

Principles of Programming Languages

CS496

Expressions of Basic Types

Variables

Simple Aggregate Types: Functions and Tuples

Polymorphism

Higher-Order Functions

Lists – Basics

Recursion and Pattern Matching on Numbers

Recursion and Pattern Matching on Lists

Function Schemes

Summary

Basic Types

- ▶ Next we will begin experimenting with
 - ▶ basic types and
 - ▶ expressions of basic types
- ▶ These types include
 - ▶ `int`
 - ▶ `bool`
 - ▶ `float`
 - ▶ `string`
 - ▶ `char`
 - ▶ `unit`
- ▶ There are other types, we'll see them later

int – integers

```
# 1;;  
2 - : int = 1  
# 12345 + 1;;  
4 - : int = 12346  
# 12345 - 1;;  
6 - : int = 12344  
# 3+4;;  
8 - : int = 7  
# 8/3;;  
10 - : int = 2  
# 30_000_000 / 300_000;;  
12 - : int = 100
```

float – floating point numbers

```
# 3.5 +. 6.;; (* notice the dot after the + *)  
2 - : float = 9.5  
# sqrt 9.;;  
4 - : float = 3.  
# 1 + 2.0;;  
6   ^^^  
   This expression has type float but is here used with type
```

Note the last expression:

- ▶ The + function operates on integers, but 2.0 is not an integer

float – floating point numbers

- ▶ It is possible to convert from integers to floats and back
- ▶ `float_of_int` is called a **function**; we will study functions later

```
# float_of_int 1;;  
2 - : float = 1.  
# int_of_float 1.2;;  
4 - : int = 1  
# 1 + int_of_float 2.0;;  
6 - : int = 3
```

char – characters

```
# 'a';;  
2 - : char = 'a'  
# 'x';;  
4 - : char = 'x'  
# "hello".[1];;  
6 - : char = 'e'
```

char – characters

- ▶ OCaml provides a set of built-in [modules](#)
- ▶ Modules define useful operations on numerous types
- ▶ An example is the [Char](#) module which provides useful operations on chars

```
1 # Char.uppercase 'z';;  
2 - : char = 'Z'  
3 # Char.uppercase '[';;  
4 - : char = '['  
5 # Char.chr 97;;  
6 - : char = 'a'  
7 # Char.code 'a';;  
8 - : int = 97
```


string — strings

```
# "Hello";;  
2 - : string = "Hello\n"  
# "Hello " ^ " world\n";;  
4 - : string = "Hello world\n"  
# "The character '\000' is not a terminator";;  
6 - : string = "The character '\000' is not a terminator"  
# "\072\105";;  
8 - : string = "Hi "  
# "Hello".[1];;  
10 - : char = 'e'
```

string – strings

The `String` module provides many useful functions on strings

```
# String.length "Ab\000cd";;  
2 - : int = 5  
# String.sub "Abcd" 1 2;;  
4 - : string = "bc"
```

bool - booleans

```
2  # 2 < 4;;  
   - : bool = true  
4  # "A good job" > "All the tea in China";;  
   - : bool = false  
6  # 2 + 6 = 8;;  
   - : bool = true  
8  # 1.0 = 1.0;;  
   - : bool = true  
10 # 2!=4;;  
   - : bool = true  
   # true && false;;  
12 - : bool = false  
   # true || false;;  
14 - : bool = true
```

use = for equality checking

bool - booleans

```
2  # if 1 < 2
   then 3+7
   else 4;;
4  - : int = 10
   # if 3!=4 then 1 else 2;;
6  - : int = 1
   # if 2 then 3 else 4;;
8      ^^^
Error: This expression has type int but an expression was expected
10 type bool
```

unit - unit type

- ▶ Special type typically assigned to expressions that cause effects
- ▶ Example: `print_string` for printing a string

```
# ();;  
2 - : unit = ()  
# print_string "hello";;  
4 hello- : unit = ()  
# print_char 'a';;  
6 a- : unit = ()  
# print_int 3;;  
8 3- : unit = ()
```

unit - unit type

- Expressions of unit type can be composed using “;”

```
# print_string "hello"; print_string "bye";;  
2 hellobye- : unit = ()
```

