Modelling the Milky Way Galaxy & the Large Magellanic Cloud

Introduction

1 Introduction

In this section some common quantities useful for describe the density profiles are defined and explained.

1.1 Critical density of the Universe:

The Critical density of the universe is defined as:

$$\rho_c = \frac{3H^2}{8\pi G} \tag{1}$$

Where H is the Hubble parameter and this parameter depends on the cosmological parameters. This density ...

1.2 Virialization

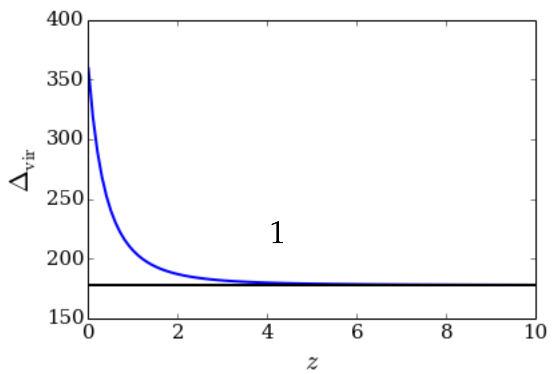
A dark matter halo is virialized when it is in equilibrium, such an equilibrium occurs after the dark matter have collapsed and the force of gravity equals the **relaxation** processes [Binney and Tremaine, 2008]. This happens when the dark matter reach an overdensity value Δ_{vir}. This overdensity corresponds to a radius and a mass r_{vir} & M_{vir} respectively.

$\Delta_{vir} = \frac{\rho_{vir}}{\rho_c}$. For a cosmolgy with $(\Omega_m + \Omega_\Lambda = 1)$ the solution for the **Top Hat** model can be approximated by:

$$\Delta_{vir} = (18\pi^2 + 82x - 39x^2)/\Omega(z) \tag{2}$$

[Eke et al., 1996, Bryan and Norman, 1998] Where $x = \Omega(z) - 1$. For the present time $(z = 0)$ $\Delta_{vir} = 360$.

The behavior of this function is shown in Fig.??



1.3 r_{200} & M_{200}

There is another radious and mass of particular interest. This is the radius that enclosed a density of 200 times the density of the Universe. M_{200} is defined as:

$$M_{200} = 200\rho_c \frac{4}{3}\pi r_{200}^3 \tag{5}$$

In the same way M_{vir} is defined as:

$$M_{vir} = \Delta_{vir}\Omega_m \rho_c \frac{4}{3}\pi r_{vir}^3 \tag{6}$$

The critial density ρ_c is the same for both cases, then it is possible to relate both masses from Eq5 and Eq6 as follows:

$$\frac{M_{200}}{M_{vir}} = \left(\frac{200}{\Delta_{vir}\Omega_m}\right)\left(\frac{r_{200}}{r_{vir}}\right)^3 \tag{7}$$

Here is common to call $q = \left(\dfrac{200}{\Delta_{vir}\Omega_m}\right)$, at $z = 0$ $q = 2.053$.

$$\frac{M_{200}}{M_{vir}} = q\left(\frac{r_{200}}{r_{vir}}\right)^3 \tag{8}$$

This Eq.8 relates M_{vir} and M_{200} for a given $rvir$ and r_{200}.
Spherical

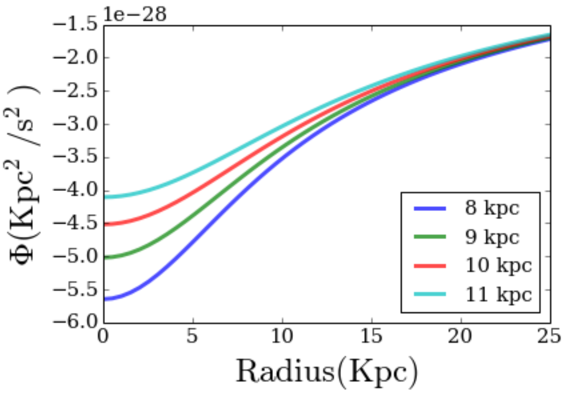
2 Spherical Potentials

2.1 Plummer

The plumer density profile is one of the simplest models which describes a constant density near the center and falls at large radius.

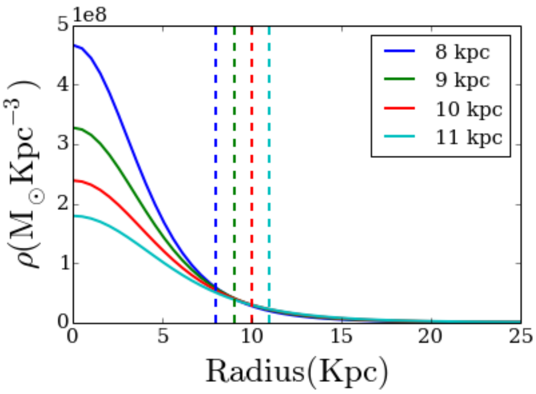
$$\Phi_P(r) = -\frac{GM}{\sqrt{r^2 + a^2}} \tag{9}$$

Where a is call the scale length. The scale length set the length a in which the mayority of the density is enclosed. Note that if a is cero the plummer potential would be exactly as the potential of a point mass. In the other hand if a goes to infty the potential is rewpresenting a very extended mass source. In other words the scale length set up the size of the volume in which the mass M is enclosed.



$$\nabla^2 \Phi_P(r) = 4\pi G \rho_P(r) = \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\Phi_P(r)}{dr}\right) \tag{10}$$

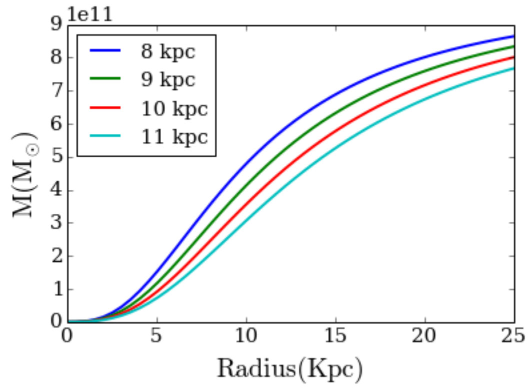
$$\rho_P(r) = \frac{3M}{4\pi a^3}(1 + \frac{r^2}{a^2})^{-5/2} \tag{11}$$



The enclosed mass can be derived from the density by integrating over the volume.

$$M_P(<r) = 4\pi \int_0^r r'^2 \frac{3M}{4\pi a^3}(1 + \frac{r'^2}{a^2})^{-5/2} dr' = \frac{3M}{a^3}\left(\frac{a^4 r^3 \sqrt{r^2/a^2 + 1}}{3(r^2 + a^2)^2}\right) \tag{12}$$

