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# Absolute measurements of trigonometric parallaxes with astrometric satellites

**V.V. Makarov**

Copenhagen University Observatory, Juliane Maries Vej 30, DK-2100 Copenhagen Ø, Denmark

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**Abstract.** The concept of absolute trigonometric parallax measurements is formulated in strict mathematical terms, in connection with the widely discussed plans for a future astrometric space mission. The parallaxes determined for a large number of stars spread all over the sky are absolute if the expected error (accidental or systematic) of the zero–point, as well as other large–scale distortions are orders of magnitude smaller than the average random error for a given star. It is shown by a very general mathematical consideration that the Hipparcos strategy of one–dimensional angular measurements and revolving mode of scanning the sky provides absolute determinations of parallaxes, as far as accidental errors are concerned. As demonstrated by means of a spherical harmonics technique, an even higher absoluteness of parallaxes would be achieved in a future astrometric mission with a larger basic angle between the two viewing directions than that of Hipparcos ($58°$), and a larger revolving scanning angle between the Sun direction and the spin axis of the satellite.

**Key words:** space vehicles – stars: distances – astrometry

## 1. Introduction

One of the prime motivations for carrying out the Hipparcos project was the possibility of absolute trigonometric parallax measurements for $100\,000$ stars. Apart from the tremendous increase in the number of stars with directly measured distances, a radical improvement of the astrometric quality of this type of data was envisaged.

The important difference between the traditional method of parallax determination from on–ground observations and the one of Hipparcos, was discussed in some detail by Perryman et al. (1995). The difference comes from the fact that in the on–ground method, the parallax for a target star can be determined only relatively to reference stars within a photographic plate or CCD frame, the average parallax of which is not known or too uncertain. If neglected, the average parallax of the background stars tends to diminish the estimated value for the target star. It is therefore a common practice to estimate the average reference parallax by independent means, e.g. photometrically, which may introduce systematic errors.

The fundamental advantage of the Hipparcos method stems from the capability of the instrument to directly measure large angular distances. Hence,

- the expected parallax displacements are quite different in the two widely separated viewing directions of the telescope in each observation (one of the displacements can even occasionally be nil, as depicted in Fig. 1 of the cited paper),
- each star is linked by many observations to a great number of other objects with various instantaneous parallax shifts,
- all observations are eventually tied up into a coherent system of parallaxes in a global solution, thus no a priori knowledge of parallaxes is required when all target stars serve as reference objects to each other.

Comprehensible as it may seem, the concept of absolute parallax measurements lacks a mathematical framework and means of analysis. What is the measure of the absoluteness of parallaxes? How is it related to the notions of astrometric precision and accuracy? Under which conditions is this desirable quality achieved? The present work is an attempt at answering these questions.

The revival of the interest in the principles of space astrometry is prompted by the development of a few projects, posing more ambitious aims than Hipparcos. Most of them are however based on the same observational principle, which proved to be highly efficient in the Hipparcos mission. The GAIA project (Global Astrometric Interferometer for Astrophysics, Lindegren & Perryman 1996), obtained a broad support of the astronomical community due to excellent prospects in various fields of astrophysics and astrometry and is presently under consideration by ESA. Some smaller national projects, such as Struve (Ilin et al., 1997) and DIVA (Röser et al., 1997) should also be mentioned.

## 2. Basic parameters

The observations of a scanning astrometry satellite of the Hipparcos type are performed by a telescope or perhaps two separate telescopes providing two viewing directions separated by a ‘basic angle’ on the sky. The basic angle is designed to be very

*Send offprint requests to*: V.V.Makarov@astro.ku.dk

constant in time, yet precise calibration from the instrument's own observations should also be used. The satellite spins slowly, with a period of revolution of a few hours, so that the two fields scan approximately along a great circle. Accurate astrometric observation is performed only in the direction of the great circle, not perpendicularly to the circle.

