Johns Hopkins Engineering

Power Electronics 525.725

Module 12 Lecture 12

DCM Flyback Large Signal Model/Final Project

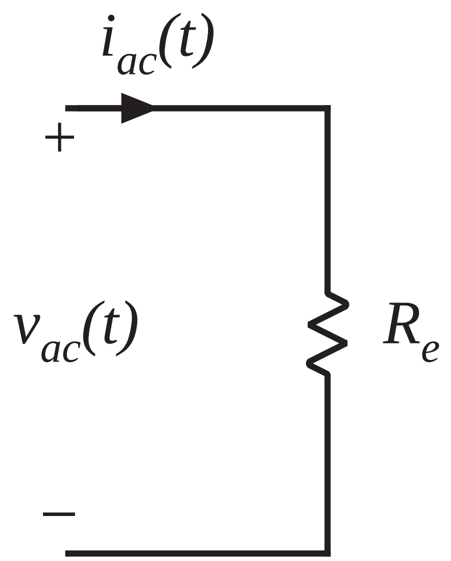
17.1 Properties of the ideal rectifier

It is desired that the rectifier present a resistive load to the ac power system. This leads to

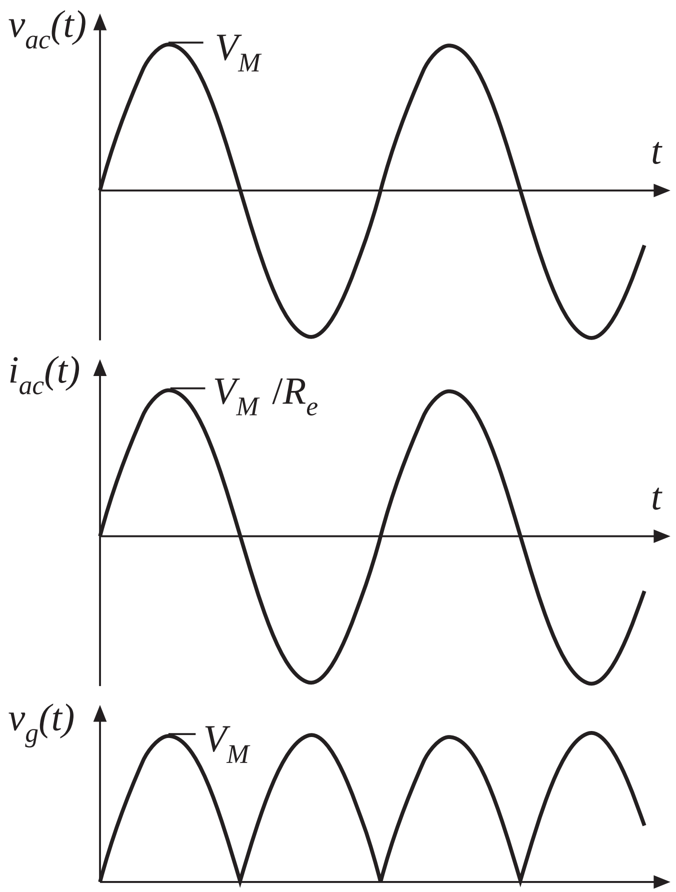
- unity power factor
- ac line current has same waveshape as voltage

