07_Backprop_Jim

December 18, 2019

1 Assignment 7: Backpropagation

Read the Rojas book (https://page.mi.fu-berlin.de/rojas/neural/neuron.pdf), chapter 7.3.3 and learn about the "matrix way" of implementing backprop.

1.1 Ex. 7.1 XOR

Implement a two-layer artificial neural network with two input neurons and one output neuron. Choose the number of hidden neurons to your liking and add an error "neuron" to your network. Our goal is to learn the XOR function. What does the network return for random weights of all combinations of (binary) inputs? **(RESULT)**

In [1]:
```
# WTF is an ERROR NEURON? :D
```

In [2]:
```
import numpy as np


# Helper functions
def sigmoid(x):
    return 1 / (1 + np.exp(-x))

def sigmoid_prime(x):
    v = sigmoid(x)
    return v * (1 - v)

def relu(x):
    x[x <= 0.0] = 0.0
    return x

def augmented(array):
    """Add ones to 0-axis."""
    shape = array.shape
    ones = np.ones((1, *shape[1:]))
    items = (array, ones)
    return np.concatenate(items, axis=0)
```

```
    def unaugmented(array):
        """Inverse operation to 'augmented'."""
        return array[:-1]
```

In [3]:
```
# Datasets
X = np.array([[0, 0], [0, 1], [1, 0], [1, 1]])

y_and = [a & b for a, b in X]
y_or = [a | b for a, b in X]
y_xor = [a ^ b for a, b in X]
```

In [4]:
```
from typing import List

import numpy as np


class GradientDescent:
    def __init__(self, lr):
        self.lr = lr

    def __call__(self, weights: List[np.array], gradients: List) -> List:
        return [
            weight - self.lr * gradient
            for weight, gradient in zip(weights, gradients)
        ]
```