The spin axis of the satellite moves in a predetermined way across the sky, so that the entire sky is covered rather uniformly by scans, and each patch of the sky is observed at many different position angles. A specific mode or schedule of scanning is called a scanning law in the Hipparcos literature. The limiting requirement for that kind of satellites is that the spin axis should never move too far from the sun direction, for the sake of constant power supply with solar panels and to limit straylight from the sun. The Hipparcos revolving scanning law is characterized by an ever constant angle, $\xi = 43°$, between the sun direction and the spin axis, the latter rotating around the former at a frequency $K = 6.4$ revolutions per year. More details about the Hipparcos scanning law can be found in ESA (1997), Vol. 2, p. 145. Although a few more sophisticated scanning schemes were studied for the Struve project (Yershov et al., 1995), no significant advantages were found versus the revolving law of Hipparcos. Nonetheless, it leaves some room for further improvement of the astrometric performance by adjusting the basic parameters $\xi$ and $K$. The choice of $\xi$ is in fact decisive, while $K$ is selected in order to provide a uniform sky coverage by scans. A small variation of $K$ hardly changes the pattern of parallax errors at large scales, therefore this parameter is not considered much in the following.

The importance of the choice of basic angle ($\gamma = 58°$ for Hipparcos) was realized at the early stages of the Hipparcos mission development (see Perryman & Vaghi, 1989). The problem is closely related to the instability of one–dimensional solutions to the problem of angular coordinate determinations along a great circle with a fixed standard angle. The instability has been known empirically by astrometrists from the classical task of divided circle calibration. It comes into full effect in the baseline method of Hipparcos reductions, where an intermediate step of 'great circle reductions' is used (Lindegren & Kovalevsky, 1989) on the way from one–dimensional angular measurements in the scanning direction to a coherent system of astrometric parameters on the sphere. In this approach, the set of observations from about five consecutive revolutions is first treated in a great circle reduction providing an estimate of each star's 'abscissa', i.e., the projection of its position onto a fixed 'reference great circle'. The solution provides in addition several instrument parameters such as the basic angle, field rotation and scale value, which are assumed to remain constant during an observation set. In a later stage of the astrometric data processing the abscissae are combined to yield, among other quantities, the celestial coordinates $\lambda$ and $\beta$, proper motions and parallaxes of the stars.

An analytical approach to the error propagation estimation for the star abscissae in the Fourier spectral domain was found by Høyer et al. (1981). Using this technique, it was shown by Makarov (1992) and Makarov et al. (1995) that the instability of the great circle solution manifests itself in the Fourier domain

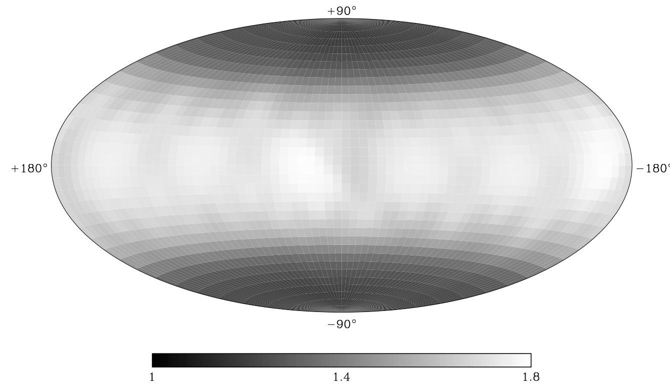


**Fig. 1.** Simulated distribution of relative parallax errors over the sky with 256 spherical harmonics, for the Hipparcos scanning law ($\gamma = 58°, \xi = 43°, K = 6.4$). Two years of observations were simulated.

as a rank deficiency which depends critically on the value of the basic angle $\gamma$. Wild amplifications of abscissae errors at certain harmonics take place, when $\gamma$ is a simple fraction of $360°$ (like $\frac{1}{4}$ or $\frac{2}{5}$), which badly affects the overall astrometric precision (cf. Fig. 2.1 in Perryman & Vaghi, 1989). Using two incommensurable basic angles for a future satellite was recommended as a possible remedy.

