# Density profiles

August, 2015

## 1 Useful Quantities and definitions

In this section some common quantities useful for describe the denstites profile are defined and explained.

### 1.1 Critical density of the Universe:

### 1.2 Virialization

A dark matter halo is virialized when its in equilibrium, such an equilibrium occurs after the dark matter have collapsed and the force of gravity equals the **relaxtion** processes (Binney & Tremaine pag 380).

**How is related the virialization with the radius, since what redshift you can define a** $r_{vir}$

A halo can be characterized using and overdensity $\Delta vir$ defined as ratio of the density of a virialized halo over the critical density of the Universe $\Delta_{vir} = \frac{\rho_{vir}}{\rho_c}$. For a cosmolgy with $(\Omega_m + \Omega_\Lambda = 1)$

$$\Delta_{vir} = (18\pi^2 + 82x - 39x^2)/\Omega(z) \tag{1}$$

(Bryan & Norman 1998) it's a good approximation, here $x = \Omega(z) - 1$. For the present time ($z = 0$) $\Delta_{vir} = 360$.

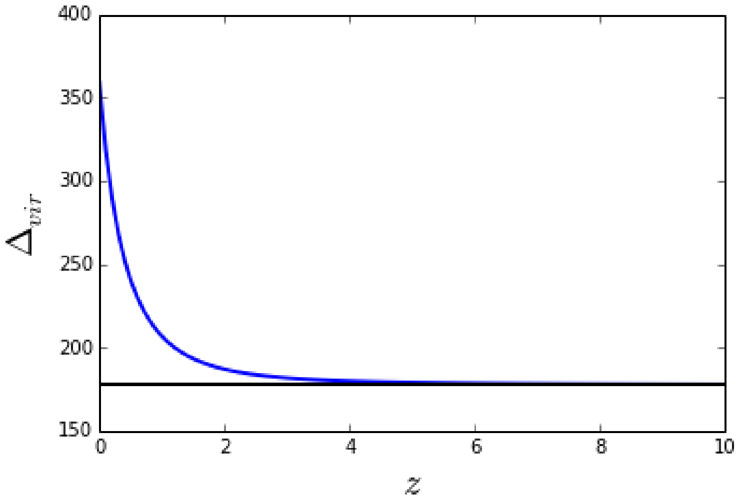
http://arxiv.org/pdf/astro-ph/9710107v1.pdf
http://arxiv.org/pdf/astro-ph/9601088v1.pdf

This overdensity is enclosed in a volume which can be charcterized with a radius $r_{vir}$ which correspond to a $M_{vir}$ given $\Delta_{rvir}$

$$\rho_{vir} = \frac{3M_{vir}}{4\pi r_{vir}^3} = \Delta_{vir}\Omega_m\rho_{crit} \tag{2}$$

$$r_{vir} = \left(\frac{3M_{vir}}{4\pi\Delta_{vir}\Omega_m\rho_{crit}}\right)^{1/3} \tag{3}$$

For example for a halo of mass $M = 1 \times 10^{12} M_\odot$ the corresponding radius is $r_{vir} = 262.4$ Kpc




### 1.3 $r_{200}$ & $M_{200}$

$$M_{200} = 200\rho_c \frac{4}{3}\pi r_{200}^3 \tag{4}$$

$$M_{vir} = \Delta_{vir}\Omega_m\rho_c \frac{4}{3}\pi r_{vir}^3 \tag{5}$$

Matching $rho_c$ for the above two equations we get.

$$\frac{M_{200}}{M_{vir}} = \left(\frac{200}{\Delta_{vir}\Omega_m}\right)\left(\frac{r_{200}}{r_{vir}}\right)^3 \tag{6}$$

Here is common to call $q = \left(\frac{200}{\Delta_{vir}\Omega_m}\right)$ at $z = 0$ $q = 2.053$

$$\frac{M_{200}}{M_{vir}} = q\left(\frac{r_{200}}{r_{vir}}\right)^3 \tag{7}$$

# 2 Densities profiles

## 2.1 Plummer

The plumer density profile is one of the simplest models which describes a constant density near the center and falls at large radius.

