Johns Hopkins Engineering

# Power Electronics 525.725

Module 3 Lecture 3a

Discontinuous Conduction Mode (DCM)

# Introduction to Discontinuous Conduction Mode (DCM)

- Occurs because switching ripple in inductor current or capacitor voltage causes polarity of applied switch current or voltage to reverse, such that the current- or voltage-unidirectional assumptions made in realizing the switch are violated.
- Commonly occurs in dc-dc converters and rectifiers, having single-quadrant switches. May also occur in converters having two-quadrant switches.
- Typical example: dc-dc converter operating at light load (small load current). Sometimes, dc-dc converters and rectifiers are purposely designed to operate in DCM at all loads.
- Properties of converters change radically when DCM is entered:

  $M$ becomes load-dependent

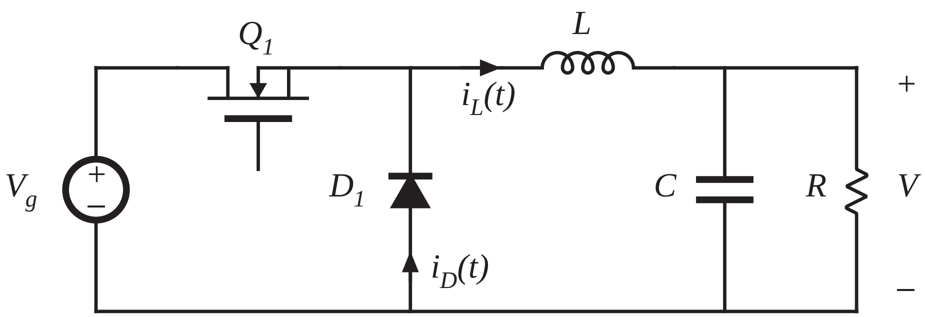
  Output impedance is increased

  Dynamics are altered

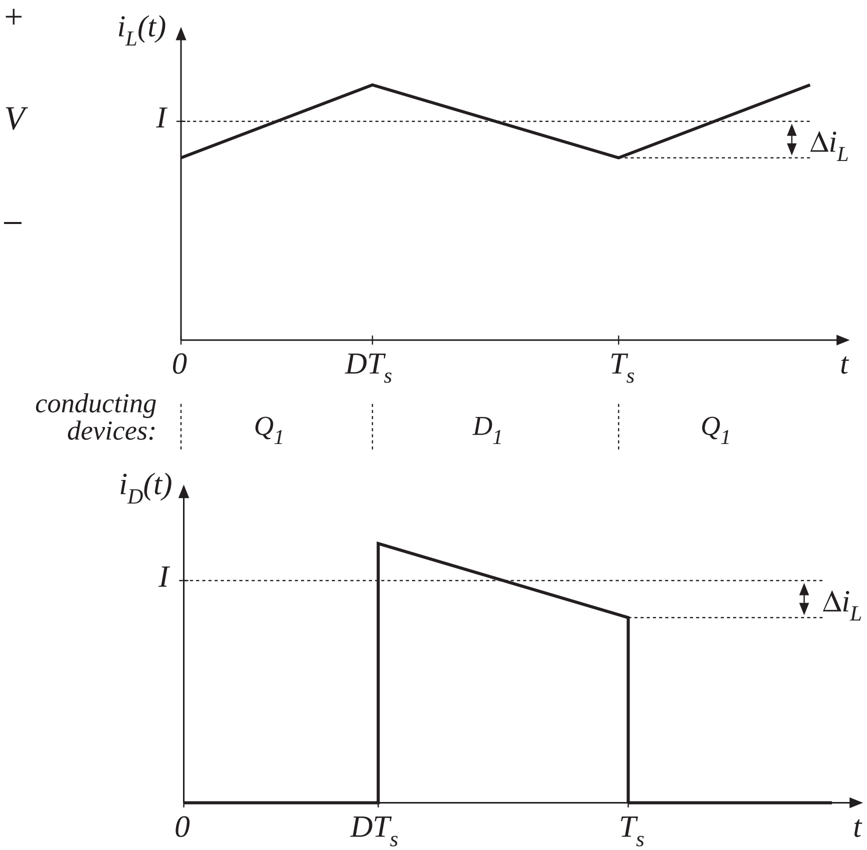
  Control of output voltage may be lost when load is removed

# 5.1. Origin of the discontinuous conduction mode, and mode boundary

Buck converter example, with single-quadrant switches




*continuous conduction mode (CCM)*



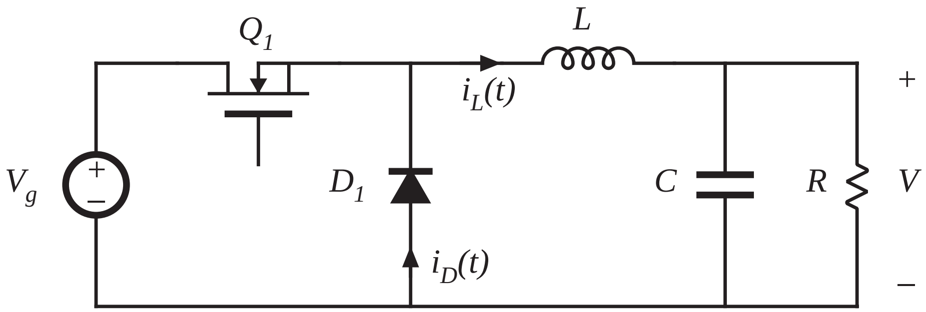

Minimum diode current is $(I - \Delta i_L)$
Dc component $I = V/R$
Current ripple is