It is widely believed however, that a more straightforward method of solution is to be implemented for a future astrometric mission, which is called a 'global solution'. The complete set of observations during the mission can be thought of as a great number of arcs, connecting pairs of programme stars. The length of each arc is close to the value of the basic angle, within the width of the field of view of the telescope. The core of the global solution idea is that this set of one-dimensional measurements along great circles can be directly tied up into a rigid system of astrometric parameters on the sphere, by a least squares method, for example. The method was tested in numerical simulations by Bucciarelli et al. (1991), and it was adopted as an alternative for the Hipparcos data reductions, attempted by the NDAC data reduction consortium (ESA 1997, Vol. 3, pp. 490–494) but never used in the actual catalogue construction. It should be noted that the consideration of the global solution method in the present paper is vastly simplified compared to any practical implementation. For example, we do not consider at all the intervening tasks of the instrumental calibration and attitude parameters determination.

Bucciarelli et al. demonstrated that the global solution is slightly superior to the baseline method w.r.t. the final precision of the resulting astrometric catalogue. Another possible, and very important advantage was indicated by Makarov (1997). Solving the coordinate problem in terms of spherical orthogonal functions, it was shown that the global solution is free of the rank deficiency problem, i.e. it seems to be well conditioned at any basic angle in the range of $30°$ to $150°$. The better conditioning of the global solution with respect to the great circle solution, if confirmed by direct numerical simulations, might also account for the better astrometric quality mentioned above.

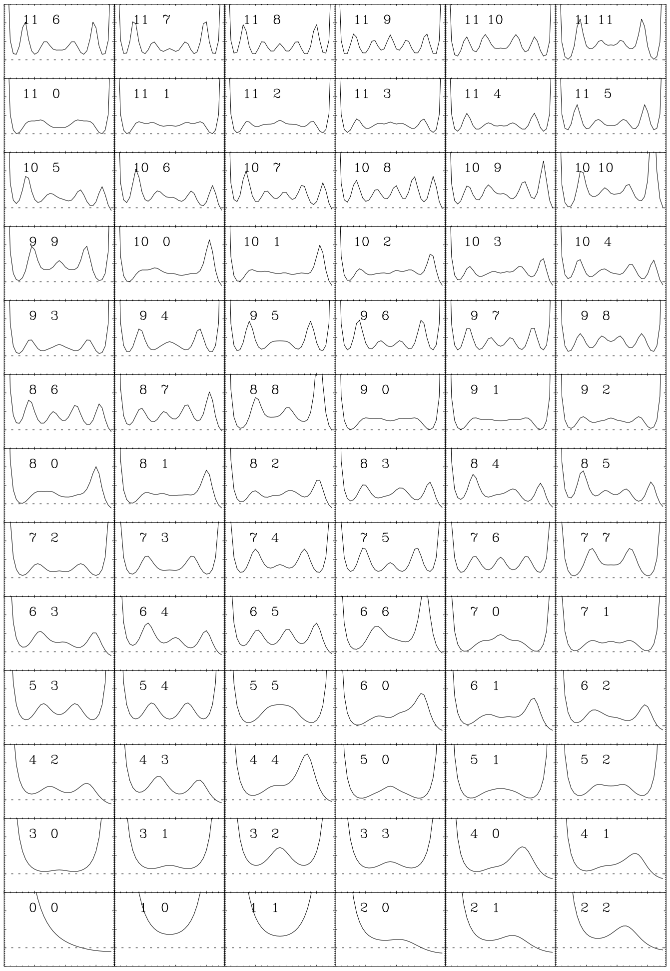


**Fig. 2.** Power spectrum coefficients $Q_j$ for the $\sin$-terms of $\Delta\pi$ for $m \neq 0$ and $\cos$-terms for $m = 0$ (cf. Eq. 2) as functions of basic angle. The range of each plot is $0^\circ$ to $180^\circ$ horizontally for $\gamma$ and $0$ to $4$ vertically. The indices of spherical functions are given inside each plot.

