HW2 ISM, Radiative transfer and processes

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1.

If the source function can be approximated as:

$$S(\tau) \approx S(\tau_*) + S'(\tau_*)(\tau - \tau_*) + \frac{1}{2}S''(\tau_*)(\tau - \tau_*)^2 \tag{1}$$

Then the full general solution for the radiative transfer can be writen as:

$$I_\nu(\tau_1, \mu) = I_\nu(\tau_2, \mu)e^{-(\tau_2-\tau_1)/\mu} + \frac{1}{\mu}\int_{\tau_1}^{\tau_2} S_\nu(\tau')e^{-(\tau'-\tau_1)/\mu}d\tau' \tag{2}$$

$$I_\nu(\tau_1, \mu) = I_\nu(\tau_2, \mu)e^{-(\tau_2-\tau_1)/\mu} + \frac{1}{\mu}\int_{\tau_1}^{\tau_2} e^{-(\tau'-\tau_1)/\mu}d\tau' \left[S(\tau_*) + S'(\tau_*)(\tau' - \tau_*) + \frac{1}{2}S''(\tau_*)(\tau' - \tau_*)^2\right] d\tau' \tag{3}$$

Now I treat the 3 integrals separately:

The first integral involving the term $S(\tau_*)$ is:

$$\int_{\tau_1}^{\tau_2} e^{-(\tau'-\tau_1)/\mu}S(\tau_*)d\tau' = S(\tau_*)\int_{\tau_1}^{\tau_2} e^{-(\tau'-\tau_1)/\mu}d\tau' = S(\tau_*)(-\mu)\left[e^{-(\tau_2-\tau_1)/\mu} - 1\right] \tag{4}$$

The second integral corresponding to the $S'(\tau_*)$ term:

$$\int_{\tau_1}^{\tau_2} e^{-(\tau'-\tau_1)/\mu}S'(\tau_*)(\tau'-\tau_*)d\tau' = S'(\tau_*)\int_{\tau_1}^{\tau_2} e^{-(\tau'-\tau_1)/\mu}(\tau'-\tau_*)d\tau' = -\mu S'(\tau_*)(\mu-\tau_*+\tau')e^{-(\tau'-\tau_1)/\mu}$$

$$= -\mu S'(\tau_*)\left[(\mu - \tau_* + \tau_2)e^{-(\tau_2-\tau_1)/\mu} - (\mu - \tau_* + \tau_1)\right] \tag{5}$$

Finally the third integral correspoding to the $S''(\tau_*)$ term is:

$$\frac{1}{2}\int_{\tau_1}^{\tau_2} e^{-(\tau'-\tau_1)/\mu}S''(\tau_*)(\tau' - \tau_*)^2d\tau' = \frac{S''(\tau_*)}{2}\int_{\tau_1}^{\tau_2} e^{-(\tau'-\tau_1)/\mu}(\tau' - \tau_*)^2d\tau'$$

$$= \frac{S''(\tau_*)}{2}\left[-\mu(\tau_*^2 - 2\tau_*(\mu + \tau') + 2\mu^2 + 2\mu\tau' + \tau'^2)e^{-(\tau'-\tau_1)/\mu}\right]$$

$$= \frac{-\mu S''(\tau_*)}{2}\left[(\tau_*^2 - 2\tau_*(\mu + \tau_2) + 2\mu^2 + 2\mu\tau_2 + \tau_2^2)e^{-(\tau_2-\tau_1)/\mu} - (\tau_*^2 - 2\tau_*(\mu + \tau_1) + 2\mu^2 + 2\mu\tau_1 + \tau_1^2)\right] \tag{6}$$

If $\tau_* = \mu$ and $\tau_1 = 0$ the terms that would be affected are Eq.5 & Eq.6 correspondly.

$$= -\mu S'(\tau_*)\left[\tau_2 e^{-(\tau_2-\tau_1)/\mu} - \tau_1\right] = -\mu S'(\tau_*)\tau_2 e^{-\tau_2/\mu} \approx 0 \tag{7}$$

$$= \frac{-\mu S''(\tau_*)}{2}\left[(\mu^2+\tau_2^2)e^{-(\tau_2-\tau_1)/\mu}-(\mu^2+\tau_1^2)\right] = \frac{-\mu S''(\tau_*)}{2}(-\mu^2) = \frac{\mu^3 S''(\tau_*)}{2} \tag{8}$$

2.

$$I_\nu = I_\nu(0)e^{-\tau_\nu} + B_\nu(T)\left[1-e^{-\tau_\nu}\right] \tag{9}$$

When the source is observed trough the nebula:

$$I_{\nu,1} = I_\nu(T_s)e^{-\tau_\nu} + I_\nu(T_n)\left[1-e^{-\tau_\nu}\right] \tag{10}$$

$$I_{\nu,2} = I_\nu(T_n)\left[1-e^{-\tau_\nu}\right] \tag{11}$$

Substracting Eq.10 & Eq.11

$$I_{\nu,1} - I_{\nu,2} = I_\nu(T_s)e^{-\tau_\nu} \tag{12}$$

$$-\tau_\nu = Ln\left(\frac{I_{\nu,1}-I_{\nu,2}}{I_\nu(T_s)}\right) \tag{13}$$

3.