In [15]:
```
class Network():
    W = None
    biases = None

    def __init__(self, hidden, m, optimizer=GradientDescent(5.), output_dim=None):
        """
        :param hidden: number of hidden layers
        :param m: number of nodes per hidden layer
        """
        self.hidden = hidden
        self.m = m
        self.optimizer = optimizer
        self.output_dim = output_dim

    @staticmethod
    def init_layer(input_dim, output_dim):
        shape = (input_dim, output_dim)
        # return np.random.uniform(0, 1, shape)
        # return np.random.random(shape)
        # return np.random.random(shape) - 0.5
        return np.random.random(shape) * np.sqrt(2.0 / (input_dim + output_dim))
```

```
def initialize(self, x, y):
    if self.output_dim is None:
        self.output_dim = len(np.unique(y))

    input_dim = len(x[0])
    self.W = self.init_weights(input_dim, self.output_dim, self.m)
    # The number of biases equals each weight matrix'es output_dim.
    self.biases = [np.ones(W_i.shape[1]) for W_i in self.W]

def init_weights(self, input_dim, output_dim, m):
    W = []
    prev_dim = input_dim
    layer_dims = ([m] * self.hidden) + [output_dim]

    for layer_dim in layer_dims:
        W.append(self.init_layer(prev_dim, layer_dim))
        prev_dim = layer_dim
    return W

def feed_forward(self, x):
    """
    DIFFERENT WAY OF ADDING BIASES:
    (aug = augmentation function)

    Rojas' book:
    O.T * W = y.T

    O1   ~> aug(O1) * aug(W_1) = y1 ~> s(y1) = O2
    40x1    41x1      41x50     50x1    50x1    50x1


    This is equal to
    O1   ~>  O1 * W_1   =  y1 + bias ~> s(y1) = O2
    40x     40x1 40x50    50x1  50x1     50x1
    because due to the vector-matrix multiplication
    each entry of the last row of W_1
    is added to each value of y1:
    y1 = [
        O1[0]*W_1[0,0] + O1[1]*W_1[1,0] + ... + O1[n]*W_1[n,0] ( + 1*W_1[n+1,0] )
        ...
        O1[m]*W_1[0,m] + O1[1]*W_1[1,m] + ... + O1[n]*W_1[n,m] ( + 1*W_1[n+1,m] )
                                                                           |
                                                               This is where the bi
                                                               would be added when
                                                               doing it like Rojas.
    ]
    As we can see adding the bias as an extra vector does the same thing.
    """
```

```python
        out_last = x
        outputs = [out_last]

        for W_i, bias in zip(self.W, self.biases):
            out_last = sigmoid((out_last @ W_i) + bias)
            outputs.append(out_last)
        return outputs

    def backprop(self, outputs, y_i) -> List:
        """
        HOW TO UPDATE BIASES:
        (aug = augmentation function)

        Rojas suggests: LR * delta * aug(O)

        Thus, the new bias for a given layer is the entry of the augmented
        output vector multiplied with 'delta'. Since the last entry is always 1
        'delta' itself describes the change of the biases. For that reason,
        they are appended and returned by this method as well.
        """
        gradients = []
        bias_gradients = []
        out_last = outputs[-1]
        out_last_prev = outputs[-2]

        e = out_last - y_i

        D = np.diag(sigmoid_prime(out_last))
        delta = D.dot(e)

        gradient = np.outer(delta, out_last_prev).T
        gradients.append(gradient)
        bias_gradients.append(delta)

        for i in reversed(range(self.hidden)):
            output_idx = i + 1
            output = outputs[output_idx]
            D = np.diag(sigmoid_prime(output))
            delta = D.dot(self.W[output_idx]).dot(delta)
            gradients.append(np.outer(delta, outputs[output_idx - 1]).T)
            bias_gradients.append(delta)

        gradients.reverse()
        bias_gradients.reverse()
        return gradients, bias_gradients

    def step(self, x_i, y_i):
```

```
            """
            :parma y_i: The value for y_i itself
                        or a function the retrieve the value from the outputs.
                        The latter is used for one-hot encoding.
            """
            outputs = self.feed_forward(x_i)
            if callable(y_i):
                y_i = y_i(outputs)
            gradients, bias_gradients = self.backprop(outputs, y_i)
            self.W = self.optimizer(self.W, gradients)
            self.biases = self.optimizer(self.biases, bias_gradients)
            return outputs, gradients, bias_gradients

        def predict(self, x):
            out_last = self.feed_forward(x)[-1]
            out_last[out_last > .5] = 1
            out_last[out_last <= .5] = 0
            return out_last

        def predict_onehot(self, x):
            out_last = self.feed_forward(x)[-1]
            out_last = np.array(np.argmax(out_last) + 1)
            out_last = out_last.astype(np.float64)
            return out_last
```

```
In [6]: class NetXor(Network):
            def fit_wo_backprop(self, x, y, epochs=1):
                self.initialize(x, y)
                for epoch in range(epochs):
                    for x_i, y_i in zip(x, y):
                        prediction = self.predict(x_i)
                        print("Input: " + str(x_i) + " , Label: " + str(y_i) + " , Prediction:


        'param1 : number of hidden layers, param2: m -> number of nodes per hidden layer'
        net = NetXor(2, 2, GradientDescent(.55), output_dim=1)

        print("####  Exercise 7.1  ####")
        net.fit_wo_backprop(X, y_xor)
```

```
####  Exercise 7.1  ####
Input: [0 0] , Label: 0 , Prediction: [1.]
Input: [0 1] , Label: 1 , Prediction: [1.]
Input: [1 0] , Label: 1 , Prediction: [1.]
Input: [1 1] , Label: 0 , Prediction: [1.]
```

1.2 Ex. 7.2 Backpropagation

Implement Backpropagation and optimize the weights of your neural network using the XOR training set:

x, y (0,0), 0
(0,1), 1
(1,0), 1
(1,1), 0
How many training iterations do you need? Plot the network error over the number of iterations! **(RESULT)**
We needed a few iterations (1 or 2) with a standard gradient descent learning rate of 3.0
Error = 1 - Accuracy

