Johns Hopkins Engineering

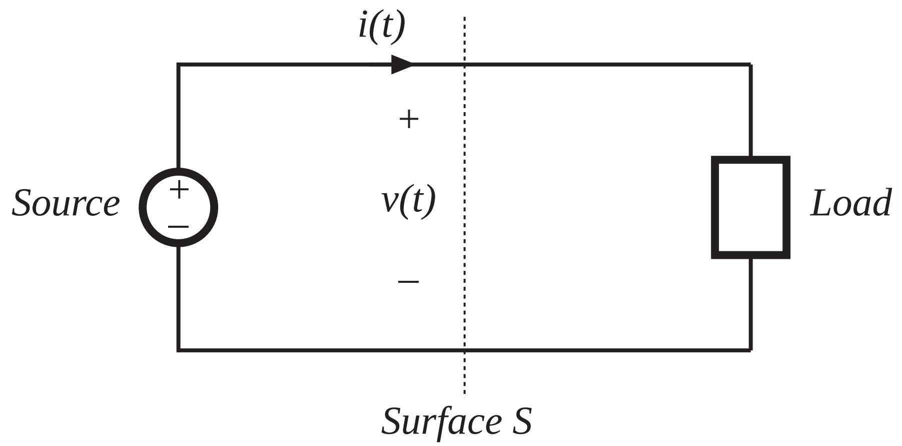
Power Electronics 525.725

Module 11 Lecture 11

Power Analysis/diode rectifiers

15.1. Average power

Observe transmission of energy through surface S



Express voltage and current as Fourier series:

$$v(t) = V_0 + \sum_{n=1}^{\infty} V_n \cos\left(n\omega t - \varphi_n\right)$$

$$i(t) = I_0 + \sum_{n=1}^{\infty} I_n \cos\left(n\omega t - \theta_n\right)$$

relate energy transmission to harmonics

Energy transmittted to load, per cycle

$$W_{cycle} = \int_0^T v(t)i(t)dt$$

This is related to average power as follows:

$$P_{av} = \frac{W_{cycle}}{T} = \frac{1}{T}\int_0^T v(t)i(t)dt$$

Investigate influence of harmonics on average power: substitute Fourier series

$$P_{av} = \frac{1}{T}\int_0^T \left(V_0 + \sum_{n=1}^{\infty} V_n \cos\left(n\omega t - \varphi_n\right)\right)\left(I_0 + \sum_{n=1}^{\infty} I_n \cos\left(n\omega t - \theta_n\right)\right)dt$$

Evaluation of integral

Orthogonality of harmonics: Integrals of cross-product terms are zero

$$\int_0^T \left(V_n \cos\left(n\omega t - \varphi_n\right)\right)\left(I_m \cos\left(m\omega t - \theta_m\right)\right)dt = \begin{cases} 0 & \text{if } n \neq m \\ \dfrac{V_n I_n}{2}\cos\left(\varphi_n - \theta_n\right) & \text{if } n = m \end{cases}$$

Expression for average power becomes

$$P_{av} = V_0 I_0 + \sum_{n=1}^{\infty} \frac{V_n I_n}{2} \cos\left(\varphi_n - \theta_n\right)$$

So net energy is transmitted to the load only when the Fourier series of $v(t)$ and $i(t)$ contain terms at the same frequency. For example, if the voltage and current both contain third harmonic, then they lead to the average power

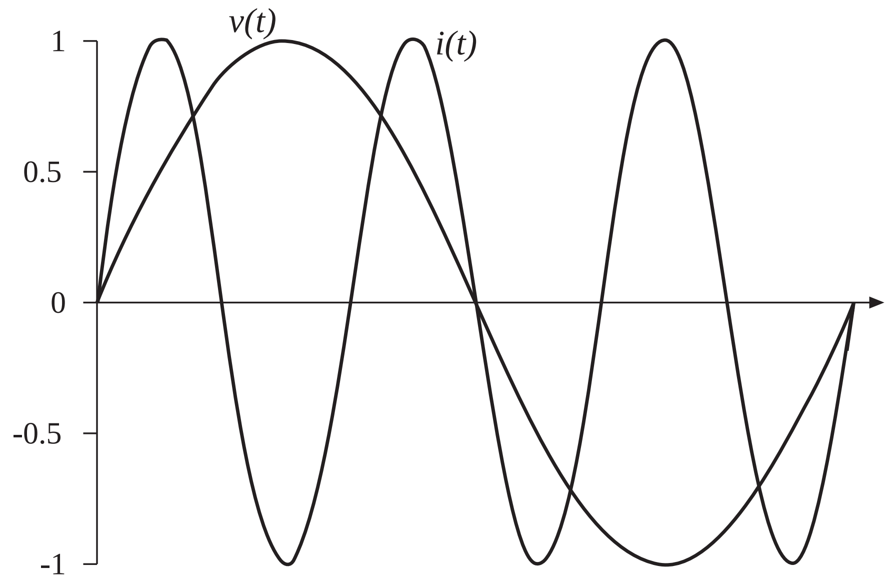
$$\frac{V_3 I_3}{2} \cos\left(\varphi_3 - \theta_3\right)$$