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Variables

- ▶ Variables are names given to values
 - ▶ These names always start with lowercase
- ▶ Variables allow these values to be reused
- ▶ Variables are declared using: `let identifier = expression`

```
# let x = 1;;  
2 val x : int = 1  
# let y = 2;;  
4 val y : int = 2  
# let z = x + y;;  
6 val z : int = 3
```


Nesting Declarations

Declarations can be nested using the form

`let variable=expression in expression`

```
1  # let x = 1
2      in let y = 2
3          in x + y;;
4  - : int = 3
5
6  # let z =
7      let x = 1
8          in let y = 2
9              in x + y;;
10 val z : int = 3
11
12 # let x =
13     let y = 2 in y
14     in x+1;;
15 - : int = 3
```

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Basic Types vs Agregate Types

- ▶ Basic types seen so far
 - ▶ `int`
 - ▶ `bool`
 - ▶ `float`
 - ▶ `string`
 - ▶ `char`
 - ▶ `unit`
- ▶ Agregate types we shall see now
 - ▶ They are built out of simple types by composing them
 - ▶ We will see two composite type constructors (more, eg. lists, later)
 - ▶ Functions
 - ▶ Tuples

Functions

```
# let succ i = i + 1;;
2 val succ : int -> int = <fun>
# succ 1;;
4 - : int = 2
# succ (succ 1);;
6 - : int = 3
# succ;;
8 - : int -> int = <fun>
```

Functions

An alternative definition using [anonymous functions](#)

```
# let succ i = i + 1;;  
2 val succ : int -> int = <fun>  
# let succ2 = fun i -> i + 1;;  
4 val succ2 : int -> int = <fun>  
# succ2 1;;  
6 - : int = 2
```

[fun](#), used above, allows [anonymous functions](#) to be defined

```
# fun x -> x+1;;  
2 - : int -> int = <fun>
```

Function Types

Lets take a closer look at the type of `succ`

```
# let succ i = i + 1;;  
2 val succ : int -> int = <fun>
```

The type of `succ` is `int -> int`

What does this function do and what is its type?

```
# let f i = i>0;;
```

What happens if you evaluate `f 3.5`?

Exercise

- ▶ Define the function `sign` which given an integer returns 1 if it is positive, -1 if it is negative and 0 if it is zero.
- ▶ What is the type of `sign`?

Exercise

- ▶ Suppose we use a function of type `int -> bool` to denote a subset of integers, namely those for which the function returns `true`
- ▶ Such functions are called **characteristic function of a set**
- ▶ For example, `let f x = x mod 2` denotes the set of even numbers
- ▶ Define union and intersection of sets represented through their characteristic functions
- ▶ union and intersection should have type
`(int->bool) -> (int->bool)-> int -> bool`

Functions with Multiple Arguments

```
# let add i j = i + j;;
2 val add : int -> int -> int = <fun>
# add 2 3;;
4 - : int = 5
# add 2 3 4;;
6 Error: This function has type int -> int -> int
      It is applied to too many arguments; maybe you forgot
```

Functions with Multiple Arguments

An alternative definition using anonymous functions

```
1 # let add2 = fun i j -> i + j;;  
  val add2 : int -> int -> int = <fun>  
3 # add2 2 3;;  
  - : int = 5
```

Function Types

- ▶ Lets take a closer look at the type of add

```
# let add i j = i + j;;  
2 val add : int -> int -> int = <fun>
```

- ▶ The type of succ is `int -> int -> int`
- ▶ How do we read this type?
 - ▶ succ is a function that
 - given an integer `i`, returns a **function** that
 - given an integer `j`, returns `i+j`

Partial Application

- ▶ `succ` is a function that
 - given an integer `i`, returns a **function** that
 - given an integer `j`, returns `i+j`

This means we can apply `add` to just ONE argument and get back a function

```
# add 1;;  
2 - : int -> int = <fun>
```

A new way to define successor!

```
# let succ3 = add 1;;  
2 val succ3 : int -> int = <fun>  
# succ3 4;;  
4 - : int = 5
```

Exercise

- ▶ Define the function `min3` that given three integers returns the smallest one.
 - ▶ Use if-then-else and conjunction
- ▶ What is the type of `min3`?

Exercise

- ▶ Define the functions `and'`, `or'`, `not'` and `xor'` which implement the standard boolean operations.
- ▶ What is the type of each of these functions?

