exercise2 (Score: 13.0 / 13.0)

1. Test cell (Score: 2.0 / 2.0)
2. Test cell (Score: 2.0 / 2.0)
3. Coding free-response (Score: 2.0 / 2.0)
4. Written response (Score: 2.0 / 2.0)
5. Test cell (Score: 1.0 / 1.0)
6. Coding free-response (Score: 2.0 / 2.0)
7. Written response (Score: 2.0 / 2.0)

Lab 3

1. 提交作業之前，建議可以先點選上方工具列的**Kernel**，再選擇**Restart & Run All**，檢查一下是否程式跑起來都沒有問題，最後記得儲存。
2. 請先填上下方的姓名(name)及學號(stduent_id)再開始作答，例如：

```
name = "我的名字"
student_id= "B06201000"
```

3. 演算法的實作可以參考lab-3 (https://yuanyuyuan.github.io/itcm/lab-3.html), 有任何問題歡迎找助教詢問。
4. **Deadline: 10/30(Wed.)**

In [1]:

```
name = "李澤諺"
student_id = "B05902023"
```

Exercise 2

It is known that when interpolating a function $f(x)$ with a polynomial p_{m+1} of degree m that using x_j for $j = 0, 1, \ldots, m$ as interpolation points the error has the form

$$|f(x) - p_{m+1}(x)| = \frac{\left|f^{(m+1)}(\xi_x)\right|}{(m+1)!} \left|\prod_{k=0}^{m} (x - x_k)\right|,$$

where $\xi_x \in [x_0, x_m]$.

Therefore, the polynomial $\omega_m(t) := \prod_{k=0}^{m} (t - x_k)$ influences the size of the interpolation error.

---

1. Put $m+1$ *distinct equidistant points* in the interval $[-1, 1]$, and plot $\omega_m(t)$ for $m = 5, 10, 15, 20$.

Part 0. Import libraries.

In [2]:

```
import matplotlib.pyplot as plt
import numpy as np
```

Part 1. Define $\omega_m(t)$ function.

In [3]:

(Top)

```
def omega_m(t, x):
    return np.prod(t - x)
```

In [4]:

omega (Top)

```
# Test
print('w_5(0.5) =', omega_m(0.5, np.linspace(-1, 1, 6)))

### BEGIN HIDDEN TESTS
from random import random

rd_number = random()
x = np.linspace(-1, 1, 11)

m = len(x)
product = 1

for i in range(m):
    product *= (rd_number - x[i])

assert omega_m(rd_number, np.linspace(-1, 1, 11)) == product, 'omega_m is wrong!'
### END HIDDEN TESTS
```

```
w_5(0.5) = 0.017325000000000007
```

Part 2. Define the equidistant points function.

For example, if $m = 4$, then $m + 1$ distinct equidistant points in the interval $[-1, 1]$ should be $[-1, -0.5, 0, 0.5, 1]$.

So the results of `equidistant_points(4)` will be `[-1.  -0.5  0.   0.5  1. ]`.

In [5]:

(Top)

```
def equidistant_points(m):
    return np.linspace(-1 , 1 , m + 1)
```

In [6]:

points (Top)

```
# Test
m = 4
print("Equidistant points:", equidistant_points(m))

### BEGIN HIDDEN TESTS
m = 10
assert np.mean(np.array(equidistant_points(m)) - np.linspace(-1, 1, m+1)) < 1e-7, 'equidistant_points is wrong!'
### END HIDDEN TESTS
```

```
Equidistant points: [-1.  -0.5  0.   0.5  1. ]
```

