Julia Nelson Problem Set 6

"I pledge my honor that I have abided by the Stevens Honor System." jnelson6 J Nelson

Problem 1

gcd(1529, 14039) as linear combination of 1529 and 14039

$1529a + 14039b = \gcd(1529, 14039)$

Find GCD:

$14039 = 9 * 1529 + 278$

$1529 = 5 * 278 + 139$

$278 = 2 * 139 + 0$

last non-zero GCD = 139

extended

$278 = 2 * 139 + 0$ (1)

$278 + 139 * (-1) = 139$ (2)

$139 = 278 + (-1)(139)$ (3)

$1529 = 5 * 278 + 139$

$139 = 1529 + (-5)(278)$

$139 = 278 + (-1)(1529 + (-5)(278))$

$= (-1)(1529) + (6)(278)$

$= (-1)(1529) + (6)(14039 + (-9)(1529))$

$= (6)(14039) + (-55)(1529)$

$\boxed{b = 6 \quad a = -55}$

Problem 2

(a) let $F_1 = 1$ $F_2 = 1$ $\gcd(F_1, F_2) = 1$

hypoth: $\gcd(F_{n+1}, F_n) = 1$

Induction: prove $\gcd(F_{n+2}, F_{n+1}) = 1$

Proof:

$$\gcd(F_{n+2}, F_{n+1}) = \gcd(F_{n+1}, F_{n+2}) =$$
$$= \gcd(F_{n+1}, F_{n+2} - F_{n+1})$$

let $\gcd(F_{n+1}, F_{n+2}) = d$

$d \mid F_{n+1}$ $d \mid F_{n+2}$

let $F_{n+1} = xd$

$F_{n+2} = yd$ since $F_{n+2} > F_{n+1} \Rightarrow y > x$

$F_{n+2} - F_{n+1} = yd - xd = (y - x)d$ (divisable by d)

$$\gcd(F_{n+1}, F_{n+2}) = \gcd(F_{n+1}, F_{n+2} - F_{n+1})$$

$$\gcd(F_{n+2}, F_{n+1}) = \gcd(F_{n+1}, F_{n+2}) = \gcd(F_{n+1}, F_{n+2} - F_{n+1}) =$$
$$= \gcd(F_{n+1}, F_n) = 1 \text{ (hypothesis)}$$

(b)

From part (a) we have $\gcd(F_{n-1}, F_n) = 1$
$\gcd(F_n, F_{n+1}) = 1$

(any 2 consec fib # is 1)

Proof:

$$\gcd(F_{n-1}, F_{n+1}) = \gcd(F_{n-1}, F_{n+1} - F_{n-1})$$

because

let $\gcd(F_{n-1}, F_{n+1}) = d$

$\Rightarrow$ $d \mid F_{n-1}$ and $d \mid F_{n+1}$

let $F_{n+1} = yd$ and $F_{n-1} = xd$

since $F_{n+1} > F_{n-1}$ $\Rightarrow$ $y > x$

$\Rightarrow F_{n+1} - F_{n-1} = yd - xd = (y-x)d$ (which is divisible by d)

$\Rightarrow \gcd(F_{n-1}, F_{n+1}) = \gcd(F_{n-1}, F_{n+1} - F_{n-1})$

So $\gcd(F_{n-1}, F_{n+1}) = \gcd(F_{n-1}, F_{n+1} - F_{n-1}) = \gcd(F_{n-1}, F_n) = 1$

(gcd of 2 consec is 1)

$\Rightarrow$ 3 consecutive fib #s are always pairwise relatively prime

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, $\boxed{55, 89, 144}$ ...

55 | 89 | 1
55
34 | 55 | 1
34
21 | 34 | 1
21
13 | 21 | 1
13
8 | 13 | 1
8
5 | 8 | 1
5
3 | 5 | 1
3
2 | 3 | 1
2
1

$\gcd(55, 89) = \gcd(89, 144) = \gcd(55, 144)$

$= 1$