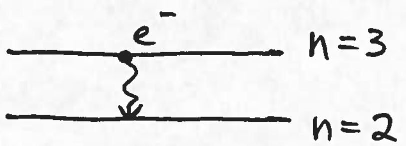
Rad Transfer I

The transfer eq'n : Interaction of Radiation with Matter

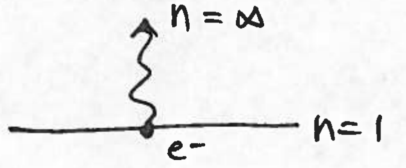
Photons can be absorbed, emitted, and scattered by matter

matter: free electrons, atoms, molecules

Ex:

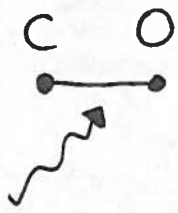


emission process
denote cross section η

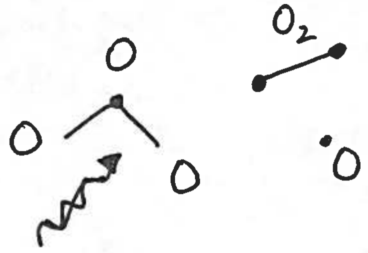


absorption process
cross section κ

scattering
cross section σ



rotational -
vibrational
excitation

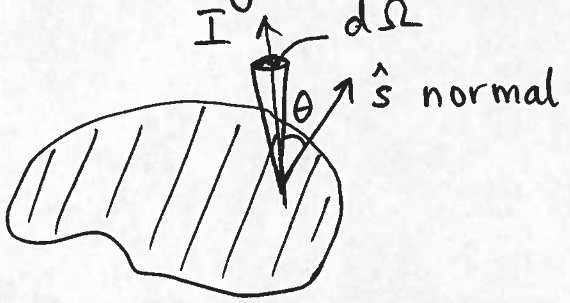


photodissociation

We have a "beam" of photons passing through matter and encountering these processes. How does the number of photons in the beam change?

How do we describe the "beam"?

The basic quantity used to describe radiation is the specific intensity I.



ds: surface element

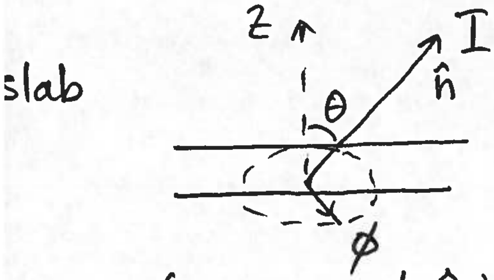
$$I(\bar{r},\hat{n},\nu,t) = \frac{d\mathcal{E}}{d\bar{S}\cdot\hat{n}\;d\Omega\,d\nu\,dt}$$

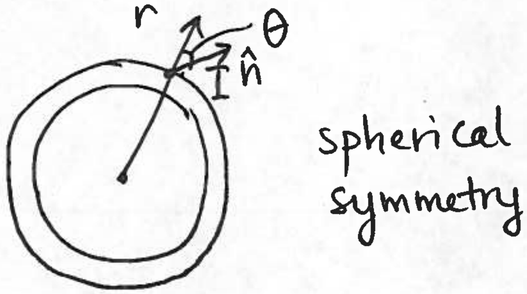
amount of energy passing through surface element dS, in direction $\hat{n}$, within a solid angle dΩ, in frequency interval dν, in time dt.

$$\hat{n}.\,d\bar{S} = dS\cos\theta$$

$$d\mathcal{E} = I(\bar{r},\hat{n},\nu,t)\,dS\cos\theta\,d\Omega\,d\nu\,dt$$

Two very frequently used geometries: Planar + spherically symm.





surface normal $\hat{s}$ in the z direction; call $\hat{k}$

no φ dependence in any variable

$\Rightarrow \hat{k}\cdot\hat{n} = \cos\theta$

$I = I(z,\theta,\nu,t)$

$\bar{r} = (r,\Theta,\Phi)$ location on the surface. No Θ and Φ dependence.

