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HW 8

"I pledge my honor that I have abided by Stevens Honor System."

Problem 1

Let $Z = X_1 + X_2$

$$P(Z=z) = \sum_{k=-\infty}^{\infty} P(X_1=k) \cdot P(X_2 = z-k)$$

$$P(Z = X_1 + X_2 = 0) = P(X_1=0) \cdot P(X_2=0)$$

$$P(Z = X_1 + X_2 = 0) = P(X_1 = 0) \cdot P(X_2=0) =$$

$$= 1/8 \cdot 1/8 = 1/64$$

$$P(X_1 + X_2 = 1) = P(X_1=0) \cdot P(X_2=1) + P(X_1=1) \cdot P(X_2=0)$$

$$= 1/8 \cdot 3/8 + (3/8 \cdot 1/8) = 3/32$$

$$P(Z = X_1 + X_2 = 2) = P(X_1=0) \cdot P(X_2=2) + P(X_1=1) \cdot P(X_2=1) +$$

$$P(X_1=2) \cdot P(X_2=0)$$

$$= 1/8 \cdot 1/2 + (3/8 \cdot 3/8) + (1/2 \cdot 1/8) = 17/64$$

Problem 2

r.v. X $\quad P(x=0) = P(x=1) = P(x=2) = 1/3$

r.v. Y $\quad P(Y=3) = 1/4$

$P(Y=4) = 3/4$

a) $Y+X = \{3, 4, 5, 4\}$

$P(3) = 1/3 \cdot 1/4 = 1/12$

$P(4) = 1/3 \cdot 1/4 + (3/4 \cdot 1/3) = 1/3$

$P(5) = 1/4 \cdot 1/3 + (3/4 \cdot 1/3) = 1/3$

$P(6) = 3/4 \cdot 1/3 = 1/4$

$Y+X$	3	4	5	6
$P(Y+X)$	$1/12$	$1/3$	$1/3$	$1/4$

b) $Y-X = \{1, 2, 3, 4\}$

$P(1) = 1/4 \cdot 1/3 = 1/12$

$P(2) = 1/3 \cdot 3/4 + (1/3 \cdot 1/4) = 1/3$

$P(3) = 1/3 \cdot 3/4 + (1/3 \cdot 1/4) = 1/3$

$P(4) = 1/3 \cdot 3/4 = 1/4$

$Y-X$	1	2	3	4
$P(Y-X)$	$\frac{1}{12}$	$1/3$	$1/3$	$1/4$

Problem 3

$X \sim U[0,1]$

$Y \sim U[0,1]$

$X+Y \sim U[0,1] + U[0,1]$

$X+Y \sim U[0+0, 1+1]$

$X+Y \sim U[0,2]$

pdf of sum $[0,2]$

$$pdf \begin{cases} f(x) & 0<x<1 \\ & 0<y<1 \\ 0 & \text{otherwises} \end{cases}$$

$$f(x) = \frac{1}{2-0} = 1/2$$

$$pdf = \begin{cases} 1/2 & 0<x<1 \\ & 0<y<1 \\ 0 & \text{otherwise} \end{cases}$$

Problem 4

let X, Y, and Z = X + Y

f_X f_Y f_Z

densities

$$f_X(x) = f_Y(x) = \begin{cases} \lambda(e^{-\lambda x}) & \text{if } x \geq 0, \\ 0 & \text{otherwise} \end{cases}$$

~~Problem 5~~

Problem 5

$X \sim \text{Gama}(\lambda, s)$

$Y \sim \text{Gama}(\lambda, t)$

joint pdf

$$f_{XY}(x, y) = f_x(x) \cdot f_Y(y)$$

$$= \frac{\lambda^{s+t}}{\Gamma(s)\Gamma(t)} e^{-\lambda(x+y)} x^{s-1} \cdot y^{t-1} \; ; \; x, y > 0$$