Johns Hopkins Engineering

Power Electronics 525.725

Module 5 Lecture 5

Large Signal Modeling

7.1. Introduction

Objective: maintain $v(t)$ equal to an accurate, constant value V.

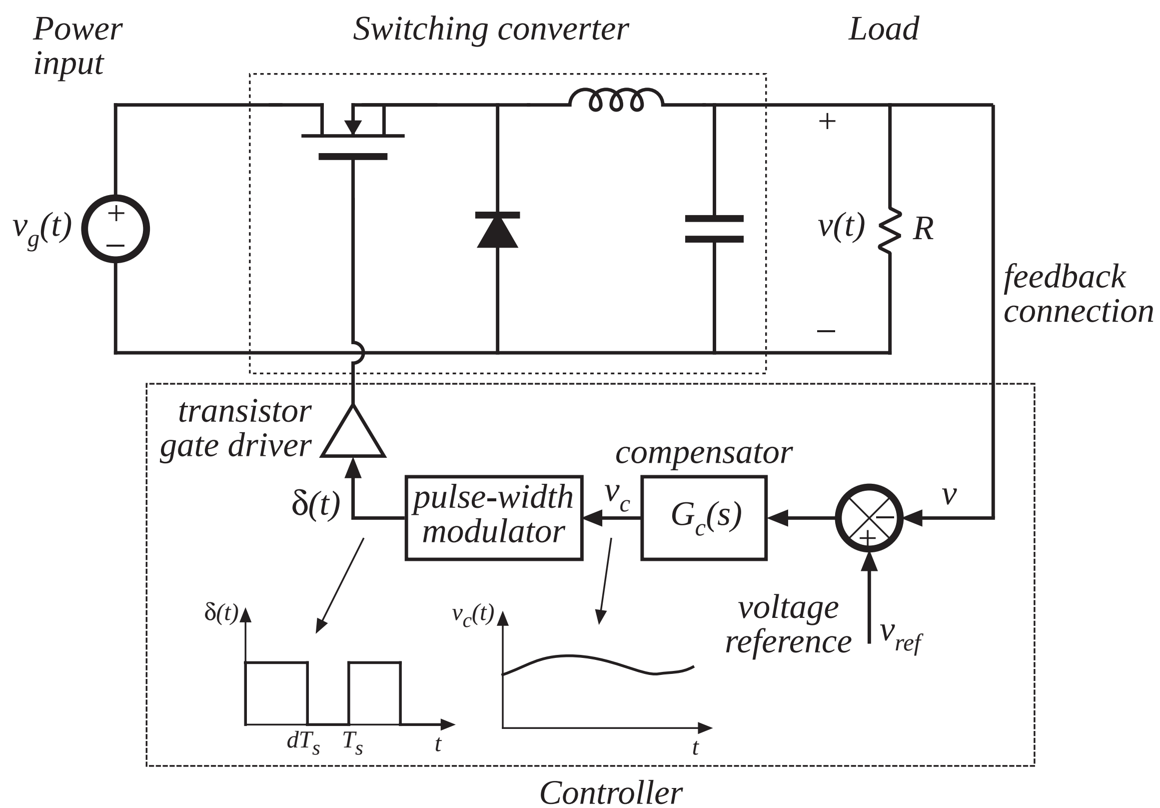
There are disturbances:

- in $v_g(t)$
- in R

There are uncertainties:

- in element values
- in V_g
- in R

A simple dc-dc regulator system, employing a buck converter



Modeling

- Representation of physical behavior by mathematical means
- Model dominant behavior of system, ignore other insignificant phenomena
- Simplified model yields physical insight, allowing engineer to design system to operate in specified manner
- Approximations neglect small but complicating phenomena
- After basic insight has been gained, model can be refined (if it is judged worthwhile to expend the engineering effort to do so), to account for some of the previously neglected phenomena

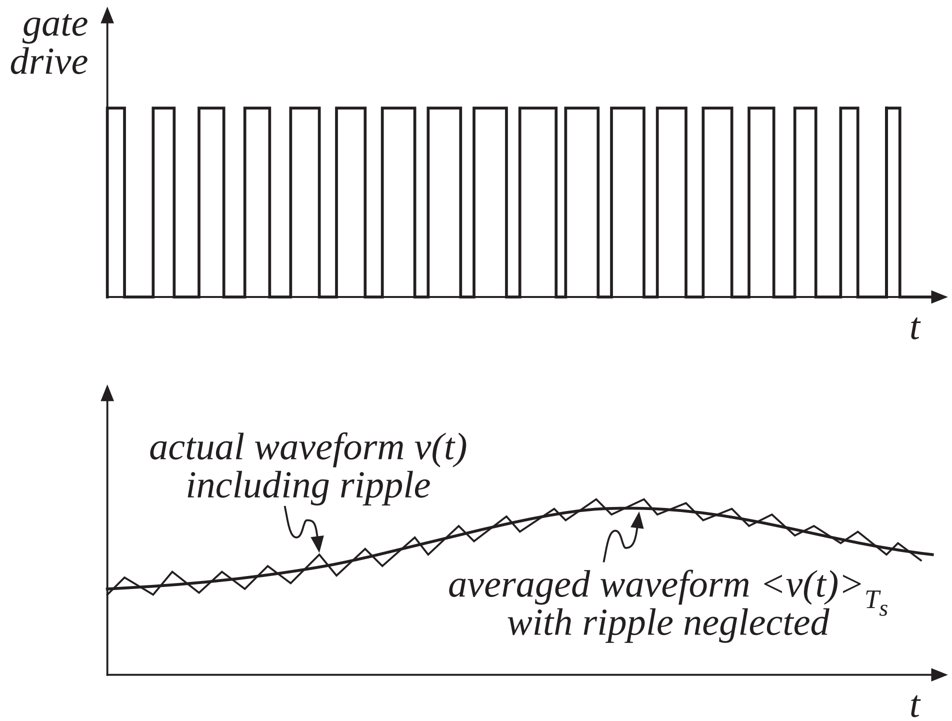
Neglecting the switching ripple

Suppose the duty cycle is modulated sinusoidally:

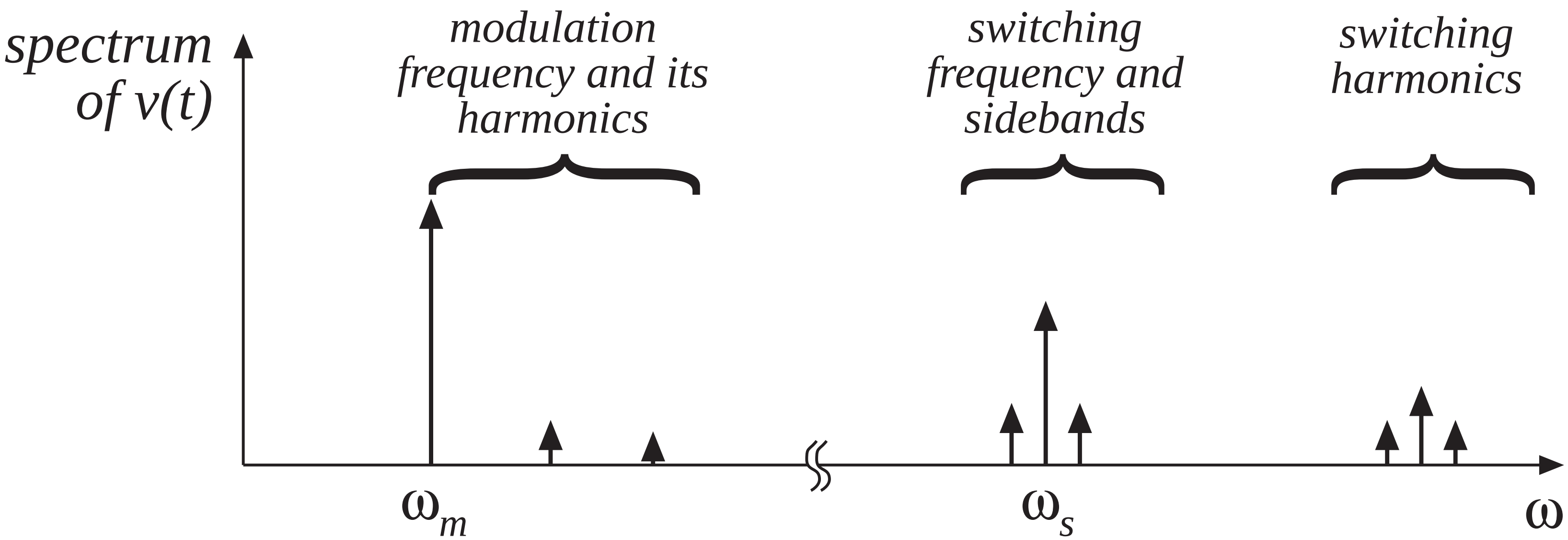
$$d(t) = D + D_m \cos \omega_m t$$

where D and D_m are constants, $| D_m | << D$, and the modulation frequency ω_m is much smaller than the converter switching frequency $\omega_s = 2\pi f_s$.

The resulting variations in transistor gate drive signal and converter output voltage:



Output voltage spectrum
with sinusoidal modulation of duty cycle



Contains frequency components at:

- Modulation frequency and its harmonics
- Switching frequency and its harmonics
- Sidebands of switching frequency

With small switching ripple, high-frequency components (switching harmonics and sidebands) are small.

If ripple is neglected, then only low-frequency components (modulation frequency and harmonics) remain.

Objective of Large Signal modeling

- Predict how low-frequency variations in duty cycle induce low-frequency variations in the converter voltages and currents
- Ignore the switching ripple
- Ignore complicated switching harmonics and sidebands

Approach:

- Remove switching harmonics by averaging all waveforms over one switching period

Averaging to remove switching ripple

Average over one switching period to remove switching ripple:

$$L \frac{d\left\langle i_L(t)\right\rangle_{T_s}}{dt} = \left\langle v_L(t)\right\rangle_{T_s}$$

$$C \frac{d\left\langle v_C(t)\right\rangle_{T_s}}{dt} = \left\langle i_C(t)\right\rangle_{T_s}$$

where

$$\left\langle x_L(t)\right\rangle_{T_s} = \frac{1}{T_s}\int_t^{t+T_s} x(\tau)\, d\tau$$

Note that, in steady-state,

$$\left\langle v_L(t)\right\rangle_{T_s} = 0$$
$$\left\langle i_C(t)\right\rangle_{T_s} = 0$$

by inductor volt-second balance and capacitor charge balance.

Nonlinear averaged equations

The averaged voltages and currents are, in general, nonlinear functions of the converter duty cycle, voltages, and currents. Hence, the averaged equations

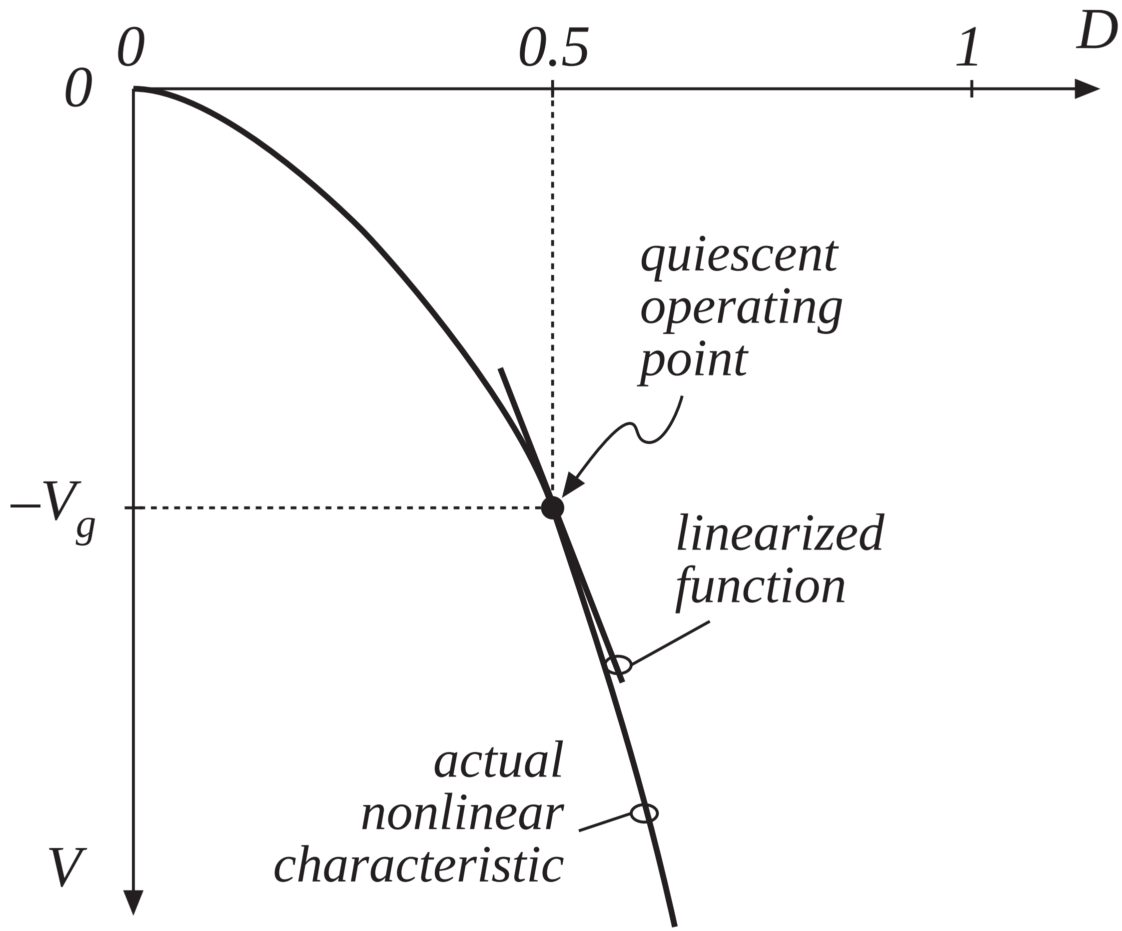
$$L \frac{d\left\langle i_L(t) \right\rangle_{T_s}}{dt} = \left\langle v_L(t) \right\rangle_{T_s}$$

$$C \frac{d\left\langle v_C(t) \right\rangle_{T_s}}{dt} = \left\langle i_C(t) \right\rangle_{T_s}$$

constitute a system of nonlinear differential equations.

Hence, must linearize by constructing a small-signal converter model.