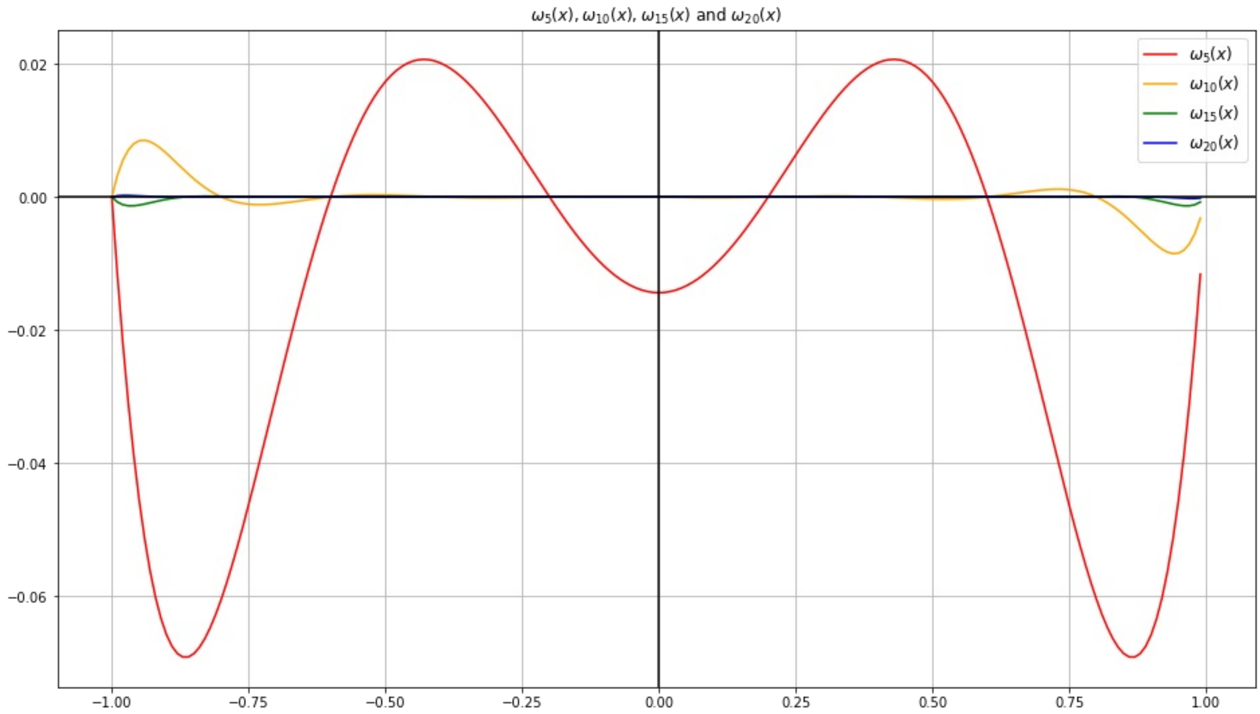
Part 3. plot $\omega_m(t)$ for $m = 5, 10, 15, 20$.

Please refer parts of plotting in " *lagrange.ipynb* ".

In [7]:

(Top)

```
x_range = np.arange(-1, 1, 0.01)
fig, ax = plt.subplots(figsize=(16, 9))
ax.plot(x_range , [omega_m(i , equidistant_points(5)) for i in x_range] , color = 'red' , label = r'$\omega_{5}(x)$')
ax.plot(x_range , [omega_m(i , equidistant_points(10)) for i in x_range] , color = 'orange' , label = r'$\omega_{10}(x)$')
ax.plot(x_range , [omega_m(i , equidistant_points(15)) for i in x_range] , color = 'green' , label = r'$\omega_{15}(x)$')
ax.plot(x_range , [omega_m(i , equidistant_points(20)) for i in x_range] , color = 'blue' , label = r'$\omega_{20}(x)$')
ax.set_title(r'$\omega_{5}(x), \omega_{10}(x), \omega_{15}(x)$ and $\omega_{20}(x)$')
plt.legend(loc='upper right' , fontsize = 12)
ax.grid(True)
ax.axhline(y=0 , color = 'black')
ax.axvline(x=0 , color = 'black')
plt.show()
```



Part 4. What's your observation of the above figure?

(Top)

當 m 越大時，$\omega_m(x)$ 在 $[-1, 1]$ 上的震盪幅度會越來越小，$\omega_m(x)$ 在 $[-1, 1]$ 上會逐漸趨近於 0。
此外，$\omega_m(x)$ 在接近 $[-1, 1]$ 邊界處的震盪較大。

---

2. Redo " *Problem 1.* " using *zeros of the Chebyshev polynomial (Chebyshev nodes)* as the interpolation points.

Part 1. Define Chebyshev nodes.

Please refer the part of Chebyshev nodes in " *lagrange.ipynb* ".

In [8]:

(Top)

```
def chebv_nodes(m):
    return np.array([np.cos(np.pi - i * (np.pi / m)) for i in range(m + 1)])
```

In [9]:

chebv_nodes (Top)

```
# Test
m = 5
print("Chebyshev nodes:", chebv_nodes(m))

### BEGIN HIDDEN TESTS
m = 10
assert np.mean(np.array(chebv_nodes(m)) - np.cos(np.linspace(0, np.pi, m+1))) < 1e-7, 'chebv_nodes is wro
ng!'
### END HIDDEN TESTS
```

```
Chebyshev nodes: [-1.         -0.80901699 -0.30901699  0.30901699  0.80901699  1.        ]
```