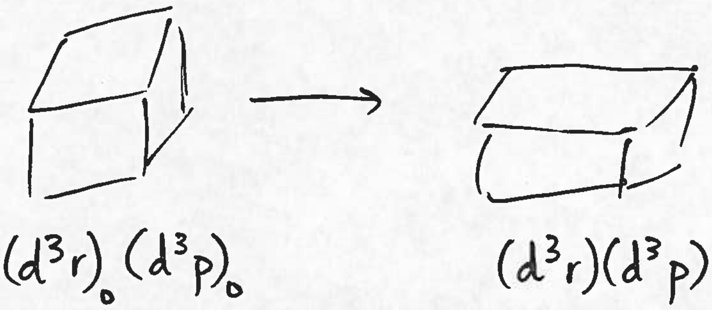
But can have angle θ betwee $\hat{n}$ and $\bar{r}$.

$I = I(r,\theta,\nu,t)$

often: $\mu = \cos\theta$ is used as a variable.

Boltzmann eq'n for photons

Particle distribution function $f(\bar{r}, \bar{p}, t)$ = number density in phase element $(\bar{r} + d\bar{r}), (\bar{p} + d\bar{p})$



$d\bar{r} = \bar{v}\, dt \quad \bar{r} \rightarrow \bar{r} + \bar{v}\, dt$

$d\bar{p} = \bar{F}\, dt \quad \bar{p} \rightarrow \bar{p} + \bar{F}\, dt$

If there were no interactions btw photons and external particles (scattering, emission, absorption) phase space could be distorted but the volume unchanged. Generically, we call this the "collision" term.

$$\frac{Df}{Dt} = \left(\frac{Df}{Dt}\right)_{coll}$$

$$\frac{\partial f}{\partial t} + \left(\frac{\partial f}{\partial x}\right)\left(\frac{\partial x}{\partial t}\right) + \left(\frac{\partial f}{\partial y}\right)\left(\frac{\partial y}{\partial t}\right) + \left(\frac{\partial f}{\partial z}\right)\left(\frac{\partial z}{\partial t}\right) + \left(\frac{\partial f}{\partial \partial p_x}\right)\left(\frac{\partial p_x}{\partial t}\right) + \left(\frac{\partial f}{\partial p_y}\right)\left(\frac{\partial p_y}{\partial t}\right) + \left(\frac{\partial f}{\partial p_z}\right)\left(\frac{\partial}{\partial t}\right.$$

$$= \left(\frac{Df}{Dt}\right)_{coll}$$

$$\Rightarrow \frac{\partial f}{\partial t} + \bar{v} \cdot \bar{\nabla} f + \bar{F} \cdot \bar{\nabla}_p f = \left(\frac{Df}{Dt}\right)_{coll}$$

for photons: $\bar{v} = c\bar{n}$

$\bar{F} = 0$ (no mass, no charge)

The coll term includes emission η, and "extinction"

$\chi = \kappa + \sigma$ absorption + scattering

Ⓘ The emission and extinction coefficients have different units.

$$\eta(\bar{r},\bar{n},\nu,t) = \frac{dE}{dV\,d\Omega\,d\nu\,dt}$$

energy emitted from vol dV
into solid angle dΩ
within a freq. band dν
in a time interval dt
[along direction $\bar{n}$]

$$[\eta] = cm^{-3}\,sr^{-1}\,Hz^{-1}\,s^{-1}$$

The extinction coefficient:

$$I(\bar{r},\bar{n},\nu,t)\cdot\chi(\bar{r},\bar{n},\nu,t) = \frac{dE}{dV\,d\Omega\,d\nu\,dt}$$

amount of energy removed from a beam with specific intensity I in volume dV, into solid angle dΩ, within a freq. band dν, in a time interval dt, along direction $\hat{n}$.

The extinction coefficient is the product of atomic absorption cross section [cm²] and the number density of absorbers (cm^{-3})

$$[\chi] = cm^{-1}.$$

$\frac{1}{\chi}$ is a measure of the distance a photon can propagate before it is removed from the beam. (mean free path)

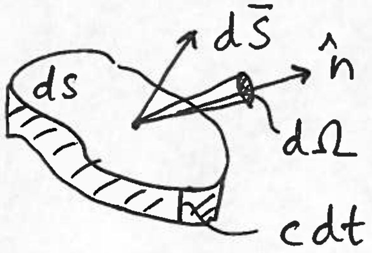
(II) What is the relation btw I and the photon distribution function f?