Buck-boost converter: nonlinear static control-to-output characteristic

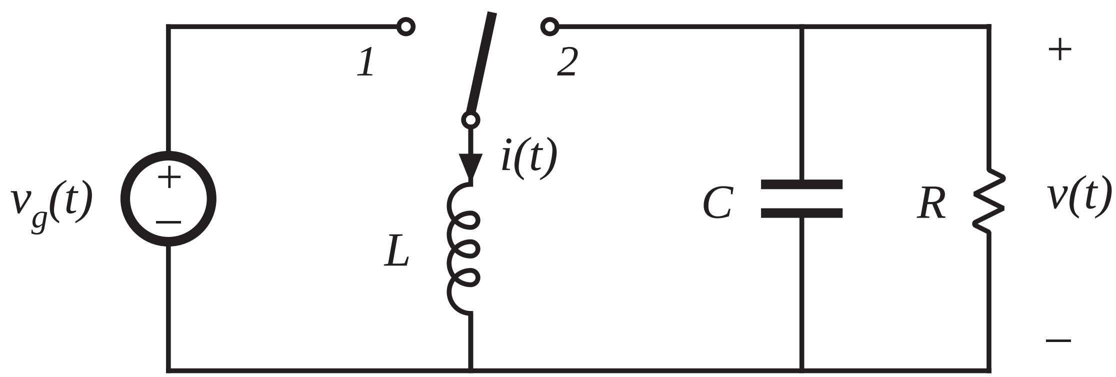


Example: linearization at the quiescent operating point

$D = 0.5$

7.2. The basic large signal modeling approach

Buck-boost converter example

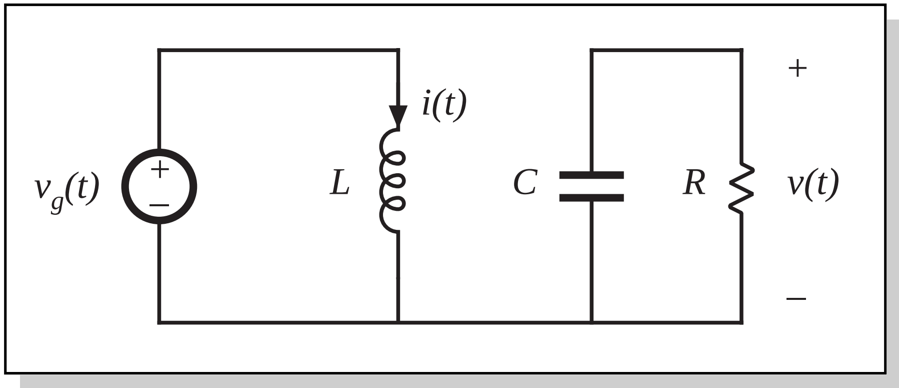


Switch in position 1

Inductor voltage and capacitor current are:

$$v_L(t) = L\,\frac{di(t)}{dt} = v_g(t)$$

$$i_C(t) = C\,\frac{dv(t)}{dt} = -\,\frac{v(t)}{R}$$



Small ripple approximation: replace waveforms with their low-frequency averaged values:

$$v_L(t) = L\,\frac{di(t)}{dt} \approx \left\langle v_g(t) \right\rangle_{T_s}$$

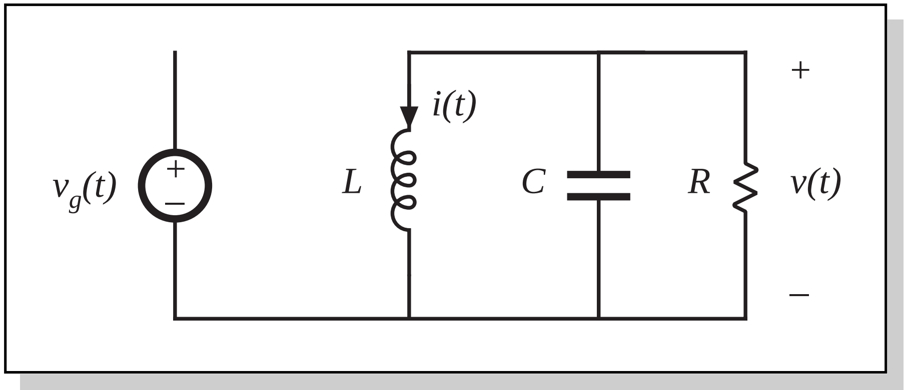
$$i_C(t) = C\,\frac{dv(t)}{dt} \approx -\,\frac{\left\langle v(t) \right\rangle_{T_s}}{R}$$

Switch in position 2

Inductor voltage and capacitor current are:

$$v_L(t) = L\,\frac{di(t)}{dt} = v(t)$$

$$i_C(t) = C\,\frac{dv(t)}{dt} = -\,i(t) - \frac{v(t)}{R}$$



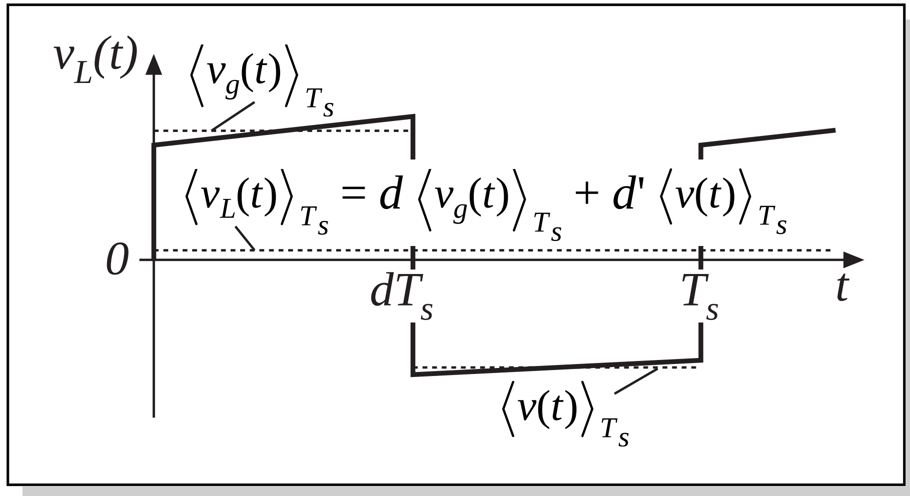
Small ripple approximation: replace waveforms with their low-frequency averaged values:

$$v_L(t) = L\,\frac{di(t)}{dt} \approx \left\langle v(t) \right\rangle_{T_s}$$

$$i_C(t) = C\,\frac{dv(t)}{dt} \approx -\left\langle i(t) \right\rangle_{T_s} - \frac{\left\langle v(t) \right\rangle_{T_s}}{R}$$

7.2.1 Averaging the inductor waveforms

Inductor voltage waveform



Low-frequency average is found by evaluation of

$$\left\langle x_L(t) \right\rangle_{T_s} = \frac{1}{T_s} \int_t^{t+T_s} x(\tau) d\tau$$

Average the inductor voltage in this manner:

$$\left\langle v_L(t) \right\rangle_{T_s} = \frac{1}{T_s} \int_t^{t+T_s} v_L(\tau) d\tau \approx d(t) \left\langle v_g(t) \right\rangle_{T_s} + d'(t) \left\langle v(t) \right\rangle_{T_s}$$

Insert into Eq. (7.2):

$$L \frac{d\left\langle i(t) \right\rangle_{T_s}}{dt} = d(t) \left\langle v_g(t) \right\rangle_{T_s} + d'(t) \left\langle v(t) \right\rangle_{T_s}$$

