Johns Hopkins Engineering

Power Electronics 525.725

Module 9 Lecture 9

Control and regulator design II

Determination of stability directly from $T(s)$

- Nyquist stability theorem: general result.
- A special case of the Nyquist stability theorem: the phase margin test

  Allows determination of closed-loop stability (i.e., whether $1/(1+T(s))$ contains RHP poles) directly from the magnitude and phase of $T(s)$.

  A good design tool: yields insight into how $T(s)$ should be shaped, to obtain good performance in transfer functions containing $1/(1+T(s))$ terms.

9.4.1. The phase margin test

A test on $T(s)$, to determine whether $1/(1+T(s))$ contains RHP poles.

The crossover frequency f_c is defined as the frequency where

$$\| T(j2\pi f_c) \| = 1 \Rightarrow 0\text{dB}$$

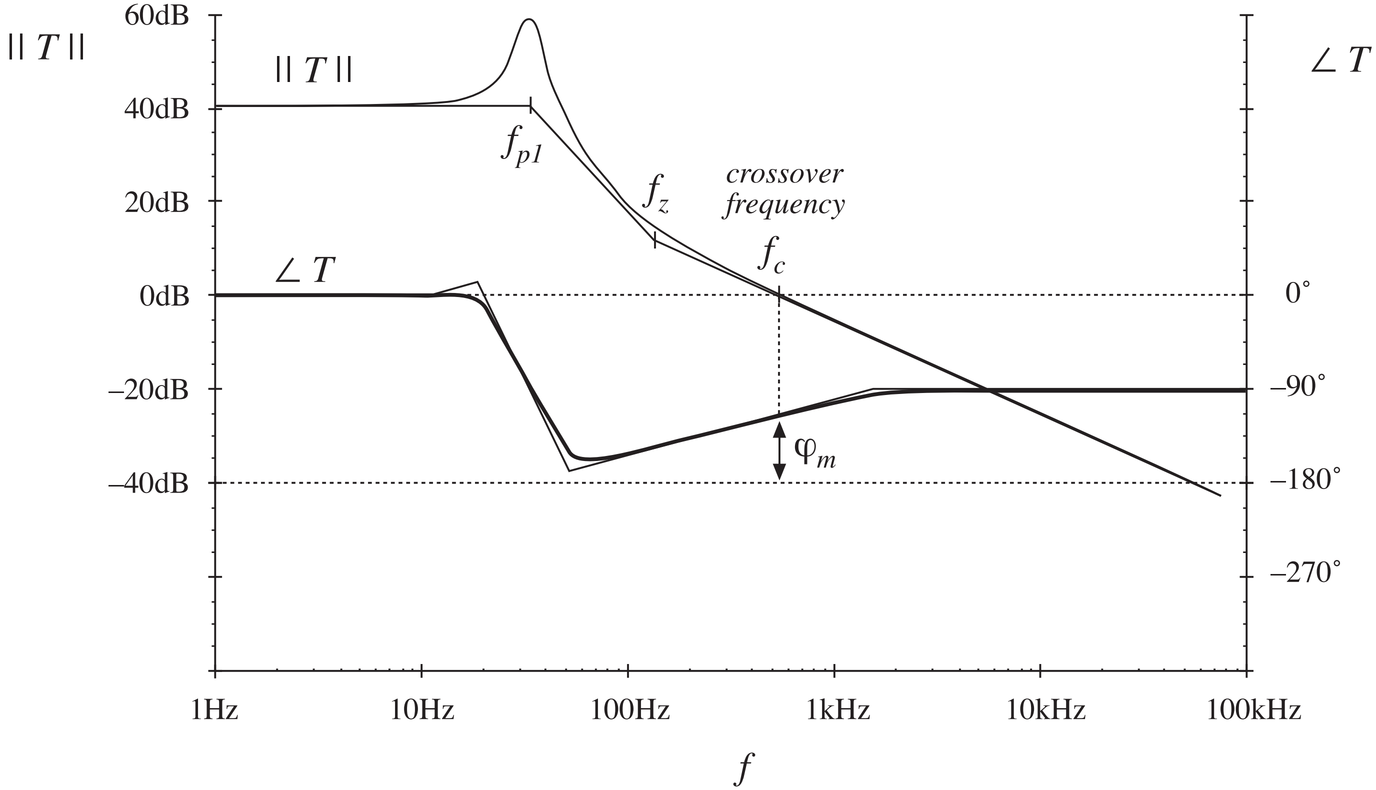
The phase margin φ_m is determined from the phase of $T(s)$ at f_c , as follows:

$$\varphi_m = 180^\circ + \angle T(j2\pi f_c)$$

If there is exactly one crossover frequency, and if $T(s)$ contains no RHP poles, then

> the quantities $T(s)/(1+T(s))$ and $1/(1+T(s))$ contain no RHP poles whenever the phase margin φ_m is positive.

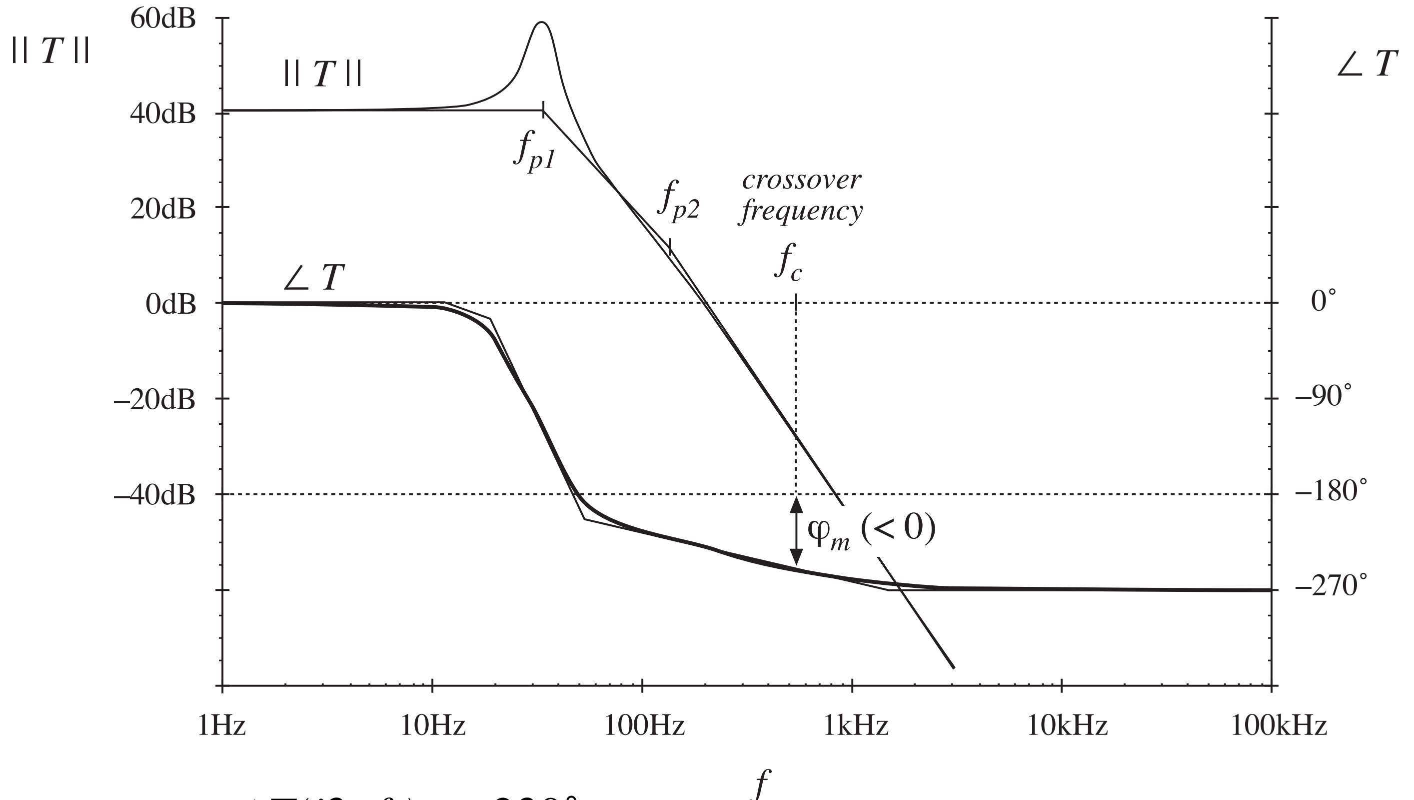
Example: a loop gain leading to a stable closed-loop system



