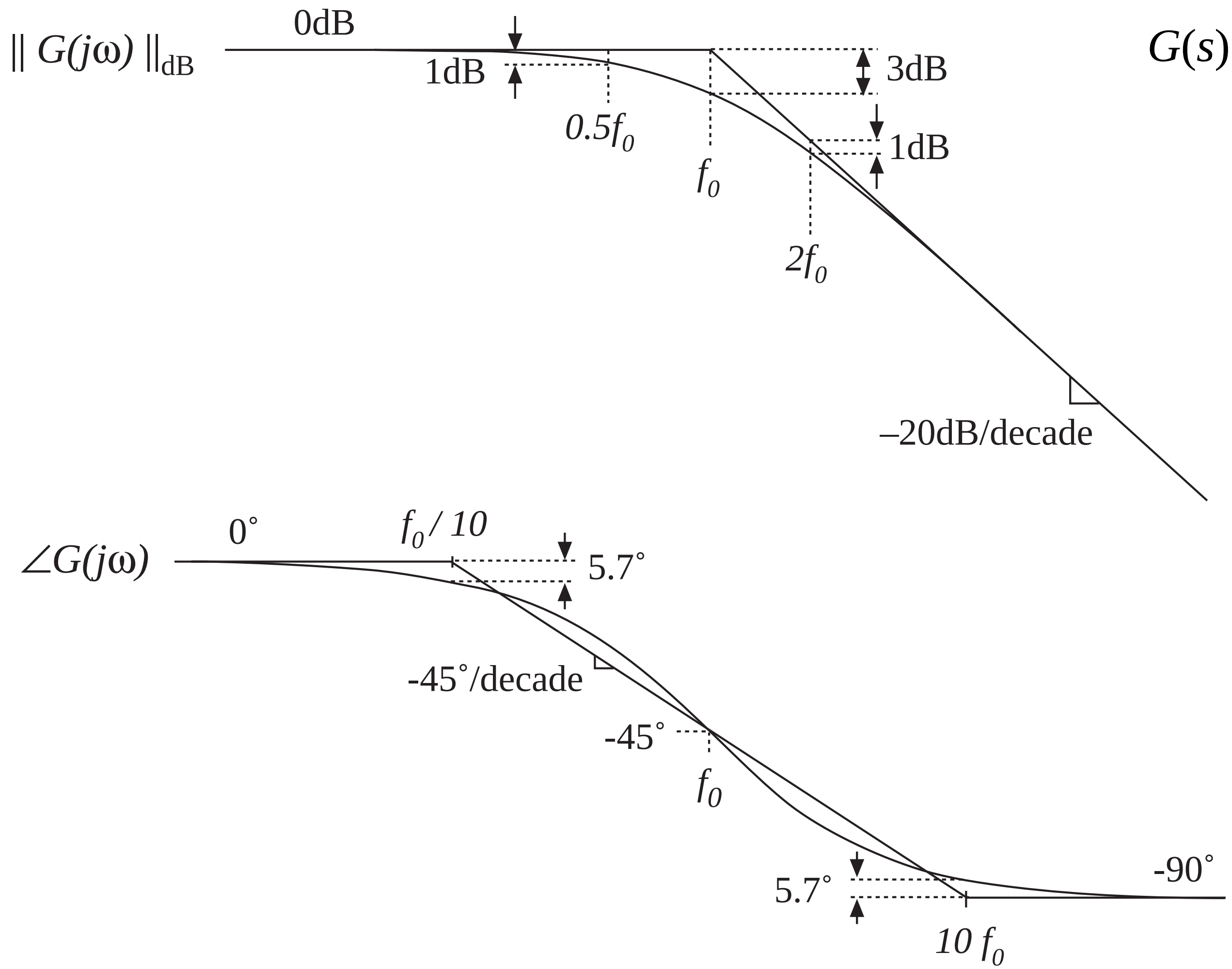
Phase asymptotes: a simpler choice

$f_a = f_0 / 10$
$f_b = 10\ f_0$



Summary: Bode plot of real pole

$$G(s) = \frac{1}{\left(1 + \frac{s}{\omega_0}\right)}$$



8.1.2. Single zero response

Normalized form:

$$G(s) = \left(1 + \frac{s}{\omega_0}\right)$$

Magnitude:

$$\left\| G(j\omega) \right\| = \sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}$$

Use arguments similar to those used for the simple pole, to derive asymptotes:

0dB at low frequency, $\omega << \omega_0$

+20dB/decade slope at high frequency, $\omega >> \omega_0$

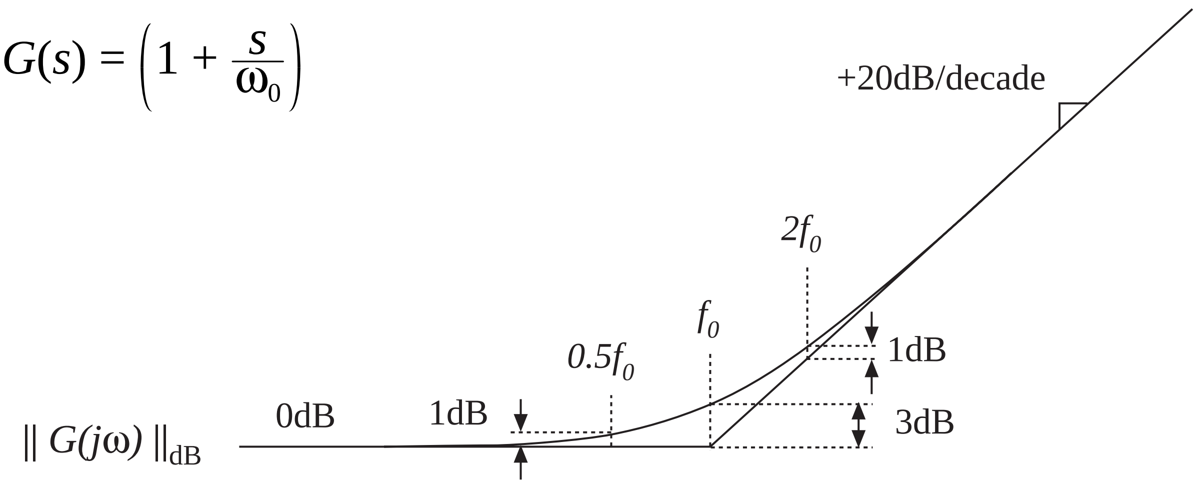
Phase:

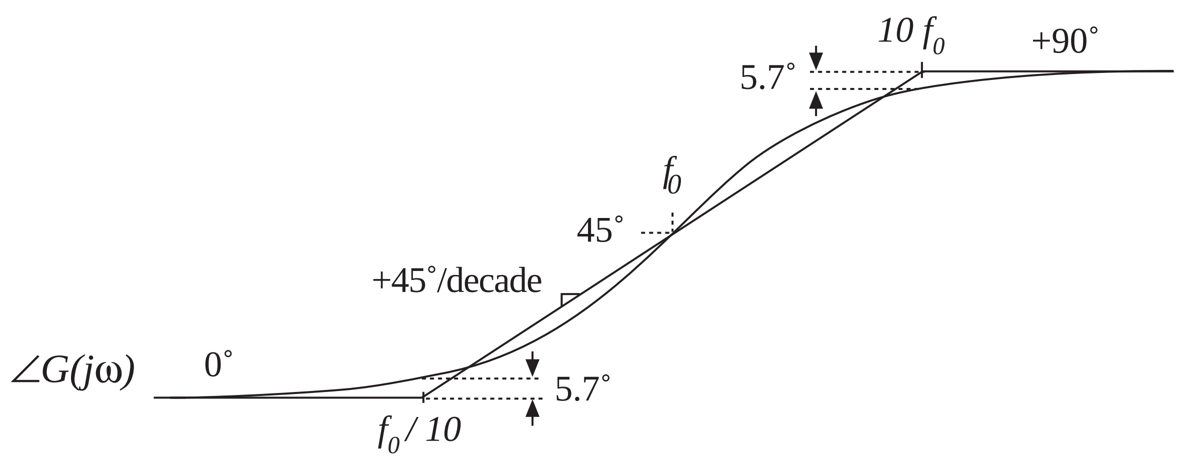
$$\angle G(j\omega) = \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

—with the exception of a missing minus sign, same as simple pole

Summary: Bode plot, real zero

$$G(s) = \left(1 + \frac{s}{\omega_0}\right)$$





8.1.3. Right half-plane zero

Normalized form:

$$G(s) = \left(1 - \frac{s}{\omega_0}\right)$$

Magnitude:

$$\left\| G(j\omega) \right\| = \sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}$$

—same as conventional (left half-plane) zero. Hence, magnitude asymptotes are identical to those of LHP zero.