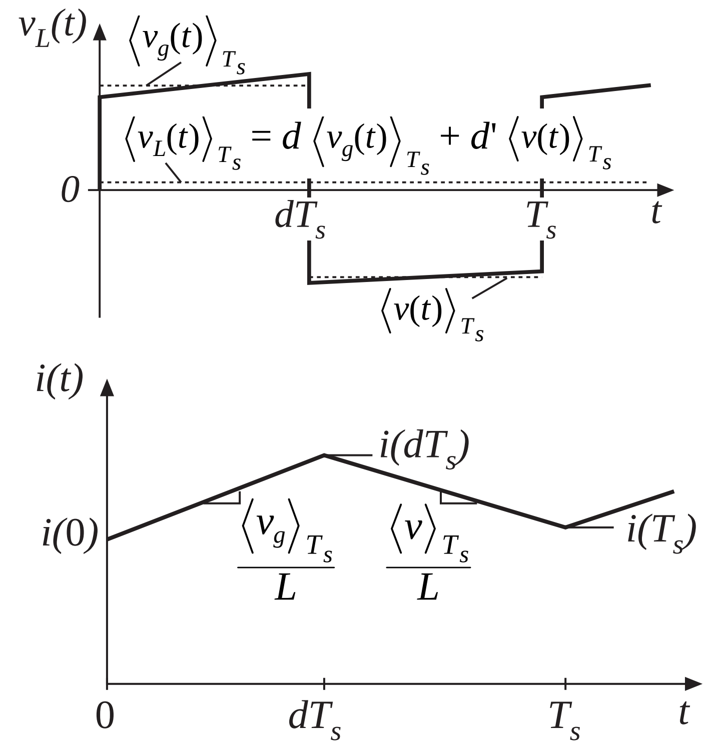
This equation describes how the low-frequency components of the inductor waveforms evolve in time.

7.2.2 Discussion of the averaging approximation

Use of the average inductor voltage allows us to determine the net change in inductor current over one switching period, while neglecting the switching ripple.

In steady-state, the average inductor voltage is zero (volt-second balance), and hence the inductor current waveform is periodic: $i(t + T_s) = i(t)$. There is no net change in inductor current over one switching period.

During transients or ac variations, the average inductor voltage is not zero in general, and this leads to net variations in inductor current.



Inductor voltage and current waveforms

Net change in inductor current is correctly predicted by the average inductor voltage

Inductor equation:

$$L \frac{di(t)}{dt} = v_L(t)$$

Divide by L and integrate over one switching period:

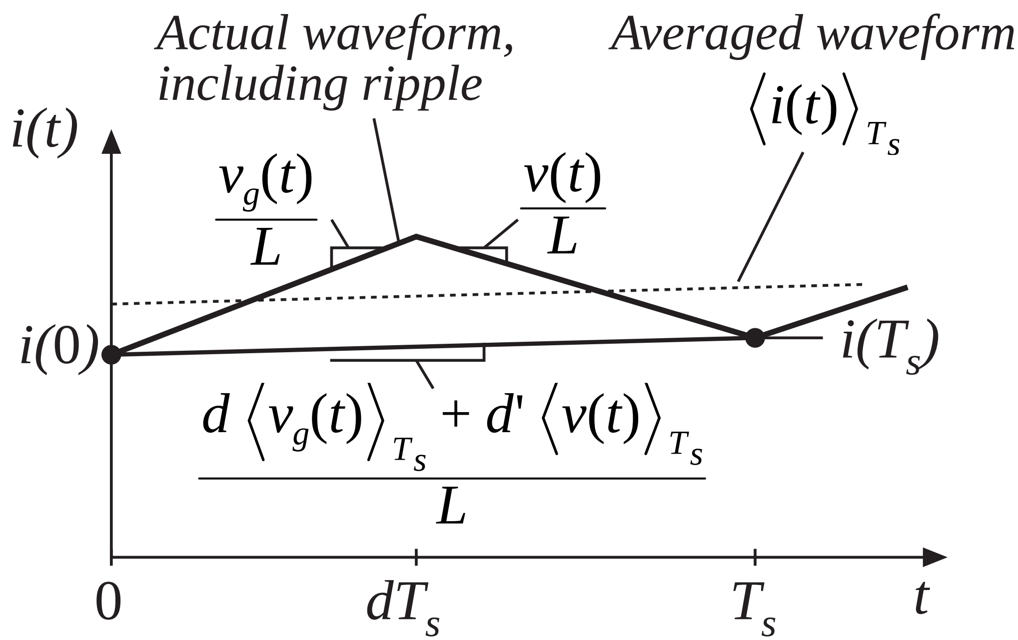
$$\int_t^{t+T_s} di = \frac{1}{L} \int_t^{t+T_s} v_L(\tau) d\tau$$

Left-hand side is the change in inductor current. Right-hand side can be related to average inductor voltage by multiplying and dividing by T_s as follows:

$$i(t+T_s) - i(t) = \frac{1}{L} T_s \left\langle v_L(t) \right\rangle_{T_s}$$

So the net change in inductor current over one switching period is exactly equal to the period T_s multiplied by the average slope $\langle v_L \rangle_{T_s} /L$.

Average inductor voltage correctly predicts average slope of $i_L(t)$



The net change in inductor current over one switching period is exactly equal to the period T_s multiplied by the average slope $\langle v_L \rangle_{T_s} /L$.

$$\frac{d\left\langle i(t) \right\rangle_{T_s}}{dt}$$

We have

$$i(t+T_s) - i(t) = \frac{1}{L}\, T_s \left\langle v_L(t) \right\rangle_{T_s}$$

Rearrange:

$$L\,\frac{i(t+T_s) - i(t)}{T_s} = \left\langle v_L(t) \right\rangle_{T_s}$$

Define the derivative of $\langle\, i \,\rangle_{T_s}$ as (Euler formula):

$$\frac{d\left\langle i(t) \right\rangle_{T_s}}{dt} = \frac{i(t+T_s) - i(t)}{T_s}$$

Hence,

$$L\,\frac{d\left\langle i(t) \right\rangle_{T_s}}{dt} = \left\langle v_L(t) \right\rangle_{T_s}$$

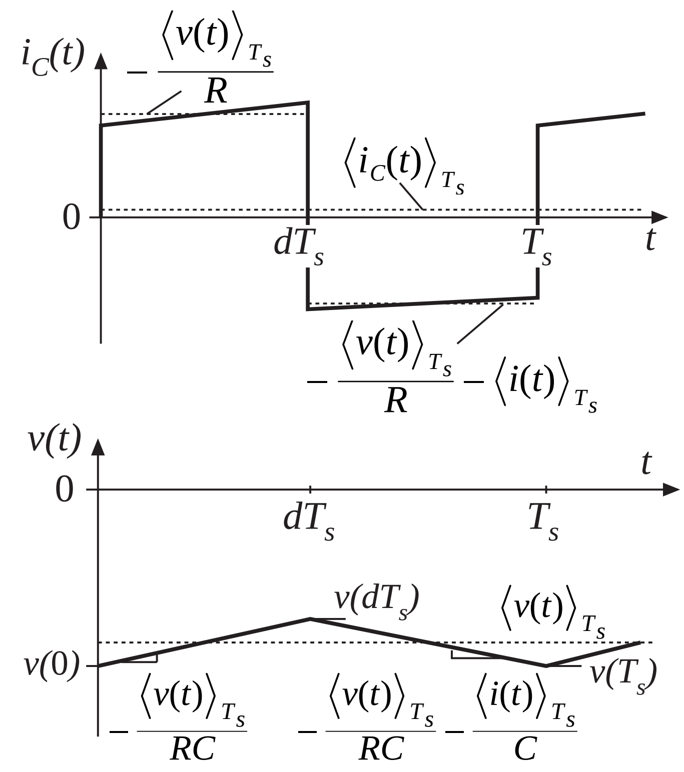
7.2.3 Averaging the capacitor waveforms

Average capacitor current:

$$\left\langle i_C(t) \right\rangle_{T_s} = d(t) \left(-\frac{\left\langle v(t) \right\rangle_{T_s}}{R} \right) + d'(t) \left(-\left\langle i(t) \right\rangle_{T_s} - \frac{\left\langle v(t) \right\rangle_{T_s}}{R} \right)$$

Collect terms, and equate to $C\, d\langle\, v \,\rangle_{T_s} /dt$:

$$C \frac{d\left\langle v(t) \right\rangle_{T_s}}{dt} = -\, d'(t) \left\langle i(t) \right\rangle_{T_s} - \frac{\left\langle v(t) \right\rangle_{T_s}}{R}$$



Capacitor voltage and current waveforms

7.2.4 The average input current

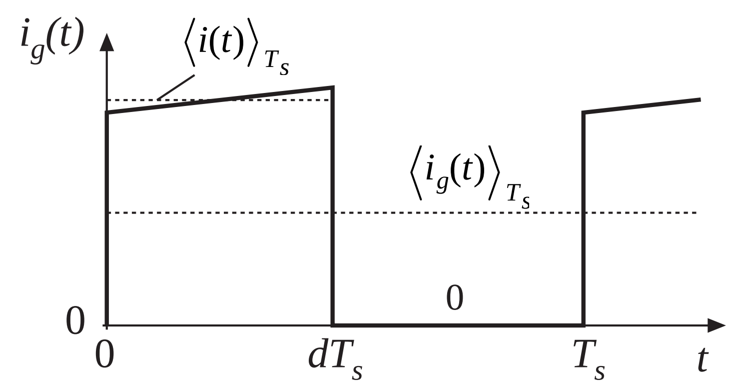
We found in Chapter 3 that it was sometimes necessary to write an equation for the average converter input current, to derive a complete dc equivalent circuit model. It is likewise necessary to do this for the ac model.

Buck-boost input current waveform is

$$i_g(t) = \begin{cases} \left\langle i(t) \right\rangle_{T_s} & \text{during subinterval 1} \\ 0 & \text{during subinterval 2} \end{cases}$$

Average value:

$$\left\langle i_g(t) \right\rangle_{T_s} = d(t) \left\langle i(t) \right\rangle_{T_s}$$



Converter input current waveform

Buck Boost Large Signal Model Equations

Converter averaged equations:

$$L \frac{d\left\langle i(t) \right\rangle_{T_s}}{dt} = d(t) \left\langle v_g(t) \right\rangle_{T_s} + d'(t) \left\langle v(t) \right\rangle_{T_s}$$

$$C \frac{d\left\langle v(t) \right\rangle_{T_s}}{dt} = - d'(t) \left\langle i(t) \right\rangle_{T_s} - \frac{\left\langle v(t) \right\rangle_{T_s}}{R}$$

$$\left\langle i_g(t) \right\rangle_{T_s} = d(t) \left\langle i(t) \right\rangle_{T_s}$$

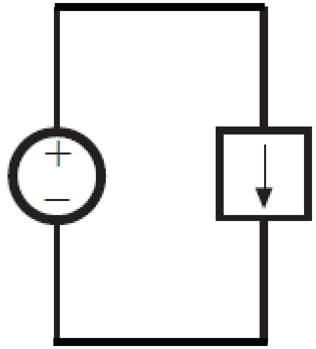
—nonlinear because of multiplication of the time-varying quantity $d(t)$ with other time-varying quantities such as $i(t)$ and $v(t)$.

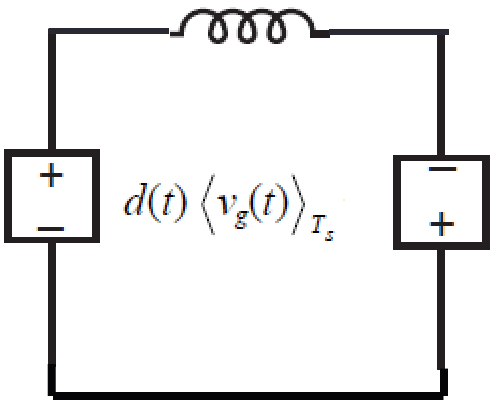
Large Signal Equivalent Circuit – (nonlinear)

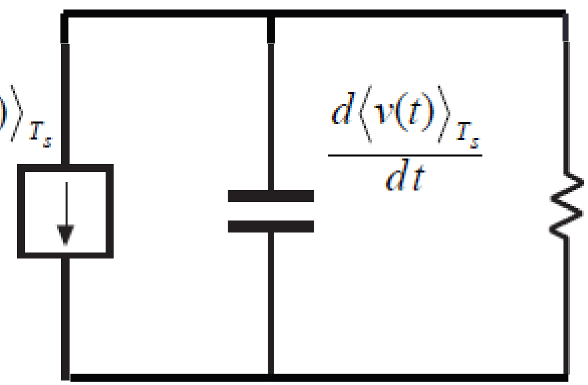
$$L\frac{d\langle i(t)\rangle_{T_s}}{dt} = d(t)\,\langle v_g(t)\rangle_{T_s} + d'(t)\,\langle v(t)\rangle_{T_s}$$

$$C\frac{d\langle v(t)\rangle_{T_s}}{dt} = -\,d'(t)\,\langle i(t)\rangle_{T_s} - \frac{\langle v(t)\rangle_{T_s}}{R}$$