$$\angle T(j2\pi f_c) = -112^\circ$$

$$\varphi_m = 180^\circ - 112^\circ = +68^\circ$$

Example: a loop gain leading to an unstable closed-loop system



$$\angle T(j2\pi f_c) = -230^\circ$$

$$\varphi_m = 180^\circ - 230^\circ = -50^\circ$$

9.4.2. The relation between phase margin and closed-loop damping factor

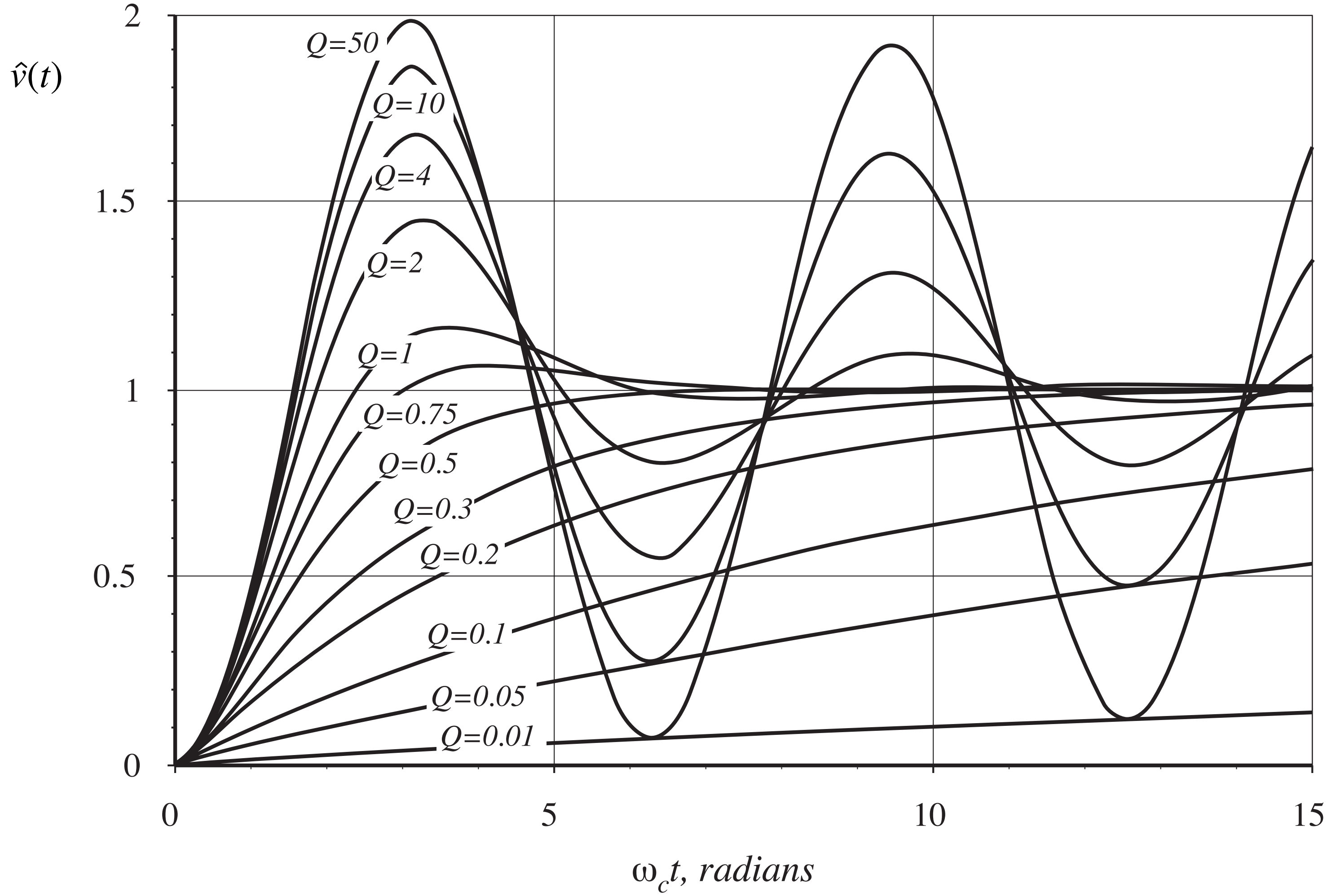
How much phase margin is required?

A small positive phase margin leads to a stable closed-loop system having complex poles near the crossover frequency with high Q. The transient response exhibits overshoot and ringing.

Increasing the phase margin reduces the Q. Obtaining real poles, with no overshoot and ringing, requires a large phase margin.

The relation between phase margin and closed-loop Q is quantified in this section.

Transient response vs. damping factor



9.5. Regulator design

Typical specifications:

- Effect of load current variations on output voltage regulation

  This is a limit on the maximum allowable output impedance

- Effect of input voltage variations on the output voltage regulation

  This limits the maximum allowable line-to-output transfer function

- Transient response time

  This requires a sufficiently high crossover frequency

- Overshoot and ringing

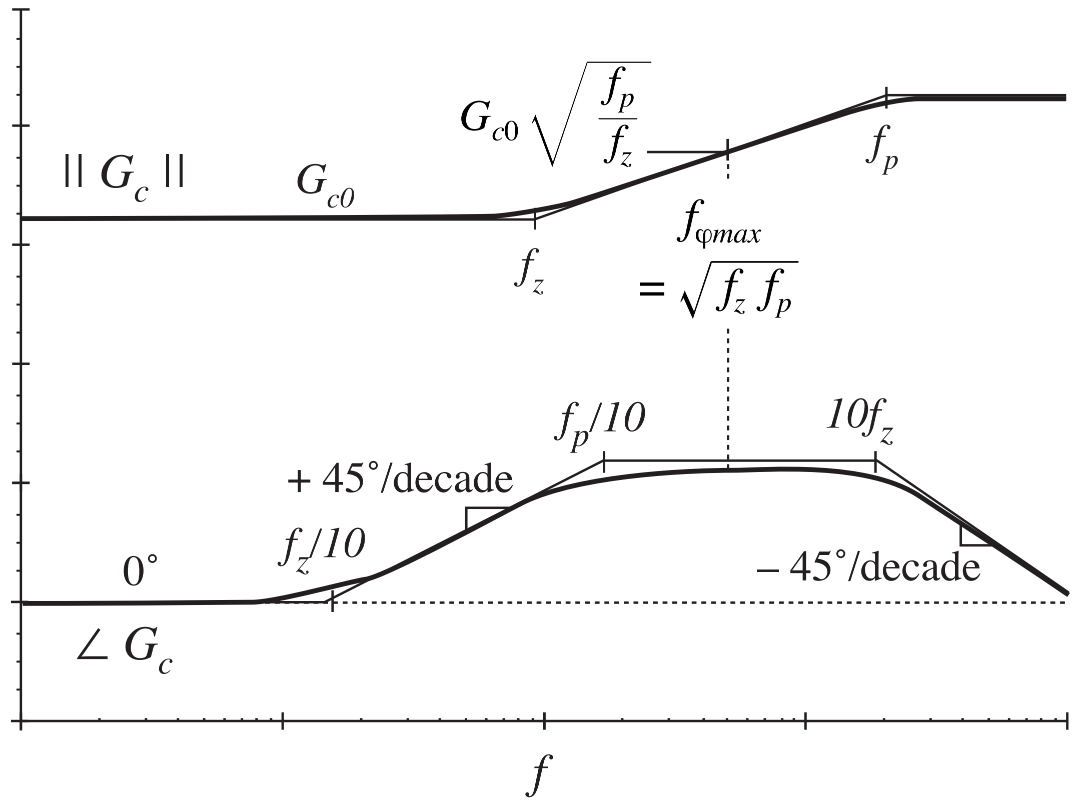
  An adequate phase margin must be obtained

The regulator design problem: add compensator network $G_c(s)$ to modify $T(s)$ such that all specifications are met.

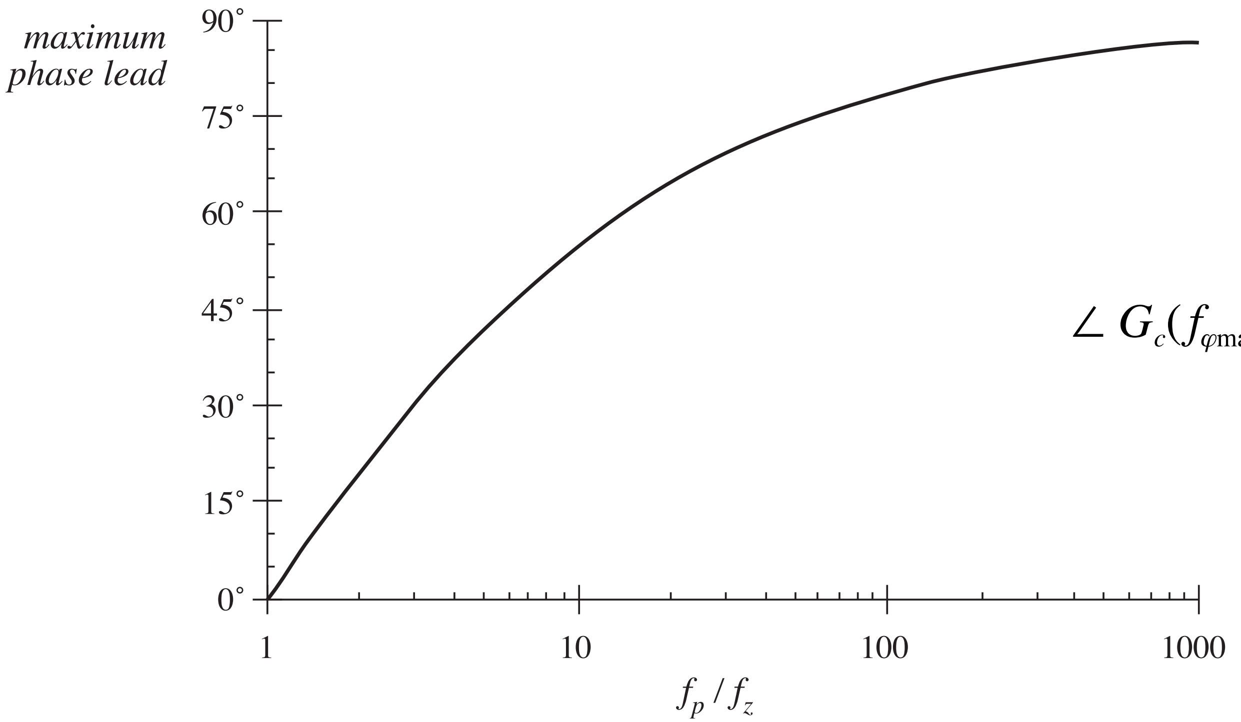
9.5.1. Lead (PD) compensator

$$G_c(s) = G_{c0} \frac{\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_p}\right)}$$

Improves phase margin



Lead compensator: maximum phase lead



$$f_{\varphi\max} = \sqrt{f_z f_p}$$

$$\angle\, G_c(f_{\varphi\max}) = \tan^{-1}\left(\frac{\sqrt{\frac{f_p}{f_z}} \pm \sqrt{\frac{f_z}{f_p}}}{2} \right)$$

$$\frac{f_p}{f_z} = \frac{1 + \sin(\theta)}{1 \pm \sin(\theta)}$$

Lead compensator design

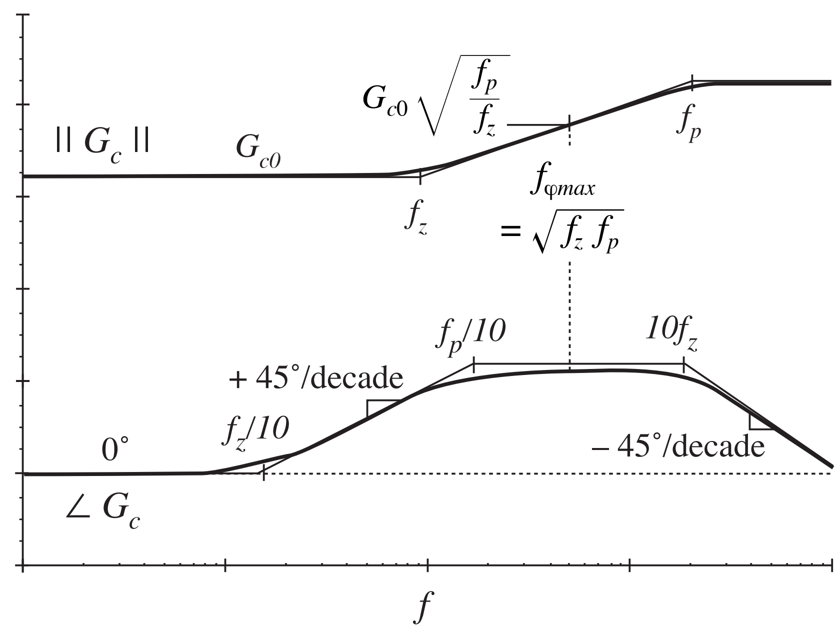
To optimally obtain a compensator phase lead of θ at frequency f_c, the pole and zero frequencies should be chosen as follows:

$$f_z = f_c \sqrt{\frac{1 \pm \sin(\theta)}{1 + \sin(\theta)}}$$

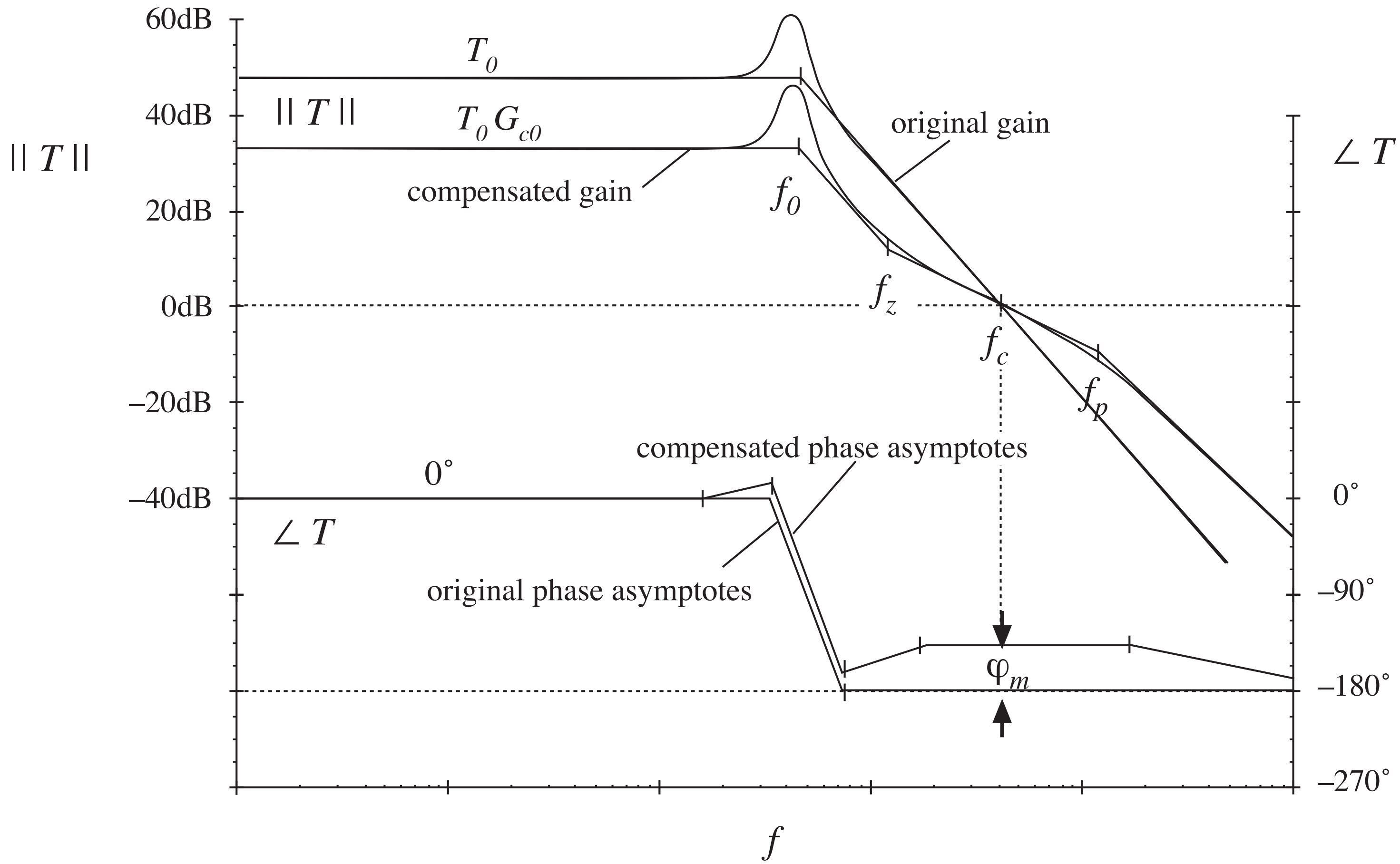
$$f_p = f_c \sqrt{\frac{1 + \sin(\theta)}{1 \pm \sin(\theta)}}$$

If it is desired that the magnitude of the compensator gain at f_c be unity, then G_{c0} should be chosen as

$$G_{c0} = \sqrt{\frac{f_z}{f_p}}$$



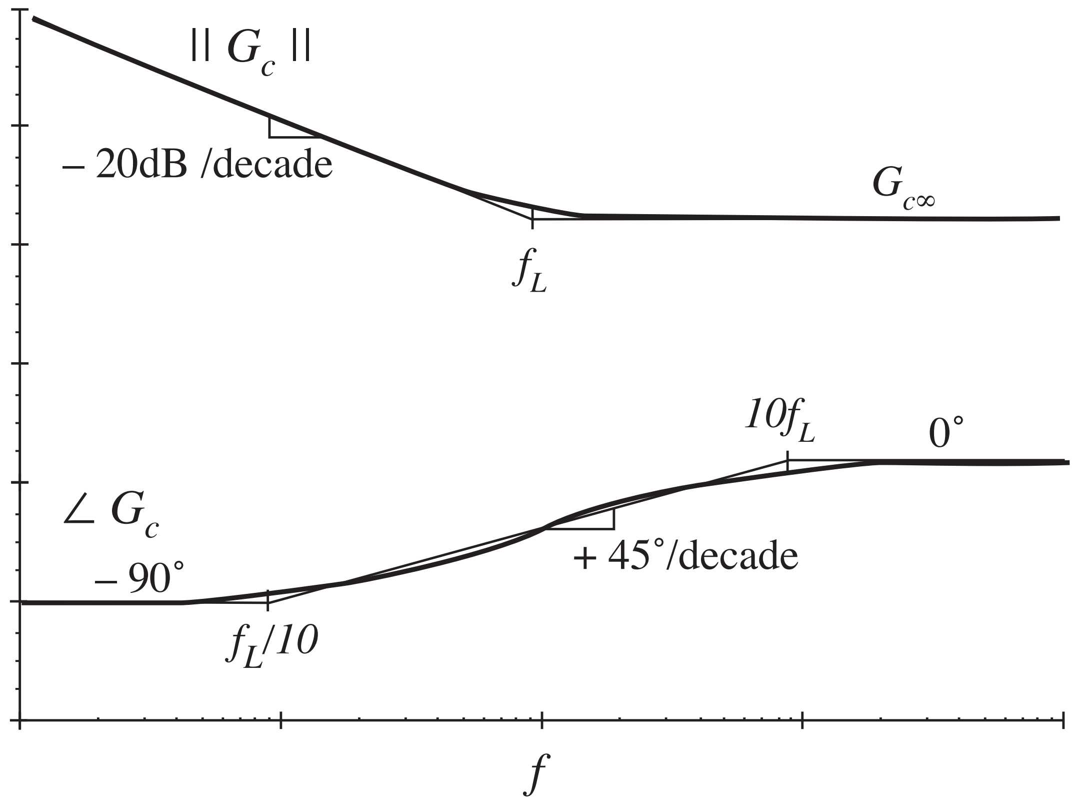
Example: lead compensation



9.5.2. Lag (PI) compensation

$$G_c(s) = G_{c\infty}\left(1 + \frac{\omega_L}{s}\right)$$

Improves low-frequency loop gain and regulation



Example: lag compensation

original (uncompensated) loop gain is

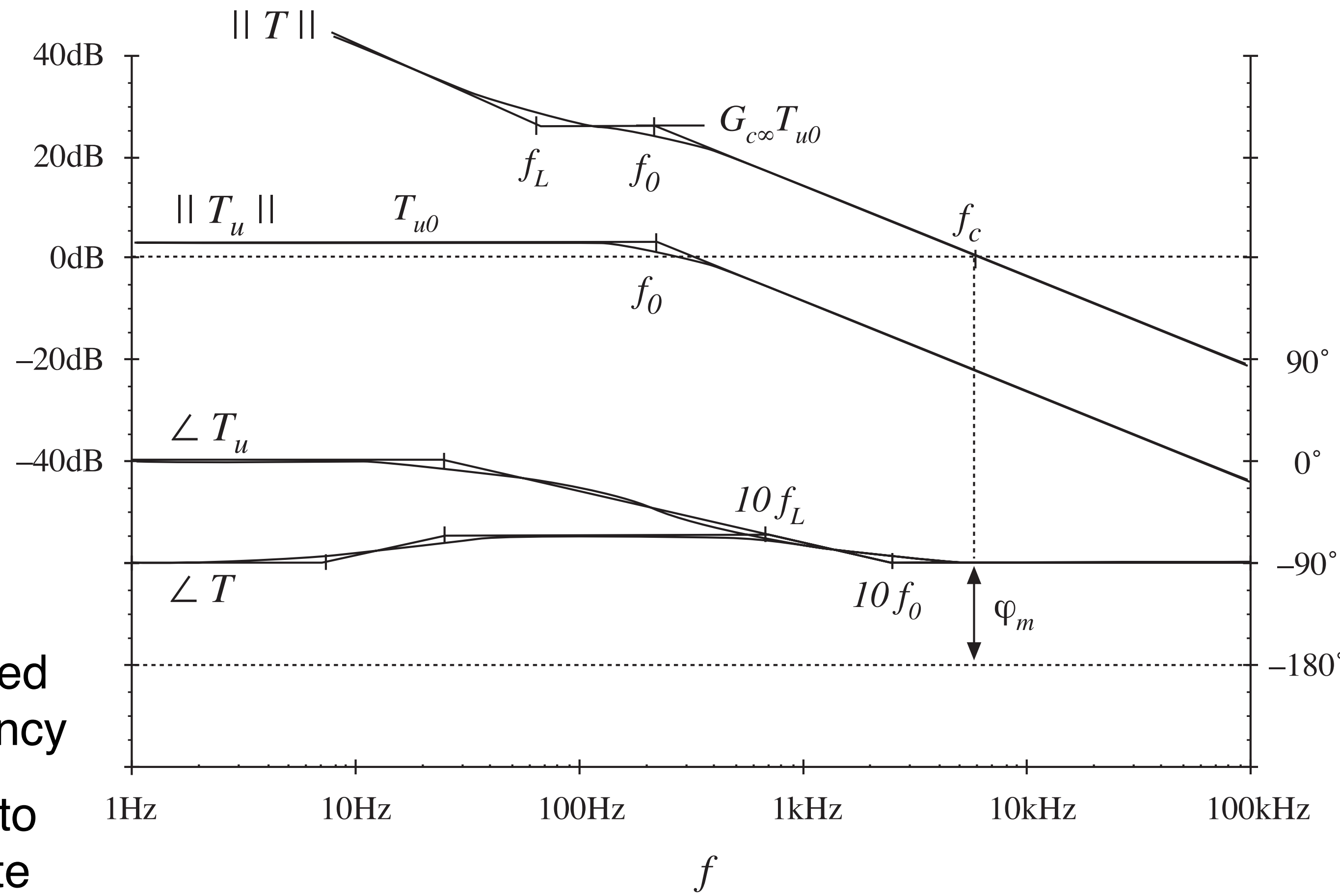
$$T_u(s) = \frac{T_{u0}}{\left(1 + \frac{s}{\omega_0}\right)}$$