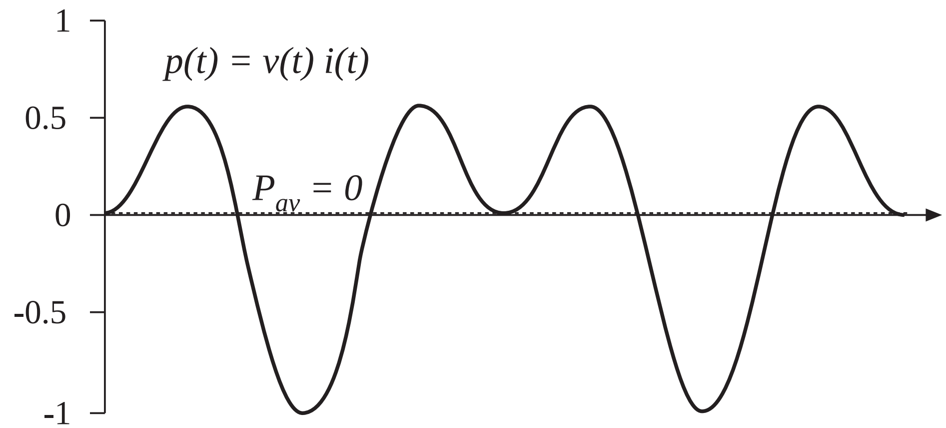
Example 1

Voltage: fundamental only

Current: third harmonic only

Power: zero average



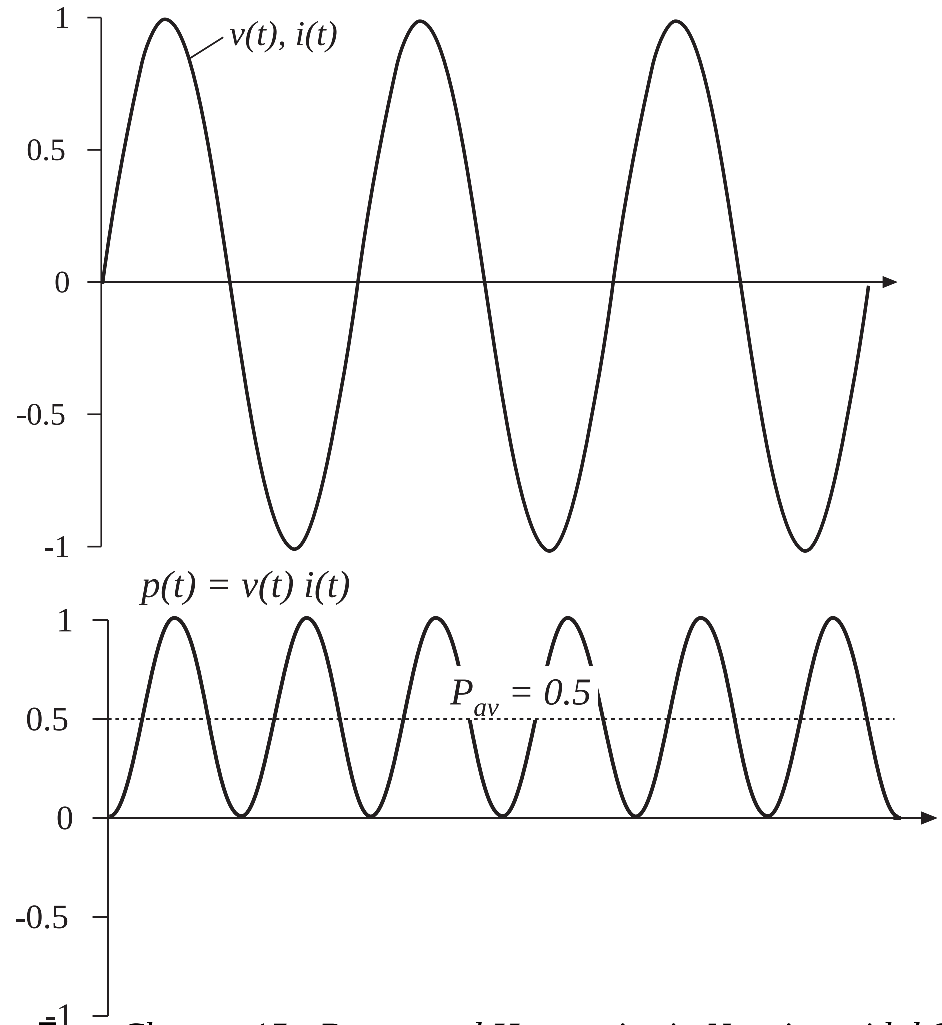


Example 2

Voltage: third harmonic only

Current: third harmonic only, in phase with voltage

Power: nonzero average



Example 3

Fourier series:

$$v(t) = 1.2 \cos(\omega t) + 0.33 \cos(3\omega t) + 0.2 \cos(5\omega t)$$

$$i(t) = 0.6 \cos(\omega t + 30°) + 0.1 \cos(5\omega t + 45°) + 0.1 \cos(7\omega t + 60°)$$

Average power calculation:

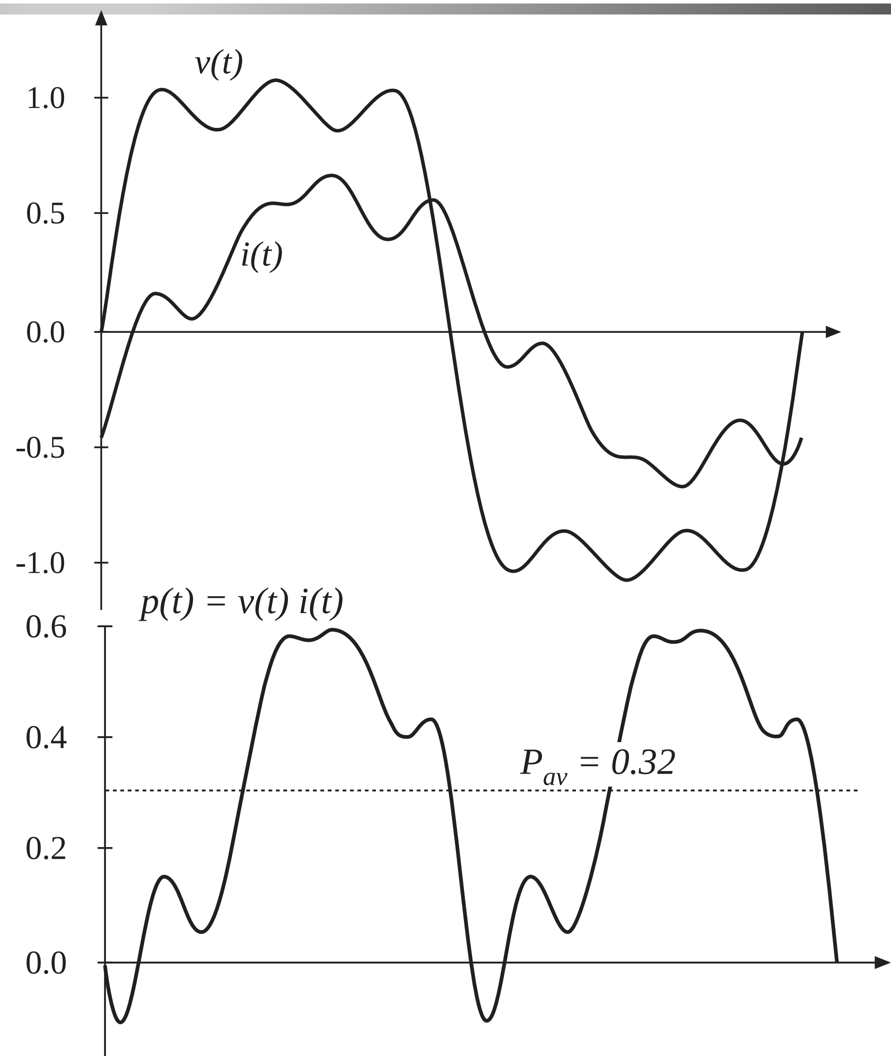
$$P_{av} = \frac{(1.2)(0.6)}{2} \cos(30°) + \frac{(0.2)(0.1)}{2} \cos(45°) = 0.32$$

Example 3

Voltage: 1st, 3rd, 5th

Current: 1st, 5th, 7th

Power: net energy is transmitted at fundamental and fifth harmonic frequencies



15.2. Root-mean-square (RMS) value of a waveform, in terms of Fourier series

$$(\text{rms value}) = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

Insert Fourier series. Again, cross-multiplication terms have zero average. Result is

$$(\text{rms value}) = \sqrt{V_0^2 + \sum_{n=1}^{\infty} \frac{V_n^2}{2}}$$

- Similar expression for current
- Harmonics always increase rms value
- Harmonics do not necessarily increase average power
- Increased rms values mean increased losses

15.3. Power factor

For efficient transmission of energy from a source to a load, it is desired to maximize average power, while minimizing rms current and voltage (and hence minimizing losses).

