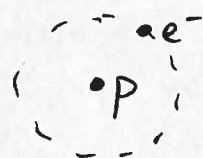


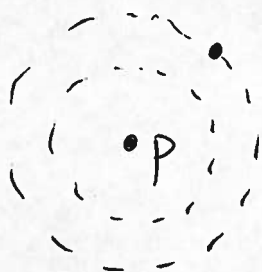
## State of the gas in the ISM

Statistical equilibrium, Dissociation, Ionization, LTE, Saha Equation, Excitation Processes. [Ref: Mihalas Ch 5]

The state of the gas: Distribution of the particles over all available bound and free energy levels.  $\rightarrow$  occupation numbers of these levels.



ground state H



$E_n = H$  excited to the  $n^{\text{th}}$  level



$H_2$ : molecular hydrogen

The state of the material depends upon the radiation field, which in turn depends on the occupation numbers ( $\eta, \chi$  depend on these). Need simultaneous, self-consistent sol'n of the radiative transfer and statistical equilibrium eq'ns.

This is indeed what you do for non-equilibrium ISM, IGM problems.

Alternative: Make assumptions about the state of the gas.

"ideal" "equilibrium - LTE"; in more detail "atomic"  
"ionized"  
etc.

## Back of the Envelope Approach to Dissociation & Ionization

1. It is generally easier to dissociate a molecule than to ionize an atom. It requires less  $E$ , so first the molecules go.
2.  $E_n \propto \frac{1}{n^2}$ , so the lower the electronic state you're trying to ionize, the more energy you need.
3. H is most abundant, so it is most frequently needed (and used to determine phase of ISM)

$$\Delta E_{\text{ionization of H from ground state}} = 13.6 \text{ eV} = 912 \text{ \AA}^\circ$$

Lyman limit.  
 $n=1 \rightarrow n=\infty$

[ Lyman- $\alpha$  :  $n=2 \rightarrow n=1$   
Lyman- $\beta$  :  $n=3 \rightarrow n=1$  etc.

H- $\alpha$  :  $n=3 \rightarrow n=2$   
H- $\beta$  :  $n=4 \rightarrow n=2$  etc. } a.k.a. Balmer series

For Helium: He I from ground state: 24.6 eV  
He II from ground state: 54.4 eV

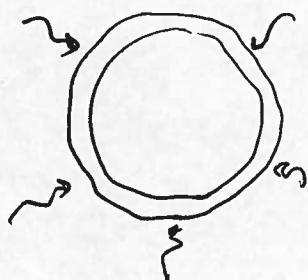
4. Use QM to predict ratios of lines as a function of  $n$ ,  $T$ , and  $F_\nu$ .

# Temperature and Ionization Regimes in the ISM

(3)

## (A) Very dense region

- T low, N high
- gas is mostly molecular
- no dissociating radiation: photon E low  
extinction very high
- no significant photoionization

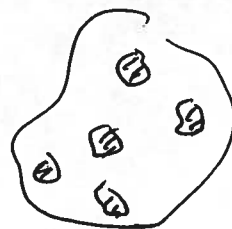


can have thin skin of atomic (dissociated) gas from external  $\gamma$  bath.

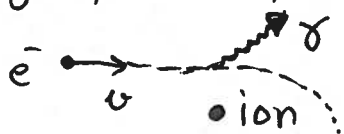
## (B) Photon dominated region

- The physics is governed by the interaction of photons with the gas.
- The gas is primarily ionized: HII and numerous other ionized elements, due to high energy photons
- There is evidence that HII regions are clumpy

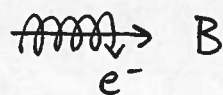
$$\text{filling factor} = \frac{\text{ionized volume}}{\text{total volume}}$$



- Dust is present  
scatters and reddens the star light
- Significant free-free (Bremsstrahlung) emission

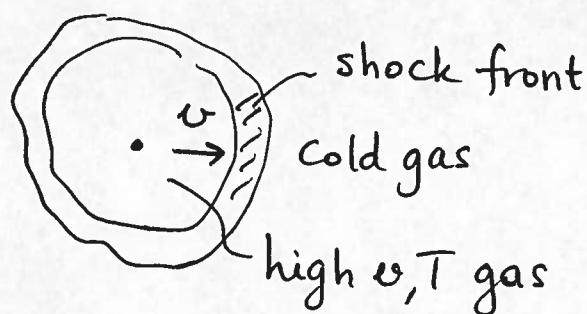


- Also observe synchrotron emission



- Observe recombination lines
- These regions may typically not be in equilibrium  
ionization levels change with  $n, T, F_\nu$ , and time.