compensator:

$$G_c(s) = G_{c\infty}\left(1 + \frac{\omega_L}{s}\right)$$

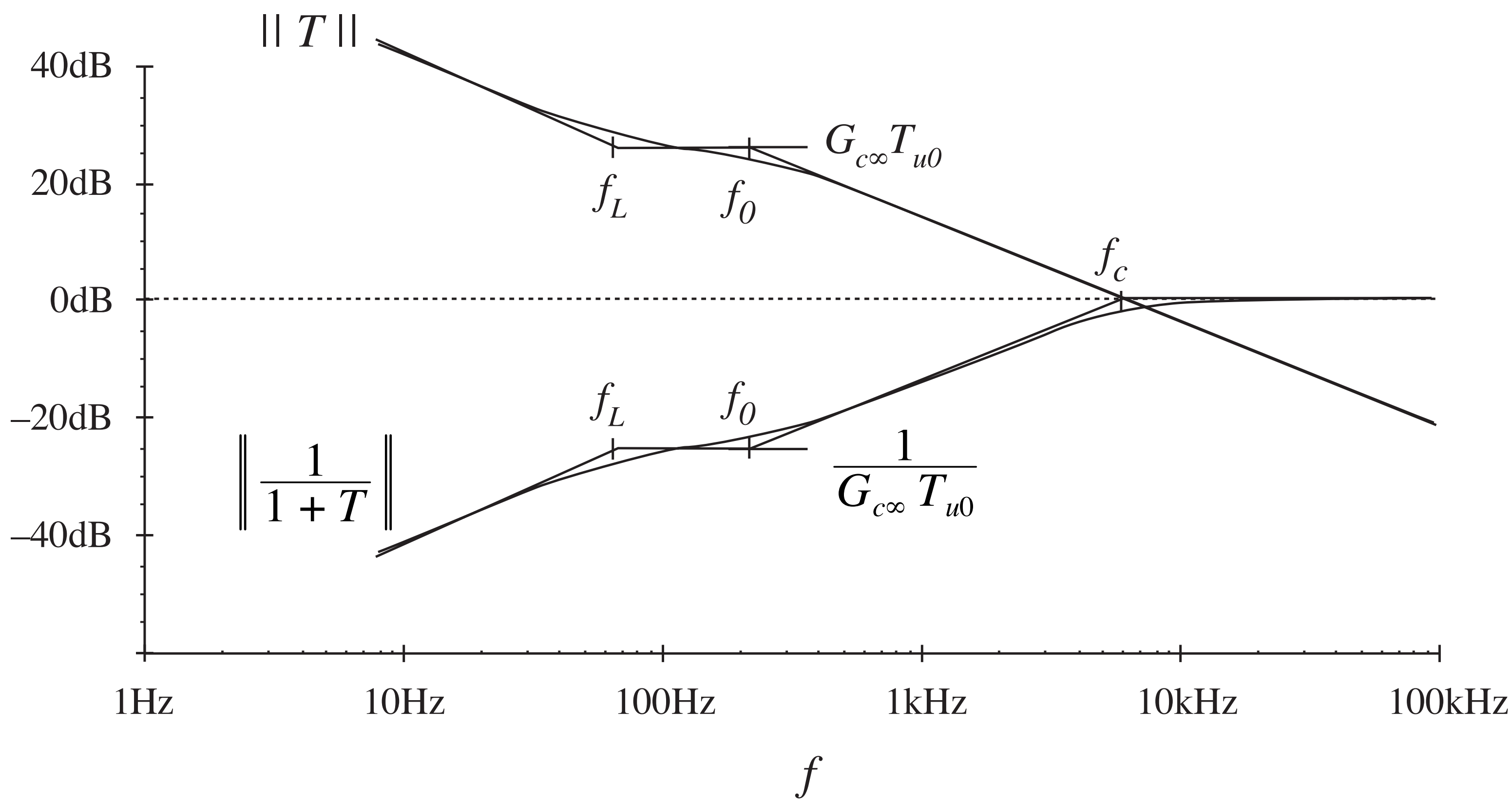
Design strategy: choose

- $G_{c\infty}$ to obtain desired crossover frequency
- ω_L sufficiently low to maintain adequate phase margin



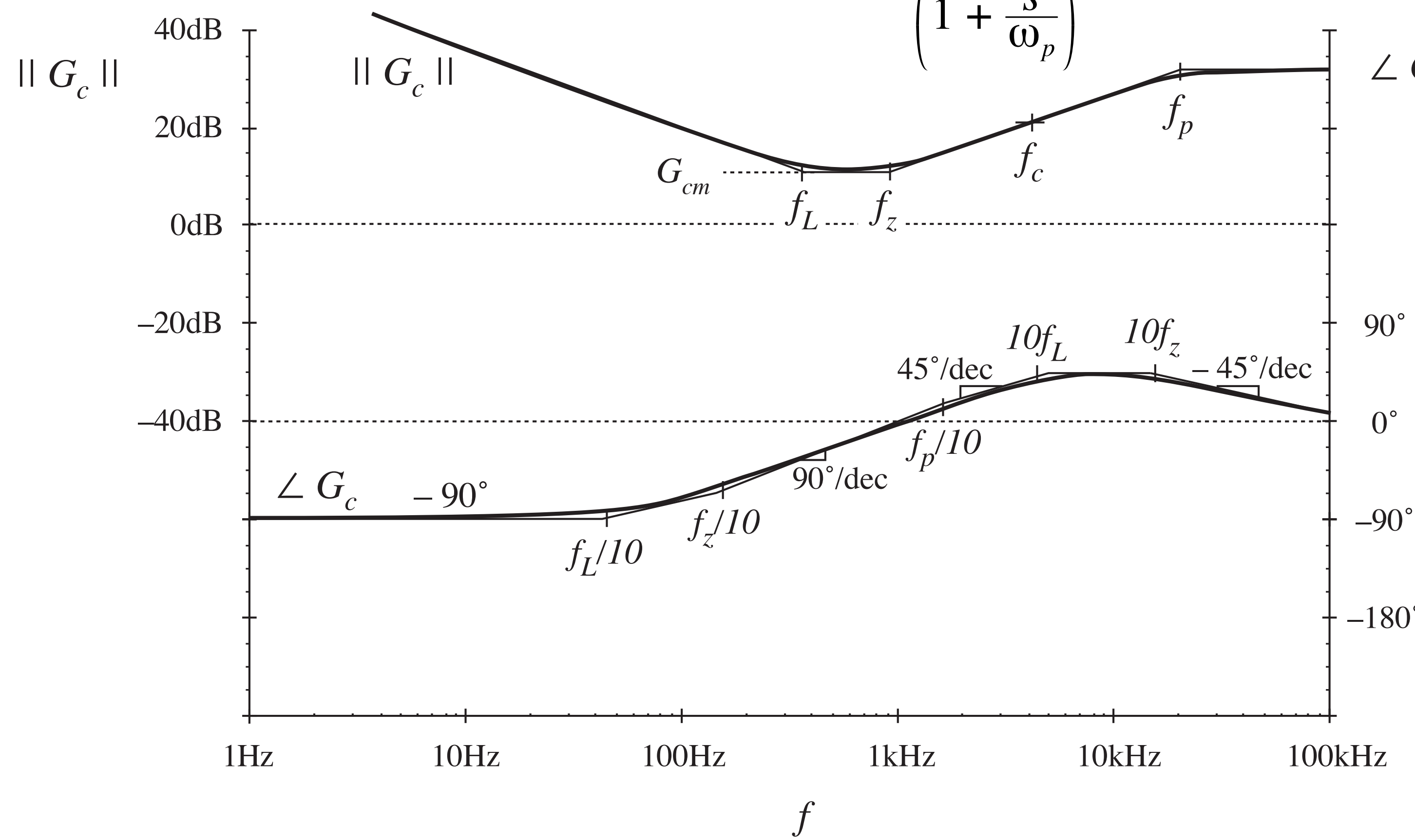
Example, continued

Construction of *1/(1+T)*, lag compensator example:



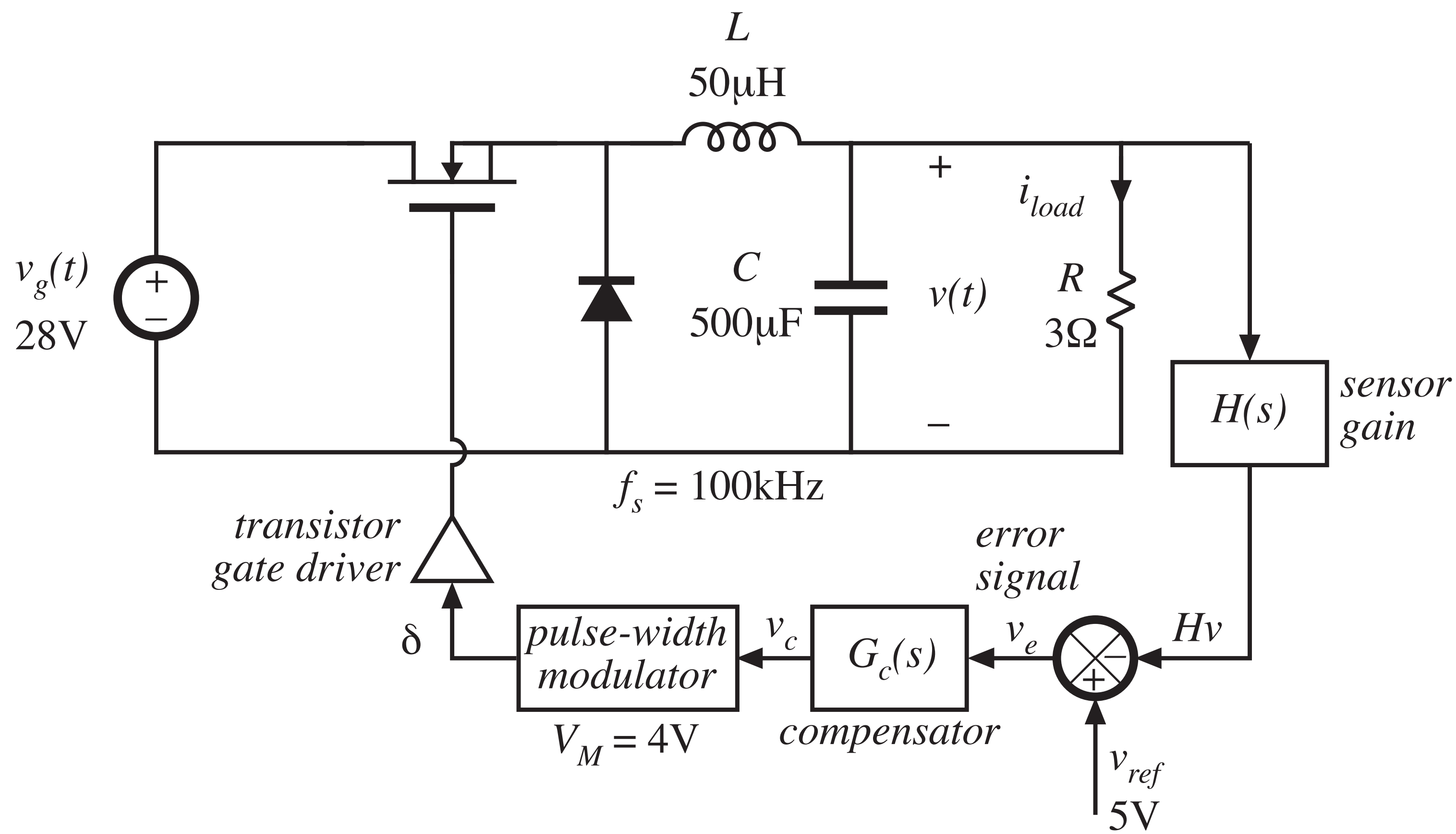
Improved compensator (PID)

$$G_c(s) = G_{cm} \frac{\left(1 + \frac{s}{\omega_z}\right)\left(1 + \frac{\omega_L}{s}\right)}{\left(1 + \frac{s}{\omega_p}\right)}$$



- add inverted zero to PD compensator, without changing dc gain or corner frequencies
- choose f_L to be $f_c/10$, so that phase margin is unchanged

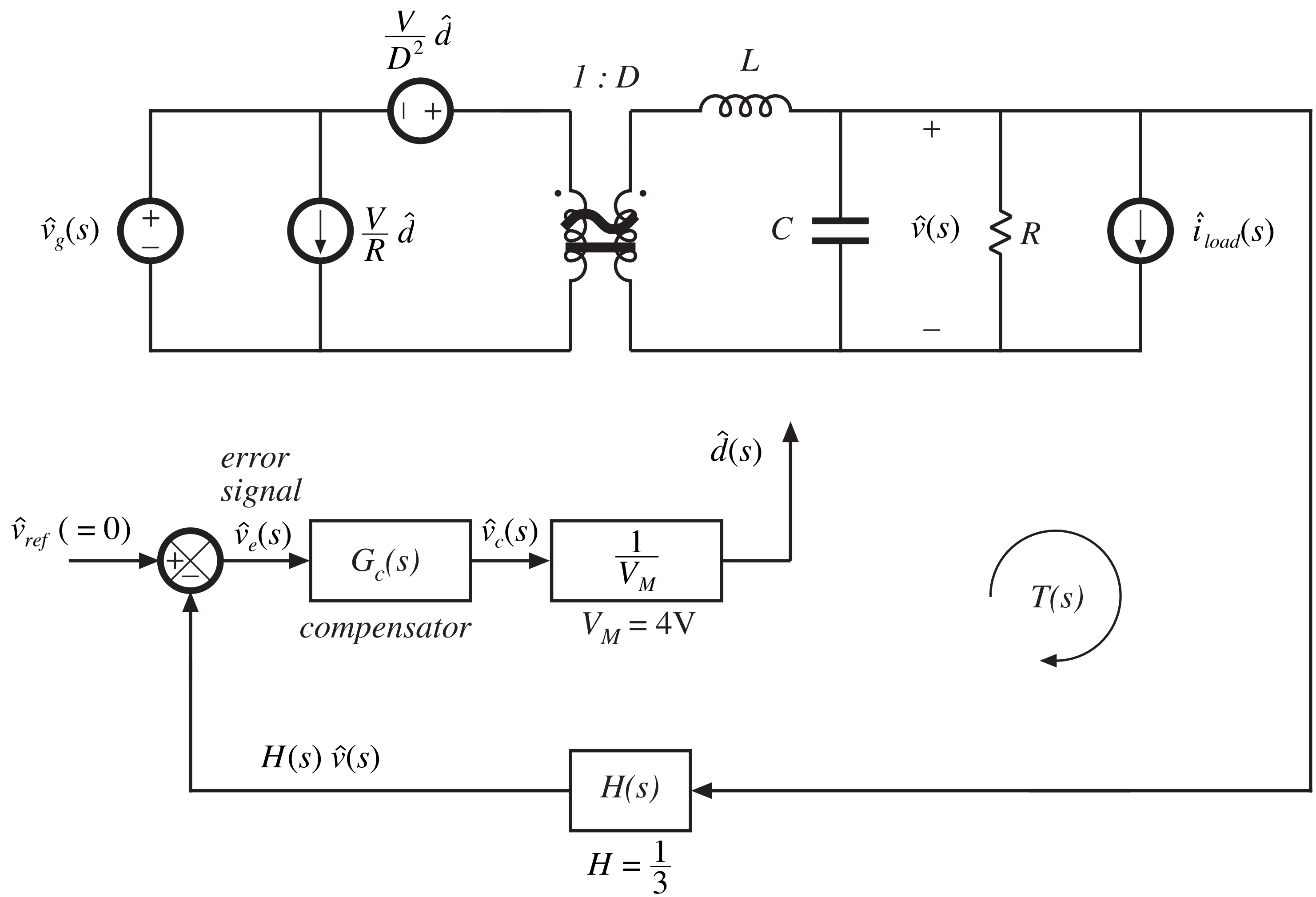
9.5.4. Design example



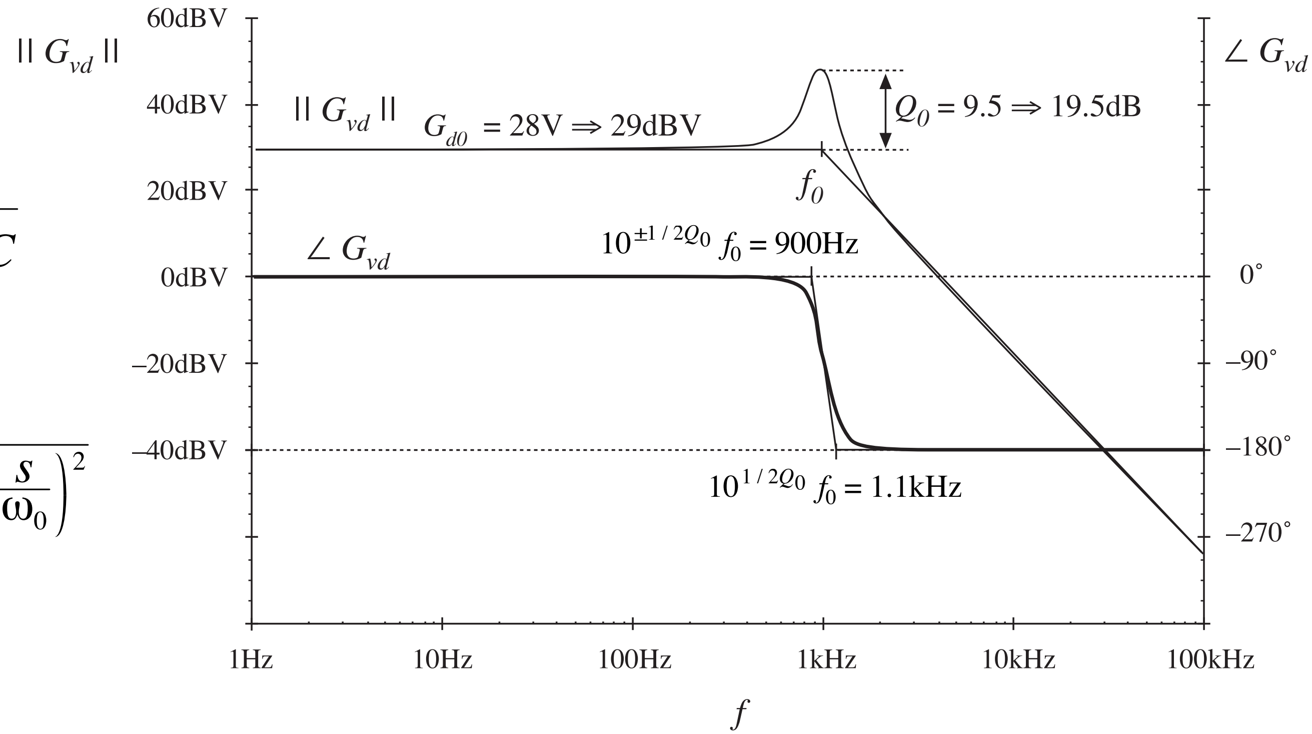
Quiescent operating point

Input voltage	$V_g = 28\text{V}$
Output	$V = 15\text{V}$, $I_{load} = 5\text{A}$, $R = 3\Omega$
Quiescent duty cycle	$D = 15/28 = 0.536$
Reference voltage	$V_{ref} = 5\text{V}$
Quiescent value of control voltage	$V_c = DV_M = 2.14\text{V}$
Gain *H(s)*	$H = V_{ref}/V = 5/15 = 1/3$