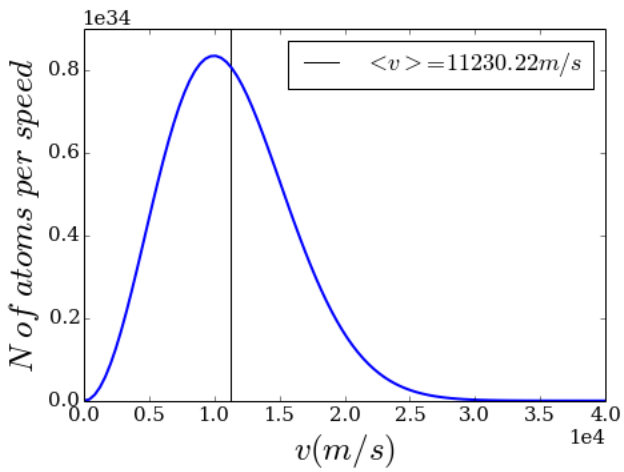

Figure 1: Velocity distribution for 10^{38}Hydrogen atoms in the solar photosphere

1.

Figure 1 show the velocity distribution of thw 10^{38} atoms in the solar photosphere. The black vertical line shows the typical speed of a Hydrogen atom, which was computed as follows:

$$<v>= 2\int_0^\infty \left(\frac{m}{2\pi KT}\right)^{3/2} 4\pi v^3 e^{-mv^2/KT} \tag{14}$$

$$<v>= 8\pi \left(\frac{m}{2\pi KT}\right)^{3/2}\left(\frac{KT}{m}\right)^4 = 2\left(\frac{2}{\pi}\right)^{1/2}\left(\frac{KT}{m}\right)^{1/2} \tag{15}$$

$$<v>= 11203.22 m/s \tag{16}$$

2.

The number of photons (N1) within a 1% of $<v>$ can be computed with the CDF as follows:

$$N1 = erf(v/\sqrt{2}a) - \sqrt{\frac{2}{\pi}}\frac{ve^{-v^2/2a^2}}{a}\Bigg|_{0.99<v>}^{1.01<v>} = 9.07 \times 10^{35} \tag{17}$$

3.

The Doppler shifht due to the speed $< v >$ would be:

$$\frac{\nu}{\nu_0} = (1+ < v > /c) = 1.000037 \tag{18}$$

4.

The velocity that would produce a doppler shift twice as the previous is:

$$v2 = 22406.44 \tag{19}$$

And the number of photons that would be within the 1% of $v2$ are:

$$N2 = erf(2v/\sqrt{2}a) - \sqrt{\frac{2}{\pi}}\frac{2ve^{-v^2/a^2}}{a}\Bigg|_{0.99<2v>}^{1.01<2v>} = 6.48 \times 10^{35} \tag{20}$$

5.

And finally for a doppler shift fourth times:

$$v4 = 44812.88 \tag{21}$$

And the number of photons that would be within the 1% of $v4$ are:

$$N4 = erf(4v/\sqrt{2}a) - \sqrt{\frac{2}{\pi}}\frac{4ve^{-2v^2/a^2}}{a}\Bigg|_{0.99<4v>}^{1.01<4v>} = 2.56 \times 10^{35} \tag{22}$$

4.

To show that $h\nu << KT$ for HII regions we select the extreme case that corresponds to $\lambda = 1mm$. Using the fact the typical temperature of a HII region is 10^4K we found that:

$$h\nu = 1.98 \times 10^{28} J \tag{23}$$

$$KT_{HII} = 1.38 \times 10^{-19} J \tag{24}$$

Then for radio observations it is valid to work in the Rayleigh-Jeans limit.