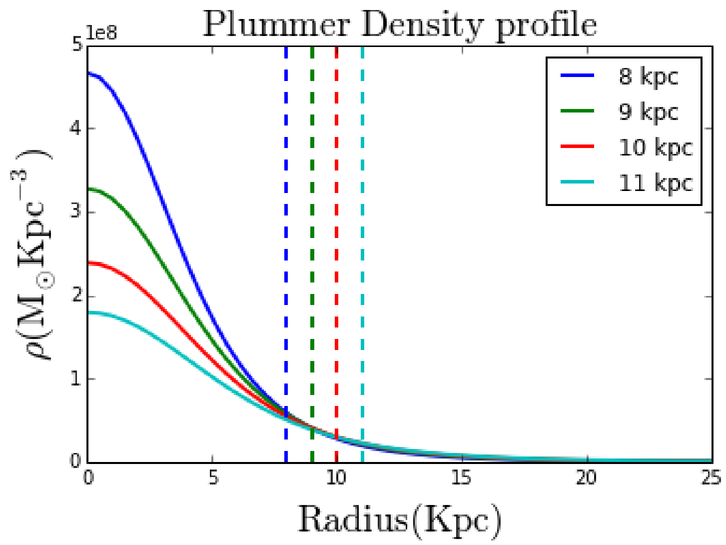
$$\rho_P(r) = \frac{3M}{4\pi a^3}(1 + \frac{r^2}{a^2})^{-5/2} \tag{8}$$

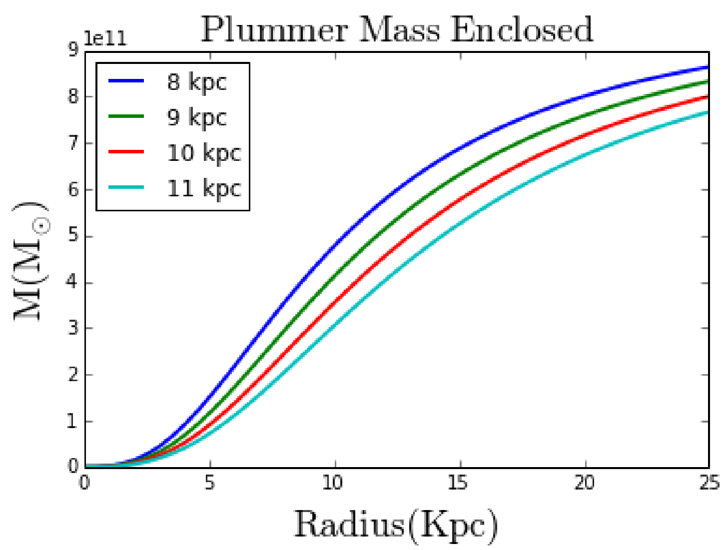
Where $a$ is call the scale length. The scale length set the length $a$ in which the mayority of the density is enclosed. Note that if $a$ is cero the plummer potential would be exactly as the potential of a point mass. In the other hand if $a$ goes to infty the potential is rewpresenting a very extended mass source. In other words the scale length set up the size of the volume in which the mass $M$ is enclosed.

