Problem Set 9

Problem 1

Problem 2

$L = \{\langle M_1, M_2\rangle \mid M_1 \text{ and } M_2 \text{ are TMs such that } L(M_1) \cap L(M_2) = \emptyset\}$

Proof by $A_{TM} \leq M_L$

Reduction

Input $\langle M, w\rangle$

Construct machine M_1

→ ignores input

→ simulates M on w

→ if M accepts w → ACCEPTS

($L(M_1) = \Sigma^*$ if M Accepts w,

otherwise $L(M_1) = \text{NULL}$)

Constructs machine M_2

input $L(M_2) = \Sigma^*$

→ Accepts input

outputs $\langle M_2, M_1\rangle$

Correctness

if M accepts w then,

$L(M_1) = \Sigma^*$ AND $L(M_1) \cap L(M_2) = \Sigma^* \neq$.

otherwise,

$\neg M$ accepts w

$L(M_1) \cap L(M_2) = \text{NULL}$ so M does not Accept w

$L(M_1) \cap L(M_2) = \text{NULL}$

A_{TM} is undecidable → L is also undecidable

L and $DISJOINT_{TM}$ is NOT TM recognizable

Problem 3

in P if decided by polynomial alg TM

on input $\langle G\rangle$

- examines all triples $(u,v), (v,w) (u,w)$

$|V|$ = # vertices

$\frac{|V|(|V|-1)(|V|-2)}{3 \cdot 2 \cdot 1} \rightarrow O(|V|^3)$

- for each triple, check all edges if connected (TRIANGLE)

if triangle found → ACCEPT

if No triangle found → REJECT

$O(|V|^3)$ steps polynomial time