# HW2 ISM, Radiative transfer and processes

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## 1.

If the source function can be approximated as:

$$S(\tau) \approx S(\tau_*) + S'(\tau_*)(\tau - \tau_*) + \frac{1}{2}S''(\tau_*)(\tau - \tau_*)^2 \tag{1}$$

Then the full general solution for the radiative transfer can be writen as:

$$I_\nu(\tau_1, \mu) = I_\nu(\tau_2, \mu)e^{-(\tau_2-\tau_1)/\mu} + \frac{1}{\mu}\int_{\tau_1}^{\tau_2} S_\nu(\tau')e^{-(\tau'-\tau_1)/\mu}d\tau' \tag{2}$$

$$I_\nu(\tau_1, \mu) = I_\nu(\tau_2, \mu)e^{-(\tau_2-\tau_1)/\mu} + \frac{1}{\mu}\int_{\tau_1}^{\tau_2} e^{-(\tau'-\tau_1)/\mu}d\tau' \left[S(\tau_*) + S'(\tau_*)(\tau' - \tau_*) + \frac{1}{2}S''(\tau_*)(\tau' - \tau_*)^2\right] d\tau' \tag{3}$$

Now I treat the 3 integrals separately:

The first integral involving the term $S(\tau_*)$ is:

$$\int_{\tau_1}^{\tau_2} e^{-(\tau'-\tau_1)/\mu} S(\tau_*)d\tau' = S(\tau_*)\int_{\tau_1}^{\tau_2} e^{-(\tau'-\tau_1)/\mu}d\tau' = S(\tau_*)(-\mu)\left[e^{-(\tau_2-\tau_1)/\mu} - 1\right] \tag{4}$$

The second integral corresponding to the $S'(\tau_*)$ term:

$$\int_{\tau_1}^{\tau_2} e^{-(\tau'-\tau_1)/\mu} S'(\tau_*)(\tau' - \tau_*)d\tau' = S'(\tau_*)\int_{\tau_1}^{\tau_2} e^{-(\tau'-\tau_1)/\mu}(\tau' - \tau_*)d\tau' = -\mu S'(\tau_*)(\mu - \tau_* + \tau')e^{-(\tau'-\tau_1)/\mu}$$

$$= -\mu S'(\tau_*)\left[(\mu - \tau_* + \tau_2)e^{-(\tau_2-\tau_1)/\mu} - (\mu - \tau_* + \tau_1)\right] \tag{5}$$

Finally the third integral correspoding to the $S''(\tau_*)$ term is:

$$\frac{1}{2}\int_{\tau_1}^{\tau_2} e^{-(\tau'-\tau_1)/\mu} S''(\tau_*)(\tau' - \tau_*)^2 d\tau' = \frac{S''(\tau_*)}{2}\int_{\tau_1}^{\tau_2} e^{-(\tau'-\tau_1)/\mu}(\tau' - \tau_*)^2 d\tau'$$

$$= \frac{S''(\tau_*)}{2}\left[-\mu(\tau_*^2 - 2\tau_*(\mu + \tau') + 2\mu^2 + 2\mu\tau' + \tau'^2)e^{-(\tau'-\tau_1)/\mu}\right]$$

$$= \frac{-\mu S''(\tau_*)}{2}\left[(\tau_*^2 - 2\tau_*(\mu + \tau_2) + 2\mu^2 + 2\mu\tau_2 + \tau_2^2)e^{-(\tau_2-\tau_1)/\mu} - (\tau_*^2 - 2\tau_*(\mu + \tau_1) + 2\mu^2 + 2\mu\tau_1 + \tau_1^2)\right] \tag{6}$$

If $\tau_* = \mu$ and $\tau_1 = 0$ the terms that would be affected are Eq.5 & Eq.6 correspondly.

$$= -\mu S'(\tau_*)\left[\tau_2 e^{-(\tau_2-\tau_1)/\mu} - \tau_1\right] = -\mu S'(\tau_*)\tau_2 e^{-\tau_2/\mu} \approx 0 \tag{7}$$

$$= \frac{-\mu S''(\tau_*)}{2} \left[ (\mu^2 + \tau_2^2) e^{-(\tau_2 - \tau_1)/\mu} - (\mu^2 + \tau_1^2) \right] = \frac{-\mu S''(\tau_*)}{2} (-\mu^2) = \frac{\mu^3 S''(\tau_*)}{2} \tag{8}$$

# 2.

$$I_\nu = I_\nu(0) e^{-\tau_\nu} + B_\nu(T) \left[ 1 - e^{-\tau_\nu} \right] \tag{9}$$

When the source is observed trough the nebula:

$$I_\nu = I_\nu(T_s) e^{-\tau_\nu} + I_\nu(T_n) \left[ 1 - e^{-\tau_\nu} \right] \tag{10}$$

$$I_\nu = I_\nu(T_s) \tag{11}$$

$$I_\nu(T_s) = I_\nu(T_s) e^{-\tau_\nu} + I_\nu(T_n) \left[ 1 - e^{-\tau_\nu} \right] \tag{12}$$

$$I_\nu(T_s)(1 - e^{-\tau_\nu}) = I_\nu(T_n) \left[ 1 - e^{-\tau_\nu} \right] \tag{13}$$