PEP 151 Lab 1
Determining the Mass of Jupiter

In this lab exercise, we will use the observed orbital properties of Jupiter's moons to determine Jupiter's mass. You will be expected to do the following:
I) Understand the method.
II) Gather information on the orbital period and orbital distance of Jupiter's moons.
III) Organize, format, and analyze data gathered using any plotting/programming tool of your choosing.
IV) Document what you did with brief descriptions into a report.

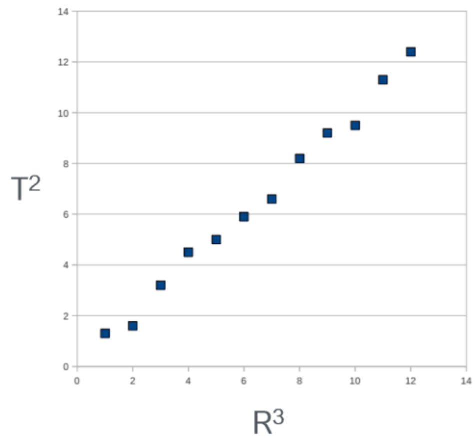
Below we will discuss each task in detail.

Section I. Method

In class we discussed the Newtonian version of Kepler's third law that relates the orbital period T, ***semimajor*** axis of the orbit R (we're using the notation "R" instead of "a" here to avoid confusion of another "a" that you will see later), and mass: $T^2 = \frac{4\pi^2}{G(M1+M2)} R^3$. For a low mass object orbiting a much more massive object, the mass of the low mass object can be ignored, then the equation can be reduced to $T^2 = \frac{4\pi^2}{GM} R^3$. If you then make observations of the satellite's orbital period and distance, you can easily find out the mass of the central object using the equation discussed here. We will apply this method to figure out the mass of Jupiter in this lab.

In theory, it seems you only need to observe one moon of Jupiter, and calculate the mass of the Jupiter from this single moon. However, in practice, when you observe a sample of Jupiter's moons and calculate the mass of Jupiter from each moon, you won't get exactly the same mass due to perturbations, experimental uncertainties etc. Therefore, we need to figure out a way to estimate the mass of Jupiter from a sample of moons instead of just one.

Let's say we have observed a set of N moons of Jupiter's. Each of them would obey the equation mentioned above, i.e. $T_i^2 = \frac{4\pi^2}{GM} R_i^3$. If we were to plot T_i^2 vs. R_i^3, we would see something like this:



As mentioned before, in an ideal world every data point will give you exactly the same mass, which means all the data points would fall exactly on a straight line (y=ax+b) and the slope of the line “a” would be $\frac{4\pi^2}{GM}$.

Now, since in reality not all data points fall on a straight line exactly, we will fit a straight line through the data points so that the slope of the best-fitted line provides the best mass estimate. We will use a method called the “least square fit”. The idea is to find the values of “a” (slope) and “b” (intercept) so that the deviation of the data points from the fitted line is minimized. The mathematical detail of this method is provided in a separate file (“LeastSquareFit.pdf”). If you are not math inclined, you don’t have to understand every single detail in that file – you only need to have a general idea so that you can perform the fit using the functions in excel in a later step. But you are a math fan, feel free to read through the details and write your own code to do the fitting.

Section II. Data gathering and organization

Gather data on Jupiter’s moons from books or resources on the internet (you need to cite the source). In order to get a good line fit, you need at least 15 moons. You can pick any moons you like, but make sure to include the 4 Galileo’s moons - Ganymede, Callisto, Io and Europa.

If you are comfortable with scripting/coding, feel free to format and analyze data with any tool you prefer. Otherwise, proceed with the following instructions on how to perform the task in excel.

Create an excel file, enter the name of each moon in column A. Enter the ***semimajor*** axis in column B; make sure they’re all in the same unit and note down the unit. Enter the orbital period in column C; again make sure they’re all in the same unit and note down the unit. If the period from the source you used is given as a negative number, disregard the minus sign as it just means the revolution is retrograde.

In order to use the equation discussed, all quantities need to be in **SI units**. If the quantities you enter in columns B and C are already in SI units, no unit conversion is needed. However, mostly likely they are not. Therefore, let's proceed under the assumption that unit conversion is needed. ***In the following operations, we'll assume the semimajor axes in column B are all in km, and the orbital periods in column C are all in days. If the units from your source are different, make adjustments to the conversion factors.***

In column D (currently unoccupied), in cell D1 (corresponding to the first moon) type

$$= \mathrm{B1} * 1000$$

and press enter. This performs the conversion from km (column B) to m (column D) for the first moon. Now click on cell D1 so that the cell is highlighted with a thick rectangle. Hover the cursor over the cell until you see a little cross sign, then move the cursor towards the lower right corner of the cell, hold down the left mouse button and drag the rectangle down column D all the way to the last row that contains your data. All the cells in column D now should be filled with values corresponding to cells in column B with the conversion from km to m.