In [7]:
```
class NetBackProp(Network):
    'TODO: fix to old version'
    def fit_with_backprop(self, x, y, epochs):
        self.initialize(x, y)

        for epoch in range(epochs):
            print("Epoch: " + str(epoch))
            predict_true = 0
            predict_false = 0

            for x_i, y_i in zip(x, y):
                self.step(x_i, y_i)
                prediction = self.predict(x_i)

                if prediction == y_i:
                    predict_true += 1
                else:
                    predict_false += 1

                print("Input: " + str(x_i) + ", Label: " + str(y_i) + ", Prediction: "

            accuracy = predict_true / (predict_true + predict_false)
            print("predicttrue: " + str(predict_true) + "; predictfalse: " + str(predi
            print("Accuracy after epoch {}: {}".format(epoch, accuracy))


print("####  Exercise 7.2  ####")
net = NetBackProp(2, 2, GradientDescent(3.), output_dim=1)
net.fit_with_backprop(X, y_xor, 10)
```

```
####  Exercise 7.2  ####
Epoch: 0
Input: [0 0], Label: 0, Prediction: [1.]
Input: [0 1], Label: 1, Prediction: [1.]
```

```
Input: [1 0], Label: 1, Prediction: [1.]
Input: [1 1], Label: 0, Prediction: [1.]
predicttrue: 2; predictfalse: 2
Accuracy after epoch 0: 0.5
Epoch: 1
Input: [0 0], Label: 0, Prediction: [0.]
Input: [0 1], Label: 1, Prediction: [1.]
Input: [1 0], Label: 1, Prediction: [1.]
Input: [1 1], Label: 0, Prediction: [0.]
predicttrue: 4; predictfalse: 0
Accuracy after epoch 1: 1.0
Epoch: 2
Input: [0 0], Label: 0, Prediction: [0.]
Input: [0 1], Label: 1, Prediction: [1.]
Input: [1 0], Label: 1, Prediction: [1.]
Input: [1 1], Label: 0, Prediction: [0.]
predicttrue: 4; predictfalse: 0
Accuracy after epoch 2: 1.0
Epoch: 3
Input: [0 0], Label: 0, Prediction: [0.]
Input: [0 1], Label: 1, Prediction: [1.]
Input: [1 0], Label: 1, Prediction: [1.]
Input: [1 1], Label: 0, Prediction: [0.]
predicttrue: 4; predictfalse: 0
Accuracy after epoch 3: 1.0
Epoch: 4
Input: [0 0], Label: 0, Prediction: [0.]
Input: [0 1], Label: 1, Prediction: [1.]
Input: [1 0], Label: 1, Prediction: [1.]
Input: [1 1], Label: 0, Prediction: [0.]
predicttrue: 4; predictfalse: 0
Accuracy after epoch 4: 1.0
Epoch: 5
Input: [0 0], Label: 0, Prediction: [0.]
Input: [0 1], Label: 1, Prediction: [1.]
Input: [1 0], Label: 1, Prediction: [1.]
Input: [1 1], Label: 0, Prediction: [0.]
predicttrue: 4; predictfalse: 0
Accuracy after epoch 5: 1.0
Epoch: 6
Input: [0 0], Label: 0, Prediction: [0.]
Input: [0 1], Label: 1, Prediction: [1.]
Input: [1 0], Label: 1, Prediction: [1.]
Input: [1 1], Label: 0, Prediction: [0.]
predicttrue: 4; predictfalse: 0
Accuracy after epoch 6: 1.0
Epoch: 7
Input: [0 0], Label: 0, Prediction: [0.]
```

```
Input: [0 1], Label: 1, Prediction: [1.]
Input: [1 0], Label: 1, Prediction: [1.]
Input: [1 1], Label: 0, Prediction: [0.]
predicttrue: 4; predictfalse: 0
Accuracy after epoch 7: 1.0
Epoch: 8
Input: [0 0], Label: 0, Prediction: [0.]
Input: [0 1], Label: 1, Prediction: [1.]
Input: [1 0], Label: 1, Prediction: [1.]
Input: [1 1], Label: 0, Prediction: [0.]
predicttrue: 4; predictfalse: 0
Accuracy after epoch 8: 1.0
Epoch: 9
Input: [0 0], Label: 0, Prediction: [0.]
Input: [0 1], Label: 1, Prediction: [1.]
Input: [1 0], Label: 1, Prediction: [1.]
Input: [1 1], Label: 0, Prediction: [0.]
predicttrue: 4; predictfalse: 0
Accuracy after epoch 9: 1.0
```