$$\langle i_g(t)\rangle_{T_s} = d(t)\,\langle i(t)\rangle_{T_s}$$

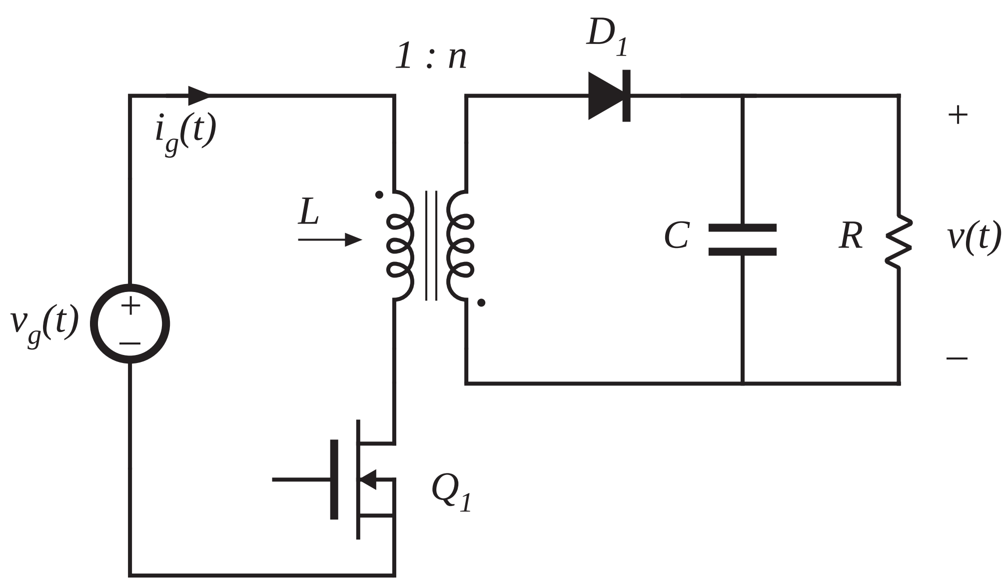






7.3. Example: a nonideal flyback converter

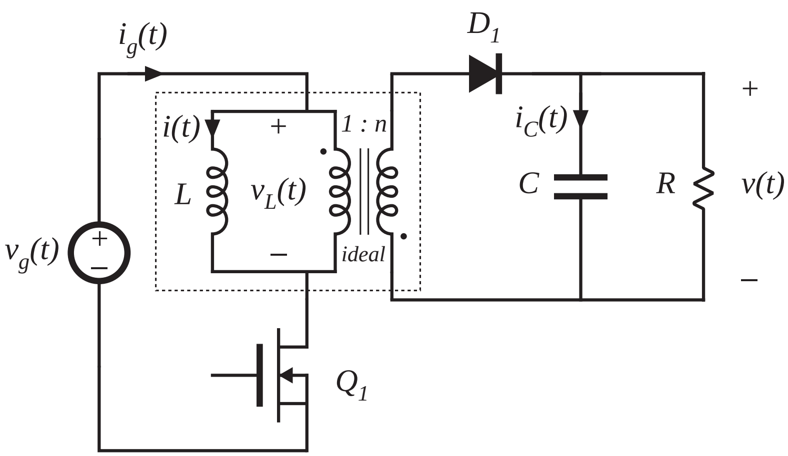
Flyback converter example



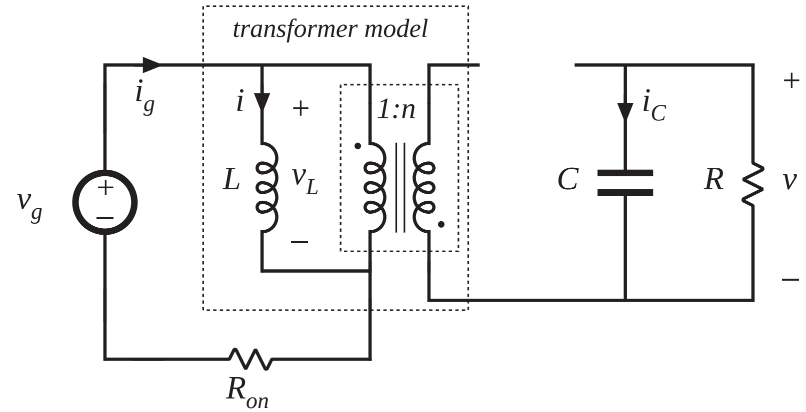
- MOSFET has on-resistance R_{on}
- Flyback transformer has magnetizing inductance L, referred to primary

Circuits during subintervals 1 and 2

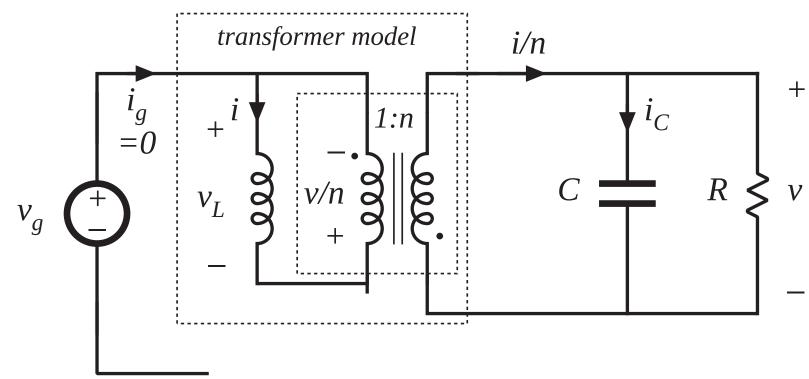
Flyback converter, with transformer equivalent circuit



Subinterval 1



Subinterval 2



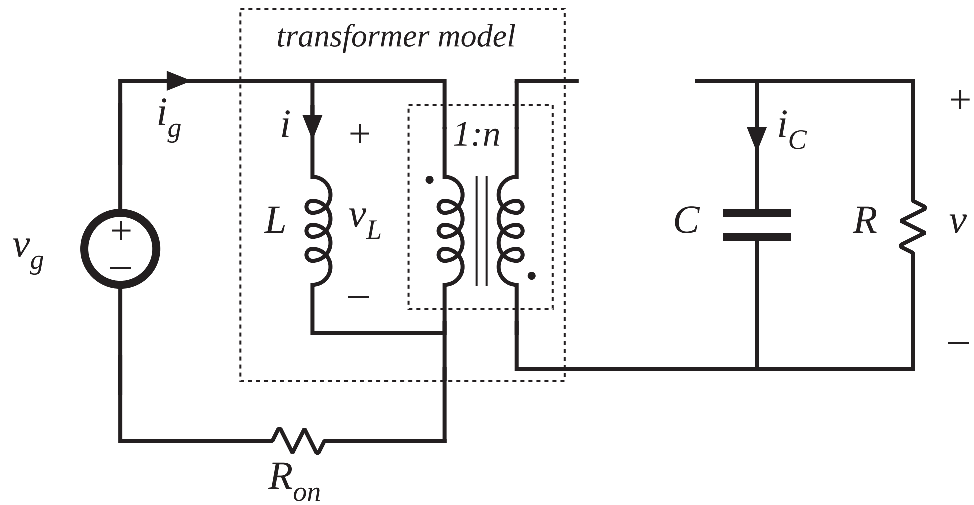
Subinterval 1

Circuit equations:

$$v_L(t) = v_g(t) - i(t)\, R_{on}$$
$$i_C(t) = -\frac{v(t)}{R}$$
$$i_g(t) = i(t)$$

Small ripple approximation:

$$v_L(t) = \left\langle v_g(t) \right\rangle_{T_s} - \left\langle i(t) \right\rangle_{T_s} R_{on}$$
$$i_C(t) = -\frac{\left\langle v(t) \right\rangle_{T_s}}{R}$$
$$i_g(t) = \left\langle i(t) \right\rangle_{T_s}$$



MOSFET conducts, diode is reverse-biased

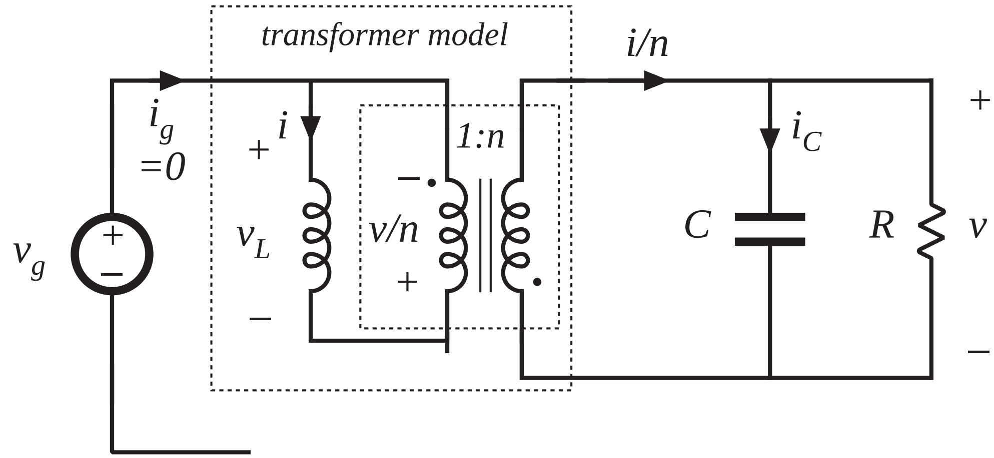
Subinterval 2

Circuit equations:

$$v_L(t) = -\frac{v(t)}{n}$$
$$i_C(t) = -\frac{i(t)}{n} - \frac{v(t)}{R}$$
$$i_g(t) = 0$$