## 3. Spherical orthogonal functions

The (small) errors of parallaxes are represented in terms of spherical functions by the expansion,

$$\Delta\pi = \sum_{j=1}^{J} a_j\, Y_j(\lambda, \beta) + \epsilon, \tag{1}$$

where $Y_j$ are the spherical orthogonal functions, $\Delta\pi$ is the difference between the observed parallax $\pi_o$ and the true parallax $\pi_t$, and $\epsilon$ is the high-frequency component of the noise. The spherical functions are related to associated Legendre polynomials by the equations

$$\begin{aligned} Y_{nms} &= R_{nm} P_n^m(\sin\beta)\ \sin m\lambda, \qquad m = 1, \ldots, n \\ Y_{nmc} &= R_{nm} P_n^m(\sin\beta)\ \cos m\lambda, \qquad m = 0, \ldots, n \\ &\qquad\qquad\qquad\qquad\qquad\qquad\quad n = 0, \ldots, N \end{aligned} \tag{2}$$

where $R_{nm}$ are normalization coefficients (see e.g. Arfken & Weber, 1995). The index $j$ counts all different spherical orthogonal functions from 1 to $J = (N+1)^2$.

In this application, we have to consider a discrete set of points (target stars) on the sphere, for which the corrections $\Delta\pi$ to the input catalogue parallaxes should be determined. The condition of orthogonality in the discrete case is

$$\sum_{i\in\Omega} Y_j(\lambda_i, \beta_i) Y_l(\lambda_i, \beta_i) = I \cdot \delta_{jl} \tag{3}$$

where $\Omega$ is the discrete set of points, and $I$ is the total number of points. This is approximately fulfilled provided the stars are uniformly distributed over the sky.

When the number of terms $J$ in Eq. (1) is close to the number of stars $I$, the representation by the spherical functions is fairly complete, and the high-frequency term $\epsilon$ is relatively small. The average standard error over the sky can then be evaluated as

$$\bar{\sigma}_\pi^2 \approx \sum_{j=1}^{J} \mathrm{var}[a_j]. \tag{4}$$

The covariances do not come into this expression, thanks to the orthogonality of the basic functions. For any individual star, however, the covariances do play a significant role, as the variance of its parallax is

$$\sigma_i^2 \approx \mathbf{Y}_i' \mathbf{Cov} \mathbf{Y}_i, \tag{5}$$

where $\mathbf{Y}_i$ is the column vector of $Y_j(\lambda_i, \beta_i), j = 1, ..., J$, and **Cov** is the full covariance matrix of the $a_j$ coefficients. The covariance matrix describes how the average standard error is distributed over the sky. Such a distribution can be far from uniform due to the inevitable observational irregularities of the scanning law, as it actually is the case with Hipparcos (see Fig. 3.2.24 in ESA 1997, Vol. 1).

The numbers $Q_j = L \cdot \mathrm{var}[a_j]/\sigma_0^2$, where $\sigma_0^2$ is the variance of an elementary observation and $L$ is the total number of observations, can be interpreted as a power spectrum of the random error, in analogy with the Fourier power spectrum of star abscissae errors, derived in Makarov et al. (1995). The power spectrum $Q_j$ describes how the parallax variance is divided among the spherical harmonics.

## 4. Normal equations

Under the above-mentioned simplifying assumptions, an observation equation follows directly from the geometry of an elementary observation. The fact is disregarded here, that not just a single pair of stars, but rather all stars within the two simultaneous fields of view are bridged in one observation. In fact, the distance between two simultaneously observed stars can differ a little from the value of basic angle $\gamma$. Besides, there is always some averaging of photon noise errors on the scale of the field of view size, that brings about additional smoothing at high-order accidental harmonics. It is intuitively clear, and can be proven by numerical simulations that the overall astrometric performance benefits from a wide field of view. These effects are, however, not relevant to the purpose of this paper.

In the small-angle approximation, a linearized observation equation for a given pair of objects $(p, q)$ is written as

$$\Delta d \;=\; \Delta\lambda_p^* \sin\phi_p - \Delta\lambda_q^* \sin\phi_q + \Delta\beta_p \cos\phi_p -$$

$$\Delta\beta_q \cos\phi_q, \tag{6}$$

where $\Delta d$ is the correction to the pre-calculated distance $d \approx \gamma$, and $\phi_r$, $r = p, q$ is the position angle of the scan direction. Low precision formulae can be used for the coordinate change due to parallax

$$\begin{aligned} \Delta\lambda\cos\beta &= -\Delta\pi\sin(\lambda-\lambda_\odot) \\ \Delta\beta &= -\Delta\pi\sin\beta\cos(\lambda-\lambda_\odot). \end{aligned} \tag{7}$$

Using Eq. (1), the observation Eq. (6) can be rewritten in terms of spherical functions. The $I$ unknown corrections to parallaxes are then replaced by $J$ unknown coefficients $a_j$, provided $J \le I$. A set of $L$ elementary observation equations can be solved by the least squares method. The normal matrix $\mathbf{N}$ for the unknowns $a_j$ can be evaluated for small $J$.