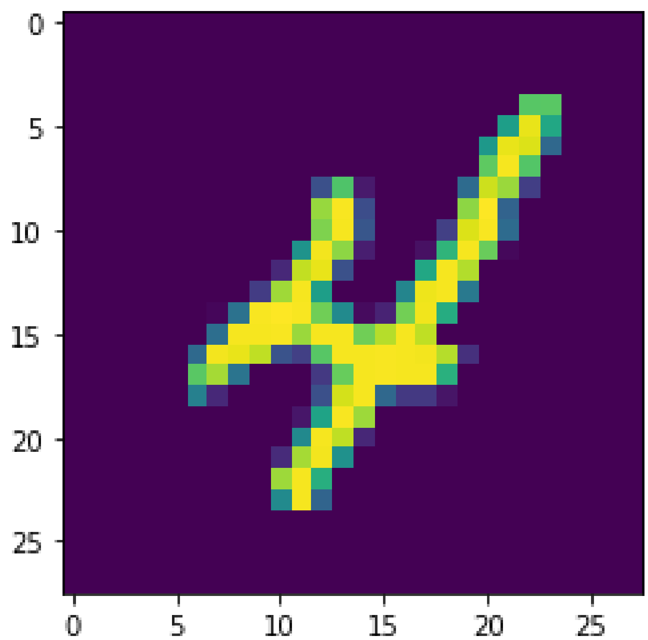
2 Ex. 7.3 MNIST (BONUS)

Train your network on the MNIST dataset and state the model accuracy (or the model error) for the training and test sets. **(RESULT)** Compare to this list

In [12]:
```
import torch
import torchvision
from PIL import Image
from matplotlib import pyplot as plt

mnist_data = torchvision.datasets.MNIST('./MNIST', train=True, transform=None, target_
#data_loader = torch.utils.data.DataLoader(mnist_data, batch_size=4, shuffle=True, nu


# Get data as numpy
np_images = np.empty([len(mnist_data), 28, 28])
np_labels = np.empty([len(mnist_data)])

for i, (image, label) in enumerate(mnist_data):
    data = (image, label)
    np_images[i] = np.array(image)
    np_labels[i] = label

print(np_images.shape)
print(np_labels.shape)
```

```
#Flatten images first
images_flat = np_images.reshape(-1, 28*28)

#Normalize
images_flat = images_flat[:] / 255

# Show one example
print(np_labels[9])
plt.imshow(np_images[9])
plt.show()
```

```
(60000, 28, 28)
(60000,)
4.0
```



In [21]:
```
import sys

import numpy as np


def one_hot_encode(outputs):
    max_value = np.argmax(outputs[-1])
    y_i_oh = np.zeros((10,))
    y_i_oh[max_value] = 1.0
    return y_i_oh
```

```
class NetMnist(Network):
    def fit_mnist(self, x, y, epochs):
        self.initialize(x, y)
        N = len(x)

        for epoch in range(epochs):
            i = 0
            print("Epoch: " + str(epoch))
            predict_true = 0
            predict_false = 0

            for x_i, y_i in zip(x, y):
                outputs, gradients, bias_gradients = self.step(x_i, y_i=one_hot_encode
                prediction = self.predict_onehot(x_i)

                if prediction == y_i:
                    predict_true += 1
                else:
                    predict_false += 1

                i += 1

            accuracy = predict_true / N
            print("predicttrue: " + str(predict_true) + "; predictfalse: " + str(predi
            print("Accuracy after epoch {}: {}".format(epoch, accuracy))

print("####  Exercise 7.3  ####")
net_mnist = NetMnist(hidden=2, m=3, optimizer=GradientDescent(3.), output_dim=10)
net_mnist.fit_mnist(images_flat, np_labels, epochs=3)
```

```
####  Exercise 7.3  ####
Epoch: 0
predicttrue: 6265; predictfalse: 53735
Accuracy after epoch 0: 0.10441666666666667
Epoch: 1
predicttrue: 6265; predictfalse: 53735
Accuracy after epoch 1: 0.10441666666666667
Epoch: 2
predicttrue: 6265; predictfalse: 53735
Accuracy after epoch 2: 0.10441666666666667
```