Phase:

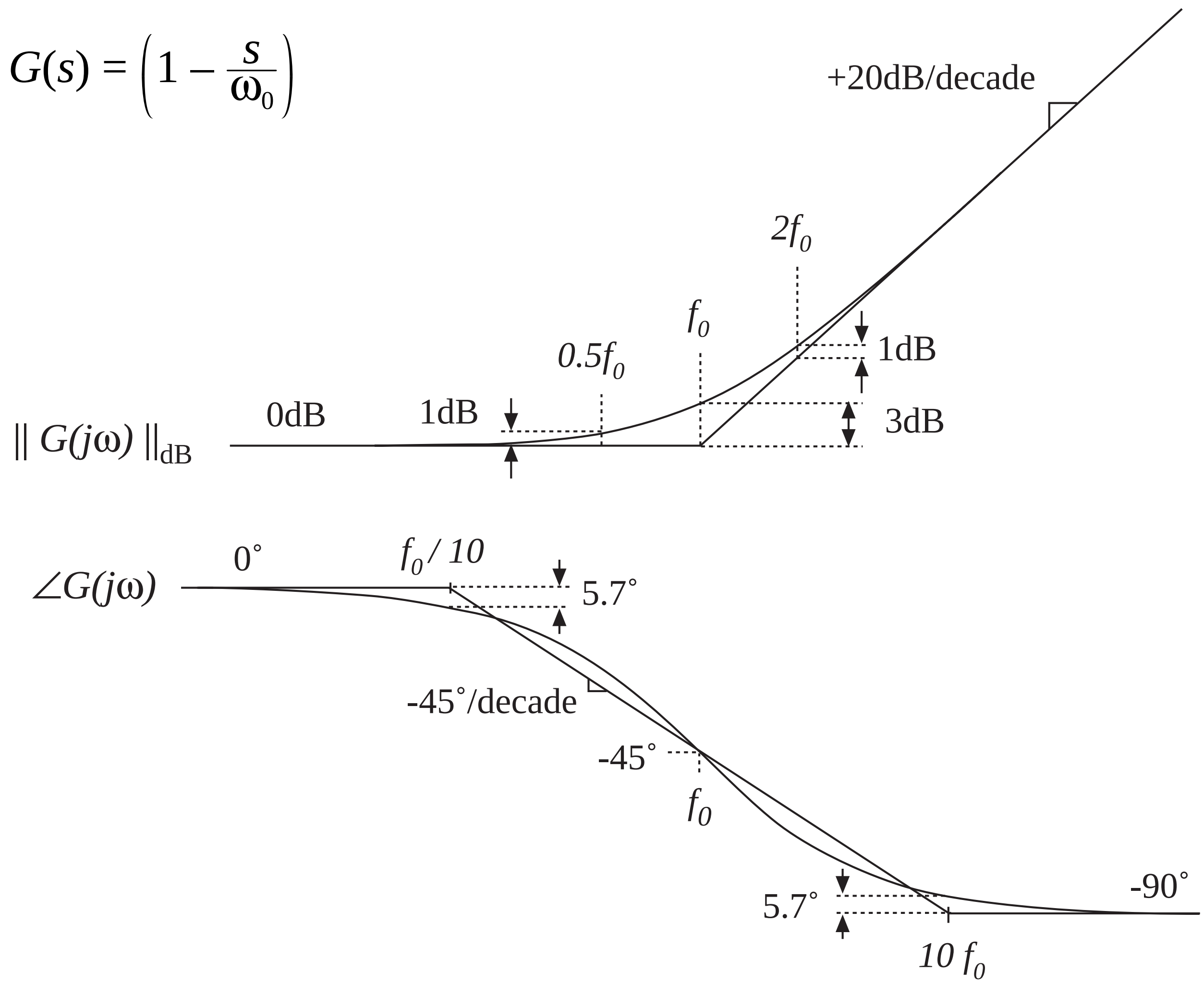
$$\angle G(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

—same as real pole.

The RHP zero exhibits the magnitude asymptotes of the LHP zero, and the phase asymptotes of the pole