Small-signal model



Open-loop control-to-output transfer function $G_{vd}(s)$



$$G_{vd}(s) = \frac{V}{D} \frac{1}{1 + s\frac{L}{R} + s^2 LC}$$

standard form:

$$G_{vd}(s) = G_{d0} \frac{1}{1 + \frac{s}{Q_0 \omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

salient features:

$$G_{d0} = \frac{V}{D} = 28\text{V}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = 1\text{kHz}$$

$$Q_0 = R\sqrt{\frac{C}{L}} = 9.5 \Rightarrow 19.5\text{dB}$$

Open-loop line-to-output transfer function and output impedance

$$G_{vg}(s) = D \frac{1}{1 + s\frac{L}{R} + s^2 LC}$$

—same poles as control-to-output transfer function
standard form:

$$G_{vg}(s) = G_{g0} \frac{1}{1 + \frac{s}{Q_0 \omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

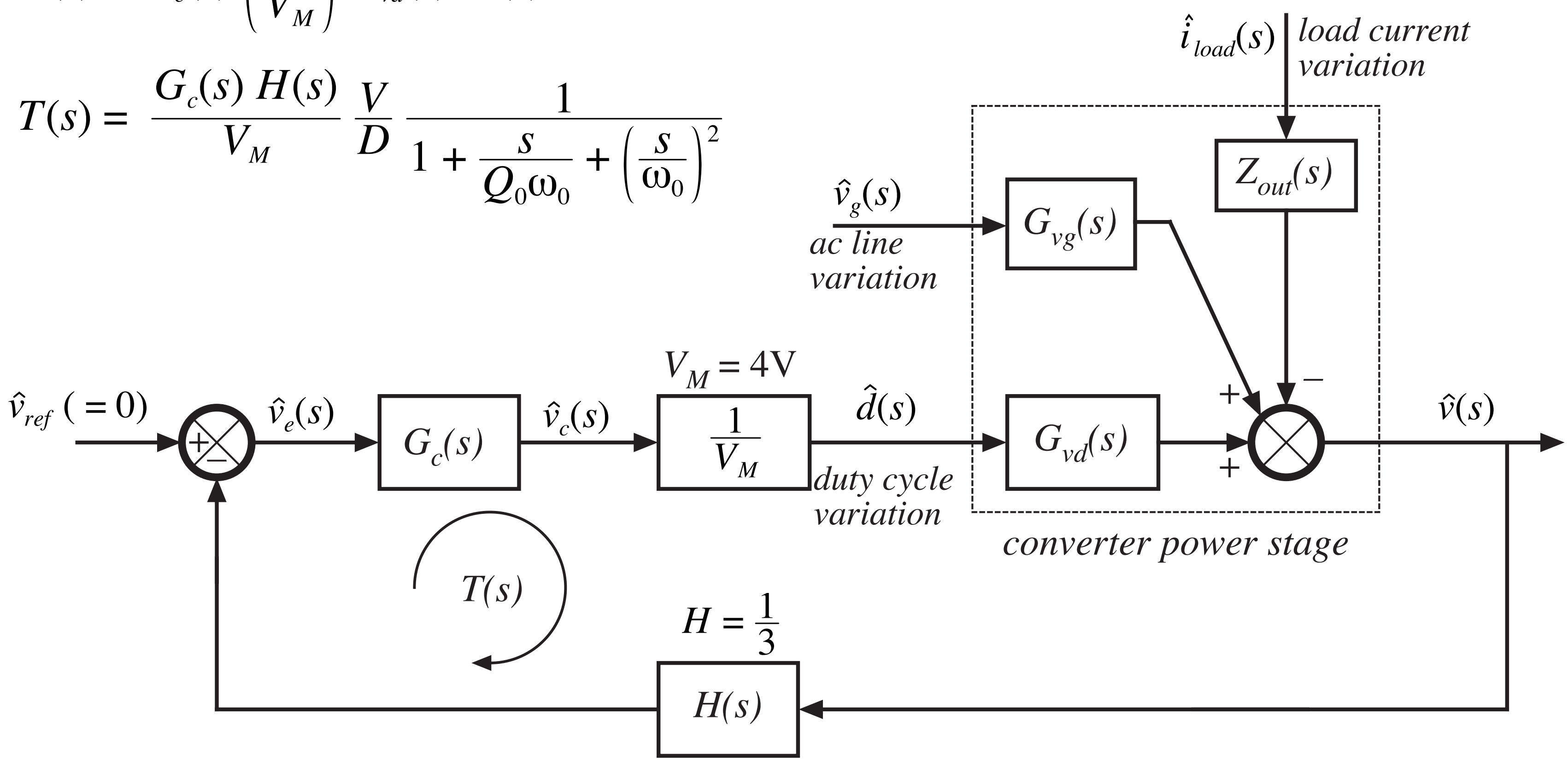
Output impedance:

$$Z_{out}(s) = R \parallel \frac{1}{sC} \parallel sL = \frac{sL}{1 + s\frac{L}{R} + s^2 LC}$$

System block diagram

$$T(s) = G_c(s) \left(\frac{1}{V_M} \right) G_{vd}(s)\, H(s)$$

$$T(s) = \frac{G_c(s)\, H(s)}{V_M} \frac{V}{D} \frac{1}{1 + \frac{s}{Q_0 \omega_0} + \left(\frac{s}{\omega_0} \right)^2}$$

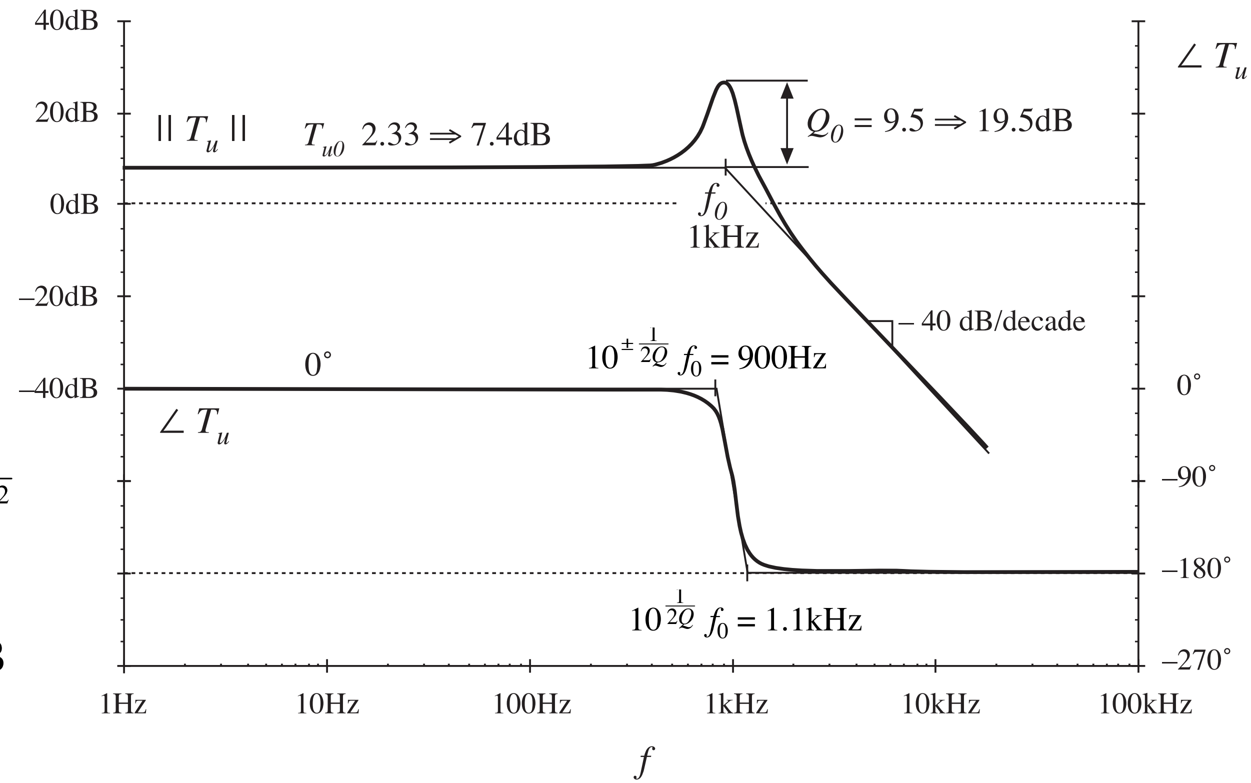


Uncompensated loop gain (with $G_c = 1$)

With $G_c = 1$, the loop gain is

$$T_u(s) = T_{u0} \frac{1}{1 + \frac{s}{Q_0 \omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