In [35]:
```
#PROBLEM: Gradients[0] sind alle null. => Daher kein Learning.
```

2.1 Alternative Solution (closer to Rojas)

In [36]:
```
# UTILS
from collections import Counter
import math

import numpy as np
import numpy.linalg
import numpy.matlib


def hot_one_encode_ints(num_classes, ints):
    """
    'ints' May also be a single int.
    """
    # See https://stackoverflow.com/a/42874726/6928824
    targets = np.array(ints).reshape(-1)
    one_hot_targets = np.eye(num_classes)[targets]
    return one_hot_targets.reshape(-1)


def hot_one_decode_int(encoded):
    return np.argmax(encoded, axis=0)


def sigmoid(x):
    return 1.0 / (1 + np.exp(-x))


def sigmoid_d(x):
    return sigmoid(x) * (1 - sigmoid(x))


def relu(x):
    return np.clip(x, 0, np.inf)


def relu_d(x):
    return (x >= 0).astype(float)


def augmented(array, append=True):
    """Add ones to 0-axis."""
    shape = array.shape
    ones = np.ones((1, *shape[1:]))
    if append:
        items = (array, ones)
    else:
```

```
        items = (ones, array)
    return np.concatenate(items, axis=0)


def unaugmented(array, appended=True):
    """Inverse operation to 'augmented'."""
    if appended:
        s = np.s_[:-1]
    else:
        s = np.s_[1:]
    return array[s]




# CLASSIFIER
from abc import ABC, abstractmethod

import numpy as np


class Classifier(ABC):
    """
    Abstract superclass for all classifiers
    """

    def __init__(self, X, y, num_classes=None):
        self.X = X
        self.y = y
        self.num_classes = num_classes or len(set(y))

    @classmethod
    def trained(cls, X, y):
        instance = cls(X, y)
        instance.train(X, y)
        return instance

    @abstractmethod
    def train(self, X, y):
        pass

    @abstractmethod
    def predict_label(self, x_test):
        pass

    def get_confusion_matrix(self, X_test, y_test, shape=None, **kwargs):
        if shape is None:
            shape = (self.num_classes, self.num_classes)
```

```python
        matrix = np.zeros(shape=shape)
        for i, x in enumerate(X_test):
            true_label = int(y_test[i])
            predicted_label = self.predict_label(x, **kwargs)
            matrix[true_label][predicted_label] += 1
        return matrix

    def print_confusion_matrix(self, X_test, y_test):
        matrix = self.get_confusion_matrix(X_test, y_test)
        print(matrix)
        print('accuracy: {}'.format(self.accuracy(X_test, y_test, matrix)))
        return matrix

    def accuracy(self, X_test, y_test, matrix=None, **kwargs):
        if matrix is None:
            matrix = self.get_confusion_matrix(X_test, y_test, **kwargs)
        return np.sum(np.diag(matrix)) / len(X_test)
```

```python
# NETWORK
import math
from typing import Any, Callable, List, Tuple

import numpy as np


class BatchMethod:
    BATCH = 0
    MINI_BATCH = 1
    ONLINE_BATCH = 2


class NeuralNetwork(Classifier):
    learning_constant = 1e-3

    def __init__(self, X, y,
                 size_in: int, size_out: int,
                 hidden_layers: List[int],
                 hot_one_encode_y: Callable[[int, Any], np.ndarray] = hot_one_encode_i
                 hot_one_decode: Callable[[np.ndarray], int] = hot_one_decode_int):
        """
        'size_in' Number of features.
        'size_out' Number of classes.
        'hidden_layers' Defines how many nodes each layer has.
        'hot_one_encode_y' Hot-one encodes a label.
        """
```

```
        super().__init__(X, y, num_classes=size_out)
        assert len(hidden_layers) > 0, 'Need at least 1 hidden layer.'

        self.size_in = size_in
        self.size_out = size_out
        self.hidden_layers = hidden_layers
        self.hot_one_encode_y = hot_one_encode_y
        self.hot_one_decode = hot_one_decode

    def train(self, X, y, *,
              num_epochs=10,
              batch_method=BatchMethod.MINI_BATCH,
              batch_size=None,
              learning_constant=1e-3,
              callback=None):
        N = len(X)
        if batch_method == BatchMethod.MINI_BATCH:
            if batch_size is None:
                batch_size = N // 20
        elif batch_method == BatchMethod.BATCH:
            batch_size = N
            if batch_size is not None:
                print('WARNING: batch_size given but ignored.')
        elif batch_method == BatchMethod.ONLINE_BATCH:
            batch_size = 1
            if batch_size is not None:
                print('WARNING: batch_size given but ignored.')
        else:
            raise ValueError('Invalid batch method.')

        X_shuffled = X[:]
        np.random.shuffle(X_shuffled)

        weights, augmented_weights = self._initialize_weight_matrices()
        num_batches = math.ceil(N / batch_size)
        for epoch in range(num_epochs):
            for batch_index in range(num_batches):
                batch = X_shuffled[batch_index:(batch_index + batch_size)]
                corrections = [
                    np.zeros(matrix.shape)
                    for matrix in augmented_weights
                ]
                for i, x in enumerate(batch):
                    new_corrections = self.backpropagation(
                        weights,
                        *self.feed_forward(augmented_weights, x, y[i]),
                        learning_constant=learning_constant,
                    )
```

```python
                corrections = self._sum_matrix_lists(
                    corrections,
                    new_corrections
                )

            weights, augmented_weights = self._apply_weight_corrections(
                augmented_weights,
                corrections
            )
        print(f'epoch {epoch + 1} done')
        if callable(callback):
            callback(augmented_weights)

    self.weights = augmented_weights

def feed_forward(self, weights, x, y_i):
    outputs = [self._O_hat(x)]
    diagonals = []
    s = sigmoid
    sd = sigmoid_d

    # Start at 1 to match math notation.
    for i, augmented_matrix in enumerate(weights, start=1):
        O_hat_prev = outputs[i - 1]
        W = augmented_matrix
        O = s(O_hat_prev.T @ W)
        D = np.diag(sd(O))

        outputs.append(self._O_hat(O))
        diagonals.append(D)
    try:
        t = self.hot_one_encode_y(self.num_classes, int(y_i))
    except IndexError as e:
        raise ValueError((
            'Cannot hot one encode "{}" because too few outputs '
            '(change "size_out" argument for "__init__")'
        ).format(int(y_i))) from e

    # O is the final (unaugmented) output.
    error = O - t
    return outputs, diagonals, error

def backpropagation(self, weights, outputs, diagonals, error,
                    *,
                    learning_constant):
    """
    'weights' Weight matrices W_i.
    'outputs' Augmented output vectors.
```