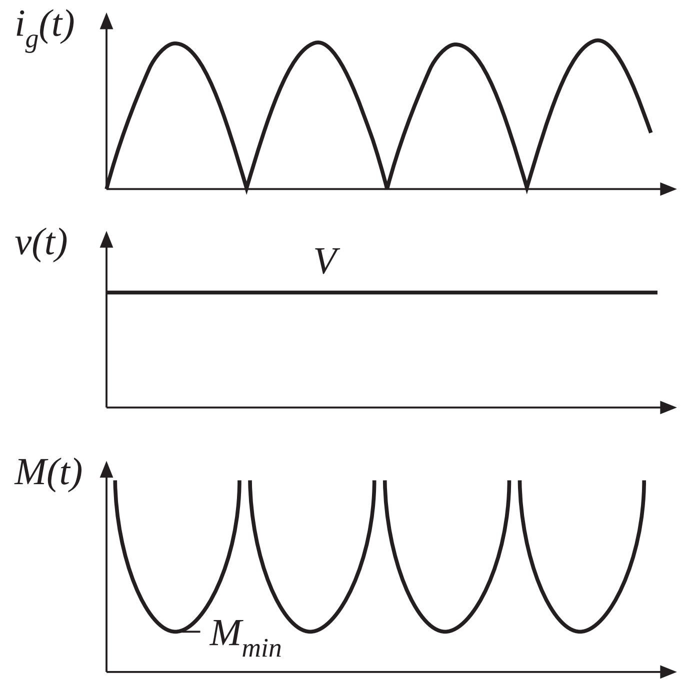
$$i_{ac}(t) = \frac{v_{ac}(t)}{R_e}$$

R_e is called the *emulated resistance*



Waveforms





$$v_{ac}(t) = V_M \sin(\omega t)$$

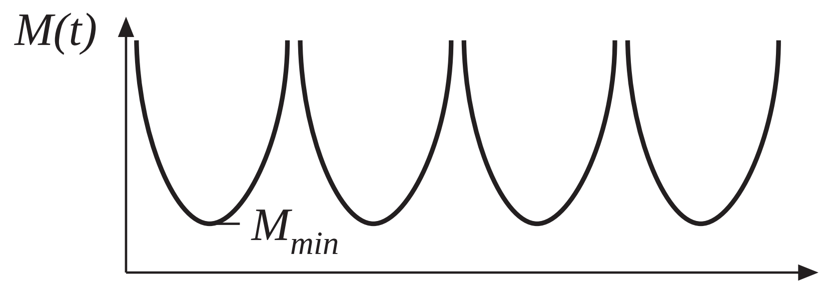
$$v_g(t) = V_M \left| \sin(\omega t) \right|$$

$$M(d(t)) = \frac{v(t)}{v_g(t)} = \frac{V}{V_M \left| \sin(\omega t) \right|}$$

$$M_{min} = \frac{V}{V_M}$$

Choice of converter

$$M(d(t)) = \frac{v(t)}{v_g(t)} = \frac{V}{V_M \left| \sin(\omega t) \right|}$$



- To avoid distortion near line voltage zero crossings, converter should be capable of producing $M(d(t))$ approaching infinity
- Above expression neglects converter dynamics
- Boost, buck-boost, Cuk, SEPIC, and other converters with similar conversion ratios are suitable
- We will see that the boost converter exhibits lowest transistor stresses. For this reason, it is most often chosen

DCM Flyback:

Motivation:

As discussed in last class, it is desired to have a resistive load hanging on the output of a full wave diode rectifier to reduce current harmonics and increase power factor. This can be accomplished by several methods:

1. Large passive LC filter
2. Active current mode control of BOOST derived topology
3. Utilizing a converter in DCM operation to look resistive

The drawback of 1 is cost and size. Large passive filters are generally large and bulky requiring damping. Also, they come at a large cost. Number 2 is the most popular method and relies on current mode control which is not discussed in this class. Input current is controlled to follow a sinusoidal reference in phase with the input voltage resulting in high power factor. Number 3 provides a simple way to achieve high power factor without requiring complicated control methods. And if we use a flyback converter, we also get the benefit of isolation. However DCM flyback converters are typical only used in lower power applications (<200W). This is due to the large rms currents seen by the main switch.

Large Signal Model:

The KVL and KCL equations During Mode 1:

$$v_L(t) = \mathrm{v}_g(t) \tag{1.1}$$

$$i_c(t) = -i_{load}(t) \tag{1.2}$$

The KVL and KCL equations During Mode 2:

$$v_L(t) = -\frac{v(t)n_p}{n_1} \text{ where np and n1 are the primary and secondary turns ratio} \tag{1.3}$$

$$i_c(t) = \frac{i_m(t)n_\mathrm{p}}{n_1} - i_{load}(t) \tag{1.4}$$

The KVL and KCL equations During Mode 3 (inductor current is zero):

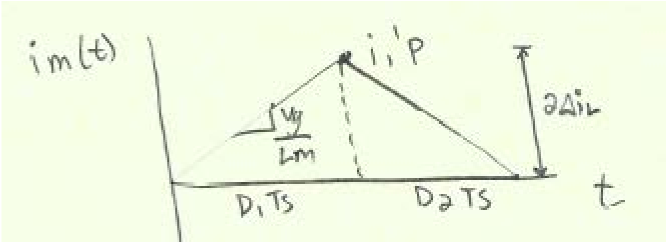
$$v_L(t) = 0 \tag{1.5}$$

$$i_c(t) = -i_{load}(t) \tag{1.6}$$

The average inductor voltage is:

$$\left\langle v_L(t)\right\rangle_{T_s} = d_1(t)\left\langle v_g(t)\right\rangle_{T_s} - d_2(t)\left\langle v(t)\right\rangle_{T_s}\frac{n_p}{n_1} + d_3(t)\cdot 0 \qquad (1.7)$$

Recall the inductor current in DCM for a flyback converter from HW 4:



We can calculate the average inductor current as follows:

$$\left\langle i_m(t)\right\rangle_{T_s} = \frac{1}{T_s}\left[\frac{1}{2}\left(d_1(t)+d_2(t)\right)\mathrm{i}_{1\,\mathrm{p}}'(\mathrm{t})\mathrm{T_s}\right] \qquad (1.8)$$

