Johns Hopkins Engineering

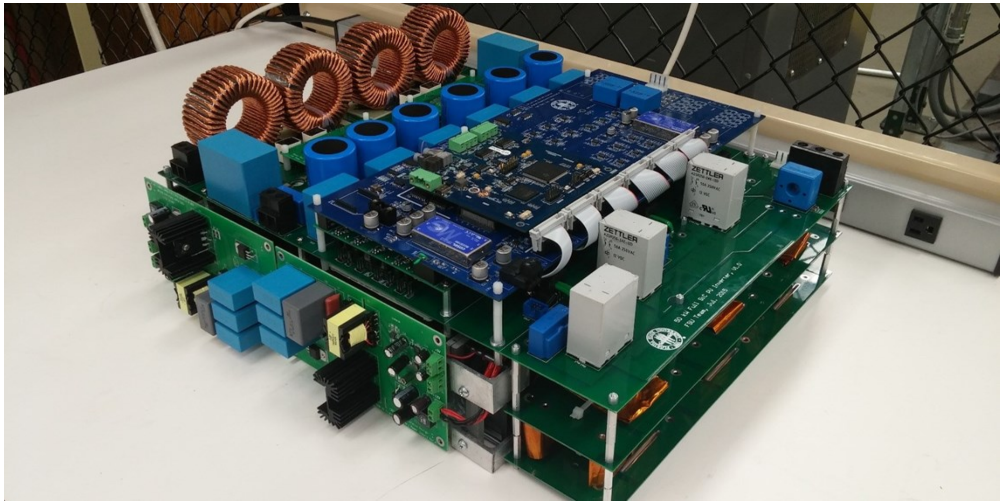
Power Electronics 525.725

Module 1 Lecture 1a

Introduction

What is Power Electronics?

- Power Electronics is the study of switching electronic circuits in order to control the flow of electrical energy
- *https://en.wikibooks.org/wiki/Power_Electronics*



Power Electronics is Everywhere

Advanced missile defense

Electric Vehicles

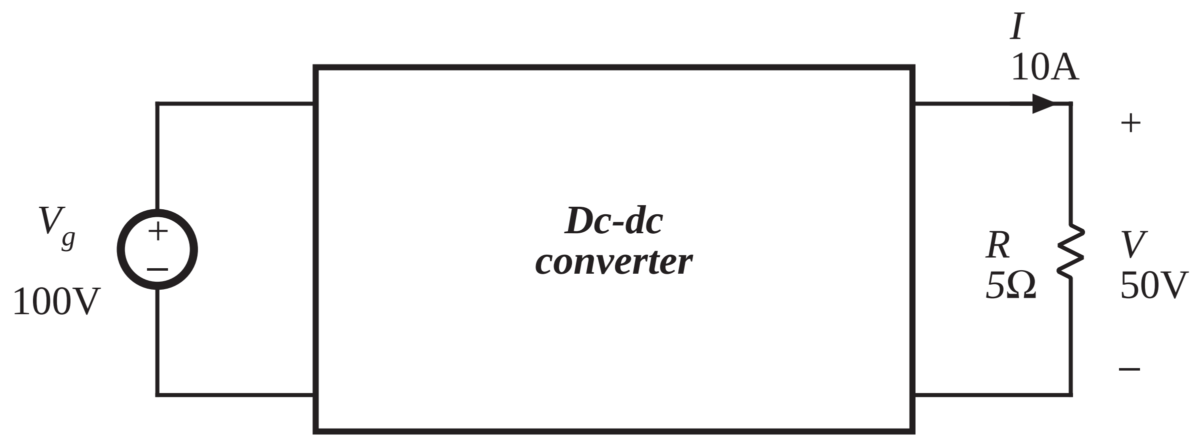
Space Exploration

Consumer Electronics

Why is Power Electronics Important?

- As the trend towards electrification and renewable energies increases, enabling technologies such as power electronics are becoming ever more important
- *www.abb.com/cawp/seitp202/85b14cadbc1d544bc1257b5b003de5af.aspx*

A simple dc-dc converter example

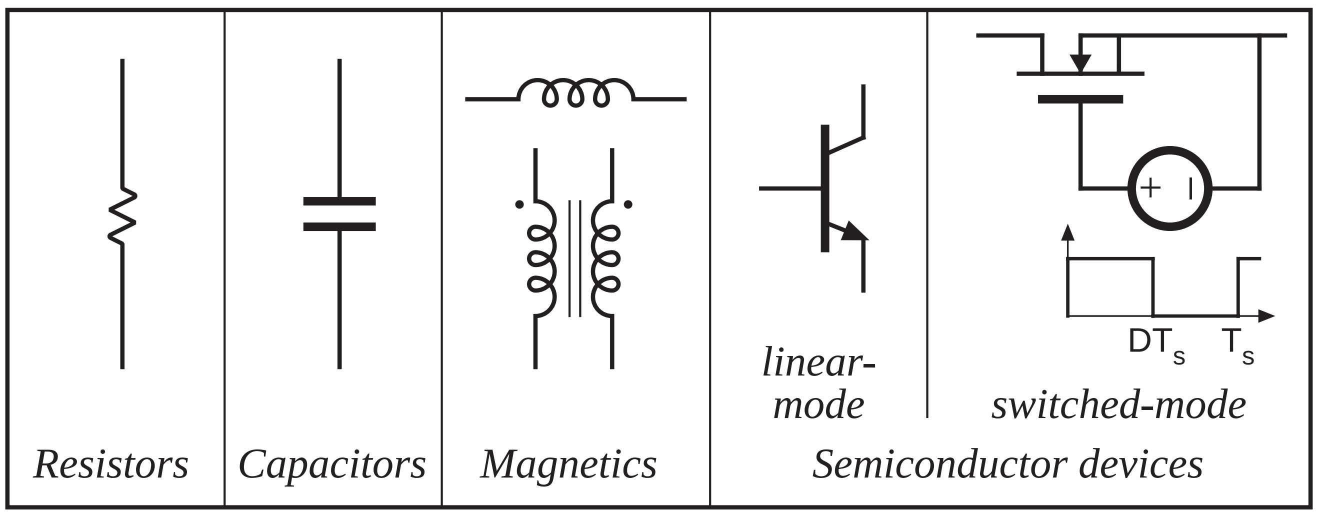


Input source: 100V

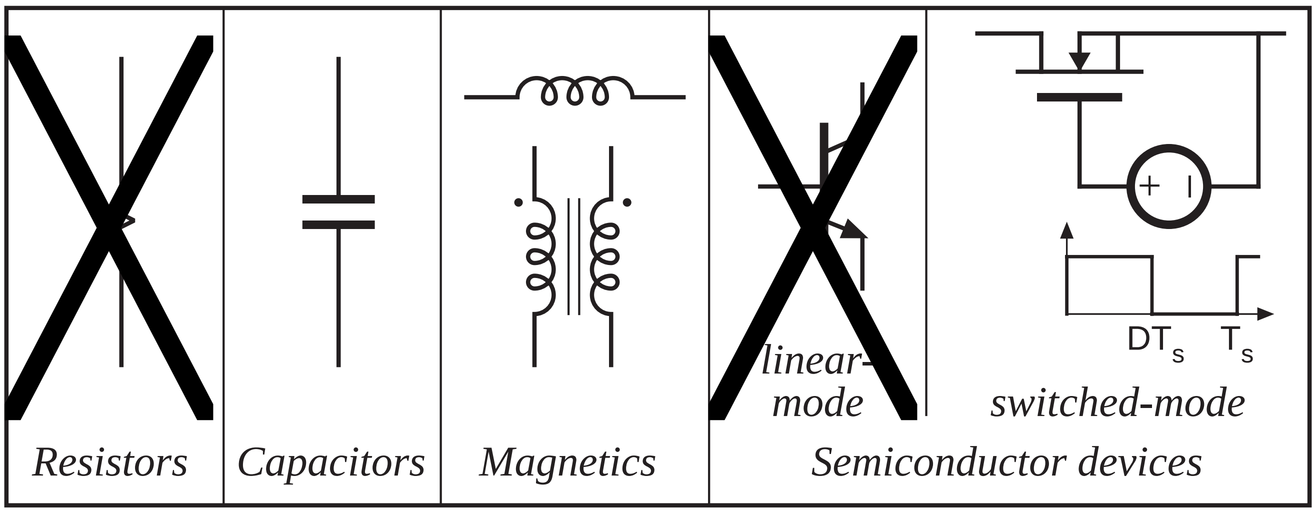
Output load: 50V, 10A, 500W

How can this converter be realized?

Devices available to the circuit designer



Devices available to the circuit designer



Power processing: avoid lossy elements

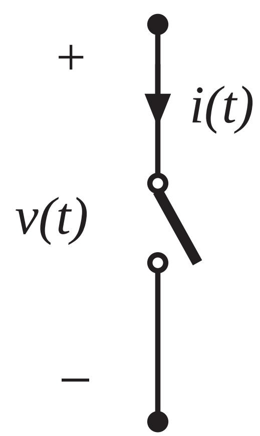
Power loss in an ideal switch

Switch closed: $v(t) = 0$

Switch open: $i(t) = 0$

In either event: $p(t) = v(t)\, i(t) = 0$

Ideal switch consumes zero power



High efficiency is essential

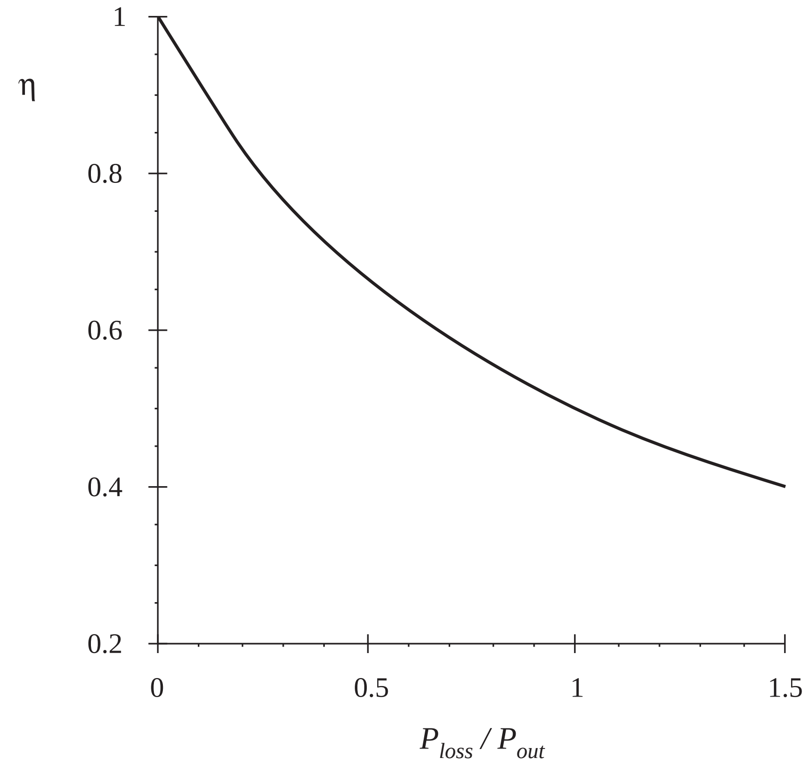
$$\eta = \frac{P_{out}}{P_{in}}$$

$$P_{loss} = P_{in} - P_{out} = P_{out}\left(\frac{1}{\eta} - 1\right)$$

High efficiency leads to low power loss within converter

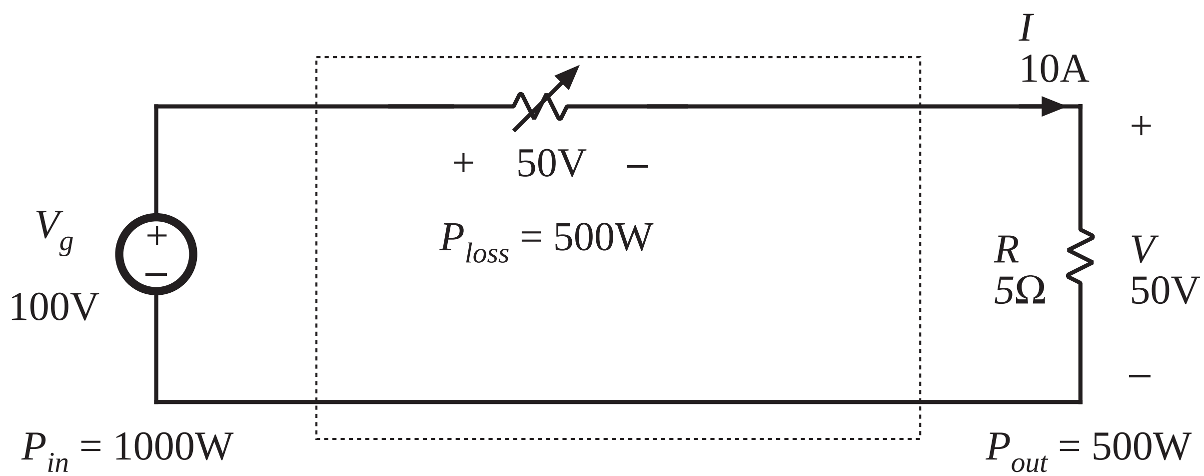
Small size and reliable operation is then feasible