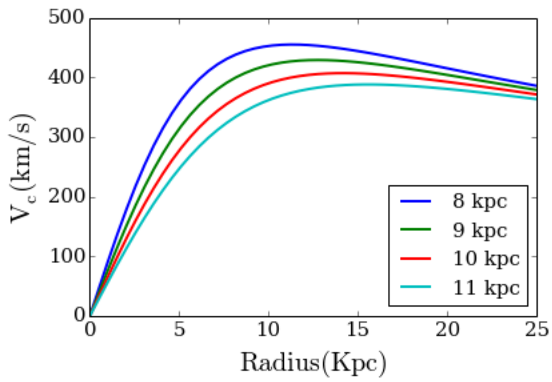
$$M_P(<r) = M \frac{r^3}{(a^2 + r^2)^{3/2}} \tag{13}$$



$$F_g = \frac{GmM}{r^2} = ma_c = m\frac{v_c^2}{r} \tag{14}$$

$$v_c = \sqrt{\frac{GM(<r)}{r}} \tag{15}$$

$$v_c = \sqrt{GM(\frac{r^2}{(r^2+a^2)^{3/2}})} \tag{16}$$



2.2 Hernquist profile

The Hernquist profile is derived in such a way that it follows the

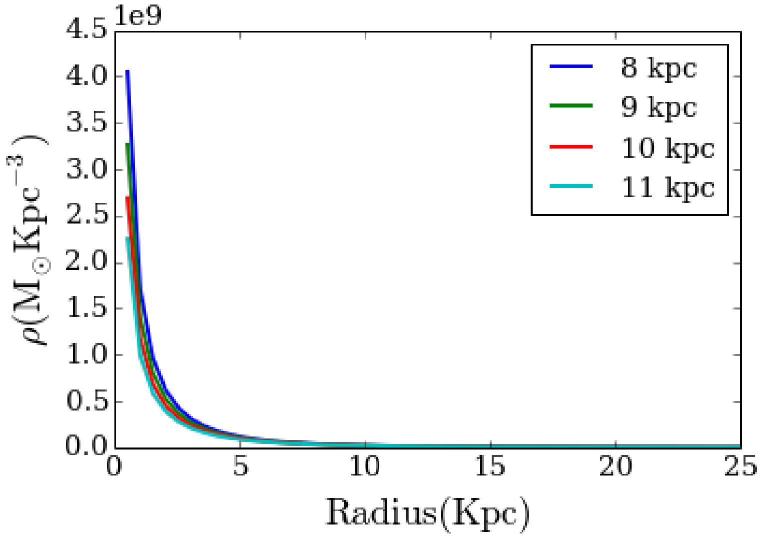
$$\rho_{Hernquist}(r) = \frac{M}{2\pi}\frac{a}{r(r+a)^3} \tag{17}$$

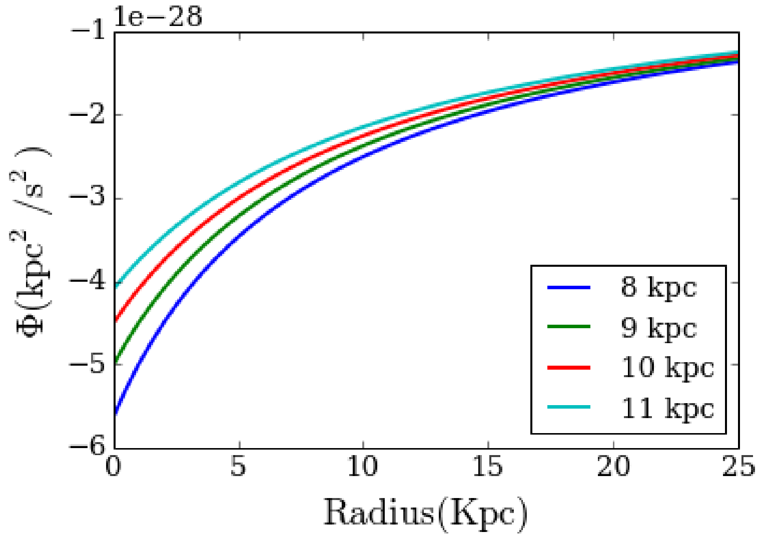
$$M_{Hernquist}(<r) = 2aM\int\frac{r}{(r+a)^3}dr \tag{18}$$

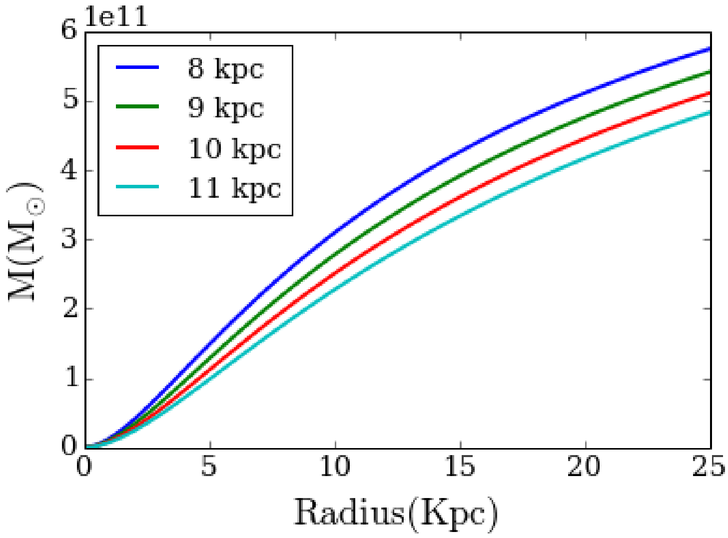
$$M_{Hernquist}(<r) = M\frac{r^2}{(r+a)^2} \tag{19}$$

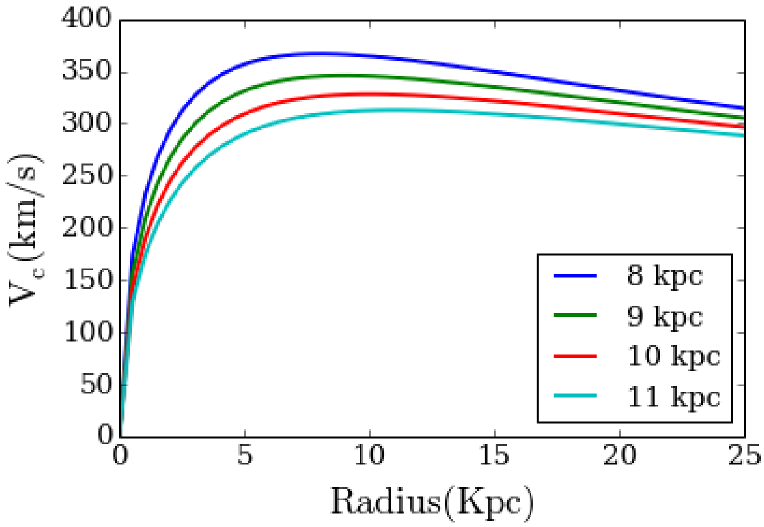
$$\Phi = -\frac{GM}{r+a} \tag{20}$$

$$v_c(r) = GM\frac{r}{(r+a)^2} \tag{21}$$









2.3 Singular Isothermal Sphere

The Singular Isothermal Sphere (**SIS**) describes a system in which the particles follow a Maxwellian density distribution. With this distribution and the Poisson equation the follow density profiles could be derived.

$$\rho_{iso}(r) = \frac{\sigma^2}{2\pi G r^2} \tag{22}$$

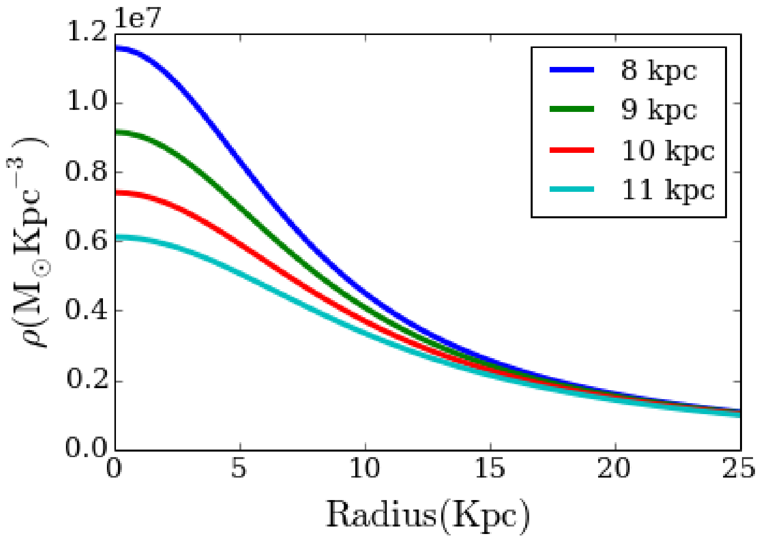
Following the same procedure as with the previuos profiles we find M, Φ and v_c.

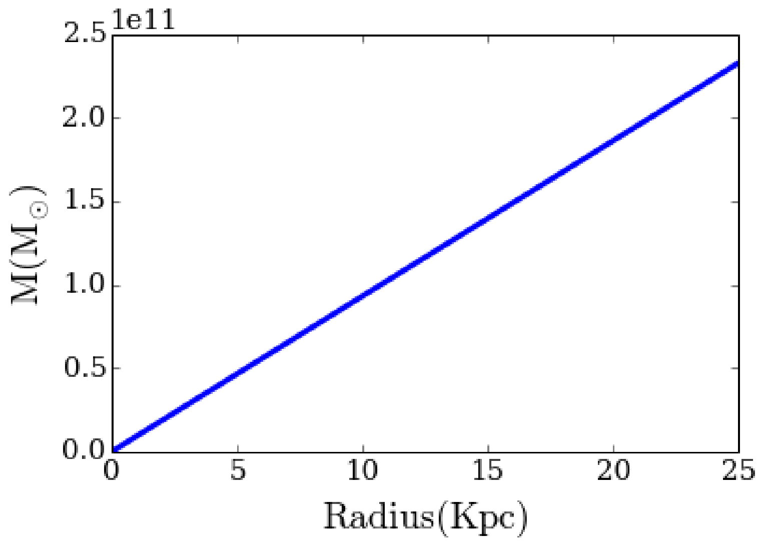
$$M_{iso}(< r) = \frac{2\sigma r}{G} \tag{23}$$

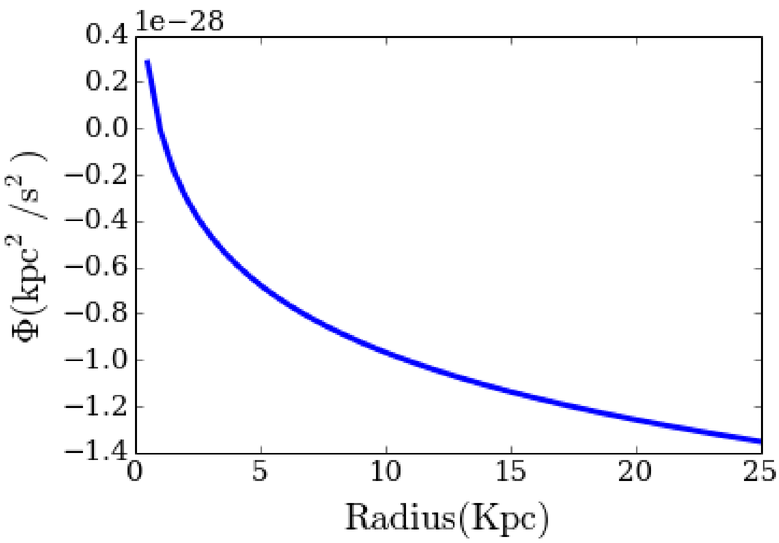
$$\Phi_{iso}(r) = 2\sigma^2 ln(r) + const. \tag{24}$$

$$v_c(r) = \sqrt{2}\sigma \tag{25}$$

This profile is quite different to the previous ones due to the fact that here the input is the velocity instead of the total Mass.







2.4 NFW

$$\rho_{NFW}(r) = \frac{M}{2\pi a^3 (r/a)(1+r/a)^2} \tag{26}$$

$$M_{NFW}(r) = M\left(ln(1+x) - \frac{x}{1+x}\right) \tag{27}$$

Where $x = r/a$, is useful to define the function $f(x)$ as:

$$f(x) = ln(1+x) - \frac{x}{1+x} \tag{28}$$

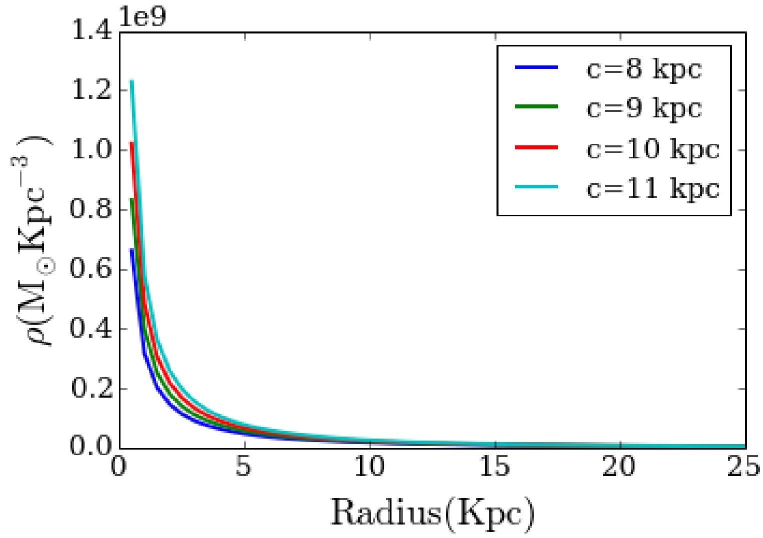
Then 27 can be expresed as:

$$M_{NFW} = 4\pi\rho_a a^3 f(x) \tag{29}$$

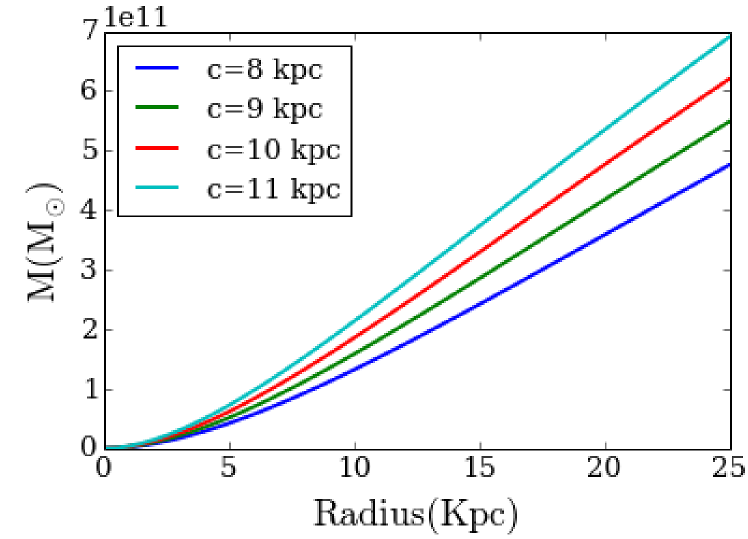
$$\Phi_{NFW} = -4\pi GM \frac{ln(1+r/a)}{r} \tag{30}$$

$$c(M_{vir}) = 9.60\left(\frac{M_{vir}}{10^{12}h^{-1}M_\odot}\right)^{-0.075} \tag{31}$$

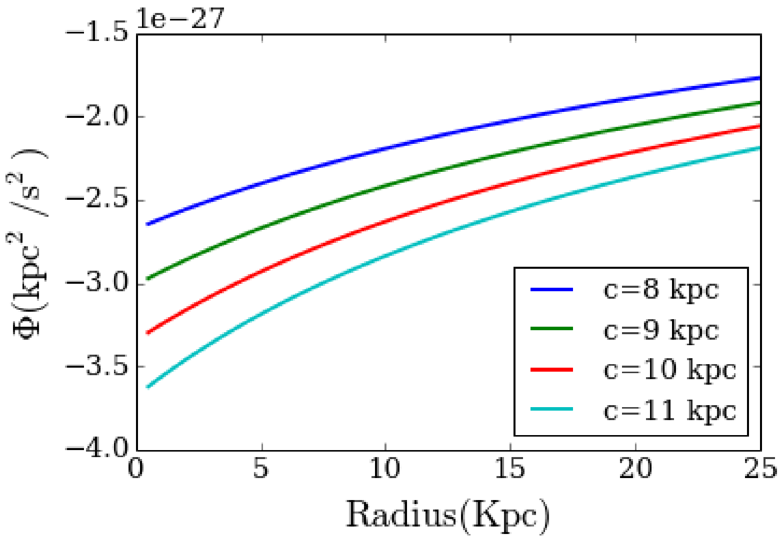
$$v_c(r) = \sqrt{\left(\frac{M(r)G}{r}\right)} = \sqrt{\left(\frac{2M\left(ln(1+c) - \frac{c}{1+c}\right)}{r}\right)} \tag{32}$$



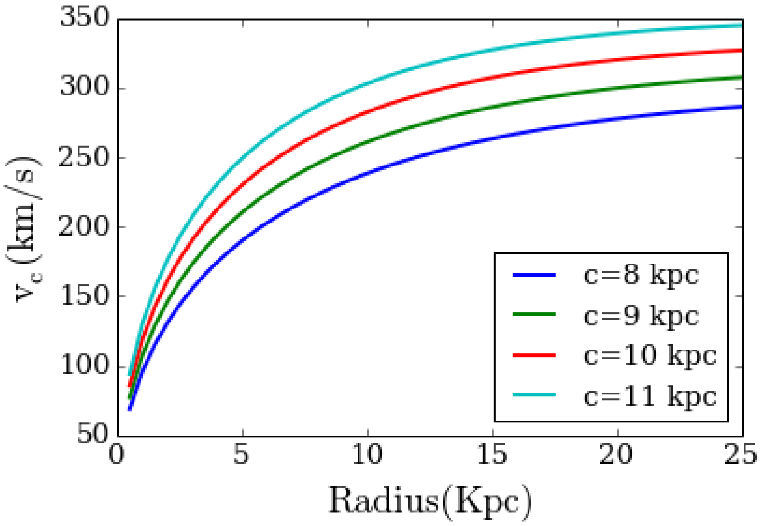
1e9
$\rho(M_\odot Kpc^{-3})$
Radius(Kpc)
c=8 kpc
c=9 kpc
c=10 kpc
c=11 kpc



1e11
$M(M_\odot)$
Radius(Kpc)
c=8 kpc
c=9 kpc
c=10 kpc
c=11 kpc



1e−27
$\Phi(kpc^2/s^2)$
Radius(Kpc)
c=8 kpc
c=9 kpc
c=10 kpc
c=11 kpc



3 Conversion from NFW to the Hernquist profile

The average density of the NFW distribution can be expressed as:

$$\bar{\rho}_{NFW}(r) = \frac{3M_{NFW}(r)}{4\pi r^3} \tag{33}$$

And with eq.29 the $\bar{\rho_{NFW}}(r)$ takes de form:

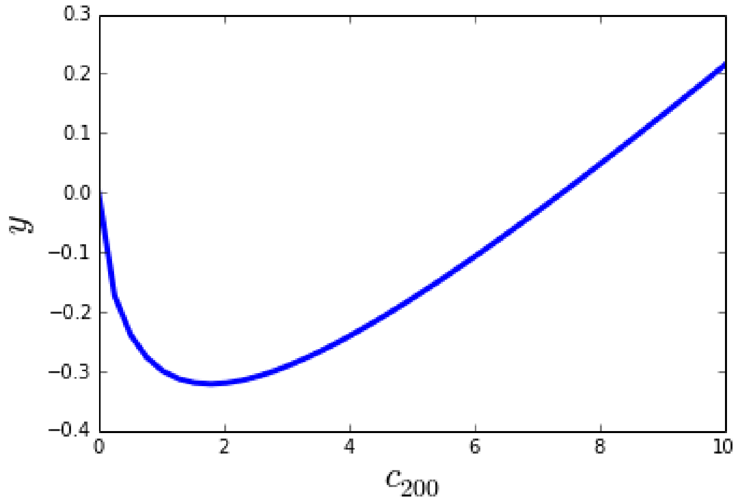
$$\bar{\rho}_{NFW}(r) = 3\rho_a \left(\frac{a}{r}\right)^3 f(x) \tag{34}$$

Now if we want to find the relationship betwee r_{200} and r_{vir} for the NFW profile we have to apply eq8.

$$q = \frac{3\rho_a \frac{a}{r_{200}} f(c_{200})}{3\rho_a \frac{a}{r_{vir}} f(c)} = \frac{c_{200}^3 f(c_{200})}{c_{vir^3} f(c_{vir})} \tag{35}$$

$$\frac{c_{200}}{c_{vir}} = \left(\frac{f(c_{200})}{q f(c_{vir})}\right)^{1/3} \tag{36}$$

For $c_{vir} = 10$ this function is shown in Fig.??, where $y = \frac{c_{200}}{c_{vir}} - \left(\frac{f(c_{200})}{q f(c_{vir})}\right)^{1/3}$



Note that the solution of Eq.36 is when $y = 0$, one solution is $c_{200} = 0$ but this is not of particular interest for us.

The other solution is computed analytically using the bisection algorithm. $c_{200} = 7.4$

In order to seek the equivalence between the NFW and the Hernquist profile, We have to match the same enclosed mass of both profiles at a given radius. To his end we have to find M_H in terms of r_s.

$$M_H(r) = M_{NFW}(r) \tag{37}$$

$$\frac{M_H r^2}{a^2 + r^2} = 4\pi\rho_s r_s^3 \left[Ln(1+x) - \frac{x}{1+x}\right] \tag{38}$$