f is the number of photons propagating with velocity c in direction $\hat{n}$ into solid angle $d\Omega$ per unit volume

$$I(\bar{r},\bar{n},\nu,t) = ch\nu \, f(\bar{r},\bar{n},\nu,t)$$

because $E = h\nu$; in time dt, volume covered is $d\bar{S}\cdot\hat{n}\,c\,dt$

$$f = \frac{d\mathcal{E}/h\nu}{\underbrace{\hat{n}\cdot d\bar{S}\,c\,dt}_{dV}\,d\Omega\,d\nu}$$



OK, back to the Boltzmann eq'n: write in terms of specific intensity I.

$$\frac{1}{ch\nu}\left[\frac{\partial I}{\partial t} + c\bar{n}\cdot\bar{\nabla} I\right] = \frac{\eta - \chi I}{h\nu}$$

(because η = energy emitted; χI = energy absorbed, $\frac{\eta}{h\nu}$ = number of photons emitted)

$$\Rightarrow \boxed{\frac{1}{c}\frac{\partial I}{\partial t} + \bar{n}\cdot\bar{\nabla} I = \eta - \chi I}$$

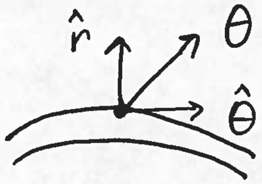
Eq'n of radiative transfer.

Typical simplifications: no explicit time dependence (e.g., emitters, absorbers, or density is not changing over time)

$$\Rightarrow \frac{\partial I}{\partial t} = 0$$

Spherical geometry:

$$I(\bar{r},\bar{n},\nu,t) \longrightarrow I(r,\theta,\nu,t)$$



$$\bar{\nabla} = \frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial\theta}\hat{\theta}$$

$$\hat{n} = (\cos\theta, \sin\theta, 0)$$

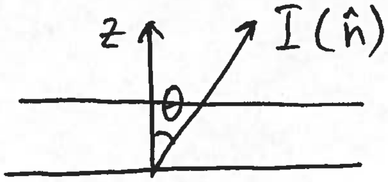
$$\hat{n}\cdot\bar{\nabla} = \cos\theta\frac{\partial}{\partial r} + \frac{1}{r}\sin\theta\frac{\partial}{\partial\theta}$$

$$\frac{1}{c}\frac{\partial I}{\partial t} + \left[\cos\theta\frac{\partial}{\partial r} + \frac{1}{r}(1-\cos^2\theta)^{1/2}\frac{\partial}{\partial\theta}\right]I = \eta - \chi I$$

Note: The tr. eq'n even in the spherically symm. case is a PDE, $\frac{\partial}{\partial r}$ and $\frac{\partial}{\partial\theta}$ term

Planar geometry:

variables function of z only



$$\frac{\partial}{\partial x} = 0 \quad \frac{\partial}{\partial y} = 0$$

$$\hat{n}\cdot\bar{\nabla} = n_z\frac{\partial}{\partial z}$$

$$= \cos\theta\frac{\partial}{\partial z}$$

$$= \mu\frac{\partial}{\partial z}$$

$$\frac{1}{c}\left(\frac{\partial I}{\partial t}\right) + \mu\frac{\partial I}{\partial z} = \eta - \chi I$$

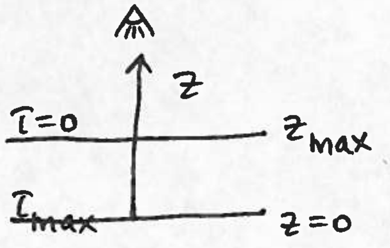
In planar geometry (1 Cartesian dimension), this is an O.D.E.

Rewrite the transfer eq'n in terms of optical depth τ, which serves as a dimensionless depth variable.

In 1-D:

$$d\tau_\nu \equiv -\chi(z,\nu)\, dz$$

$$\tau_\nu = \int_z^{z_{max}} \chi(z',\nu)\, dz'$$ – sign conventional such that



$$= \int_z^{z_{max}} \frac{1}{\ell}\, dz$$

ℓ = mean free path

$\tau \Rightarrow$ the number of mean-free paths for a photon of freq. ν btw z and z_{max}.

