# Problem Set 2

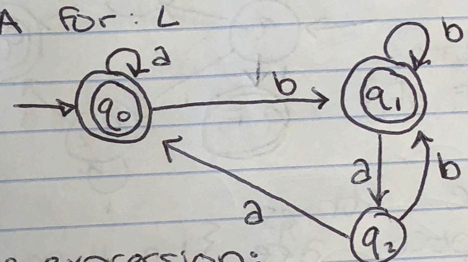
## Problem 1:

Give ~~a~~ regular ~~lang~~ expressions to generate the following langs.

a) $\{w \in \{a,b\}^* : w \text{ does not end in } ba\}$

regular expression

DFA for: L




reg expression:

$$a^*(bb^*(abb^*)^*(aa^*))^*$$

b) $\{w \in \{0,1\}^* : w = \alpha \circ \beta$ and $\alpha$ has an even # of 1's and $\beta$ has an even number of 0's$\}$

$\alpha \to$ even # 1's

$\beta \to$ even # 0's

$\alpha \circ \beta = \omega$

regular expression $(0^*10^*10^*)^* \circ (1^*01^*0)^*$

$\alpha$ $\circ$ $\beta$

Problem 2:

Prove OFAs recognize all and only regular languages

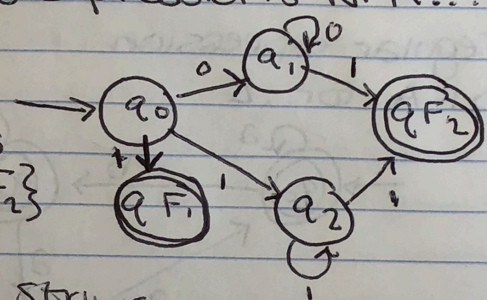
Consider the regular expression...

$(0+1)^*1$ (all set of strings tht end w/ 1)

this expressions NFA...

2 final states

$F = \{qF_1, qF_2\}$




Possible strings accepted are $\{1, 01, 11, 001, \ldots\}$

so,

For 1:

$q_0 \xrightarrow{1} qF_1$ (final state reached)

for 01:

$q_0 \xrightarrow{0} q_1 \xrightarrow{1} qF_2$ (Final state)

For 11:

$q_0 \xrightarrow{1} q_2 \xrightarrow{1} qF_2$

all possible final states after scanning the i/p string are also final state of machine, such strings are generated via regular expressio only.

Therefore, OFAs recognize all obssesive finite state machines

Problem 3:

True/False w/ counterexamples

a) True - all finite langs are regular because they can be expressed as a reg exp. but all regular langs are not finite langs (although being expressed ~~as~~ by reg expression) but does not mean finite

ex $a^*$ (Empty, a, aa, aaa, ...) Reg but not finite

$\{a+b+ab\} = \{a, b, ab\}$ Lang finite and regular

b) False - no mention of transition b/w states. There may exist a case where b/w 2 states there's no transition defined and no further move defined for that particular symbol and string not completely traversed then will not be accepted so it may not be $\Sigma$.

d) True -

Reg langs are closed under...

union

L1 and L2 are reg langs / can be expressed by reg exp. and reg exp. are closed under union so $L1 \cup L2$ is too

L1 and L2 reg langs / expressed by RE

intersection

reg langs are closed under complement so $L1'$ and $L2'$ will be regular as $L1' \cup L2'$ will also be regular because union (above) and $L1' \cup L2'$ is reg so its comp will be too.

$L_1 \cap L_2$ is also reg (set theory)

$L1 \cap L2 = (L' \cup L2')'$ so $(L1 - L2)$ will also be reg → so closed under intersection

difference

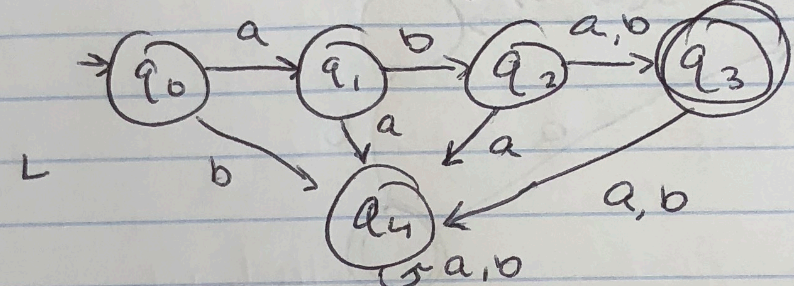
reg langs closed under complement so $L1'$ or $L2'$ will also be reg (from above) reg langs are closed under intersection so $L1 \cap L2'$ will be reg, we know