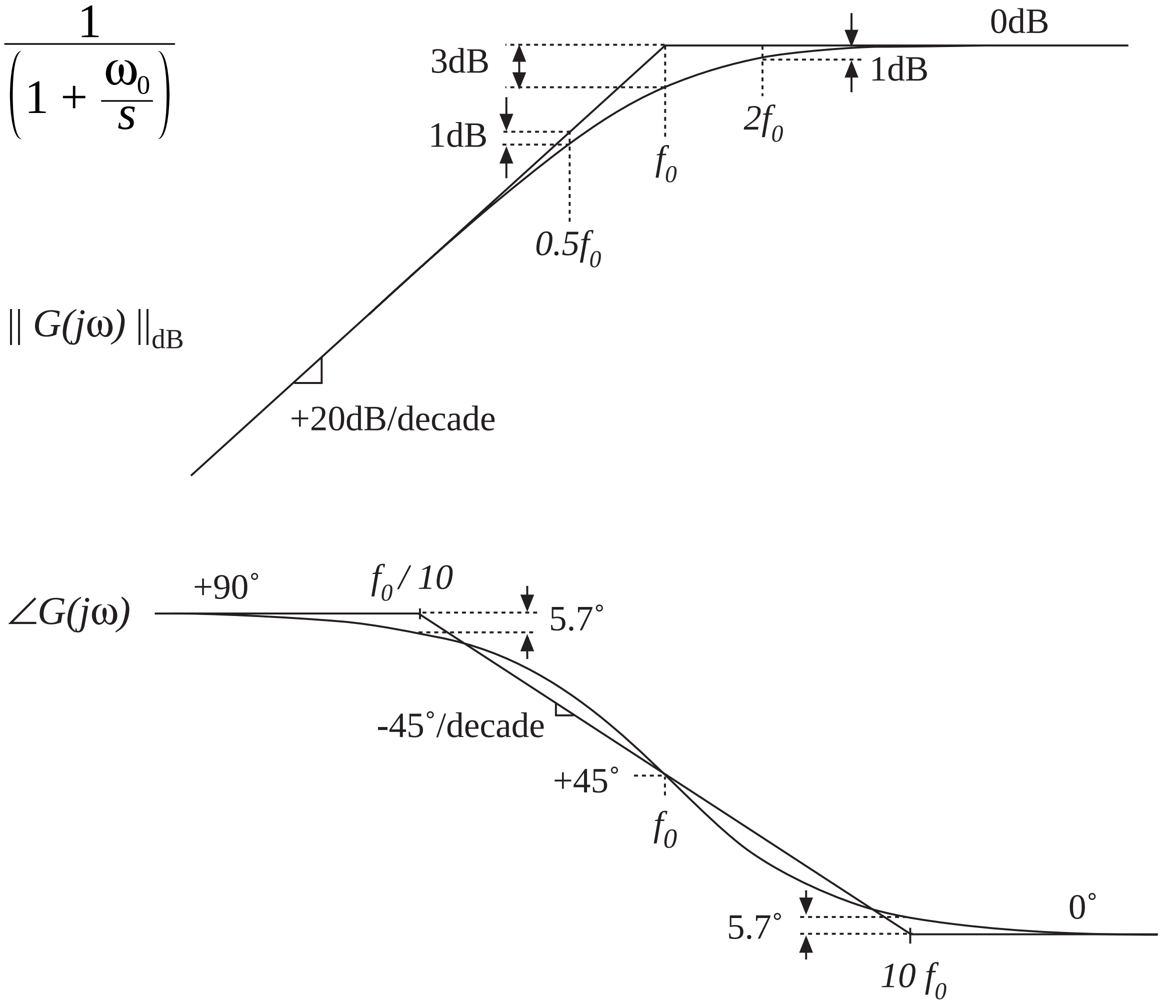
Summary: Bode plot, RHP zero

$$G(s) = \left(1 - \frac{s}{\omega_0}\right)$$



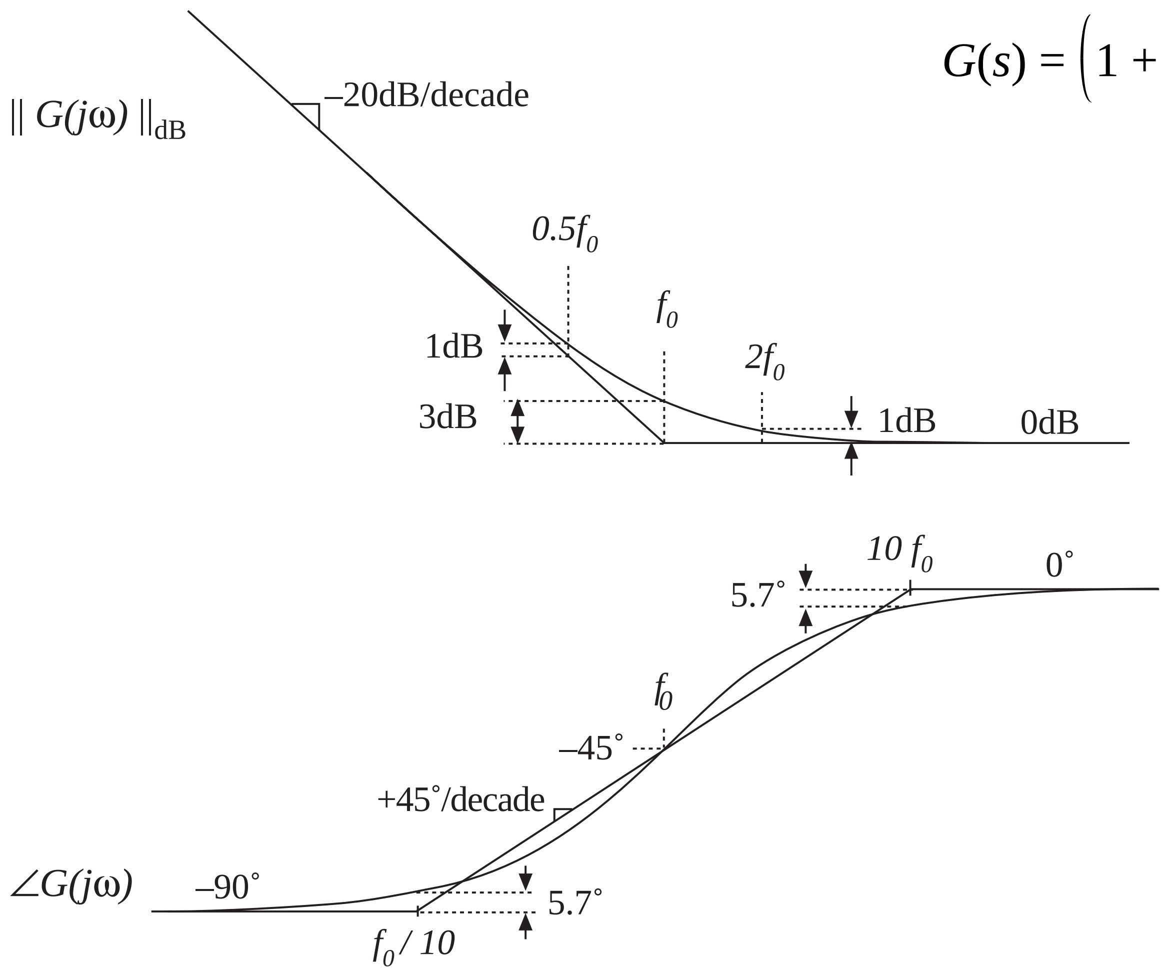
Asymptotes, inverted pole

$$G(s) = \frac{1}{\left(1 + \frac{\omega_0}{s}\right)}$$



Asymptotes, inverted zero

$$G(s) = \left(1 + \frac{\omega_0}{s}\right)$$



8.1.5. Combinations

Suppose that we have constructed the Bode diagrams of two complex-values functions of frequency, $G_1(\omega)$ and $G_2(\omega)$. It is desired to construct the Bode diagram of the product, $G_3(\omega) = G_1(\omega)\, G_2(\omega)$.

Express the complex-valued functions in polar form:

$$G_1(\omega) = R_1(\omega)\, e^{j\theta_1(\omega)}$$
$$G_2(\omega) = R_2(\omega)\, e^{j\theta_2(\omega)}$$
$$G_3(\omega) = R_3(\omega)\, e^{j\theta_3(\omega)}$$

The product $G_3(\omega)$ can then be written

$$G_3(\omega) = G_1(\omega)\, G_2(\omega) = R_1(\omega)\, e^{j\theta_1(\omega)}\, R_2(\omega)\, e^{j\theta_2(\omega)}$$

$$G_3(\omega) = \left(R_1(\omega)\, R_2(\omega)\right) e^{j(\theta_1(\omega) + \theta_2(\omega))}$$

Combinations

$$G_3(\omega) = \left(R_1(\omega)\ R_2(\omega)\right) e^{j(\theta_1(\omega) + \theta_2(\omega))}$$

