Density profiles

1 Useful Quantities and definitions

In this section some common quantities useful for describe the denstites profile are defined and explained.

1.1 Critical density of the Universe:

1.2 Virialization

A dark matter halo is virialized when its in equilibrium, such an equilibrium occurs after the dark matter have collapsed and the force of gravity equals the **relaxtion** processes (Binney & Tremaine pag 380).

How is related the virialization with the radius, since what redshift you can define a r_{vir}

A halo can be characterized using and overdensity Δvir defined as ratio of the density of a virialized halo over the critical density of the Universe $\Delta_{vir} = \frac{\rho_{vir}}{\rho_c}$. For a cosmolgy with $(\Omega_m + \Omega_\Lambda = 1)$

$$\Delta_{vir} = (18\pi^2 + 82x - 39x^2)/\Omega(z) \tag{1}$$

(Bryan & Norman 1998) it's a good approximation, here $x = \Omega(z) - 1$. For the present time $(z = 0)$ $\Delta_{vir} = 360$.

http://arxiv.org/pdf/astro-ph/9710107v1.pdf
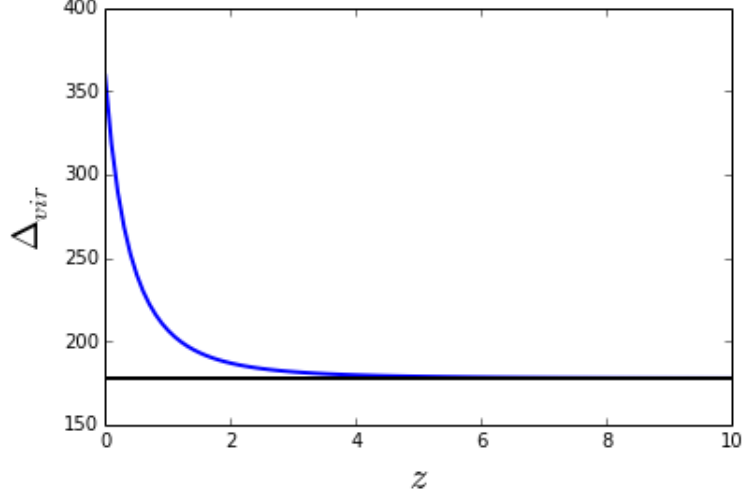http://arxiv.org/pdf/astro-ph/9601088v1.pdf

This overdensity is enclosed in a volume which can be charcterized with a radius r_{vir} which correspond to a M_{vir} given Δ_{rvir}

$$\rho_{vir} = \frac{3M_{vir}}{4\pi r_{vir}^3} = \Delta_{vir}\Omega_m\rho_{crit} \tag{2}$$

$$r_{vir} = \left(\frac{3M_{vir}}{4\pi\Delta_{vir}\Omega_m\rho_{crit}}\right)^{1/3} \tag{3}$$

For example for a halo of mass $M = 1 \times 10^{12} M_\odot$ the corresponding radius is $r_{vir} = 262.4$ Kpc

1.3 r_{200} & M_{200}

$$M_{200} = 200\rho_c \frac{4}{3}\pi r_{200}^3 \tag{4}$$

$$M_{vir} = \Delta_{vir}\Omega_m \rho_c \frac{4}{3}\pi r_{vir}^3 \tag{5}$$

Matching rho_c for the above two equations we get.

$$\frac{M_{200}}{M_{vir}} = \left(\frac{200}{\Delta_{vir}\Omega_m}\right)\left(\frac{r_{200}}{r_{vir}}\right)^3 \tag{6}$$

Here is common to call $q = \left(\dfrac{200}{\Delta_{vir}\Omega_m}\right)$ at $z = 0$ $q = 2.053$

$$\frac{M_{200}}{M_{vir}} = q\left(\frac{r_{200}}{r_{vir}}\right)^3 \tag{7}$$

2 Densities profiles

2.1 Plummer

The plumer density profile is one of the simplest models which describes a constant density near the center and falls at large radius.

$$\rho_P(r) = \frac{3M}{4\pi a^3}(1 + \frac{r^2}{a^2})^{-5/2} \tag{8}$$

Where a is call the scale length. The scale length set the length a in which the mayority of the density is enclosed. Note that if a is cero the plummer potential would be exactly as the potential of a point mass. In the other hand if a goes to infty the potential is rewpresenting a very extended mass source. In other words the scale length set up the size of the volume in which the mass M is enclosed.