Efficiency is a good measure of converter performance



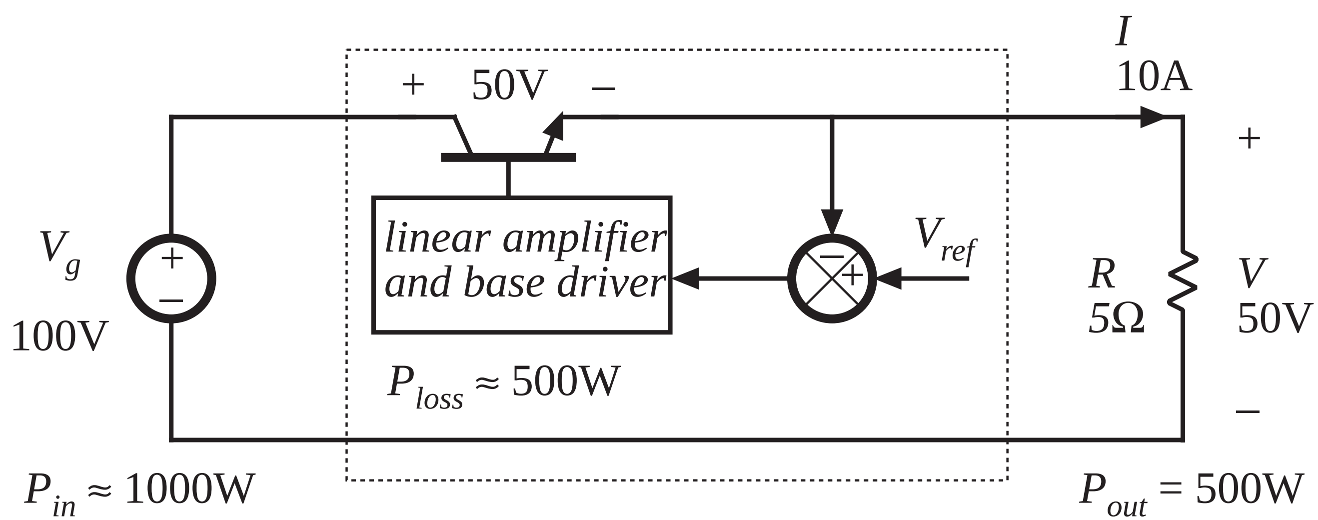
Dissipative realization

Resistive voltage divider

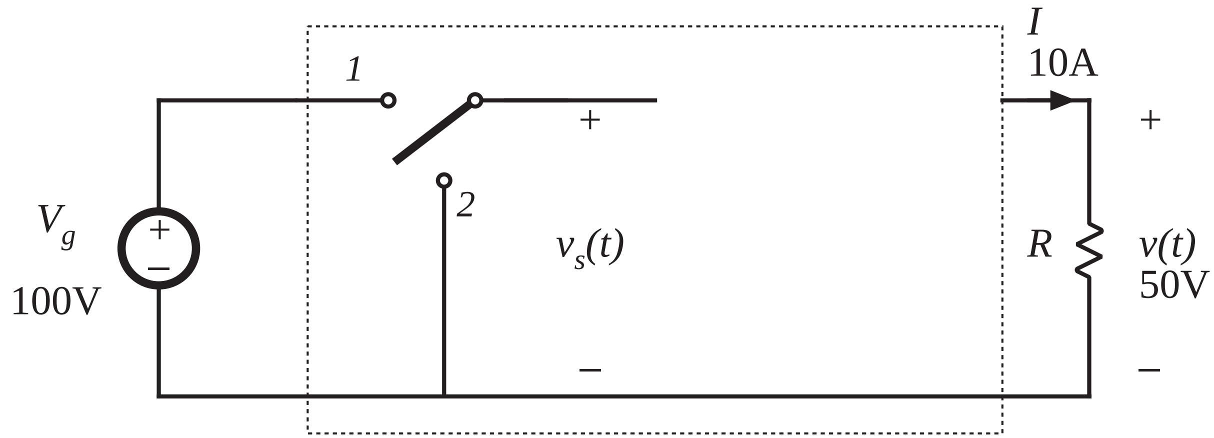


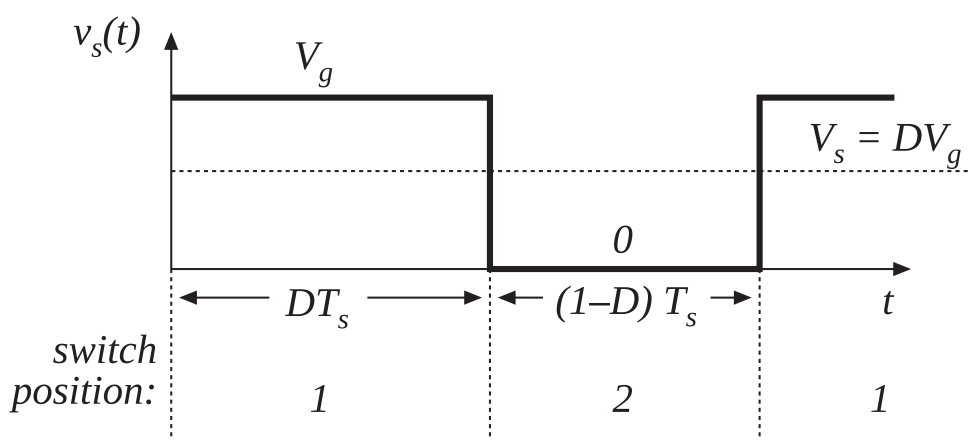
Dissipative realization

Series pass regulator: transistor operates in active region

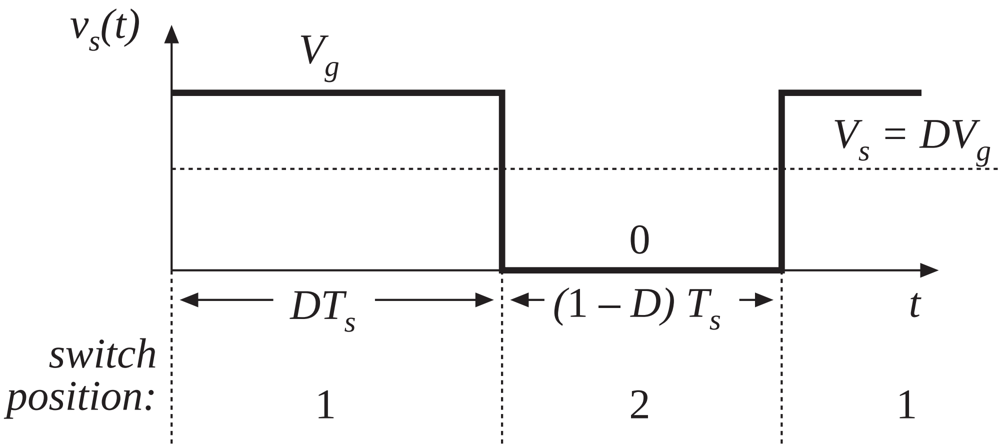


Use of a SPDT switch





The switch changes the dc voltage level



D = switch duty cycle

$0 \le D \le 1$

T_s = switching period

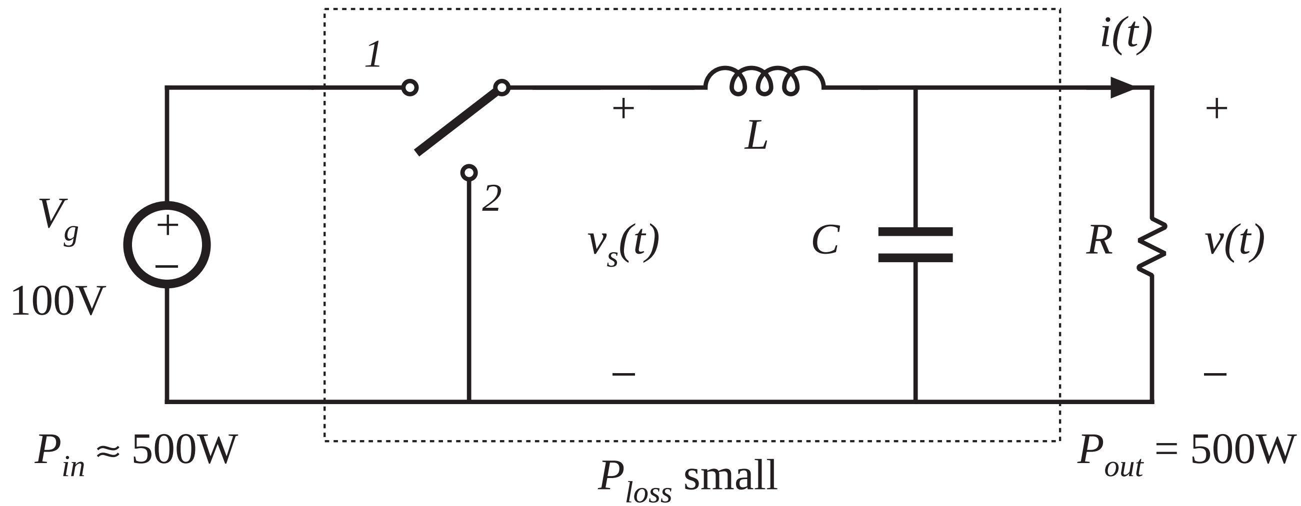
f_s = switching frequency
$= 1 / T_s$

DC component of $v_s(t)$ = average value:

$$V_s = \frac{1}{T_s}\int_0^{T_s} v_s(t)\,dt = DV_g$$

Addition of low pass filter

Addition of (ideally lossless) L-C low-pass filter, for removal of switching harmonics:



- Choose filter cutoff frequency f_0 much smaller than switching frequency f_s
- This circuit is known as the "buck converter"

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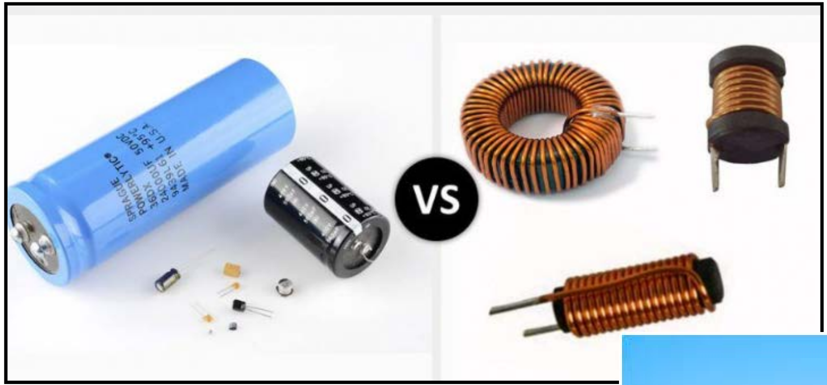
Module 1 Lecture 1b

Principles of Steady State Converter Analysis

What is Steady State Analysis

- The analysis of a switching converter under steady state condition to determine the salient features of a converter such as:
  - o DC output voltage
  - o Voltage ripple
  - o Current ripple
  - o DC inductor current



Defining the converter salient features during steady state allows the designer to size components

