Motivation:

The aim of this project is to study oscillations in thin disks, in particular the dynamics of the oscillations and the what are the causes of the excitations. A deep understanding of the physics behind these oscillations would allow to study effect as the quasi-periodic oscillations. Also variations in the X-ray flux from black holes and AGNs might be explained. To this aim we follow mainly the review article by **Kato 2001**.

In accretion disks the main forces are the gravitational force and the centrifugal force. When these two forces are in equilibrium the disk is stable. The gravitational force is the one from the central object of the accretion disk.

The excitation mechanisms of disk oscillations are: XXX, XXX, XXX, and viscous processes. Viscosity is the major source of heating in the disk, also the azimuthal force caused by viscosity produces angular momentum transport in the radial direction. The first one can be seen as a thermal process while the second one is a dynamical process.

Accretion disks are collisional and non-selfgravitating.

1 Hydrodynamics equations

The basic equations are those derived from Boltzmann equations:

The continuity equation is:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u_i) = 0 \tag{1}$$

Which could be expressed in cylindrical coordinates as:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{r \partial r}(r \rho v_r) + \frac{\partial}{r \partial \phi}(\rho v_\phi) + \frac{\partial}{\partial z}(\rho v_z) = 0 \tag{2}$$

Where v_r, v_ϕ and v_z are the components of the velocity in cylindrical coordinates.

The momentum equation is:

$$\frac{\partial u_j}{\partial t} + u_j \frac{\partial u_i}{\partial x_i} = -a_j - \frac{1}{\rho} \frac{\partial \psi_{ij}}{\partial x_i} \tag{3}$$

The dispersion relation of disks:

$$\tilde{\omega} = \omega - m\Omega \tag{4}$$

Epicyclic frequency:

$$\kappa^2 = 2\Omega \left(2\Omega + r \frac{d\Omega}{dr} \right) \tag{5}$$

Dispersion relation:

$$(\tilde{\omega}^2 - \kappa^2)(\tilde{\omega}^2 - n\Omega_k^2) = \tilde{\omega}^2 c_s^2 k_r^2 \tag{6}$$

To understand this dispersion relation it is better to study the limit cases:
In the case of oscillations in the plane if the disk $n = 0$ Eq.3 reduced to:

$$\tilde{\omega}^2 = \kappa^2 + k_r^2 c_s^2 \tag{7}$$

This condition is known as **inertial-acoustic** waves and corresponds to the oscillations of a fluid element that was displaced in the radial direction, the oscillations arise to the resorting forces that brings back the fluid to the initial position. The oscillation frequency is the epicyclic frequency $\kappa(r)$ first term in the right part of equation 4. While the second term corresponds to acoustic oscillations due to the restoring force from compressible fluids.

Now in the long-wavelength limit ($k_r = 0$) Eq.4 is reduced to:

$$\tilde{\omega}^2 = \kappa^2 \tag{8}$$

$$\tilde{\omega}^2 = n\Omega_K^2 \tag{9}$$

Which corresponds to vertical oscillations in the disk, due to a perturbation of a fluid element in the vertical direction. The vertical component of the gravitational force in the restore force that returns the fluid element to the plane of the disk. The frequency of this oscillations is Ω_K.

This two oscillations are coupled in the form $(\tilde{\omega}^2 - \kappa^2)(\tilde{\omega}^2 - n\Omega_k^2)$ in the dispersion relation Eq.3. Vertical oscillations induce perturbations in the radial direction due to the inhomogeneities in the disk. The coupling is stronger when the radia wavelength is shorter and the acoustic speed is faster.

The solutions for Eq.3 are:

$$\tilde{\omega}^2 = \frac{(n\Omega_k^2 + \kappa^2 + c_s^2 K_r^2) \pm \sqrt{(} - 4\kappa^2 n\Omega_k^2)}{2} \tag{10}$$

The modes with the $+$ sign in Eq.7 are called **p-modes** while the solutions with $-$ are called **g-modes**

1.1 Relativistic effects on the Dispersion Relation

When general relativistic effects are taking into account

$$\kappa^2 = \frac{GM}{r^3}\left(1 + \frac{a}{\hat{r}^{3/2}}\right)^{-2}\left(1 - \frac{6}{\hat{r}} + \frac{8a}{\hat{r}^{3/2}} - \frac{3a^2}{\hat{r}^2}\right) \tag{11}$$

Where a is a dimensionless parameter specifying the amount of angular momentum of the central object if $a = 0$ the object is rotating and if $a = 1$ the central object have the maximum rotation (the case of extreme Kerr).

$$\hat{r} = \frac{r}{GM/c^2} \tag{12}$$

$$\Omega^2 = \Omega_K^2 \left(1 - \frac{4a}{\hat{r}^{3/2} + \dfrac{3a^2}{\hat{r}^2}} \right) \tag{13}$$

$$\Omega_K^2 = \frac{GM}{r^3} \left[1 + \frac{a}{(8\hat{r}^3)^{1/2}} \right]^{-1} \tag{14}$$

$$(\tilde{\omega}^2 + \kappa^2)(\tilde{\omega}^2 - n\Omega^2) = \tilde{\omega}^2 c_s^2 k_r^2 \tag{15}$$