When only uncorrelated accidental errors are assumed for the measured corrections $\Delta d$, the normal matrix shows how such errors translate into *random* distortions of the global parallax system. Although entirely accidental by their origin, the distortions appear as systematics in the only realization of the catalogue.

## 5. Condition number

Let $\mathbf{x}$ and $\tilde{\mathbf{x}} = \mathbf{x} + \delta\mathbf{x}$ be the least squares solutions to the problems

$$\begin{aligned} \mathbf{A}\mathbf{x} &\approx \mathbf{d} \\ \mathbf{A}\tilde{\mathbf{x}} &\approx \tilde{\mathbf{d}}, \end{aligned}$$

where $\tilde{\mathbf{d}} = \mathbf{d} + \mathbf{e}$, and $\mathbf{e}$ is a (small) perturbation to the right–hand part $\mathbf{d}$. According to the perturbation theory,

$$\frac{\|\delta\mathbf{x}\|}{\|\mathbf{x}\|} \le k\,\frac{\|\mathbf{e}\|}{\|\mathbf{A}\|\,\|\mathbf{x}\|} \le k\,\frac{\|\mathbf{e}\|}{\|\mathbf{d}\|}, \tag{8}$$

where $k = \mathrm{cond}(\mathbf{A})$ is the condition number of the design matrix $\mathbf{A}$. The condition number is defined as

$$k = \frac{\max(\mathbf{s})}{\min(\mathbf{s})}, \tag{9}$$

where $\mathbf{s}$ is the vector of the singular values $s_j$ of the matrix $\mathbf{A}$, $j = 1,\ldots,r$, and the rank $r = J$. The condition number of the normal matrix $\mathbf{N} = \mathbf{A}^{\mathrm{T}}\mathbf{A}$ is $\mathrm{cond}(\mathbf{N}) = k^2$.

Eq. (8) shows the fundamental meaning of the condition number in error propagation studies. The perturbation $\mathbf{e}$ can be regarded as an error of any kind, whether accidental or systematic. The $\|\mathbf{e}\|/\|\mathbf{d}\|$ term can be interpreted as a total quadratic relative error of the right–hand part of the system (observations), then the condition number puts an upper limit on the magnification of the error in the least squares solution. Systems with large condition numbers are called *ill–conditioned*, and their full rank solutions are generally unstable. For a *well–conditioned* system the condition number is close to 1, and then there is no amplification of the relative error in the solution, which is the best one can hope for.

In the present application, we consider $\mathbf{A}$ to be a design matrix for the system of Eqs. (6) translated into the spherical harmonic terms. For an entirely accidental perturbation, one finds then a direct relation between the average variance of parallax error over the sky $\bar{\sigma}_\pi^2$ (Eq. 4) and the observation variance $\sigma_0^2$

$$\bar{\sigma}_\pi^2 \le \frac{k^2}{\|\mathbf{A}\|^2} L\,\sigma_0^2 = L\,\sigma_0^2 \cdot \min(\mathbf{s})^{-2}, \tag{10}$$

where the spectral norm $\|\mathbf{A}\| = \max(\mathbf{s})$.

