Julia Nelson | Problem Set 9 | CS334 | "I pledge my honor that I have abided by the Stevens Honor System."

Problem 1

give high level descript. of TM that decides

$\{\langle M\rangle : M \text{ is a PDA and } L(M) \text{ is an infinite language}\}$

design TM that decides the PDA

On input $\langle M\rangle$ (where M is a PDA)

1. Create CFG for M
2. Convert the CFG into Chomsky Normal Form
3. Accept $\langle M\rangle$ if for some $u, v \in \Sigma^*$ $S \to uSv$

otherwise Reject.

Machine is decidable if TM accepts

Problem 2

$\{\langle G\rangle : G \text{ is a CFG over } \{a,b\} \text{ and } a^* \cap L(G) \neq \emptyset\}$

Show is decidable ...

Create TM that decides the language

On input $\langle G\rangle$:

1. Create CFG A so the language of A, $L(A) = a^* \cap L(G)$
2. Using E_{CFG} decider R (← Theorem 4.8) check if $L(A) = \emptyset$
3. if R rejects → ACCEPT
   if R accepts → REJECT

KNOWN: $a^* \cap L(G)$ is a CFL.

if $L(G)$ is a CFL and a^* is a Reg language + a PDA recognizes the lang $a^* \cap L(G)$ if a string's final state $\in F_{PDA}$

"Pledge"

Problem 3

let A be a TM recognizable lang that encodes TMs that are Deciders
Prove theres a decidable lang not decided by any TM in A. $A = \{\langle M_1 \rangle, \langle M_2 \rangle, \ldots \langle M_i \rangle\}$

let the TM deciders of A be some M_i

→ A contains TM decider encodings therefore an Enumerator E can enumerate it

→ Consider the string S_i that encodes M_i

"On input $\langle S_i \rangle$ for Decidable language D

1. if $\langle M_i \rangle$ Accepts → S_i is not in lang D → REJECT
2. if $\langle M_i \rangle$ Rejects → S_i is in lang D → ACCEPT

D is a decidable language

and because S_i does not get accepted by $\langle M_i \rangle$ it means the decider is not in A.

There exists some decidable lang D that does not have a decider in A.

"Pledge"

Problem 4

TM M on input w ever attempts to move its head left while on the leftmost cell

Language = $\{\langle M,w\rangle : M$ is a TM that tries to move Left at LM tape cell on input $\langle w\rangle\}$

a) Prove that Language L is undecidable.

b) Prove by contradiction

Assume L is decidable and a TM H'

let H be a TM that decides for M'

$H(\langle M,w\rangle) =$ {accept if M accepts w; reject if M does not accept w}

Create TM that constructs TM #

on input that puts a symbol like # on 1st cell and then the following input on rest of tape.

Then run the tape through M

if M accepts → Accept, Else → Reject

* confused on where to go to decide at end

sorry!