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"I pledge my honor that I have abided by the Stevens Honor System."

# Homework 4

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**Problem 1** — An elevator car has three occupants, and occupants can get out of the elevator at three different floors. Assuming that each person acts independently, and that each person is equally likely to exit on any of the three floors, find the probability that exactly one person gets out on each of the floors.

$\binom{3}{3}$ = ways to chose floors for all 3

3! ways to permute people

$\binom{3}{3}3!$ have no 2 on same floor

Probability of ↗ = $\dfrac{\binom{3}{3}3!}{3^3} = \dfrac{\left(\frac{3!}{3!(3-3)!}\right)3!}{3^3} = \dfrac{\frac{3\cdot2\cdot1}{3\cdot2\cdot1\cdot1}\cdot\frac{3\cdot2\cdot1}{1}}{3\cdot3\cdot3} = \dfrac{2}{9} = 0.22$

Occupants can get off any of 3 floors

$3\cdot3\cdot3 = 3^3$

$3\cdot2\cdot1$ ways floors are all different

$\dfrac{3\cdot2\cdot1}{3\cdot3\cdot3} = \dfrac{2}{9} = 0.22$ probability all exit on different floors



$\binom{n}{k} = \dfrac{n!}{k!(n-k)!}$

**Problem 2** — Suppose you have n pairs of socks in a drawer and you pick out k socks. What is the average number of pairs of socks that you will have if you repeat this experiment a large number of times?

n pairs of socks

k socks picked

Avg # of pairs repeated?

$p = \frac{k}{n}$

Binomial $(n, p)$

$E(x) = np = n\frac{k}{n} = k$

k socks

picking k socks from n pairs = 2n total socks

$\binom{2n}{k} = \frac{2n!}{(2n-k)!\,k!}$ distinct

?

**Problem 3** — In a bag of $N$ coins one is known to be a 2-headed coin (that is, both faces are heads), and the others are all normal coins. You draw a coin from the bag at random and toss it $k$ times. You get all heads. At what $k$ you decide it is the 2-headed coin?

N coins

1 - 2headed

Toss k times

$A = P(\text{you pick 2headed coin}) = 1/N$

$B = P(k \text{ heads tossed}) = \left(\frac{1}{2}\right)^k$

$$\frac{1}{N} = \left(\frac{1}{2}\right)^k$$

$$k = \frac{\log(N)}{\log(2)} = \log_2(N)$$

if $k = 10 \rightarrow \log_2 10 = 3.3 \rightarrow$ Min $4^{th}$ toss/heads/ you can decide it's 2 head

$k = 1000 \rightarrow \log_2 1000 = 9.96 \rightarrow 10^{th}$ heads you can decide

**Problem 4** — Starting at a fixed time, we observe the gender of each newborn child at a certain hospital until a boy (B) is born. Let p = P(B), assume that successive births are independent, and define the random variable X by X = number of births observed. What is the probability mass of X?

1st child boy

$p(1) = P(X=1)$
$= P(B)$
$= p$

2nd child boy

$p(2) = P(X=2)$
$= P(GB)$
$= P(G) \cdot P(B)$
$= (1-p)p$

3rd child boy

$p(3) = P(X=3)$
$= P(GGB)$
$= P(G) \cdot P(G) \cdot P(B)$
$= (1-p)^2 p$

$$p(x) = \begin{cases} (1-p)^{x-1} p & \text{if } x = 1, 2, 3 \ldots \\ 0 & \text{otherwise} \end{cases}$$

$p = P(B)$
$1-p = P(G)$

p between 0-1

$p = 0.5$

$$p(x) = (1-p)^{x-1} p$$

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**Problem 5** – A store carries flash drives with 1, 2, 4, 8, or 16 GB of memory. The table gives the distribution of X = the amount of memory in a purchased drive:

| x | 1 | 2 | 4 | 8 | 16 |
|---|---|---|---|---|---|
| P(x) | .05 | .1 | .35 | .4 | .1 |

Determine F(x), the cumulative distribution function of X.

| x | P(x) | $F(x) = P(X \leq x)$ | |
|---|---|---|---|
| x = 1 | .05 | 0+.05 = 0.05 | $X \leq 1$ |
| x = 2 | .1 | .05+.1 = 0.15 | $X \leq 2$ |
| x = 4 | .35 | .15+.35 = 0.50 | $X \leq 4$ |
| x = 8 | .4 | .50+.4 = 0.90 | $X \leq 8$ |
| x = 16 | .1 | .90+.1 = 1.00 | $X \leq 16$ |