The dispersion relation of disks:

$$\tilde{\omega} = \omega - m\Omega \tag{1}$$

Epicyclic frequency:

$$\kappa^2 = 2\Omega \left(2\Omega + r\frac{d\Omega}{dr} \right) \tag{2}$$

Dispersion relation:

$$(\tilde{\omega}^2 - \kappa^2)(\tilde{\omega}^2 - n\Omega_k^2) = \tilde{\omega}^2 c_s^2 k_r^2 \tag{3}$$

To understand this dispersion relation it is better to study the limit cases:
In the case of oscillations in the plane if the disk $n = 0$ Eq.**??** reduced to:

$$\tilde{\omega}^2 = \kappa^2 + k_r^2 c_s^2 \tag{4}$$

This condition is known as **inertial-acoustic** waves and corresponds to the oscillations of a fluid element that was displaced in the radial direction, the oscillations arise to the resorting forces that brings back the fluid to the initial position. The oscillation frequency is the epicyclic frequency $\kappa(r)$ first term in the right part of equation **??**. While the second term corresponds to acoustic oscillations due to the restoring force from compressible fluids.

Now in the long-wavelength limit ($k_r = 0$) Eq.**??** is reduced to:

$$\tilde{\omega}^2 = \kappa^2 \tag{5}$$

$$\tilde{\omega}^2 = n\Omega_K^2 \tag{6}$$

Which corresponds to vertical oscillations in the disk, due to a perturbation of a fluid element in the vertical direction. The vertical component of the gravitational force in the restore force that returns the fluid element to the plane of the disk. The frequency of this oscillations is Ω_K.

This two oscillations are coupled in the form $(\tilde{\omega}^2 - \kappa^2)(\tilde{\omega}^2 - n\Omega_k^2)$ in the dispersion relation Eq.**??**. Vertical oscillations induce perturbations in the radial direction due to the inhomogeneities in the disk. The coupling is stronger when the radia wavelength is shorter and the acoustic speed is faster.

The solutions for Eq.**??** are:

$$\tilde{\omega}^2 = \frac{(n\Omega_k^2 + \kappa^2 + c_s^2 K_r^2) \pm \sqrt{(} - 4\kappa^2 n\Omega_k^2)}{2} \tag{7}$$

The modes with the $+$ sign in Eq.**??** are called **p-modes** while the solutions with $-$ are called **g-modes**

0.1 Relativistic effects on the Dispersion Relation

$$\kappa^2 = \frac{GM}{r^3} \left(1 + \frac{a}{\hat{r}^{3/2}} \right)^{-2} \left(1 - \frac{6}{\hat{r}} + \frac{8a}{\hat{r}^{3/2}} - \frac{3a^2}{\hat{r}^2} \right) \tag{8}$$

$$\hat{r} = \frac{r}{GM/c^2} \tag{9}$$

$$\Omega^2 = \Omega_K^2 \left(1 - \frac{4a}{\hat{r}^{3/2} + \frac{3a^2}{\hat{r}^2}} \right) \tag{10}$$

$$\Omega_K^2 = \frac{GM}{r^3}\left[1 + \frac{a}{(8\hat{r}^3)^{1/2}}\right]^{-1} \tag{11}$$

$$(\tilde{\omega}^2 + \kappa^2)(\tilde{\omega}^2 - n\Omega^2) = \tilde{\omega}^2 c_s^2 k_r^2 \tag{12}$$