Define a "source function"

$$S_\nu = S(z,\nu) = \frac{\eta(z,\nu)}{\chi(z,\nu)}$$

Transfer eq'n becomes (time indep't, planar 1-D)

$$\mu \frac{\partial I_\nu}{\partial \tau_\nu} = I_\nu - S_\nu$$ (note the – sign)

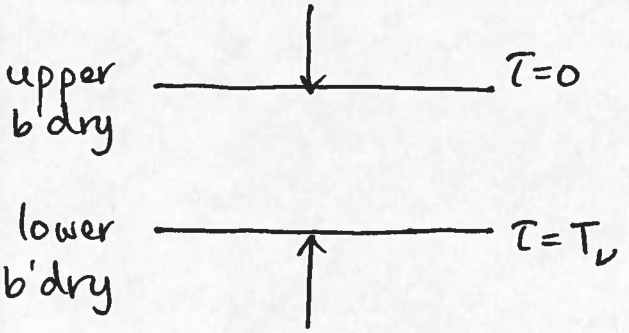
To solve the rad. transfer eq'n (an ODE, integro-differential ODE if S_ν has scattering, or a PDE), need to specify

Boundary Conditions

How many do you need? In 1-D (spatial dimension), one needs to specify I at $\tau=0$ or $\tau=\tau_{max}$ for each μ and each ν. (ODE: 1 B.C.)

You can think of it as $\mu \times \nu$ O.D.E.s (or 1 O.D.E. for each μ and each ν).

- for a finite slab of specified total optical depth T_ν



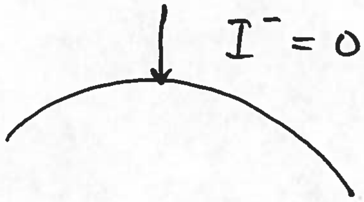
most useful to specify I^- at $\tau=0$
and I^+ at $\tau=T_\nu$

$$I(\tau_\nu=0,\mu,\nu)=I^-(\mu,\nu)$$
$$-1 \leq \mu \leq 0$$

$$I(\tau_\nu=T_\nu,\mu,\nu)=I^+(\mu,\nu)$$
$$0 \leq \mu \leq 1$$

These determine a unique solution.

- for a semi-infinite case, e.g. a star of nearly ∞ optical depth



$\downarrow$ $\tau_\nu \to \infty$ $\quad \lim\limits_{\tau \to \infty} I(\tau_\nu,\mu,\nu)\, e^{-\tau_\nu/\mu} = 0$ for all μ.

for inner boundary.

If this were truly the case, there would be no outward flux. So, in reality, you would not solve the transfer eq'n μ by μ, as we wrote it, but take averages ("moments"), as we will see. That will allow us to specify a net flux.

Simple examples and the Formal Solution of the Transfer Eq'n

1) No material is present.

$$\chi_\nu = \eta_\nu = 0$$

Tr. eq'n in 1-D becomes

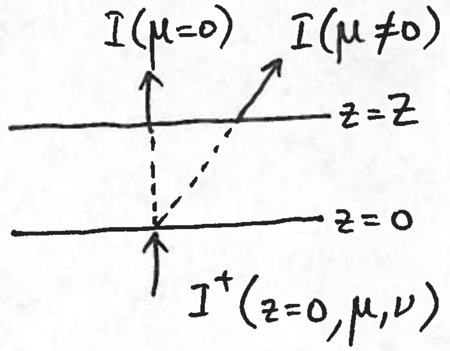
$$\frac{\partial I_\nu}{\partial z} = 0 \Rightarrow I_\nu = \text{const.}$$

Specific intensity is invariant.

2) Material emits at freq ν but cannot absorb (decay of metastable levels in low density gas, for example)

$$\mu \left(\frac{\partial I_\nu}{\partial z}\right) = \eta_\nu$$

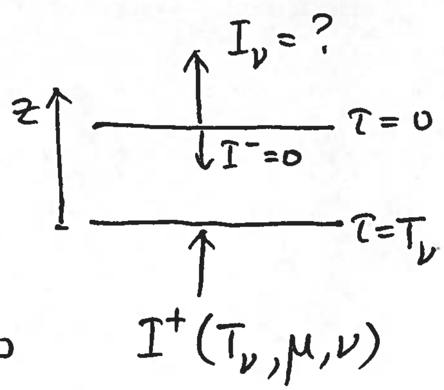
$$I_\nu(\mu) = \underset{\substack{\uparrow \\ \text{geometrical} \\ \text{pathlength factor}}}{\mu^{-1}} \int_0^z \eta(z,\nu)\,dz + I^+(0,\mu,\nu)$$



3) Radiation is absorbed but not emitted (e.g., a filter at a particular frequency)

$$\mu \frac{\partial I_\nu}{\partial z} = -\chi_\nu I_\nu \quad \text{with } d\tau_\nu = -\chi_\nu\,dz$$

$$\Rightarrow \quad I(\tau_\nu = 0, \mu, \nu) = I_\nu^+(T_\nu, \mu)\, e^{-(T_\nu/\mu)}$$



exponentially attenuated. This ties back to the def'n of optical depth as the # of mean free paths.

