Problem Set 3

"I pledge my honor that I have abided by the Stevens Honor system" J Willson

Problem 1

$\Pi_0 = \{\{1,2,3,5,6,7,8,9\}\{4\}\}$
G1 G2

G_1	a	b
1	G_1	G_1
2	G_1	G_2
3	G_2	G_1
5	G_1	G_1
6	G_2	G_1
7	G_1	G_1
8	G_1	G_2
9	G_1	G_1

$\Pi_1 = \{\{1,5,7,9\}\{2,8\}\{3,6\}\{4\}\}$

G_1	a	b
1	G_2	G_3
5	G_2	G_3
7	G_2	G_3
9	G_2	G_3

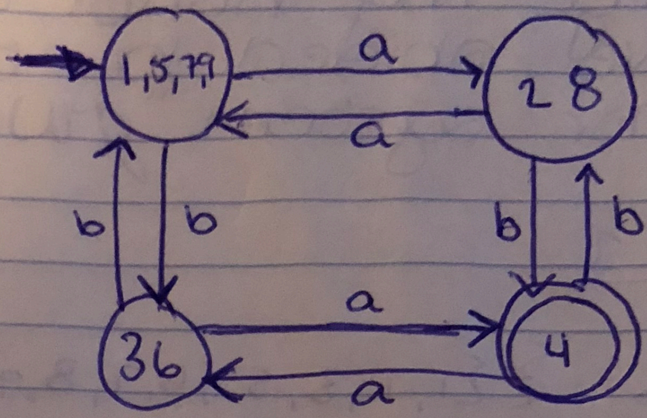
G_2	a	b
2	G_1	G_4
8	G_1	G_4

G_3	a	b
3	G_4	G_1
6	G_4	G_1

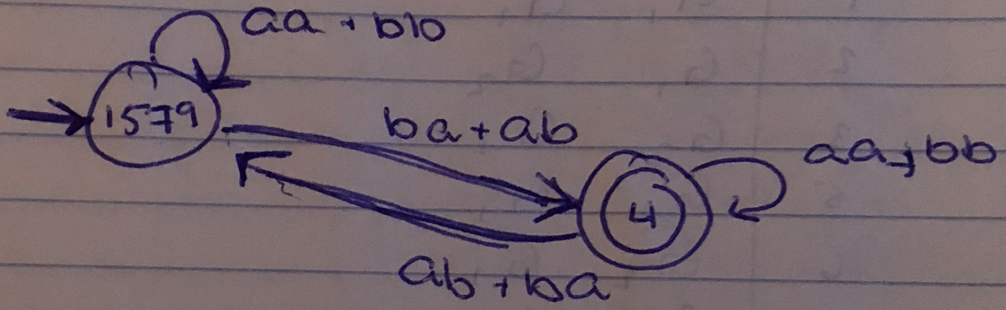
$\Pi_2 = \{\{1,5,7,9\},\{2,8\},\{3,6\}\{4\}\}$

$\Pi = \Pi_2$ So stop here

DFA:



DFA to RegEx:



{ strings that contain an odd # of a's and odd number of b's }

RegExp

$((bb+aa)^*(ba+ab)(aa+bb)^*(ab+ba))^*(bb+aa)^*(ba+ab)(aa+bb)$

Problem 2

given $X = \{0^k w : k \geq 1,\ w \in \{0,1\}^* \text{ and } w \text{ contain atleast } k \text{ 0's}\}$

let z be a string that belongs to X

let $z = uv^i k = \underbrace{000}_{u\ v^i}\ \underbrace{110100}_{k} \in X$

when $i = 2$ we have

$$z = uv^2 w = \underbrace{0}_{u}\ \underbrace{0000}_{v^2}\ \underbrace{110100}_{k}$$

but string $\underbrace{0\ 0000}_{0^k}\ \underbrace{110100}_{w} \notin X$

becaus $k = 5$ and # of 0's in $w = 3$
so w does not contain atleest k zeros for $i = 2$
by pumping lemma of reg langs.
L is not regular

$y = \{0^k w : k \geq 1\ w \in \{0,1\}^*$ and w contains almost k 0's$\}$

let $z = uv^i k$ be a string Which belongs to y

now

$z = \underbrace{0000}_{0^4}\underbrace{100100}_{w} \in y$

now

$\underbrace{0000}_{u}\underbrace{1001}_{v^2}\underbrace{00}_{k}$

when $i = 2$ we have..

