Computational Linear Algebra
Homework 1

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Contents

1 Question 1

See attached R code.

2 Question 2

2.a

See attached R code.

2.b

See attached R code.

2.c

Let $f(x) = \cos(x)^2 + \sin(x) - 1$, then we find that $f'(x) = -2\sin(x)\cos(x) + \cos(x)$. Then the tangent line at x_0 for any $x_0 \in \mathbb{R}$ to f is

$$\ell(x) = f'(x_0)(x - x_0) + f(x_0).$$

We now solve for x when $\ell(x) = 0$.

$$\begin{aligned}
\ell(x) &= 0 \\
f(x_0)(x - x_0) + f'(x_0) &= 0 \\
f(x_0)(x - x_0) &= -f'(x_0) \\
x - x_0 &= -\frac{f'(x_0)}{f(x_0)} \\
x &= x_0 - \frac{f'(x_0)}{f(x_0)} \\
&= x_0 + \frac{2\sin(x_0)\cos(x_0) - \cos(x_0)}{\cos(x_0)^2 + \sin(x_0) - 1}
\end{aligned}$$

This means that in newton's method for f,

$$x_{i+1} = x_i + \frac{2\sin(x_i)\cos(x_i) - \cos(x_i)}{\cos(x_i)^2 + \sin(x_i) - 1}.$$

2.d

See attached R code.

2.e

See attached R code.

3 Question 3

We have the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & x \\ 4 & 5 & x & 6 \\ 7 & x & 8 & 9 \\ x & 10 & 11 & 12 \end{bmatrix}$$

and we wish to solve for x when $\det A = 1000$. We will then let $f(x) = \det A - 1000$ and we may solve for some roots.

The rest of this problem will be doe in R; see attached R code.

4 Question 4

See attached R code.