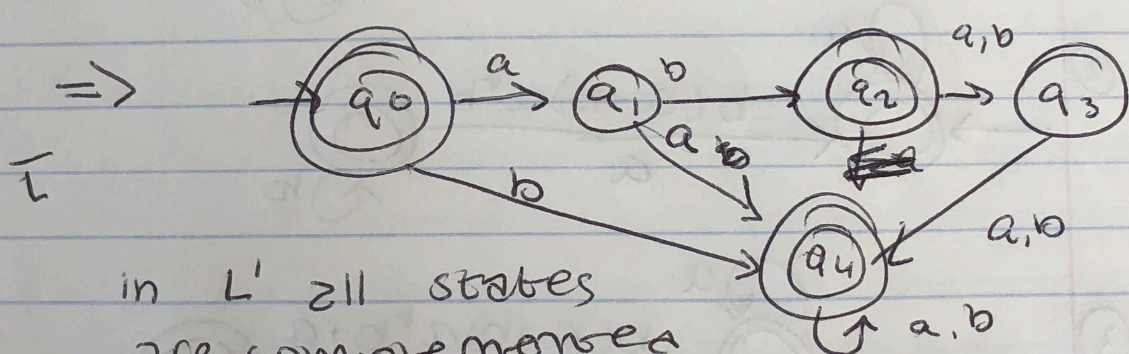
$L1 - L2 = L1 \cap L2'$ so $(L1 - L2)$ will also be reg ie closed under intersection

e) True

- reg langs closed under complement
- all final states become nonfinal States and all non final states than it will accep the comp lang

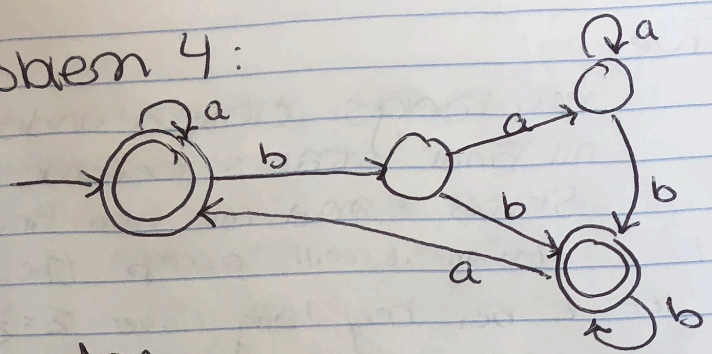
let L be reg lang over $\Sigma = \{a, b\}$ and accepts string aba and abb only



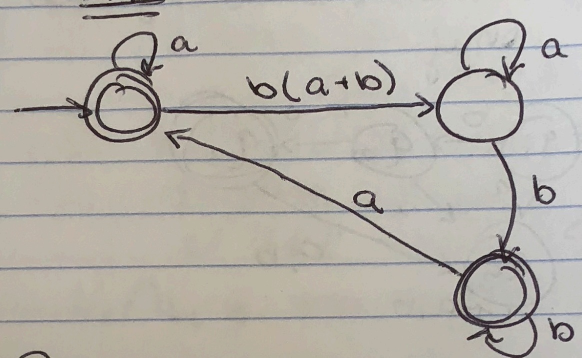

in L' all states are complemented and it accepts all strings over {a,b} except abb and aba

Problem 4:

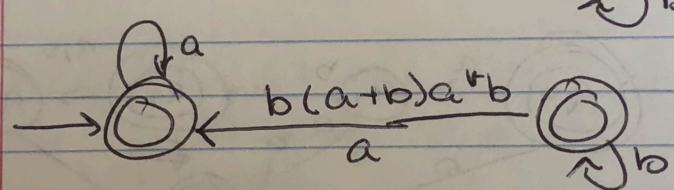



DFA

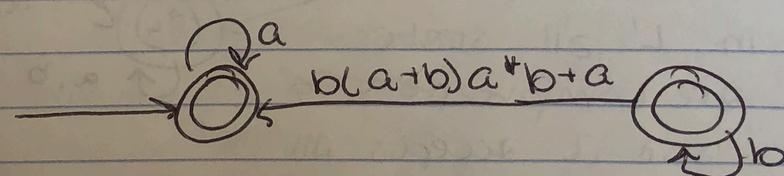
(1)



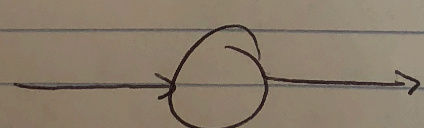

(2)




(3)




(4)



$a^*(b(a+b)a^*b+a)b^*$

Final Regular expression