The enclosed mass can be derived from the density by integrating over a volume.

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$$M_P(<r) = 4\pi \int_0^r r'^2 \frac{3M}{4\pi a^3} \left(1 + \frac{r'^2}{a^2}\right)^{-5/2} dr' = \frac{3M}{a^3} \left(\frac{a^4 r^3 \sqrt{r^2/a^2 + 1}}{3(r^2 + a^2)^2} \right) \qquad (9)$$

$$M_P(<r) = M \frac{r^3}{(a^2 + r^2)^{3/2}} \qquad (10)$$

Plummer Potential profile

Plummer Potential profile
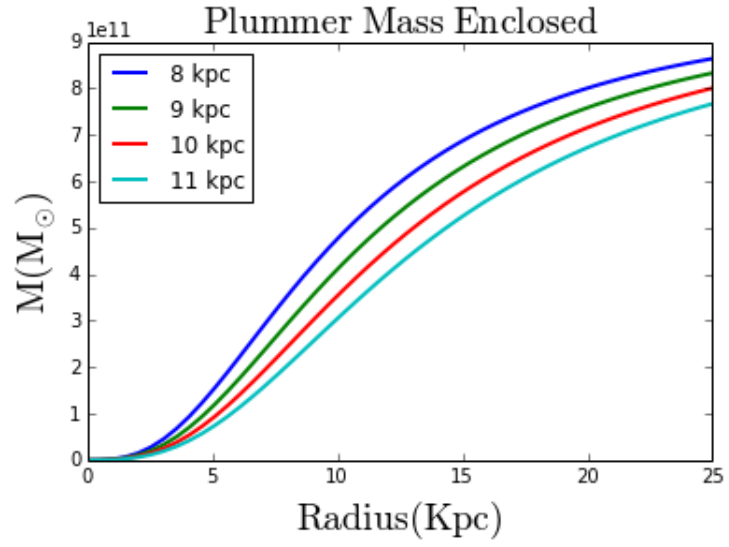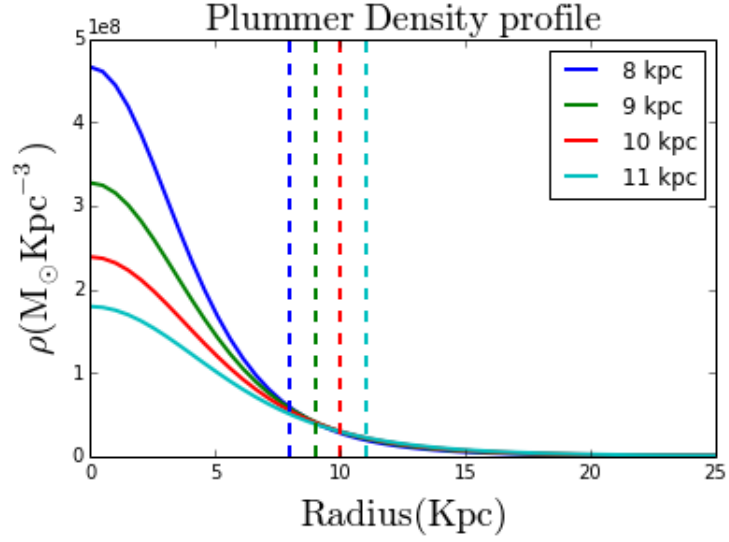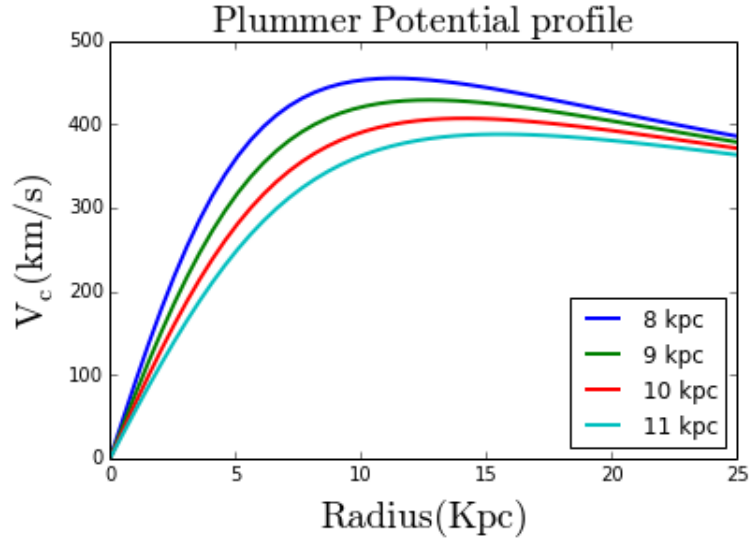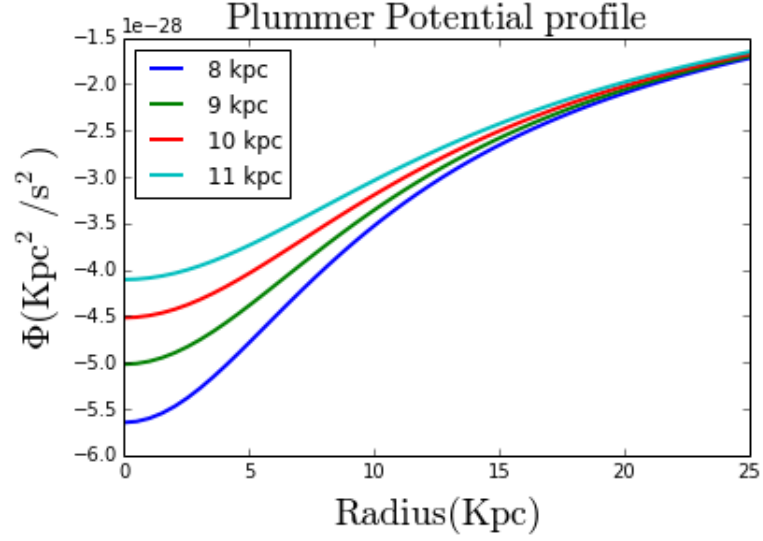
2.2 Hernquist
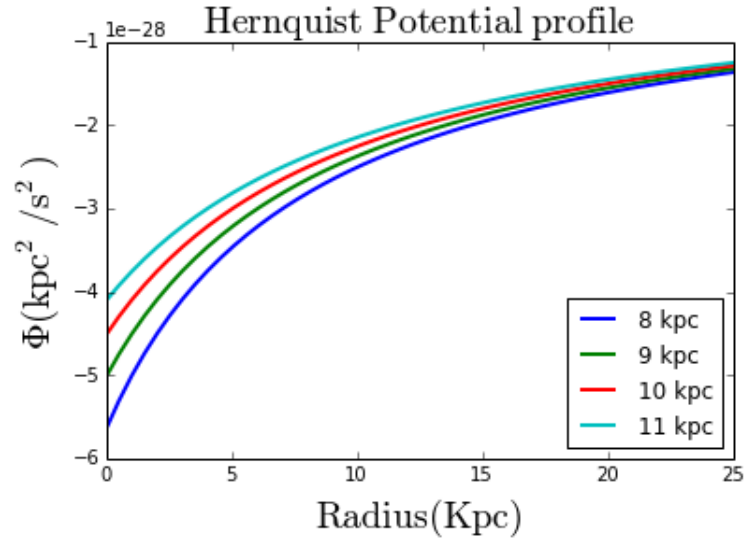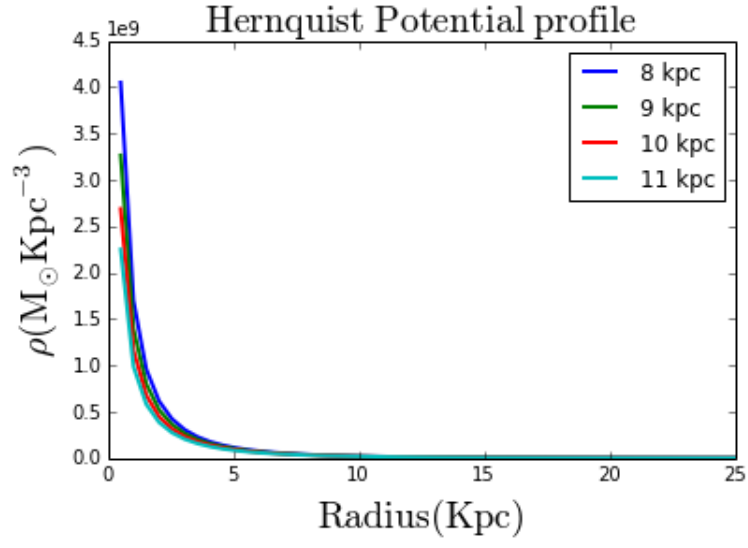
The Hernquist profile is derived in such a way that it follows the