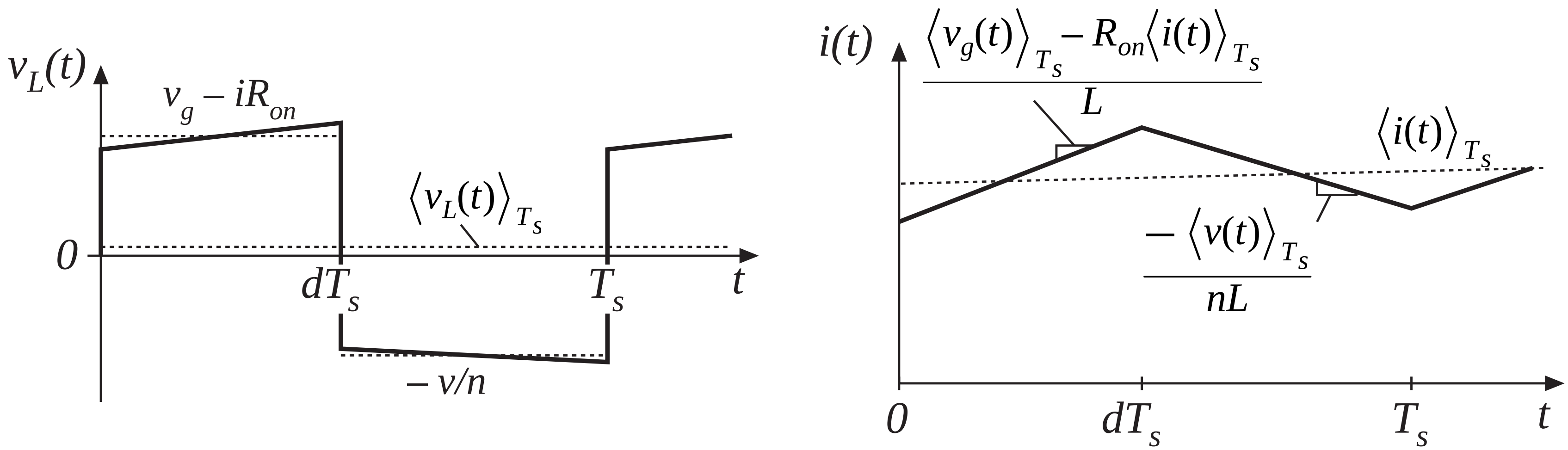
Small ripple approximation:

$$v_L(t) = -\frac{\left\langle v(t) \right\rangle_{T_s}}{n}$$
$$i_C(t) = -\frac{\left\langle i(t) \right\rangle_{T_s}}{n} - \frac{\left\langle v(t) \right\rangle_{T_s}}{R}$$
$$i_g(t) = 0$$



MOSFET is off, diode conducts

Inductor waveforms



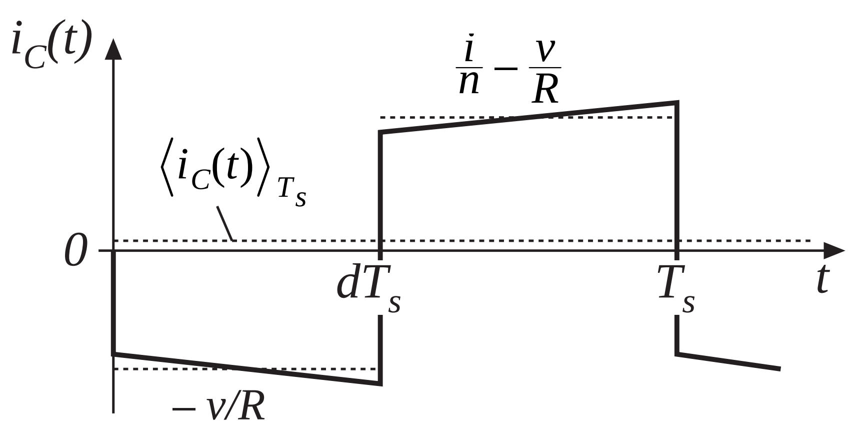
Average inductor voltage:

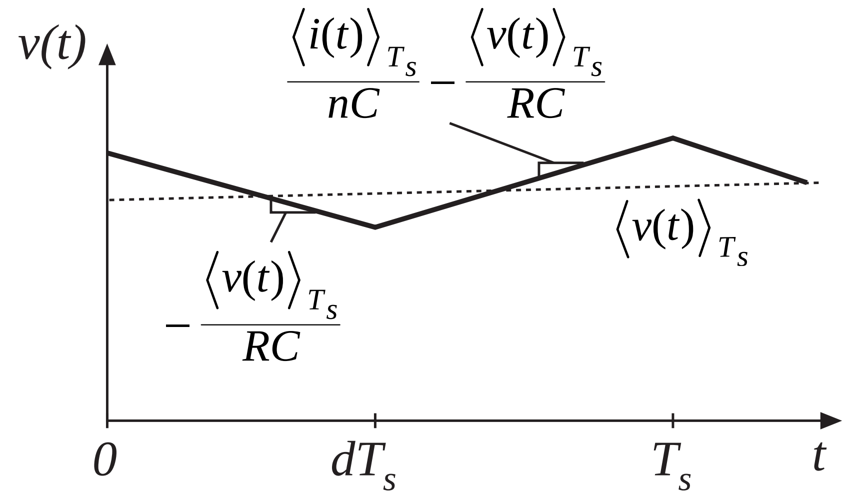
$$\left\langle v_L(t) \right\rangle_{T_s} = d(t) \left(\left\langle v_g(t) \right\rangle_{T_s} - \left\langle i(t) \right\rangle_{T_s} R_{on} \right) + d'(t) \left(\frac{-\left\langle v(t) \right\rangle_{T_s}}{n} \right)$$

Hence, we can write:

$$L \frac{d\left\langle i(t) \right\rangle_{T_s}}{dt} = d(t) \left\langle v_g(t) \right\rangle_{T_s} - d(t) \left\langle i(t) \right\rangle_{T_s} R_{on} - d'(t) \frac{\left\langle v(t) \right\rangle_{T_s}}{n}$$