The composite phase is

$$\theta_3(\omega) = \theta_1(\omega) + \theta_2(\omega)$$

The composite magnitude is

$$R_3(\omega) = R_1(\omega)\ R_2(\omega)$$

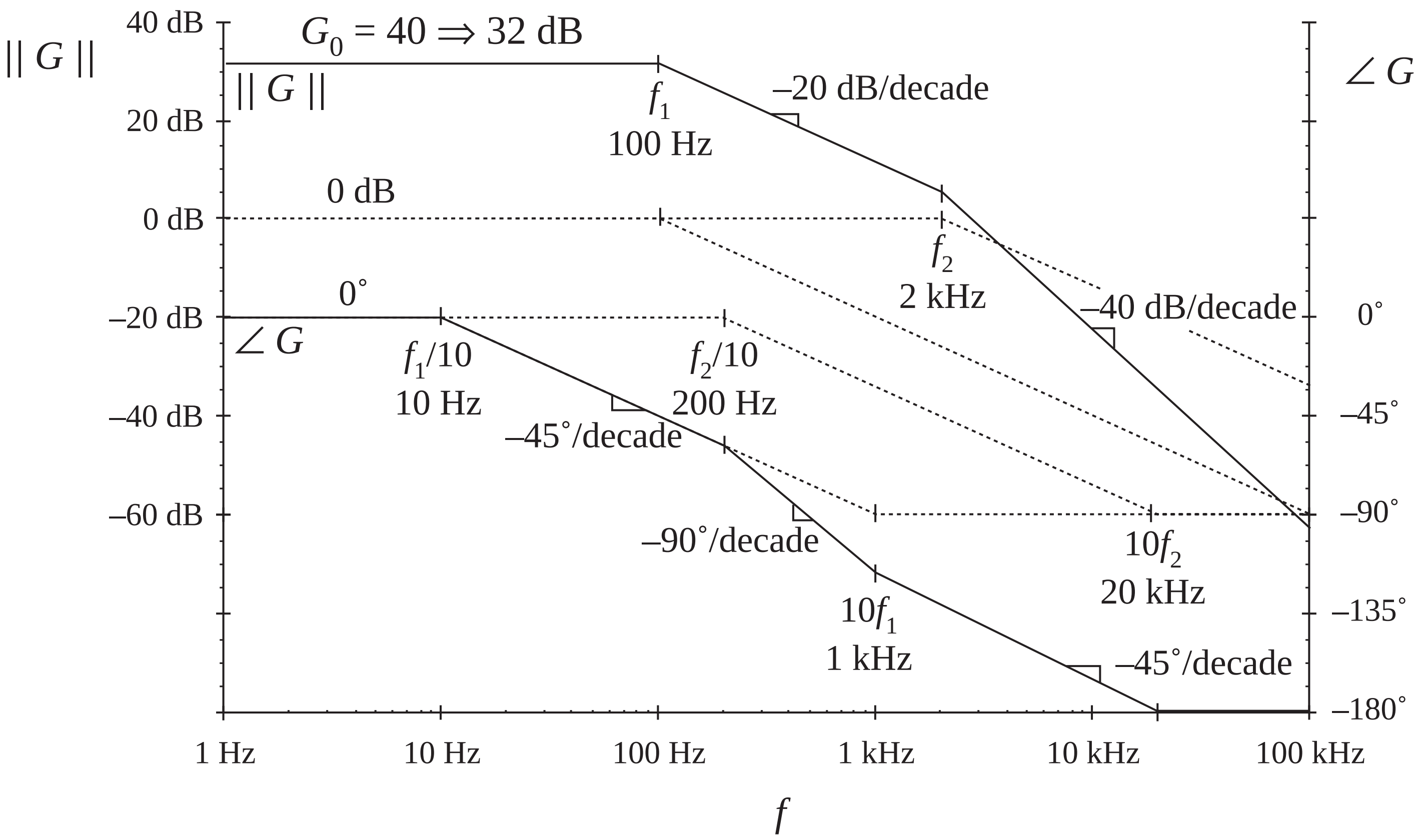
$$\left| R_3(\omega) \right|_{\mathrm{dB}} = \left| R_1(\omega) \right|_{\mathrm{dB}} + \left| R_2(\omega) \right|_{\mathrm{dB}}$$

Composite phase is sum of individual phases.

Composite magnitude, when expressed in dB, is sum of individual magnitudes.

Example 1: $G(s) = \dfrac{G_0}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)}$

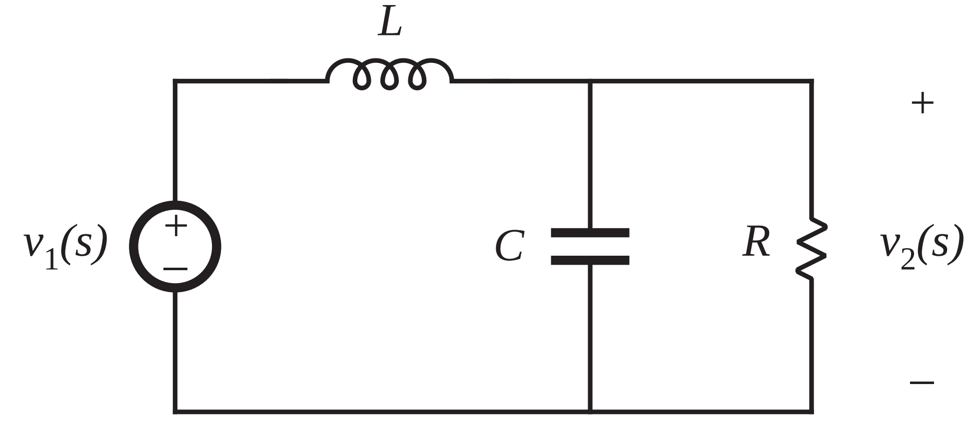
with $G_0 = 40 \Rightarrow 32$ dB, $f_1 = \omega_1/2\pi = 100$ Hz, $f_2 = \omega_2/2\pi = 2$ kHz



8.1.6 Quadratic pole response: resonance

Example

$$G(s) = \frac{v_2(s)}{v_1(s)} = \frac{1}{1 + s\frac{L}{R} + s^2 LC}$$



Two-pole low-pass filter example

Second-order denominator, of the form

$$G(s) = \frac{1}{1 + a_1 s + a_2 s^2}$$

with $a_1 = L/R$ and $a_2 = LC$

How should we construct the Bode diagram?

Approach 1: factor denominator

$$G(s) = \frac{1}{1 + a_1 s + a_2 s^2}$$

We might factor the denominator using the quadratic formula, then construct Bode diagram as the combination of two real poles:

$$G(s) = \frac{1}{\left(1 - \frac{s}{s_1}\right)\left(1 - \frac{s}{s_2}\right)} \quad \text{with} \quad s_1 = -\frac{a_1}{2a_2}\left[1 - \sqrt{1 - \frac{4a_2}{a_1^2}}\right]$$

$$s_2 = -\frac{a_1}{2a_2}\left[1 + \sqrt{1 - \frac{4a_2}{a_1^2}}\right]$$

- If $4a_2 \le a_1^2$, then the roots s_1 and s_2 are real. We can construct Bode diagram as the combination of two real poles.
- If $4a_2 > a_1^2$, then the roots are complex. In Section 8.1.1, the assumption was made that ω_0 is real; hence, the results of that section cannot be applied and we need to do some additional work.

Approach 2: Define a standard normalized form for the quadratic case

$$G(s) = \frac{1}{1 + 2\zeta \frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2} \quad \text{or} \quad G(s) = \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

- When the coefficients of s are real and positive, then the parameters ζ, ω_0, and Q are also real and positive
- The parameters ζ, ω_0, and Q are found by equating the coefficients of s
- The parameter ω_0 is the angular corner frequency, and we can define $f_0 = \omega_0/2\pi$
- The parameter ζ is called the *damping factor.* ζ controls the shape of the exact curve in the vicinity of $f = f_0$. The roots are complex when $\zeta < 1$.
- In the alternative form, the parameter Q is called the *quality factor.* Q also controls the shape of the exact curve in the vicinity of $f = f_0$. The roots are complex when $Q > 0.5$.

The Q-factor

In a second-order system, ζ and Q are related according to

$$Q = \frac{1}{2\zeta}$$

Q is a measure of the dissipation in the system. A more general definition of Q, for sinusoidal excitation of a passive element or system is

$$Q = 2\pi \frac{\text{(peak stored energy)}}{\text{(energy dissipated per cycle)}}$$

For a second-order passive system, the two equations above are equivalent. We will see that Q has a simple interpretation in the Bode diagrams of second-order transfer functions.