Power factor is a figure of merit that measures how efficiently energy is transmitted. It is defined as

$$\text{power factor} = \frac{(\text{average power})}{(\text{rms voltage})\ (\text{rms current})}$$

Power factor always lies between zero and one.

15.3.2. Nonlinear dynamical load, sinusoidal voltage

With a sinusoidal voltage, current harmonics do not lead to average power. However, current harmonics do increase the rms current, and hence they decrease the power factor.

$$P_{av} = \frac{V_1 I_1}{2} \cos(\varphi_1 - \theta_1)$$

$$(\text{rms current}) = \sqrt{I_0^2 + \sum_{n=1}^{\infty} \frac{I_n^2}{2}}$$

$$(\text{power factor}) = \left(\frac{\frac{I_1}{\sqrt{2}}}{\sqrt{I_0^2 + \sum_{n=1}^{\infty} \frac{I_n^2}{2}}} \right) \left(\cos(\varphi_1 - \theta_1) \right)$$

$$= (\text{distortion factor})\ (\text{displacement factor})$$

Distortion factor

Defined only for sinusoidal voltage.

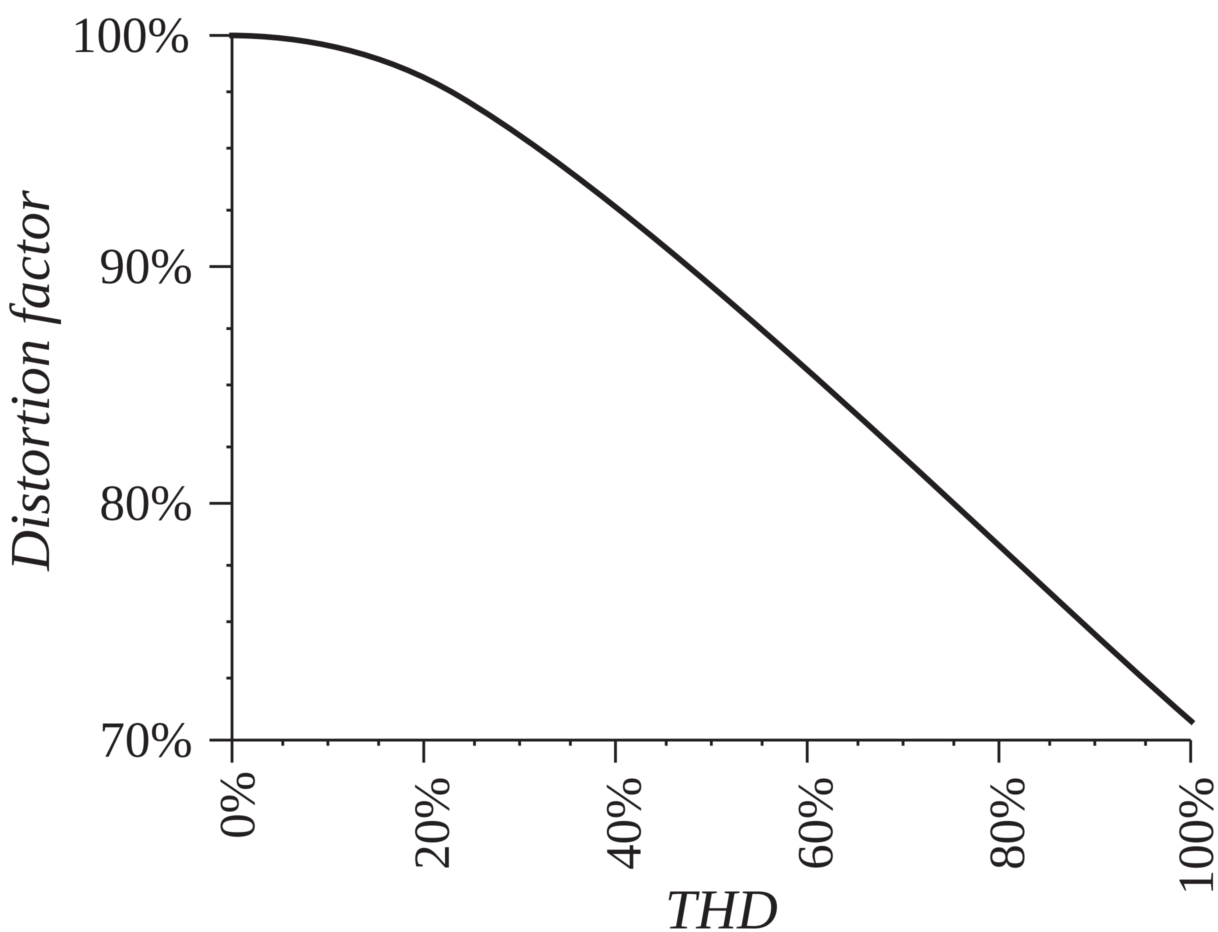
$$(\text{distortion factor}) = \left(\frac{\dfrac{I_1}{\sqrt{2}}}{\sqrt{I_0^2 + \displaystyle\sum_{n=1}^{\infty} \frac{I_n^2}{2}}} \right) = \frac{(\text{rms fundamental current})}{(\text{rms current})}$$