### C Supernova Remnants



$$v \gtrsim 1000 \text{ km s}^{-1}$$

$$T \gtrsim 10^6 \text{ K}$$

- ionization by collisions as well as by high  $E$  photons
- H and He ionized, as well as heavier elements
- production of X-rays

### D General ISM

- UV photons from stars partially ionize the ISM
- Dispersion measure (DM) from pulsars sensitive to free electrons; provide the best estimate of the ionization state.

# Back to Statistical Equilibrium.

## (Local) Thermodynamic Equilibrium

The state of the gas specified by  $T$  and particle density  $n$ .  
If there is radiation, it is also described by the same  $T$ .  
Sufficient collisions, emission, absorption to bring particles to equilibrium velocity distribution.

"Local": Even if there are large scale gradients in  $T$  and  $n$  (which there usually are), assume that locally, equilibrium is reached.

- One  $T$  applies to the velocity distribution function of atoms, ions, and electrons (Maxwell-Boltzmann)
- to the distribution of atoms and ions over all states (Saha)
- the distribution of thermal emission (Planck)

## I. The Maxwellian Velocity Distribution (non-relativistic)

$$f(v) dv_x dv_y dv_z = \left( \frac{m}{2\pi kT} \right)^{3/2} e^{-m(v_x^2 + v_y^2 + v_z^2)/2kT} dv_x dv_y dv_z$$

or

$$f(v) dv = \left( \frac{m}{2\pi kT} \right)^{3/2} e^{-\frac{mv^2}{2kT}} 4\pi v^2 dv$$

spherical coordinates in  $\vec{v}$  space

## II. The Boltzmann Equation

(6)

Fraction of atoms in excited level  $j$  of atoms  $X^{(r)}$

$$\frac{n_j(X^{(r)})}{n(X^{(r)})} = \frac{g_{rj} e^{-E_{rj}/kT}}{\sum_k g_{rk} e^{-E_{rk}/kT}}$$

define the denominator as the partition function for ion  $X^{(r)}$

$$f_r = \sum_k g_{rk} e^{-E_{rk}/kT}$$

$g_{rk}$  are the statistical weights (e.g., 2 for electrons,  $2J+1$  m-states for atomic levels, etc).

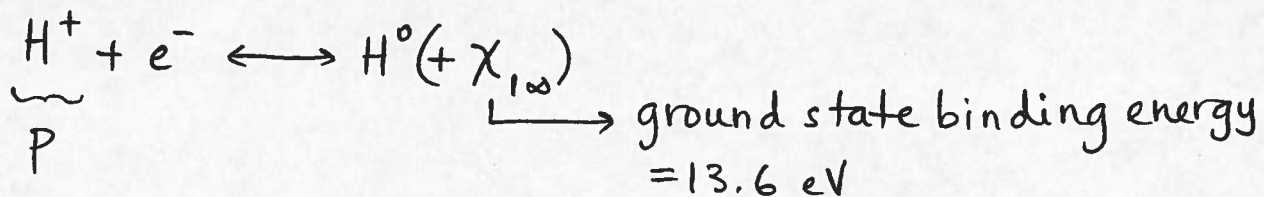
Alternatively, specify the relative population of levels

$$\frac{n_j(X^{(r)})}{n_k(X^{(r)})} = \frac{g_{rj}}{g_{rk}} e^{-\underbrace{(E_{rj} - E_{rk})/kT}_{\equiv \chi_{rjk} = h\nu_{jk}}}$$

$E_{rj}$  etc are defined with respect to the ground state.  
r defines the ionization state (neutral, singly ionized, etc.)  
→ though, of course, no ground state needs to be specified for relative populations; it only depends on  $\Delta E$ .

### III. Saha Ionization Equation

(7)



Integrating the distribution functions over speeds (or momenta) yields the number density of electrons, the protons, and the H atom:

$$n_e = \frac{2(2\pi m_e kT)^{3/2}}{h^3} e^{\mu_e/kT}$$

$$n_p = \frac{(2\pi m_p kT)^{3/2}}{h^3} e^{\mu_p/kT}$$

$$n_H = \frac{2(2\pi(m_p+m_e)kT)^{3/2}}{h^3} e^{\mu_H/kT} e^{\chi/kT}$$

Note the statistical weights: 2 for the H atom, and I put 2 only in the  $e^-$  density, combined for the proton and the electron spins  $\uparrow\uparrow$  and  $\uparrow\downarrow$ .

$\mu \equiv$  chemical potential.

Saha eq'n: Form the ratio

$$\frac{n_p n_e}{n_H} = \left( \frac{2\pi m_e kT}{h^2} \right)^{3/2} e^{(\mu_e + \mu_p - \mu_H)/kT} e^{-\chi/kT}$$

In equilibrium:  $\mu_e + \mu_p - \mu_H = 0$

$$\left( \frac{2\pi m_e k}{h^2} \right)^{3/2} = 2.4 \times 10^{15}$$



Got rid of  $\mu$ 's, but still 3 unknowns, 1 eq'n.

