HW2 ISM, Radiative transfer and processes

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September 30, 2015

1.

If the source function can be approximated as:

$$S(\tau) \approx S(\tau_*) + S'(\tau_*)(\tau - \tau_*) + \frac{1}{2}S''(\tau_*)(\tau - \tau_*)^2 \tag{1}$$

Then the full general solution for the radiative transfer can be writen as:

$$I_\nu(\tau_1, \mu) = I_\nu(\tau_2, \mu)e^{-(\tau_2-\tau_1)/\mu} + \frac{1}{\mu}\int_{\tau_1}^{\tau_2} S_\nu(\tau')e^{-(\tau'-\tau_1)/\mu}d\tau' \tag{2}$$

$$I_\nu(\tau_1, \mu) = I_\nu(\tau_2, \mu)e^{-(\tau_2-\tau_1)/\mu} + \frac{1}{\mu}\int_{\tau_1}^{\tau_2} e^{-(\tau'-\tau_1)/\mu}d\tau' \left[S(\tau_*) + S'(\tau_*)(\tau' - \tau_*) + \frac{1}{2}S''(\tau_*)(\tau' - \tau_*)^2\right] d\tau' \tag{3}$$

Now I treat the 3 integrals separately:

The first integral involving the term $S(\tau_*)$ is:

$$\int_{\tau_1}^{\tau_2} e^{-(\tau'-\tau_1)/\mu}S(\tau_*)d\tau' = S(\tau_*)\int_{\tau_1}^{\tau_2} e^{-(\tau'-\tau_1)/\mu}d\tau' = S(\tau_*)(-\mu)\left[e^{-(\tau_2-\tau_1)/\mu} - 1\right] \tag{4}$$

The second integral corresponding to the $S'(\tau_*)$ term:

$$\int_{\tau_1}^{\tau_2} e^{-(\tau'-\tau_1)/\mu}S'(\tau_*)(\tau'-\tau_*)d\tau' = S'(\tau_*)\int_{\tau_1}^{\tau_2} e^{-(\tau'-\tau_1)/\mu}(\tau'-\tau_*)d\tau' = -\mu S'(\tau_*)(\mu-\tau_*+\tau')e^{-(\tau'-\tau_1)/\mu}$$

$$= -\mu S'(\tau_*)\left[(\mu - \tau_* + \tau_2)e^{-(\tau_2-\tau_1)/\mu} - (\mu - \tau_* + \tau_1)\right] \tag{5}$$

Finally the third integral correspoding to the $S''(\tau_*)$ term is:

$$\frac{1}{2}\int_{\tau_1}^{\tau_2} e^{-(\tau'-\tau_1)/\mu}S''(\tau_*)(\tau' - \tau_*)^2d\tau' = \frac{S''(\tau_*)}{2}\int_{\tau_1}^{\tau_2} e^{-(\tau'-\tau_1)/\mu}(\tau' - \tau_*)^2d\tau'$$

$$= \frac{S''(\tau_*)}{2}\left[-\mu(\tau_*^2 - 2\tau_*(\mu + \tau') + 2\mu^2 + 2\mu\tau' + \tau'^2)e^{-(\tau'-\tau_1)/\mu}\right]$$

$$= \frac{-\mu S''(\tau_*)}{2}\left[(\tau_*^2 - 2\tau_*(\mu + \tau_2) + 2\mu^2 + 2\mu\tau_2 + \tau_2^2)e^{-(\tau_2-\tau_1)/\mu} - (\tau_*^2 - 2\tau_*(\mu + \tau_1) + 2\mu^2 + 2\mu\tau_1 + \tau_1^2)\right] \tag{6}$$

If $\tau_* = \mu$ and $\tau_1 = 0$ the terms that would be affected are Eq.5 & Eq.6 correspondly.

$$= -\mu S'(\tau_*)\left[\tau_2 e^{-(\tau_2-\tau_1)/\mu} - \tau_1\right] = -\mu S'(\tau_*)\tau_2 e^{-\tau_2/\mu} \approx 0 \tag{7}$$

$$= \frac{-\mu S''(\tau_*)}{2}\left[(\mu^2+\tau_2^2)e^{-(\tau_2-\tau_1)/\mu}-(\mu^2+\tau_1^2)\right] = \frac{-\mu S''(\tau_*)}{2}(-\mu^2) = \frac{\mu^3 S''(\tau_*)}{2} \quad (8)$$

