Julia Nelson

"I pledge my honor that I have abided by the Stevens Honor System."

Provide an example of

1. a finite path fragment.

red → red/yellow → green

2. an infinite path fragment that is not a path.

green → yellow → green → yellow → red → red/yellow → green → yellow → red → ...

3. an infinite path fragment that is a path.

red → red/yellow → green → yellow → red → red/yellow → green → yellow → ...

4. Are there any finite paths? Why?

Because the transition system is a loop there are only finite path fragments

like red → red/yellow → green is a finite fragment of the infinite path

red → red/yellow → green → yellow → red → ....

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Exercise 2

Suppose we have the following set of atomic propositions $AP = \{red, green, yellow\}$. For each of the following properties, (a) express them using set comprehension[1] and (b) state whether they are safety of liveness properties.

1. "All lights can never be on at the same time"

   property holds for all infinite paths → $\in (2AP)\omega$)

   At time 0(A0), red, green, yellow all present

   $red \in A0 \wedge green \in A0 \wedge yellow \in A0$

   set comprehension: $\{A0A1\ldots \in (2AP)\omega \mid red \in A0 \wedge green \in A0 \wedge yellow \in A0\}$

   a) $\{A0A1\ldots \in (2AP)\omega \mid A0 \cap A1 \neq \emptyset, A1 \cap A2 = \emptyset, \ldots\}$

   b) safety property

2. "The traffic light is green infinitely often"

   holds forall infinite → $\in (2AP)\omega$

   there exists an index i that green is present at Ai

   $\exists i \in N, green \in Ai$

   → $\{A0A1\ldots \in (2AP)\omega \mid \exists i \in N, green \in Ai\}$

   a) $\{A0A1\ldots \in (2AP)\omega \mid \exists i \in N, A0A1\ldots Ai = \{green\}\}$

   b) Liveness property

3. "once red, the light cannot become green immediately"

   holds for all infinite → $\in (2AP)\omega$

   for every index i in infinite path if red and green both present at Ai+1, then green must also be present at Ai

   $\forall i \in N, (red \in Ai{+}1 \wedge green \in Ai{+}1) \rightarrow green \in Ai\}$

   a) $\{A0A1\ldots \in (2AP)\omega \mid \forall i \in N, A0A1\ldots Ai \neq \{green\} \rightarrow Ai{+}1 \neq \{green\}\}$

   b) safety property

4. "once red, the light always becomes green eventually"

   holds for all infinite → $\in (2AP)\omega$

   there exists an index i that green is present at Ai+1 for all indices j less than i, red and green present at Aj and Aj+1

   $\{A0A1\ldots \in (2AP)\omega \mid \exists i \in Ai{+}1 \wedge \forall j \in N, j < i \rightarrow (red \in Aj \wedge green \in Aj{+}1)\}$

   a) $\{A0A1\ldots \in (2AP)\omega \mid \exists i \in N, A0A1\ldots Ai = \{red\} \rightarrow \exists j \geq i, Aj = \{green\}\}$

   b) Liveness property

[1] In other words, in the format $\{A_0A_1\ldots \in (2^{AP})^\omega \mid \ldots\}$.

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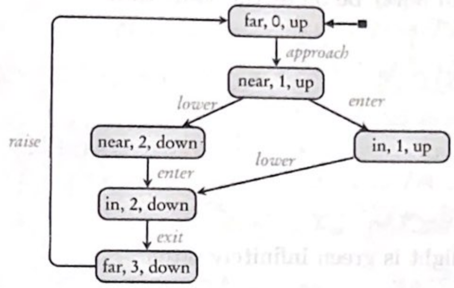
Exercise 3

Consider the following transition system depicting the operations of a simple train controller where *far, near, in* refers to a train's distance to the crossing, 0-3 refer to the controller software's internal state and *up, down* refers to whether the barrier is up or down:



Give an example of:

1. a finite path fragment

→(far, 0, up) → approach (near, 1, up) → enter (in, 1, up) —
→ lower (in, 2, down) → exit (far, 3, down)

(far, 0, up) → (near, 1, up) → (in, 1, up) → (in, 2, down) → (far, 3, down)

2. an infinite path fragment that is not a path

(far, 0, up) → (near, 1, up) → (near, 1, up) → (near, 2, down) → (in, 2, down)
→ (far, 3, down)

approach → enter → $\boxed{}$ → lower → enter → exit

3. a path

(far, 0, up) → (near, 1, up) → (near, 2, down) → (in, 2, down) → (far, 3, down)

approach → lower → enter → exit

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Exercise 4

Consider the following set $AP = \{far, near, in, up, down\}$. You are asked to describe a series of properties over AP. The labeling function is not required for this exercise, but it will be supplied all the same, so that the meaning of the atomic propositions in AP becomes clearer. $L(\langle far, 0, up\rangle) = \{far, up\}$, $L(\langle near, 1, up\rangle) = \{near, up\}$, $L(\langle in, 0, up\rangle) = \{in, up\}$, $L(\langle near, 2, down\rangle) = \{near, down\}$, $L(\langle in, 2, down\rangle) = \{in, down\}$, $L(\langle far, 3, down\rangle) = \{far, down\}$.

Next you are asked to describe the properties below using set comprehension (as seen in class). Recall that these properties may or may not hold. That is irrelevant. All you have to do is formulate them:

1. Every time the barrier goes up, it eventually goes down

   holds for all infinite $\in (2^{AP})^\omega$
   for every index i in infinite path if barrier is up at A_{i+1} then an index j such that j > i and barrier down at A_j

   $\{A_0 A_1 \ldots \in (2^{AP})^\omega \mid \forall i \in \mathbb{N}, up \in A_{i+1} \to \exists j \in \mathbb{N}, j > i \wedge down \in A_j\}$

2. It is not possible for the barrier to be up and the train to be at the crossing.

   holds all infinite $\in (2^{AP})^\omega$
   for every index i in infinite path if barrier is up at A_i then train ~~~~ is near/in/far at A_i then property does not hold

   $\forall i \in \mathbb{N}, (up \in A_i \wedge (near \in A_i \vee in \in A_i \vee far \in A_i)\} \to \perp$

   $\{A_0 A_1 \ldots \in (2^{AP})^\omega \mid \forall i \in \mathbb{N}, (up \in A_i \wedge (near \in A_i \vee in \in A_i \vee far \in \ldots$
   $A_i)) \to \perp\}$

3 Submission Instructions

Submit a zip file named `hw5.zip` through Canvas containing a picture (pdf, jpeg or png) of your handwritten solution for each exercise.