Analytical expressions for f_0 and Q

Two-pole low-pass filter example: we found that

$$G(s) = \frac{v_2(s)}{v_1(s)} = \frac{1}{1 + s\frac{L}{R} + s^2 LC}$$

Equate coefficients of like powers of s with the standard form

$$G(s) = \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

Result:

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

$$Q = R\sqrt{\frac{C}{L}}$$

Magnitude asymptotes, quadratic form

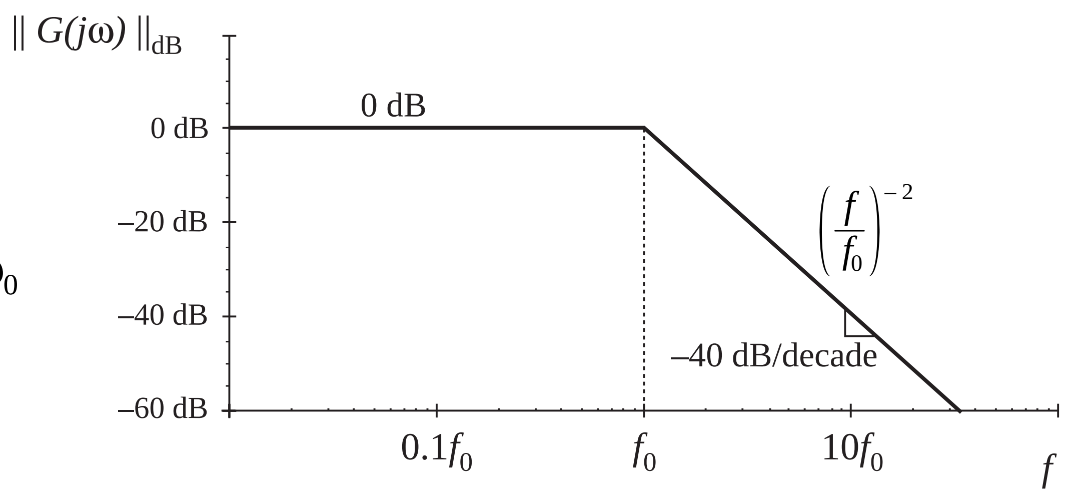
In the form $G(s) = \dfrac{1}{1 + \dfrac{s}{Q\omega_0} + \left(\dfrac{s}{\omega_0}\right)^2}$

let $s = j\omega$ and find magnitude: $\left\| G(j\omega) \right\| = \dfrac{1}{\sqrt{\left(1 - \left(\dfrac{\omega}{\omega_0}\right)^2\right)^2 + \dfrac{1}{Q^2}\left(\dfrac{\omega}{\omega_0}\right)^2}}$

Asymptotes are

$\left\| G \right\| \to 1 \quad \text{for } \omega \ll \omega_0$

$\left\| G \right\| \to \left(\dfrac{f}{f_0}\right)^{-2} \quad \text{for } \omega \gg \omega_0$



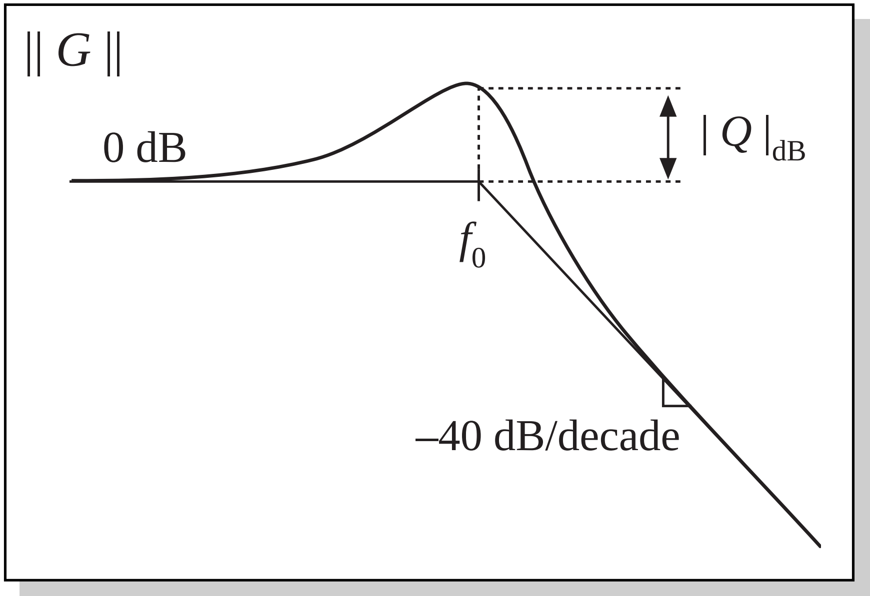
Deviation of exact curve from magnitude asymptotes

$$\left\| G(j\omega) \right\| = \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \frac{1}{Q^2}\left(\frac{\omega}{\omega_0}\right)^2}}$$

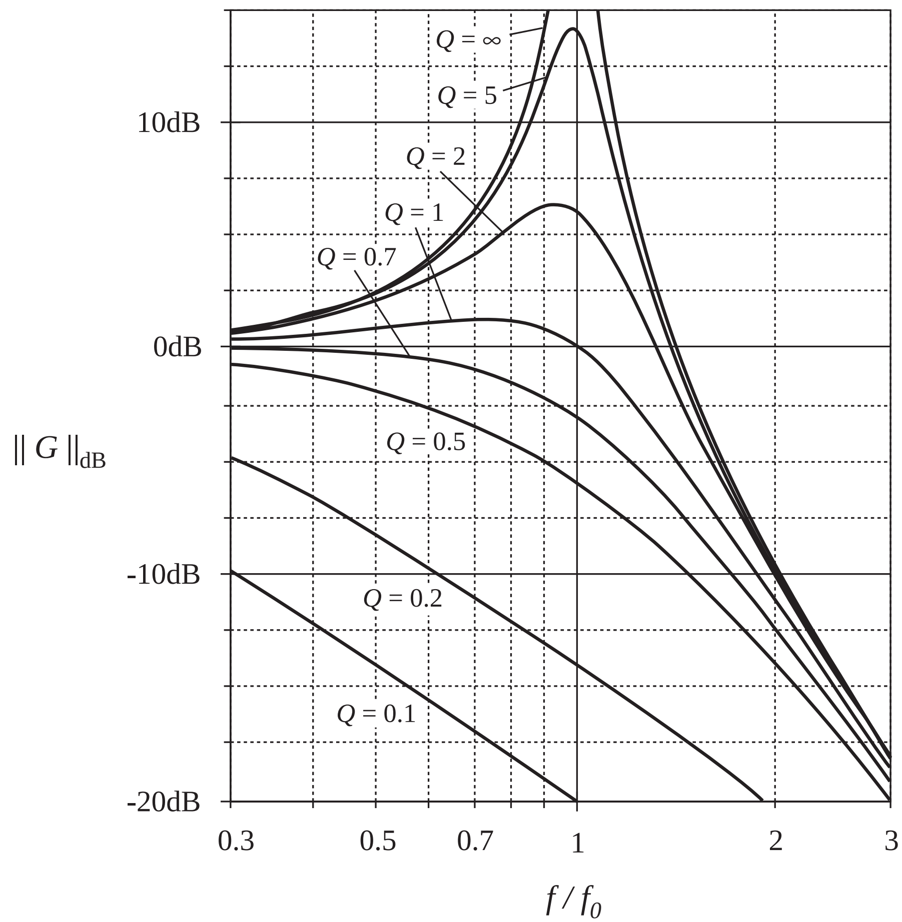
At $\omega = \omega_0$, the exact magnitude is