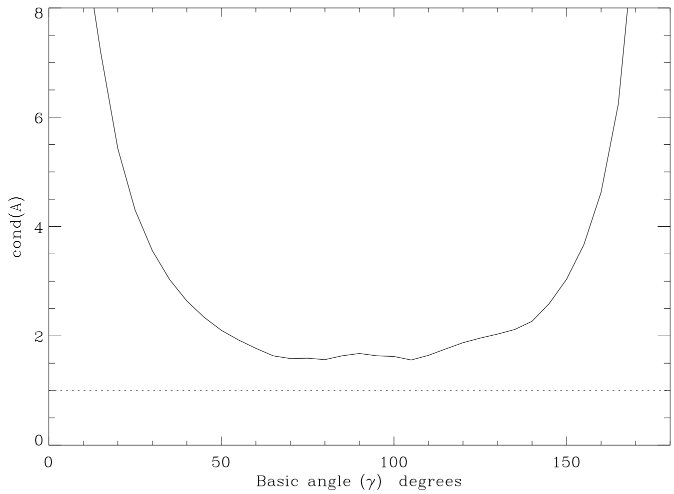


**Fig. 3.** Estimated condition number, obtained in the global solution for parallaxes in the spherical basic functions, as a function of basic angle. Assumed scanning law parameters: $\xi = 55°, K = 4.8$.

## 6. Simulations and results

The normal matrix was evaluated in the following way. For a given set of the basic parameters $(\gamma, \xi, K)$ a two–year long mission was simulated. In each year, the complete path of the Sun along the ecliptic was divided into 200 small intervals, and the instantaneous attitude of the satellite spin axis was computed at the beginning of each interval, as governed by the chosen scanning law. One revolution with a given spin axis attitude was further divided into 36 equidistant along–scan positions, at which observation equations for parallax were computed in terms of the spherical basic functions. Thus, the total number of simulated elementary observations was $L = 14\,400$. Normal matrices could then be calculated for up to the first 256 terms of expansion (1), i.e. up to $Y_{15\,15}$, for different combinations of the basic parameters.

An important issue to be addressed here is to what extent the estimation of $\mathbf{N}$ thus truncated is relevant for the full rank problem. An addition of more unknowns to the design system would change the estimated part of $\mathbf{N}$, because the columns of the design matrix are not orthogonal due to intrinsic properties of the parallactic displacement and the inevitable irregularities of the scanning law. There are, however, good reasons to believe that the approximation is accurate enough when at least a 100 unknowns are solved for. With a set of basic parameters $(\gamma = 58°, \xi = 55°, K = 4.8)$ the normal matrix $\mathbf{N}$ and the condition numbers were computed for $J = 64, 144$ and 256. The change of **N** seems to slow down exponentially with increasing degree of the expansion, being well within 0.5% between 144 and 256. It is expected that the covariances between the higher degree terms begin to oscillate around 0, thus not producing any systematic change in the truncated **N**.

The same can also be concluded from direct simulations of the mean error distribution over the celestial sphere, which provide a very realistic result with $J \geq 144$. We simulated observations with a somewhat simplified Hipparcos scanning law ($\gamma = 58°, \xi = 43°, K = 6.4$) for $J = 256$ and produced an error distribution by means of Eq. (5), using only the available 256 by 256 part of the approximated covariance matrix. The result is shown in Fig. 1, to be compared to Figs. 3.2.24 and 3.2.25 in ESA (1997), Vol. 1. The general distribution is reproduced quite well, in particular the variation with $\beta$ of the error by a factor 1.8. This confirms that the truncated part of the normal matrix represents the large scale accidental errors quite well.

The resulting power spectrum coefficients $Q_j$ for the cos–terms ($m = 0$) and the sin–terms only with $m \neq 0$ are shown in Fig. 2, as functions of $\gamma$. A sun aspect angle $\xi = 55°$ and $K = 4.8$ were assumed, and the expansion (1) was truncated to $J = 144$. The cos–terms with $m \neq 0$ are not shown but they are very similar to the sin–terms of the same order, due to the rather uniform scanning pattern along the ecliptic. All the curves in Fig. 2 are fairly smooth, and this makes an important difference with the case of one–dimensional great–circle solutions. There are no heavily dominating peaks in the spectra, notwithstanding the actually infinitely small field of view in this analysis.

The normal matrix **N** turns out to be fairly sparse when the solution is sought in terms of the spherical basic functions. For instance, all off–diagonal elements vanish, unless the sum of the orders of the corresponding functions $Y_{n_1m_1}$ and $Y_{n_2m_2}$, $m_1 + m_2$ is even, thanks to the perfectly symmetric scanning pattern around the ecliptic. The regular scanning along the ecliptic provides also negligibly small covariances between the $\sin -$ and $\cos -$ terms of tesseral and sectorial functions $Y_{nm}$ ($m \neq 0$).