Now let's repeat the procedure to do unit conversion for the orbital period in column C (in days in this example). Go to column E (unoccupied for now), in cell E1 type

$$= \mathrm{C1} * 24 * 60 * 60$$

and press enter. This performs the conversion from days (column C) to seconds (column E) for the first moon. Now just like what we did before with column D, click on cell E1, hover over, and drag the rectangle all the way down to the last useful cell in column E to convert units for all other moons.

Now we have semimajor axes R in meters stored in column D, and orbital period T in seconds stored in column E. Next we want to calculate the values of R^{3} in column F, and T^{2} in column G.

To do that, in cell F1 type

$$= \mathrm{D1} * \mathrm{D1} * \mathrm{D1}$$

and in field G1 type

$$= \mathrm{E1} * \mathrm{E1}$$

to do the calculation for the first moon. Then follow the same procedure as before to populate other cells in column F and G automatically for the other moons. Now we are ready to plot T^2 vs. R^3 and perform line fitting.

Section III. Data Analyses

As stated before, feel free to write your own code to do the line fitting etc. if you prefer. Otherwise, we will proceed with the data we formatted in the excel file. The following instructions are specifically for Excel 2010. It should be easy to figure it out for other versions, as the procedures are essentially the same except that the button names and locations might be different. Further, there are a lot of tutorials on how to do least square fit in excel on the internet (e.g. if you search for “least square fit in excel” or “linear least square regression in excel” on youtube you’ll get lots of videos).

To create a plot of T^2 vs. R^3 in your excel data sheet, use cursors to select all cells in columns F and G that contain your data. From the menu bar at the top choose Insert –→Charts –→ Scatter and choose the graph that has a bunch of points with no lines. If everything works correctly, you should now have a plot of T^2 vs. R^3 that looks like the one shown in the method section above.

On the plot you just generated, left click on any point (that should highlight all points on the plot), then right click on any point and choose “Add a trendline” from the menu that appears. A pop-up window should appear, choose “Linear” and at the bottom check “Display equation on chart” and “Display R-squared value on chart”. You should now have a line through the data points on your plot with an equation that looks like e.g. y = 3E-16x − 3E-11 (these are just example values). This is the expression of the fitted line y = ax+b, where the slope “a” is 3e-16 and the intercept “b” is -3e-11.

Now notice the slope “a” displayed on the graph is rounded off to integer and thus not very accurate. To see a more accurate value for the slope, right click on the equation and from the pop-up menu choose “Format trendline label”. Choose Category –→ Scientific and set Decimal places field to 4. Close the window and your equation on the graph will be updated with more accurate values, e.g. y = 3.1461E−16x−3.2406E+ 11 (again these are just example values; the actual values will depend on your data).

We will only focus on the value of the slope "a", since as we discussed before the slope of the line is $\frac{4\pi^2}{GM}$. In this example, the mass of Jupiter can be calculated from $\frac{4\pi^2}{GM}$ = 3.1461E−16 (just remember to use SI units for G).

Now repeat the line fitting procedure to estimate the mass of Jupiter ***using only the four Galileo's moon***. You'll get another graph with a slightly different value of the slope.

Compare the two masses you estimated from all the moons in your data and from only Galileo's moons with the established mass of Jupiter in the literature. It is quite possible that the mass you estimated using only Galileo's moon is closer to the established value. The reason is that some of the moons you chose may be very small and therefore susceptible to perturbations, but they're treated with equal importance in our line fitting procedure as those larger moons. This can be taken care of by assigning different weights to the moons -- we will not do this here though.

Section IV. Report

Gather all your data, graphs, and mass estimate results into a report with a brief description of what you did.

Report structure:

a) Brief introduction: in your own words state the goal and methodology of this lab in a few sentences.

b) Cite your data source and include a table showing all the data you gathered (a screenshot of your excel sheet containing the data would be fine).

c) Show the two plots of T^2 vs. R^3 generated in your excel sheet with the high accuracy equation displayed, one for all moons one for only Galileo's moons. You need to label the axes of the plots.

d) Show your calculation of Jupiter's mass from the slope of the fitted line – again one for all moons and one for only Galileo's moons.

e) Compare your estimates with the established value in the literature.

Convert the report into one single pdf file and upload it to canvas.