Where:

$$\mathrm{i}_{1\,\mathrm{p}}'(t) = \frac{\left\langle v_g(t)\right\rangle_{T_s}}{L_m}d_1(t)T_s \qquad (1.9)$$

Solving (1.8) and (1.9) for $d_2(t)$ and $\mathrm{i}_{1\,\mathrm{p}}'$

$$d_2(t) = \underbrace{\frac{2\left\langle i_m(t)\right\rangle_{T_s} L_m f_s}{\left\langle v_g(t)\right\rangle_{T_s} d_1(t)}}_{d1pd2} - d_1(t) \qquad (1.10)$$

Let:

$$d1pd2 = \frac{2\left\langle i_m(t)\right\rangle_{T_s} L_m f_s}{\left\langle v_g(t)\right\rangle_{T_s} d_1(t)} \qquad (1.11)$$

$$d_2(t) = d1pd2 - d_1(t) \qquad (1.12)$$

We will need to find the average capacitor current. Recall from earlier homework's on the subject of DCM that the average diode current can be related to the capacitor current:

$$\langle i_c(t)\rangle_{T_s} = \langle i_D(t)\rangle_{T_s} - \langle i_{load}(t)\rangle \tag{1.13}$$

Therefore, we need the average diode current. But we can solve for this by using the actual waveform for the diode current which is a triangle:

$$\langle i_D(t)\rangle_{Ts} = \frac{1}{T_s}\left(\frac{1}{2}d_2(t)\,\mathrm{T}_s\mathrm{i}_{Dpeak}(t)\right) \text{ where } \mathrm{i}_{\mathrm{Dpeak}}(\mathrm{t}) \text{ is the peak diode current} \tag{1.14}$$

We need to know the peak diode current $i_{Dpeak}(t)$

Recall from HW 4 and using the ideal transformer relationship:

$$\begin{aligned}
&0 = n_p i'_p(t) + n_1 i_1(t)\\
&\text{At the start of mode "2" where the peak diode current occurs}\\
&0 = -n_p i_M(t) + n_1 i_1(t)\\
&0 = -n_p i_M(t = d_1(\mathrm{t})) + n_1 i_1(t = d_1(\mathrm{t})) \therefore
\end{aligned} \tag{1.15}$$

$$0 = -n_p\, \mathrm{i_1'}_{\mathrm{p}}(t) + n_1 i_{Dpeak}(t) \tag{1.16}$$

Solving (1.14) and (1.16) for $i_{Dpeak}(t), \langle i_D(t)\rangle_{Ts}$ in terms of d1pd2

$$\langle i_D(t)\rangle_{Ts} = \left(\frac{n_p}{n_1}\right)\frac{d_2(t)\langle i_m(t)\rangle_{T_s}}{d1pd2} \tag{1.17}$$

To find the average input current, we can just compute directly from the waveform which recall is a simple triangle.

$$\langle i_g(t)\rangle_{Ts} = \frac{1}{T_s}\left(\frac{1}{2}d_1(t)\,\mathrm{T}_s\mathrm{i_1'}_{\mathrm{p}}(t)\right) \tag{1.18}$$

Solving for the average input current in terms of d1pd2:

$$\langle i_g(t)\rangle_{T_s} = \frac{d_1(t)\langle i_m(t)\rangle_{T_s}}{d1pd2} \tag{1.19}$$

Let an effective resistance:

$$\boxed{R_e = \frac{2L_m}{d_1(t)^2 T_s}} \tag{1.20}$$

Substituting (1.20) and (1.11) into (1.19)

$$\left\langle i_g(t) \right\rangle_{T_s} = \frac{\left\langle v_g(t) \right\rangle_{T_s}}{R_e} \tag{1.21}$$

In summary the large signal equations for the flyback operating in DCM mode are given by:

$$\boxed{\begin{aligned} &\left\langle i_g(t) \right\rangle_{T_s} = \frac{\left\langle v_g(t) \right\rangle_{T_s}}{R_e} \\ &\left\langle i_c(t) \right\rangle_{T_s} = \left(\frac{n_p}{n_1} \right) \frac{d_2(t) \left\langle i_m(t) \right\rangle_{T_s}}{d1pd2} - \left\langle i_{load}(t) \right\rangle \\ &\left\langle v_L(t) \right\rangle_{T_s} = d_1(t) \left\langle v_g(t) \right\rangle_{T_s} - d_2(t) \left\langle v(t) \right\rangle_{T_s} \left(\frac{n_p}{n_1} \right) \end{aligned}} \tag{1.22}$$

Where:

$$\boxed{\begin{aligned} &d1pd2 = \frac{2\left\langle i_m(t) \right\rangle_{T_s} L_m f_s}{\left\langle v_g(t) \right\rangle_{T_s} d_1(t)} \\ &d_2(t) = d1pd2 - d_1(t) \\ &R_e = \frac{2L_m}{d_1(t)^2 T_s} \end{aligned}} \tag{1.23}$$

Note that the input port to the large signal model looks just like a resistor!! This is what we desire to be seen by a front end diode rectifier. Therefore the DCM flyback can achieve high power factor! The resister is a function of duty cycle, $d_1(t)$, allowing us to control the power delivered to the output port.

