ASTR 515 – ISM and Star Formation – Fall 2015
Assignment II on Radiative Transfer and Processes
Due Date: Friday October 16

1. **Theoretical Spectrum of the Galactic Center Black Hole**
   In this problem, we are going to consider radiatively inefficient accretion flows onto black holes, i.e., the case when the density is so low that the accreting gas cannot cool by emitting radiation. For the following, assume that the gas consists only of hydrogen, and that it is fully ionized everywhere. Some constants you may (or may not) need:
   $m_p = 1.7 \times 10^{-24}$ g.
   $m_e = 9.1 \times 10^{-28}$ g.
   $c = 9 \times 10^{10}$ cm s^{-1}.
   $e = 4.8 \times 10^{-10}$ esu.
   $k = 1.4 \times 10^{-16}$ cm^2 g s^{-2} K^{-1}.
   $h = 6.6 \times 10^{-27}$ cm^2 g s^{-1}.

   a. Assuming that 50% of the gravitational energy is converted into thermal energy of the particles accreting onto a black hole with mass M, find the temperature of the particles at $5R_{\rm S}$, where the Schwarzschild radius is given by $R_{\rm S} = 2GM/c^2$.

   b. Turbulence in accretion flows generates a magnetic field such that the magnetic energy density can be comparable to the thermal energy density. Using such arguments, we can estimate the magnetic field around the black hole in the center of the Milky Way to be $B \approx 10^4$ G at $5R_{\rm S}$. Assuming that the emission is optically thin, estimate the peak of the synchrotron spectrum emitted from such a flow. *Hint: Use the temperature you found in Part (a) to find an average electron Lorentz factor. Do this in the simplest possible way.* You may also find the expression for the cutoff (or critical) frequency for synchrotron emission useful:
   $$\nu_c = \frac{3}{4\pi}\gamma^3\omega_B \sin\alpha,$$
   where
   $$\omega_B = eB/\gamma mc$$
   and α is the pitch angle.

   c. Using a characteristic scale of $\sim 5R_S$ for the synchrotron emitting region in the inner accretion flow, determine whether the assumption that the emission is optically thin valid. If not, how would the spectrum be different?

2. **Strömgren sphere.** Consider a pure hydrogen nebula surrounding a hot star of radius R. At some distance r from the star, the ionization equilibrium equation

$$n_{H^0} \int_{\nu 0}^{\infty} \frac{4\pi J_\nu}{h\nu} a_\nu d\nu = n_p n_e \alpha(T) \tag{1}$$

becomes

$$\frac{n_{H^0} R^2}{r^2} \int_{\nu 0}^{\infty} \frac{\pi F_\nu(R)}{h\nu} a_\nu e^{-\tau_\nu} d\nu = n_p n_e \alpha(T), \tag{2}$$

where the optical depth is

$$\tau_\nu(r) = \int_0^r n_{H^0}(r') a_\nu dr', \tag{3}$$

and the ionization cross section is approximately given by

$$a_\nu = 6.3 \times 10^{-18} \left(\frac{\nu_0}{\nu}\right)^3 \text{ cm}^2, \tag{4}$$

where ν_0 corresponds to the ionization threshold for hydrogen at 912 A. We will assume that the star emits like a blackbody. Define the ionization fraction x such that $n_{H^+} = x n_H$, $n_{H0} = (1-x) n_H$.

(a) Integrate numerically equation (2) to plot the ionization fraction as a function of the distance from the star in parsecs. Repeat the exercise for these two cases:

i) $T_{\text{eff}} = 45,000$ K, $R/R_\odot = 11$

ii) $T_{\text{eff}} = 40,000$ K, $R/R_\odot = 20$

Assume that $T_e = 10,000$ K in the nebular gas so that $\alpha = 2.59 \times 10^{-13}$ cm^{3}s^{-1}, and $n_H = 10$ cm^{-3} throughout the nebula. You may assume that $a_\nu = 6.3 \times 10^{-18}$ cm^2 in the calculation of the optical depth.

(b) Calculate the number of ionizing photons for the two black bodies using

$$Q(H^0) = \int_{\nu 0}^{\infty} \frac{L_\nu}{h\nu} d\nu. \tag{5}$$

(c) Calculate the Strömgren radius of both stars using $Q(H^0)$ from part (b).