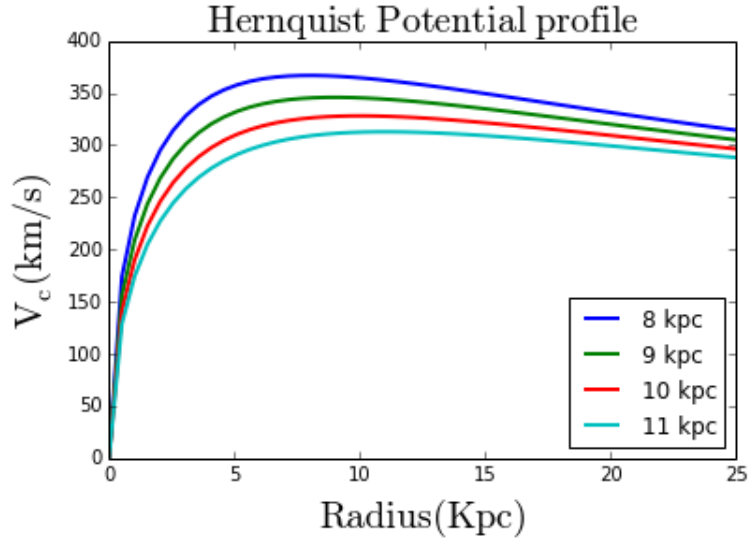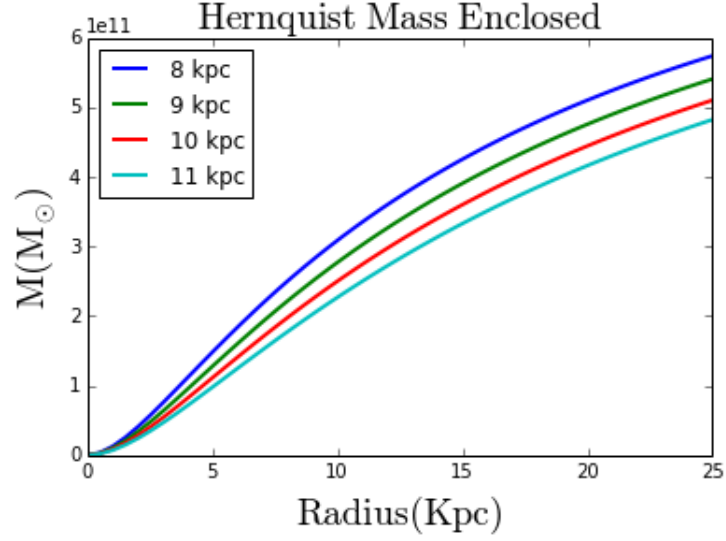$$\rho_{Hernquist}(r) = \frac{M}{2\pi} \frac{a}{r(r+a)^3} \tag{11}$$

$$M_{Hernquist}(< r) = 2aM \int \frac{r}{(r+a)^3} dr \tag{12}$$

$$M_{Hernquist}(< r) = M \frac{r^2}{(r+a)^2} \tag{13}$$

$$\Phi = -\frac{GM}{r+a} \tag{14}$$

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$$v_c(r) = GM \frac{r}{(r+a)^2} \qquad (15)$$

