Julia Nelson Problem Set 2 9/13/19

"I pledge my honor that I have abided by the Stevens Honor System" J Nelson

Problem 1

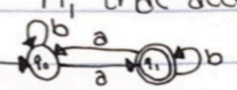
a) $\{w : w \text{ has an odd number of a's and ends with } b\}$

Two lang Intersection: $L_1 \cap L_2$

$L_{①} = \{w : w \text{ has an odd \# of a's}\}$

$L_{②} = \{w : w \text{ ends with a "b"}\}$

b) M_1 that accepts lang L_1



$\Sigma = \{a, b\}$

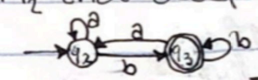
Transitions: $\delta(q_0, a) = q_1$, $\delta(q_0, b) = q_0$, $\delta(q_1, a) = q_0$, $\delta(q_1, b) = q_1$

$Q = \{q_0, q_1\}$

Start state: q_0

$F = q_1$

M_2 that accepts L_2



$\Sigma = \{a, b\}$

Transitions: $\delta(q_2, a) = q_2$, $\delta(q_2, b) = q_3$, $\delta(q_3, a) = q_2$, $\delta(q_3, b) = q_3$

$Q = \{q_2, q_3\}$

Start state: q_2

$F = q_3$

c) Cross Product of L_1 and L_2

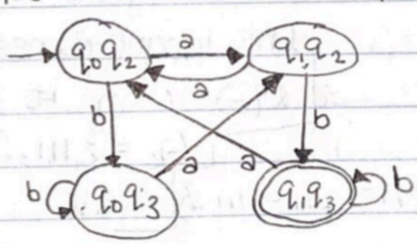
$$\{q_0, q_1\} \times \{q_2, q_3\} = \{q_0q_2, q_0q_3, q_1q_2, q_1q_3\}$$

Initial state: q_0q_2

$F = q_1q_3$

Transitions:
$\delta(q_0q_2, a) = q_1q_2$
$\delta(q_0q_2, b) = q_0q_3$
$\delta(q_0q_3, a) = q_1q_2$
$\delta(q_0q_3, b) = q_0q_3$
$\delta(q_1q_2, a) = q_0q_2$
$\delta(q_1q_2, b) = q_1q_3$
$\delta(q_1q_3, a) = q_0q_2$
$\delta(q_1q_3, b) = q_1q_3$

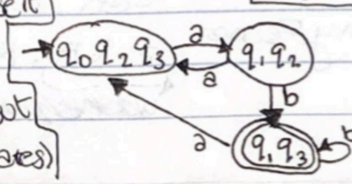
FSA: $\{w : w \text{ has odd \# a's and ends w/ } b\}$



d) You can combine states q_0q_2 and q_0q_3 together.

Not q_1q_2 with q_1q_3 because it affects the lang w/ odd a's.

q_0q_2 and q_0q_3 the states both go to the same state w/ the same input (they have the same transition states) AND don't affect the lang.



Simplified states (to read easily)

state	a	b
02	12	03
03	12	03
12	02	13
13	02	13

Problem 2

ϵ if $A=\epsilon$ $\emptyset$ if $A=\emptyset$ a for $a \in \Sigma$

To reverse:

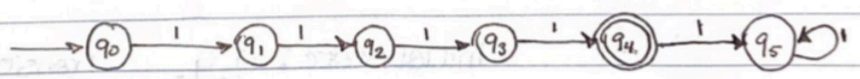
- Reverse the transition directions
- A^R accept is A start state
- Add new start state with ϵ transition to the original accept states of A
  - Shows $w^R \in A^R$

A is a reg lang (accepted by min. 1 FSA)

Reversing the states + accept reverses the language to A^R that is also regular.

Problem 3

a) 6 state FSA for $L_4 = \{1111\}$



Start $= q_0$

$F = q_4$

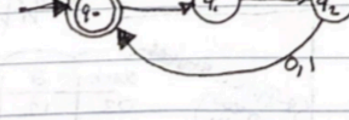
We can't reduce the states because this language needs a dead state (q_5) so it requires 6 states

$L_k = \{111\ldots1_k\}$ needs $k+2$ states

b) $\Sigma = \{0,1\}$

$k = 3$

$L = \{000, 001, 010, 100, 101, 110, 111 \ldots\}$



For any alphabet in place of $\{0,1\}$ would be accepted for $k=3$ can be accepted by a 3-state FSA. And not by any fewer states

2: X 1: X