DCM Flyback Design Parameters:

In order for the flyback to look like a resister at the input, the converter has to operate in DCM over all line and load conditions. The input source to the rectifier is ac:

$$v_{ac}(t) = \mathrm{V}_M \sin(\omega t) \tag{1.24}$$

Recall the output of a diode rectifier is the rectified version of (1.24) :

$$v_g(t) = V_M \left|\sin(\omega t)\right| \tag{1.25}$$

The converter will operate in DCM provided that $d_3(t)$ is greater than zero which implies:

$$d_2(t) < 1 - d_1(t) \tag{1.26}$$

Here we need to consider something which applies in DCM. Recall from an earlier lecture:

$$\left\langle v_L(t)\right\rangle_{T_s} = L\frac{i(t+T_s) - i(t)}{T_s} \tag{1.27}$$

But in DCM:

$$\begin{aligned} &i(t+T_s) = 0 \text{ inductor current at the end of the switching cycle} \\ &i(t) = 0 \text{ inductor current at the beggining of the switching cycle} \end{aligned} \tag{1.28}$$

Therefore, the dynamic average inductor voltage, $\left\langle v_L(t)\right\rangle_{T_s}$ is 0 even when the converter is not in equilibrium. Don't confuse this result with the steady state result where average inductor voltage is always zero!!

$$\left\langle v_L(t)\right\rangle_{T_s} = 0 \tag{1.29}$$

We can actually use this result to determine the dynamic duty cycle, $d_2(t)$ as a function of $d_1(t)$:

$$d_2(t) = d_1(t)\frac{\left\langle v_g(t)\right\rangle_{T_s}}{\left\langle v(t)\right\rangle_{T_s}}\frac{n_1}{n_p} \tag{1.30}$$

Substituting (1.30) into (1.26)

$$d_1(t) < \frac{1}{\dfrac{\langle v_g(t)\rangle_{T_s}\, n_1}{\langle v(t)\rangle_{T_s}\, n_p} + 1} \tag{1.31}$$

Therefore (1.31) must be satisfied for the converter to operate in DCM. Substituting (1.25) into (1.31):

$$d_1(t) < \frac{1}{\dfrac{V_M\,|\sin(\omega t)|\, n_1}{\langle v(t)\rangle_{T_s}\, n_p} + 1} \tag{1.32}$$

The worst case for this inequality occurs when the rectified sinusoid is at its peak value:

$$D_1 < \frac{1}{\dfrac{V_M n_1}{V n_p} + 1} \tag{1.33}$$

We need the steady state solution for the duty cycle D_1. But first, let's consider the average input power. Since the input to the flyback large signal model in DCM looks like a resister, we conclude that the apparent power drawn from the source is equal to the average power and the power factor is 1. Therefore the average input power drawn from the AC source given by:

$$P_{av} = P_{aparrent} = V_{acrms} I_{acrms} \tag{1.34}$$

Since the output of the rectifier is a linear resister, we can relate peak sine waves to rms:

$$P_{av} = \underbrace{\frac{V_M}{\sqrt{2}}}_{V_{acrms}} \underbrace{\left(\frac{\dfrac{V_M}{R_e}}{\sqrt{2}} \right)}_{I_{acrms}} \tag{1.35}$$

This average power is transferred to the output load which is represented as a constant power load:

$$P_{av} = \underbrace{\frac{V_M}{\sqrt{2}}}_{V_{acrms}} \underbrace{\left(\frac{\dfrac{V_M}{R_e}}{\sqrt{2}} \right)}_{I_{acrms}} = P_{load} \tag{1.36}$$

Substituting the effective resistance given in (1.20) into (1.36)

$$\boxed{D_1 = \frac{2\left(\sqrt{\frac{L_m P_{load}}{T_s}}\right)}{V_M}} \tag{1.37}$$

Insertion of (1.37) into (1.33), and solution for L_M yields:

$$\boxed{L_M < \frac{1}{4}\frac{\left(Vn_p V_M\right)^2 T_s}{P_{load}(V_M n_1 + Vn_p)^2}} \tag{1.38}$$