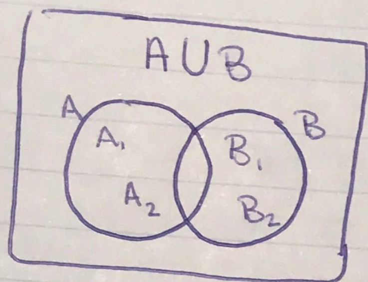
Julia Nelson Problem Set 4 CS 135

"I pledge my honor that I have abided by the Stevens Honor System" J Nelson jnelson6

Problem 1

(a)



let

if A =
$1 \to x$
$2 \to y$

if B =
$6 \to j$
$7 \to k$

The union of countably infinite sets is countably infinite

$A \cup B$ =
$1 \to x$
$2 \to y$
$6 \to j$
$7 \to k$

(b) The subset of a set has less or equal number of relation as the set. So every infinite subset of a set that is countably infinite is countably infinite.

(c) A and B are countably infinite then so is $A \times B$

Cartesian product = $A \times B$ is set of all ordered pairs (a,b) so $a \in A$ and $b \in B$

so... $A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$

let

$B_a = \{(a,b) \in A \times B \mid b \in B\}$

Since B is countable so is B_a

$\bigcup_{a \in A} B_a$ is the union (of countable sets) and is countable.

so it is countably. Since $A \times B = \bigcup_{a \in A} B_a$

$A \times B$ is countable

(d)

$\mathbb{Q}$ is set of rational # and is countable
(each rat # is a subset $\mathbb{Z} \times (\mathbb{N} - \{0\})$
so no rat # is represented twice)

map $\varphi: \mathbb{Q} \to \mathbb{Z} \times (\mathbb{N} - \{0\})$ sends
rational # a/b in the lowest term
to orderded pair (a, b).
(- ~~sign~~ sign alway in num of fraction)

The map is an injection of
a countably infinite set. So $\mathbb{Q}$ is
at most countable.
Since $\mathbb{Q}$ isn't finite it is countably
infinite

Problem 2

Find smallest relation (fewest elements) that contain the relation

$\{(1,2), (1,4), (3,3), (4,1)\}$

(a) Reflexive and transitive

if $x \in A$, and $y \in x$ then $y \in A$

$R = \{(1,1), (2,2), (3,3), (4,4), (1,2), (1,4), (4,1)\}$

(b) Symmetric and transitive

- $(1,2) \to (2,1)$
- $(1,4) \rightleftarrows (4,1)$

$(3,3) \to (3,3)$

$(1,1)$ needed

$R = \{(1,2), (2,1), (1,1), (1,4), (4,1), (3,3)\}$

(c) Reflexive and symetric and transitive

Reflexive | Symetric needed | trasitive

$R = \{(1,1), (2,2), (3,3), (4,4), (1,2), (2,1), (1,4), (4,1)\}$