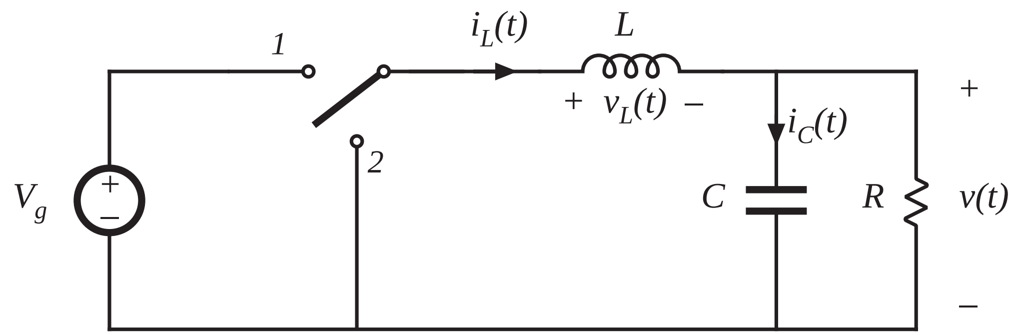
Three Important Concepts for Steady State Analysis

- Inductor volt-second balance
- Capacitor charge balance
- Small ripple approximation

2.2. Inductor volt-second balance, capacitor charge balance, and the small ripple approximation

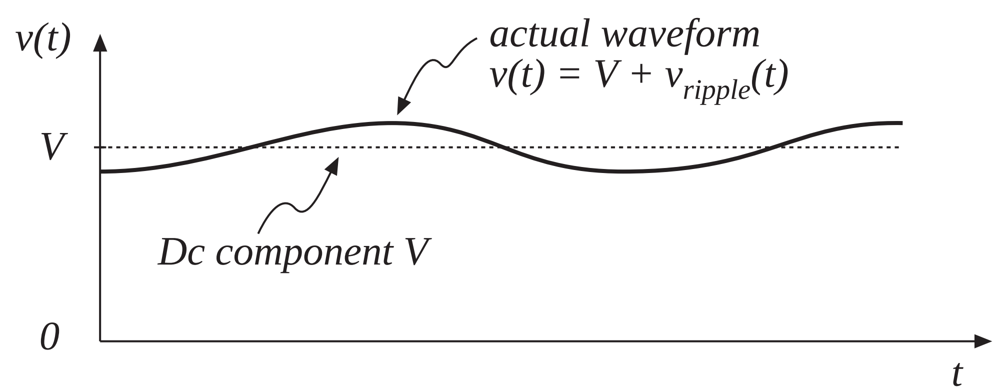
Actual output voltage waveform, buck converter

Buck converter containing practical low-pass filter



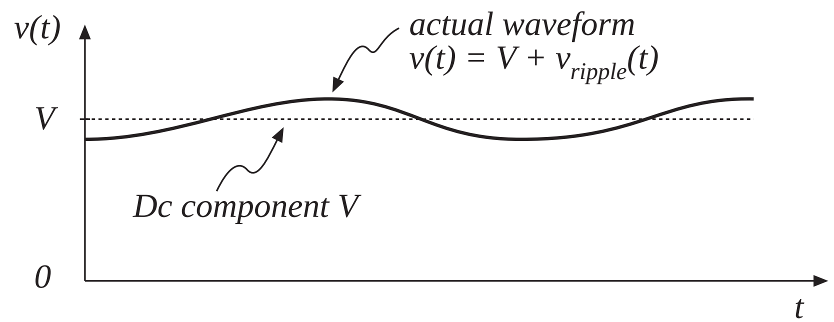
Actual output voltage waveform

$$v(t) = V + v_{ripple}(t)$$



The small ripple approximation

$$v(t) = V + v_{ripple}(t)$$



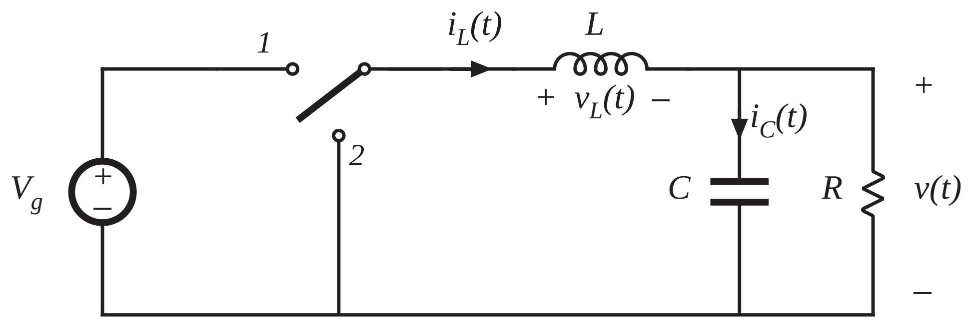
In a well-designed converter, the output voltage ripple is small. Hence, the waveforms can be easily determined by ignoring the ripple:

$$\left\| v_{ripple} \right\| << V$$

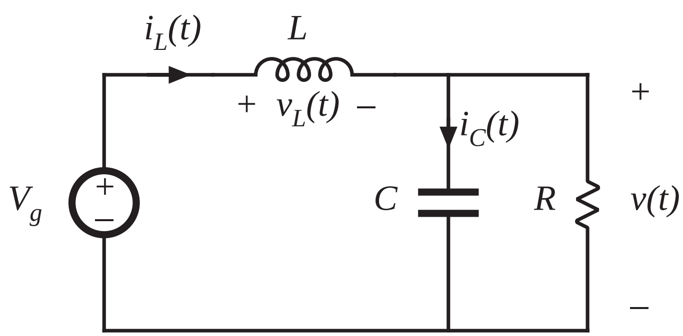
$$v(t) \approx V$$

Buck converter analysis: inductor current waveform

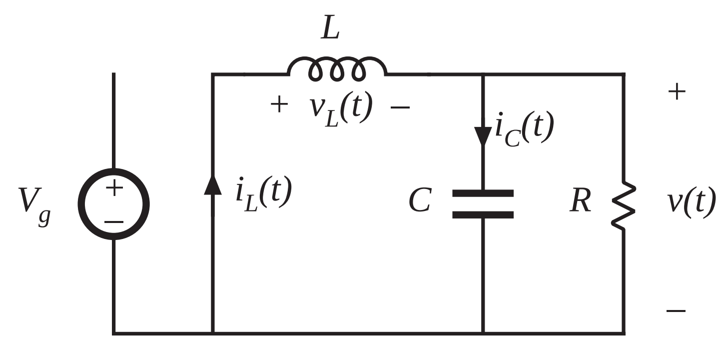
original converter



switch in position 1



switch in position 2



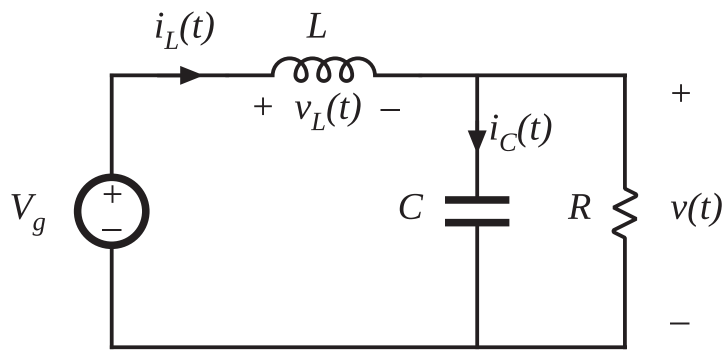
Inductor voltage and current
Subinterval 1: switch in position 1

Inductor voltage

$$v_L = V_g - v(t)$$

Small ripple approximation:

$$v_L \approx V_g - V$$



Knowing the inductor voltage, we can now find the inductor current via

$$v_L(t) = L\,\frac{di_L(t)}{dt}$$

Solve for the slope:

$$\frac{di_L(t)}{dt} = \frac{v_L(t)}{L} \approx \frac{V_g - V}{L}$$

$\Rightarrow$ *The inductor current changes with an essentially constant slope*

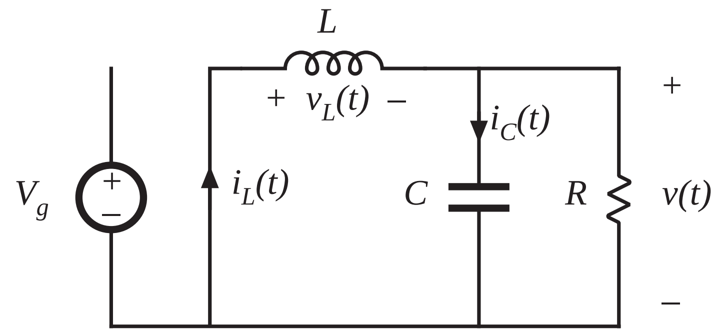
Inductor voltage and current
Subinterval 2: switch in position 2

Inductor voltage

$$v_L(t) = -\,v(t)$$

Small ripple approximation:

$$v_L(t) \approx -\,V$$



Knowing the inductor voltage, we can again find the inductor current via

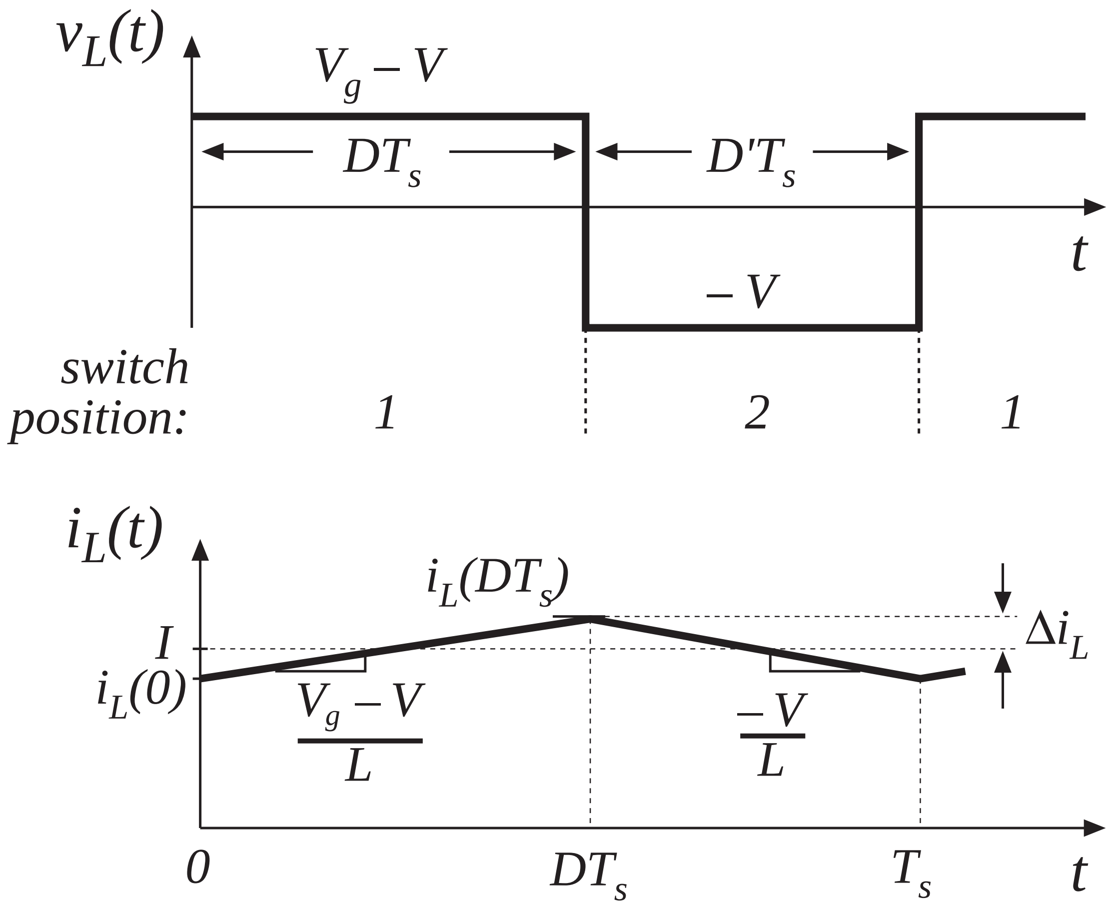
$$v_L(t) = L\,\frac{di_L(t)}{dt}$$

Solve for the slope:

$$\frac{di_L(t)}{dt} \approx -\,\frac{V}{L}$$

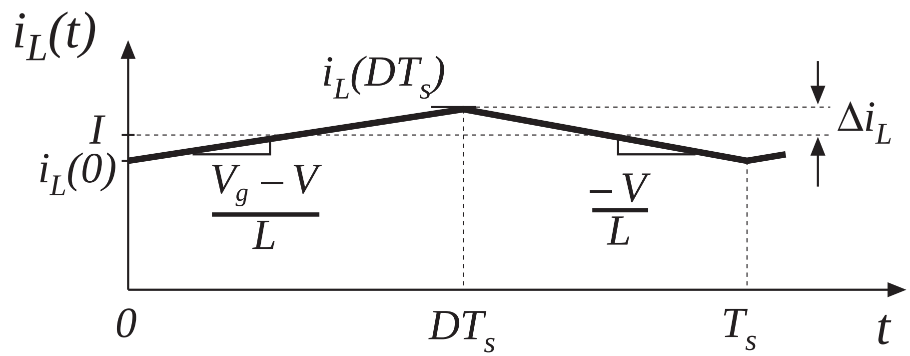
$\Rightarrow$ *The inductor current changes with an essentially constant slope*

