Julia Nelson

"I pledge my honor that I have abided by the Stevens Honor System."

Problem 1

r.v. X normally distr. 0 mean

st. d. = 1

$\mu = 0$

r.v. $Y = 3X + 2$

$\sigma^2 = 1$

probability density function of Y?

letting $X \sim N(0,1)$ ← given

$X \sim N(0,1)$

f_Y

$\varepsilon f_Y(a) \approx P\{a -$

$$F_X = P(X \leq x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$$

Theorem

if $X \sim N(\mu_x, \sigma_x^2)$ $Y = aX + b$

then $Y \sim N(\mu_Y, \sigma_Y^2)$ where

$$\mu_Y = a\mu_X + b, \quad \sigma_Y^2 = a^2 \sigma_x^2$$

$$\mu_Y = 3(0) + 2 \quad , \quad \sigma_Y^2 = 3^2(1^2)$$

$$= 2 \qquad\qquad = 9$$

$$Y \sim N(2, 9)$$

find PDF from here

(ran out of time)

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Problem 2

20 employees < 15 women, 5 men

pick 1 for Day
1 for Night
1 for Reception Desk
1 for Phones

a) How many ways can we do that?

pick 4 spots from 20 people

$$\frac{n!}{(n-r)!} = \frac{20!}{(20-4)!} = 116280 \text{ ways}$$

checking
20 ways to pick 1st
19 ways to pick 2nd
18 ways to pick 3rd
17 ways to pick 4th

using Multiplication rule we have
$20 \times 19 \times 18 \times 17$
which $= \boxed{116{,}280 \text{ ways}}$

b) How many ways if we want exactly 1 man?

Chose 1 man from 5 total
chose 4 women from 15 total

$$\binom{n}{r} = \binom{5}{1}\binom{15}{4} = \frac{5!}{1!(4!)}\left(\frac{15!}{4!(11!)}\right)$$

$$= 5(1365)$$

$\boxed{= 6825}$ ways to pick exactly 1 man

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A=

Problem 3

1) P(Win against Begginner) = 90% = .9
P(Win against Intermediate) = 50% = .5
P(win against Master) = 30% = .3

Equal chance for skill level of opponent = $\frac{1}{3}$ P(B) P(I) P(M)

whats probability you'll win?

Total probability that you Win...

$$P(\underset{\text{TOTAL}}{\text{You Win}}) = \frac{0.9 + 0.5 + 0.3}{3} = \frac{1.7}{3} = 0.567$$

checking ↓

W ⇒ event of winning
B ⇒ event opponent Begginner
I ⇒ event opp Intermed
M ⇒ event opp Maste

Bayes

$P(W|B) = 9/10$
$P(W|I) = 5/10$
$P(W|M) = 3/10$

$$P(W) = P(W|B)P(B) + P(W|I)P(I) + P(W|M)P(M)$$

$$= \left(\frac{9}{10} \cdot \frac{1}{3}\right) + \left(\frac{5}{10} \cdot \frac{1}{3}\right) + \left(\frac{3}{10} \cdot \frac{1}{3}\right)$$

$$= \frac{9}{30} + \frac{5}{30} + \frac{3}{30}$$

$$= 17/30$$

$\boxed{= 0.567}$ chance you win

56.7%

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Problem 4

Roll die until you see 6

STOP

N = # rolls before you stop NOT BINOMIAL

NO Fix # trials

a) $P(N=10)$?

$P(N=10) = P(\text{NOT } 6)^{n-1} \cdot P(\text{rolled } 6)$

$P(\text{rolling } 6) = 1/6$

$P(\overline{\text{rolling } 6}) = 5/6$

We want "No 6" (N-1) times + "yes 6" on N^{th} roll

P(9 Failures + 1 success)

$P(N=10) = \left(5/6\right)^9 \cdot \left(\frac{1}{6}\right) = 0.0323$

$\boxed{= 3.23\% \text{ that 6th appears for } 1^{st} \text{ time on } 10^{th} \text{ roll}}$

b) $P(1 \text{ on } 1^{st} \mid P(N=10))$?

$P(1 \text{ on } 1^{st} \text{ roll}) = \frac{1}{6}$

$P(N=10) = 0.0323$

rolls are independent from eachother

given that we won't get a 6 doesn't change outcome probability of rolling a die so

Bayes

$P(1 \text{ on } 1^{st} \mid N=10) = P(1 \text{ on } 1^{st})$

$\boxed{= (1/6) \text{ chance of rolling 1 on } 1^{st} \text{ roll}}$

$P(B \mid A) = \frac{P(A|B)P(A)}{P(B)}$

$= \frac{\left(\frac{1}{6}\left(\frac{5}{6}\right)^8\left(\frac{1}{6}\right)\right)0.0323}{(1/6)} =$

$\rightarrow (5/6)^8(1/6)(.0323) = 0.233 = \boxed{23.3\% \text{ chance}}$

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Problem 5

cust rate = 2 / minute $\doteq \lambda$ rate

P(No customers between 940 pm 955 pm AND 3 ARRIVE between 955 ~~to~~ 10pm

customer rate

$(9{:}40 - 9{:}55) = 15$ mins

Poission distribution

$$\frac{(\lambda t)^n (e)^{-\lambda t}}{n!} \qquad n = \#\text{ cust} \quad t = \text{time}$$

Probability event A

P(No cust b/w 9:40-9:55) =>

$$P(N(15)^0) = \frac{(2 \cdot 15 \text{mins})^0 (e)^{-2(15)}}{0!} = \frac{e^{-30}}{1}$$

$$= \frac{1}{e^{30}}$$

Probability event B

P(3 customers b/w 955 - 10) =>

$t = 5$ mins
$\lambda = 2$

$$P(N5^3) = \frac{(2 \cdot 5)^3 \cdot e^{-2(5)}}{3!} = \frac{1000 \cdot e^{-10}}{6}$$

$$= \frac{1000}{6e^{10}} = \frac{500}{3e^{10}}$$

Probability both

$$P(\text{None b/w 940-955 AND 3 b/w 955-10}) = \frac{1}{e^{30}} \cdot \frac{500}{3e^{10}} = \boxed{\frac{500}{3e^{40}} \text{ probability}}$$