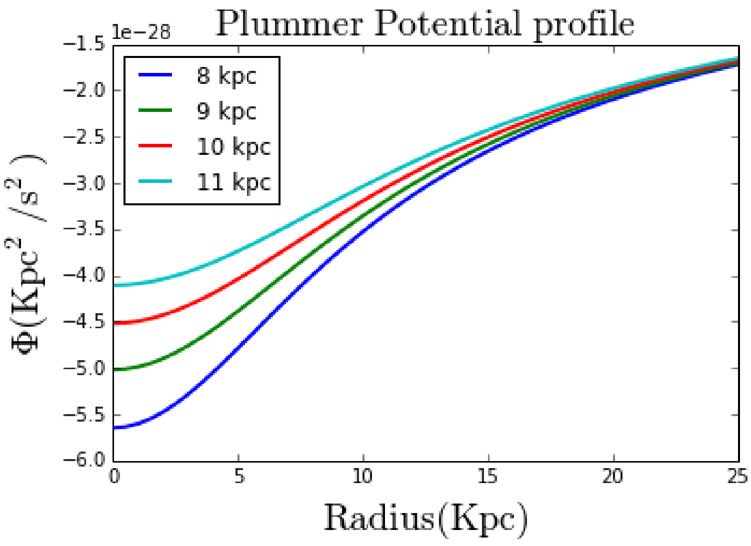
The enclosed mass can be derived from the density by integrating over a volume.

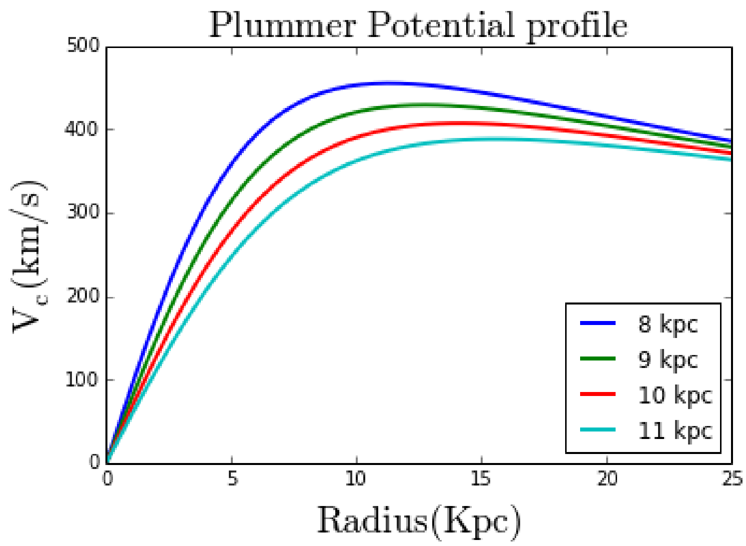
$$M_P(<r) = 4\pi \int_0^r r'^2 \frac{3M}{4\pi a^3}(1+\frac{r'^2}{a^2})^{-5/2} dr' = \frac{3M}{a^3}\left(\frac{a^4 r^3 \sqrt{r^2/a^2+1}}{3(r^2+a^2)^2}\right) \tag{9}$$

$$M_P(<r) = M\frac{r^3}{(a^2+r^2)^{3/2}} \tag{10}$$






## 2.2 Hernquist

The Hernquist profile is derived in such a way that it follows the

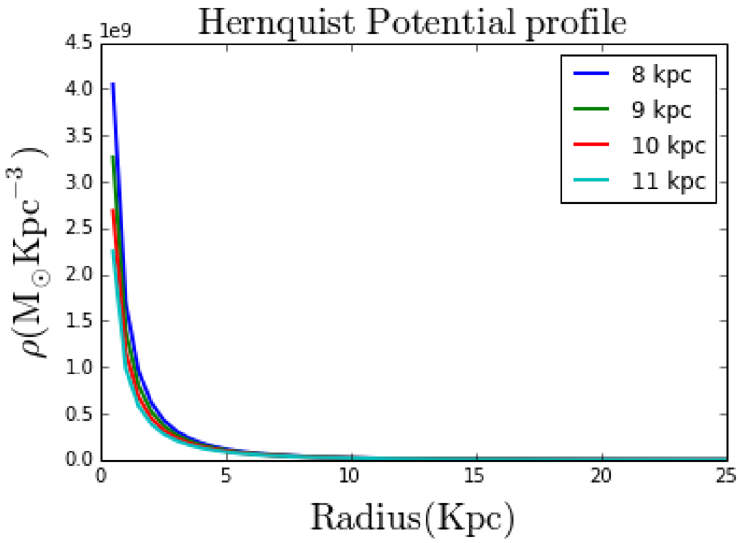
$$\rho_{Hernquist}(r) = \frac{M}{2\pi}\frac{a}{r(r+a)^3} \tag{11}$$