Inductor voltage and current waveforms



$$v_L(t) = L\,\frac{di_L(t)}{dt}$$

Determination of inductor current ripple magnitude



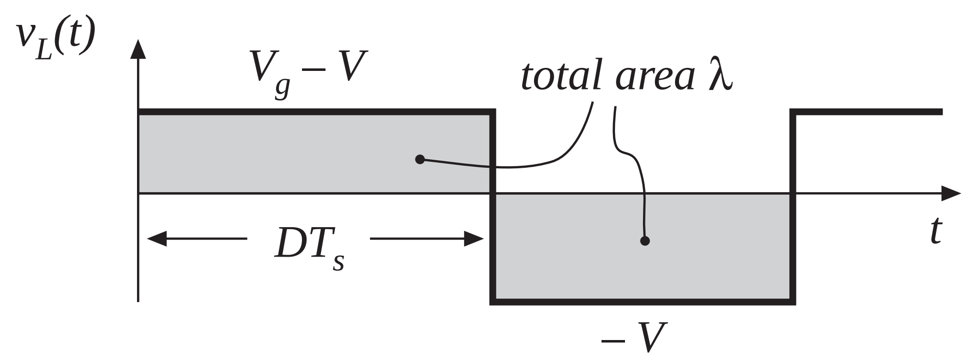
$$(change\ in\ i_L) = (slope)(length\ of\ subinterval)$$

$$\left(2\Delta i_L\right) = \left(\frac{V_g - V}{L}\right)\left(DT_s\right)$$

$$\Rightarrow \qquad \Delta i_L = \frac{V_g - V}{2L} DT_s \qquad\qquad L = \frac{V_g - V}{2\Delta i_L} DT_s$$

Inductor volt-second balance: Buck converter example

Inductor voltage waveform, previously derived:



Integral of voltage waveform is area of rectangles:

$$\lambda = \int_0^{T_s} v_L(t)\,dt = (V_g - V)(DT_s) + (-V)(D'T_s)$$

Average voltage is

$$\langle v_L \rangle = \frac{\lambda}{T_s} = D(V_g - V) + D'(-V)$$

Equate to zero and solve for *V:*

$$0 = DV_g - (D + D')V = DV_g - V \qquad \Rightarrow \qquad V = DV_g$$

The principle of capacitor charge balance: Derivation

Capacitor defining relation:

$$i_C(t) = C\,\frac{dv_C(t)}{dt}$$

Integrate over one complete switching period:

$$v_C(T_s) - v_C(0) = \frac{1}{C}\int_0^{T_s} i_C(t)\,dt$$

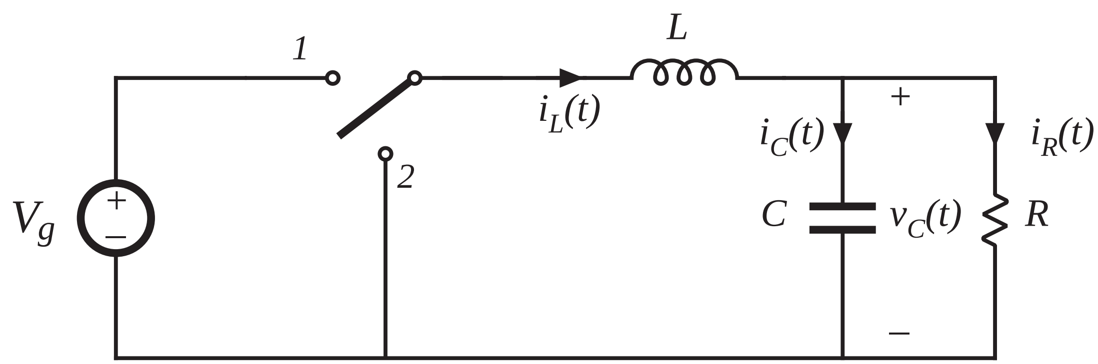
In periodic steady state, the net change in capacitor voltage is zero:

$$0 = \frac{1}{T_s}\int_0^{T_s} i_C(t)\,dt = \left\langle i_C \right\rangle$$

Hence, the total area (or charge) under the capacitor current waveform is zero whenever the converter operates in steady state. The average capacitor current is then zero.

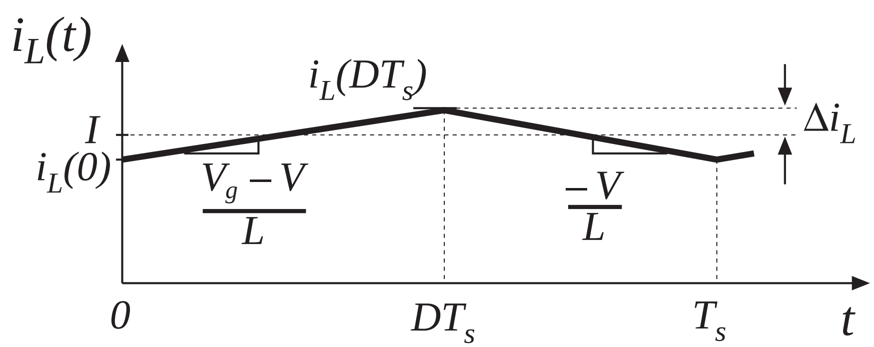
2.5 Estimating ripple in converters containing two-pole low-pass filters

Buck converter example: Determine output voltage ripple



Inductor current waveform.

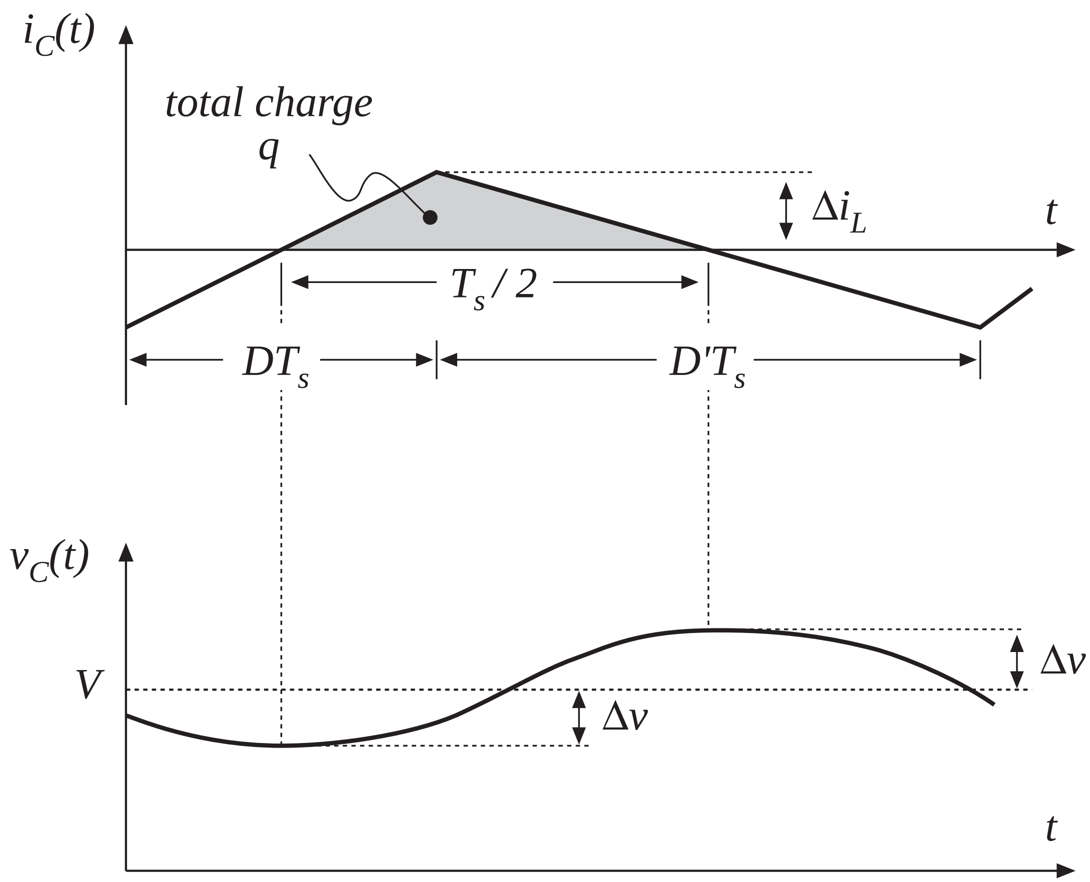
What is the capacitor current?



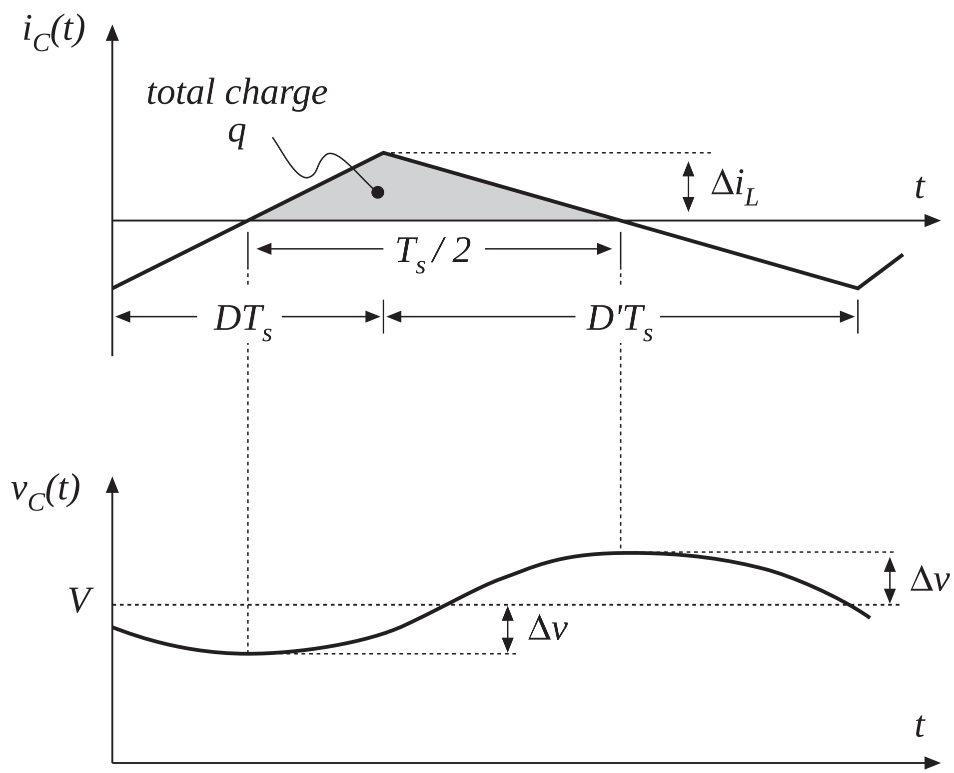
Capacitor current and voltage, buck example

Must not neglect inductor current ripple!

If the capacitor voltage ripple is small, then essentially all of the ac component of inductor current flows through the capacitor.