→ i.e., survival prob. of a photon goes down as $e^{-\tau}$.

4) The Formal Solution

$$\mu \frac{\partial I_\nu}{\partial \tau_\nu} = I_\nu - S_\nu$$

$$\frac{\partial I_\nu}{\partial \tau_\nu} - \frac{1}{\mu} I_\nu = \frac{-1}{\mu} S_\nu$$

Note: $\frac{\partial}{\partial \tau_\nu}(I_\nu e^{-\tau_\nu/\mu})$
$= \frac{\partial I_\nu}{\partial \tau_\nu} e^{-\tau_\nu/\mu} + I_\nu e^{-\tau_\nu/\mu} \cdot \frac{-1}{\mu}$
$= e^{-\tau_\nu/\mu}\left(\frac{\partial I}{\partial \tau} - \frac{1}{\mu} I_\nu\right)$

use integrating factor $e^{-\tau_\nu/\mu}$ to rewrite this as

$$\frac{\partial \left[I_\nu e^{-\tau_\nu/\mu}\right]}{\partial \tau_\nu} = -\frac{1}{\mu} S_\nu e^{-\tau_\nu/\mu}$$

and integrate w.r.t. τ_ν

$$I_\nu(\tau_1, \mu) = I_\nu(\tau_2, \mu) e^{-(\tau_2 - \tau_1)/\mu} + \frac{1}{\mu} \int_{\tau_1}^{\tau_2} S_\nu(\tau') e^{-(\tau' - \tau_1)/\mu} d\tau'$$

if S_ν is known, have a complete sol'n of the tr. eq'n.

Note: S_ν may be coupling different μ's and ν's, e.g. in scattering. Then the sol'n of these coupled O.D.E.s still quite difficult (actually they become coupled integro-differential eq'ns because an integral over I appears on the R.H.S. in the S_ν).

Often, we're not interested in specific intensity but averages over the specific intensity. This might be dictated by the observables (flux), utility (nearly isotropic or free-streaming I or the impossibility of specifying $I(\mu)$.

Before we average over the transfer eq'n, let's define the averages over specific intensity itself. These are called moments. (Note: for fluids in general, these are averages over velocity. For photons, this reduces to averages over direction).

Mean Intensity - Zeroth Moment

Average of specific intensity over angles

$$\bar{J}(\bar{r},\nu,t) = \frac{1}{4\pi}\oint I(\bar{r},\hat{n},\nu,t)\, d\Omega$$

$$= \frac{1}{4\pi}\oint I(\bar{r},\hat{n},\nu,t)\, \underbrace{\sin\theta\, d\theta\, d\phi}_{\equiv -d\mu\, d\phi}$$

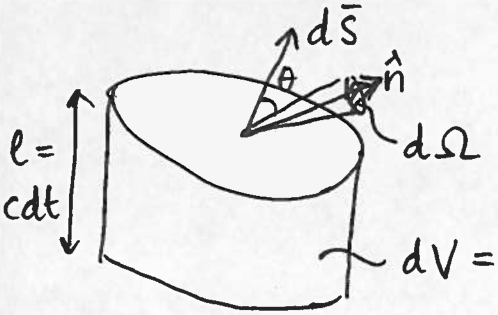
in 1-D planar, no ϕ dependence:

$$\bar{J}(z,\nu,t) = \frac{1}{4\pi}\int_0^{2\pi} d\phi \int_{-1}^{1} d\mu\; I(z,\mu,\nu,t) = \frac{1}{2}\int_{-1}^{1} I(z,\mu,\nu,t)\, d\mu$$

in spherical geometry:

$$\bar{J}(r,\nu,t) = \frac{1}{2}\int_{-1}^{1} I(r,\mu,\nu,t)\, d\mu$$

Mean intensity is closely related to the energy density of radiation. What is the energy contained in volume V in radiation?



$$d\mathcal{E} = I(\bar{r},\hat{n},\nu,t)\, ds \cos\theta \, d\Omega \, d\nu \, dt$$

$$dV = c\,dt\,\underbrace{dS\cos\theta}_{\hat{n}\cdot d\bar{s}}$$

To find the energy in volume dV:

- integrate over all $d\Omega$
- integrate over dV
- integrate over ν

$$d\mathcal{E} = I(\bar{r},\hat{n},\nu,t)\,\frac{dV}{c}\,d\Omega\,d\nu$$

$$\mathcal{E} = \int dV \int d\Omega \int d\nu \, \frac{1}{c}\, I(\vec{r},\hat{n},\nu,t)$$

assuming I is independent of $\vec{r}$ within V:

$$\mathcal{E}(\bar{r},t) = \frac{1}{c}\, V \int d\Omega\, d\nu \, I(\bar{r},\hat{n},\nu,t)$$

if we were interested in the energy density in one freq. band, we would drop the integral over ν.

$$\mathcal{E}(\bar{r},\nu,t) = \frac{1}{c}\, V \underbrace{\int d\Omega\, I(\bar{r},\hat{n},\nu,t)}_{4\pi\, J(\bar{r},\nu,t)}$$

$$\Rightarrow \quad \mathcal{E}(\bar{r},\nu,t) = \frac{4\pi}{c}\, J(\bar{r},\nu,t)\cdot V$$

so the "monochromatic energy density is:

$$E_R(\bar{r},\nu,t) = \frac{4\pi}{c}\, J(r,\nu,t)$$

Ex: In thermal equilibrium (which we have not defined yet), we'll see that

I is uniform (no $\bar{r}$ dependence)

isotropic (no $\hat{n}$ dependence)

time-indep't (no t dependence)

It has ν dependence given by Planck

$$I_\nu(T) = \frac{2h\nu^3}{c^2}\left(e^{h\nu/kT}-1\right)^{-1} \equiv B_\nu(T)$$

Monochromatic energy density:

$$E_R(\nu) = \frac{4\pi}{c} J(\bar{r},\nu,t) = \frac{4\pi}{c} B_\nu(T)$$

Total energy density: $\int d\nu \, E_R(\nu)$

$$E = \frac{8\pi h}{c^3}\int_0^\infty \left(e^{h\nu/kT}-1\right)^{-1}\nu^3 d\nu = aT^4$$

with $a \equiv \frac{8\pi^5 k^4}{15 c^3 h^3}$

Flux – First Moment

The momentum of the rad. field

Specifies the net rate of E flow through a surface in a given direction.

rate means per unit time, per unit area

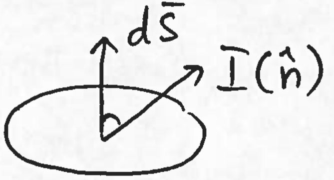
$$\bar{F}(\bar{r},\nu,t) = \frac{d\mathcal{E}}{dS\,dt\,d\nu} = \oint I(\bar{r},\hat{n},\nu,t)\,\hat{n}\,d\Omega$$

$$[F] = \text{erg cm}^{-2}\,\text{s}^{-1}\,\text{Hz}^{-1}$$

Is this indeed the correct relation of I_ν to F_ν?

$$d\varepsilon = I(\bar{r},\hat{n},\nu,t)\,\underbrace{dS\cos\theta}\,d\Omega\,d\nu\,dt$$

$$F = \frac{d\varepsilon}{dS\,d\nu\,dt} = \oint I(\bar{r},\hat{n},\nu,t)\,\hat{n}\,d\Omega$$



$$\hat{n}\cdot d\bar{S} = dS\cos\theta$$

Note: Eddington flux $H \equiv \frac{1}{4\pi}F$, in analogy with J.

Ex: Truly isotropic radiation field

$$I(\hat{n}) = I_0$$

Then $F = \int I_0\,\hat{n}\,d\Omega = I_0 \oint \cos\theta\,\sin\theta\,d\theta\,d\phi$

$$= I_0 \cdot 2\pi \int_{-1}^{1} \mu\,d\mu = 0$$

That's why at very high τ such as centers of stars, I is nearly but not completely isotropic (because there is a net outward flux).