In the limit $r \to 0$

$$M_H = \frac{4\pi\rho_s r_s^3 a^2}{r^2} \left[(x - \frac{x^2}{2}) - x\right] \tag{39}$$

$$M_H = 4\pi\rho_s r_s^3 \frac{a^2}{r^2} \left(-\frac{r^2}{2r_s^2}\right) \tag{40}$$

$$M_H = 2\pi r_s a^2 \tag{41}$$

With this relation is possible now to match both profiles at a given radius $\tilde{r}$

$$M_H(\tilde{r}) = M_{NFW}(\tilde{r}) \tag{42}$$

$$2\pi\rho_s a^2 r_s \frac{\tilde{r}^2}{a^2} \frac{1}{\left(1 + \frac{\tilde{r}}{a}\right)^2} = 4\pi\rho_s r_s^3 \left(Ln\left(1 + \frac{\tilde{r}}{r_s}\right) - \frac{\tilde{r}}{\tilde{r} + r_s}\right) \tag{43}$$

$$\frac{\tilde{r}^2 a^2}{(a + \tilde{r})^2} = 2r_s^2 \left(Ln\left(1 + \frac{\tilde{r}}{r_s}\right) - \frac{\tilde{r}}{\tilde{r} + r_s}\right) \tag{44}$$

$$\frac{\tilde{r}^2 a^2}{(a+\tilde{r})^2} = 2r_s^2 f(\tilde{x}) \tag{45}$$

$$\left(\frac{a}{r_s}\right)^2 = \frac{2}{\tilde{r}^2}(a+\tilde{r})^2 f(\tilde{x}) \tag{46}$$

$$\frac{a}{r_s} = \frac{[2f(\tilde{x})]^{1/2}}{\tilde{r}}(a+\tilde{r}) \tag{47}$$

$$\left(\frac{a}{r_s}\right)\left(1 - \frac{[2f(\tilde{x})]^{1/2}}{\tilde{x}}\right) = [2f(\tilde{x})]^{1/2} \tag{48}$$

$$\frac{a}{r_s} = \frac{[2f(\tilde{x})]^{1/2}}{\left(1 - \frac{[2f(\tilde{x})]^{1/2}}{\tilde{x}}\right)} \tag{49}$$

$$\frac{a}{r_s} = \frac{[2f(\tilde{x})]^{1/2}\tilde{x}}{\tilde{x} - (2f(\tilde{x})^{1/2})} = \frac{1}{\left([2f(\tilde{x})]^{-1/2} - \frac{1}{\tilde{x}}\right)} \tag{50}$$

Finally the ratio of the enclosed mass of the Hernquist and the NFW profiles is:

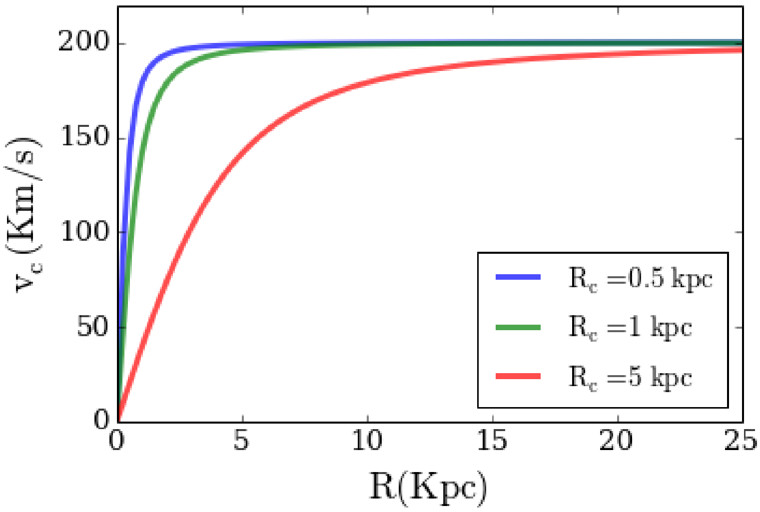
$$\frac{M_H}{M_{vir}} = \frac{2\pi\rho_s a^2 r_s}{4\pi\rho_s r_s^3 f(c_{vir})} = \frac{1}{2f(c_{vir})}\left(\frac{a}{r_s}\right)^2 \tag{51}$$

Disc profile

$$\Phi_L(R,z) = \frac{1}{2}v_0^2 ln\left(R_c^2 + R^2 + \frac{z^2}{q_\phi^2}\right) + constant \tag{52}$$

The circular velocity at $z = 0$ is $v_c^2 2(R, z = 0) = r\frac{d\Phi}{dR}$:

$$v_c(R, z = 0) = r\frac{d\Phi_L}{dr} = \frac{v_0 R}{\sqrt{r_c^2 + R^2}} \tag{53}$$

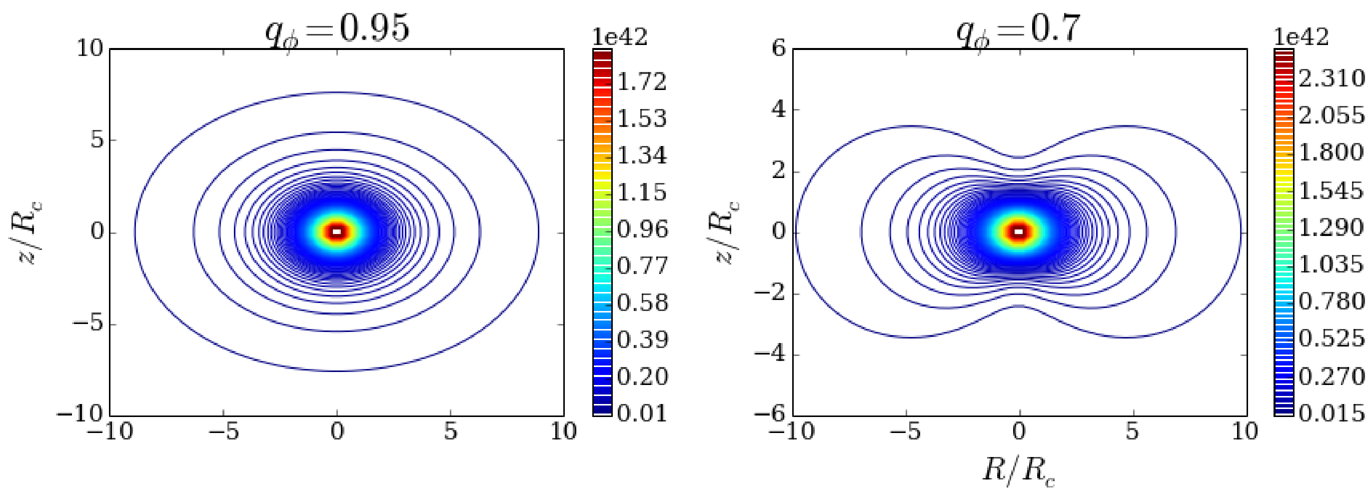


To derive the density we make use of Poisson's equation in cylindrical coordinates:

$$\rho_L(R,z) = \frac{\nabla^2\Phi_L}{4\pi G} = \frac{1}{4\pi G}\left(\frac{1}{r}\frac{d}{dR}R\frac{d}{dR} + \frac{d^2}{dz^2}\right)\Phi_L \tag{54}$$

$$\rho_L(R,z) = \frac{v_0^2}{8\pi G}\left(\frac{1}{R}\frac{4R(R_c^2+R^2+\frac{z^2}{q_\phi^2})-4R^3}{(R_c^2+R^2+\frac{z^2}{q_\phi^2})^2} + \frac{\frac{2}{q_\phi^2}(R_c^2+R^2+\frac{z^2}{q_\phi^2}) - \frac{4z^2}{q_\phi^4}}{(R_c^2+R^2+\frac{z^2}{q_\phi^2})^2}\right) \tag{55}$$

$$\rho_L(R,z) = \frac{v_0^2}{4\pi G q_\phi^2}\frac{(2q_\phi^2+1)R_c^2 + r^2 + (2-q_{\Phi^2})z^2}{(R_c^2+r^2+z^2 q_\Phi^{-2})^2} \tag{56}$$

