

CS 334 Fall 2019: Problem Set 6.

Problem 1. (10 points) Use the pumping lemma for context free languages to prove that $\{a^i b^k c^i d^k : i, k \geq 0\}$ is not context free. Be sure your proof is complete – don't leave out any important details.

Problem 2. (10 points) A 2-stack PDA is a pushdown automaton with two stacks. The input tape is read only. Describe a 2-stack PDA that accepts the language $L = \{a^i b^k c^i d^k : i, k \geq 0\}$. You do not need to give a PDA diagram – a high-level, but complete, description of how your PDA works will suffice.

Since Lemma 2.27 in the textbook proves that the language of every PDA is context-free, this problem shows that 2-stack PDAs are strictly more powerful than PDAs.

Problem 3. (20 points) Let $L_{add} = \{a^i b^{i+j} c^j : i, j \geq 0\}$ and $L_{mult} = \{a^i b^{ij} c^j : i, j \geq 0\}$. For each language, either give a CFG/PDA for it, or prove that it is not a CFL.

Problem 4. (20 points) For each of the following languages, either give a CFG generating it, or a high-level description of a PDA that recognizes it:

- a) The complement of $\{a^n b^n \mid n \geq 0\}$
- b) $\{x_1 \# x_2 \# \dots \# x_k \mid k \geq 1, \text{ each } x_i \in \{a, b\}^* \text{ and for some } i, j \ x_i = x_j^R\}$

Optional Problem 1. (10 points) Let C be a context-free language, and R be a regular language. Show that the language $C \cap R$ is context free. Start with a PDA $(Q, \Sigma, \Gamma, \delta, q_{start}, F)$ for C and a DFA $(Q', \Sigma', \delta', q'_{start}, F')$ for R , then describe a PDA for $C \cap R$. Your description may be informal, but must be precise and complete.

Optional Problem 2. (10 points) If A, B are regular languages, show that the language $A \triangle B = \{xy : x \in A, y \in B, |x| = |y|\}$ is context-free. (Hint: Think machines!)