Related to Total Harmonic Distortion (THD):

$$(\text{THD}) = \frac{\sqrt{\displaystyle\sum_{n=2}^{\infty} I_n^2}}{I_1}$$

$$(\text{distortion factor}) = \frac{1}{\sqrt{1 + (\text{THD})^2}}$$

Distortion factor vs. THD

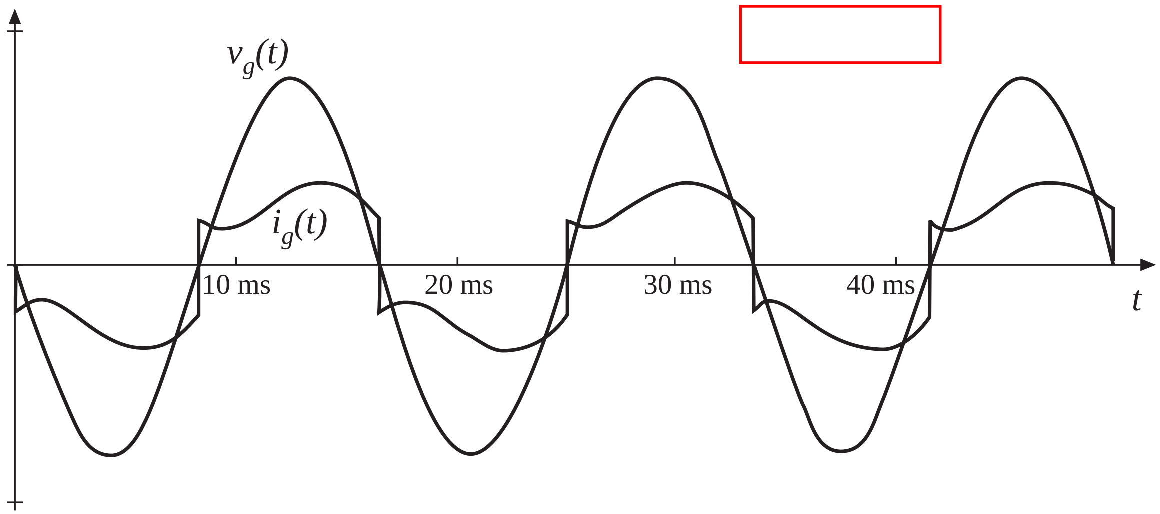


16.1.1 Continuous conduction mode

Large L

Typical ac line waveforms for CCM :

As $L \to \infty$, ac line current approaches a square wave



CCM results, for $L \to \infty$:

$$\text{distortion factor} = \frac{I_{1,\,rms}}{I_{rms}} = \frac{4}{\pi\sqrt{2}} = 90.0\%$$