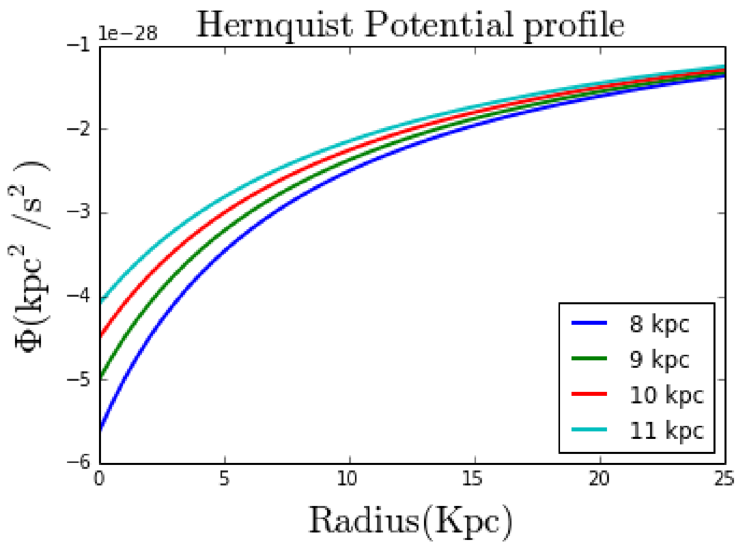
$$M_{Hernquist}(<r) = 2aM\int\frac{r}{(r+a)^3}dr \tag{12}$$

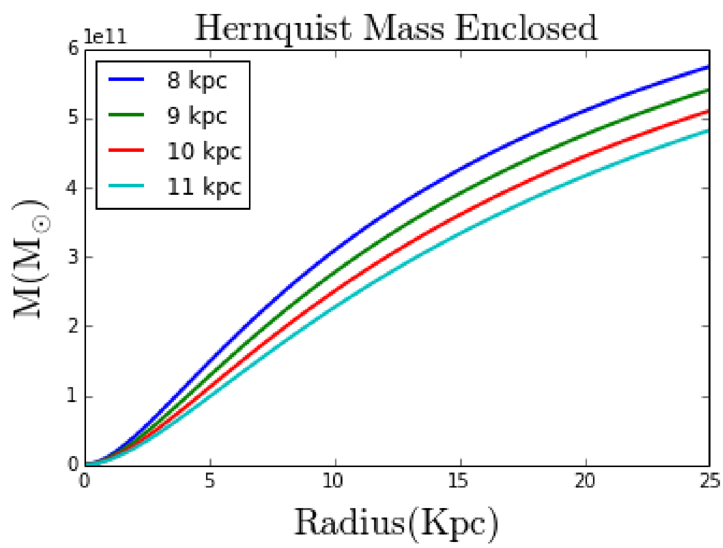
$$M_{Hernquist}(<r) = M\frac{r^2}{(r+a)^2} \tag{13}$$

