

Deriving Intrinsic Motivation from Uncertainty about Future Goals

<https://github.com/jnegrea/csc2541project1>

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 - ▶ The learner does not know the reward of each state
 - ▶ The learner does not know the transition mechanics
- ▶ The machine learns the topology of the state space, the transition mechanics, the topography of the reward function, and how to use the mechanics to optimise the specified reward.

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- ▶ For animal learners, reward signals are recieved by engaging in exploratory, 'fun' behaviours. Can machines be incited to play and explore? To have fun?
- ▶ Exploratory behaviour is important for animal development – human children learn about the world by playing. Can a machine which learns by playing outperform one that does not when performance is integrated over a wide variety of tasks?

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Qualitative Aspects of Intrinsic Motivation I

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- ▶ In order to develop such measures we need heuristics which defines what it means to 'play effectively.'

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- ▶ Schmidhuber [12] posits that an intrinsically motivated learner should have:
 - ▶ An adaptive world model
 - ▶ A learning algorithm to update the model
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 - ▶ An adaptive world model
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 - ▶ An intrinsic reward system measuring model improvement
 - ▶ A behavioural policy optimising intrinsic reward
- ▶ The kernel of this thesis is that intrinsic motivation is derived from the pursuit of a better model of the dynamics of the world.

Qualitative Aspects of Intrinsic Motivation II

- ▶ Salge et al. [11] posit that an intrinsically motivated learner should seek states of high (heuristic) *empowerment*.
- ▶ Empowerment is an intrinsic utility measure which is *Local, Universal, and Task-Independent*:

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 - ▶ *Task-Independent*: The utility is independent of any particular goal or reward function

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 - ▶ Find the reward function such that an RL agent maximising the reward has highest expected fitness
 - ▶ Find the RL strategy which maximises the optimal reward function
- ▶ By construction, performs no worse on average than RL using the the natural reward corresponding to fitness.
- ▶ Avoids greedy behaviour in favour of exploratory behaviour (when beneficial).

Quantitative Measures for Intrinsic Motivation II: Empowerment

- ▶ *Empowerment* (metric) aims to achieve the heuristic it is named for
- ▶ Quantified as the channel capacity of a state:

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- ▶ Discussed thoroughly in Singh et al. [13], applied in Mohamed and Rezende [9]
- ▶ Aims to find the state in which an actor is most able to travel to any arbitrary state
- ▶ Not obvious if the information theoretic definition is suitable
- ▶ Does not consider the how rewards may be assigned in the future

Quantitative Measures for Intrinsic Motivation III: Expected Reward

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 - ▶ Agent learns about the environment and transition mechanics as in traditional RL *and*
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 - ▶ Agent learns about the environment and transition mechanics as in traditional RL *and*
 - ▶ Agent learns about distribution of rewards and how to improve heuristic empowerment by computing the posterior distribution of rewards and updating the iER.
- ▶ Originally motivated by goal to show that the empowerment metric was Bayes-optimal the class of problems in Mohamed and Rezende [9]
 - ▶ It turns out this is false

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