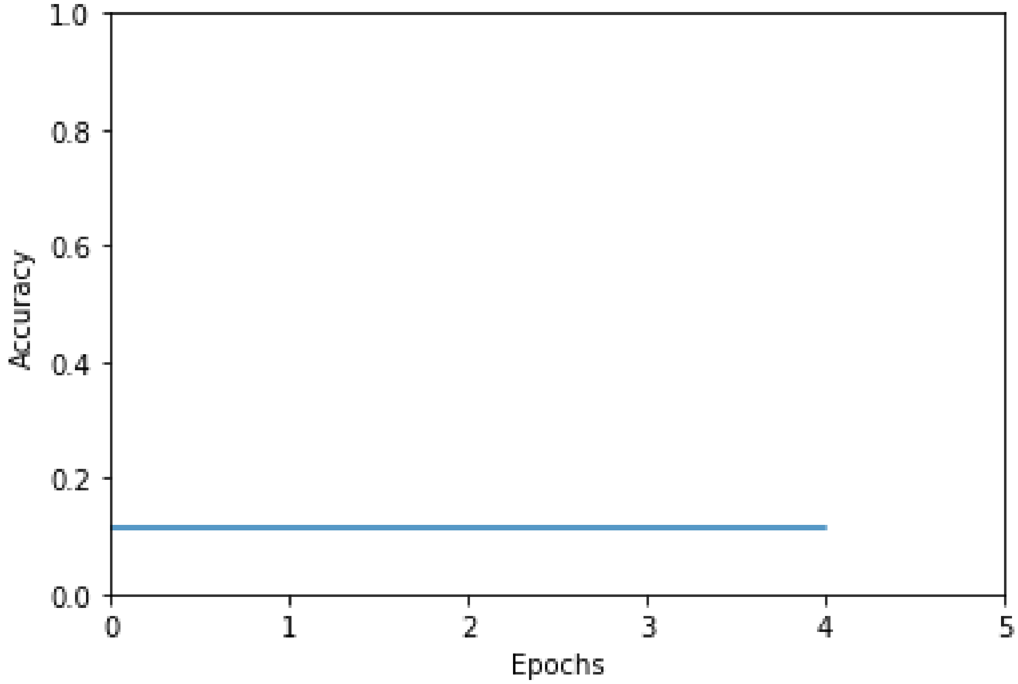
```
        'diagonals' Diagonal matrices D_i containing derivates
        'error' Error derivate vector e
        """
        N = len(diagonals)
        deltas = []
        i_max = N - 1
        for i in range(i_max, -1, -1):
            D = diagonals[i]
            if i == i_max:
                delta = D @ error
            else:
                W = weights[i + 1]
                delta = D @ W @ prev_delta
            # Prepend delta to keep order equal to the other variables.
            deltas.insert(0, delta)
            prev_delta = delta

        # The corrections' indices must be ascending
        # to match the order of weight matrices.
        return [
            -learning_constant * np.outer(delta, outputs[i]).T
            for i, delta in enumerate(deltas)
        ]

    def predict_label(self, x_test, weights=None):
        """
        'weights' Override self.weights, used for accuracy measurement.
        """
        if weights is None:
            weights = self.weights

        O_hat_prev = self._O_hat(x_test)
        s = sigmoid

        # Start at 1 to match math notation.
        for i, augmented_matrix in enumerate(weights, start=1):
            W = augmented_matrix
            O = s(O_hat_prev.T @ W)
            O_hat_prev = self._O_hat(O)

        return self.hot_one_decode(O)

    def _initialize_weight_matrices(self) -> Tuple[List[np.ndarray]]:
        matrices = []
        prev_dim_size = self.size_in
        for layer_size in self.hidden_layers:
            shape = (prev_dim_size, layer_size)
            matrix = np.random.uniform(0, 1, shape)
```

```
                matrices.append(matrix)
                prev_dim_size = layer_size

            shape = (prev_dim_size, self.size_out)
            matrix = np.random.uniform(0, 1, shape)
            matrices.append(matrix)
            return matrices, [augmented(matrix) for matrix in matrices]

        def _apply_weight_corrections(self, augmented_weights, corrections):
            corrected_weights = self._sum_matrix_lists(
                augmented_weights,
                corrections
            )
            return (
                [unaugmented(matrix) for matrix in corrected_weights],
                corrected_weights,
            )

        def _sum_matrix_lists(self, a, b):
            # TODO: Use zip
            if len(a) != len(b):
                raise ValueError('Unequally long lists.')
            return [a_i + b[i] for i, a_i in enumerate(a)]

        def _O_hat(self, O):
            return augmented(O)
```