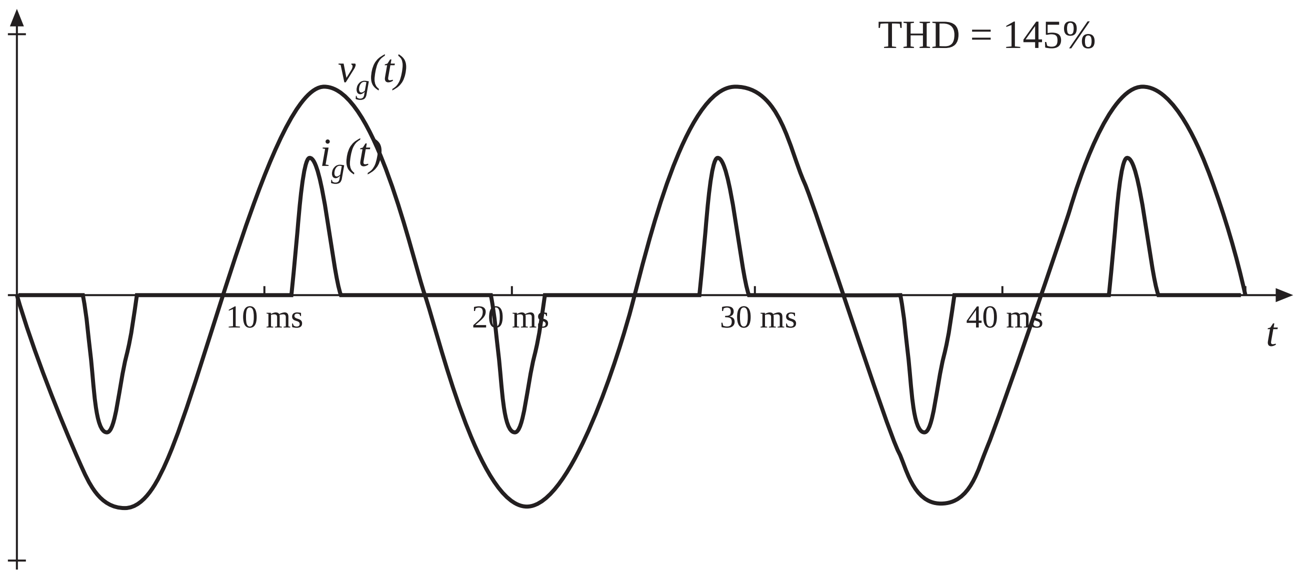
$$\text{THD} = \sqrt{\left(\frac{1}{\text{distortion factor}}\right)^2 - 1} = 48.3\%$$

16.1.2 Discontinuous conduction mode

Small L

Typical ac line waveforms for DCM :

As $L \rightarrow 0$, ac line current approaches impulse functions (peak detection)

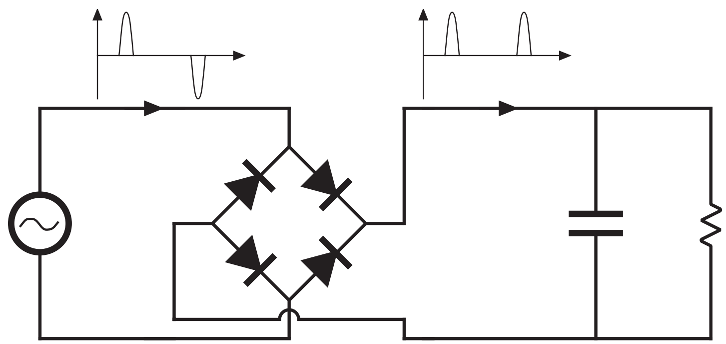


As the inductance is reduced, the THD rapidly increases, and the distortion factor decreases.

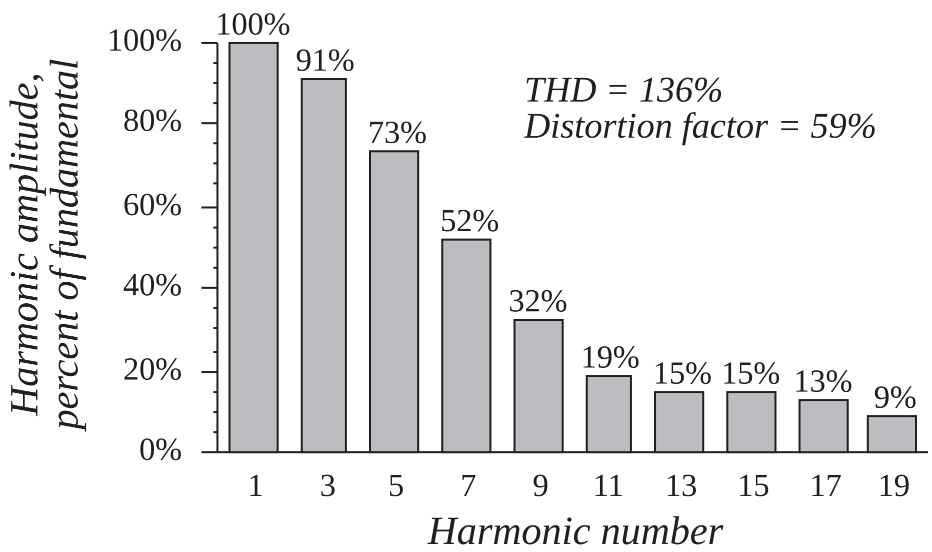
Typical distortion factor of a full-wave rectifier with no inductor is in the range 55% to 65%, and is governed by ac system inductance.