3. **Curve of Growth** In this problem, we are going to use the curve of growth to estimate the column density of sodium atoms in the solar photosphere. Figure 1 shows a section of the spectrum near the 5890 Å line of neutral sodium, which is produced when an electron jumps from the ground state 3s^1 to the excited state 3p^1. The oscillator strength for this transition is $f = 0.645$. Figure 2 shows a curve of growth for the sun.

1. Estimate the equivalent width of this sodium absorption line.
2. Find its location on the curve of growth.
3. Calculate the column density of sodium atoms in the ground state. Note that the units of your answer should be atoms cm^{-2}.
4. Now estimate the ratio of sodium atoms in the ground state to the the sodium atoms

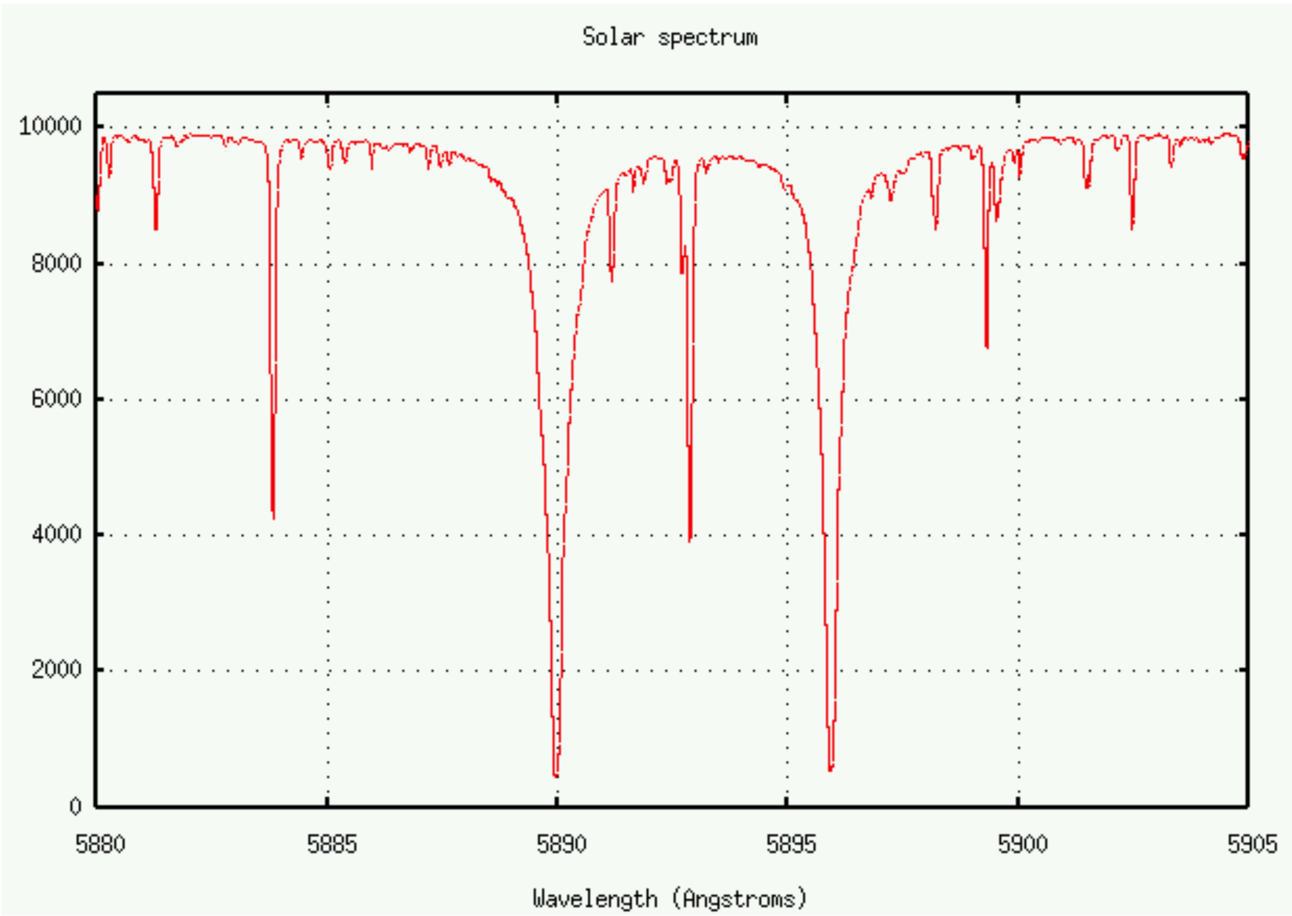

Figure 1: A section of the solar spectrum near the 5890 $\mathring{A}$ line of neutral sodium.