Tuples

Just like ordered tuples in math

```
# (2,3);;  
2 - : int * int = (2, 3)  
# (true,3);;  
4 - : bool * int = (true, 3)  
# (true,2,4);;  
6 - : bool * int * int = (true, 2, 4)  
#  
8 # (2,(true,23));;  
- : int * (bool * int) = (2, (true, 23))
```

- ▶ Tuples that have just two components are called **pairs**
- ▶ The type of a tuple is $t_1 * t_2 * \dots * t_n$ where each t_i is the type of the respective component

Tuples

How do we access the components of a tuple?

```
# let fst (x,y) = x;;
2 val fst : 'a * 'b -> 'a = <fun>
# fst (2,3);;
4 - : int = 2
# fst (2,3,4);;
6 Error: This expression has type 'a * 'b * 'c
   but an expression was expected of type 'd * 'e
```

Note that `fst` uses [pattern matching](#):

- ▶ `(x,y)` is a pattern that can only match a pair
- ▶ binds variables `x` and `y` to the first and second component of the pair, resp.

Tuples

```
# let f2 (x,_) = x;;  
2 val f2 : 'a * 'b -> 'a = <fun>  
# f2 (2,3);;  
4 - : int = 2
```

Exercise on Types

Provide expressions of the following types:

1. `bool`
2. `int * int`
3. `bool -> int`
4. `(int * int) -> bool`
5. `int -> (int -> int)`
6. `(bool -> bool) * int`

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Polymorphism

- ▶ What is the type of this function?

```
let id x = x
```

- ▶ Here are examples of its use:

```
# id 2;;  
2 - : int = 2  
# id true;;  
4 - : bool = true  
# id "hello";;  
6 - : string = "hello"
```

- ▶ Its type should be $t \rightarrow t$, for any type t
- ▶ How do we express such a type? We use [type variables](#)

```
'a -> 'a
```

Polymorphism

```
2 let id x = x;;  
   val id : 'a -> 'a = <fun>
```

- ▶ This type is read as follows:
id is a function that given a value of type 'a, returns another value of the same type 'a
- ▶ We say id is **polymorphic**
- ▶ Notice that
 - ▶ id 2 has type int and
 - ▶ id true has type bool

Polymorphism

- ▶ It is a feature of type systems
- ▶ It allows an expression to have infinite types
- ▶ The type system then adjusts these types to more concrete ones depending on the use of these expressions
 - ▶ `id: 'a -> 'a (*general type *)`
 - ▶ `id: int -> int (*more concrete type *)`
- ▶ This style of polymorphism is called **parametric polymorphism** (the parameter is the type variable)

More Examples

```
# let f x = 7;;
```

► What does `f` do and what is its type?

```
# let f x = 7;;  
2 val fst : 'a -> 'int = <fun>
```

More Examples

```
# let fst (x,y) = x;;  
2 val fst : 'a * 'b -> 'a = <fun>  
# fst (2,3);;  
4 - : int = 2
```

- ▶ `fst` takes a pair of type `'a * 'b` and returns a result of type `'a`

More Examples

```
# let f x y = x;;
```

- ▶ What does this function do?
- ▶ What is its type?

Exercise

- ▶ Write a function `swap` that takes in a pair and returns the same pair but where the components have been swapped
- ▶ For example, `swap (2,true)` should return `(true,2)`
- ▶ What is the type of this function?

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Motivating Example

What is the type of the following function?

```
# let twice f x = f (f x);;
```

Consider the following example

```
1 # let twice f x = f (f x);;  
  val t : ('a -> 'a) -> 'a -> 'a = <fun>  
3 # let sqr x = x*x;;  
  val sqr: int -> int = <fun>  
5 # twice sqr 2  
  - : int = 16
```

Higher-Order Functions

A function that takes another function as argument or that returns a function as result¹

¹The precise notion is more technical; this suffices for us.