Hernquist Potential profile

Hernquist Potential profile

Hernquist Mass Enclosed

Hernquist Potential profile

2.3   Singular Isothermal Sphere

The Singular Isothermal Sphere (**SIS**) describes a system in which the particles follow a Maxwellian density distribution. With this distribution and the Poisson equation the follow density profiles could be derived.
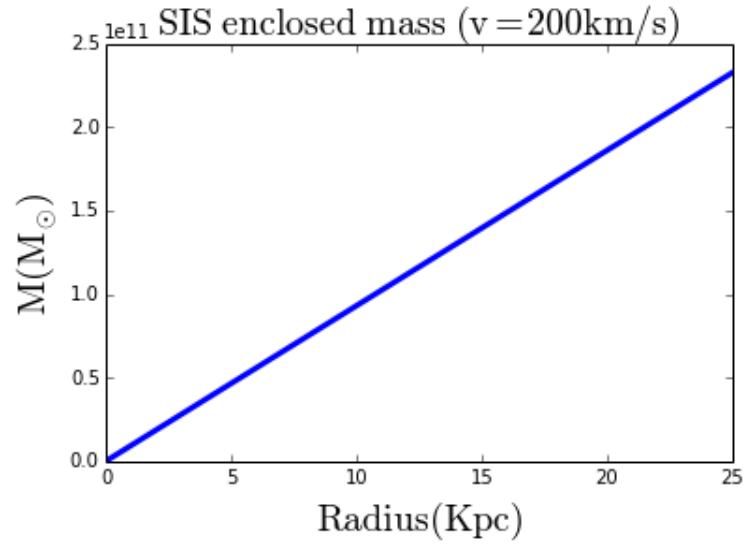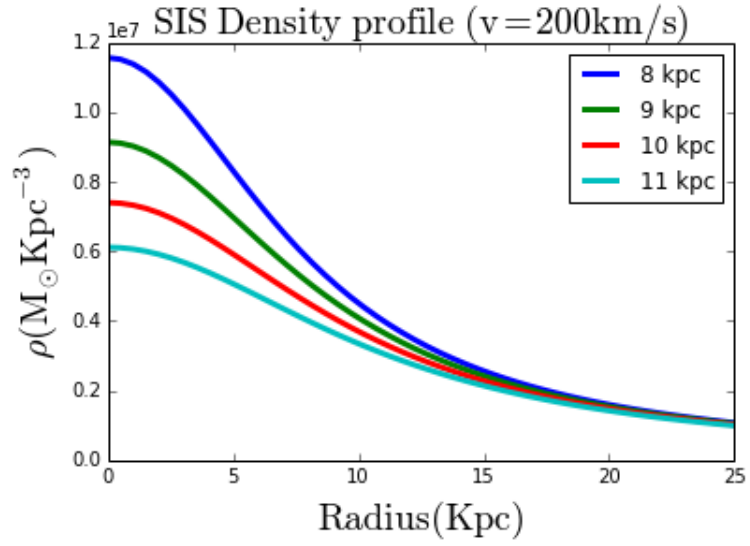
$$\rho_{iso}(r) = \frac{\sigma^2}{2\pi G r^2} \tag{16}$$

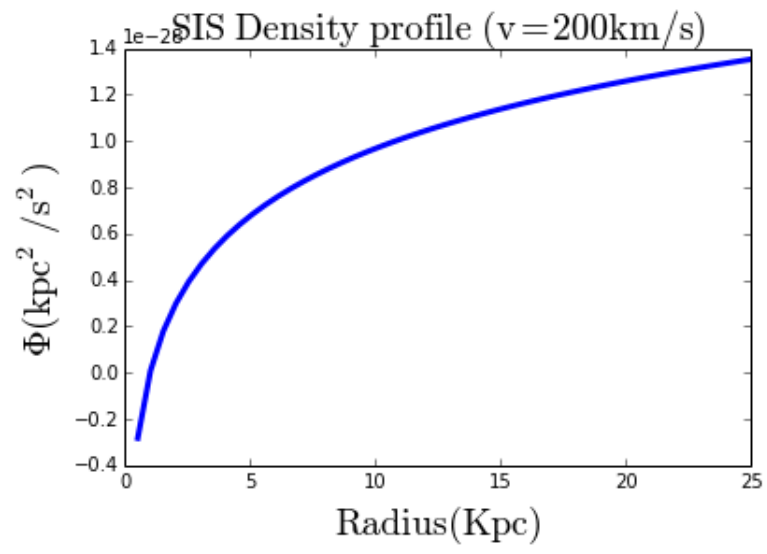Following the same procedure as with the previuos profiles we find M, Φ and v_c.

$$M_{iso}(< r) = \frac{2\sigma r}{G} \tag{17}$$

$$\Phi_{iso}(r) = 2\sigma^2 ln(r) + const. \tag{18}$$

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$$v_c(r) = \sqrt{2}\sigma \tag{19}$$

This profile is quite different to the previous ones due to the fact that here the input is the velocity instead of the total Mass.

SIS Density profile (v=200km/s)

2.4 NFW