Theorems to Prove

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We introduce the following notation:

$$\vec{x} \triangleq (\hat{\rho}_x \in \texttt{program}, \hat{\sigma}_x \in \texttt{stack}, \rho_x \in \texttt{program}, \sigma_x \in \texttt{stack})$$

Based on the linear contraints of our stack machine, we have the following property about well-formed closures:

$$\begin{aligned} \texttt{wf}\ (\vec{x}, v \in \texttt{stack\_value}) \quad \triangleq \quad & (\texttt{Apply} :: \rho, v :: \texttt{Closure}\ (\hat{\rho}, \hat{\sigma}) :: \sigma, \dots) \Longrightarrow \\ & (\hat{\rho} \mathbin{@} \rho, v :: \hat{\sigma} \mathbin{@} \sigma, \dots) \Longrightarrow^* (\rho, v' :: \sigma, \dots) \end{aligned}$$

Recall that to reverse `Apply`, we must split the program and the stack such that we restore the original closure. We define this split by the following:

$$\texttt{splits}\ (\rho', \sigma', v, \vec{x}) \quad \triangleq \quad \rho' = \hat{\rho}_x \mathbin{@} \rho_x \wedge \sigma' = \hat{\sigma}_x \mathbin{@} \sigma_x :\ \texttt{wf}\ (\vec{x}, v)$$

Using the property of the well-formed closure, if we could prove the `Unique Splits` proposition, then `Apply` would be injective.

$$\texttt{Unique Splits}: \quad \forall \rho', \sigma'.\ \exists\ \vec{x}\ \texttt{splits}\ (\rho', \sigma', \vec{x}) \wedge \exists\ \vec{y}\ \texttt{splits}\ (\rho', \sigma', \vec{y}) \Longrightarrow \vec{x} = \vec{y}$$

We introduce the following property and hypothesize that well-formed closures with the following property are uniquely reversible.

`Igloo Property`: We have program ρ with initial stack σ in a closure. Let $\hat{\rho}_i$ be the stack after executing $\hat{\rho}_i$, where $\hat{\rho}_i$ is the first i instructions of ρ. A program has the Igloo property iff

$$\forall\ i.\ 0 < i < |\rho| \Rightarrow |\hat{\sigma}_i|.$$