Homework 03 – Julia Nelson

Code

"I pledge my honor that I have abided by the Steven's Honor System."

Problem 1

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"I pledge my honor that I have abided by the Stevens Honor System." JNelson

Problem 1

Find moment estimator and max likelihood estimator for θ of the uniform dist. on $(0,\theta)$.

$$f(x_i) = \begin{cases} 1/\theta, & 0 \le x_i \le \theta \\ 0, & \text{otherwise} \end{cases}$$

Moment estimator $E_\theta(x) = \int_0^\theta x \frac{1}{\theta} dx = \theta/2$

MLE: $\prod_{i=1}^{n} f(x_i) = (1/\theta)^n = Ln(\theta)$

$$\log(Ln(\theta)) = \begin{cases} -n\log(\theta), & \text{if } \theta \ge x_i \\ -\infty, & \text{otherwise} \end{cases}$$

MLE = $\max(x_i)$ since $-n\log(\theta)$ is decreasing function of θ

derivative $= \frac{-n}{\theta} < 0$

$$\boxed{\hat{\theta} = x_{(n)} = \max(x_i)}$$

Problem 6.17

a)

The margin of error is 0.2453521.

The 95% confidence interval is (5.154648, 5.645352).

Hide

```
n <- 340
mean <- 5.4
sd <- 2.3
critical_t <- qt(.975, df <- n-1)
margin_error <- critical_t*(sd/sqrt(n))
margin_error
```

Hide

```
lower_bound <- mean - margin_error
lower_bound
upper_bound <- mean + margin_error
upper_bound
```

b)

The margin of error is 0.3221148.

The 95% confidence interval is (5.076885, 5.723115).

The interval in (b) is wider than the interval in (a).

Hide

```
critical_t <- qt(.995, df <- n-1)
margin_error <- critical_t*(sd/sqrt(n))
margin_error
```

Hide

```
lower_bound <- mean - margin_error
lower_bound
upper_bound <- mean + margin_error
upper_bound
```

Problem 6.27

a)

The 95% confidence interval is (11.02992, 11.97008).

Hide

```
n <- 1200
mean <- 11.5
sd <- 8.3
critical_t <- qt(.975, df <- n-1)
margin_error <- critical_t*(sd/sqrt(n))
margin_error
lower_bound <- mean - margin_error
lower_bound
upper_bound <- mean + margin_error
upper_bound
```

b)

No. There are 204 students who have times of 0, as given in the problem.

c)

The sample is large enough, with a high enough response rate of 83% that indicates that the use of normality is okay.

Problem 6.28

a)

The mean is 690 minutes and the sd is 498 minutes.

Hide

```
n <- 1200
mean_min <- 60*11.5
mean_min
sd_min <- 60*8.3
sd_min
```

b)

The 95% confidence interval is (661.795, 718.205).

Hide

```
critical_t <- qt(.975, df <- n-1)
margin_error <- critical_t*(sd_min/sqrt(n))
margin_error
lower_bound <- mean_min - margin_error
lower_bound
upper_bound <- mean_min + margin_error
upper_bound
```

c)

Could have multiplied the confidence interval in the previous exercise, (11.02992, 11.97008), by 60.

Problem 6.58

Hide

```
z <- 1.77
```

a)

The P-value is 0.0384.

Hide

```
    P_value <- pnorm(z, lower.tail = FALSE)
    P_value
```

b)

The P-value is 0.9616.

Hide

```
    P_value <- pnorm(z, lower.tail = TRUE)
    P_value
```

c)

The P-value is 0.0767.

Hide

```
    P_value <- 2*pnorm(-abs(z))
    P_value
```

Problem 6.59

Hide

```
z <- -1.69
```

a)

The P-value is 0.9545.

Hide

```
    P_value <- pnorm(z, lower.tail = FALSE)
    P_value
```

b)

The P-value is 0.04551.

Hide

```
P_value <- pnorm(z, lower.tail = TRUE)
    P_value
```

c)

The P-value is 0.0910.

Hide

```
    P_value <- 2*pnorm(-abs(z))
    P_value
```

Problem 6.71

a)

The P-value is 0.0217.

Since the P-value is small, less than 0.05, we reject the null hypothesis.

Thus, there is evidence that the population mean is larger than 115.

