# CS 334 Fall 2019: Problem Set 9.

**Problem 1.** (10 points) Language $A$ is mapping-reducible to language $B$, written $A \leq_m B$, if there is a computable function $f: \Sigma^* \to \Sigma^*$ such that $\forall w \in \Sigma^*$: $w \in A \; if \; and \; only \; if \; f(w) \in B$.

a) (5 points) Show that $\leq_m$ is a transitive relation.
b) (5 points) Show that if $A$ is TM-recognizable and $A \leq_m \bar{A}$, then $A$ is decidable. (Hint: first argue that $A \leq_m \bar{A}$ implies that $\bar{A} \leq_m A$.)

**Problem 2.** (10 points) Is the language

$$DISJOINT_{TM} = \{\langle M, N\rangle : M \; and \; N \; are \; TMs \; and \; L(M) \cap L(N) = \phi\}$$

decidable or undecidable? Prove your answer.

**Problem 3.** (10 points A triangle in an undirected graph is a cycle of length 3. Show that the language $TRIANGLE = \{\langle G\rangle : graph \; G \; contains \; a \; triangle\}$ is in $\boldsymbol{P}$.

**Problem 4.** (10 points) Behold, a genie appears before you! Given a formula $\phi(x_1, x_2, \ldots, x_n)$ in conjunctive normal form with $n$ boolean variables, the genie will correctly tell you (in one step) whether or not the formula is satisfiable. Unfortunately, the genie will not give you a truth assignment to the variables that makes the formula true.

Your problem is to figure out a satisfying truth assignment when the genie says the formula is satisfiable. You can present the genie with a polynomial (in $n$) number of queries.

i. (5 points) Give a high-level description of your algorithm, with enough detail.
ii. (2 points) What is the maximum number of queries made by your algorithm?
iii. (3 points) Explain why your algorithm correctly finds a satisfying assignment for a satisfiable formula.