Estimating capacitor voltage ripple Δv



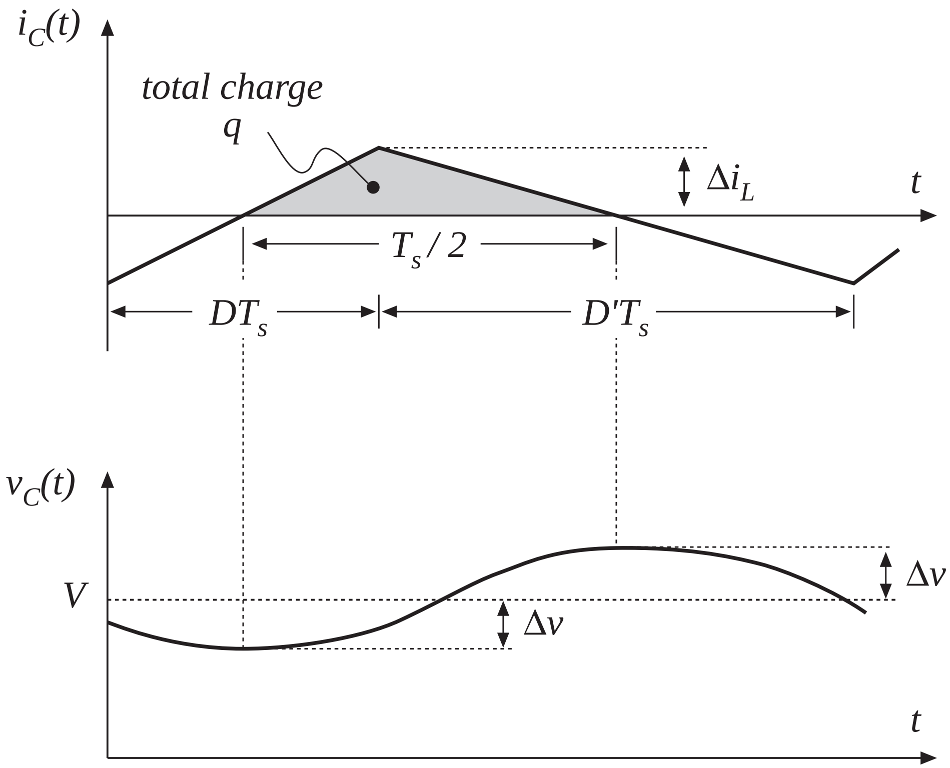
Current $i_C(t)$ is positive for half of the switching period. This positive current causes the capacitor voltage $v_C(t)$ to increase between its minimum and maximum extrema. During this time, the total charge q is deposited on the capacitor plates, where

$$q = C\,(2\Delta v)$$

$$(\textit{change in charge}) =$$

$$C\,(\textit{change in voltage})$$

Estimating capacitor voltage ripple Δv



The total charge q is the area of the triangle, as shown:

$$q = \frac{1}{2}\,\Delta i_L\,\frac{T_s}{2}$$

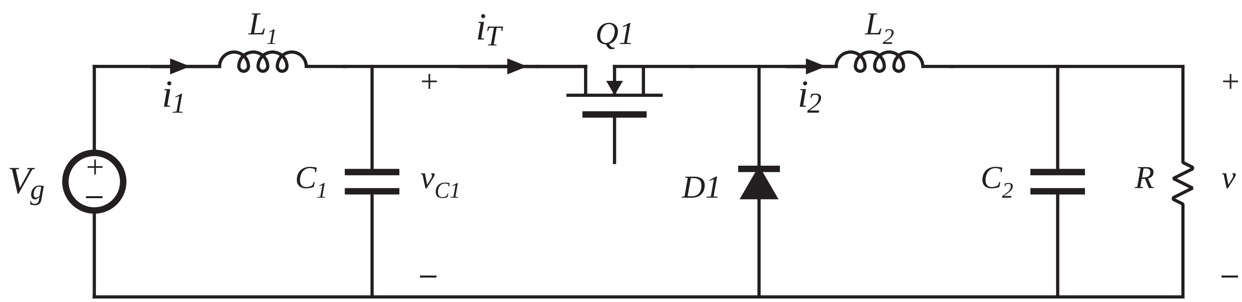
Eliminate q and solve for Δv:

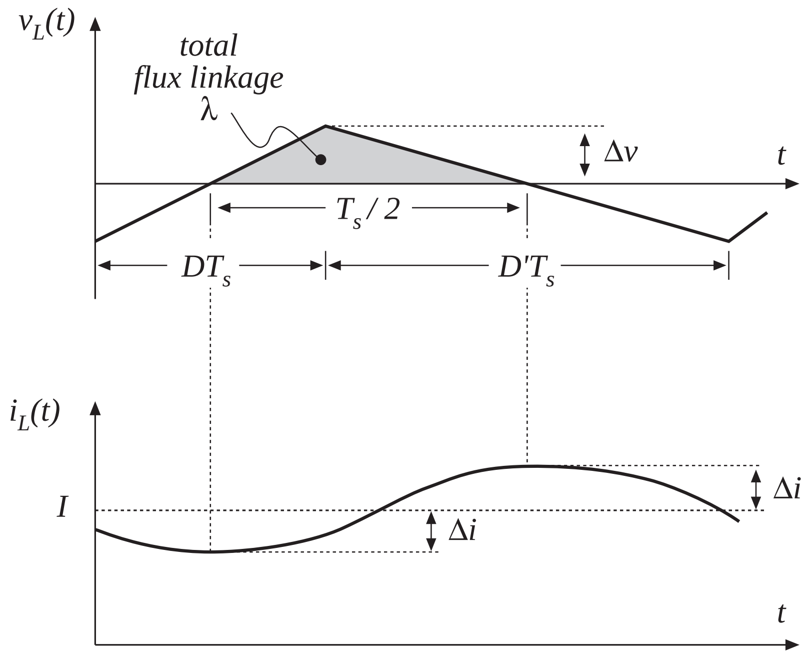
$$\Delta v = \frac{\Delta i_L\,T_s}{8\,C}$$

Note: in practice, capacitor equivalent series resistance (esr) further increases Δv.

Inductor current ripple in two-pole filters

Example:
problem 2.9





can use similar arguments, with

$$\lambda = L\,\Delta i$$

λ = inductor flux linkages

= inductor volt-seconds

Steady State Analysis and Key Concepts from Lecture 1

1. Draw equivalent circuit when switch is on and switch is off. Set up polarities for inductor voltages+ currents and capacitor currents + voltages to be consistent in both conditions
2. Write inductor voltages using KVL and capacitor currents using KCL when the switch is on and the switch is off
3. Apply small ripple approximation
    a. Capacitor voltages can be approximated as constant DC "average" equivalents with ripple neglected $v_c(t) \approx V_c$ recall average relationship $\mathrm{V}_c = \langle V_c \rangle = \frac{1}{T_s}\int_0^{T_s} v_c(t)dt$
    b. Inductor currents can be approximated as constant DC "average" equivalents with ripple neglected $i_L(t) \approx \mathrm{I}_L$ recall average relationship $\mathrm{I}_L = \langle \mathrm{I}_L \rangle = \frac{1}{T_s}\int_0^{T_s} i_L(t)dt$
4. Draw waveforms of inductor voltage and capacitor currents "should be square waves"
5. Apply volt second balance to derive voltage relationships as a function of duty. (integrating square waves)

$$0 = \frac{1}{T_s}\int_0^{T_s} v_L(t)dt = \langle v_L \rangle$$

6. Apply capacitor charge balance to derive current relationships as a function of duty. (integrating square waves)

$$0 = \frac{1}{T_s}\int_0^{T_s} i_c(t)dt = \langle i_c \rangle$$

7. Draw cap voltages and inductor currents (triangle waves) labeling the slopes during the switch on time and switch off time. Use slopes to determine capacitor voltage ripple as a function of C and inductor current ripple as a function of L.
8. If you run into a situation where the voltage ripple across the capacitor or current ripple through an inductor is predicted to be zero, since the quantity during the switch on time equals the quantity during the switch off time (typically double pole filters which are always connected- revisit Cuk example!!) ,you will need to apply charge or flux linkage to determine voltage or current ripple (see below) ** Do not confuse with small ripple approximation. Small ripple approximation does not state the actual capacitor voltage or inductor current ripple is zero!!

For voltage ripple prediction

$$q = C(2\Delta v)$$

$$\frac{dq}{dt} = i \Rightarrow \int idt = q$$

$$\int idt = C(2\Delta v) = \frac{1}{2}\Delta i_L \frac{T_s}{2}$$

For current ripple prediction

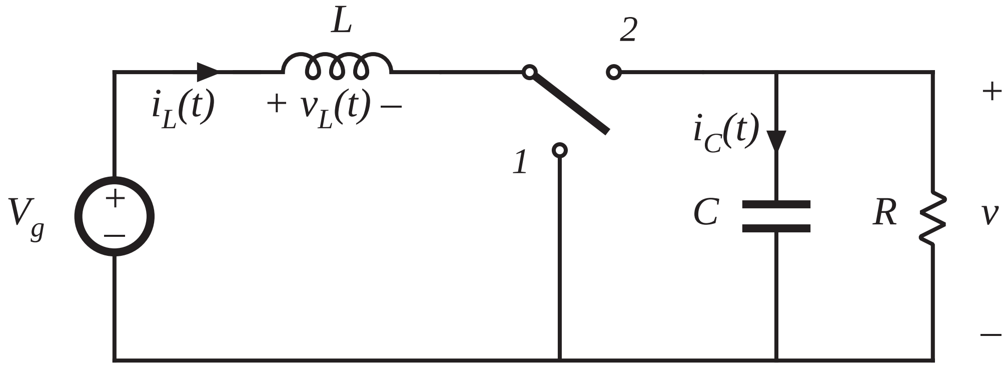
$$\lambda = L(2\Delta i)$$

$$\frac{d\lambda}{dt} = v \Rightarrow \int vdt = \lambda$$

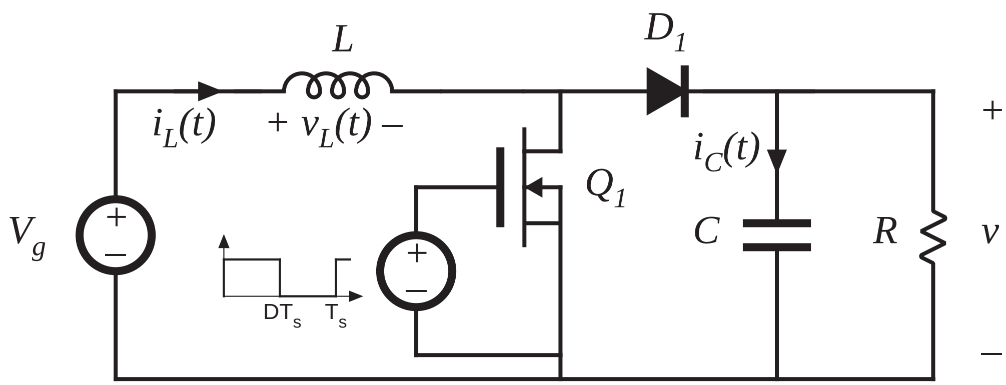
$$\int vdt = L(2\Delta i) = \frac{1}{2}\Delta v \frac{T_s}{2}$$

2.3 Boost converter example

Boost converter with ideal switch

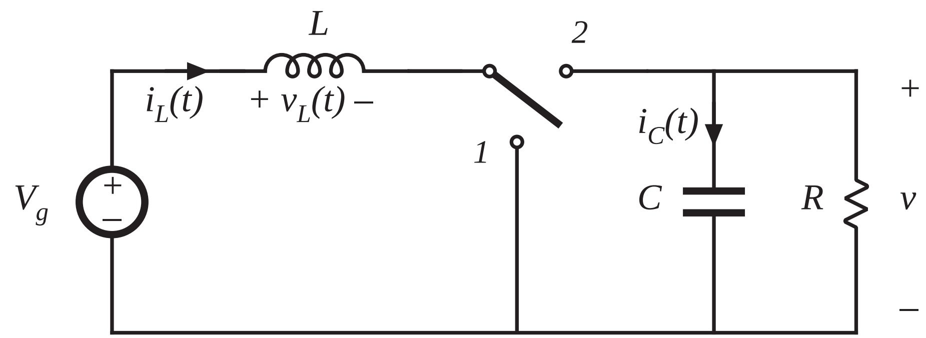


Realization using power MOSFET and diode



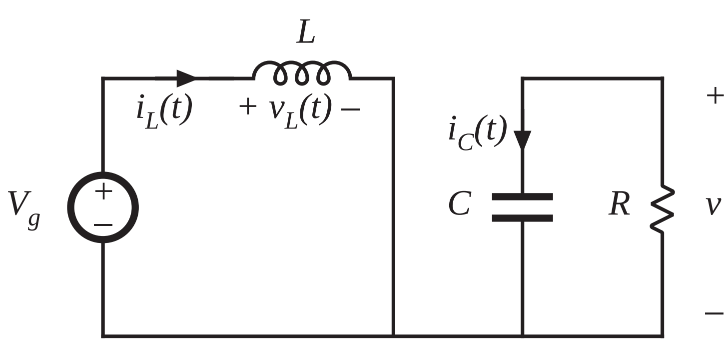
Boost converter analysis

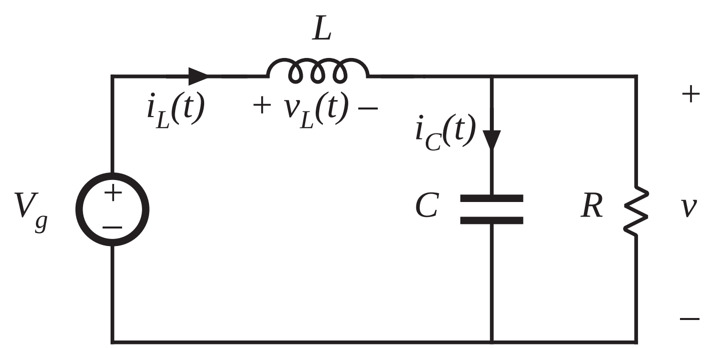
original converter



switch in position 1

switch in position 2

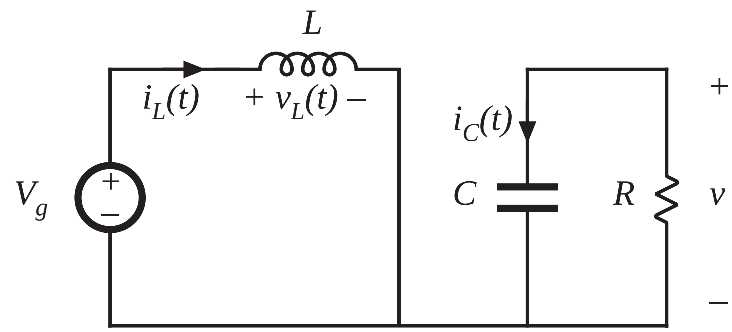




Subinterval 1: switch in position 1

Inductor voltage and capacitor current

$$v_L = V_g$$
$$i_C = -\,v\,/\,R$$



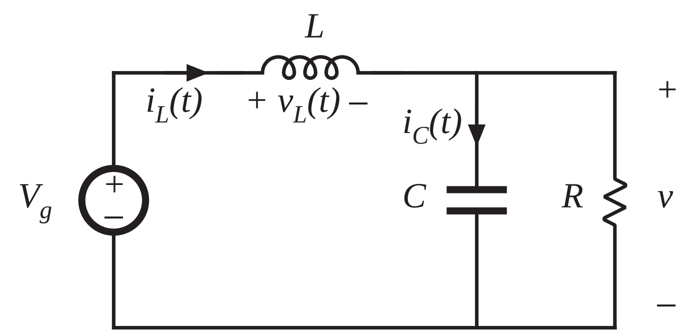
Small ripple approximation:

$$v_L = V_g$$
$$i_C = -\,V\,/\,R$$

Subinterval 2: switch in position 2

Inductor voltage and capacitor current

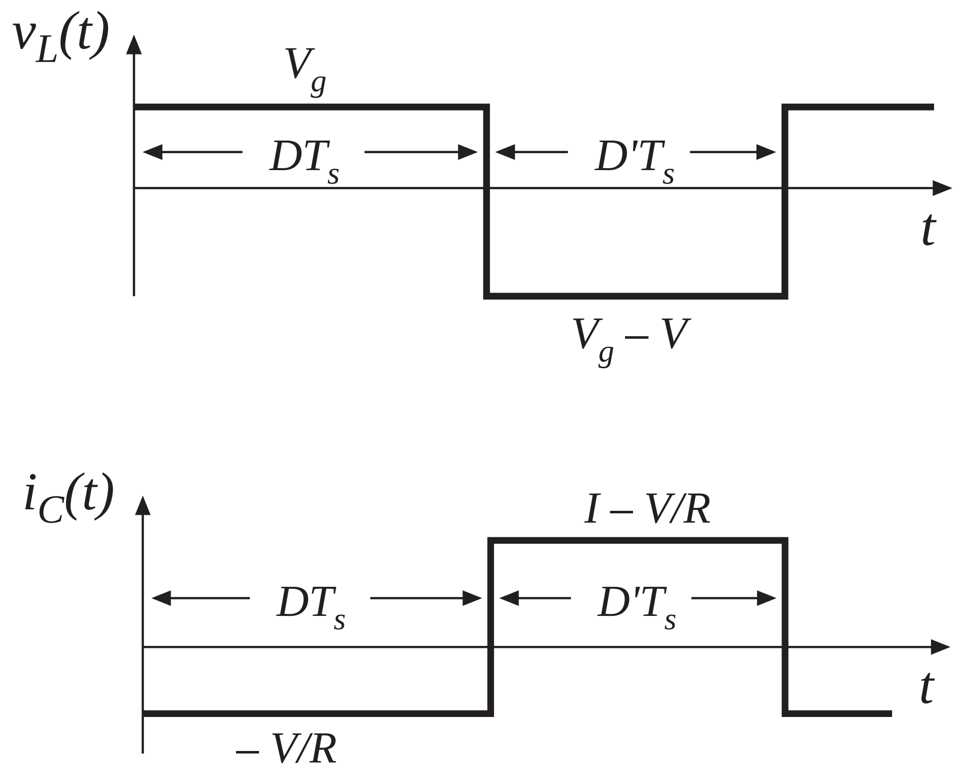
$$v_L = V_g - v$$
$$i_C = i_L - v / R$$



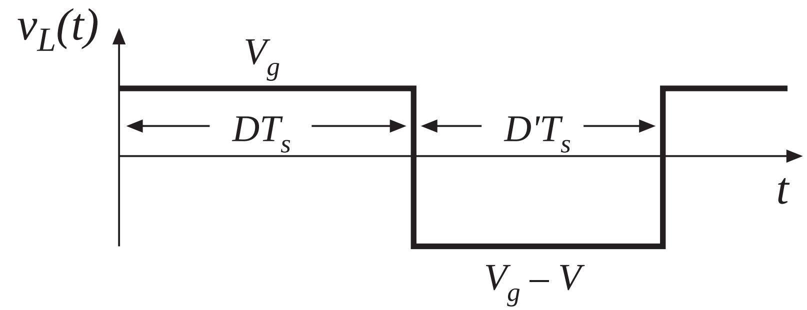
Small ripple approximation:

$$v_L = V_g - V$$
$$i_C = I - V / R$$

Inductor voltage and capacitor current waveforms



Inductor volt-second balance



Net volt-seconds applied to inductor over one switching period:

$$\int_0^{T_s} v_L(t)\,dt = (V_g)\,DT_s + (V_g - V)\,D'T_s$$

Equate to zero and collect terms:

$$V_g\,(D + D') - V\,D' = 0$$

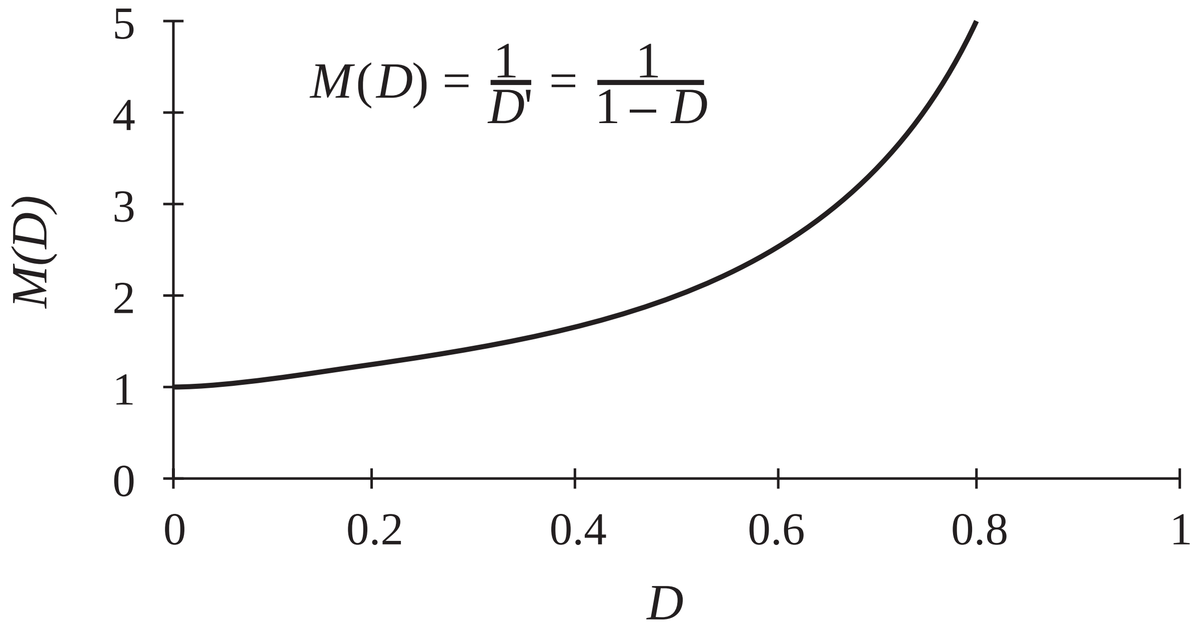
Solve for V:

$$V = \frac{V_g}{D'}$$

The voltage conversion ratio is therefore

$$M(D) = \frac{V}{V_g} = \frac{1}{D'} = \frac{1}{1 - D}$$

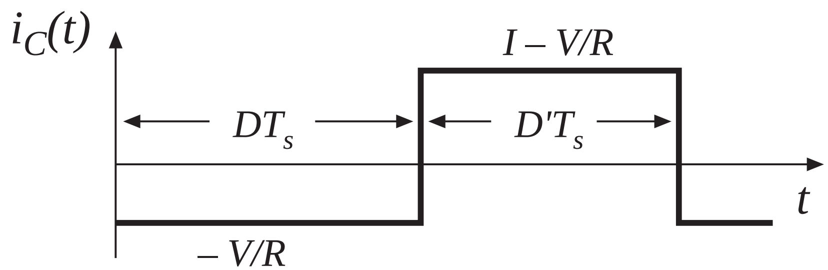
Conversion ratio *M(D)* of the boost converter



Determination of inductor current dc component

Capacitor charge balance:

$$\int_0^{T_s} i_C(t)\,dt = \left(-\frac{V}{R}\right) DT_s + \left(I - \frac{V}{R}\right) D'T_s$$



Collect terms and equate to zero:

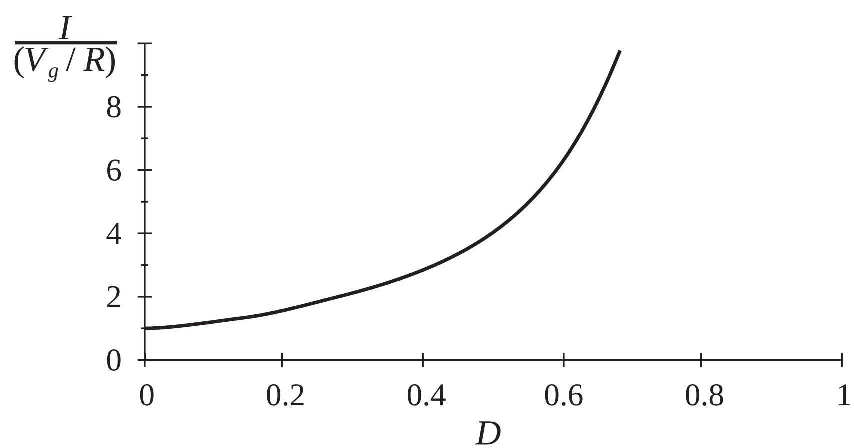
$$-\frac{V}{R}(D + D') + I\,D' = 0$$

Solve for *I*:

$$I = \frac{V}{D'\,R}$$

Eliminate *V* to express in terms of V_g:

$$I = \frac{V_g}{D'^2\,R}$$



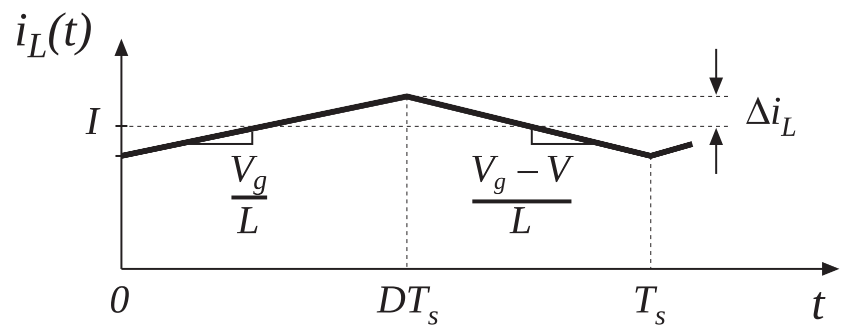
Determination of inductor current ripple