$$\Delta i_L = \frac{(V_g - V)}{2L} DT_s = \frac{V_g DD'T_s}{2L}$$

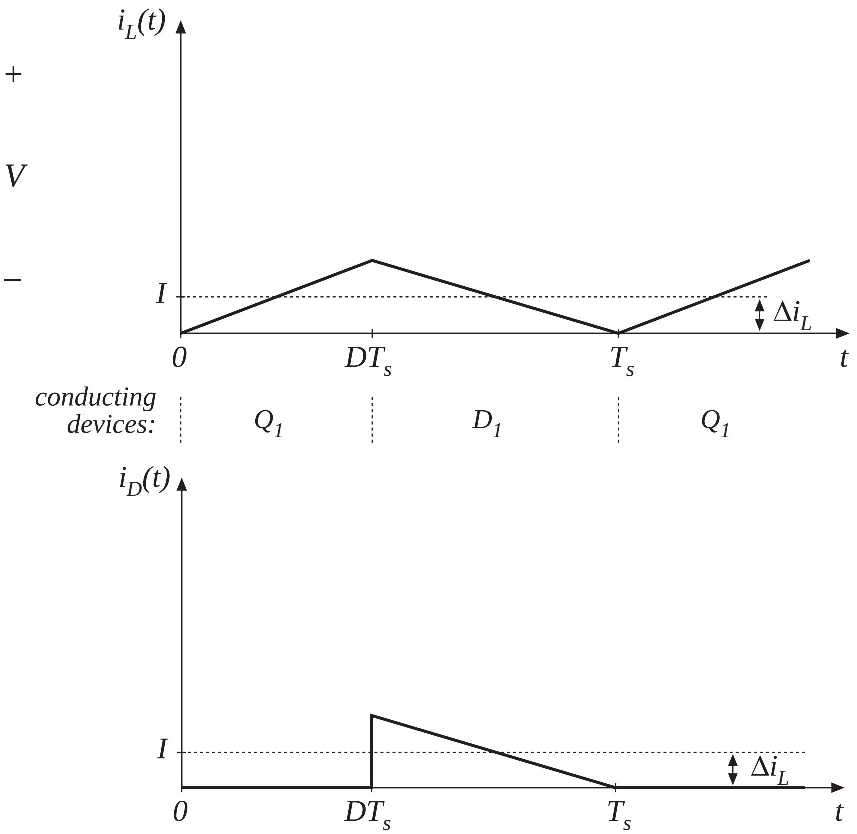
Note that $I$ depends on load, but $\Delta i_L$ does not.

# Reduction of load current

Increase $R$, until $I = \Delta i_L$




*CCM-DCM boundary*




Minimum diode current is $(I - \Delta i_L)$

Dc component $I = V/R$

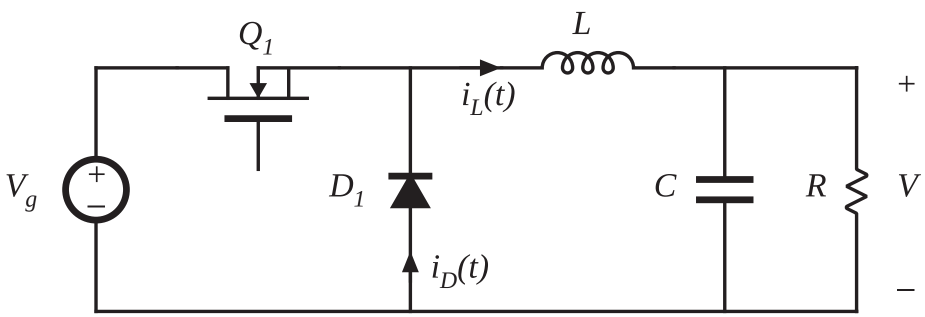
Current ripple is

$$\Delta i_L = \frac{(V_g - V)}{2L} DT_s = \frac{V_g DD'T_s}{2L}$$

Note that $I$ depends on load, but $\Delta i_L$ does not.

# Further reduce load current

Increase $R$ some more, such that $I < \Delta i_L$

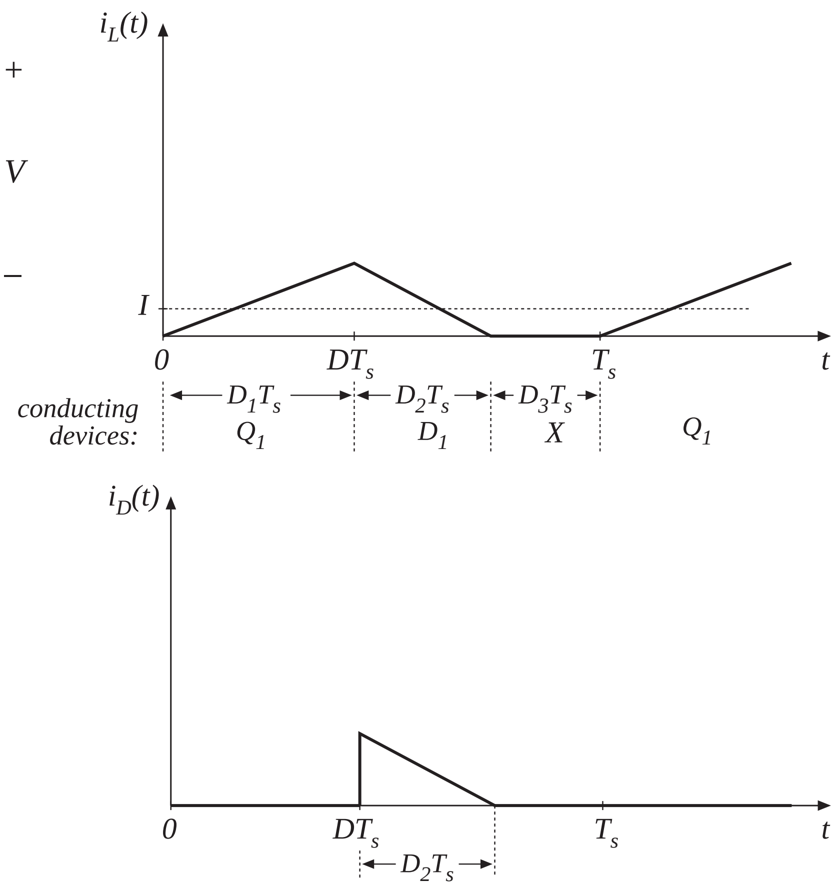



Minimum diode current is $(I - \Delta i_L)$
Dc component $I = V/R$
Current ripple is

$$\Delta i_L = \frac{(V_g - V)}{2L} DT_s = \frac{V_g DD'T_s}{2L}$$

Note that $I$ depends on load, but $\Delta i_L$ does not.
The load current continues to be positive and non-zero.

*Discontinuous conduction mode*



# Mode boundary

$$I > \Delta i_L \quad \text{for CCM}$$
$$I < \Delta i_L \quad \text{for DCM}$$

Insert buck converter expressions for $I$ and $\Delta i_L$ :

$$\frac{DV_g}{R} < \frac{DD'T_sV_g}{2L}$$

Simplify:

$$\frac{2L}{RT_s} < D'$$

This expression is of the form

$$K < K_{crit}(D) \qquad \text{for DCM}$$

$$\text{where} \quad K = \frac{2L}{RT_s} \quad \text{and} \quad K_{crit}(D) = D'$$

# Critical load resistance $R_{crit}$

Solve $K_{crit}$ equation for load resistance $R$:

$$R < R_{crit}(D) \qquad \text{for CCM}$$
$$R > R_{crit}(D) \qquad \text{for DCM}$$

where $$R_{crit}(D) = \frac{2L}{D'T_s}$$

# Summary: mode boundary

$$K > K_{crit}(D) \quad \text{or} \quad R < R_{crit}(D) \quad \text{for CCM}$$
$$K < K_{crit}(D) \quad \text{or} \quad R > R_{crit}(D) \quad \text{for DCM}$$