Hide

```
mean <- 115
sd <- 30
sample_mean <- 127.8
n <- 25
t <- (sample_mean-mean)/(sd/sqrt(n))
t
P_value <- pt(-abs(t),df=n-1)
P_value
```

b)

The sample needs to be independently and randomly chosen from the popualtion and the data needs to be normally distributed.

The nearly normal condition is the most important part to the validity of the conclusion.

If the data is not nearly normal, the probabilities will be incorrect, which is what the conclusion is based on.

Problem 6.73

a)

H0: The mean difference is 0.

Ha: The mean difference is not 0.

Hide

```
n <- 20
sigma <- 3
diff <- c(5.0, 6.5, -0.6, 1.7, 3.7, 4.5, 8, 2.2, 4.9, 3, 4.4, 0.1, 3.0, 1.1, 1.1, 5, 2.1, 3.7, -0.6, -4.2)
m <- mean(diff)
m
```

b)

The P-value is 0.00065.

Since the P-value is so small, we reject the null hypothesis.

There is evidence that the mean difference is not 0.

There is evidence to suggest a difference in the computer and driver's calculations, on average. The computer may be miscalibrated.

Hide

```
t <- (m-0)/(sigma/sqrt(n))
t
P_value <- 2*pt(-abs(t),df=n-1)
P_value
```

Problem 6.99

a)

The P-value is 0.2856.

Hide

```
mean <- 2403.7
sample_mean <- 2453.7
sd <- 880
n1 <- 100
t <- (sample_mean-mean)/(sd/sqrt(n1))
t
P_value <- pt(-abs(t),df=n1-1)
P_value
```

b)

The P-value is 0.1023.

Hide

```
n2 <- 500
t <- (sample_mean-mean)/(sd/sqrt(n2))
t
P_value <- pt(-abs(t),df=n2-1)
P_value
```

c)

The P-value is 0.0023.

Hide

```
n3 <- 2500
t <- (sample_mean-mean)/(sd/sqrt(n3))
t
P_value <- pt(-abs(t),df=n3-1)
P_value
```

Problem 6.120

Hide

```
x <- c(0, 1, 2, 3, 4, 5, 6)
p_0 <- c(0.1, 0.1, 0.2, 0.1, 0.1, 0.1, 0.3)
p_1 <- c(0.2, 0.2, 0.2, 0.1, 0.1, 0.1, 0.1)
data <- data.frame(cbind(x, p_0, p_1))
```

a)

- X <= 2 means X = 0 or 1 or 2
- P(Type 1 Error) = P(X = 0 or X = 1 or X = 2) = P(X = 0) + P(X = 1) + P(X = 2)
- using probabilities from p0

There is a probability of type 1 error of 0.40, or 40%

Hide

```
    P_type1 <- 0.1 + 0.1 + 0.2
    P_type1
```

b)

- P(Type 2 Error) = P(X = 3 or X = 4 or X = 5 or X = 6) = P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6)
- Using probabilities from p1.

There is a probability of type 2 error of 0.40, or 40%.

Hide

```
    P_type2 <- 0.1 + 0.1 + 0.1 + 0.1
    P_type2
```

Problem 7.22

a)

There are 15 degrees of freedom.

Hide

```r
n <- 16
t <- 2.15
mean <- 8
df <- n - 1
df
```

b)

$2.131 < t < 2.602$

The critical t-values are 2.131 and 2.602.

c)

$0.01 < P < 0.025$

The P-value is between 0.01 and 0.025.

d)

At the 5% level, since P-value < 5%, the result IS statistically significant.

At the 1% level, since P-value > 1%, the result IS NOT statistically significant.

e)

The P-value is 0.0241.

Hide

```r
P_value <- pt(-abs(t),df=n-1)
P_value
```

Problem 7.23

a)

There are 26 degrees of freedom.

Hide

```
n <- 27
t <- 2.01
mean <- 40
df <- n - 1
df
```

b)

$1.706 < t < 2.056$

The critical t-values are 1.706 and 2.056.

c)

$0.05 < P < 0.10$

The P-value is between 0.05 and 0.10.

d)

At the 5% level, since P-value > 5%, the result IS NOT statistically significant.

At the 1% level, since P-value > 1%, the result IS NOT statistically significant.

e)

The P-value is 0.0549.

Hide

```
P_value <- 2*pt(-abs(t),df=n-1)
P_value
```