Dynamics

$$\underbrace{\begin{bmatrix} \mathbf{H} & -\mathbf{J}_c^\top \\ -\mathbf{J}_c & 0 \end{bmatrix}}_{\mathbf{K}} \underbrace{\begin{bmatrix} \ddot{\mathbf{q}} \\ \lambda \end{bmatrix}}_{\nu} = \underbrace{\begin{bmatrix} \mathbf{S}^\top \tau - \mathbf{h} \\ \dot{\mathbf{J}}_c \dot{\mathbf{q}} \end{bmatrix}}_{\Psi}$$

By Definition

$$\boxed{\nu = \mathbf{K}^{-1}\Psi}$$

$$\boxed{\begin{aligned} &\mathbf{H}\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{J}_c^\top \lambda = \mathbf{S}^\top \tau \\ &\mathbf{J}_c \ddot{\mathbf{q}} + \dot{\mathbf{J}}_\mathbf{c} \dot{\mathbf{q}} = 0 \end{aligned}}$$

EMRNEA $(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}, \mathbf{a}_g, \mu, \lambda)$

$$\begin{bmatrix} \mu\tau \\ \mathbf{v} \\ \mathbf{a} \end{bmatrix} = \begin{bmatrix} \mu^\top \left[\mathbf{H}(\mathbf{q})\dot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{J}_c^\top \lambda\right] \\ \mathbf{J}_c \dot{\mathbf{q}} \\ \dot{\mathbf{J}}_\mathbf{c} \dot{\mathbf{q}} + \mathbf{J}_c \dot{\mathbf{q}} \end{bmatrix}$$

where μ is any vector

Conventional iLQR

AD tools once: $\dfrac{\partial \nu}{\partial \mathbf{q}}$ $\quad \mathcal{O}(n^2)$

Conventional DDP

AD tools twice: $\dfrac{\partial^2 \nu}{\partial^2 \mathbf{q}}$ $\quad \mathcal{O}(n^3)$

$\xi^\top \dfrac{\partial^2 \nu}{\partial^2 \mathbf{q}}$ $\quad \mathcal{O}(n^3)$

EMRNEA DDP

With $\xi^\top = \gamma^\top \mathbf{K}^{-1}$

$$[\mathsf{T}_1 \ \mathbf{V}_1 \ \mathbf{A}_1]^\top = \text{EMRNEA}\left(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}, \mathbf{a}_g, \xi_1, \lambda\right) \quad \mathcal{O}(n)$$

$$[\mathsf{T}_2 \ \mathbf{V}_2 \ \mathbf{A}_2]^\top = \text{EMRNEA}\left(\mathbf{q}*0, \dot{\mathbf{q}}*0, \frac{\partial \dot{\mathbf{q}}}{\partial \mathbf{q}}, \mathbf{a}_g*0, \xi_1, \frac{\partial \lambda}{\partial \mathbf{q}}\right) \quad \mathcal{O}(n)$$

$$[\mathsf{T}_3 \ \mathbf{V}_3 \ \mathbf{A}_3]^\top = \text{EMRNEA}\left(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}, \mathbf{a}_g, \xi_2, \lambda\right) \quad \mathcal{O}(n)$$

$$[\mathsf{T}_4 \ \mathbf{V}_4 \ \mathbf{A}_4]^\top = \text{EMRNEA}\left(\mathbf{q}*0, \dot{\mathbf{q}}*0, \frac{\partial \dot{\mathbf{q}}}{\partial \mathbf{q}}, \mathbf{a}_g*0, \xi_2, \frac{\partial \lambda}{\partial \mathbf{q}}\right) \quad \mathcal{O}(n)$$

$$\begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}^\top \frac{\partial^2 \nu}{\partial^2 \mathbf{q}} = \begin{aligned} &[\gamma_1^\top] \frac{\partial^2 \ddot{\mathbf{q}}}{\partial \mathbf{q} \partial \mathbf{q}} = \frac{\partial^2 \mathsf{T}_1}{\partial^2 \mathbf{q}} - \frac{\partial^2 \mathsf{T}_2}{\partial^2 \mathbf{q}} \quad \mathcal{O}(n^2) \\ &[\gamma_2^\top] \frac{\partial^2 \lambda}{\partial \mathbf{q} \partial \mathbf{q}} = \frac{\partial^2 \mathbf{A}_3}{\partial^2 \mathbf{q}} + \frac{\partial \mathbf{A}_4}{\partial \mathbf{q}} \quad \mathcal{O}(n^2) \end{aligned}$$