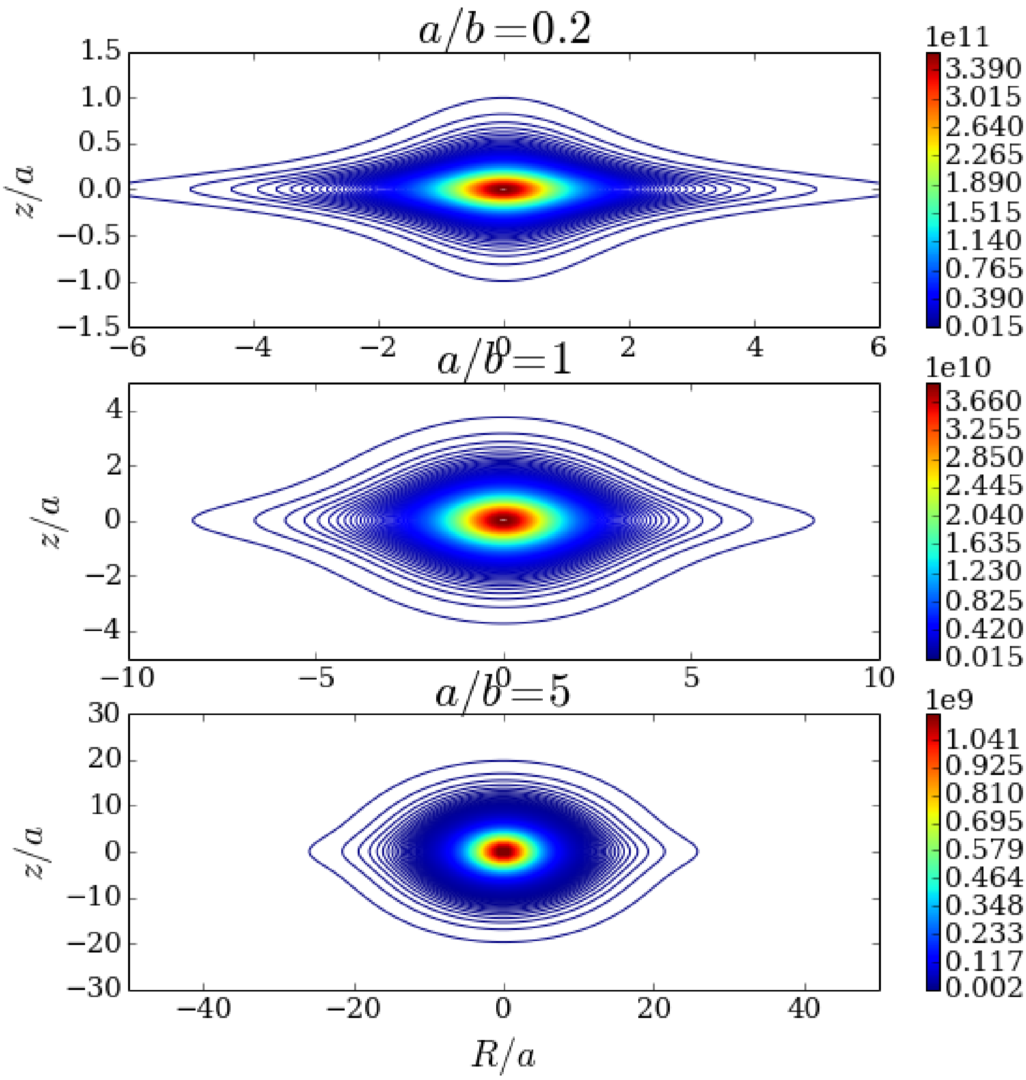


Discs

4 Discs Potentials

$$\Phi_M(R,z) = -\frac{GM}{\sqrt{R^2 + (a + \sqrt{(}z^2 + b^2))^2}} \tag{57}$$

$$\rho_M(R,Z) = \left(\frac{b^2 M}{4\pi}\right) \frac{aR^2 + (a + 3\sqrt{z^2 + b^2})(a + \sqrt{z^2 + b^2})^2}{[R^2 + (a^2 + \sqrt{z^2 + b^2})^2]^{5/2}(z^2 + b^2)^{3/2}} \tag{58}$$



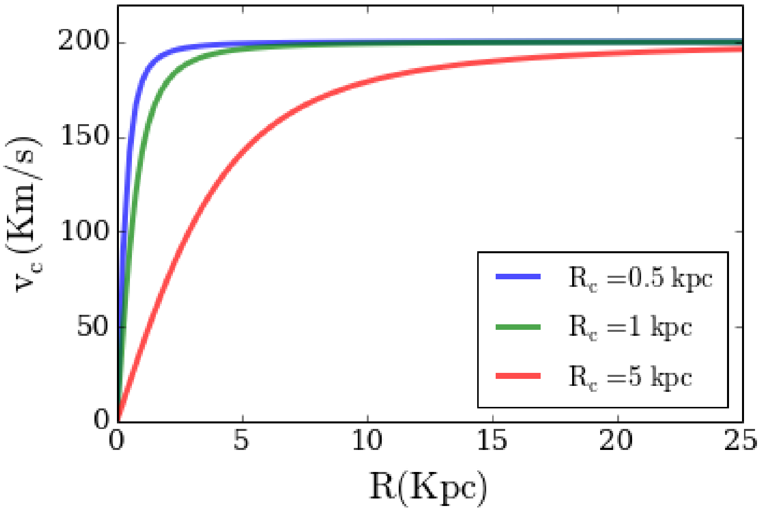
5 Logarithmic Profile

Disc profile

$$\Phi_L(R,z) = \frac{1}{2} v_0^2 ln \left(R_c^2 + R^2 + \frac{z^2}{q_\phi^2} \right) + constant \tag{59}$$

The circular velocity at $z = 0$ is $v_c^2 2(R, z = 0) = r\frac{d\Phi}{dR}$:

$$v_c(R, z=0) = r\frac{d\Phi_L}{dr} = \frac{v_0 R}{\sqrt{r_c^2 + R^2}} \tag{60}$$

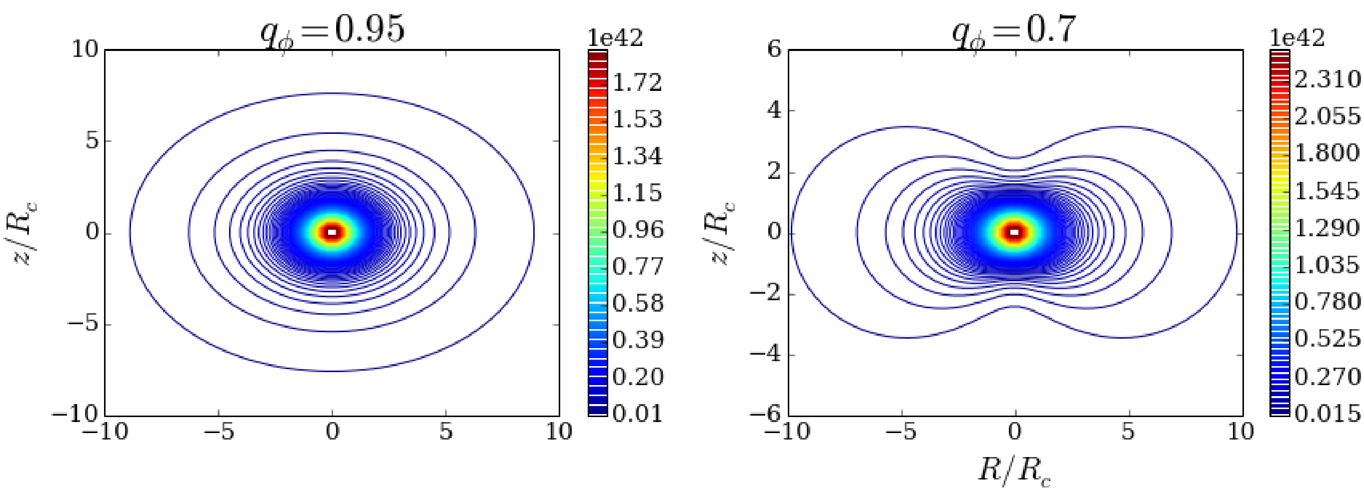


To derive the density we make use of Poisson's equation in cylindrical coordinates:

$$\rho_L(R, z) = \frac{\nabla^2 \Phi_L}{4\pi G} = \frac{1}{4\pi G}\left(\frac{1}{r}\frac{d}{dR}R\frac{d}{dR} + \frac{d^2}{dz^2}\right)\Phi_L \tag{61}$$

$$\rho_L(R, z) = \frac{v_0^2}{8\pi G}\left(\frac{1}{R}\frac{4R(R_c^2 + R^2 + \frac{z^2}{q_\phi^2}) - 4R^3}{(R_c^2 + R^2 + \frac{z^2}{q_\phi^2})^2} + \frac{\frac{2}{q_\phi^2}(R_c^2 + R^2 + \frac{z^2}{q_\phi^2}) - \frac{4z^2}{q_\phi^4}}{(R_c^2 + R^2 + \frac{z^2}{q_\phi^2})^2}\right) \tag{62}$$

$$\rho_L(R, z) = \frac{v_0^2}{4\pi G q_\phi^2}\frac{(2q_\phi^2 + 1)R_c^2 + r^2 + (2 - q_{\Phi^2})z^2}{(R_c^2 + r^2 + z^2 q_\Phi^{-2})^2} \tag{63}$$



6 Triaxial Potentials

6.1 Logarithmic Triaxial Density Profile

Motivated by the fact that the models of the Milky Way Dark Matter halo at that time can not reproduce the Sgr stream. Mainly because in axisietric potentials it is not possible to fit the angular precession and the the distance of the stream. Law et al. [2009] propose a triaxial halo Eq.64 which is also in agreement with the CDM paradigm that predicts triaxial halos rather than spherical Lee and Suto [2003]. Law et al. [2009] report results of the orbital integration in which they find that the best fit in their triaxial halo is for $q_z = 1.25$, $q_1 = 1.5$ and $\phi = 90$. Law and Majewski [2010] made N-body simulations in which they include the triaxial halo potential.

$$\Phi = v_{halo}^2 ln(C_1x^2 + C_2y^2 + C_3xy + (z/q_z)^2 + r_{halo}^2) \tag{64}$$

Where the constants C_1, C_2, C_3 are defined as:

$$C_1 = \left(\frac{cos^2\phi}{q_1^2} + \frac{sin^2\phi}{q_2^2}\right) \tag{65}$$

$$C_2 = \left(\frac{cos^2\phi}{q_2^2} + \frac{sin^2\phi}{q_1^2}\right) \tag{66}$$

$$C_3 = 2sin\phi cos\phi\left(\frac{1}{q_1^2} - \frac{1}{q_2^2}\right) \tag{67}$$

From Eq.64 we can derive the circular velocity v_c.

$$v_c^2 = r\nabla\Phi \tag{68}$$

$$\nabla\Phi = \frac{d}{dx}\Phi\hat{i} + \frac{d}{dy}\Phi\hat{j} + \frac{d}{dz}\Phi\hat{k} \tag{69}$$

$$\nabla\phi = v_{halo}^2\left(\frac{(2C_1x + C_3y)\hat{i} + (2C_2y + C_3x)\hat{j} + (2z/q_z^2)\hat{k}}{(C_1x^2 + C_2y^2 + C_3xy + (z/q_z)^2 + r_{halo}^2)}\right) \tag{70}$$

$$v_c = v_{halo}\sqrt{\frac{r((2C_1x + C_3y)^2 + (2C_2y + C_3x)^2 + (2z/q_z^2)^2)^{1/2}}{(C_1x^2 + C_2y^2 + C_3xy + (z/q_z)^2 + r_{halo}^2)}} \tag{71}$$

[?] fixed the value of v_{halo} in order that the MW rotation curve reproduce the obervsed value of $v_{LSR} = 220km/s$ at $R_\odot$ this implies that $v_{halo} = 135km/s$. The rotation curve of is presented in Fig.**??**, the MW rotation curve is also presented in Fig.**??** all the paremeters of this rotation curve are summarized in Table**??**.