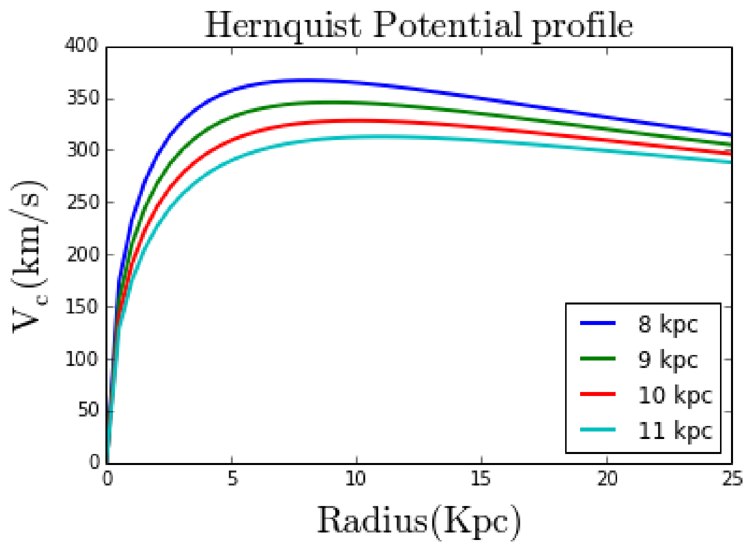
$$\Phi = -\frac{GM}{r+a} \tag{14}$$

$$v_c(r) = GM\frac{r}{(r+a)^2} \tag{15}$$






## 2.3 Singular Isothermal Sphere

The Singular Isothermal Sphere (**SIS**) describes a system in which the particles follow a Maxwellian density distribution. With this distribution and the Poisson equation the follow density profiles could be derived.

$$\rho_{iso}(r) = \frac{\sigma^2}{2\pi G r^2} \tag{16}$$

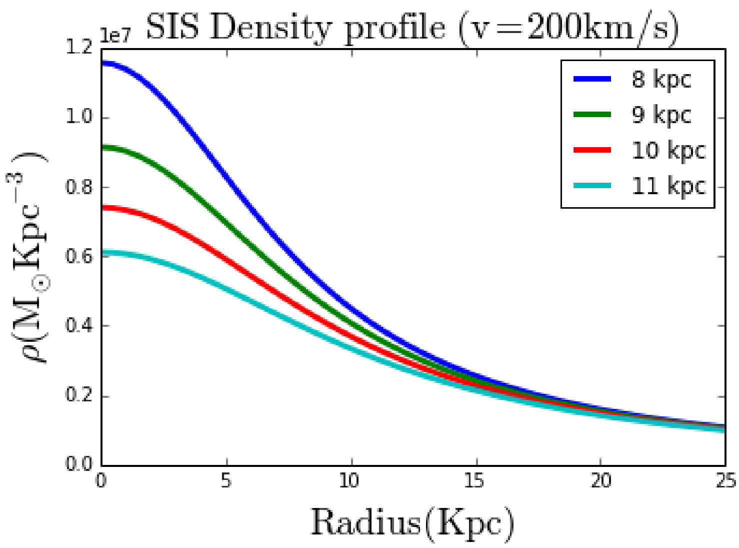
Following the same procedure as with the previuos profiles we find $M, \Phi$ and $v_c$.

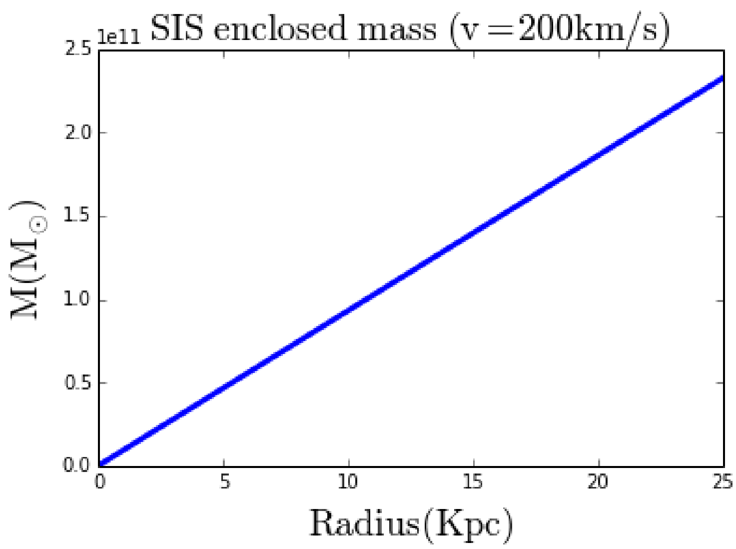
$$M_{iso}(< r) = \frac{2\sigma r}{G} \tag{17}$$