On the other hand, there are significant non–zero correlations between other terms, in particular between the zonal harmonic coefficients of even order, which are responsible for the distinctive pattern in Fig. 1. The low–order zonal harmonic terms are correlated in such a way, that the average error of parallax is much larger near the ecliptic than at the poles. The non–zero correlations of parallax errors have also some bearing on the problem of star–to–star correlations of parallax errors in compact groups on the sky. If one wants to determine the distance to an open cluster of stars by averaging the $n$ available Hipparcos parallaxes, the resulting error would not at all be $\sqrt{n}$ times smaller than the average individual error. In case of Hipparcos, this effect was evaluated by Lindegren (1989), who found that the average of $n$ stars improves roughly as $n^{-0.35}$. Ideally, however, one would like to know the propagation of errors depending on the size and configuration of the cluster, as well as its ecliptic latitude. The average parallax of an open cluster near the ecliptic should be less precise than that of a similar cluster elsewhere with similar individual errors, due to the presence of the large scale zonal correlations. Theoretically, the error of the average for any cluster could be computed by Eq. (5), if the full covariance matrix was known.

**Table 1.** Condition numbers of the design matrix for different scanning law parameters. The approximation was computed with $J = 256$.

| $\xi$ degrees | cond(**A**) $\gamma = 58°$ | $\gamma = 105°$ |
|---|---|---|
| 35 | 2.58 | 2.03 |
| 45 | 2.28 | 1.76 |
| 55 | 2.00 | 1.56 |
| 65 | 1.76 | 1.55 |

The estimated condition number of the design matrix as a function of basic angle is given in Fig. 3, with $\xi = 55°$ and $J = 256$. The condition number varies slowly and smoothly with basic angle in the range $50°$ to $130°$. Yet, a basic angle of about $75°$ or $105°$ seems to provide a slightly better overall precision, all other conditions being the same. For the Hipparcos set of parameters the estimated condition number is 2.34. It is interesting to note that the smallest singular value in the approximated spectra was always found to correspond to the zero–point term $Y_{00}$.

Further improvement can be achieved by a larger sun aspect angle $\xi$, as is seen from Table 1. This was known from other studies (e.g., Mignard & Falin, 1997) and explained by a better distribution of scan directions in the ecliptic zone and more uniform sky coverage. The improvement can now be quantified by means of the condition number. Generally, by using $\gamma = 105°$ and $\xi = 55°$ instead of the Hipparcos $\gamma = 58°$ and $\xi = 43°$, the absoluteness of parallaxes would improve by roughly 30 percent.

## 7. Conclusions

We have shown, that the expected pattern of large-scale distortions of Hipparcos-like parallax systems can be evaluated without actually solving the observation equations, in the most general global solution. The truncated covariance matrix for the distortions expressed in terms of the spherical basic functions provides a reasonable approximation to the full rank problem, and the condition number of the design matrix can be estimated in this way. The condition number is a natural measure of the *absoluteness* of parallaxes, as it describes the relative accuracy of the parallax zero–point, as well as the other low-order spherical terms in relation to the total parallax error. It proves to be nicely small with the basic parameters and scanning law of Hipparcos, and can still be improved for a future astrometric satellite mission by increasing the spin axis angle to the sun up to $55°$ and by using a larger basic angle between the two viewing directions, e.g., $105°$. In this way, the condition number, and thus the expected quality of the parallax system would be improved by roughly 30 percent, by just changing these two parameters, having all other conditions the same. Surely, a much greater additional improvement is anticipated in the GAIA project due to a more advanced instrumental design.

It appears from this study that the choice of the basic angle for a future astrometric space mission can be somewhat relaxed as far as the random parallax errors are concerned. This might not be the case with systematic errors. Systematic distortions of the parallax system can be caused by e.g. periodic variations of the basic angle within one revolution of the satellite depending on the attitude of the optical system with respect to the sun direction. The present concept is general enough to cope with this problem, too. The estimated condition number puts a limit on the possible increase of such variations in the resulting parallax system. A detailed picture of the resulting distortions can be obtained, given a specific model of systematic instrumental errors.

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