*Table 5.1. CCM-DCM mode boundaries for the buck, boost, and buck-boost converters*

| *Converter* | $K_{crit}(D)$ | $\max_{0 \le D \le 1}(K_{crit})$ | $R_{crit}(D)$ | $\min_{0 \le D \le 1}(R_{crit})$ |
|---|---|---|---|---|
| Buck | $(1 - D)$ | $1$ | $\dfrac{2L}{(1-D)T_s}$ | $2\dfrac{L}{T_s}$ |
| Boost | $D(1 - D)^2$ | $\dfrac{4}{27}$ | $\dfrac{2L}{D(1-D)^2 T_s}$ | $\dfrac{27}{2}\dfrac{L}{T_s}$ |
| Buck-boost | $(1 - D)^2$ | $1$ | $\dfrac{2L}{(1-D)^2 T_s}$ | $2\dfrac{L}{T_s}$ |

# 5.2. Analysis of the conversion ratio *M(D,K)*

Analysis techniques for the discontinuous conduction mode:

Inductor volt-second balance

$$\left\langle v_L \right\rangle = \frac{1}{T_s} \int_0^{T_s} v_L(t)\, dt = 0$$

Capacitor charge balance

$$\left\langle i_C \right\rangle = \frac{1}{T_s} \int_0^{T_s} i_C(t)\, dt = 0$$

Small ripple approximation sometimes applies:

$$v(t) \approx V \quad \text{because} \quad \Delta v \ll V$$

$$i(t) \approx I \ \text{ is a poor approximation when } \ \Delta i > I$$

Converter steady-state equations obtained via charge balance on each capacitor and volt-second balance on each inductor. Use care in applying small ripple approximation.

Steady State Analysis DCM

1. Draw equivalent circuit when switch is on and switch is off and when the diode stops conducting. Set up polarities for inductor voltages+ currents and capacitor currents + voltages to be consistent in both conditions
2. Write inductor voltages using KVL and capacitor currents using KCL when the switch is on and the switch is off
3. Apply small ripple approximation
    a. Capacitor voltages can be approximated as constant DC "average" equivalents with ripple neglected $v_c(t) \approx V_c$ recall average relationship $\mathrm{V}_c = \langle V_c \rangle = \frac{1}{T_s}\int_0^{T_s} v_c(t)dt$
    b. Inductor currents CANNOT be approximated as constant DC "average" equivalents with ripple neglected $i_L(t) \neq \mathrm{I}_L$
4. Draw waveforms of inductor voltage "should be a square wave"
5. Apply volt second balance to derive voltage relationships as a function of D1 and D2. (Integrating square waves). D2 is unknown

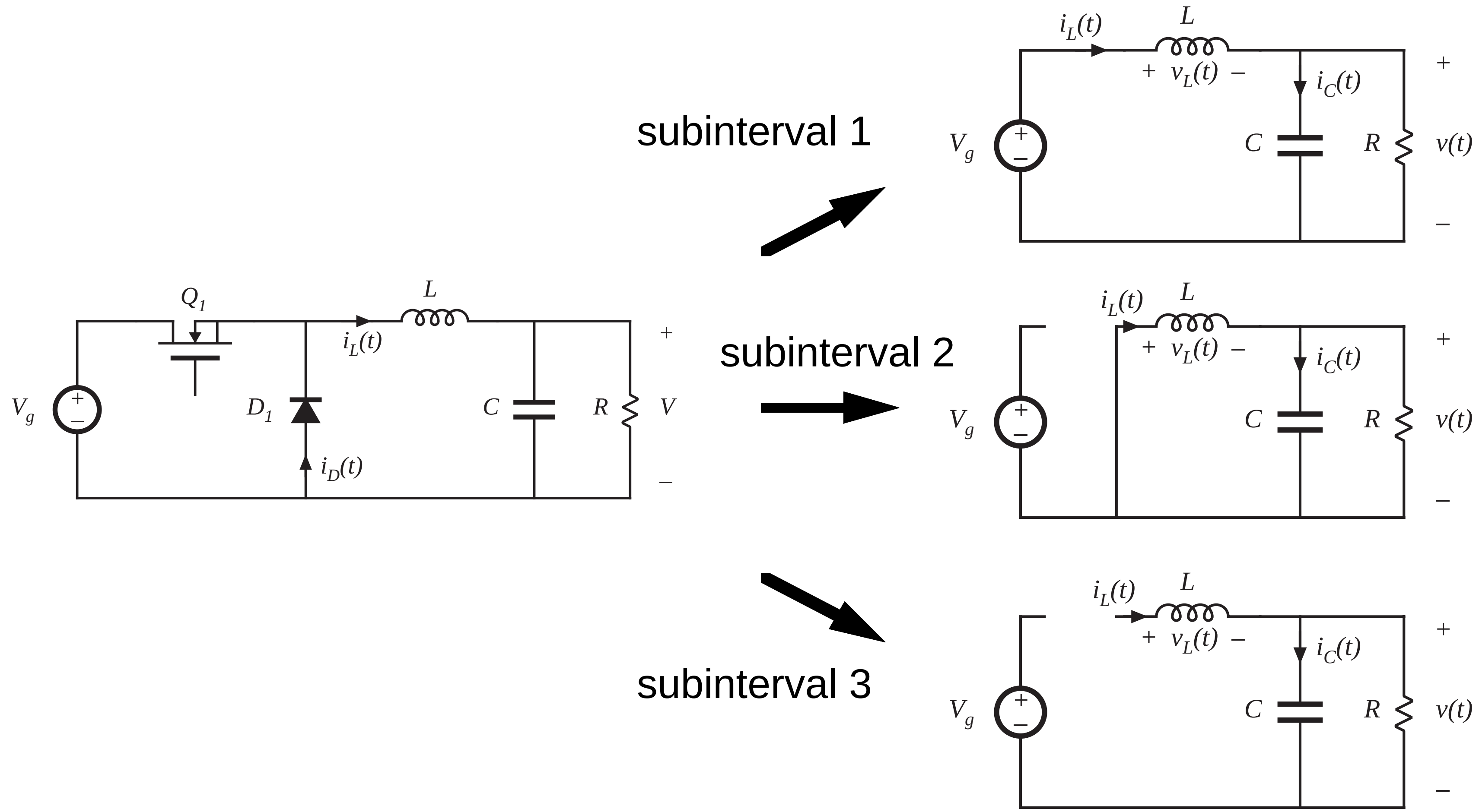
$$0 = \frac{1}{T_s}\int_0^{T_s} v_L(t)dt = \langle v_L \rangle$$

6. Write a current nodal equation at capacitor terminal.
7. Apply capacitor charge balance to derive a current relationship as a function of load current. (may be the average diode current or inductor current depending on topology)

$$0 = \frac{1}{T_s}\int_0^{T_s} i_c(t)dt = \langle i_c \rangle$$

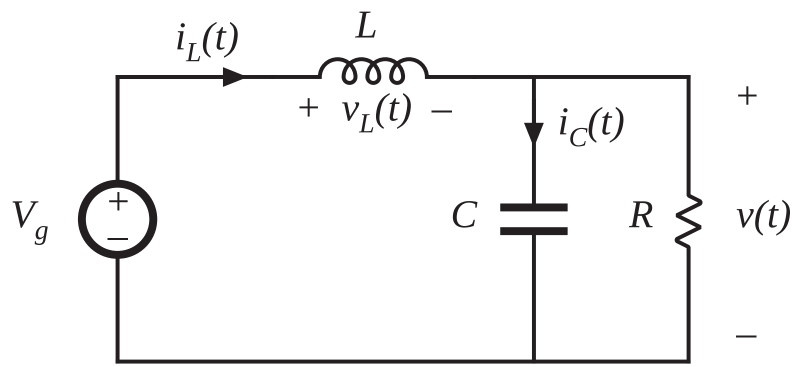
8. Sketch the current resulting from the capacitor charge balance and equate this current to the average output current which can be computed from the area of a triangular current waveform in DCM. The resulting expression should be a function of D1 and D2.
9. Solve two equations and two unknowns. One equation resulting from the volt second balance and the other from equating the diode or inductor current to the output load current. The two unknowns should be the output voltage and D2.

# Example: Analysis of DCM buck converter *M(D,K)*



# Subinterval 1

$$v_L(t) = V_g - v(t)$$
$$i_C(t) = i_L(t) - v(t) / R$$

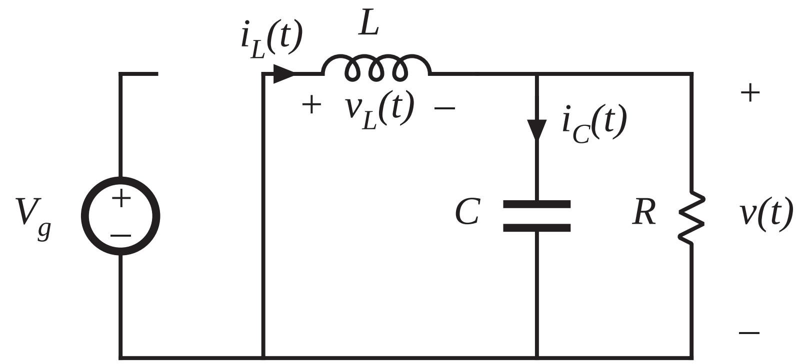



Small ripple approximation for *v(t)* (but not for *i(t)*!):

$$v_L(t) \approx V_g - V$$
$$i_C(t) \approx i_L(t) - V / R$$

# Subinterval 2

$$v_L(t) = -\,v(t)$$
$$i_C(t) = i_L(t) - v(t)\,/\,R$$




Small ripple approximation for *v(t)* but not for *i(t)*:

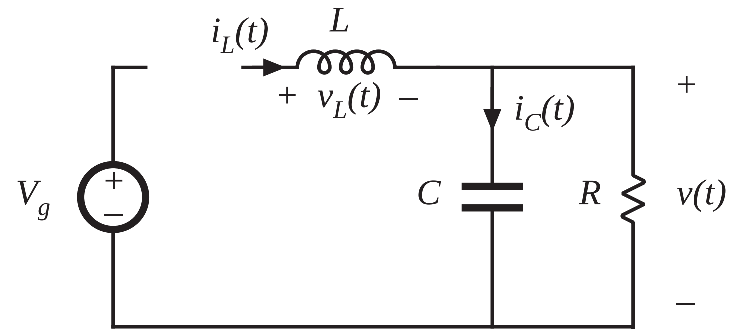
$$v_L(t) \approx -\,V$$
$$i_C(t) \approx i_L(t) - V\,/\,R$$

# Subinterval 3

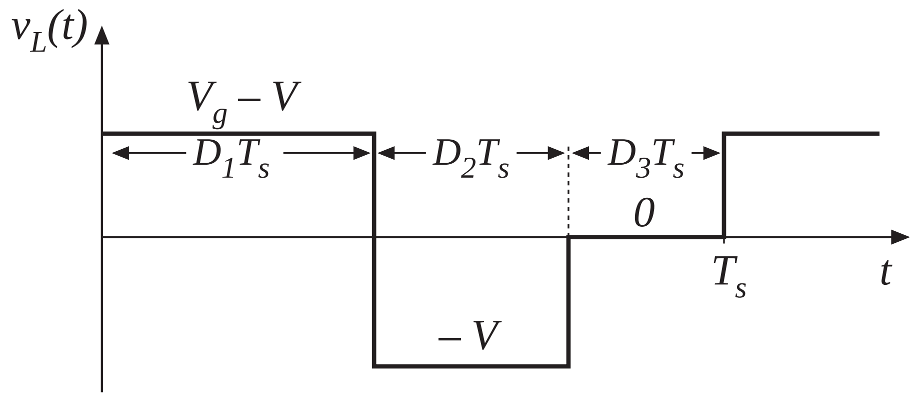
$$v_L = 0, \quad i_L = 0$$
$$i_C(t) = i_L(t) - v(t) \,/\, R$$

Small ripple approximation:

$$v_L(t) = 0$$
$$i_C(t) = -\, V \,/\, R$$



# Inductor volt-second balance




Volt-second balance:

$$\left\langle v_L(t) \right\rangle = D_1(V_g - V) + D_2(-V) + D_3(0) = 0$$

Solve for $V$:

$$V = V_g \frac{D_1}{D_1 + D_2}$$

note that $D_2$ is unknown

# Capacitor charge balance

node equation:

$$i_L(t) = i_C(t) + V / R$$

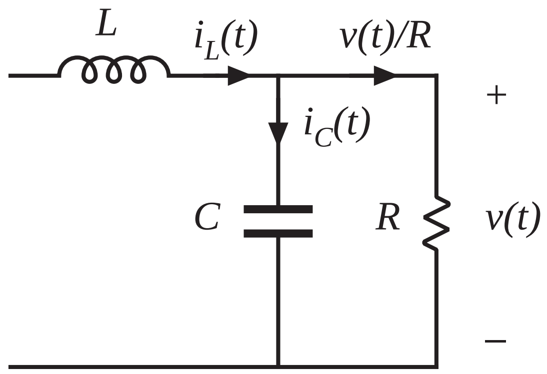
capacitor charge balance:

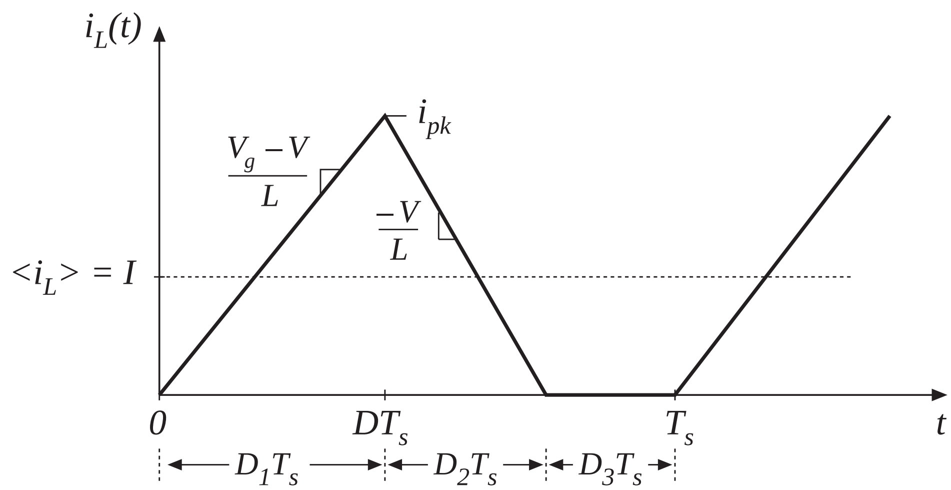
$$\langle i_C \rangle = 0$$

hence

$$\langle i_L \rangle = V / R$$

must compute dc component of inductor current and equate to load current (for this buck converter example)



# Inductor current waveform

peak current:

$$i_L(D_1T_s) = i_{pk} = \frac{V_g - V}{L}\, D_1T_s$$

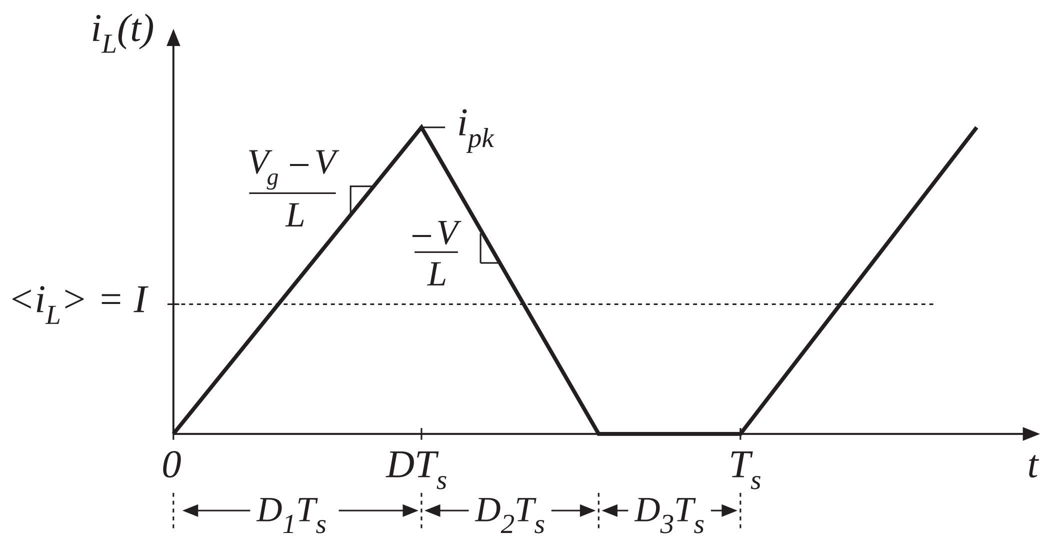
average current:

$$\left\langle i_L \right\rangle = \frac{1}{T_s}\int_0^{T_s} i_L(t)\, dt$$

triangle area formula:

$$\int_0^{T_s} i_L(t)\, dt = \frac{1}{2}\, i_{pk}\,(D_1 + D_2)T_s$$

$$\left\langle i_L \right\rangle = (V_g - V)\,\frac{D_1T_s}{2L}\,(D_1 + D_2)$$




equate dc component to dc load current:

$$\frac{V}{R} = \frac{D_1T_s}{2L}\,(D_1 + D_2)\,(V_g - V)$$

# Solution for $V$

Two equations and two unknowns ($V$ and $D_2$):

$$V = V_g \frac{D_1}{D_1 + D_2}$$ (from inductor volt-second balance)

$$\frac{V}{R} = \frac{D_1 T_s}{2L} (D_1 + D_2)(V_g - V)$$ (from capacitor charge balance)

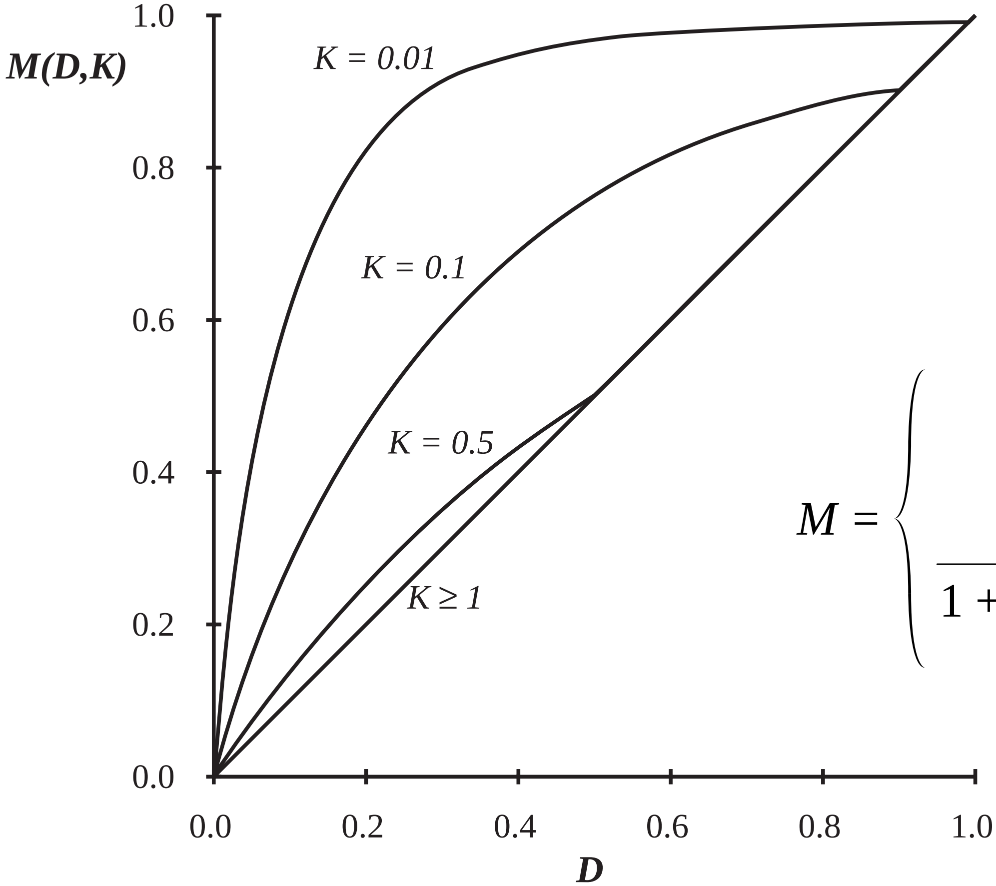
Eliminate $D_2$ , solve for $V$ :

$$\frac{V}{V_g} = \frac{2}{1 + \sqrt{1 + 4K / D_1^2}}$$

where $K = 2L / RT_s$

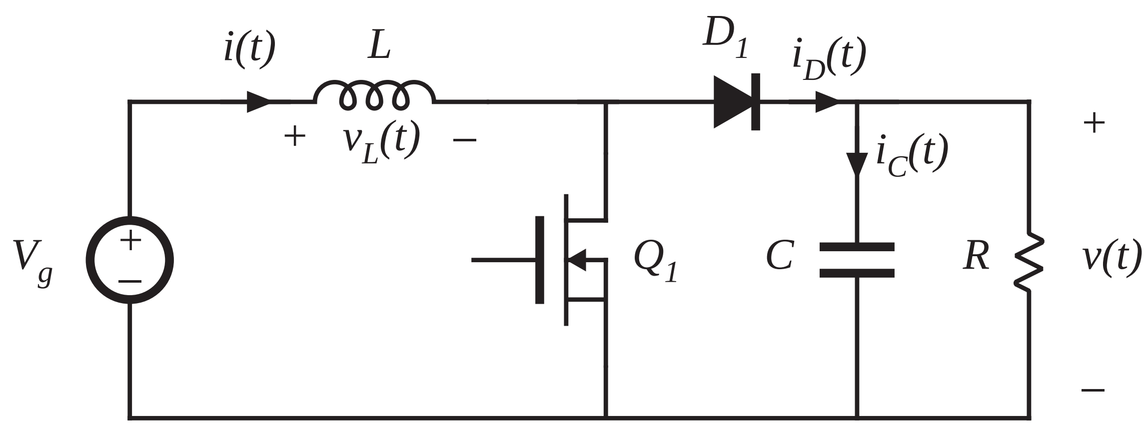
valid for $K < K_{crit}$

# Buck converter $M(D,K)$




$$M = \begin{cases} D & \text{for } K > K_{crit} \\ \dfrac{2}{1 + \sqrt{1 + 4K / D^2}} & \text{for } K < K_{crit} \end{cases}$$

# 5.3. Boost converter example



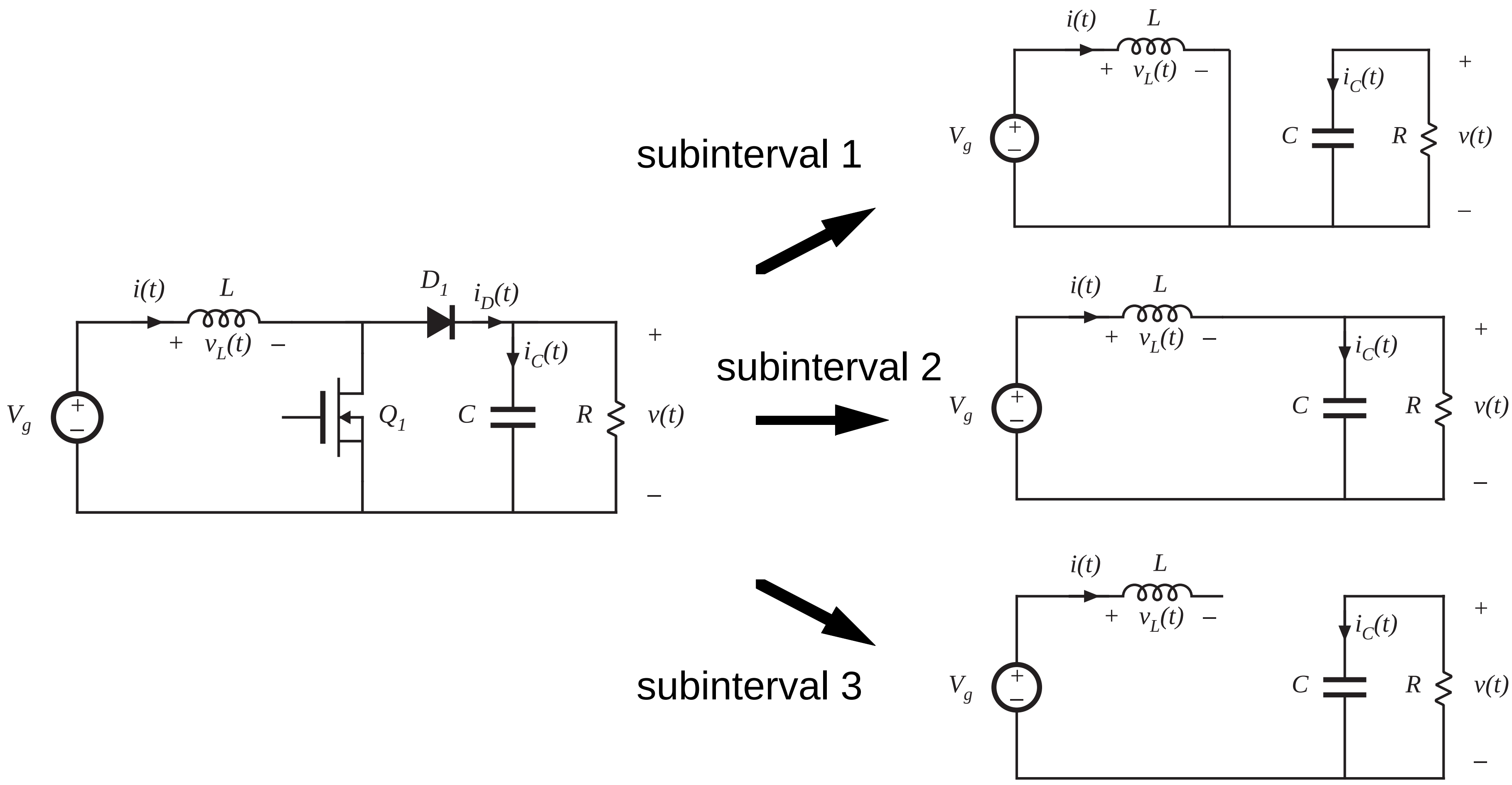

Mode boundary:

$$I > \Delta i_L \quad \text{for CCM}$$
$$I < \Delta i_L \quad \text{for DCM}$$

Previous CCM soln:

$$I = \frac{V_g}{D'^2 R} \qquad \Delta i_L = \frac{V_g}{2L} DT_s$$

# Conversion ratio: DCM boost

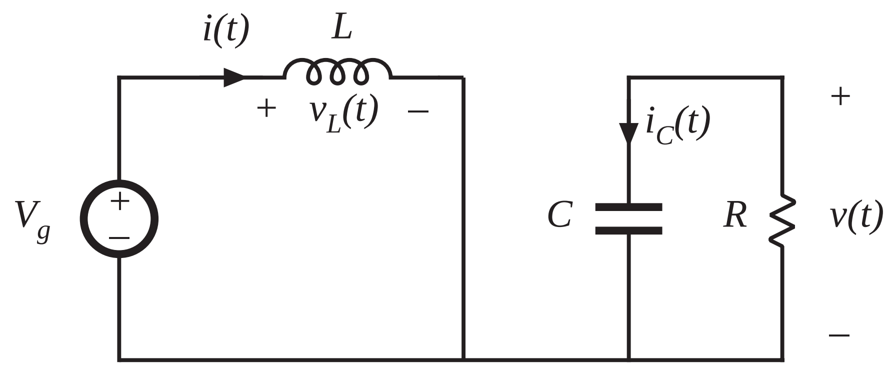


# Subinterval 1

$$v_L(t) = V_g$$
$$i_C(t) = -\,v(t)\,/\,R$$

Small ripple approximation for *v(t)* (but not for *i(t)*!):

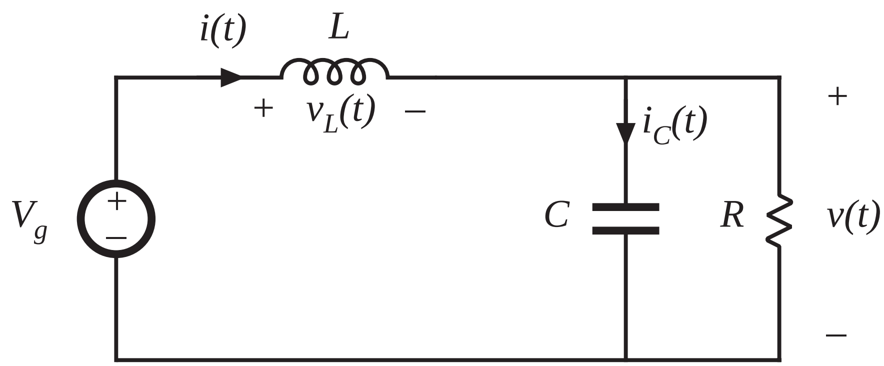
$$v_L(t) \approx V_g$$
$$i_C(t) \approx -\,V\,/\,R$$




$$0 < t < D_1T_s$$

# Subinterval 2

$$v_L(t) = V_g - v(t)$$
$$i_C(t) = i(t) - v(t) / R$$




Small ripple approximation for *v(t)* but not for *i(t)*:

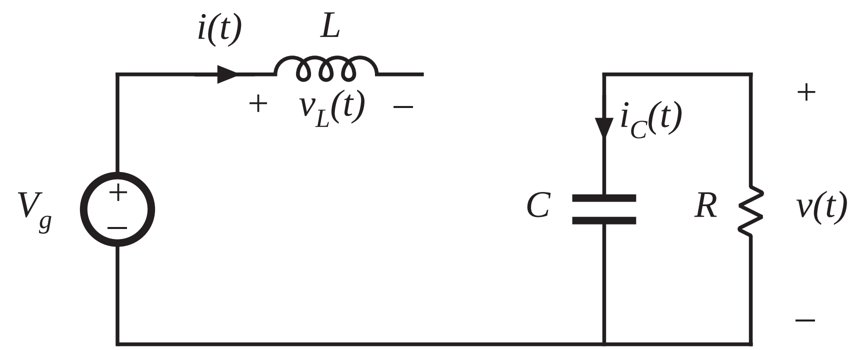
$$v_L(t) \approx V_g - V$$
$$i_C(t) \approx i(t) - V / R$$

$$D_1T_s < t < (D_1 + D_2)T_s$$

# Subinterval 3

$v_L = 0, \quad i = 0$

$i_C(t) = -\,v(t)\,/\,R$



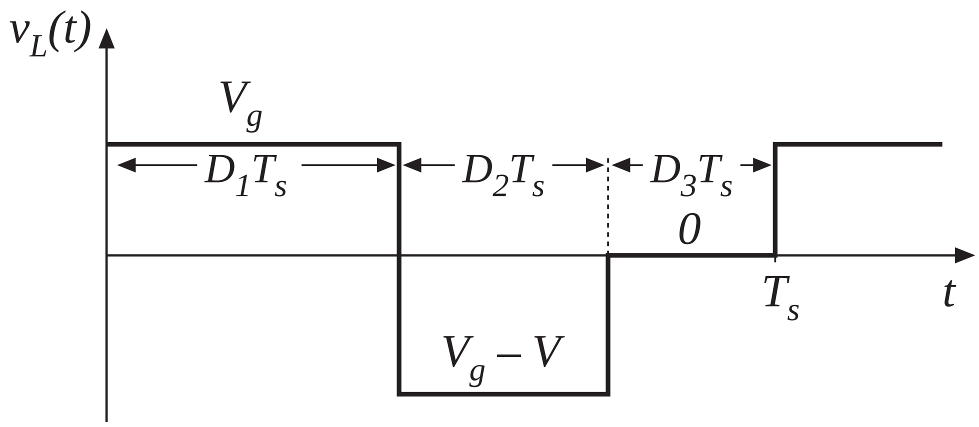

Small ripple approximation:

$v_L(t) = 0$

$i_C(t) = -\,V\,/\,R$

$(D_1 + D_2)T_s \;<\; t \;<\; T_s$

# Inductor volt-second balance




Volt-second balance:

$$D_1 V_g + D_2 (V_g - V) + D_3 (0) = 0$$

Solve for $V$:

$$V = \frac{D_1 + D_2}{D_2} V_g$$

note that $D_2$ is unknown

# Capacitor charge balance

node equation:

$$i_D(t) = i_C(t) + v(t) / R$$

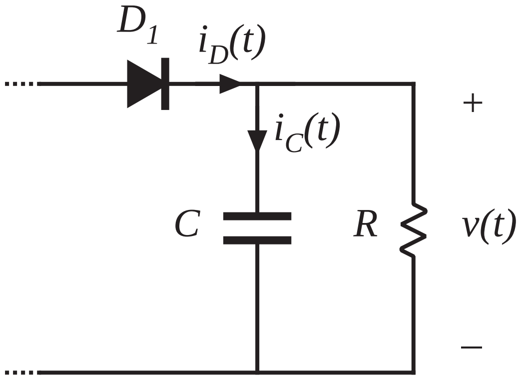
capacitor charge balance:

$$\left\langle i_C \right\rangle = 0$$

hence

$$\left\langle i_D \right\rangle = V / R$$

must compute dc component of diode current and equate to load current (for this boost converter example)



# Inductor and diode current waveforms

peak current:

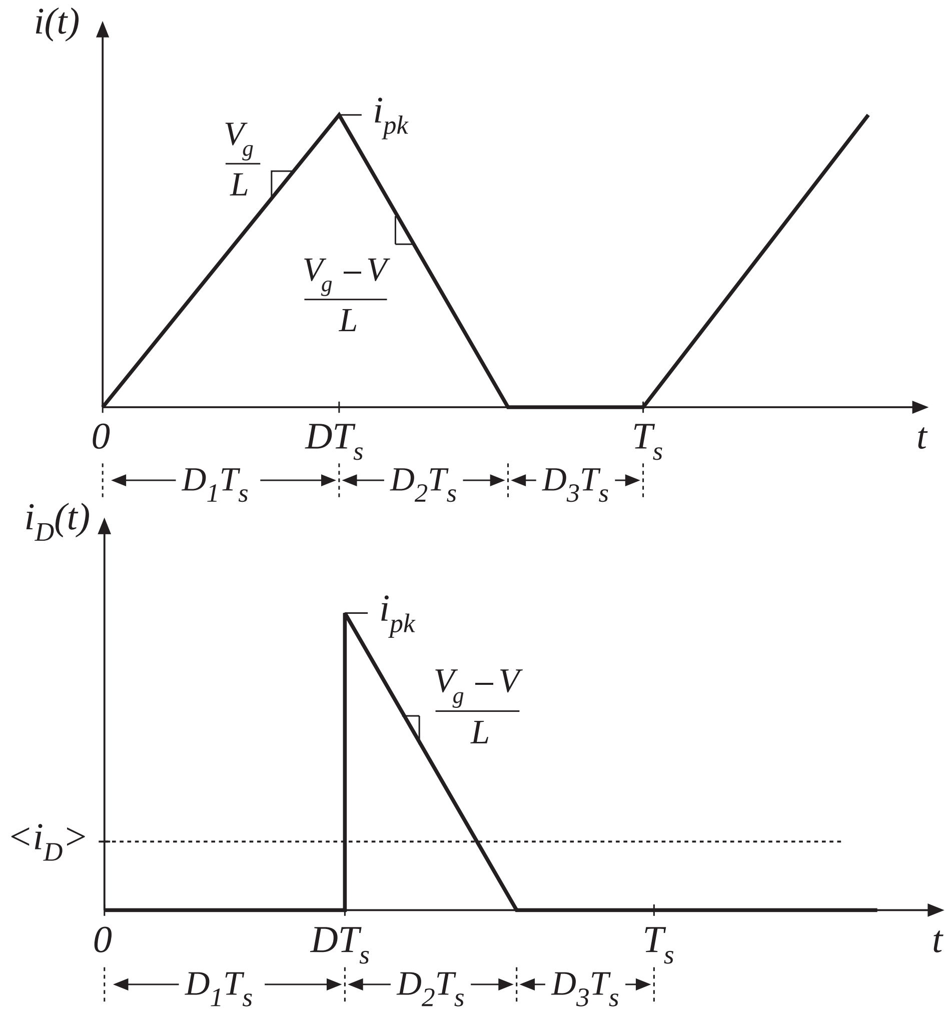
$$i_{pk} = \frac{V_g}{L} D_1 T_s$$

average diode current:

$$\left\langle i_D \right\rangle = \frac{1}{T_s} \int_0^{T_s} i_D(t)\, dt$$

triangle area formula:

$$\int_0^{T_s} i_D(t)\, dt = \frac{1}{2}\, i_{pk}\, D_2 T_s$$



# Equate diode current to load current

average diode current:

$$\left\langle i_D \right\rangle = \frac{1}{T_s}\left(\frac{1}{2}\, i_{pk}\, D_2 T_s\right) = \frac{V_g D_1 D_2 T_s}{2L}$$

equate to dc load current:

$$\frac{V_g D_1 D_2 T_s}{2L} = \frac{V}{R}$$

# Solution for $V$

Two equations and two unknowns ($V$ and $D_2$):

$$V = \frac{D_1 + D_2}{D_2} V_g$$ (from inductor volt-second balance)

$$\frac{V_g D_1 D_2 T_s}{2L} = \frac{V}{R}$$ (from capacitor charge balance)

Eliminate $D_2$ , solve for $V$. From volt-sec balance eqn:

$$D_2 = D_1 \frac{V_g}{V - V_g}$$

Substitute into charge balance eqn, rearrange terms:

$$V^2 - V V_g - \frac{V_g^2 D_1^2}{K} = 0$$

# Solution for $V$

$$V^2 - VV_g - \frac{V_g^2 D_1^2}{K} = 0$$

Use quadratic formula:

$$\frac{V}{V_g} = \frac{1 \pm \sqrt{1 + 4D_1^2 / K}}{2}$$

Note that one root leads to positive $V$, while other leads to negative $V$. Select positive root:

$$\frac{V}{V_g} = M(D_1,K) = \frac{1 + \sqrt{1 + 4D_1^2 / K}}{2}$$

where $K = 2L / RT_s$
valid for $K < K_{crit}(D)$

Transistor duty cycle $D$ = interval *1* duty cycle $D_1$

# Summary of key points

1. The discontinuous conduction mode occurs in converters containing current- or voltage-unidirectional switches, when the inductor current or capacitor voltage ripple is large enough to cause the switch current or voltage to reverse polarity.
2. Conditions for operation in the discontinuous conduction mode can be found by determining when the inductor current or capacitor voltage ripples and dc components cause the switch on-state current or off-state voltage to reverse polarity.
3. The dc conversion ratio $M$ of converters operating in the discontinuous conduction mode can be found by application of the principles of inductor volt-second and capacitor charge balance.

# Summary of key points

4. Extra care is required when applying the small-ripple approximation. Some waveforms, such as the output voltage, should have small ripple which can be neglected. Other waveforms, such as one or more inductor currents, may have large ripple that cannot be ignored.
5. The characteristics of a converter changes significantly when the converter enters DCM. The output voltage becomes load-dependent, resulting in an increase in the converter output impedance.