Response to Reviewers:

ALDSC-22-1022: Accelerating hybrid systems differential dynamic programming

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Associate Editor:

We thank the Associate Editor and the reviewers for their evaluation of our manuscript. We have included original reviews and response to reviewers below. We have also edited the paper to reflect suggested recommendations from the reviewers, and those changes are highlighted in blue in the revised manuscript. This additional round of edits also brought our own changes (e.g., to streamline notation in a few places), and those changes are noted in blue as well.

Reviewer 1:

R1.A *This is very valuable scientific research. Also, it is very relevant to ALDSC. The only issue that I see is that this paper needs extensive grammar checking.*

We thank the reviewer for taking the time to review this paper and for the comments. We have extensively checked the paper for grammatical errors and edited accordingly. These changes have been highlighted in blue.

Reviewer 2:

R2.A *This letter proposes to reduce the computational burden of second-order dynamic partials via the proposed mRNEAc algorithm and reverse-mode Automatic Differentiation tool, which avoids the direct derivative of tensor. The computational time is compared to the iLQR and traditional DDP and validated through simulation results. Overall, this is a well written letters with articulated logical organization. Several minor changes as follows are recommended.*

We thank the reviewer for taking the time to review this paper and for the comments.

R2.B *m seems to have multiple meanings, i.e., number of modes and slope in Fig. 4, please check and change.*

We have since modified the paper to ensure that m is only used once to refer to the number of the modes.

R2.C *The abbreviation should be defined at its first appearance. For example, RNEA in the introduction, ID algorithm.*

We thank the reviewer for noting this missed introduction of an abbreviation. We also checked the whole document for other missing introductions of abbreviations.

R2.D *It would be interesting if the author can provide results comparison with/without uncertainties of the proposed algorithms. The presented results seem do not cover this topic yet.*

In terms of uncertainty (e.g., from sensors, terrain, modeling error, etc.), the iLQR/DDP framework that our research is built upon only considers deterministic dynamics. While there are extensions (e.g., iLQG) that consider interplay with state estimation, a derivation with such a probabilistic iLQR/DDP formulation is beyond the scope of this short letter. We think this challenge is interesting future work and thank the reviewer for the suggestion. We have added call out to this future work in the conclusion.

R2.E *What is the optimization cost function for the planar quadruped?*

A detailed description of the running cost function has been included in the paper as follows:

> The bounding sequence is encoded via a design of running and terminal costs in each mode that aim to track a reference configuration, and are given as
>
> $$\ell_i(x_k, u_k, \lambda_k) = \left[x - x_{ref,i}\right]^T Q_i \left[x - x_{ref,i}\right] + u^T R_i\, u + \left[\lambda - \lambda_{ref,i}\right]^T S_i \left[\lambda - \lambda_{ref,i}\right]$$
>
> $$\varphi_i(x_{N_{i+1}^-}) = \left[x - x_{ref,i}\right]^T Q_{fi} \left[x - x_{ref,i}\right]$$
>
> Here, $x_{ref,i}$ refers to a pre-defined reference configuration in mode i and $\lambda_{ref,i}$ refers to reference ground reaction forces. The terms Q, R, and S are weighting matrices for the state, control, and contact forces, respectively, in each mode. As such, this formulation aims to attain a pre-defined configuration while reducing energy consumption and attaining the requisite ground reaction forces. The state weighting matrix, Q_{fi}, in the terminal cost penalizes deviations that the robot should attain in its final state in each mode.

R2.F *Will the ground terrain affect the trajectory optimization results for planar quadruped?*

In short, yes, the ground terrain geometry will affect the optimization results if it is known in advance. We are currently modeling the ground as hard contact, and so the ground terrain properties (e.g., sand vs. mud) are not currently addressed in the formulation, though this omission is common in other work.

If there are uncertainties in the terrain geometry or properties, these will manifest as external disturbances. The DDP formulation features a feedback correction term (equation (10) added in the revised manuscript) that allows us to account for disturbances and would therefore tailor the optimal control to differing terrains. The output of our proposed algorithm would therefore provide some robustness to those differing terrains. A hardware verification of this theory is planned in our future work to show these benefits in practice.

R2.G *There are several grammar issues in the paper, please read through carefully and correct them. To name a few, '... its partials with respect...', '...in lower complexity computational complexity...'*

We thank the reviewer for the suggested revisions. We have extensively reviewed the paper for grammatical errors. These changes have been highlighted in blue.