In [37]:
```
import matplotlib.pyplot as plt


print("####  Exercise 7.3b  ####")

mnist_data_test = torchvision.datasets.MNIST('./MNIST', train=False, transform=None, 
# Get data as numpy
np_images_test = np.empty([len(mnist_data_test), 28, 28])
np_labels_test = np.empty([len(mnist_data_test)])

for i, (image, label) in enumerate(mnist_data_test):
    data = (image, label)
    np_images_test[i] = np.array(image)
    np_labels_test[i] = label

print(np_images_test.shape)
print(np_labels_test.shape)

#Flatten images first
images_flat_test = np_images_test.reshape(-1, 28*28)
```

```python
#Normalize
images_flat_test = images_flat_test[:] / 255


accuracies = []
NUM_EPOCHS=5

def cb(weights):
    acc = net_mnist_b.accuracy(images_flat_test, np_labels_test, weights=weights)
    print('accuracy', acc)
    accuracies.append(acc)

net_mnist_b = NeuralNetwork(images_flat, np_labels, 28*28, 10, [30, 20])
net_mnist_b.train(
    images_flat, np_labels,
    batch_size=32,
    learning_constant=1e-2,
    num_epochs=NUM_EPOCHS,
    callback=cb,
)
plt.plot(accuracies)
plt.axis([0, NUM_EPOCHS, 0, 1])
plt.xlabel('Epochs')
plt.ylabel('Accuracy')
plt.show()
```

```
####  Exercise 7.3b  ####
(10000, 28, 28)
(10000,)
epoch 1 done
accuracy 0.1135
epoch 2 done
accuracy 0.1135
epoch 3 done
accuracy 0.1135
epoch 4 done
accuracy 0.1135
epoch 5 done
accuracy 0.1135
```



1.0
0.8
0.6
0.4
0.2
0.0
Accuracy
0
1
2
3
4
5
Epochs