

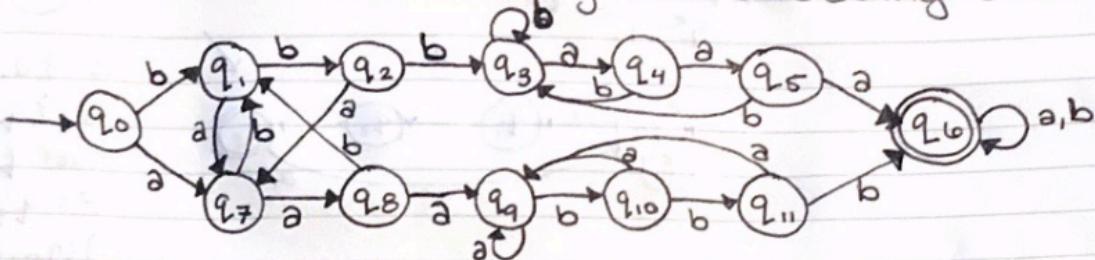
## Problem Set 1

$$\Sigma = \{a, b\}$$

Problem 1

1.

$L_1 = \{\omega : \omega \text{ contains one string } aaa \text{ and a string } bbb\}$



$$\Sigma = \{a, b\}$$

$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}, q_{11}\}$$

Start State:  $q_0$

$$F = q_6$$

$$\text{Transitions: } \delta(q_0, a) = q_7$$

$$\delta(q_0, b) = q_1$$

$$\delta(q_1, a) = q_7$$

$$\delta(q_1, b) = q_2$$

$$\delta(q_2, a) = q_7$$

$$\delta(q_2, b) = q_3$$

$$\delta(q_3, a) = q_4$$

$$\delta(q_3, b) = q_5$$

$$\delta(q_4, a) = q_5$$

$$\delta(q_4, b) = q_3$$

$$\delta(q_5, a) = q_6$$

$$\delta(q_5, b) = q_3$$

$$\delta(q_6, a) = q_6$$

$$\delta(q_6, b) = q_6$$

$$\delta(q_7, a) = q_8$$

$$\delta(q_7, b) = q_9$$

$$\delta(q_8, a) = q_9$$

$$\delta(q_8, b) = q_9$$

$$\delta(q_9, a) = q_{10}$$

$$\delta(q_9, b) = q_9$$

$$\delta(q_{10}, a) = q_9$$

$$\delta(q_{10}, b) = q_{11}$$

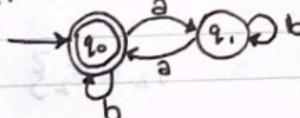
$$\delta(q_{11}, a) = q_9$$

$$\delta(q_{11}, b) = q_6$$

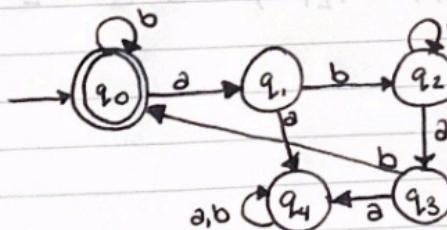
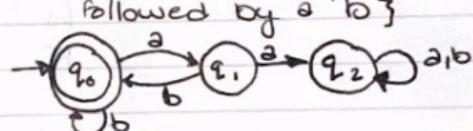
2.

$L_2 = \{\omega : \omega \text{ contains an even # of } a's \text{ and each } a \text{ is followed immediately by at least 1 } b\}$

$$L_{2,1} = \{\omega : \omega \text{ contains even # } a's\}$$



$$L_{2,2} = \{\omega : \text{each } a \text{ is immediately followed by a } "b"\}$$



$$\Sigma = \{a, b\}$$

$$Q = \{q_0, q_1, q_2, q_3, q_4\}$$

Start State:  $q_0$

$$F = q_4$$

Transitions

$$\delta(q_0, a) = q_1$$

$$\delta(q_0, b) = q_0$$

$$\delta(q_1, a) = q_4$$

$$\delta(q_1, b) = q_2$$

$$\delta(q_2, a) = q_3$$

$$\delta(q_2, b) = q_2$$

$$\delta(q_3, a) = q_4$$

$$\delta(q_3, b) = q_0$$

$$\delta(q_4, a) = q_1$$

$$\delta(q_4, b) = q_2$$

$$\delta(q_5, a) = q_6$$

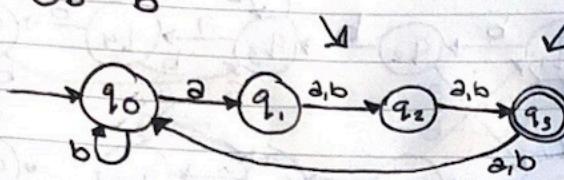
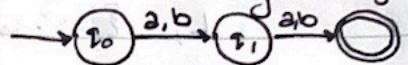
$$\delta(q_5, b) = q_4$$

3.  $L_3 = \{w : \text{one 3rd last symbol in } w \text{ is } a\}$

$L_{3,1} = \{w : \text{1st symbol is } a\}$



$L_{3,2} = \{w : \text{exactly 2 symbols}\}$



$$\Sigma = \{a, b\}$$

$$Q = \{q_0, q_1, q_2, q_3\}$$

Start:  $q_0$

$$F = q_3$$

$$\delta(q_0, a) = q_1 \quad \delta(q_3, a) = q_0$$

$$\delta(q_0, b) = q_0 \quad \delta(q_3, b) = q_0$$

$$\delta(q_1, a) = q_2$$

$$\delta(q_1, b) = q_2$$

$$\delta(q_2, a) = q_3$$

$$\delta(q_2, b) = q_3$$

$$\delta(q_3, a) = q_0$$

$$\delta(q_3, b) = q_0$$

## Problem 2

Modify proof of Theorem 1.25.

PROOF: Let  $A_1, A_2$  be regular languages over respectively  $\Sigma_1$  and  $\Sigma_2$  recognized by FSA  $M_1$  and  $M_2$ .  $M_1$  recognizes  $A_1$ , where  $M_1 = (Q_1, \Sigma_1, \delta_1, q_1, F_1)$ .  $M_2$  recognizes  $A_2$ , where  $M_2 = (Q_2, \Sigma_2, \delta_2, q_2, F_2)$ .

Define a new FSA  $M = (Q, \Sigma, \delta, q_0, F)$ :

$$Q = Q_1 \times Q_2$$

$$\Sigma = \Sigma_1 \cup \Sigma_2$$

$$q_0 = (q_1, q_2)$$

$$F = \{(\delta_1, \delta_2) \mid \delta_1 \in F_1 \text{ or } \delta_2 \in F_2\}$$

$$\begin{aligned} \delta((\delta_1, \delta_2), a) &= (\delta_1(a), \delta_2) \text{ if } a \in \Sigma_1, \\ &= (q_1, \delta_2(a)) \text{ if } a \in \Sigma_2 \end{aligned}$$