$$T_{u0} = \frac{H}{D} \frac{V}{V_M} = 2.33 \Rightarrow 7.4\text{dB}$$



$f_c = 1.8\text{kHz}$, $\varphi_m = 5^\circ$

Lead compensator design

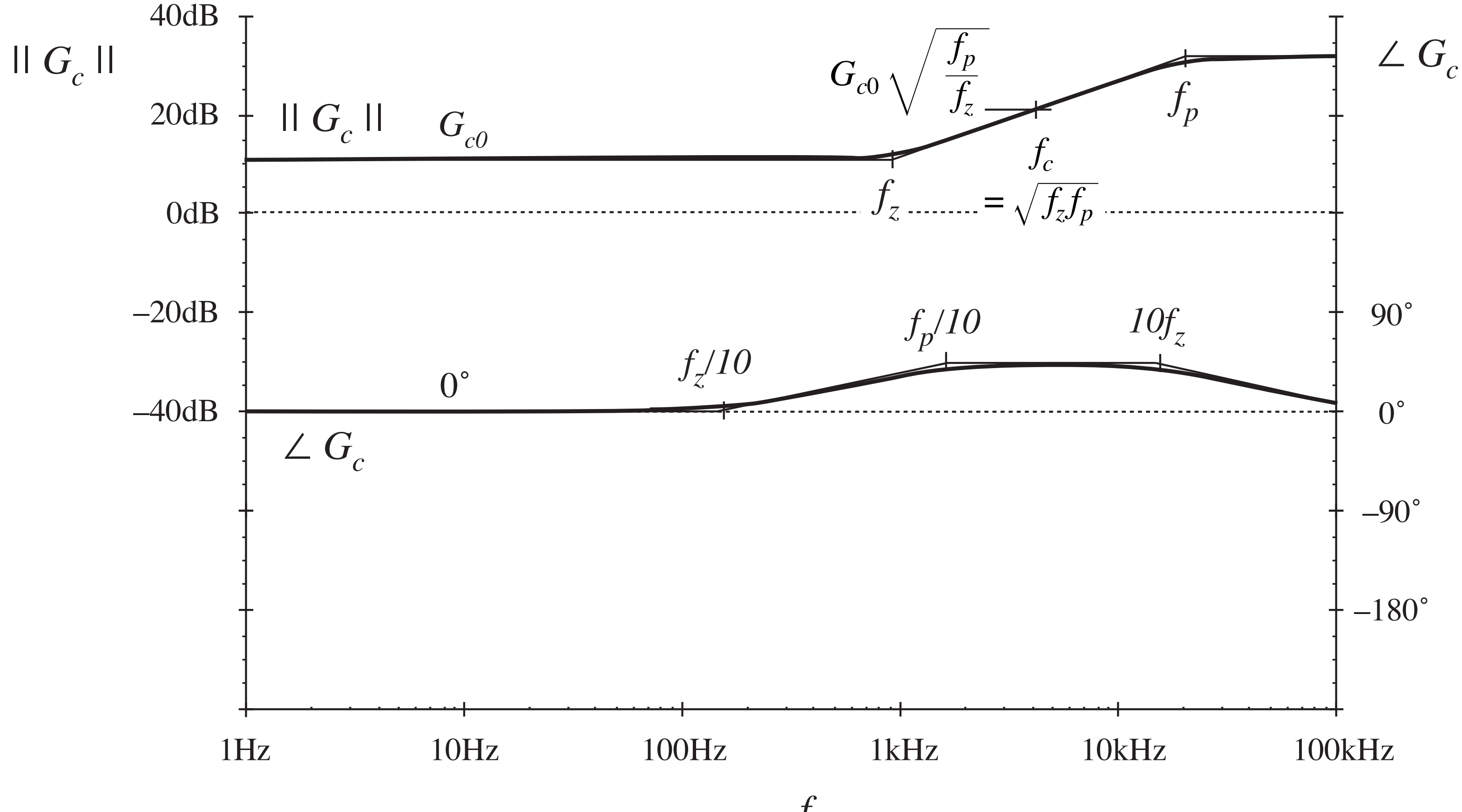
- Obtain a crossover frequency of 5kHz, with phase margin of 52°
- T_u has phase of approximately -180° at 5kHz, hence lead (PD) compensator is needed to increase phase margin.
- Lead compensator should have phase of +52° at 5kHz
- T_u has magnitude of -20.6dB at 5kHz
- Lead compensator gain should have magnitude of +20.6dB at 5kHz
- Lead compensator pole and zero frequencies should be

$$f_z = (5\text{kHz})\sqrt{\frac{1 \pm \sin(52°)}{1 + \sin(52°)}} = 1.7\text{kHz}$$

$$f_p = (5\text{kHz})\sqrt{\frac{1 + \sin(52°)}{1 \pm \sin(52°)}} = 14.5\text{kHz}$$

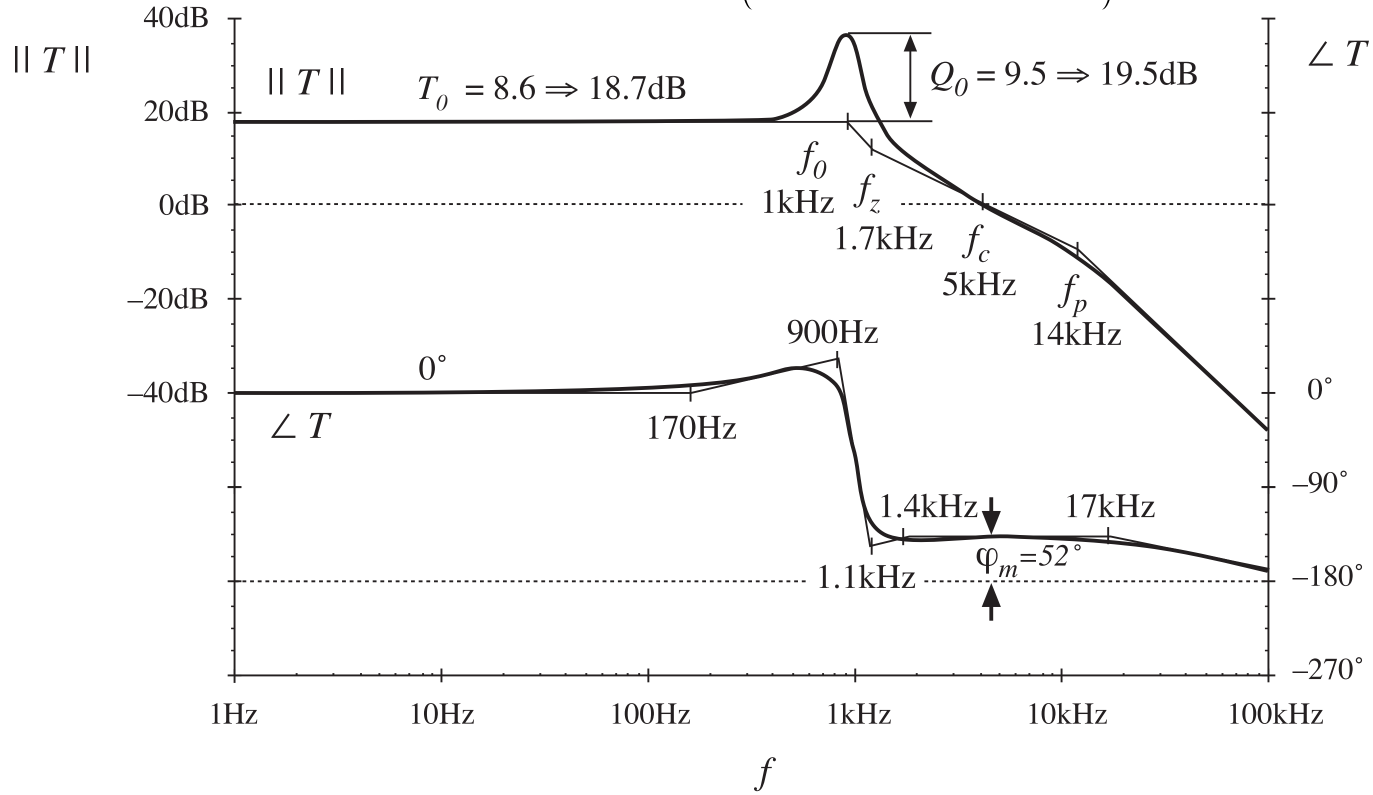
- Compensator dc gain should be $G_{c0} = \left(\frac{f_c}{f_0}\right)^2 \frac{1}{T_{u0}} \sqrt{\frac{f_z}{f_p}} = 3.7 \Rightarrow 11.3\text{dB}$

Lead compensator Bode plot

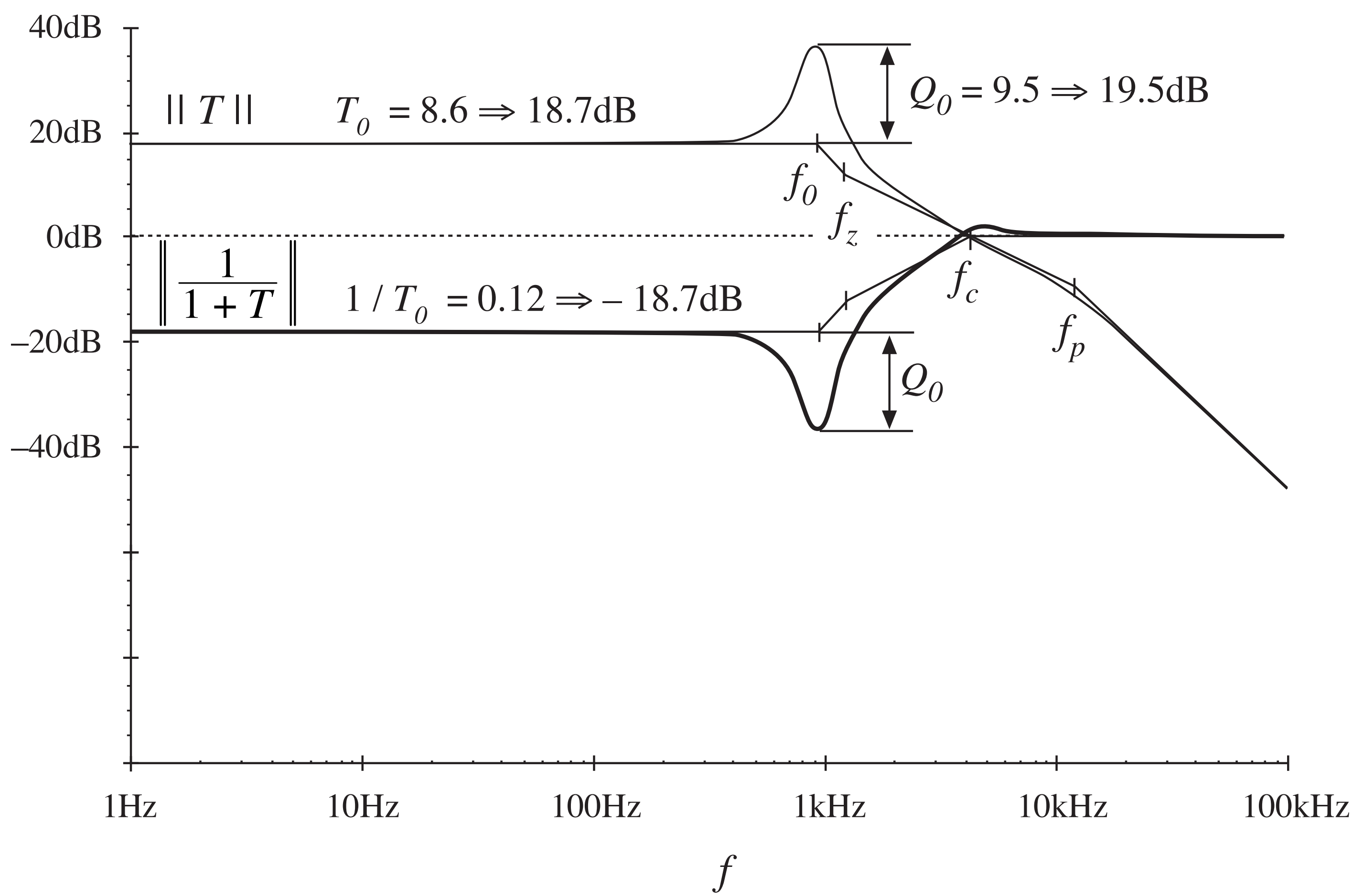


Loop gain, with lead compensator

$$T(s) = T_{u0}\, G_{c0} \frac{\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_p}\right)\left(1 + \frac{s}{Q_0\omega_0} + \left(\frac{s}{\omega_0}\right)^2\right)}$$