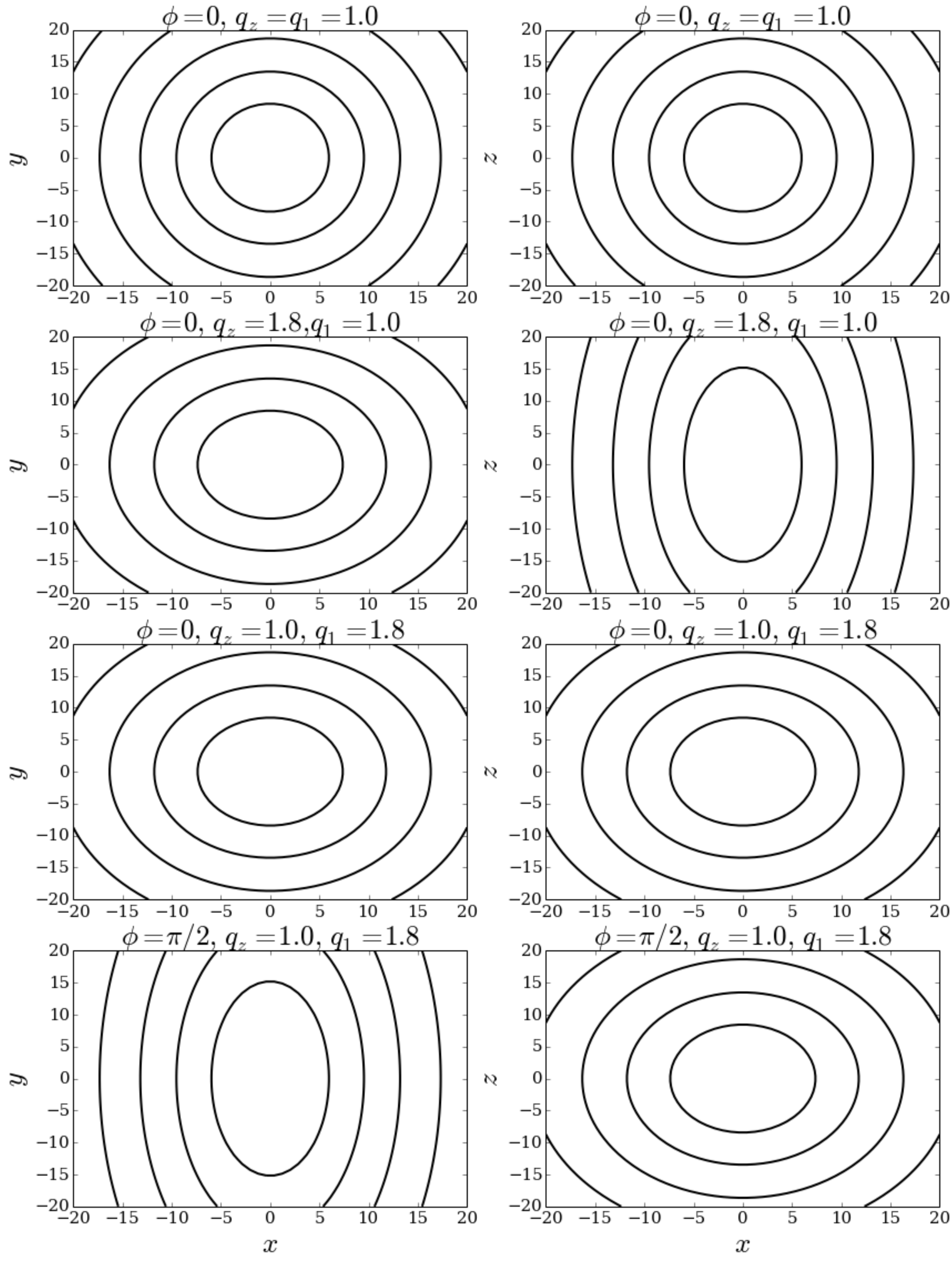
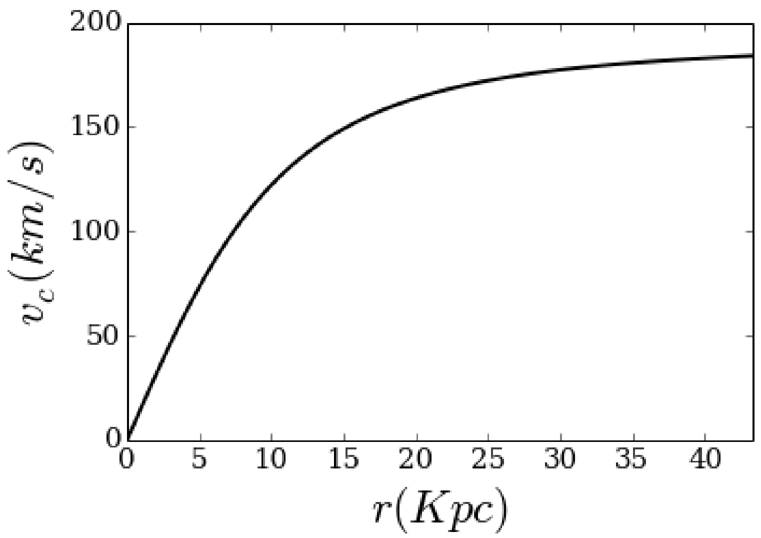
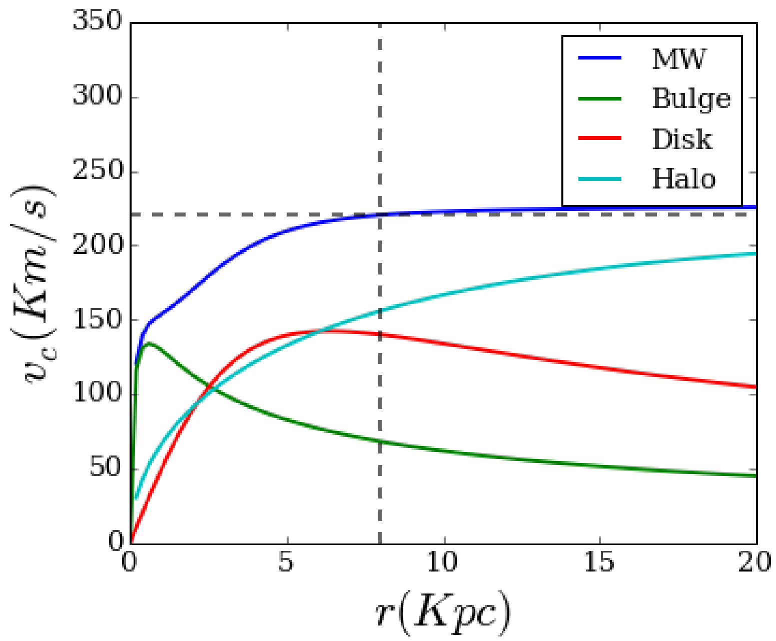


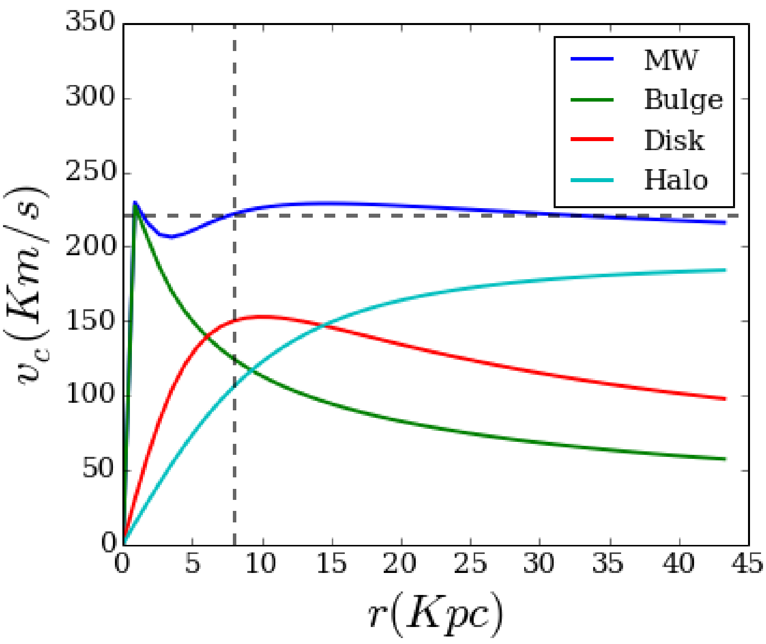
Figure 2: Logarithmic Triaxial Potential. In the top panel



ectionModelling the MW

Component	Besla07	LM2010	Roeland12	Gomez15
Disk Model	Miyamoto-Nagai	Miyamoto-Nagai		
Disk Mass($M_\odot$)	5.5^{10}	1.0×10^{11}		
Disk Param	$R_d = 3.5, z = r_{disk}/5.0$	$\alpha = 1, a = 6.5kpc, b = 0.6kpc$		$, a = 6.5kpc, b = 0.26Kpc$
Bulge Model	Hernquist	Hernquist		Hernquist
Bulge Mass($M_\odot$)	10^{10}	3.4×10^{10}		
Bulge Param	$0.6kpc$	$c = 0.7kpc$	$0.6Kpc$	
DM halo Model	NFW	Triaxial 6.1	Hernquist(NFW)	
DM halo mass($M_\odot$)	10^{12}	$\times 10^{10}$		
Halo Param	$c = 11, r_{vir} = 258Kpc$	$r_{halo} = 12Kpc$		
Solar distance $R_\odot$ (kpc)	8.0	8.0		
reference	Besla07	LM2010		





7 TO-DO:

- Study the properties and different parameters (M, $/pho, v_c, a$)
- Put all the plots and explnations in the text
- note: Klypin relation between c and M_{vir} Doesn't take into account adiabatic contraction.
- work in the code that integrates the orbits using the accelerations. (Viernes)

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