(8)

Need additional constraints:  $n_p = n_e$  (for Hydrogen)

and we can also usually specify the total number density

$$n = n_H + n_p$$

Define ionized fraction  $y \equiv \frac{n_e}{n} = \frac{n_p}{n}$  and combine these equations to solve for  $y$ :

$$\frac{y^2}{1-y} = \frac{4 \times 10^{-9}}{\rho} T^{3/2} e^{-1.6 \times 10^5 / T} \quad \text{in cgs units.}$$

For all atoms: A quick guesstimate of the ionized fraction comes from realizing that  $e^{-X/kT}$  is the dominant factor in the Saha eq'n. Compare  $X_{ij}$  with  $kT$  to determine if the gas is mostly ionized or neutral - can also do the same for the dissociation energy.

Pitfall: We have not accounted for pressure ionization, which dominates at high densities. In essence, what we write down for individual free atom distribution function is no longer accurate at high density, small particle separation and continuum states need to be accounted for.

One more warning: Here, we considered ionization from ground state only. In reality, sum over all excited states.



(9)

What does LTE mean for the radiation?

- The radiation is also specified by a single temperature
- in equilibrium with the gas
- The source function has a specific form

$$S_\nu = B_\nu(T) = \frac{\eta_\nu}{\alpha_\nu}$$

- The Formal Solution for thermal radiation from a single uniform  $T$  in the emitting region:

$$I_\nu = I_\nu(0)e^{-\tau_\nu} + B_\nu(T)[1 - e^{-\tau_\nu}]$$

which means that for  $I_\nu = B_\nu$  (as opposed to  $S_\nu = B_\nu$ ), one needs  $\tau \gg 1$  and a single  $T$  to describe the source function. [If there is a significant  $T$  gradient,  $I_\nu$  is distorted away from a BB even if  $S_\nu = B_\nu$ ].

- The Rayleigh-Jeans limit and Brightness Temperature

$$B_\nu(T_B) = \frac{2h\nu^3}{c^2} [\exp(h\nu/kT_B) - 1]^{-1} = I_\nu(T_B)$$

set this equal to the intensity

$T_B$  = brightness temperature

$T$  of an equivalent BB at that freq  $\nu$ , even if the whole spectrum is not  $B_\nu(T)$ .

In the Rayleigh-Jeans limit:

$$h\nu \ll kT \Rightarrow I_\nu^{RJ}(T) = \frac{2\nu^2}{c^2} kT$$

$$I^{RJ}(T) \propto T$$

(10)

In this limit (thermal radiation + R-J), can write the formal solution as

$$T_B = T_B(0) e^{-\tau_\nu} + T(1 - e^{-\tau_\nu})$$

again for a single uniform  $T$  in the emitting region,

$$T_B \rightarrow \text{true } T \text{ of the material when } \tau \gg 1.$$

We'll return to thermal radiation when we talk about specific absorption/emission processes.

## Excitation Processes

What excites atoms into higher energy states? (or molecules)  
States can refer to electronic, rotational, vibrational, spin, magnetic energy/quantum levels or combinations of these.

- Collisions (neutral-neutral and neutral-ion)
- Interactions with electrons
- Interactions with photons

## Neutral-neutral interactions

- short range forces
- interaction cross section is essentially the geometric cross section: actual overlap

$$\sigma_{nn} \approx \pi (r_1^2 + r_2^2) \sim 10^{-15} \text{ cm}^2$$

$\uparrow \quad \uparrow$   
 atom 1   atom 2

- characteristic distance: mean free path

$$\text{m.f.p.} = \ell_c \approx (n_n \sigma_{nn})^{-1}$$

$$[\ell_c] = [\text{cm}^{-3} \cdot \text{cm}^2]^{-1} = \text{cm}$$

$$\ell_c \approx \frac{10^{15}}{n_H} \text{ cm}$$

$$\text{if } n_H = 1, \ell_c = 3 \times 10^{-4} \text{ pc}$$

- characteristic velocity - assuming thermal distr.