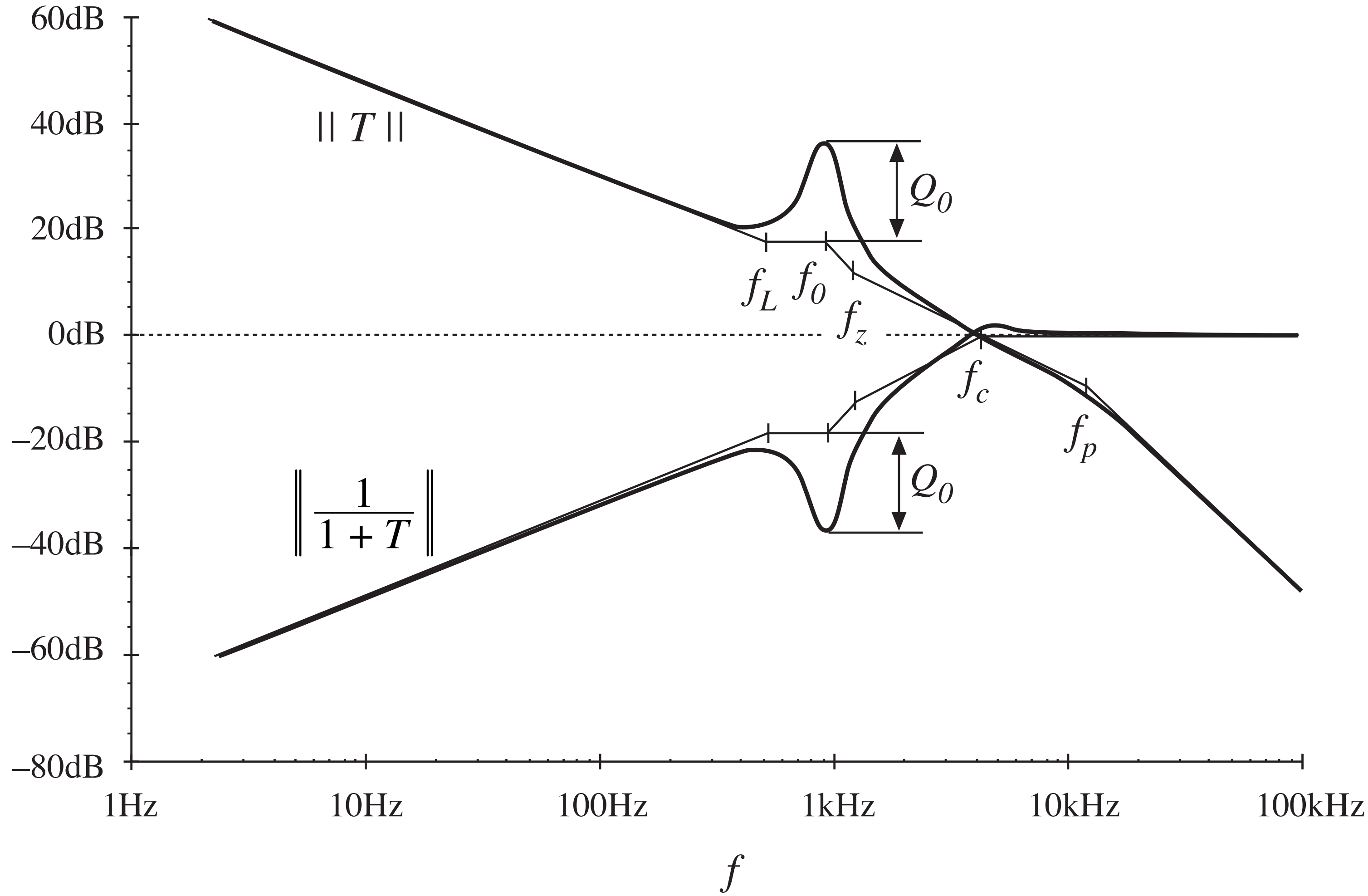


$1/(1+T)$, with lead compensator

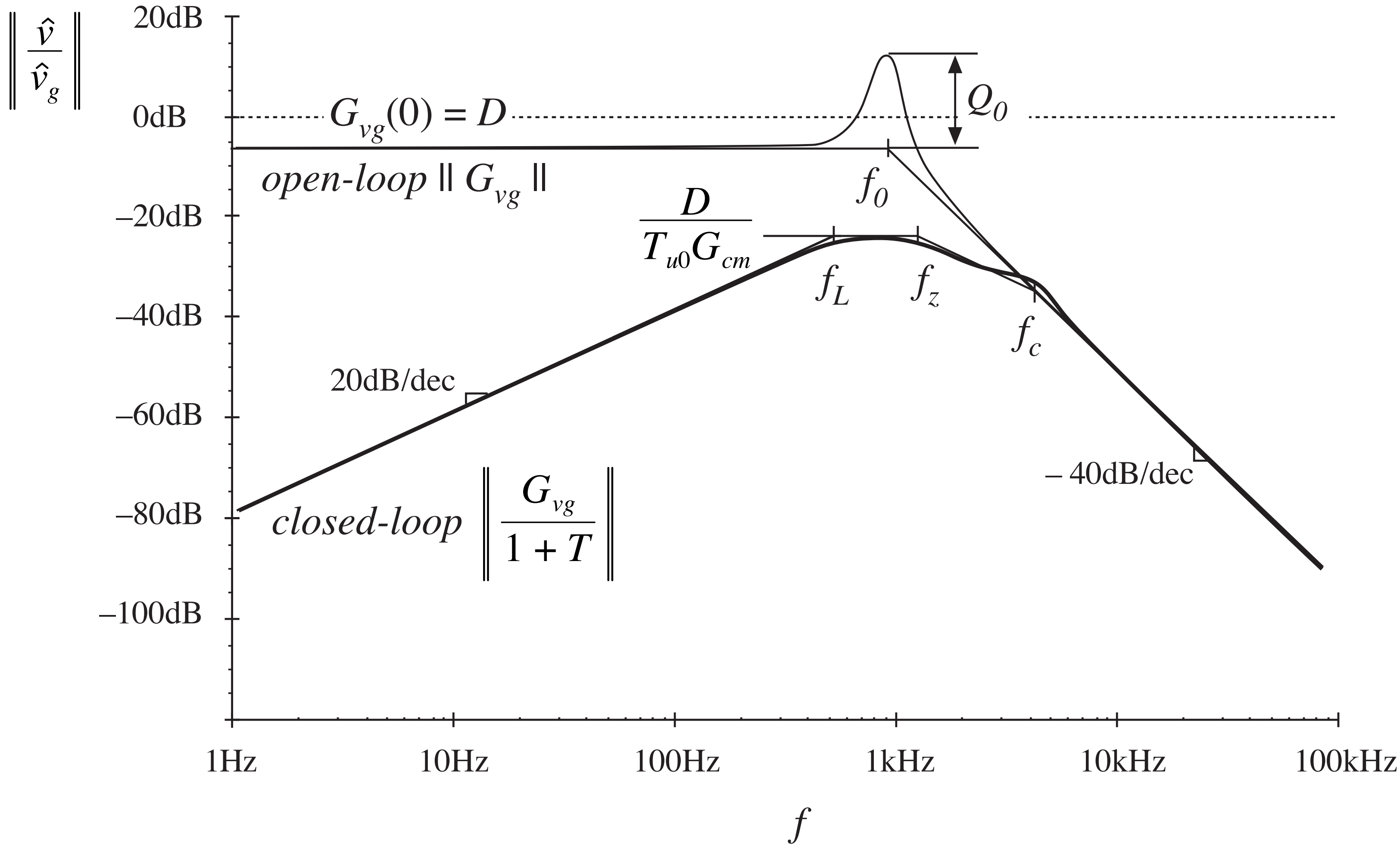


- need more low-frequency loop gain
- hence, add inverted zero (PID controller)

$T(s)$ and $1/(1+T(s))$, with PID compensator



Line-to-output transfer function



9.7. Summary of key points

1. Negative feedback causes the system output to closely follow the reference input, according to the gain $1 / H(s)$. The influence on the output of disturbances and variation of gains in the forward path is reduced.

2. The loop gain $T(s)$ is equal to the products of the gains in the forward and feedback paths. The loop gain is a measure of how well the feedback system works: a large loop gain leads to better regulation of the output. The crossover frequency f_c is the frequency at which the loop gain T has unity magnitude, and is a measure of the bandwidth of the control system.

Summary of key points

3. The introduction of feedback causes the transfer functions from disturbances to the output to be multiplied by the factor $1/(1+T(s))$. At frequencies where T is large in magnitude (i.e., below the crossover frequency), this factor is approximately equal to $1/T(s)$. Hence, the influence of low-frequency disturbances on the output is reduced by a factor of $1/T(s)$. At frequencies where T is small in magnitude (i.e., above the crossover frequency), the factor is approximately equal to 1. The feedback loop then has no effect. Closed-loop disturbance-to-output transfer functions, such as the line-to-output transfer function or the output impedance, can easily be constructed using the algebra-on-the-graph method.

4. Stability can be assessed using the phase margin test. The phase of T is evaluated at the crossover frequency, and the stability of the important closed-loop quantities $T/(1+T)$ and $1/(1+T)$ is then deduced. Inadequate phase margin leads to ringing and overshoot in the system transient response, and peaking in the closed-loop transfer functions.

Summary of key points

5. Compensators are added in the forward paths of feedback loops to shape the loop gain, such that desired performance is obtained. Lead compensators, or PD controllers, are added to improve the phase margin and extend the control system bandwidth. PI controllers are used to increase the low-frequency loop gain, to improve the rejection of low-frequency disturbances and reduce the steady-state error.

6. Loop gains can be experimentally measured by use of voltage or current injection. This approach avoids the problem of establishing the correct quiescent operating conditions in the system, a common difficulty in systems having a large dc loop gain. An injection point must be found where interstage loading is not significant. Unstable loop gains can also be measured.