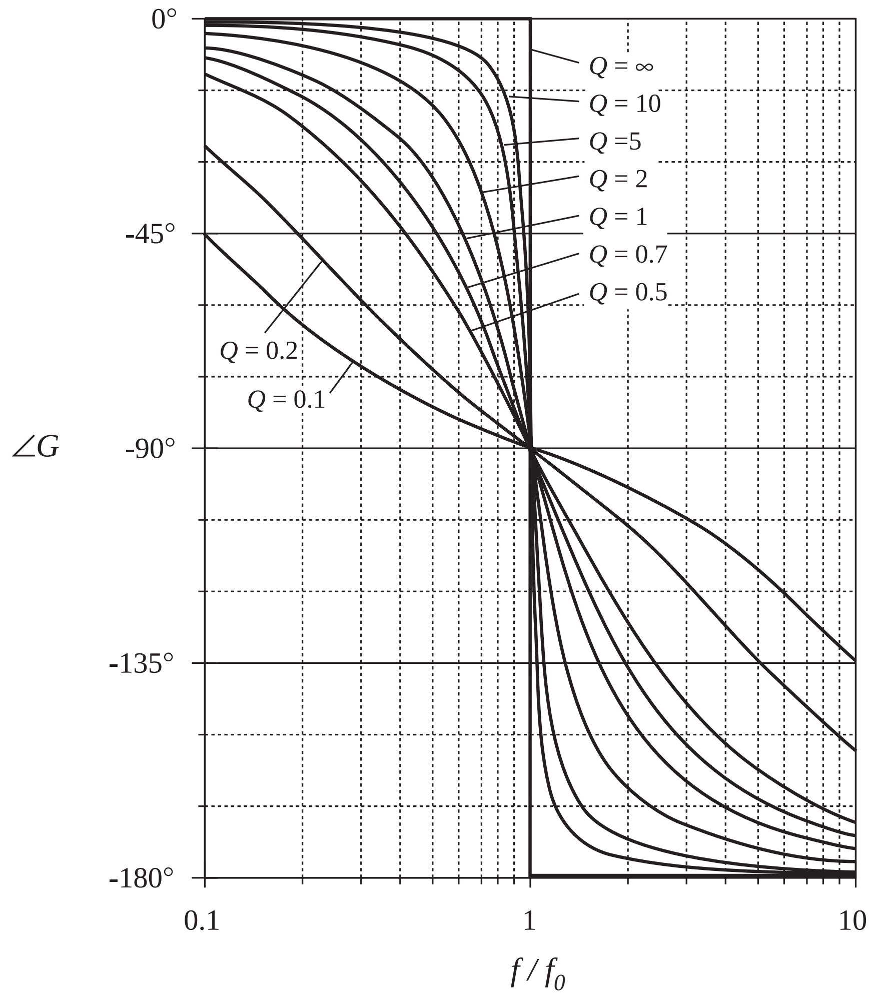
$$\left\| G(j\omega_0) \right\| = Q \qquad \text{or, in dB:} \qquad \left\| G(j\omega_0) \right\|_{dB} = \left| Q \right|_{dB}$$

The exact curve has magnitude Q at $f = f_0$. The deviation of the exact curve from the asymptotes is $| Q |_{dB}$



Two-pole response: exact curves





8.2.1. Example: transfer functions of the buck-boost converter

Small-signal ac equations of the buck-boost converter, derived in section 7.2:

$$L\,\frac{d\hat{i}(t)}{dt} = D\hat{v}_g(t) + D'\hat{v}(t) + \left(V_g - V\right)\hat{d}(t)$$

$$C\,\frac{d\hat{v}(t)}{dt} = -\,D'\hat{i}(t) - \frac{\hat{v}(t)}{R} + I\hat{d}(t)$$

$$\hat{i}_g(t) = D\hat{i}(t) + I\hat{d}(t)$$

Definition of transfer functions

The converter contains two inputs, $\hat{d}(s)$ and $\hat{v}_g(s)$ and one output, $\hat{v}(s)$

Hence, the ac output voltage variations can be expressed as the superposition of terms arising from the two inputs:

$$\hat{v}(s) = G_{vd}(s)\, \hat{d}(s) + G_{vg}(s)\, \hat{v}_g(s)$$

The control-to-output and line-to-output transfer functions can be defined as

$$G_{vd}(s) = \left. \frac{\hat{v}(s)}{\hat{d}(s)} \right|_{\hat{v}_g(s) = 0} \quad \text{and} \quad G_{vg}(s) = \left. \frac{\hat{v}(s)}{\hat{v}_g(s)} \right|_{\hat{d}(s) = 0}$$

Derivation of transfer functions

Algebraic approach

Take Laplace transform of converter equations, letting initial conditions be zero:

$$sL\hat{i}(s) = D\hat{v}_g(s) + D'\hat{v}(s) + \left(V_g - V\right)\hat{d}(s)$$

$$sC\hat{v}(s) = -D'\hat{i}(s) - \frac{\hat{v}(s)}{R} + I\hat{d}(s)$$

Eliminate $\hat{i}(s)$, and solve for $\hat{v}(s)$

$$\hat{i}(s) = \frac{D\hat{v}_g(s) + D'\hat{v}(s) + \left(V_g - V\right)\hat{d}(s)}{sL}$$

Derivation of transfer functions

$$sC\hat{v}(s) = -\frac{D'}{sL}\left(D\hat{v}_g(s) + D'\hat{v}(s) + \left(V_g - V\right)\hat{d}(s)\right) - \frac{\hat{v}(s)}{R} + I\hat{d}(s)$$

$$\hat{v}(s) = \frac{-DD'}{D'^2 + s\frac{L}{R} + s^2 LC}\,\hat{v}_g(s) - \frac{V_g - V - s\,LI}{D'^2 + s\frac{L}{R} + s^2 LC}\,\hat{d}(s)$$

write in normalized form:

$$\hat{v}(s) = \left(-\frac{D}{D'}\right)\frac{1}{1 + s\frac{L}{D'^2 R} + s^2\frac{LC}{D'^2}}\,\hat{v}_g(s) - \left(\frac{V_g - V}{D'^2}\right)\frac{\left(1 - s\frac{LI}{V_g - V}\right)}{1 + s\frac{L}{D'^2 R} + s^2\frac{LC}{D'^2}}\,\hat{d}(s)$$

Derivation of transfer functions

Hence, the line-to-output transfer function is

$$G_{vg}(s) = \left.\frac{\hat{v}(s)}{\hat{v}_g(s)}\right|_{\hat{d}(s) = 0} = \left(-\frac{D}{D'}\right) \frac{1}{1 + s\,\frac{L}{D'^2 R} + s^2\,\frac{LC}{D'^2}}$$

which is of the following standard form:

$$G_{vg}(s) = G_{g0} \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

Salient features of the line-to-output transfer function

Equate standard form to derived transfer function, to determine expressions for the salient features:

$$G_{g0} = -\frac{D}{D'}$$

$$\frac{1}{\omega_0^2} = \frac{LC}{D'^2} \qquad \omega_0 = \frac{D'}{\sqrt{LC}}$$

$$\frac{1}{Q\omega_0} = \frac{L}{D'^2 R} \qquad Q = D'R\sqrt{\frac{C}{L}}$$

Control-to-output transfer function

$$G_{vd}(s) = \left.\frac{\hat{v}(s)}{\hat{d}(s)}\right|_{\hat{v}_g(s)=0} = \left(-\frac{V_g - V}{D'}\right) \frac{\left(1 - s\,\frac{LI}{V_g - V}\right)}{\left(1 + s\,\frac{L}{D'^2 R} + s^2\,\frac{LC}{D'^2}\right)}$$

Standard form:

$$G_{vd}(s) = G_{d0}\,\frac{\left(1 - \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2\right)}$$

Salient features of control-to-output transfer function

$$G_{d0} = -\frac{V_g - V}{D'} = -\frac{V_g}{D'^2} = \frac{V}{D D'}$$

$$\omega_z = \frac{V_g - V}{L I} = \frac{D'^2 R}{D L} \qquad \text{(RHP)}$$

$$\omega_0 = \frac{D'}{\sqrt{LC}}$$

$$Q = D'R \sqrt{\frac{C}{L}}$$

— Simplified using the dc relations:

$$V = -\frac{D}{D'} V_g$$

$$I = -\frac{V}{D' R}$$

Plug in numerical values

Suppose we are given the following numerical values:

$$D = 0.6$$
$$R = 10\Omega$$
$$V_g = 30\text{V}$$
$$L = 160\mu\text{H}$$
$$C = 160\mu\text{F}$$

