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HW 9

"I pledge my honor that I have abided by the Stevens Honor System."

Problem 1

coin tossed 9 times → HHH TTTT H T

3H = Run

4T = Run

H = Run

T = Run

let X = r.v = # of Runs

Tossed 3 times

total outcomes = $2^3 = 8$

probability 1/8

outcome	# Runs x	Prob
HHH	1	1/8
HHT	2	1/8
HTH	3	1/8
HTT	2	1/8
THH	2	1/8
THT	3	1/8
TTH	2	1/8
TTT	1	1/8

$P(X=1) = 1/8 + 1/8 = 1/4$

$P(X=2) = 1/8 + 1/8 + 1/8 + 1/8 = 1/2$

$P(X=3) = 1/8 + 1/8 = 1/4$

x	1	2	3
$P(X=x)$	1/4	1/2	1/4

Expected # Runs

$$E(X) = \sum_{x=1}^{3} x\,P(X=x)$$

$$= 1(1/4) + 2(1/2) + 3(1/4)$$

$$= 1/4 + 1 + 3/4$$

$\boxed{= 2}$ expected # of Runs

Problem 2

Dice

X appears $\quad$ variance(x) = ?

$$\sum_{i=1}^{6} \frac{(x_i - \bar{x})^2}{6}$$

mean = 3.5

$$\frac{(1-3.5)^2 + (2-3.5)^2 + (3-3.5)^2 + (4-3.5)^2 + (5-3.5)^2 + (6-3.5)^2}{6}$$

$= 17.5/6$

$$Var(X) = E(x^2) - E(X)^2$$

$$E(x^2) = \sum_{k=1}^{6} k^2/6 = \frac{1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2}{6}$$

$= 91/6$

$$Var(X) = E(x^2) - E(x)^2$$

$$= 91/6 - E(x)^2$$

$$E(x)^2 = 1(1/6) + 2(1/6) + 3(1/6) + 4(1/6) + 5(1/6) + 6(1/6)$$

$$= (21/6)^2$$

$$Var(x) = 91/6 - (21/6)^2 = \frac{105}{36}$$

$$\boxed{\text{Variance of } X = \frac{105}{36} = 2.9167}$$

Problem 3

$Z = (X, Y)$

r.v. $A = X^2$ $B = Y^2$

$A = ?$
$B = ?$
$AB = ?$

joint density

$f(x,y) = 1$ $0 < x < 1$, $0 < y < 1$

Marginal of x

$f_x(x) = \int_0^1 f(x,y)\,dy$

$= \int_0^1 1\,dy = 1$

Marginal of y

$f_y(y) = \int_0^1 f(x,y)\,dx$

$= \int_0^1 1\,dx = 1$

$E(A) = E(x^2) = \int_0^1 x^2 f_x(x)\,dx$

A $\boxed{= 1/3}$

$E(B) = E(y^2) = \int_0^1 y^2 f_y(y)\,dy$

B $\boxed{= 1/3}$

$E(AB) = E(x^2 y^2)$

$= \int_0^1 \int_0^1 x^2 y^2 f(x,y)\,dx\,dy$

AB $\boxed{= 1/9}$

Problem 4

X - exponent dist. r.v. λ param

Expected X = ?

$X \sim \exp(\lambda)$

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

expected value

$$E(x) = \int x f(x)\, dx$$

$$= \int_0^\infty x \lambda e^{-\lambda x}\, dx \qquad : \int_0^\infty e^{ax} x^{n-1}\, dx = \frac{\Gamma n}{a^n}$$

$$= \lambda \int_0^\infty x^{2-1} e^{-\lambda x}\, dx$$

$$= \lambda \frac{\Gamma 2}{\lambda^2} \qquad : \text{property}$$

$$= \frac{\lambda \times 1!}{\lambda^2} = \lambda / \lambda^2$$

$$E(x) = 1/\lambda$$

$$\boxed{\text{expected value of } X = 1/\lambda}$$

Problem 5

r.v. X

Cauchy pdf

$f_X(x) = \frac{a}{\pi} \frac{1}{a^2+x^2}$

a = density paramt

expected X = ?

$F(x) = \frac{a}{\pi} \cdot \frac{1}{a^2+x^2} \quad -\infty < x < \infty$

$E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \int_{-\infty}^{\infty} x \cdot \frac{a}{\pi} \cdot \frac{1}{a^2+x^2}\,dx$

$= \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{x}{a^2+x^2}\,dx$

$u = x^2 + a^2 \rightarrow \frac{du}{dx} = 2x \rightarrow dx = \frac{1}{2x}\,du$

limit

$u = -\infty$ to ∞

$E(x) = \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{x}{u} \cdot \frac{1}{2x}\,du = \frac{a}{2\pi} \int_{-\infty}^{\infty} \frac{1}{u}\,du$

$= \frac{a}{2\pi} \left[\log_e u\right]_{-\infty}^{\infty} = \frac{a}{2\pi} \left[\log(x^2+a^2)\right]_{-\infty}^{\infty}$

$= \frac{a}{2\pi} \left[\log \infty - \{\log(-\infty)\}\right]$

undefined

$E(x) = \frac{a}{2\pi} [\infty - \text{undefined}]$

$\boxed{\text{expected value of } X \text{ is undefined}}$