Peak detection rectifier example

Conventional single-phase peak detection rectifier



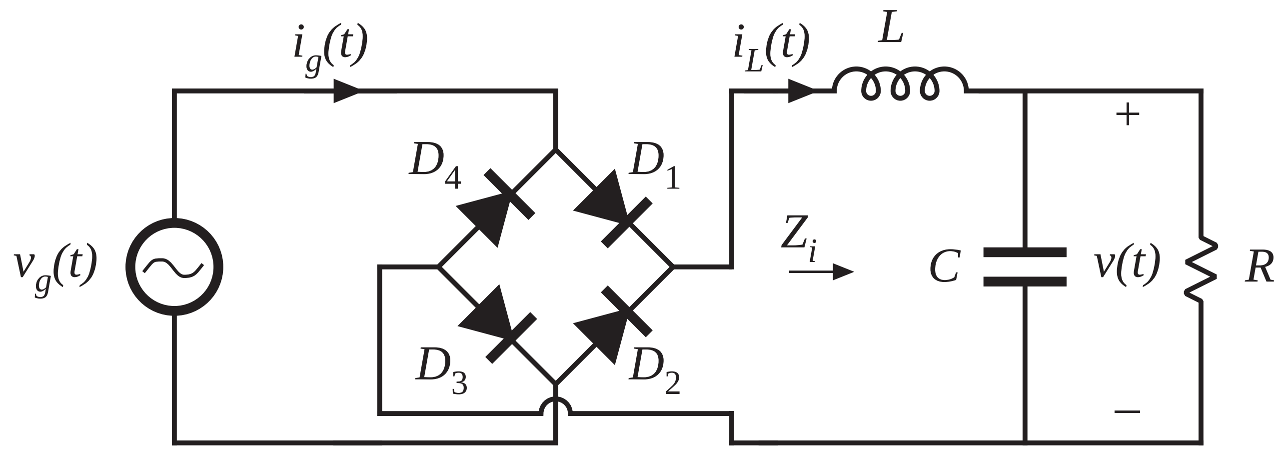
Typical ac line current spectrum



16.1.4 Minimizing *THD* when *C* is small

Sometimes the L-C filter is present only to remove high-frequency conducted EMI generated by the dc load, and is not intended to modify the ac line current waveform. If L and C are both zero, then the load resistor is connected directly to the output of the diode bridge, and the ac line current waveform is purely sinusoidal.

An approximate argument: the L-C filter has negligible effect on the ac line current waveform provided that the filter input impedance Z_i has zero phase shift at the second harmonic of the ac line frequency, $2\ f_L$.