$z = uv^2k = \underbrace{\underbrace{0000}_{u}\underbrace{10010100101}_{v^2}}_{w}\underbrace{0}_{k}$

$0^4 w$ but w has 7 zeros

So z for $i = 2 \notin y$

so by pumping lemma of reg lang

y is not Regular

Problem 3

CFG:

$X \to XX \mid Y$

$Y \to 0Y1 \mid 01$

a) the language generated by the grammar is equal number of 0's and 1's. minimum 1 zero and one.

set notation

$$L = \{0^n 1^n (01)^m \mid n \geq 0, m \geq 0, n+m \geq 1\}$$

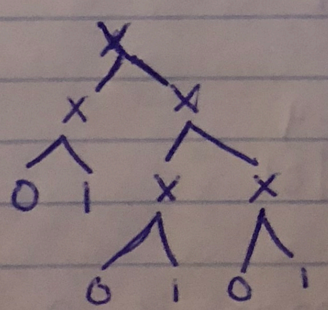
b)

Ambiguous grammar

more than one parse tree = ambiguous

derive 010101

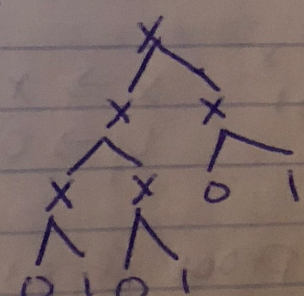
ptree 1



derive 010101

∴ its ambig

ptree 2



derives 010101

c) ~~$X \to XX4/4$~~
~~$Y \to$~~

$X \to XX4/4$
$Y \to 0\cancel{4}4\cancel{1}/01$

if grammar is ambiguous if it has single left recursion or associative should follow

$\boxed{X \to XX/4}$ → double associative so need to change the second X with the non terminal

$X \to 4$
$Y \to 01$
so $X \to 01$

replace second X w/ 01

$X \to X01/4$
$Y \to 041/01$

d) proof of grammar is ambiguous

$X \to X01/4$
$Y \to 041/01$

→ if grammar has unique parse tree to derive the particular string
Here only unique parse trees for above grammars

→

→ ~~here~~ these exist single left associative

$\boxed{X \to x} \; 01 / Y$ → single associative

~~Y → 0Y1~~

$\boxed{Y} \to 0\boxed{Y}1$

single left associative

∴ the grammar is ambiguous

Problem 4

CFG for palindromes lang over {0,1}

$G_{palindromes} = (\{P\}, \{0,1\}, A, P)$

A represents the production rules:

$P \to \epsilon$
$P \to 0$
$P \to 1$
$P \to 0P0$
$P \to 1P1$

~~we can also write~~

$P \to \epsilon / 0 / 1 / 0P0 / 1P1$

CFG palindrome with equal # of 0s and 1s

$G_{even\text{-}pal} = (\{P\}, \{0,1\}, A, P)$ where A denotes Production Rule

P is start symbol

$P \to$ ~~ep~~ $\epsilon \mid 01P10 \mid 10P01$

so $P \to \epsilon$ (# of 0s = # of 1s = 0)

$P \to 01P10 \to 0110$ palindrome

$P \to 10P01 \to 1001$ Palindrome

Prob 5

a) $(a^x b^{xy} c^y)$ $x, y \geq 0$

push down automata can count only in forward direction ie it can count the occurence a but it cant count occurence of c.

Hence it cant compare the occurence of b w/ the occurance of a and c

$a^x b^{xy} c^y$ is not context free and theres not any CFG for this language

b) $a^n b^{x+y} c^y$ context free
$a^n b^n b^y c^y$ grammar for this lang

$S \to AB$
$A \to aAb \mid \epsilon$
$B \to bBc \mid \epsilon$

$a^n b^{n+y} c^y$ is a context free grammar

we can compare x occurence of a with x occurence of b and also similarly y occurences of b and c

Problem 6

$\{a^i b^i c^k d^k : i, k \geq 0\} \cup \{a^i b^k c^k d^i : i, k \geq 0\}$

$S \to A \mid B$

$A \to CD$

$C \to aCb \mid \epsilon$

$D \to cDd \mid \epsilon$

$B \to aBd \mid aEd \mid \epsilon$

$E \to bEc \mid \epsilon$