Inductor current slope during subinterval 1:

$$\frac{di_L(t)}{dt} = \frac{v_L(t)}{L} = \frac{V_g}{L}$$

Inductor current slope during subinterval 2:

$$\frac{di_L(t)}{dt} = \frac{v_L(t)}{L} = \frac{V_g - V}{L}$$



Change in inductor current during subinterval 1 is *(slope) (length of subinterval):*

$$2\Delta i_L = \frac{V_g}{L} DT_s$$

Solve for peak ripple:

$$\Delta i_L = \frac{V_g}{2L} DT_s$$

- Choose *L* such that desired ripple magnitude is obtained

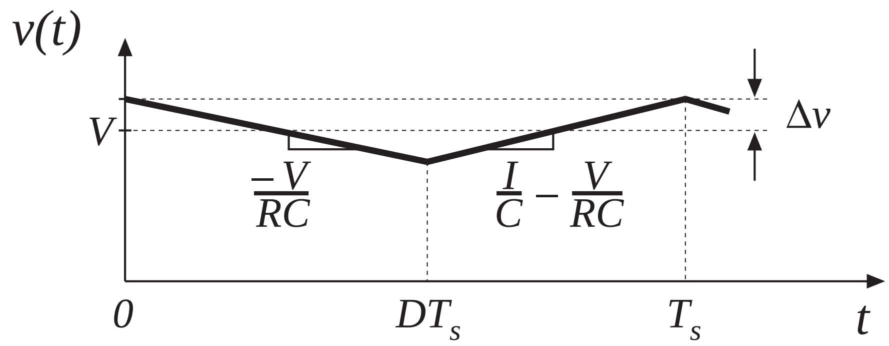
Determination of capacitor voltage ripple

Capacitor voltage slope during subinterval 1:

$$\frac{dv_C(t)}{dt} = \frac{i_C(t)}{C} = \frac{-V}{RC}$$

Capacitor voltage slope during subinterval 2:

$$\frac{dv_C(t)}{dt} = \frac{i_C(t)}{C} = \frac{I}{C} - \frac{V}{RC}$$



Change in capacitor voltage during subinterval 1 is *(slope) (length of subinterval):*

$$-2\Delta v = \frac{-V}{RC}\, DT_s$$

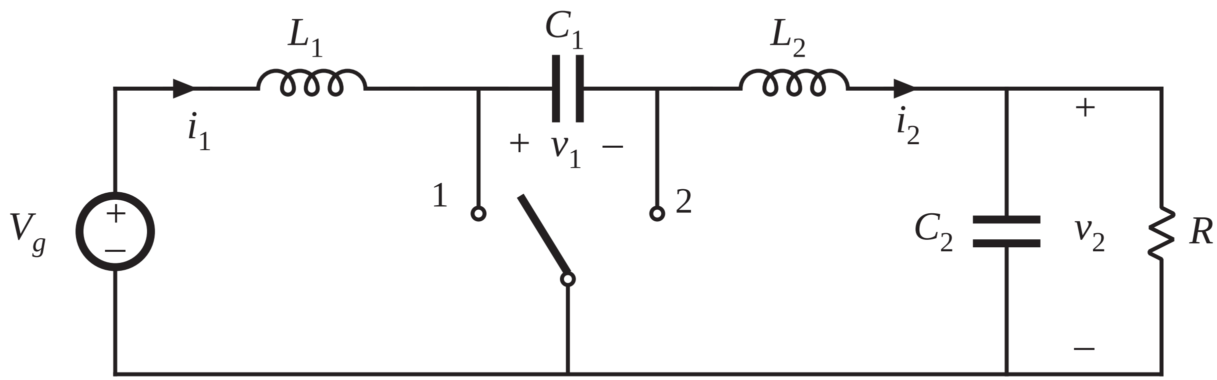
Solve for peak ripple:

$$\Delta v = \frac{V}{2RC}\, DT_s$$

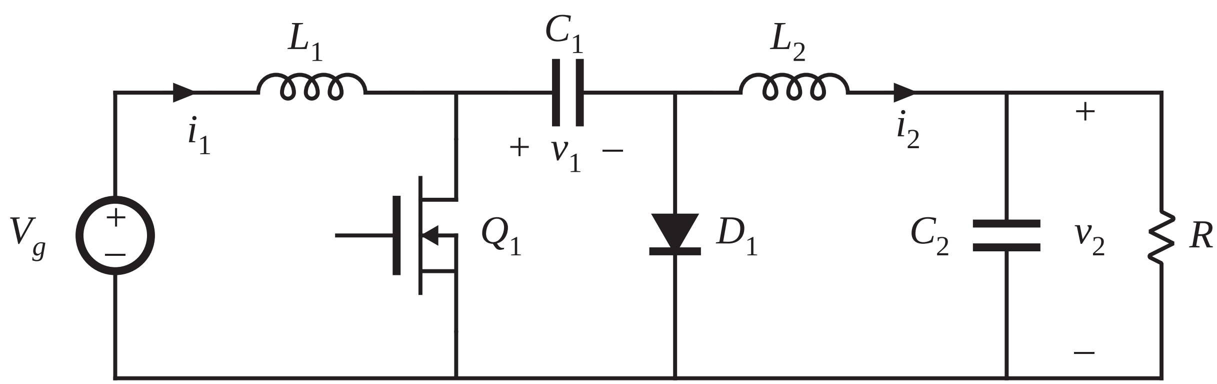
- Choose *C* such that desired voltage ripple magnitude is obtained
- In practice, capacitor *equivalent series resistance* (esr) leads to increased voltage ripple

2.4 Cuk converter example

Cuk converter, with ideal switch



Cuk converter: practical realization using MOSFET and diode

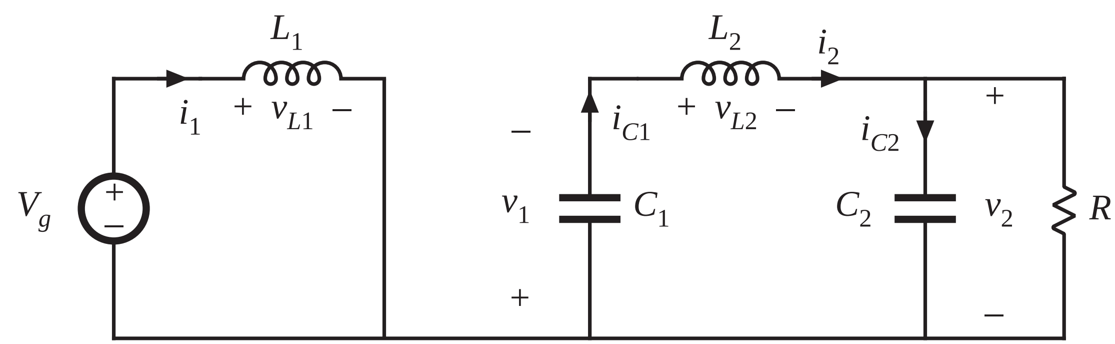


Cuk converter circuit

with switch in positions 1 and 2

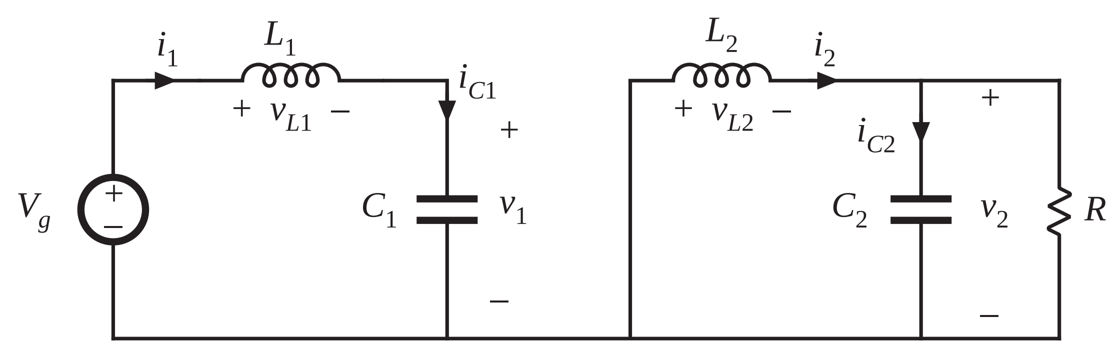
Switch in position 1: MOSFET conducts

Capacitor C_1 releases energy to output



Switch in position 2: diode conducts

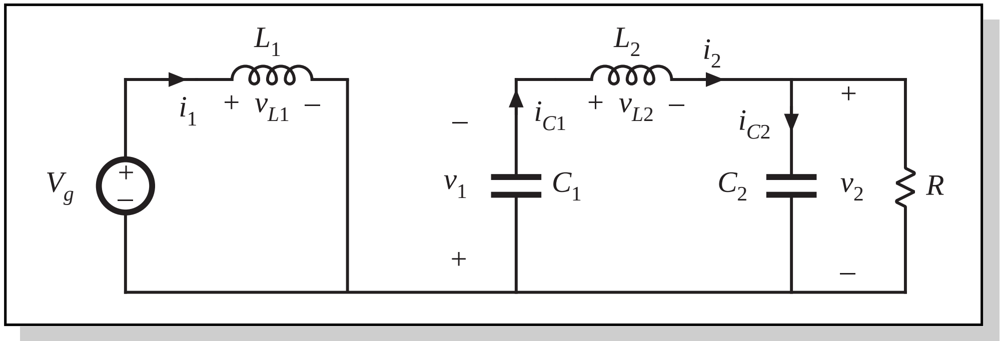
Capacitor C_1 is charged from input



Waveforms during subinterval 1
MOSFET conduction interval

Inductor voltages and capacitor currents:

$$v_{L1} = V_g$$
$$v_{L2} = -v_1 - v_2$$
$$i_{C1} = i_2$$
$$i_{C2} = i_2 - \frac{v_2}{R}$$



Small ripple approximation for subinterval 1:

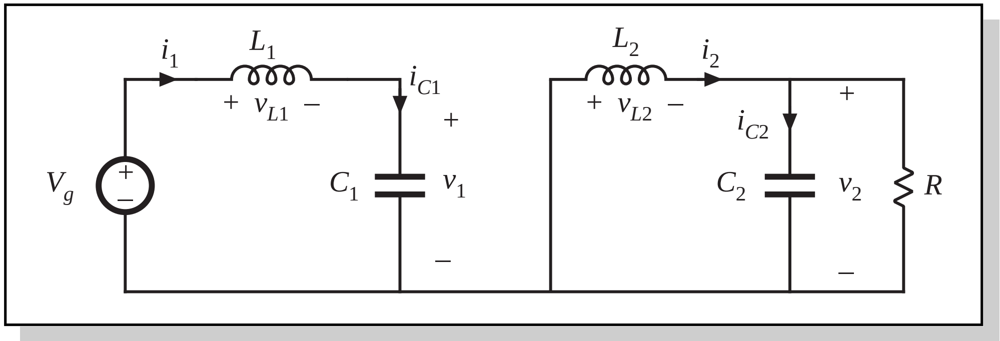
$$v_{L1} = V_g$$
$$v_{L2} = -V_1 - V_2$$
$$i_{C1} = I_2$$
$$i_{C2} = I_2 - \frac{V_2}{R}$$

Waveforms during subinterval 2

Diode conduction interval

Inductor voltages and capacitor currents:

$$v_{L1} = V_g - v_1$$
$$v_{L2} = - v_2$$
$$i_{C1} = i_1$$
$$i_{C2} = i_2 - \frac{v_2}{R}$$



Small ripple approximation for subinterval 2:

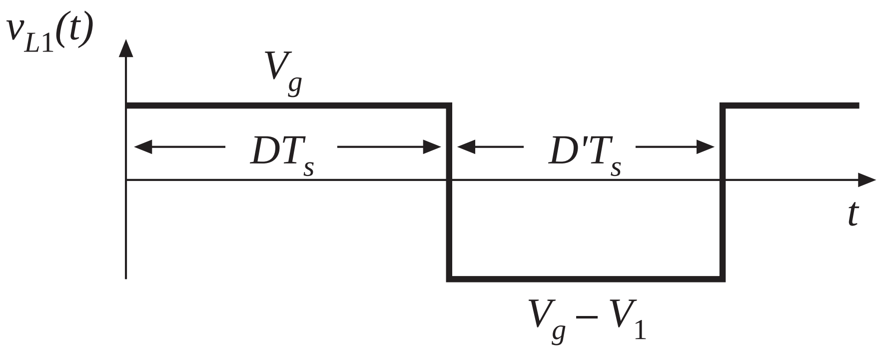
$$v_{L1} = V_g - V_1$$
$$v_{L2} = - V_2$$
$$i_{C1} = I_1$$
$$i_{C2} = I_2 - \frac{V_2}{R}$$

Equate average values to zero

The principles of inductor volt-second and capacitor charge balance state that the average values of the periodic inductor voltage and capacitor current waveforms are zero, when the converter operates in steady state. Hence, to determine the steady-state conditions in the converter, let us sketch the inductor voltage and capacitor current waveforms, and equate their average values to zero.