Then the salient features have the following numerical values:

$$\left\| G_{g0} \right\| = \frac{D}{D'} = 1.5 \Rightarrow 3.5\text{dB}$$

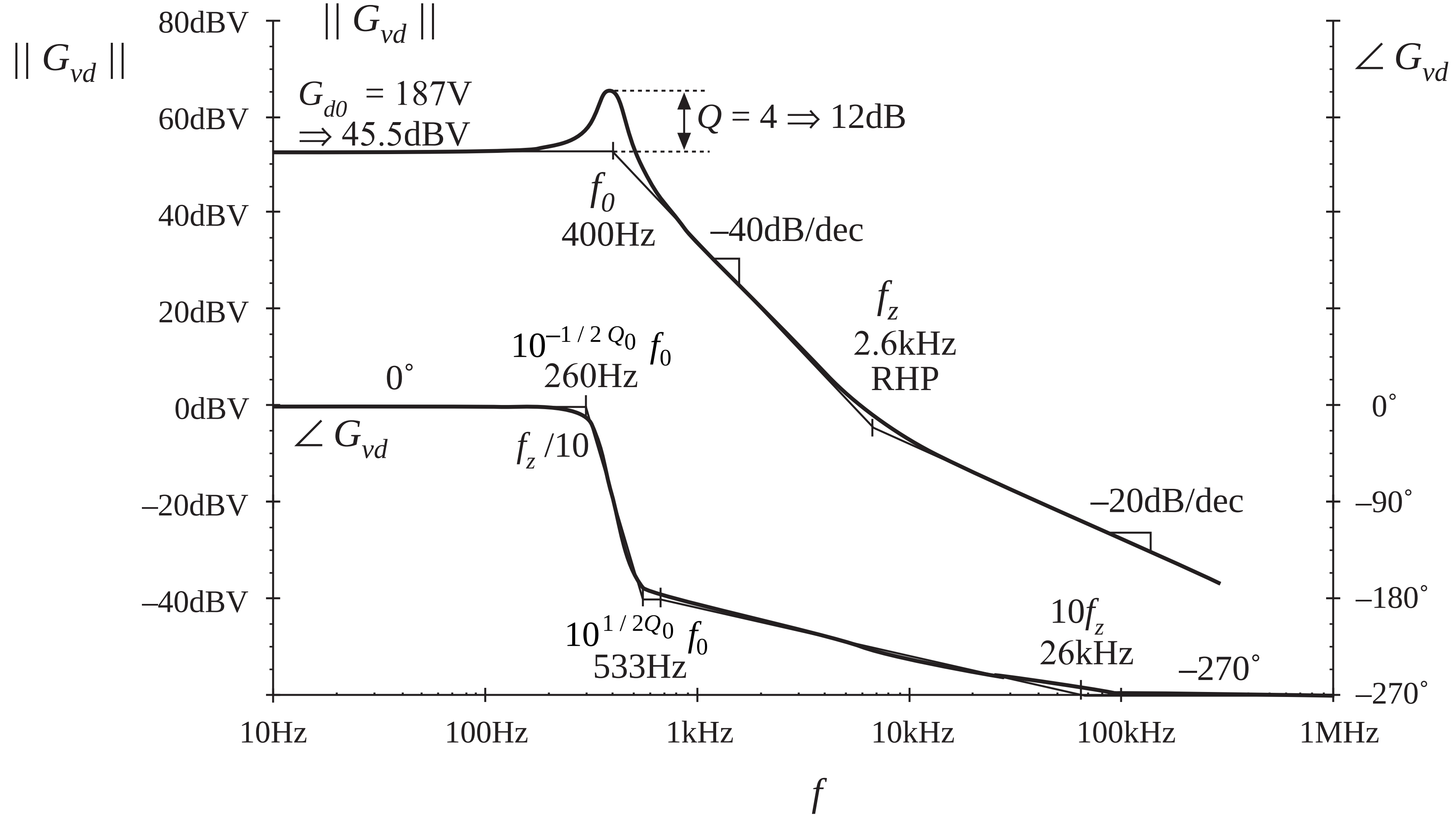
$$\left\| G_{d0} \right\| = \frac{\left\| V \right\|}{D\,D'} = 187.5\text{V} \Rightarrow 45.5\text{dBV}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{D'}{2\pi\sqrt{LC}} = 400\text{Hz}$$

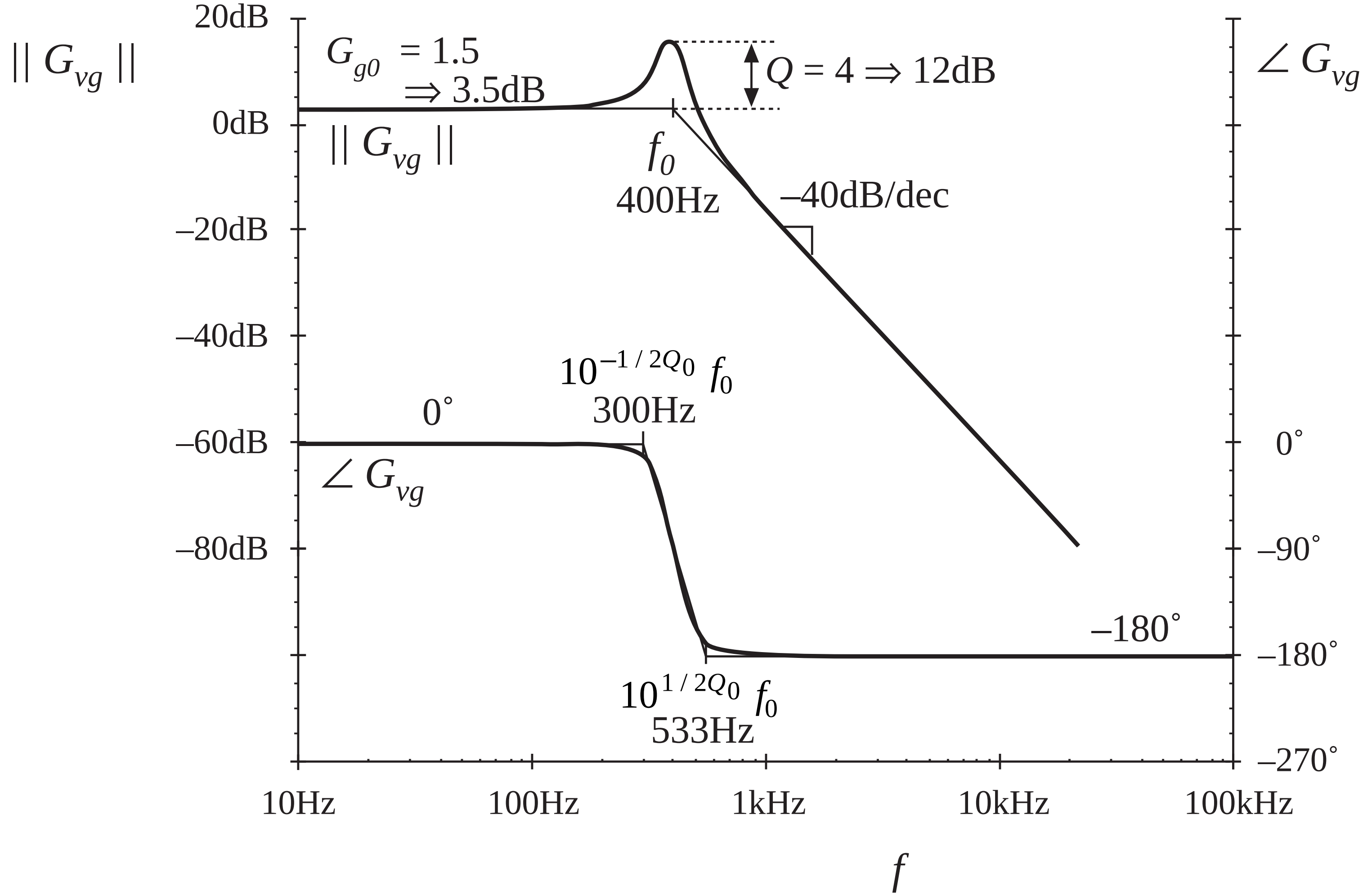
$$Q = D'R\sqrt{\frac{C}{L}} = 4 \Rightarrow 12\text{dB}$$

$$f_z = \frac{\omega_z}{2\pi} = \frac{D'^2 R}{2\pi DL} = 2.65\text{kHz}$$

Bode plot: control-to-output transfer function



Bode plot: line-to-output transfer function



8.2.2. Transfer functions of some basic CCM converters

Table 8.2. Salient features of the small-signal CCM transfer functions of some basic dc-dc converters

