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Problem Set 6

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"I pledge my honor that I have abided by the Stevens Honor System."

① if G is CFG in Chomsky Normal form

show that a string of Terminal symbols length $n \geq 1$

is from $2n-1$ rules of G

We need 1. NonTerm → Terminal ($A \to a$)

2. NonTerm → NonTerm NonTerm

3. Start Symb → Empty ($S \to \varepsilon$)

→ let String be length $n \geq 1$

String: S → String starts with (NonTerminal) start symbol S. ⟹ $n = 1$ length

S → n → → Add a NonTerminal symbol "n" to string by using non-term ~~...~~ rule ⟹ S → n → n → length $= n = 2n$ $n-1$ Rule because applied on part of string

→ Apply (non-Term → Term) rule on all Non-Term symbols of length n ⟹ applied to ~~...~~ n # rules

Applied $n-1$ Rules with (Non-Term → NonTerm NonTerm)

Applied n Rules with (Non-Term → Term)

G is in CNF in $\underbrace{n-1+n}$ $2n-1$ Rules Applied

(2)

$S \to TT \mid U$

$T \to 0T \mid T0 \mid \#$

$U \to 0U00 \mid \#$

TT
0T0T
0#0T
0#0#

{ ##, 0##, 0##0, 0#0#, #0#, ..., 0#00, 00#0000 }

L(G) ~~string~~ → Must contain 2 #'s or must be $0^n \# 0^{2n}$

A grammar is NOT Reg if both sides are linear (Left/Right)

Because $T \to 0T \mid T0$, the grammar is not Regular

(0T|T0 ← left linear
Right Linear
... ← 0
0 → ...)

(3) Give CFG for $\{a^i b^i c^k d^k : i,k \geq 0\} \cup \{a^i b^k c^k d^i : i,k \geq 0\}$

Is it Ambiguous?

$\{a^i b^k c^k d^i : i,k \geq 0\}$

CFG
$$S_1 \to a S_1 d \mid X \mid \varepsilon$$
$$X \to b X c \mid \varepsilon$$

$\downarrow$

$G_1 = (V_1, \Sigma_1, P_1, S_1)$

$V_1 = \{S_1, X\}$

$\Sigma_1 = \{a\,b\,c\,d\}$

$S = S_1$

$P_1 = \{S_1 \to a S_1 d, S_1 \to X$
$S_1 \to \varepsilon, X \to \ldots \}$

$\{a^i b^i c^k d^k : i,k \geq 0\}$

CFG $S_2 \to YZ$
$$Y \to aYb \mid \varepsilon$$
$$Z \to cZd \mid \varepsilon$$

$\downarrow$

$G_2 = (V_2, \Sigma, P_2, S_2)$

$V_2 = \{S_2, Y, Z\}$

$S_2 = \{S_2\}$

$P_2 = \{S_2 \to YZ, Y \to aYb, \ldots\}$

G = The Union of G_2 and G_1 = $G_2 \cup G_1$

$V = V_2 \cup V_1$
$= \{S_2, Y, Z, S_1, X\}$

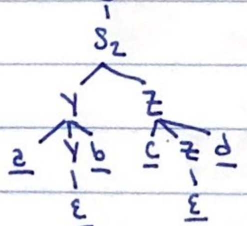
$\Sigma = \{a, b, c, d\}$

$S = \{S \to S_2 \mid S_1\} = \{S\}$

$P = P_2 \cup P_1 = \{ S \to S_2 \mid S_1,$
$S_2 \to YZ,$
$S_1 \to a S_1 d \mid X \mid \varepsilon,$
$X \to bXc \mid \varepsilon,$
$Y \to aYb \mid \varepsilon,$
$Z \to cZd \mid \varepsilon \}$

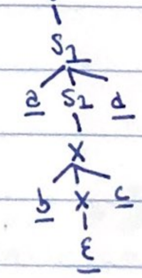
Parse Trees for abcd

#1



$a \varepsilon b c \varepsilon d$
$= abcd$

#2



$a b \varepsilon c d$
$= abcd$

More than 1 parse tree for same String. Grammar is Ambiguous

(4) $L_{add} = \{a^i b^{i+j} c^j : i,j \geq 0\}$ and $L_{mult} = \{a^i b^{ij} c^j : i,j \geq 0\}$

→ give CFG or Prove not CFL

$a^i b^{i+j} c^j$ is the same as...

$a^i b^i b^j c^j : i,j \geq 0$

CFG: $S \to XY$

$X \to aXb \mid \varepsilon$

$Y \to bYc \mid \varepsilon$

represents $\{b^j c^j : j \geq 0\}$

represents $\{a^i b^i : i \geq 0\}$

represents

$\{X \circ Y\} = \{a^i b^i : i \geq 0\} \circ \{b^j c^j : j \geq 0\}$

$= \{a^i b^{i+j} c^j : i,j \geq 0\}$

Because concatenation is closed and produces a CFG L_{add} is [redacted] a CFL

→ Assume L_{mult} is a CFL

→ use Pumping Lema For some P exists [redacted] a string s, $|s| \geq P$

→ let $s = \{a^P b^{P^2} c^P\}$ and P be the Pumping length

by splitting s into uvxyz

→ we must satisfy

1. $uv^i xy^i z \in A \ \forall i \geq 0$
2. $|vy| > 0$
3. $|vxy| \leq P$

$s = a^P b^{P^2} c^P = \underbrace{aa\ldots a}_{P}\underbrace{bbbb\ldots bb}_{P^2}\underbrace{c\ldots c}_{P}$

[redacted]

let $P = 3$

$s = \underbrace{aab}_{vxy}$bb bbb bbb ccc

$a^3 b^9 c^3$

(a and b combo)

$|vxy| \leq P \to 4 \leq 3$? No

does not satisfy

choosing ~~vx~~ vxy to satisfy condition...

$s = \underbrace{aaa}_{vxy} b\ldots\overset{x9}{b}ccc$ (all a's)

does not exist in lang when $uv^i xy^i z$.. too many a's compared to b's/c's

$s = a^3 \underbrace{b}_{vxy}{}^9 c^3 \to$ #b ≠ #a's + #c's (all b's)

so string not in lang

[redacted]

[redacted]

sorry!

I ran out of time to show but I believe that no matter where you pick vxy to be in s, it will never satisfy all of the conditions in order to be a Context Free Lang

Therefore, L_{mult} is NOT a CFL

missing choosing vxy as [redacted] all c's and [redacted] b's and c's