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Give the complexity of the following functions. Choose the most appropriate notation from among O , θ , and Ω .

```
1. void function1(int n) {
    for (int i = 1; i <= n; i++) {
        for (int j = i; j <= n; j += 2) {
            cout << "*";
        }
    }
}
```

Answer: $\theta(n^2)$... $(n \times \lfloor n/2 \rfloor)$

```
2. void function2(int n) {
    int count = 0;
    for (int i = 1; i * i <= n; i++) {
        count++;
    }
    cout << count;
}
```

Answer: $\theta(\sqrt{n})$... $\lceil \sqrt{n} \rceil$ steps

```
3. void function3(int n) {
    int count = 0;
    for (int i = n/2; i <= n; i++) {
        for (int j = 1; j + n/2 <= n; j++) {
            for (int k = 1; k <= n; k *= 2) {
                count++;
            }
        }
    }
    cout << count;
}
```

Answer: $\theta((n/2)^2 \log n)$

```
4. void function4(int n) {
    int count = 0;
    for (int i = n/2; i <= n; i++) {
        for (int j = 1; j <= n; j *= 2) {
            for (int k = 1; k <= n; k *= 2) {
                count++;
            }
        }
    }
    cout << count;
}
```

Answer: $\theta((n/2)(\log n)^2)$

5. **void function5(int n) {**
 if (n % 2 == 0) {
 return;
 }
 for (int i = 1; i <= n; i++) {
 for (int j = 1; j <= n; j++) {
 cout << "*";
 break;
 }
 }
}

Answer: $O(n^2)$

6. **void function6(int n) {**
 int count = 0;
 for (int i = 1; i <= n/2; i++) {
 for (int j = 1; j <= n/3; j++) {
 for (int k = 1; k <= n/4; k++) {
 count++;
 }
 }
 }
 cout << count;
}

Answer: $\theta(n^3)$

7. **void function7(int n) {**
 for (int i = 1; i <= n; i++) {
 for (int j = 1; j <= n; j += i) {
 cout << "*";
 }
 }
}

Answer: $\theta(n^2)$

8. **void function8(int n) {**
 int i = 1, s = 1;
 while (s <= n) {
 i++;
 s += i;
 cout << "*";
 }
}

Answer: $\theta(n)$

9. Processing Arrays

- Suppose you have an unsorted array of integers of length n and want to sum all the elements inside it. What is the running time of your algorithm? $O(n)$
- Suppose you have an unsorted array of integers of length n and want to determine if all the values inside are positive. What is the running time of your algorithm? worst: $O(n)$ best: $O(1)$
- Suppose you have a sorted array of integers of length n and want to determine the median value. What is the running time of your algorithm? $O(n/2)$ (half is median.. length/2)

10. TRUE T/F $f(n) = 3n^2 + 4n + 2 \in \theta(n^2)$

If true, prove it by giving *integral* values for the required constants c_1 , c_2 , and n_0 . Choose the tightest values possible for the c_1 and c_2 constants. If false, show the contradiction.

True

$$3n^2 + 4n + 2 \leq cn^2 \quad \text{for } n \geq n_0$$

~~$$3n^2 + 4n + 2 \leq cn^2$$~~
$$f(n) = 3n^2 + 4n + 2$$

$$3 + \frac{4}{n} + \frac{2}{n^2} \leq c$$

$$c_1 n^2 \leq f(n) \leq c_2 n^2$$

 c_1 :

$$3 + \frac{4}{n} + \frac{2}{n^2} \geq c_1$$

As $n \rightarrow \infty$

$$\frac{4}{n} \rightarrow 0 \text{ and } \frac{2}{n^2} \rightarrow 0$$

So 3 is tightest

value of c_1 c_2 :

$$3 + \frac{4}{n} + \frac{2}{n^2} \leq c_2$$

As $n \rightarrow \infty$ $\frac{4}{n} \rightarrow 0$ and $\frac{2}{n^2} \rightarrow 0$ max val when $n = n_0$

$$c_2 = 3 + \frac{4}{n} + \frac{2}{n^2}$$

$$= 3 + \frac{4}{n_0} + \frac{2}{n_0^2}$$

 $n_0 = 1$

$$= 3 + \frac{4}{1} + \frac{2}{1^2} = 9$$

$$c_1 = 3 \quad c_2 = 9 \quad n_0 = 1$$