Converter	G_{g0}	G_{d0}	ω_0	Q	ω_z
buck	D	$\dfrac{V}{D}$	$\dfrac{1}{\sqrt{LC}}$	$R\sqrt{\dfrac{C}{L}}$	∞
boost	$\dfrac{1}{D'}$	$\dfrac{V}{D'}$	$\dfrac{D'}{\sqrt{LC}}$	$D'R\sqrt{\dfrac{C}{L}}$	$\dfrac{D'^2R}{L}$
buck-boost	$-\dfrac{D}{D'}$	$\dfrac{V}{D\,D'^2}$	$\dfrac{D'}{\sqrt{LC}}$	$D'R\sqrt{\dfrac{C}{L}}$	$\dfrac{D'^2\,R}{D\,L}$

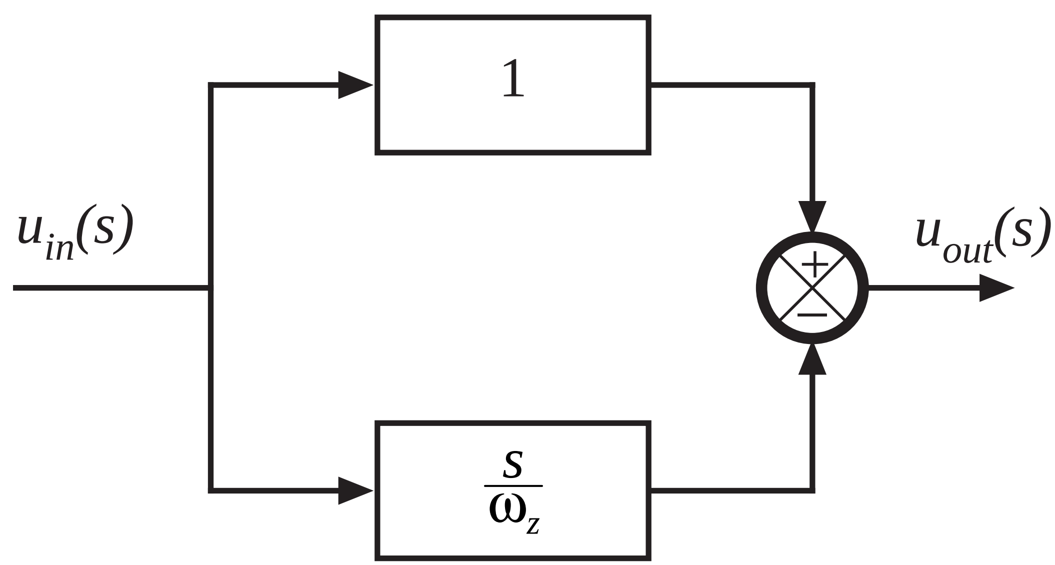
where the transfer functions are written in the standard forms

$$G_{vd}(s) = G_{d0}\,\frac{\left(1 - \dfrac{s}{\omega_z}\right)}{\left(1 + \dfrac{s}{Q\omega_0} + \left(\dfrac{s}{\omega_0}\right)^2\right)}$$

$$G_{vg}(s) = G_{g0}\,\frac{1}{1 + \dfrac{s}{Q\omega_0} + \left(\dfrac{s}{\omega_0}\right)^2}$$

8.2.3. Physical origins of the right half-plane zero

$$G(s) = \left(1 - \frac{s}{\omega_0}\right)$$

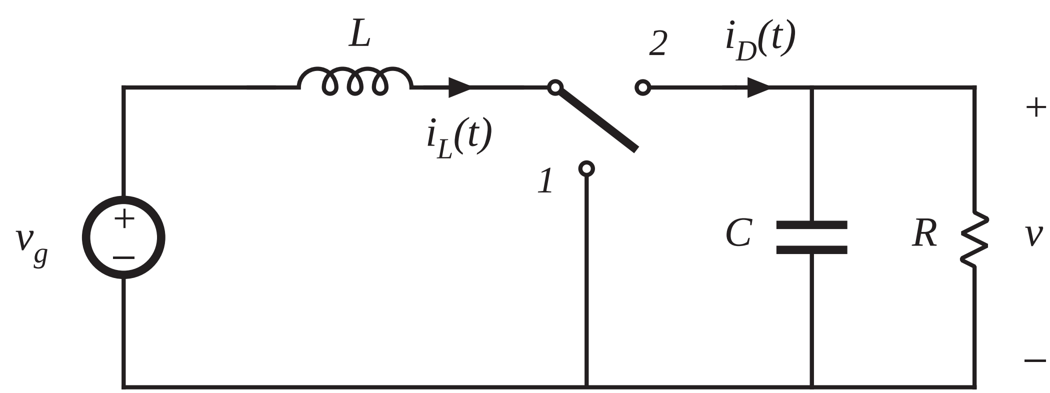


- *phase reversal at high frequency*
- *transient response: output initially tends in wrong direction*

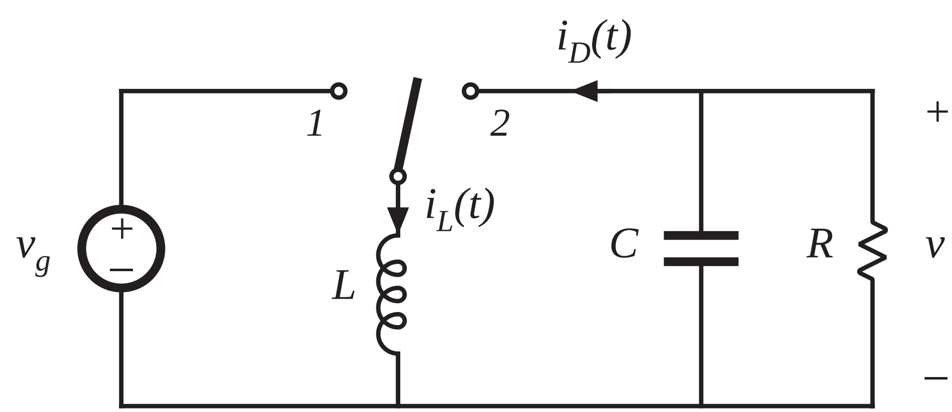
Two converters whose CCM control-to-output transfer functions exhibit RHP zeroes

$$\left\langle i_D \right\rangle_{T_s} = d' \left\langle i_L \right\rangle_{T_s}$$

Boost



Buck-boost



Waveforms, step increase in duty cycle

$$\left\langle i_D \right\rangle_{T_s} = d' \left\langle i_L \right\rangle_{T_s}$$

- Increasing *d(t)* causes the average diode current to initially decrease
- As inductor current increases to its new equilibrium value, average diode current eventually increases