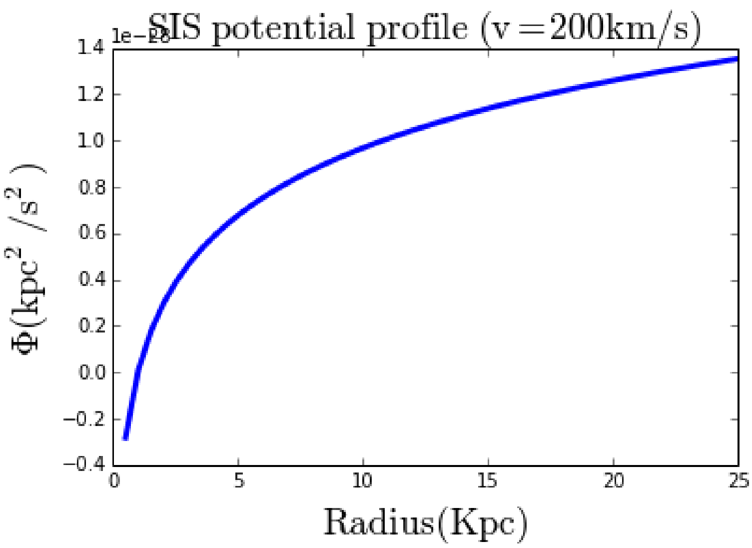
$$\Phi_{iso}(r) = 2\sigma^2 ln(r) + const. \tag{18}$$

$$v_c(r) = \sqrt{2}\sigma \tag{19}$$

This profile is quite different to the previous ones due to the fact that here the input is the velocity instead of the total Mass.






## 2.4 NFW

$$\rho_{NFW}(r) = \frac{M}{2\pi a^3 (r/a)(1 + r/a)^2} \tag{20}$$

$$M_{NFW}(r) = M\left(ln(1 + x) - \frac{x}{1 + x}\right) \tag{21}$$

Where $x = r/a$, is useful to define the function $f(x)$ as:

$$f(x) = ln(1 + x) - \frac{x}{1 + x} \tag{22}$$

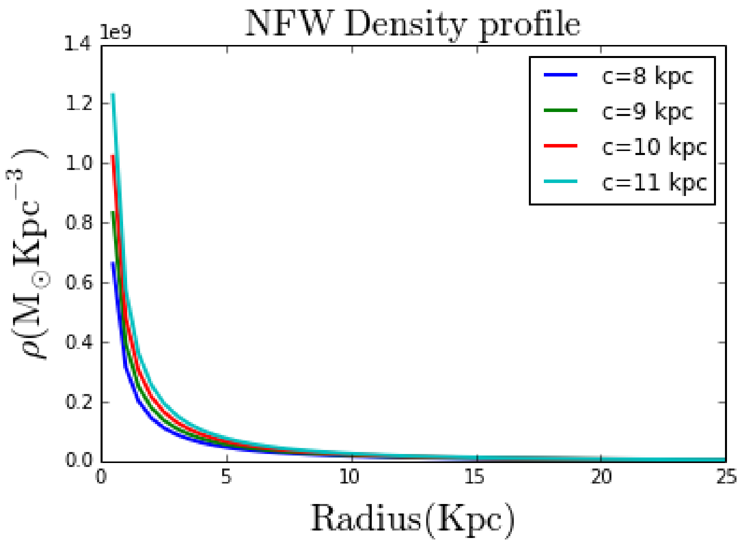
Then 21 can be expresed as:

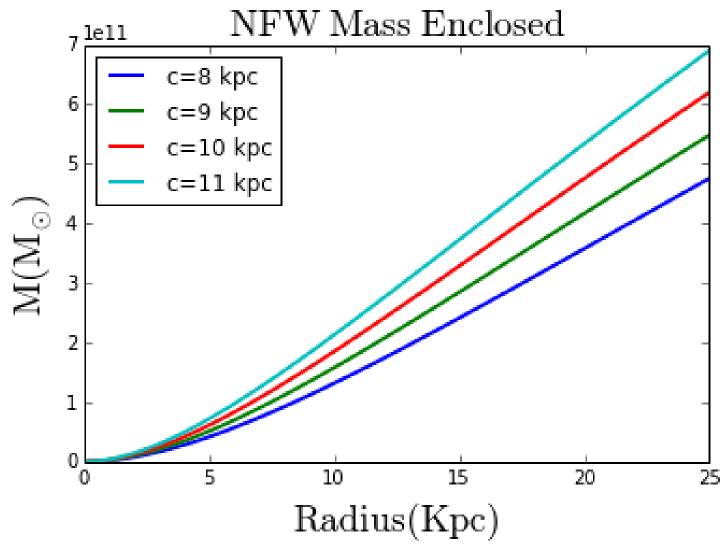
$$M_{NFW} = 4\pi \rho_a a^3 f(x) \tag{23}$$

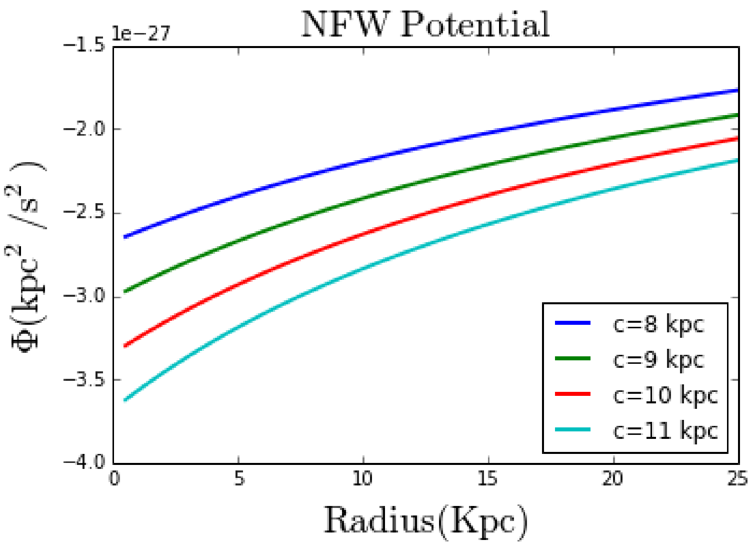
$$\Phi_{NFW} = -4\pi G M \frac{ln(1 + r/a)}{r} \tag{24}$$

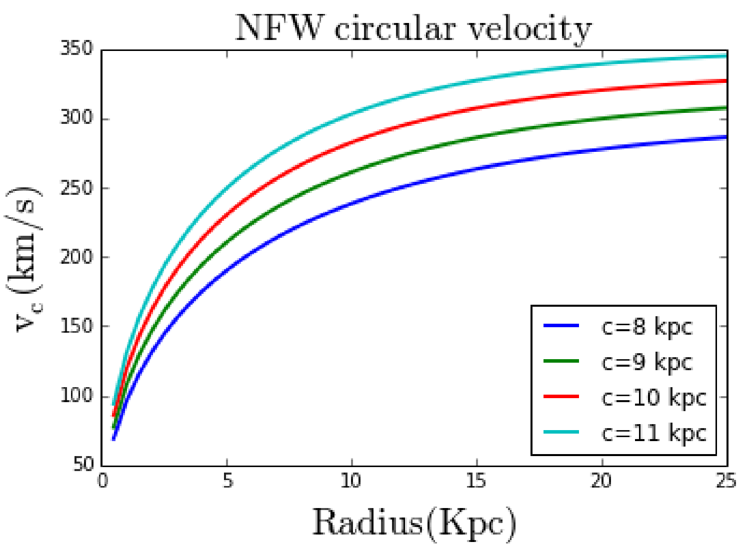
$$c(M_{vir}) = 9.60 \left(\frac{M_{vir}}{10^{12} h^{-1} M_{\odot}}\right)^{-0.075} \tag{25}$$

$$v_c(r) = \sqrt{\left(\frac{M(r)G}{r}\right)} = \sqrt{\left(\frac{2M\left(ln(1 + c) - \frac{c}{1+c}\right)}{r}\right)} \tag{26}$$


NFW Density profile
1e9
$\rho(M_\odot Kpc^{-3})$
Radius(Kpc)
c=8 kpc
c=9 kpc
c=10 kpc
c=11 kpc
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0
5
10
15
20
25


NFW Mass Enclosed
1e11
$M(M_\odot)$
Radius(Kpc)
c=8 kpc
c=9 kpc
c=10 kpc
c=11 kpc
0
1
2
3
4
5
6
7
0
5
10
15
20
25


NFW Potential
1e−27
$\Phi(kpc^2/s^2)$
Radius(Kpc)
c=8 kpc
c=9 kpc
c=10 kpc
c=11 kpc
−1.5
−2.0
−2.5
−3.0
−3.5
−4.0
0
5
10
15
20
25




# 3 Conversion from NFW to the Hernquist profile

The average density of the NFW distribution can be expressed as:

$$\bar{\rho}_{NFW}(r) = \frac{3M_{NFW}(r)}{4\pi r^3} \tag{27}$$

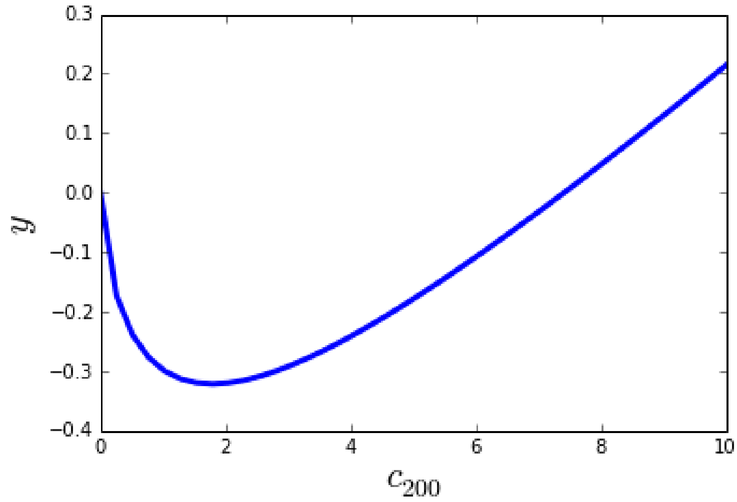
And with eq.23 the $\bar{\rho}_{NFW}(r)$ takes de form:

$$\bar{\rho}_{NFW}(r) = 3\rho_a \left(\frac{a}{r}\right)^3 f(x) \tag{28}$$

Now if we want to find the relationship betwee $r_{200}$ and $r_{vir}$ for the NFW profile we have to apply eq7.

$$q = \frac{3\rho_a \frac{a}{r_{200}} f(c_{200})}{3\rho_a \frac{a}{r_{vir}} f(c)} = \frac{c_{200}^3 f(c_{200})}{c_{vir^3} f(c_{vir})} \tag{29}$$

$$\frac{c_{200}}{c_{vir}} = \left(\frac{f(c_{200})}{q f(c_{vir})}\right)^{1/3} \tag{30}$$




# 4 Miyamoto-Nagai Disk