Waveforms:

Inductor voltage $v_{L1}(t)$

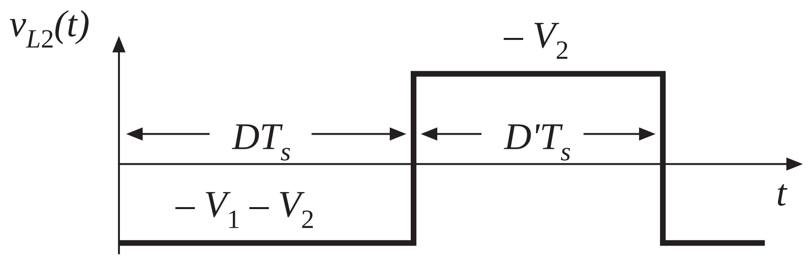


Volt-second balance on L_1:

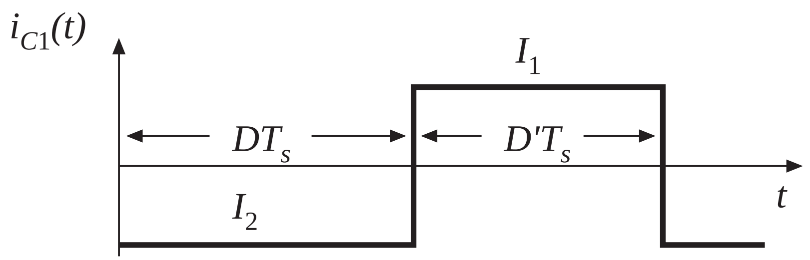
$$\langle v_{L1} \rangle = DV_g + D'(V_g - V_1) = 0$$

Equate average values to zero

Inductor L_2 *voltage*



Capacitor C_1 *current*



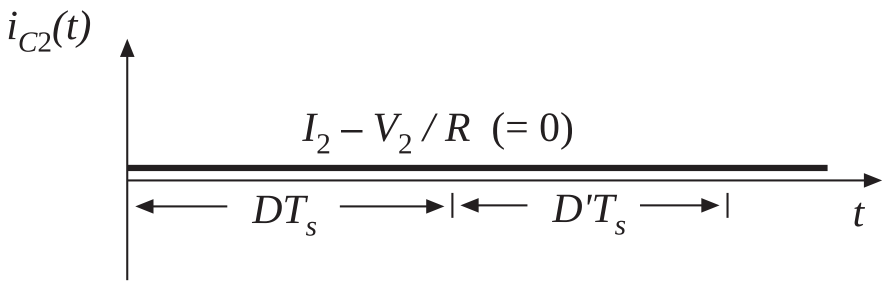
Average the waveforms:

$$\left\langle v_{L2} \right\rangle = D(-V_1 - V_2) + D'(-V_2) = 0$$

$$\left\langle i_{C1} \right\rangle = DI_2 + D'I_1 = 0$$

Equate average values to zero

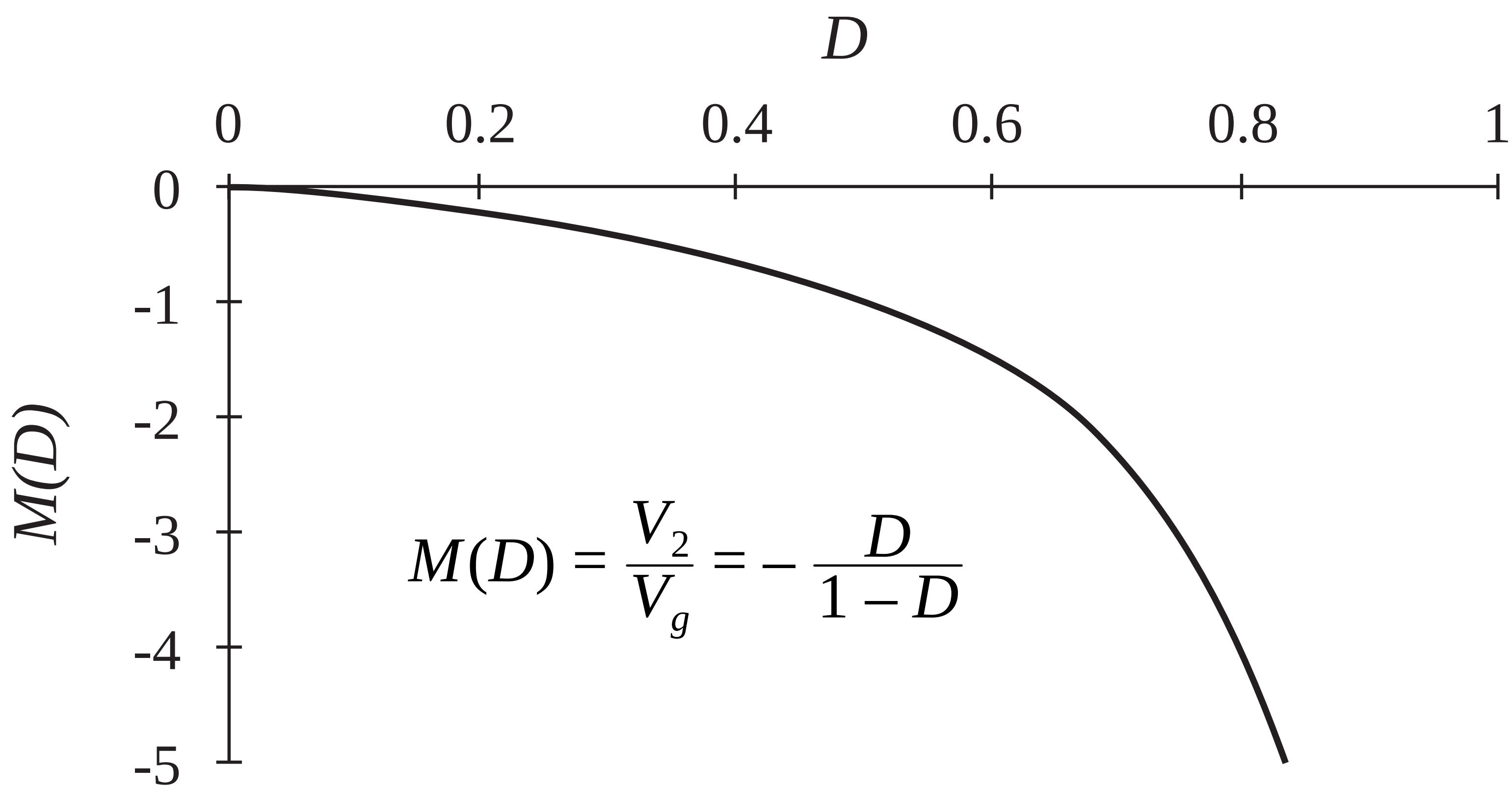
Capacitor current $i_{C2}(t)$ waveform



$$\langle i_{C2} \rangle = I_2 - \frac{V_2}{R} = 0$$

Note: during both subintervals, the capacitor current i_{C2} is equal to the difference between the inductor current i_2 and the load current V_2/R. When ripple is neglected, i_{C2} is constant and equal to zero.

Cuk converter conversion ratio $M = V / V_g$



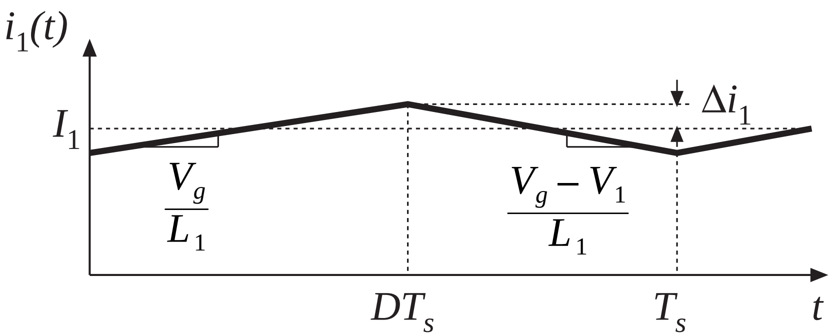
Inductor current waveforms

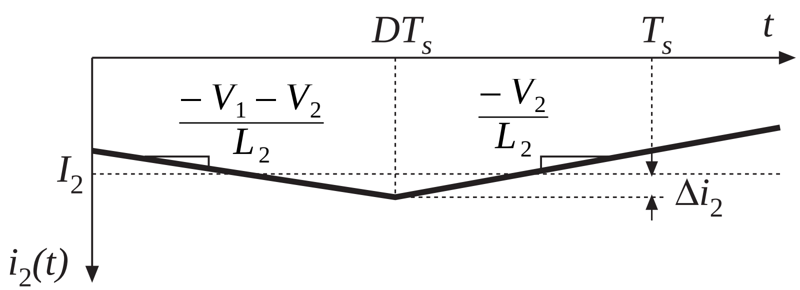
Interval 1 slopes, using small ripple approximation:

$$\frac{di_1(t)}{dt} = \frac{v_{L1}(t)}{L_1} = \frac{V_g}{L_1}$$
$$\frac{di_2(t)}{dt} = \frac{v_{L2}(t)}{L_2} = \frac{-V_1 - V_2}{L_2}$$

Interval 2 slopes:

$$\frac{di_1(t)}{dt} = \frac{v_{L1}(t)}{L_1} = \frac{V_g - V_1}{L_1}$$
$$\frac{di_2(t)}{dt} = \frac{v_{L2}(t)}{L_2} = \frac{-V_2}{L_2}$$





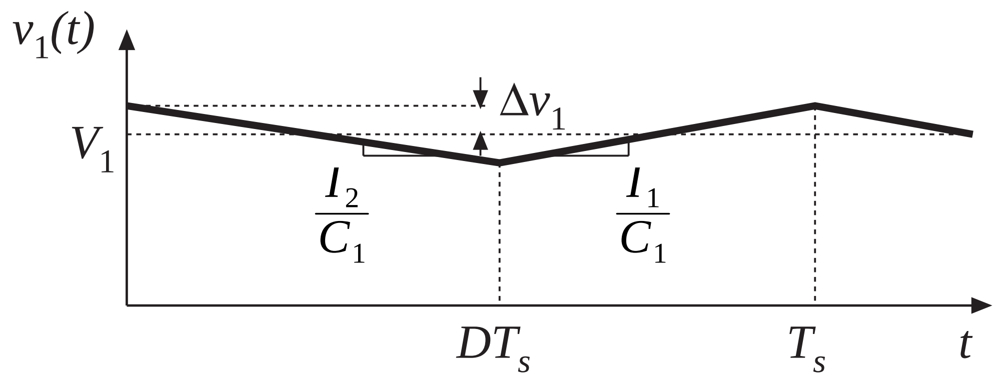
Capacitor C_1 waveform

Subinterval 1:

$$\frac{dv_1(t)}{dt} = \frac{i_{C1}(t)}{C_1} = \frac{I_2}{C_1}$$

Subinterval 2:

$$\frac{dv_1(t)}{dt} = \frac{i_{C1}(t)}{C_1} = \frac{I_1}{C_1}$$



Ripple magnitudes

Analysis results

$$\Delta i_1 = \frac{V_g D T_s}{2L_1}$$

$$\Delta i_2 = \frac{V_1 + V_2}{2L_2} D T_s$$

$$\Delta v_1 = \frac{-I_2 D T_s}{2C_1}$$

Use dc converter solution to simplify:

$$\Delta i_1 = \frac{V_g D T_s}{2L_1}$$

$$\Delta i_2 = \frac{V_g D T_s}{2L_2}$$

$$\Delta v_1 = \frac{V_g D^2 T_s}{2D'RC_1}$$

Q: How large is the output voltage ripple?