Higher-Order Functions

Are these functions higher-order?

```
# let add x y = x + y;;  
2 val add : int -> int -> int = <fun>  
# let myAnd x y = x && y;;  
4 val myAnd : bool -> bool -> bool = <fun>
```

- ▶ According to our definition, they are
 - ▶ add, given an integer, returns a function
- ▶ Note: `int -> int -> int` is the same as writing `int -> (int -> int)`
 - ▶ `->` associates to the right

Higher-Order Functions

A function that takes another function as argument or that returns a function as result²

Some more examples

```
# let apply f x = f x;;
2 val apply : ('a -> 'b) -> 'a -> 'b = <fun>
# apply (fun x -> (x,x)) 3;;
4 - : int * int = (3, 3)
# let apply' = fun f -> (fun x -> f x);;
6 val apply' : ('a -> 'b) -> 'a -> 'b = <fun>
# apply' (fun x -> (x,x)) 3;;
8 - : int * int = (3, 3)
```

²The precise notion is more technical; this suffices for us.

More Examples

```
# let compose f g x = f (g x);;
2 val compose : ('a -> 'b) -> ('c -> 'a) -> 'c -> 'b = <fun>
# let sqr x = x*x;;
4 val sqr : int -> int = <fun>
# compose sqr sqr 2;;
6 - : int = 16
```


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Lists

A list is an ordered sequence of values of the same type

```
1 # [1;2;3];;
2 - : int list = [1; 2; 3]
3 # [1;1;3];;
4 - : int list = [1; 1; 3]
5 # ["hello"; "bye"];;
6 - : string list = ["hello"; "bye"]
7 # [1;"hello"];;
8 Error: This expression has type string but an expression was
   of type int
9
10 # 1::[2;3];;
11 - : int list = [1; 2; 3]
12 # [];;
13 - : 'a list = []
14 # 1::(2::(3::([])));;
15 - : int list = [1; 2; 3]
```

`::` is called **cons**, it adds an element to the beginning of a list

Lists – `cons` Operator

`::` is called `cons`

- ▶ it adds an element to the beginning of a list
- ▶ its type is `'a -> 'a list -> 'a list`

```
# 1::[2;3];;  
2 - : int list = [1; 2; 3]  
# [];;  
4 - : 'a list = []  
# 1::(2::(3::[]));;  
6 - : int list = [1; 2; 3]
```

Lists – Append

```
# [1;2;3] @ [4;5];;  
2 - : int list = [1; 2; 3; 4; 5]  
# [1;2;3] @ [];;  
4 - : int list = [1; 2; 3]  
# [1;2] @ ["hello";"bye"];;  
6 Error: This expression has type string but an expression was  
of type int
```

Recall: use = for equality checking

Concatenating Lists

- ▶ Which of these are true and which are false?
- ▶ Under what assumptions?

```
1  [[]] @ xs      = xs
2  [[]] @ [xs]    = [[] , xs]
3  [[]] @ xs      = [xs]
4  [] :: xs       = xs
5  [[]] @ [xs]    = [xs]
6  [[]] @ xs      = [] :: xs
7  [xs] @ [xs]    = [xs , xs]
8  [] @ xs        = [] :: xs
9  [[]] @ xs      = [[] , xs]
10 [xs] @ []      = [xs]
```

List Module

- ▶ Contains many useful operations on lists
- ▶ One example is `length`

```
# List.length [1;1;2];;  
2 - : int = 3  
# length [1;2;3];;  
4 Error: Unbound value length  
# open List;;  
6 # length [1;2;3];;  
- : int = 3
```

Note: Browsable sources of OCaml libraries

[http://caml.inria.fr/cgi-bin/viewvc.cgi/ocaml/trunk/
stdlib/](http://caml.inria.fr/cgi-bin/viewvc.cgi/ocaml/trunk/stdlib/)

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Recursion

- ▶ Problem: Write a program that, given an integer n , adds the first n integers
- ▶ Example: if $n = 10$ then we want to add

$$0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$$

```
# let rec sum n =  
2   match n with  
    0 -> 0  
4   | n -> n + sum (n-1);;  
val sum : int -> int = <fun>  
6 # sum 0;;  
- : int = 0  
8 # sum 10;;  
- : int = 55
```


Recursion

```
1 # let rec sum n =  
    match n with  
3     0 -> 0  
    | n -> n + sum (n-1);;
```

- ▶ `rec` says that we are defining a recursive function
 - ▶ A recursive function is a function that can call itself
- ▶ `match` is used for pattern matching on `n`
 - ▶ It is typically used in combination with `rec` but doesn't have to
- ▶ Lets follow the execution of couple of uses of `sum`

Recursion

On the board:

▶ sum 0

▶ sum 1

▶ sum 2

▶ sum 3

Recursion

```
1 # let rec sum n =
2   match n with
3     0 -> 0
4   | n -> n + sum (n-1);;
5 # sum (-3);;
6 Stack overflow during evaluation (looping recursion?).
7 # let rec sum n =
8   match n with
9     0 -> 0
10  | n when n>0 -> n + sum (n-1)
11  | _ -> failwith "sum:: argument must be non-negative";;
12 val sum : int -> int = <fun>
13 # sum (-3);;
14 Exception: Failure "sum:: argument must be non-negative".
15 # sum 10;;
16 - : int = 55
```

Another Example of Recursion

- ▶ Problem: Write a program that, given an integer n , multiplies the first n integers
- ▶ Note: if $n = 0$ it should return 1
- ▶ Example: if $n = 10$ then we want to return

$$1 * 2 * 3 * 4 * 5 * 6 * 7 * 8 * 9 * 10$$

```
# let rec fact n =  
2   match n with  
    0 -> 1  
4   | n -> n * fact (n-1);;  
val fact : int -> int = <fun>  
6 # fact 0;;  
- : int = 0  
8 # fact 10;;  
- : int = 3628800
```

Exercise

- ▶ Write a function `list_enum` that given a positive number `n` returns the list `[n;n-1;...;1;0]`
- ▶ For example, `list_enum 5` should return `[5;4;3;2;1;0]`
- ▶ What is the type of `list_enum`?

Exercise

- ▶ Write a function `repeat` that given an argument `x` and a positive number `n` returns the list

`[x;x;...;x]`

where `x` is repeated `n` times

- ▶ For example, `repeat "hello" 4` should return
`["hello"; "hello"; "hello"; "hello"]`
- ▶ What is the type of `repeat`?

Exercise

- ▶ Write a function `stutter` that given two positive numbers `n` and `m` returns a new list of the form
$$[[n;n;\dots;n]; [n-1;n-1;\dots;n-1]; \dots; [0;0;\dots;0]]$$
 where each nested list has `m` items
- ▶ For example, `stutter 3 2` should return
$$[[3;3]; [2;2]; [1;1]; [0;0]]$$
- ▶ What is the type of `stutter`?

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The Length of a List

```
let rec length l =  
2   match l with  
    [] -> 0  
4   | x::xs -> 1 + length xs
```

- ▶ Note the two cases in the definition:
 - ▶ the empty list `[]` – called the base case
 - ▶ the non-empty list `x::xs` – called the inductive case
- ▶ Run this function on a sample list
- ▶ What is the type of `length`?

Sum of a List of Numbers

```
let rec sum l =  
2   match l with  
    [] -> 0  
4   | x::xs -> x + sum xs
```

Alternatively,

```
let rec sum = function  
2   [] -> 0  
    | x::xs -> x + sum xs
```

Exercise

- ▶ Write a function that multiplies all the numbers in a list

Functions that Construct Lists

```
1 let rec incr l =  
    match l with  
3   [] -> []  
   | x::xs -> (x+1)::incr xs
```

Note: Function application has precedence over ::

Stutter

What does this function do?

```
let rec stutter l =  
2   match l with  
    [] -> []  
4   | (x::xs) -> x::x::(stutter xs)
```

Exercise

- ▶ Define a function `is_zero_list` that given a list of numbers returns a list of booleans indicating whether each number is 0 or not.
- ▶ For example,

```
> is_zero_list [3;0;7;0;0];;  
2 - : bool list = [false; true; false; true; true]
```

- ▶ What is the type of this function?

Functions that Filter Elements from a List

What does this function do?

```
let rec even l =  
2   match l with  
    [] -> []  
4   | (x::xs) ->  
    if (x mod 2=0)  
6   then x :: (even xs)  
    else even xs
```

► Try it out on an example

Functions that Filter Elements from a List

What does this function do?

```
1 let rec even l =  
    match l with  
3     [] -> []  
    | (x::xs) ->  
5         if (x!=[])  
           then x :: (g xs)  
7         else even xs
```

► Try it out on an example

Functions that Filter Elements from a List

- ▶ Define a function that given a list of strings and a number n , filters (i.e. keeps) those strings whose length is smaller or equal to n
- ▶ What is the type of this function?

Functions on Lists that Deviate from Standard Patterns

- ▶ The standard patterns when defining a recursive function f over lists are:
 - ▶ Define f over the empty list $[]$ (called base case)
 - ▶ Define f over the non-empty list $x::xs$ (called inductive case)
- ▶ Some functions however don't fall in that scheme
- ▶ Here are some examples that we will develop on the board:
 - ▶ `head`
 - ▶ `tail`
 - ▶ `maximum`
 - ▶ `last`
 - ▶ `remove_adjacents`

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Motivating Examples

- ▶ Let us implement the following functions
 - ▶ `succ1 : int list -> int list`
 - ▶ `to_upper1 : char list -> char list`
 - ▶ `all_zero : int list -> bool list`
- ▶ What do you notice in common among all these implementations?

Map

```
let rec map f l =  
2   match l with  
    | [] -> []  
4   | (x::xs) -> (f x)::(map f xs)
```

- ▶ What does `map` do?
- ▶ What is its type?
- ▶ How can we use it to define `succl`, `to_upperl` and `all_zero`?

```
let succl' = map (fun x -> x+1)  
2 let to_upperl' = map Char.uppercase_ascii  
let all_zero = map (fun x -> x=0)
```

Filter

- ▶ Lets implement the following functions:
 - ▶ `greater_than_zero : int list -> int list`
 - ▶ `uppercase : char list -> char list`
 - ▶ `non_empty : 'a list list -> 'a list list`
- ▶ What do you notice that they have in common?

Filter

```
let rec filter p l =  
2   match l with  
    | [] -> []  
4   | (x::xs) -> if (p x)  
                  then x::(filter p xs)  
6                  else filter p xs
```

- ▶ What does `filter` do and what is its type?
- ▶ How can we use `filter` to implement `greater_than_zero`, `uppercase` and `non_empty`?

```
let greater_than_zero = filter (fun x -> x>0)  
2 let uppercase = filter (fun x -> x=Char.uppercase_ascii x)  
let non_empty = filter (fun x -> x!=[])
```

Iterate

- ▶ Suppose we want to print out all the strings in a list of strings
- ▶ Here is one possible implementation of `print_list_of_strings`

```
let rec print_list_of_strings l =  
2 match l with  
  | [] -> ()  
4  | (x::xs) -> print_string x;  
                  print_list_of_strings xs
```


Iterate

- ▶ OCaml provides `List.Iter`

```
List.iter print_string
```

Fold

Consider the implementation of the following functions

- ▶ `sum_list : int list -> int`, that adds all the elements in a list of integers
- ▶ `and_list : bool list -> bool`, that indicates whether all the booleans in the list are true
- ▶ `concat : 'a list list -> 'a list`, that concatenates all the lists in a list

What do you notice in common among their implementations?

Fold

```
let rec fold_right f l a =  
2 match l with  
| [] -> a  
4 | (x::xs) -> f x (fold_right f xs a)
```

- ▶ Here is a description of the result of `fold_right f [x1; ...; xn] a`:
$$f\ x1\ (f\ x2\ (\dots\ (f\ xn\ a)\ \dots))$$
- ▶ What is its type?
- ▶ How can we define `all_fives`, `all` and `concat` in terms of `fold_right`?

Function Schemes

- ▶ map, filter, iter and fold are known as function schemes
- ▶ They abstract common patterns of behaviour
- ▶ Also, they allow for code reuse
- ▶ Finally, they help better understand the problem

Higher-Order Function Schemes

2

```
take  
append
```

- ▶ Function schemes over function types

Expressions of Basic Types

Variables

Simple Aggregate Types: Functions and Tuples

Polymorphism

Higher-Order Functions

Lists – Basics

Recursion and Pattern Matching on Numbers

Recursion and Pattern Matching on Lists

Function Schemes

Summary

Summary

- ▶ OCaml expressions
 - ▶ Every expression has a unique type
- ▶ Types:
 - ▶ Basic types such as `int` and `bool`
 - ▶ Agregate types such as `int -> int` and `int * int`
- ▶ We've learned to evaluate programs in `utop`
- ▶ A word on style:
https://www.seas.upenn.edu/~cis341/current/programming_style.shtml
- ▶ Polymorphism
- ▶ Higher-order functions
- ▶ Recursion on numbers and lists
- ▶ Functions on lists
- ▶ Function schemes (`map`, `filter`, `fold`)