Capacitor waveforms





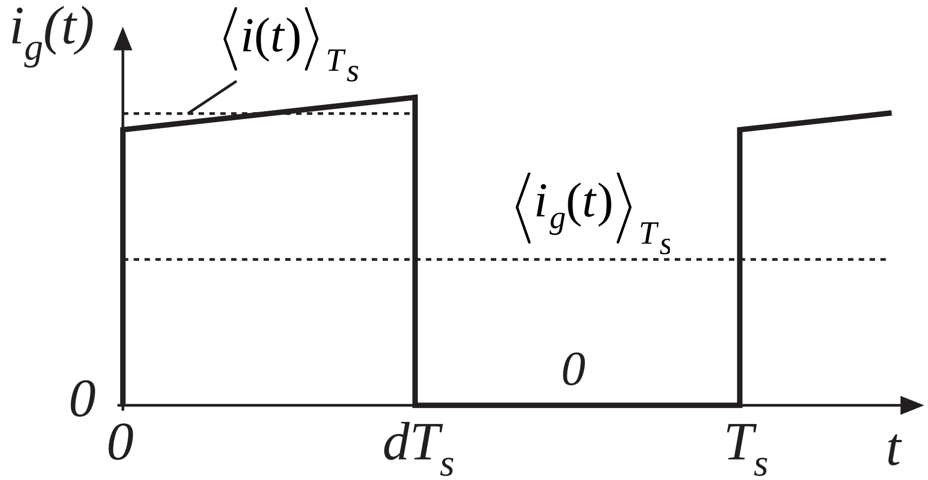
Average capacitor current:

$$\left\langle i_C(t) \right\rangle_{T_s} = d(t)\left(\frac{-\left\langle v(t) \right\rangle_{T_s}}{R} \right) + d'(t)\left(\frac{\left\langle i(t) \right\rangle_{T_s}}{n} - \frac{\left\langle v(t) \right\rangle_{T_s}}{R} \right)$$

Hence, we can write:

$$C\,\frac{d\left\langle v(t) \right\rangle_{T_s}}{dt} = d'(t)\,\frac{\left\langle i(t) \right\rangle_{T_s}}{n} - \frac{\left\langle v(t) \right\rangle_{T_s}}{R}$$

Input current waveform



Average input current:

$$\left\langle i_g(t) \right\rangle_{T_s} = d(t) \left\langle i(t) \right\rangle_{T_s}$$

The averaged converter equations

$$L\frac{d\left\langle i(t)\right\rangle_{T_s}}{dt} = d(t)\left\langle v_g(t)\right\rangle_{T_s} - d(t)\left\langle i(t)\right\rangle_{T_s} R_{on} - d'(t)\frac{\left\langle v(t)\right\rangle_{T_s}}{n}$$

$$C\frac{d\left\langle v(t)\right\rangle_{T_s}}{dt} = d'(t)\frac{\left\langle i(t)\right\rangle_{T_s}}{n} - \frac{\left\langle v(t)\right\rangle_{T_s}}{R}$$

$$\left\langle i_g(t)\right\rangle_{T_s} = d(t)\left\langle i(t)\right\rangle_{T_s}$$

LARGE SIGNAL MODEL

— a system of nonlinear differential equations

SIMULINK SIMULATION FLYBACK CONVERTER LARGE SIGNAL MODEL

$$L\,\frac{d\langle i(t)\rangle_{T_s}}{dt} = d(t)\,\langle v_g(t)\rangle_{T_s} - d(t)\,\langle i(t)\rangle_{T_s}\,R_{on} - d'(t)\,\frac{\langle v(t)\rangle_{T_s}}{n}$$

$$C\,\frac{d\langle v(t)\rangle_{T_s}}{dt} = d'(t)\,\frac{\langle i(t)\rangle_{T_s}}{n} - \frac{\langle v(t)\rangle_{T_s}}{R}$$

LARGE SIGNAL MODEL

$$\langle i_g(t)\rangle_{T_s} = d(t)\,\langle i(t)\rangle_{T_s}$$

$V_g = 48\ \mathrm{V}$

$V = 12\ \mathrm{V}$

$P_{out} = 150\ \mathrm{W}$

$f_{sw} = 100\ \mathrm{kHz}$

$L_m = 250\ \mathrm{uH}$

$R_{on} = 25\ \mathrm{m\Omega}$

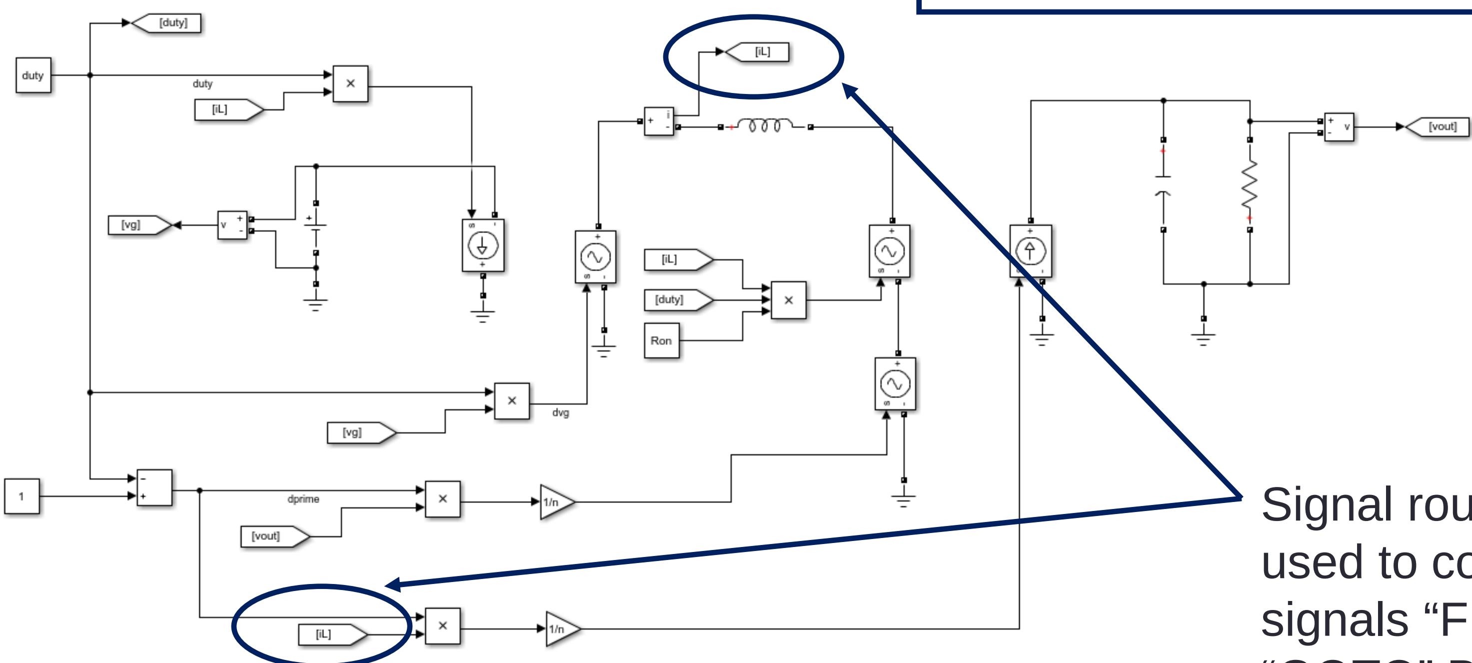
Flyback Large Signal Model

$$L\frac{d\langle i(t)\rangle_{T_s}}{dt} = d(t)\,\langle v_g(t)\rangle_{T_s} - d(t)\,\langle i(t)\rangle_{T_s} R_{on} - d'(t)\,\frac{\langle v(t)\rangle_{T_s}}{n}$$

$$C\frac{d\langle v(t)\rangle_{T_s}}{dt} = d'(t)\,\frac{\langle i(t)\rangle_{T_s}}{n} - \frac{\langle v(t)\rangle_{T_s}}{R}$$

$$\langle i_g(t)\rangle_{T_s} = d(t)\,\langle i(t)\rangle_{T_s}$$

LARGE SIGNAL MODEL



Signal routers used to connect signals “FROM” “GOTO” Blocks