Homework 6

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23.8 First we will use a theorem from real analysis

Theorem 0.1. *Let (a_n) be a convergent sequence in $\mathbb{R}$ with standard topology and $(b_n)0$ be a sequence in $\mathbb{R}$. The sequence $(a_n + b_n)$ converges iff (b_n) converges.*

A proof for this may be provided if desired, however this is considered to be outside the scope of this course.

Let us choose $x \in \mathbb{R}^\omega$ and let us consider this to be (b_n). Let us also choose $\epsilon \in (0, 1)$, then the open ball around x is defined as $\left\{(s_n) \mid \forall_{n\in\mathbb{N}} |s_n - b_n| < \frac{\epsilon}{n}\right\}$. This means that any element s in our open ball may be written as $\left(b_n + \frac{\delta}{n}\right)$ for some $\delta \in (0, \epsilon)$. We now use our theorem from above consider $\frac{\delta}{n}$ to be our sequence (a_n). Thus we may cover $\mathbb{R}^\omega$ with two open non-intersecting sets, The set of all convergent sequences in $\mathbb{R}$ with standard topology and the set of all non-convergent sequences in $\mathbb{R}$ with the standard topology.

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