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Homework 2

"I pledge my honor that I have abided by the Stevens Honor System."

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```
Mystery(n)
  // input non-neg # int n
  S ← 0
  for i ← 1 to n do
      S ← S + i*i
  return S
```

(a) What does this alg compute?

sum of all ints 0→n squared

$\sum_{i=1}^{n} i^2$

(b) Whats the basic operation?

Multiplication

(c) How many time basic op executed?

n times

(d) Whats the efficiency class?

$\Theta(n)$ → loop executes n times

(e) Suggest impovement?

instead of loop use sum of squares formula → $\frac{n(n+1)(2n+1)}{6}$

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ⓐ $x(n) = x(n-1)+5 \quad \forall n > 1$

$x(1) = 0$

$= x(n-2)+5+5$

$= x(n-i)+5i$

$x(n-2)+5+5 = x(n-2)+2(5)$

$x(n-3)+5+5+5 = x(n-3)+3(5)$

$x(n-i)+5i = 0$ if $i = n-1$

$= x(n-(n-1))+5(n-1)$

$= x(1)+5(n)-5$

$= x(1)+5n-5$

$= x(1)+(n-1)\cdot 5$

$= 0+5(n-1)$

$x(n) = 5(n-1)$

ⓑ $x(n) = 3x(n-1) \quad \forall n > 1$

$x(1) = 4$

$= 3(3)x(n-2)$

$= 3^2 x(n-2)$

$= 3^2(3x(n-3))$

$= 3^3 x(n-3)$

$3^i x(n-i)$ for $i = n-1$

$3^{(n-1)} x(n-(n-1))$

$= 3^{n-1} x(1)$

$= 3^{n-1}(4)$

$= 4(3^{n-1})$

ⓒ $x(n) = x(n-1)+n \quad \forall n > 0$

$x(0) = 0$

$= x(n-1)+n$

$= x(n-2)+(n-1)+n$

$= x(n-3)+(n-2)+n-1)+n$

$= x(n-i)+(n-i+1)+(n-i+2)+\dots+n$

$= x(0)+1+2+\dots+n$

$\frac{n(n+1)}{2} = \frac{n^2+n}{2}$

(d) $x(n) = x\left(\frac{n}{2}\right) + n \quad \forall n > 1$

$x(1) = 1$

solve for $n = 2^k$

$x(2^k) = x(2^{k-1}) + 1$

$x(2^{k-1}) = x(2^{k-2}) + 1$

$x(2^k) = x(2^{k-2}) + 2$

$x(2^{k-2}) = x(2^{k-3}) + 1$

$x(2^k) = x(2^{k-3}) + 3$

$x(2^k) = x(2^{k-i}) + i$

$2^{k-i} = 1$

$2^{k-i} = 2^0$

$k - i = 0$

$i = k$

$x(2^k) = x(2^{k-k}) + k$

$x(2^k) = 1 + k$

$x(n) = 1 + \lg(n)$

$= x(2^{k-k}) + 2^1 + 2^2 + \dots + 2^k = 2^{k+1} - 1$

$= 2(2^k) - 1$

$= 2n - 1$

(e)

$x(n) = x\left(\frac{n}{3}\right) + 1 \quad \forall n > 1$

$x(1) = 1$

solve for $n = 3^k$

$x(3^k) = x\left(\frac{3^k}{3}\right) + 1$

$= x(3^{k-1}) + 1$

$= x(3^{k-2}) + 1 + 1 \quad \rightarrow x(3^{k-2}) + 2$

$= x(3^{k-3}) + 1 + 1 + 1 \quad \rightarrow x(3^{k-3}) + 3$

$= x(3^{k-i}) + i$

$n = 3^k$

$k = \log_3 n$

$\frac{n(n+1)(2n+1)}{6}$

?

$i = k$

$x(3^{k-k}) + k$

$= x(3^0) + k$

$= x(1) + k$

$= 1 + k$

$= 1 + \log_3 n$

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a

\# times Mult executed

$$x(n) = x(n-1) + 2 \qquad x(1) = 0$$

$$= x(n-2) + 2 + 2$$

$$= x(n-3) + 2 + 2 + 2$$

$$\Rightarrow x(n-i) + 2i$$

$$i = n-1$$

$$x(n-(n-1)) + 2(n-1)$$

$$= x(1) + 2(n-1)$$

$$= 0 + 2(n-1)$$

$$= 2(n-1)$$

b

Compared to the non-rec alg for this sum, both have same # of multiplications.
The recursive has more constants of calling than the straight forward algorithm, making it the worse performer.