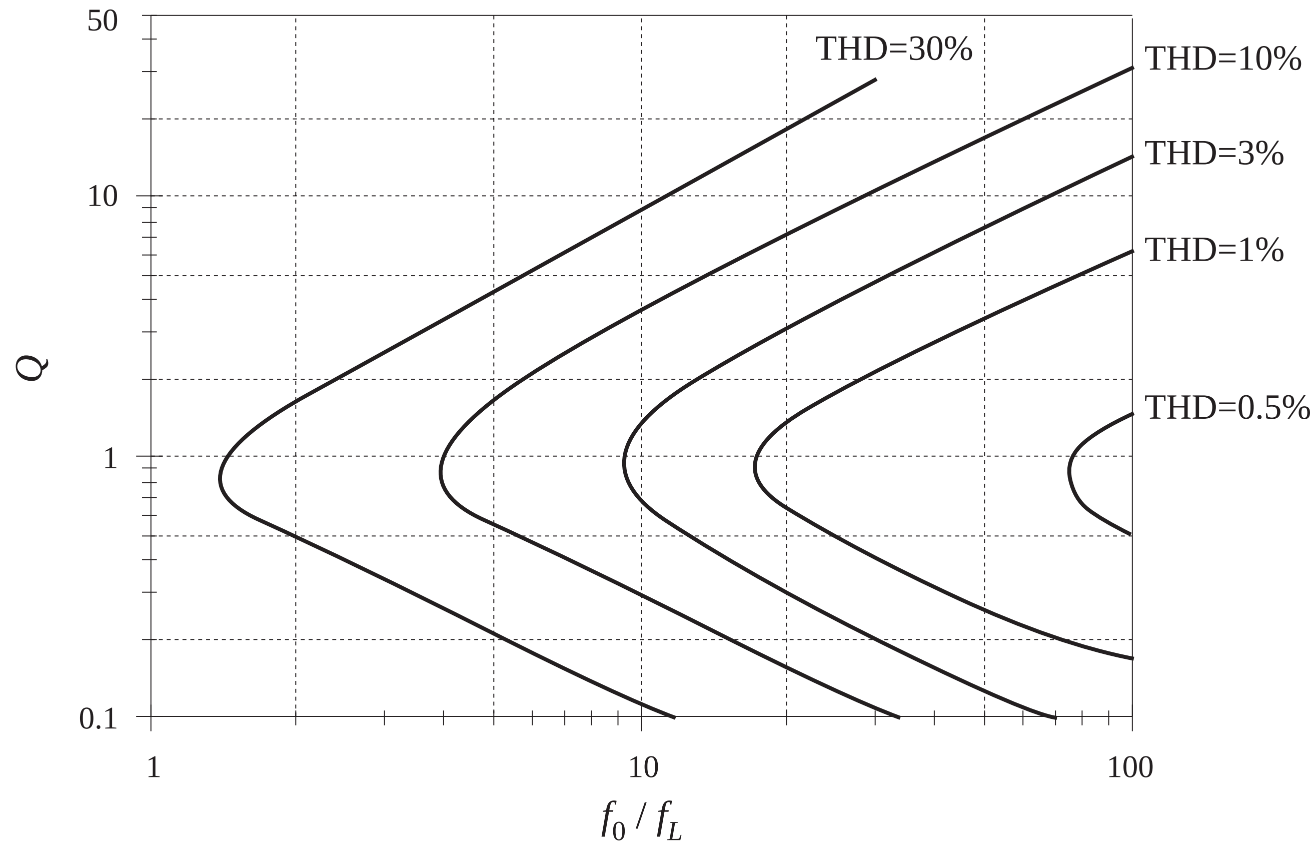
Approximate THD

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

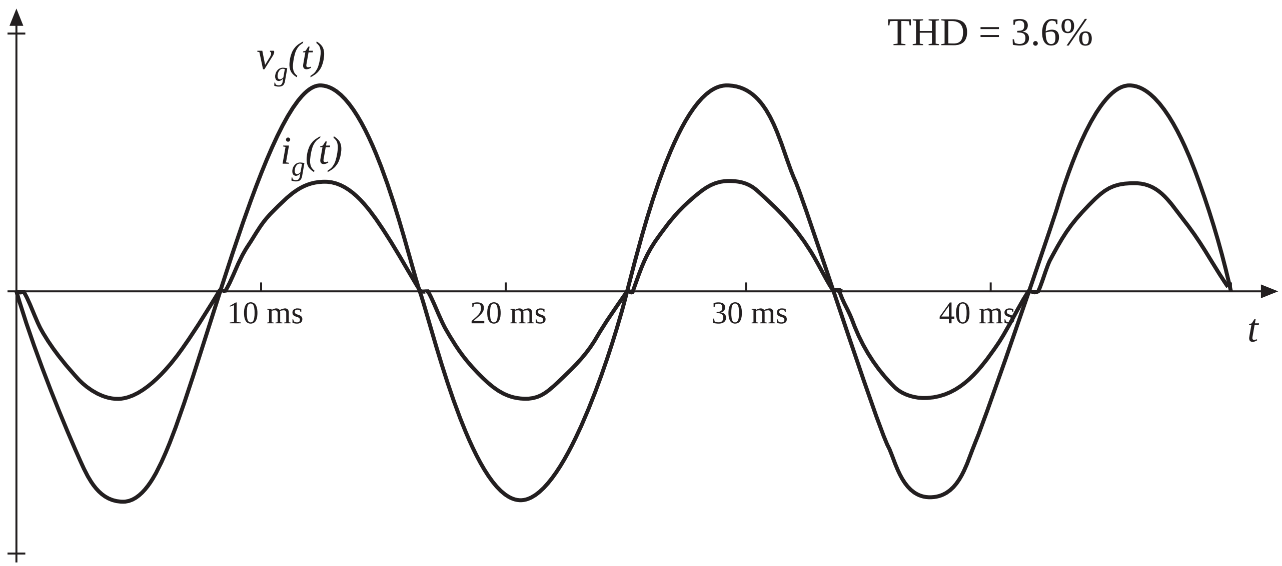
$$R_0 = \sqrt{\frac{L}{C}}$$

$$Q = \frac{R}{R_0}$$

$$f_p = \frac{1}{2\pi RC} = \frac{f_0}{Q}$$



Example



Typical ac line current and voltage waveforms, near the boundary between continuous and discontinuous modes and with small dc filter capacitor. $f_0/f_L = 10$, $Q = 1$