Estimating Model-based WCPM for ORF Assessment

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Outline

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Introduction

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Introduction

- Why Model-based WCPM?
  - Standard errors of model-based WCPM can be computed for each observation with a single test administration.
  - Missing data are allowed.
    - Accordingly, we can collect data and “equate” model parameters with “common-passage non-equivalent group” design.
    - Passages between multiple grade levels can be equated.

Form Assembly Design for Grade 3										
Form	On-Grade Passages[1]								Off-Grade Passages[2]	
									Grade 2	Grade 4
1	1L	2L	3L	1M	2M	3M	4M	5M	2L 3M	2L 3M
2	3L	4L	5L	4M	5M	6M	7M	8M	4L 6M	4L 6M
3	5L	6L	7L	7M	8M	9M	10M	11M	6L 9M	6L 9M
4	7L	8L	9L	10M	11M	12M	13M	14M	8L 12M	8L 12M
5	9L	10L	11L	13M	14M	15M	16M	17M	10L 15M	10L 15M
6	11L	12L	13L	16M	17M	18M	19M	20M	12L 18M	12L 18M
7	13L	14L	15L	19M	20M	21M	22M	23M	14L 21M	14L 21M
8	15L	16L	17L	22M	23M	24M	25M	26M	16L 24M	16L 24M
9	17L	18L	19L	25M	26M	27M	28M	29M	18L 27M	18L 27M
10	19L	20L	1L	28M	29M	30M	1M	2M	20L 30M	20L 30M

[1] Note. L = long passage; M = medium passage.

[2] All Grade 3 students received two Grade 2 and two Grade 4 off-grade passages.

https://jnese.github.io/core-blog/

Introduction

- With equated model parameters, estimated model-based WCPM will be naturally in a common scale.
    - Post-equting is not necessary.
    - This works for ANY set of passages in the equated passage pool.
        - Useful for longitudinal observations.

Latent Model for Speed and Accuracy

	Data	*Operationalized Model*	*Parameters*
Accuracy	U_{ij}: Accuracy measured by a count of words correctly read by person j for passage i.	*Binomial-count factor model* $U_{ij} \sim Bin\left[n_i, p_i(\theta_j)\right]$, where n_i is the # of words in passage i, and $p_i(\theta_j) = \frac{exp\left[a_i(\theta_j - b_i)\right]}{1+exp\left[a_i(\theta_j - b_i)\right]}$.	a_i: Discrimination of passage i related accuracy b_i: Difficulty of passage related accuracy θ_j: Accuracy of person j
Speed	T_{ij}: Speed measured by seconds took to read passage i by person j.	*Log-normal factor model* $\log(T_{ij}) \sim N\left(\mu_{T_{ij}}, \sigma^2_{T_j}\right)$, where $\mu_{T_{ij}} = \beta_i - \tau_j$ $\sigma^2_{T_j} = \frac{1}{\alpha_i^2}$.	α_i: Discrimination of passage i related speed β_i: Difficulty of passage related speed θ_j: Speed of person j

Model-based WCPM (1)

The model-based WCPM f_j for person j is obtained by

$$f_j = \frac{E[U_{.j}]}{E[T_{.j}]} \times 60.$$

- $E[U_{.j}]$ is the expected value of the total number of words read correctly
- $E[T_{.j}]$ is the expected value of the total reading time
- The rate is further multiplied by a constant 60, such that it will have the same scale as the observed WCPM

Model-based WCPM (2)

$$E[U_{.j}] = \sum_{i=1}^{k} n_i p_{ij} = \sum_{i=1}^{k} \left[n_i \times \frac{\exp[a_i(\theta_j - b_i)]}{1 + \exp[a_i(\theta_j - b_i)]} \right].$$

- n_i is the number of words in the ith passage.
- p_{ij} is the probability of reading each word correctly in the ith passage by person j.
- θ_j is accuracy person parameter.
- a_i and b_i are accuracy passage parameters.

Model-based WCPM (3)

$$E[T_j] = \sum_{i=1}^{k} \exp(\mu_{ij} + \frac{1}{2}\sigma_i^2) = \sum_{i=1}^{k} \exp[\beta_{0i} + \log(\frac{n_i}{10}) - \tau_j + \frac{1}{2\alpha_i}].$$

- μ_{ij} is the reading time (in natural logarithm scale) for the ith passage by person j
- $\mu_{ij} = \beta_i - \tau_j = \beta_{0i} + \log(n_i/10) - \tau_j$
  - β_{0i} is the time intensity parameter based on the rescaled reading time (seconds per 10 words).
  - τ_j is the speed person parameter.

Example: Data

- Data were collected as part of the CORE project.
  - 58 3rd and 4th grade students in a Pacific Northwest state of the U.S.
- 4 passages
  - A total of 260 words (47, 47, 80, and 86)
  - All 58 students read all 4 passages.
    - Observed WCPM
      - M = 110.37, SD = 35.50, min = 43.04, and max = 182.94

Example: Analysis

- The data were fitted to the model by JAGS software via Bayesian MCMC algorithm.
- Obtained:
    - Passage parameters
    - Model-based WCPM
        - Mean of the posterior distribution (analogous to the point estimate)
        - SD of the posterior distribution (analogous to the CSEM)
        - 95% highest density interval (analogous to the 95% CI)

Example: Results - Passage Parameters

Table 1

Estimated passage parameters for real data analysis.

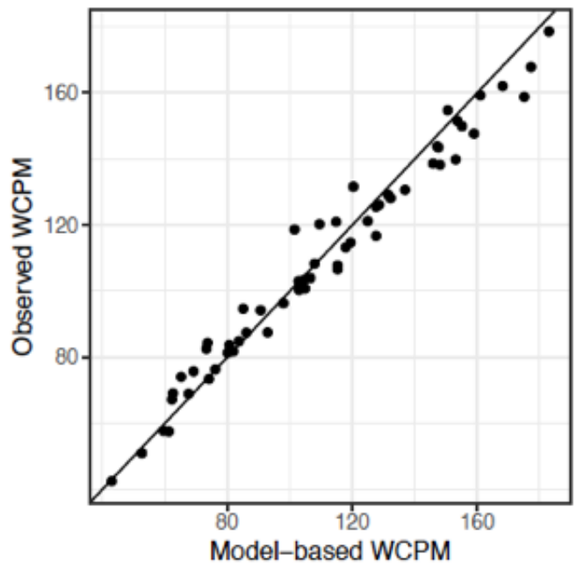
Passage	Word Counts	a	b	α	β
1	47	0.96	-2.99	6.76	1.68
2	47	0.57	-4.40	9.21	1.63
3	80	1.00	-2.45	5.23	1.76
4	86	1.02	-2.80	6.43	1.53