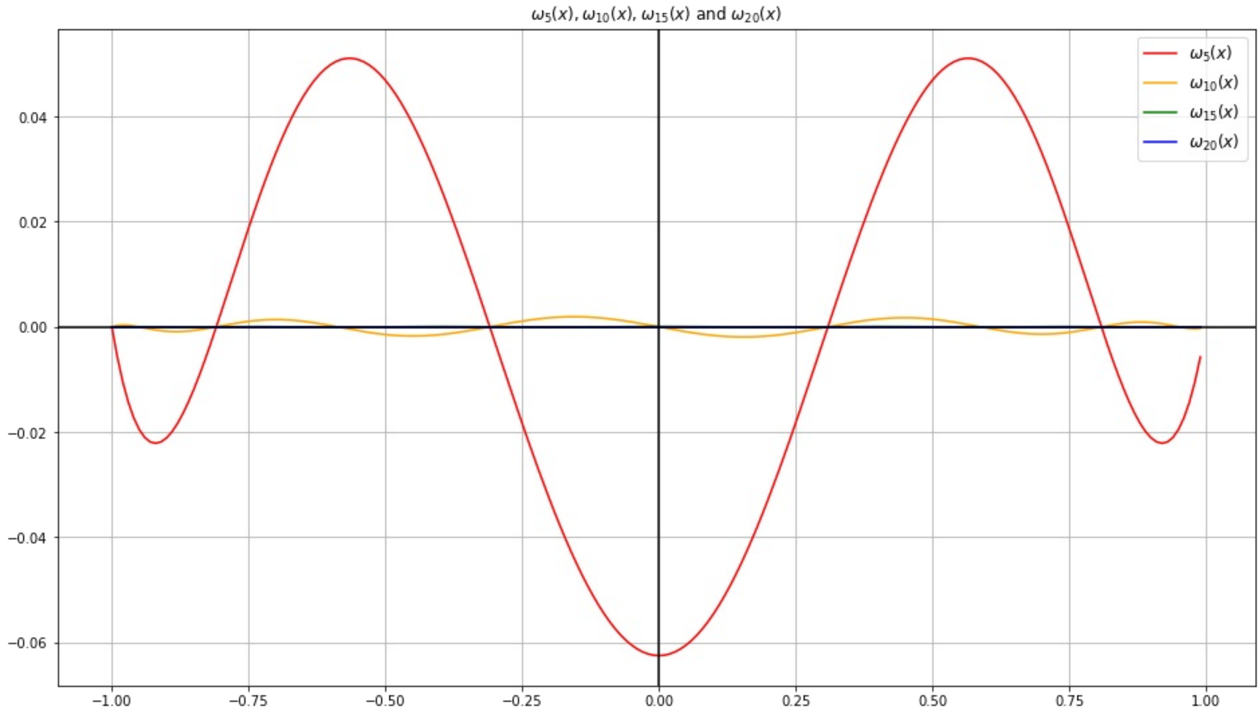
Part 2. plot $\omega(t)$ for $m = 5, 10, 15, 20$.

Please refer parts of plotting in " *lagrange.ipynb* ".

In [10]:

(Top)

```
x_range = np.arange(-1, 1, 0.01)
fig, ax = plt.subplots(figsize=(16, 9))
ax.plot(x_range , [omega_m(i , chebv_nodes(5)) for i in x_range] , color = 'red' , label = r'$\omega_{5}(x)$')
ax.plot(x_range , [omega_m(i , chebv_nodes(10)) for i in x_range] , color = 'orange' , label = r'$\omega_{10}(x)$')
ax.plot(x_range , [omega_m(i , chebv_nodes(15)) for i in x_range] , color = 'green' , label = r'$\omega_{15}(x)$')
ax.plot(x_range , [omega_m(i , chebv_nodes(20)) for i in x_range] , color = 'blue' , label = r'$\omega_{20}(x)$')
ax.set_title(r'$\omega_{5}(x), \omega_{10}(x), \omega_{15}(x)$ and $\omega_{20}(x)$')
plt.legend(loc='upper right' , fontsize = 12)
ax.grid(True)
ax.axhline(y=0 , color = 'black')
ax.axvline(x=0 , color = 'black')
plt.show()
```



Part 3. What's your observation of the above figure?

(Top)

當 m 越大時，$\omega_m(x)$ 在 $[-1, 1]$ 上的震盪幅度會越來越小，$\omega_m(x)$ 在 $[-1, 1]$ 上會逐漸趨近於 0。
與使用equidistance points相比，使用Chebyshez nodes似乎會使 $\omega_m(x)$ 在 $[-1, 1]$ 上趨近到 0 的速度較快。
此外，與使用equidistance points相比，使用Chebyshez nodes時 $\omega_m(x)$ 在接近 $[-1, 1]$ 邊界處的震盪似乎反而較小，不過其實震盪幅度似乎較為平均。

In [ ]: