

AST 515 – ISM and Star Formation – Fall 2015

Homework #1 Due: Wednesday September 09

1. Hydrogen Problems.

- (a) Suppose an electron recombines into the $n=5, l=4$ level of hydrogen. What is the probability that an $H\alpha$ photon will be emitted during the radiative cascade starting from $(n,l) = (5,4)$?
- (b) Radio Recombination Lines (RRLs) are a dust extinction free way to probe physical conditions in a H plasma. When an electron recombines, it can do so in a very high principle quantum number in rarefied plasmas. The notation $Hn\alpha$ is shorthand for a transition of Hydrogen (“H”) from $n + \Delta n \rightarrow n$ where α implies $\Delta n = 1$, β implies $\Delta n = 2$, γ implies $\Delta n = 3$, etc. Which radio recombination line $Hn\alpha$ transition is closest in frequency to the HI hyperfine transitions (what is n)? What is the frequency of that RRL?

2. The ground state of neutral Carbon (CI) is $1s^2 2s^2 2p^2$.

- (a) The lowest energy ground state fine structure transition is observable at submm wavelengths. What are the terms for the transitions and which radiation multipole best describes this transition?
- (b) If one of the p electrons becomes excited into the 3s orbital, what are the electronic terms of this state?
- (c) Order the terms in increasing energy
- (d) What is the parity of the electronic terms in the ground electronic state and the excited electronic state in part (a)
- (e) Which resonance transitions are allowed between the excited electronic state energy levels and the ground electronic state lowest energy levels (3P_J)? Explain using the selection rules for electric dipole transitions.
- (f) Calculate the Einstein A for the lowest energy ground state fine structure transition of CI. [Hint: Check out Sections 2.8 and 5.5 of Quantum Mechanics in Astrophysics by Neal Evans (link on class webpage).]

3. Molecular Spin Statistics

- (a) Which rotational energy levels are allowed for the linear molecular CO₂ by spin statistics in the ground electronic state? [Hint: the inversion symmetry of the ground electronic state is even.]
- (b) Which rotational levels of the CO molecule are ortho and para? Explain.

4. Ozone (O₃) is a bent, asymmetric top molecule that is a minor species in the Earth's atmosphere.

- (a) In the ALMA 1mm band (216-275 GHz), which frequencies (list quantum numbers and frequencies in GHz) might have a contribution from terrestrial O₃? [Hint: Only consider transitions with $E_u < \sim 300$ K.]
- (b) Given the quantum numbers for allowed transitions, which inertial axis (A, B, or C) must the electric dipole moment lie along?
- (c) Check out the online "CSO atmospheric transmission plotter" – are most of the absorption features in the transmission curve due to O₃ in this band?
- (d) Do ozone rotational lines have hyperfine splitting? Explain.

5. Observations of shocked H₂ in Orion via infrared ro-vibrational emission have the following intensities:

$$I = 6.9 \times 10^{-3} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ sr}^{-1} \text{ for } v = 1-0 \text{ S}(1)$$

$$I = 5.7 \times 10^{-4} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ sr}^{-1} \text{ for } v = 2-1 \text{ S}(1)$$

- (a) Give the full quantum numbers for each observed transition and calculate their total statistical weights.
- (b) Using the values of the Einstein A's found in Turner et al. 1977, ApJS, 35, 281 and the energy levels in Black & van Dishoeck 1987 ApJ, 322, 412, compute the gas kinetic temperature of the emitting region. You may assume that the emission is optically thin, isotropic, and thermalized. [Hint: Boltzmann is your friend.]