Julia Nelson Problem Set 8

"I pledge my honor that I have abided by the Stevens Honor System."

Problem 1

given $L = \{$ all palindromes where the # of 0s = # 1's$\}$ $\Sigma = \{0,1\}$ → Possible strings generated = $\{$ ~~…~~, 0110, 1001, 11000011 … $\}$ 01011010

Prove L is Not context free.

Proof:

For sake of of contradiction, assume L is context free in which case the pumping lemma for CFL's applies.

Let p be the pumping length the pumping lemma garuntees

Consider the string $s = 0^p 1^p 1^p 0^p \in L$

Let $s = uvxyz$ where $|vxy| \leq p$ and $|vy| > 0$

since $|vxy| \leq p$ possible case are:

case 1 (for both 0s and 1s)

vxy is contained of a single continuous block of 0's, or 1's only.

ex: 0000110

When only the string is pumped, the # of 0's will not be = to the # of 1s. and $> p$. The pumped string is not in the language L.

block #s
#1 #2 #3 #4
$0^p 1^p 1^p 0^p$

Case 2 Since $|vxy| \leq p$, vxy can contain symbols from at most 2 contiguous blocks. It can contain from the 1st and 2nd, or 3rd and 4th blocks. In all these cases when we pump the string the # of 0s in block 1 and 4 or the # of 1s in blocks 2 and 3 will be different, so that resulting string is not in the language. (not a palindrome)

→ $0^5 1^5 0^4$
0000011111 0000

Thus, in every case, pumping the string s results in a string not in the language. This contradicts the pumping lemma for CFL's therefore, L is not context free

Problem 2

The class of TM-decidable languages is closed under...

Union:

Let L_1 and L_2 be decidable languages
TMs M_1 and M_2 decide them
let TM M decide the language $L : L_1 \cup L_2$

On input ω:
1. Run M_1 on $\omega \rightarrow$ if accept then ACCEPT
2. Run M_2 on $\omega \rightarrow$ if accept then ACCEPT
   otherwise $\rightarrow$ REJECT

TM M ACCEPTS ω if
$\rightarrow M_1$ or M_2 ACCEPT
TM M REJECTS ω if
$\rightarrow M_1$ and M_2 both REJECT
M decides $L_1 \cup L_2$

if $\omega \in L_1$ or L_2
M will Accept ω
$L_1 \cup L_2$ is Decidable

Concatenation:

Let L_1 and L_2 be decidable language
concatenation $\rightarrow$ $L_1 L_2 = \{xy \mid x \in L_1 \text{ and } y \in L_2\}$
let TMs M_1 and M_2 decide L_1 and L_2
let TM M decide $L_1 L_2$

On input ω:
1. partition ω into 2 substrings xy
2. Run M_1 on x
3. Run M_2 on y
   if both accept $\rightarrow M$ ACCEPTs ω

(Partition must be adjusted + rerun through M,
if none accept after trying all $\rightarrow$ REJECT)

TM $\rightarrow$ accepts when both M_1 and M_2
accept.

$L_1 L_2$ is decidable

Star:

for language L, $L^* = \{x \in L \cup LL \cup LLL \cup \ldots\}$ let M_1 decide L
let M decide L^*

On input ω:
1. For each possible partition of
   ω, $\omega_1 \omega_2 \omega_3 \omega_4 \cdots \omega_n$
2. Run M_1 on $\omega_1 \ldots \omega_n$ (each subsection)
3. if M_1 accepts each $\omega_i \rightarrow$ ACCEPT
   otherwise $\rightarrow$ reject

(similar to last, must partition each
possible way to check)

L^* is decidable

Intersection:

let L_1 and L_2 be decidable languages
TMs M_1 and M_2 decide them
let M decide $L_1 \cap L_2$

on input ω:
1. Run M_1 on $\omega \rightarrow$ if rejects, REJECT
2. If doesnt reject, run M_2 on ω
   if M_2 accepts $\rightarrow$ ACCEPT
   otherwise $\rightarrow$ REJECT

$L_1 \cap L_2$ is decidable

Complement:

let language L be decidable
TM M_1 decides L
TM M decides $\overline{L}$
on input ω:
1. Run M_1 on $\omega \rightarrow$ if rejects, ACCEPT
   otherwise $\rightarrow$ REJECT

$\overline{L}$ is decidable

(if one TM or L rejects
it means ω is $\overline{L}$.)

Problem 3

"Pledge"

The class of TM-recognizable languages is closed under...

Union:

let L_1 and L_2 be recognizable languages
TMs M_1 and M_2 recognize them
TM M recognizes $L_1 \cup L_2$

On input ω:
① Alternate running M_1 and M_2 on ω each step
② if M_1 or M_2 accept → ACCEPT
if M_1 and M_2 reject/halt → REJECT
(looping) → M loops

M accepts when it reaches Accept state after all the steps. → $L_1 \cup L_2$ is Recognizable
M rejects when both M_1 and M_2 reject

Concatenation:

let L_1 and L_2 be recognizable languages
TMs M_1 and M_2 recognize them
TM M recognizes $L_1 L_2$

On input ω:
① Partition ω into 2 parts ω_1 and ω_2
② Run M_1 on ω_1 → if halt/reject, REJECT
③ Run M_2 on ω_2 → if accept, ACCEPT
otherwise → REJECT

$L_1 L_2$ is recognizable

Star:

let L be a recognizable language
TM M_1 recognizes L
TM M recognizes L^*

On input ω:
① partition ω into parts $\omega_1, \omega_2 \dots \omega_n$
② Run M_1 on ω_i (all parts)
if M_1 accepts ALL ω_i → ACCEPT
if M_1 halt/rejects any ω_i → REJECT
(must try different partitions)
if $\omega_i \in L$ then there exists a TM M that recognizes L^*

Intersection:

let L_1 and L_2 be recognizable languages
TMs M_1 and M_2 recognize them
let TM M recognize $L_1 \cap L_2$

On input ω:
① Run M_1 on ω. → if halt/reject, REJECT
② Then Run M_2 on ω → if halt/reject, REJECT
otherwise → ACCEPT

if M_1 and M_2 accept ω → M accepts ω
$\omega \in L_1 \cap L_2$

$L_1 \cap L_2$ is recognizable

Complement:

if L and $\overline{L}$ are TM-recognizable
the L is decidable
from theorem, ~~[illegible]~~
A lang is Decidable iff both it and its complement are recognizable
TM would NOT Halt for strings not in the language

Problem 4

"Pledge"

Proposition: Every infinite TM-recognizable language has an infinite Decidable subset.

let L be an infinite Recognizable language and has an infinite decidable subset.
There exists an enumerator E for all strings in L

let $L' = \{w_1, w_2, w_3 \dots w_n\}$ $i > 1$
w_i is **next string** in order after w_{i-1}

Consider:

① <u>L' is infinite and $L' \subseteq L$</u>

let L' be finite

→ E generates all strings $< w_i$, the largest in L'

→ because E generates finite # strings $< w_i$

L is finite → <u>CONTRADICTION!</u>

Also: M only ACCEPTS if in Standard Order

→ L' is NOT finite → L' is Infinite

Because E. printed all strings $\in L'$,

→ $L' \subseteq L$

② <u>L' is decidable</u>

Can construct E'

on no input:

① prints out strings accepted by M

→ Meaning all in Standard Order

Because L is infinite, E' is infinitely enumerating

E' is decidable because $\boxed{\text{prints Std order}}$

→ Therefore, the language L', of E' is decidable

→ L' is an infinite decidable subset of L, an infinite Recognizable language