$$\frac{3}{2} m_n v^2 = kT$$

$$v = \left( \frac{2kT}{3m_n} \right)^{1/2}$$

- characteristic collision timescale

$$t_{nn} \approx \frac{\ell_c}{v} = (n_n \sigma_{nn})^{-1} \left( \frac{2kT}{3m_n} \right)^{-1/2}$$

$$\frac{1}{t_{nn}} \approx 7 \times 10^{-12} n_n T^{1/2} s^{-1}$$

$$n_n = 1$$

$$T = 80 \text{ K}$$

$$t_{nn} \approx 500 \text{ yr}$$

$$n_n = 10^4$$

$$T = 10 \text{ K}$$

$$t_{nn} \approx 1.5 \text{ mo}$$

$$n_n = 1$$

$$T = 10^4 \text{ K}$$

$$t_{nn} \approx 45 \text{ yrs.}$$

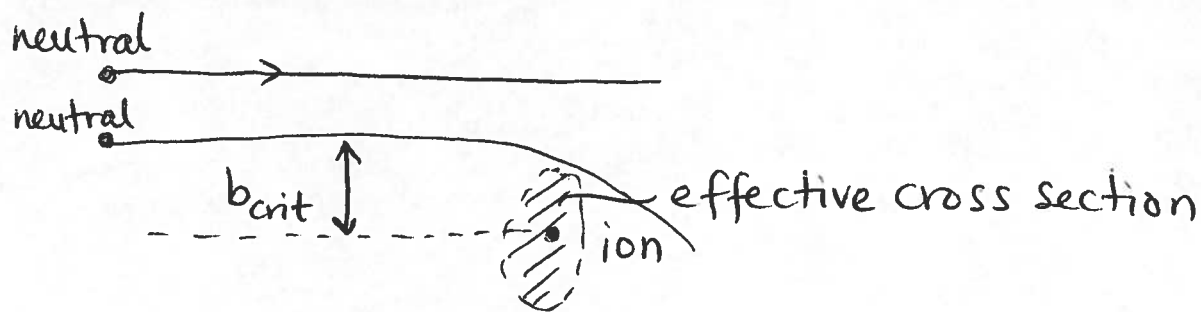
$\Rightarrow$  density  $n$  is more important than  $T$  in determining collision rates.

### Collision Rates

$$\frac{\text{Rate}}{\text{vol}} = n_n n_n \sigma_{nn} \langle v \rangle$$

If one species of neutrals, coll rate  $\propto n^2$ . Can also be two different species.

### Ion-Neutral Collisions and Collision Rate



$b$  = impact parameter

Interaction energy:

$$U(r) = \vec{E} \cdot \underset{\uparrow}{\vec{p}} = \frac{Ze}{r^2} \left( \propto \frac{Ze}{r^2} \right) = \propto \frac{Z^2 e^2}{r^4}$$

E field induced in neutral atom  
due to its polarizability

Compare this potential energy with the kinetic energy of the system:

$$\mu = \text{reduced mass} = \frac{m_1 m_2}{m_1 + m_2}$$

$$\text{if } \frac{\propto Z^2 e^2}{r^4} \ll \frac{\mu v^2}{2} \quad \text{weak interaction}$$

$$\frac{\propto Z^2 e^2}{r^4} \gg \frac{\mu v^2}{2} \quad \text{strong interaction}$$

Equate them to find the critical impact parameter  $b_{\text{crit}}$

$$\frac{\propto Z^2 e^2}{r^4} = \frac{\mu v^2}{2}$$

Effective collision cross section:

$$\sigma_{\text{ni}} \cong \pi b_{\text{crit}}^2 = \pi Z e \left( \frac{2 \propto}{\mu} \right)^{1/2} \frac{1}{v}$$

Can do this in a more detailed way by integrating/averaging over impact parameters.

Rate coefficient:

$$\frac{\text{Rate}}{\text{vol}} = n_i n_n \underbrace{\langle \sigma_{ni} v \rangle}_{\text{indicates avg over } b \text{ and } v} \text{ cm}^{-3} \text{ s}^{-1}$$

$\equiv k = \text{"rate coefficient"}$

$$k \approx \pi z e \left( \frac{2\alpha}{\mu} \right)^{1/2} \approx 2 \times 10^{-9} \text{ cm}^3 \text{ s}^{-1}$$

ion-neutral scattering rate coefficient.

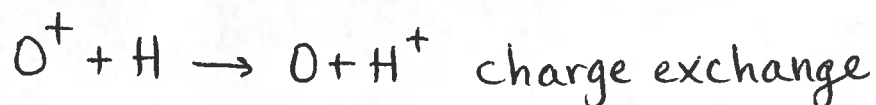
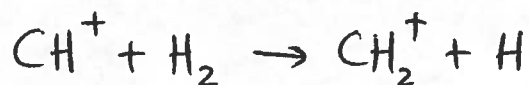
Ex: If  $n_i = n_n = 1$ , Coll rate =  $2 \times 10^{-9} \text{ cm}^{-3} \text{ s}^{-1}$  for ion-neutral

$$\text{Coll timescale } \frac{1}{\text{rate}} \approx 15 \text{ yrs.}$$

whereas  $t_{nn} = 500 \text{ yrs}$  for  $n_n = 1$

$\Rightarrow$  ion-neutral collisions 30x more frequent.

Examples of ion-neutral collisions:



We'll do ion-ion collisions when we talk about Bremsstrahlung.