Radiation Pressure Tensor – Second Moment

$$P(\bar{r},\nu,t) = \frac{1}{c}\oint I(\bar{r},\hat{n},\nu,t)\,\hat{n}\hat{n}\,d\Omega$$

or

$$P_{ij}(\bar{r},\nu,t) = \frac{1}{c}\oint I(\bar{r},\hat{n},\nu,t)\,n_i n_j\,d\Omega$$

Diagonal terms P_{ii} are related to energy density because for a fluid, they have a form

$$\int \rho\,v_i v_i\,d\Omega \Rightarrow \text{energy density}$$

The sum of the diagonals

$$\Sigma P_{ii} = P_{xx} + P_{yy} + P_{zz} = \frac{1}{c}\int I\,\underbrace{(n_x^2 + n_y^2 + n_z^2)}_{=1}\,d\Omega$$

$$= \frac{1}{c}\int I\,d\Omega = E(\bar{r},\nu,t)$$

energy density

If the radiation field is isotropic, $P_{ij} = 0$

$$P_{ii} = P_{jj} = P_{kk} = \frac{1}{3}E(\bar{r},\nu,t)$$

We rarely use P_{ij} in its full generality. In plane-parallel or spherically symmetric case, we compute the kk or rr component (and we drop $\frac{1}{c}$ from the definition and call it "K", as in K_{rr} or K_{zz})

Let's now look again at the transfer equation to see what simplifications we can make under what circumstances.

Radiative Transfer Eq'n in 1D (planar or spherical)

$$\mu \frac{\partial I}{\partial z} = \eta - \chi I \qquad \text{O.D.E.}$$

$$\left[\mu\frac{\partial}{\partial r} + \frac{1}{r}(1-\mu^2)^{1/2}\frac{\partial}{\partial \theta}\right] I = \eta - \chi I \qquad \text{P.D.E. (even in 1-D!)}$$

$$\frac{1}{c}\frac{\partial I}{\partial t} + \mu\frac{\partial I}{\partial z} = \eta - \chi I \qquad \text{P.D.E.}$$

$$\frac{1}{c}\frac{\partial I(\mu,\nu)}{\partial t}+\mu\frac{\partial I(\mu,\nu)}{\partial z}=\eta-\kappa I(\mu,\nu)-\sigma I(\mu,\nu)+\int P(\mu'\to\mu,\nu'\to\nu)\,I(\mu',\nu')\,d\mu'd\nu'$$

$$\text{where } \sigma=\int P(\mu\to\mu';\nu\to\nu')\,d\mu'\,d\nu'$$

$\Rightarrow$ integro-differential eq'n.

To simplify, make assumptions or educated guesses about

① timescales; time dependent or independent

② photon mean free path

③ isotropy of the radiation field

I. Timescales

Compare the timescale of the propagation of radiation

$t_R \sim l/c$ free-streaming

or

$t_R \sim \lambda_p/c$ λ_p = photon mean free path

to other timescales in the problem.

$$\left.\begin{array}{ll} t_f \sim l/v & \text{fluid flow} \\ t_I - & \text{ionization} \\ t_{th} - & \text{thermal} \end{array}\right\} \text{timescale over which the fluid moves; its ionization state or thermal properties change.}$$

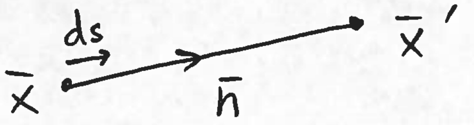
When $t_R \ll t_f, t_I, t_{th}$, ignore the time dependence of the radiation field and treat the problem as quasi-static.

Exceptions:

1) If you need to follow the propagation of the rad. front
2) $\lambda_p \gg \ell$ (optically thin; near boundary; free-streaming)
3) global coupling, time retardation is important

Then you may not be able to ignore time dependence.

II. Remember the mean free path



$\ell = |\bar{x}' - \bar{x}|$

$$\tau_\nu = \int_0^\ell \chi_\nu \, ds \qquad [\chi] = \text{cm}^{-1}$$

$$\lambda_\nu \equiv \frac{1}{\chi_\nu} \quad (\text{cm}) \quad \text{photon m.f.p.}$$

optical depth $\tau = \frac{\ell}{\lambda}$ number of interactions (on average)

optically thin $\longleftrightarrow$ optically thick

free-streaming limit diffusion limit

What happens in the diffusion limit? In random walk

$$d_*^2 = N d^2 \qquad d_* = \sqrt{N}\, d$$

$\uparrow$ net displacement $\uparrow$ mean free path

We want $d_* = \ell$

$d = \lambda_{mfp}$

# of interactions $N = \frac{\ell^2}{\lambda_{mfp}^2}$

Diffusion time:

$$t_d \sim \left(\frac{\ell}{\lambda_{mfp}}\right)^2 \left(\frac{\lambda_p}{c}\right) = \frac{\ell^2}{c\lambda_{mfp}} = \frac{\ell}{c} \cdot \frac{\ell}{\lambda_{mfp}} = \tau \cdot \underset{\substack{\uparrow \\ \text{light} \\ \text{crossing} \\ \text{time}}}{t_{cross}}$$

$t_d \approx \tau \cdot t_{cross}$

Suppose $\frac{\lambda_p}{\ell} \ll 1$ very optically thick

and $t_d \ll t_f$ static

$\Rightarrow$ static diffusion limit.

