Julia Nelson Problem Set 8.

"I pledge my honor that I have abided by the Stevens Honor System." - JNelson

Problem 1

a) i. One-to-One but not onto

$\mathbb{Z} \to \mathbb{N}$ $f(x) = \begin{cases} 3x+3 & x \geq 0 \\ -3x+2 & x < 0 \end{cases}$

ii Onto not one-to-one

$\mathbb{Z} \to \mathbb{N}$ $f(x) = |x| + 1$

iii. One-to-one and onto

$\mathbb{Z} \to \mathbb{N}$ $f(x) = \begin{cases} 2x+2 & x \geq 0 \\ -2x-1 & x < 0 \end{cases}$

iv. Neither one-to-one nor onto

$\mathbb{Z} \to \mathbb{N}$ $f(x) = 3$

b) No.

counter example:

$\{6\} = j$
$\{7,8\} = k$
$\{5\} = l$

$g(x): j \to k$
$g(6) = 7$
$f(x): k \to l$
$f(7) = f(8) = 5$

j is only $\{6\}$ so it can only be $\{7$ or $8\}$ making it not onto.
But $f(7)$ and $f(8) = l$ making it onto.

c) $f: B \to C$ $g: A \to B$ $h: A \to C = f \circ g$

i. if h is surjective, then f <u>must</u> be surjective because of h, all $c \in C$ there is an $a \in A$

so $g(f(a)) = c$ so $b = g(a)$. $b \in B$ and $f(b) = g(f(a)) = c$ $\Rightarrow$ $f(b) = c$ so f is subjunctive

ii. same as the counter example given in part b).

$\{6\} = j$
$\{7,8\} = k$
$\{5\} = l$

eg $g(x): j \to k$
$g(6) = 7$
$f(x): k \to l$
$f(7) = f(8) = 5$

~~iii. counter example:~~

iii counter example

$j = \{1\}$ $k = \{2,3\}$ $l = \{4\}$

define:

$g(1) = 2$, $f(2) = f(3) = 4$

so $f \circ g$ is injective

f is <u>not</u> injective

Problem 2

$$\begin{array}{l} P(0) \\ \underline{\forall k \in \mathbb{N} : (P(k) \Rightarrow P(k+1))} \\ \therefore \forall n \in \mathbb{N} : P(n) \end{array}$$

$\forall n \in \mathbb{N} : P(n)$ is True

then, $\exists n \in \mathbb{N} : \neg P(n)$

so, $\exists n \in \mathbb{N} : P(n)$ is not true

so let subset $S : \{ k \in \mathbb{N} : P(k) \}$ be not true

then S is non empty subset of integers (well-ordering), $(n \in S)$

so its smallest element, k_0
(by well ordering principle)

$\Rightarrow$ then $P(k_0 - 1)$ is true because $k_0 - 1 \notin S$ when k_0 is the smallest in S

Contradiction! because hypothesis says $P(k)$ is true

so argument is valid