in the excited state of this transition. Remember that the wavelength of this transition is 5890 $\mathring{A}$, so you may use this to calculate the energy difference between these two levels. In addition, you may assume that the solar photosphere is at $T = 6000$ K.
One more hint: You will need the statistical weights of the sodium atom in the 3s1 and 3p1 states. Identify the allowed m and s quantum numbers in each state and multiply them to get the total degeneracy of each state, which will give you the statistical weight of each state.
5. Now estimate the ratio of neutral sodium atoms to ionized sodium atoms. The pressure at the depth where the line is formed is $P \simeq 200$ dyne cm^{-2}.
Hints: The ionization energy for sodium is about 5.1 eV.
You may consider only singly ionized sodium as the ionized state. (In other words, consider NaI and NaII only in the Saha equation).
You may assume the ideal gas law to find the total particle density.
6. Compute the total column density of sodium atoms in the solar photosphere.

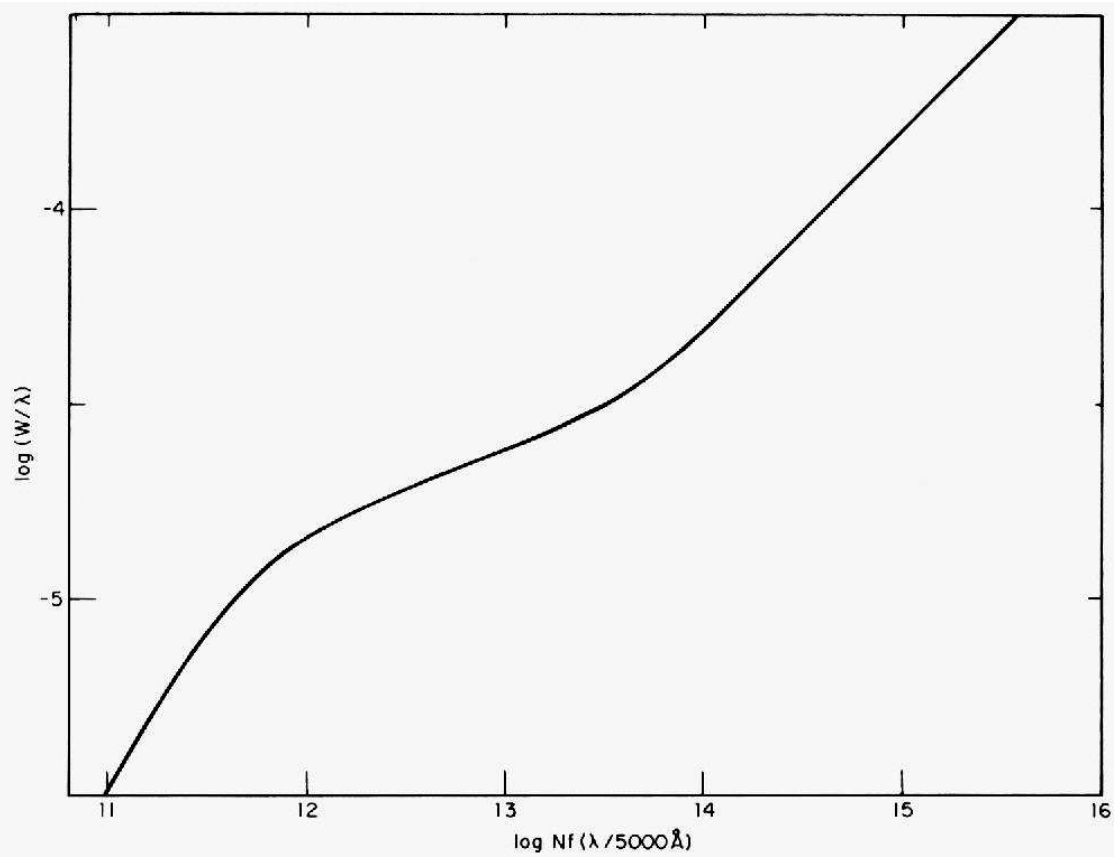

Figure 9.22 A general curve of growth for the Sun. (Figure from Aller, *Atoms, Stars, and Nebulae*, Revised Edition, Harvard University Press, Cambridge, MA, 1971.)

Figure 2: A curve of growth for the sun.