Homework II

Benji Altman

February 5, 2018

Problem 1

Part a:

The nth degree taylor polynomial for a function f about some point x_0, given that f is n times differentiable at x_0, is

$$\sum_{i=0}^{n} \left[\frac{f^{(i)}(x_0)}{i!}(x - x_0)^i \right]$$

so the degree 5 Taylor polynomial of $\cos(x)$,

$$\begin{aligned}
P(x) &= \sum_{i=0}^{5} \left[\frac{\cos^{(i)}(0)}{i!}(x)^i \right] \\
&= \cos(0) - x\sin(0) - x^2\frac{\cos(0)}{2} + x^3\frac{\sin(0)}{6} + x^4\frac{\cos(0)}{24} - x^5\frac{\sin(0)}{120} \\
&= 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4
\end{aligned}$$

Part b:

Let $g(x) = |P(x) - cos(x)|$ and solve for the maxima on the range $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$. We can do this with some calculus, we first find $g'(x) = -x + \frac{1}{6}x^3 - \sin(x)$, and we then wish to solve for roots