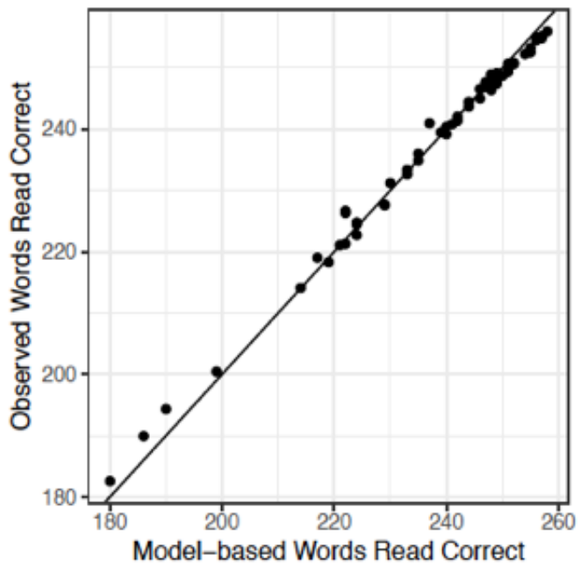
Example: Results - Model-based WCPM

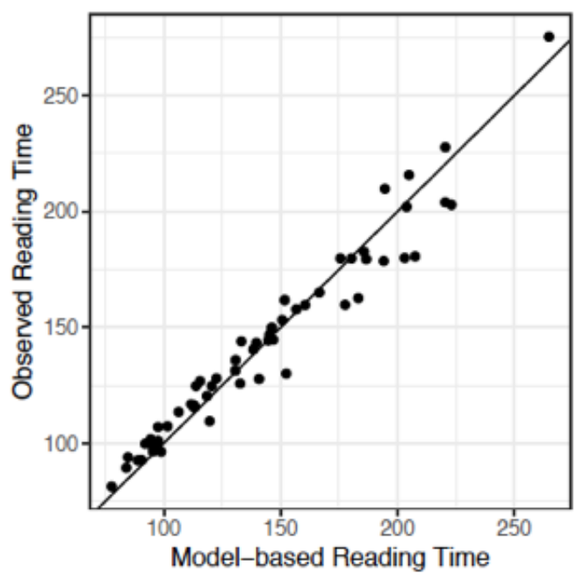
Observed WCPM	Model-based WCPM	CSEM	LB	UB	Range
43.04	42.55	3.34	36.10	49.14	13.04
52.70	51.03	4.17	42.81	59.05	16.24
59.60	57.79	4.31	49.75	66.46	16.72
61.35	57.62	4.62	47.82	66.20	18.39
62.33	67.26	4.91	58.09	77.39	19.31
62.66	69.18	5.38	58.95	79.93	20.98
65.29	74.10	5.76	62.89	85.39	22.50
67.67	69.00	5.07	59.99	79.45	19.46
69.23	75.77	5.87	64.86	87.07	22.21
73.34	82.61	6.36	70.40	94.81	24.41
73.71	84.38	6.56	71.07	96.70	25.63
74.15	73.49	5.81	62.79	85.48	22.69
76.21	76.38	5.66	66.17	88.09	21.92
80.16	81.37	5.92	69.43	92.91	23.48
80.70	83.76	5.85	72.86	96.03	23.18
81.91	81.81	6.43	68.98	94.31	25.33
83.76	84.89	6.31	72.35	96.76	24.40
85.12	94.71	6.98	81.44	108.75	27.32
86.10	87.49	6.42	75.45	100.47	25.02
90.70	94.26	6.57	81.58	106.72	25.14

Observed WCPM	Model-based WCPM	CSEM	LB	UB	Range
127.61	125.57	9.27	107.62	144.48	36.86
127.61	116.75	8.70	99.83	133.95	34.12
128.60	126.15	8.80	108.95	142.93	33.99
131.44	129.25	9.33	111.69	148.41	36.72
132.22	128.20	9.14	110.53	146.34	35.81
136.90	130.67	9.35	111.90	148.70	36.80
145.84	138.63	9.73	121.67	159.31	37.64
147.17	143.86	10.87	121.59	164.47	42.88
147.54	143.52	10.39	123.61	164.44	40.83
148.09	138.25	10.08	118.44	158.23	39.79
150.56	154.76	11.36	133.53	178.00	44.46
153.09	139.87	10.27	119.98	159.55	39.57
153.58	151.60	10.83	130.84	173.75	42.91
155.14	149.90	10.62	129.70	170.94	41.24
158.94	147.69	10.58	125.83	167.32	41.49
160.99	159.28	11.37	134.92	180.04	45.12
168.11	162.09	11.59	140.98	185.98	45.00
175.05	158.79	11.80	136.47	182.53	46.05
177.19	167.86	12.09	144.18	191.11	46.93
182.94	178.60	12.75	154.05	203.58	49.53

Example: Results - Model-based vs. Observed Quantities







What Else Have Been Done?

- Algorithms to estimate model-based WCPM by treating estimated passage parameters as known parameters have been developed.
  - Bayesian MCMC
  - MLE and its SE
- Equating on a larger data set has been implemented.
  - 150 passages in an equated passage pool
  - Model-based WCPM for approx. 1,000 students on up to 4 measurement occations

Next Steps

- Improve SE estimation
- Improve Bayesian estimation
- Extend to modeling with sentence-level data
    - Need to incorporate dependency between sentences within passage
- Develop a suite of data analysis algorithms