III. Look at isotropy of I. This is connected to, but not exclusively determined by, the mean free path.

Ex: Interior of a star

$\frac{\lambda_p}{\ell} \ll 1$; properties of the plasma (η, χ, B etc) isotropic

then $I(\mu) = I_0$

$$J = \frac{1}{4\pi} \int I \, d\mu \, d\phi = I_0$$

$$\bar{H} = \frac{1}{4\pi}\int I\mu\, d\mu\, d\phi = 0$$

$$K_{rr} = \frac{1}{4\pi}\int I\mu^2 d\mu\, d\phi = \frac{1}{2}\int_{-1}^{1} I\mu^2 d\mu = \frac{1}{2}\cdot\frac{2}{3}\cdot I_0 = \frac{J}{3}$$

in isotropy, $K/J = \frac{1}{3}$

In reality, a small deviation from isotropy

$$I_\nu(\tau,\mu) = a_\nu(\tau) + b_\nu(\tau)\mu \qquad \text{linear in } \mu.$$

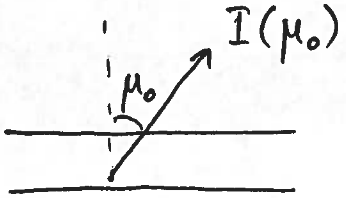
$$J = \frac{1}{2}\int_{-1}^{1} I\, d\mu = a$$

$$\bar{H} = \frac{1}{2}\int_{-1}^{1} I\mu\, d\mu = \frac{b}{3} \qquad (\text{in the } \hat{r} \text{ direction})$$

$$K_{rr} = \frac{1}{2}\int_{-1}^{1} I\mu^2 d\mu = \frac{a}{3}$$

$\Rightarrow$ $K = \frac{J}{3}$ but $H \neq 0$; net flux.

<u>Ex:</u> Free-streaming limit



$$I(\mu) = I_0\,\delta(\mu-\mu_0)$$

$$J = \frac{1}{2}\int I_0\,\delta(\mu-\mu_0)\, d\mu = \frac{I_0}{2}$$

$$H = \frac{1}{2}\int I_0\mu\,\delta(\mu-\mu_0)\, d\mu = \frac{\mu_0 I_0}{2}$$

$$K = J$$

Moments of the Transfer Eq'n and the Eddington factors

We want to solve

$$\frac{1}{c}\frac{\partial I}{\partial t} + \mu \frac{\partial I}{\partial z} = \eta - \chi I \qquad \text{(ignore } \nu \text{ dependence)}$$

Zeroth moment:

$$\frac{1}{2}\int \left[\frac{1}{c}\frac{\partial I}{\partial t} + \mu \frac{\partial I}{\partial z}\right] d\mu = \frac{1}{2}\int [\eta - \chi I]\, d\mu$$

$$\frac{1}{c}\frac{\partial J}{\partial t} + \frac{\partial H}{\partial z} = \eta - \chi J$$

becomes an equation for energy density (that also involves the flux H)

First moment:

$$\frac{1}{2}\int \left[\frac{1}{c}\frac{\partial I}{\partial t}\mu + \frac{\partial I}{\partial z}\mu^2\right] d\mu = \frac{1}{2}\int [\eta - \chi I]\mu\, d\mu$$

$$\frac{1}{c}\frac{\partial H}{\partial t} + \frac{\partial K}{\partial z} = 0 - \chi I$$

becomes the momentum equation (that also involves K)

An approach to solving the moment eq'ns: Variable Eddington factor:

$$f = \frac{K}{J}$$

fitting formula that goes from $\underset{\text{small } \lambda_{mfp}}{\frac{1}{3}} \longrightarrow \underset{\text{large } \lambda_{mfp}}{1}$

(or can calculate f approximately).