$$B_\nu(T) = \frac{2\nu^2}{c^2}KT \tag{25}$$

$$T_b = \frac{c^2}{2\nu^2 K}I_\nu \tag{26}$$

$$I_\nu = T_\nu(0)e^{-\tau_\nu} + B_\nu(T)(1 - e^{-\tau_\nu}) \tag{27}$$

$$\frac{2\nu^2 K}{c^2}T_\nu = \frac{2\nu K}{c^2}T_b(0)e^{-\tau_\nu} + \frac{2\nu^2 K}{c^2}T(1 - e^{-\tau_\nu}) \tag{28}$$

$$T_b = T_b(0)e^{-\tau_\nu} + T(1-e^{-\tau_\nu}) \tag{29}$$

Eq.(15.29) of the text says that:

$$\rho\kappa_\nu^{ff} = \sum_i n(Z_i)n_e \left(\frac{2m_e}{3\pi KT}\right)\left[\frac{4\pi Z_i^2 e^6}{3m_e^2 ch\nu^3}\right]\bar{g}_{ff}(\nu)\left(1-e^{-h\nu/KT}\right) \tag{30}$$

For $h\nu << KT$ that we have already shown that it is the case for the radio wavelengths we can expand the last term in Eq.30 as:

$$\left(1-e^{-h\nu/KT}\right) \approx 1 - \left(1-\frac{h\nu}{KT}\right) = \frac{h\nu}{KT} \tag{31}$$

Now we can express Eq.30 as:

$$\rho\kappa_\nu^{ff} = \sum_i n(Z_i)n_e \left(\frac{2m_e}{3\pi KT}\right)\left[\frac{4\pi Z_i^2 e^6}{3m_e^2 ch\nu^3}\right]\bar{g}_{ff}(\nu)\frac{h\nu}{KT} \tag{32}$$

And using the definition of the C constant as:

$$C = \left(\frac{2m_e}{3\pi KT}\right)^{1/2}\left[\frac{4\pi e^6}{3m_e^2 cK}\right] \tag{33}$$

Then Eq.32 can be expressed as:

$$\rho\kappa_\nu^{ff} = \sum_i n(Z_i)n_e C Z_i^2 T^{-3/2}\nu^{-2} bar g_{ff}(\nu) \tag{34}$$

And for a pure Hydrogen plasma $Z_i = 1$ and $\sum_i n(Z_1) = n_e$ we found the following expression:

$$\rho\kappa_\nu^{ff} = n_e^2 C T^{-3/2}\nu^{-2} bar g_{ff}(\nu) \tag{35}$$

The Gaunt factor $bar g_{ff}(\nu)$ for in the radio regime is computed using:

$$bar g_{ff}(\nu) = \frac{\sqrt{3}}{2\pi}\left[ln\left(\frac{8K^3T^3}{\pi^2 e^4 m_e \nu^2} - 5\gamma\right)\right] \tag{36}$$

We compute the Gaunt factor using the following values:

- $\gamma = 0.5772$
- $T = 10^4 K$
- $\nu = 10^9 Hz$
- $K = 1.38e-23 J/K$
- $m_e = 9.10e-31 Kg$
- $e = 4.8e-10 Fr$

With this values we get:

$$\bar{g}_{ff}(\nu) = 5.96 \tag{37}$$

Now using the definition of EM:

$$EM = \int n_e^2 ds \tag{38}$$

The optical depth can de expressed as:

$$\tau_\nu = \int \rho \kappa_\nu^{ff} ds = \int C n_e^2 T^{-3/2} \nu^{-2} bar g_{ff}(\nu) = C T^{-3/2} \nu^{-2} bar g_{ff}(\nu) \int n_e^2 ds = C T^{-3/2} \nu^{-2} bar g_{ff}(\nu)(EM) \tag{39}$$

Now at low frequencies when $\tau_\nu >> 1$ and without background source T_b from Eq.29 can be expressed as:

$$T_b = T(1 - e^{-\tau_\nu}) = T(1 - 0) = T \tag{40}$$

While for low frequencies $\tau_\nu << 1$ T_b can be expressed as:

$$T_b = T(1 - e^{-\tau_\nu}) = T(1 - (1 - \tau_\nu)) = T\tau_\nu \tag{41}$$

The flux can be simply approximated by the Intrinsic intensity I_ν per solid angle that for a distant espherical source can be approzimated as πR_s^2 where R_s is the radius of the source, and divided per the distance r squared of the source. Then:

$$F_\nu = \pi I_\nu \left(\frac{R_s^2}{r^2} \right) \tag{42}$$

Wich in temrs of the brightness temperature can be expressed as:

$$F_\nu = \frac{2\pi K}{c^2} \left(\frac{R_s^2}{r^2} \right) \nu^2 T_b \tag{43}$$

If the exciting source would be a O5 star the radius of the HII region wouldn't be consistence with the Stromgren's radius that is $R_s = 1pc$ for a density of $n_0 = 1155.72 cm^{-3}$, and for a density of $n_0 = 2000 cm^{-3}$ $R_s = 0.69pc$.