

# Problem Set 6

10/24/19

## Problem 1

Prove  $L = \{a^i b^k c^l d^k \mid i, k \geq 0\}$  is NOT Context Free

Let  $L$  be a CFL

String  $s = a^p b^p c^p d^p \in L \rightarrow$  since  $\dots |s| \geq p$

$s = uvwxy$  where  $\forall i \geq 0$

let  $v$  be  $i \in a^p$   
and  $y \in b^p$   
1st Pump  
 $a^{p+n} b^{p+m} a^p b^p$   
which  $\notin L$

$uv^i wx^i y \in L$

$|uvx| \leq p$

$|vx| \geq 0$

$s = z \rightarrow uv^i xy^i z \in L$

$|vw| \leq p$  contains no  $a$ 's +  $c$ 's (both)  
or  $b$ 's and  $d$ 's (both)

$|vx| > 0$

so  $uv^0 xy^0 z = uvw$

which  $\notin L$

because either

$a$ 's  $<$   $c$ 's or  $c$ 's  $<$   $a$ 's

OR

$b$ 's  $<$   $d$ 's or  $d$ 's  $<$   $b$ 's

Not CFL

because CFLs cannot  
match 2 substrings of  
arbitrary length over  
alphabet of 2 or more symbols

## Problem 2

$$L = \{a^i b^k c^l d^k : i, k \geq 0\}$$

### PDA Description:

$i = \# \text{ of } a\text{'s}$

~~if~~

$k = \# \text{ of } b\text{'s}$

While reading  $i$  # of  $a$ 's

push  $i$  #  $a$ 's onto the Stack 1

Then read  $k$  # of  $b$ 's and push them onto Stack 2

Read # of  $c$ 's and for each  $c$ , pop 1  $a$  off of Stack 1.

If Stack 1 now =  $\emptyset$  Empty  
→ Move on to read  $d$ 's  
Else → Reject

Reading  $d$ 's, pop 1  $b$  from Stack 2 for each  $d$ .

If Stack 2 =  $\emptyset$ . Empty after all  $d$ 's read (popped) ~~reject~~.

→ Accept

if Stack 1 or 2 not empty  
→ Reject



### Problem 3

$$L_{add} = \{a^i b^{i+j} c^j : i, j \geq 0\}$$

CFG

~~Not CFL~~

$$S \rightarrow AB$$

$$A \rightarrow aAb | \epsilon$$

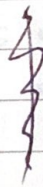
$$B \rightarrow bBc | \epsilon$$

~~CFG~~  $CFG = \{ (S, A, B), (a, b, c), (S \rightarrow AB, A \rightarrow aAb | \epsilon, B \rightarrow bBc | \epsilon) \}$

~~Not CFL~~

$$L_{mult} = \{a^i b^{ij} c^j : i, j \geq 0\}$$

Not CFL because  
cant multiply i and j  
to get correct output  
of b's.



Seq AB

# Problem 4

a)  $L = \text{The complement of } \{a^n b^n \mid n \geq 0\}$

$$L = \{a^i b^j \mid i > j\} \cup \{a^i b^j \mid i < j\} \cup \dots$$

$$\dots (a+b)^* b (a+b)^* a (a+b)^*$$

$$\Rightarrow L = \underbrace{\{a^i b^j \mid i > j\}}_A + \underbrace{\{a^i b^j \mid i < j\}}_B + \underbrace{(a+b)^* b (a+b)^* a (a+b)^*}_C$$

$$S \rightarrow ABC$$

$$A \rightarrow aAb \mid aA \mid a$$

$$B \rightarrow aBb \mid Bb \mid b$$

$$C \rightarrow XbXaX$$

$$X \rightarrow aX \mid bX \mid \epsilon$$

$$CFG = \{(S, A, B, C, X), (a, b), S, (S \rightarrow ABC, A \rightarrow \dots)\}$$

b)

$$L = \{x_1 \# x_2 \# \dots \# x_k \mid k \geq 1 \text{ for each } x_i \in \{a, b\}^* \text{ and for some } i \text{ and } j, x_i = x_j^R\}$$

Palindrome

or distinct  $i/j$  so  $x_i = x_j^R$

$$S \rightarrow S_1 \mid S_2$$

$$S_1 \rightarrow A \mid X \# A \mid A \# X$$

$$A \rightarrow aA \mid bAb \mid b \mid a \mid \epsilon$$

$$S_2 \rightarrow B \mid X \# B \mid B \# X$$

$$B \rightarrow aBa \mid bBb \mid \#X \#$$

$$X \rightarrow a \mid b \mid \# \mid \epsilon$$