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Give the complexity of the following functions. Choose the most appropriate notation from among O , θ , and Ω .

```
1. void function1(int n) {
    for (int i = 1; i <= n; i++) {
        for (int j = i; j <= n; j += 2) {
            cout << "*";
        }
    }
}
```

Answer: $\theta(n^2)$

```
2. void function2(int n) {
    int count = 0;
    for (int i = 1; i * i <= n; i++) {
        count++;
    }
    cout << count;
}
```

Answer: $\theta(\text{sqrt}(n))$

```
3. void function3(int n) {
    int count = 0;
    for (int i = n/2; i <= n; i++) {
        for (int j = 1; j + n/2 <= n; j++) {
            for (int k = 1; k <= n; k *= 2) {
                count++;
            }
        }
    }
    cout << count;
}
```

Answer: $\theta((n^2) * \lg(n))$

```
4. void function4(int n) {
    int count = 0;
    for (int i = n/2; i <= n; i++) {
        for (int j = 1; j <= n; j *= 2) {
            for (int k = 1; k <= n; k *= 2) {
                count++;
            }
        }
    }
    cout << count;
}
```

Answer: $\theta(n(\lg(n))^2)$

```

5. void function5(int n) {
    if (n % 2 == 0) {
        return;
    }
    for (int i = 1; i <= n; i++) {
        for (int j = 1; j <= n; j++) {
            cout << "*";
            break;
        }
    }
}

```

Answer: $O(n)$

```

6. void function6(int n) {
    int count = 0;
    for (int i = 1; i <= n/2; i++) {
        for (int j = 1; j <= n/3; j++) {
            for (int k = 1; k <= n/4; k++) {
                count++;
            }
        }
    }
    cout << count;
}

```

Answer: $O(n^2)$

```

7. void function7(int n) {
    for (int i = 1; i <= n; i++) {
        for (int j = 1; j <= n; j += i) {
            cout << "*";
        }
    }
}

```

Answer: $\theta(n \log(n))$

```

8. void function8(int n) {
    int i = 1, s = 1;
    while (s <= n) {
        i++;
        s += i;
        cout << "*";
    }
}

```

Answer: $\theta(\text{sqrt}(n))$

9. Processing Arrays

- Suppose you have an unsorted array of integers of length n and want to sum all the elements inside it. What is the running time of your algorithm? $\theta(n)$
- Suppose you have an unsorted array of integers of length n and want to determine if all the values inside are positive. What is the running time of your algorithm? $O(n)$

- c. Suppose you have a sorted array of integers of length n and want to determine the median value. What is the running time of your algorithm? $\theta(1)$

10. TRUE T/F $f(n) = 3n^2 + 4n + 2 \in \theta(n^2)$

If true, prove it by giving *integral* values for the required constants c_1 , c_2 , and n_0 . Choose the tightest values possible for the c_1 and c_2 constants. If false, show the contradiction.

#10

True. $f(n) = 3n^2 + 4n + 2 \in \theta(n^2)$

$$c_1 \cdot n^2 \leq 3n^2 + 4n + 2 \leq c_2 \cdot n^2$$

Tightest constraints

$$c_1 \cdot n^2 \leq 3n^2 + 4n + 2$$

$$\text{let } c_1 = 3$$

$$3n^2 \leq 3n^2 + 4n + 2$$

$$0 \leq \cancel{3n^2} 4n + 2$$

$$0 \leq 4n + 2 \quad (\text{for } \forall n > 0)$$

$$c_2 \cdot n^2 \geq 3n^2 + 4n + 2$$

$$\text{let } c_2 = 4$$

$$4n^2 \geq 3n^2 + 4n + 2$$

$$n^2 \geq 4n + 2$$

$$\text{if } n=1 \quad 1 \geq 4+2 \quad X$$

$$n=2 \quad 4 \geq 8+2 \quad X$$

$$n=3 \quad 9 \geq 12+2 \quad X$$

$$n=4 \quad 16 \geq 16+2 \quad X$$

$$n=5 \quad 25 \geq 20+2$$

$$25 \geq 22 \quad \checkmark$$

$$n^2 \geq 4n + 2 \quad (\forall n \geq 5)$$

$$n_0 = 0 \text{ or } \textcircled{5}$$

$$\boxed{c_1 = 3 \quad c_2 = 4 \quad n_0 = 5}$$