2.

$$I_\nu = I_\nu(0)e^{-\tau_\nu} + B_\nu(T)\left[1-e^{-\tau_\nu}\right] \quad (9)$$

When the source is observed trough the nebula:

$$I_{\nu,1} = I_\nu(T_s)e^{-\tau_\nu} + I_\nu(T_n)\left[1-e^{-\tau_\nu}\right] \quad (10)$$

$$I_{\nu,2} = I_\nu(T_n)\left[1-e^{-\tau_\nu}\right] \quad (11)$$

Substracting Eq.10 & Eq.11

$$I_{\nu,1} - I_{\nu,2} = I_\nu(T_s)e^{-\tau_\nu} \quad (12)$$

$$-\tau_\nu = Ln\left(\frac{I_{\nu,1}-I_{\nu,2}}{I_\nu(T_s)}\right) \quad (13)$$

3.

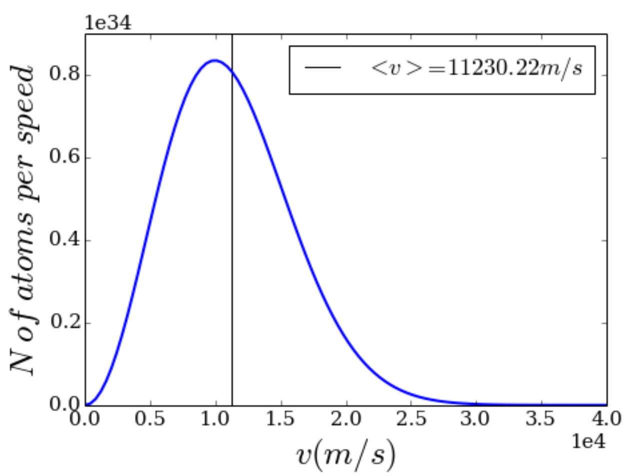

Figure 1: Velocity distribution for 10^{38}Hydrogen atoms in the solar photosphere

1.

Figure 1 show the velocity distribution of thw 10^{38} atoms in the solar photosphere. The black vertical line shows the typical speed of a Hydrogen atom, which was computed as follows:

$$< v >= 2\int_0^\infty \left(\frac{m}{2\pi KT}\right)^{3/2} 4\pi v^3 e^{-mv^2/KT} \quad (14)$$

$$< v >= 8\pi\left(\frac{m}{2\pi KT}\right)^{3/2}\left(\frac{KT}{m}\right)^4 = 2\left(\frac{2}{\pi}\right)^{1/2}\left(\frac{KT}{m}\right)^{1/2} \quad (15)$$

$$< v >= 11203.22 m/s \quad (16)$$

2.

The number of photons (N1) within a 1% of $< v >$ can be computed with the CDF as follows:

$$N1 = erf(v/\sqrt{2}a) - \sqrt{\frac{2}{\pi}}\frac{ve^{-v^2/2a^2}}{a}\Bigg|_{0.99<v>}^{1.01<v>} = 9.07 \times 10^{35} \quad (17)$$

3.

The Doppler shifht due to the speed $< v >$ would be:

$$\frac{\nu}{\nu_0} = (1+ < v > /c) = 1.000037 \quad (18)$$

4.

4.

To show that $h\nu << KT$ for HII regions we select the extreme case that corresponds to $\lambda = 1mm$. Using the fact the typical temperature of a HII region is 10^4K we found that:

$$h\nu = 1.98 \times 10^{28} J \quad (19)$$

$$KT_{HII} = 1.38 \times 10^{-19} J \quad (20)$$

Then for radio observations it is valid to work in the Rayleigh-Jeans limit.

$$B_\nu(T) = \frac{2\nu^2}{c^2}KT \quad (21)$$

$$T_b = \frac{c^2}{2\nu^2 K}I_\nu \quad (22)$$

$$I_\nu = T_\nu(0)e^{-\tau_\nu} + B_\nu(T)(1 - e^{-\tau_\nu}) \quad (23)$$

$$\frac{2\nu^2 K}{c^2}T_\nu = \frac{2\nu K}{c^2}T_b(0)e^{-\tau_\nu} + \frac{2\nu^2 K}{c^2}T(1 - e^{-\tau_\nu}) \quad (24)$$

$$T_\nu = T_b(0)e^{-\tau_\nu} + T(1 - e^{-\tau_\nu}) \quad (25)$$