Analytical work

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1 Gas density profile derivation

Makino et al (1997) found an analytical expresion (Eq.8 in that paper) for the gas profile assuming that the gas is isothermal and it is inside a DM halo with a NFW profile. They argue that this equation it's well approximated by the following profile:

$$\rho_g(r) = \frac{\rho_{g,0} A}{\left[1 + \left(\frac{r}{r_{c,eff}}\right)^2\right]^{3\beta_{eff}/2}} \tag{1}$$

Where $A(b) = -0.178b + 0.982$

In order to compute $\rho_{g,0}$ we used Eq.14 in that paper:

$$\rho_{g0} = \frac{f_{gas}\Omega_b\rho_{c0}\delta_c}{\Omega_0} e^{27b/2}\left[Ln(1+c) - \frac{c}{1+c}\right]\left[\int_0^c x^2(1+x)^{27b/2x}dx\right] \tag{2}$$

This integral can be resolved by numerical methods, but doesn't have an analytical solution. For our purpose we will follow the procedure made in Makino & Asano 98.

For this aim we recall Eq.6 of Makino et al (97) which is the condition of hydristatic equilibrium.

$$\frac{KT_X}{\mu m_p}\frac{dln\rho_g}{dr} = -\frac{GM(r)}{r^2} \tag{3}$$

And recalling that:

$$M = \int \rho(r)dv = \int \rho(r)r^2 sin(\theta)drd\theta d\phi = 4\pi \int \rho(r)r^2 dr \tag{4}$$

Taking the derivative of M in Eq.3 we get:

$$-\frac{KT_X}{G\mu m_p}\frac{d}{dr}\left(r^2 ln\rho_g\right) = 4\pi\rho(r) \tag{5}$$

Which can be expressed as:

$$\rho(r) = -\frac{KT_X}{G\mu m_p 4\pi}\left[2r\frac{d}{dr}(Ln\rho_g) + r^2\frac{d^2}{dr^2}(Ln\rho_g)\right] \tag{6}$$

Let's take a look of the derivative terms involving $Ln\rho_g$:

$$\frac{d}{dr}(Ln\rho_g) = \frac{d}{dr}\left[\frac{-3\beta}{2}Ln\left(1 + (\frac{r}{r_c})^2\right) + Ln(A) + Ln(\rho_{g0})\right] \tag{7}$$

$$\frac{d}{dr}(Ln\rho_g) = \frac{-3\beta}{2(1+(\frac{r}{r_c})^2)}\frac{2r}{r_c^2} = \frac{-3\beta r}{r_c^2\left(1+\left(\frac{r}{r_c}\right)^2\right)} \tag{8}$$

And the second derivate would be:

$$\frac{d^2}{dr^2}(Ln(\rho_g)) = \frac{-3\beta}{r_c^2}\frac{\left(1+(\frac{r}{r_c})^2\right) - 2\left(\frac{r}{r_c^2}\right)^2}{\left(1+\left(\frac{r}{r_c}\right)^2\right)^2} \tag{9}$$

Replacing Eq.8 and Eq.9 in Eq.6 we obtain:

$$\rho(r) = \frac{KT}{4\pi r^2 \mu_m m_p G}\left[2r\left(\frac{3\beta r}{r_c^2(1+(\frac{r}{r_c})^2)}\right) + \frac{r^2}{r_c^2}\left(\frac{3\beta(1+(\frac{r}{r_c})^2)2\frac{r^2}{r_c^2}}{(1+\left(\frac{r}{r_c}\right)^2)^2}\right)\right] \tag{10}$$

Making some algebra we get:

$$\rho(r) = \frac{3\beta KT}{4\pi G\mu_n m_p r_c^2}\left[\frac{3+(r/r_c)^2}{(1+(r/r_c)^2)^2}\right] \tag{11}$$

Which is the same result obtained by Makino & Asano (98).

2 Column Density derivation:

In order to compute the column density we integrate $\rho(r)$ over a surface element ds of radius b which is the impact parameter.

$$N_{HI} = \int \rho(r)ds = \int_0^{2\pi}\int_0^b \rho(r)rdrd\theta \tag{12}$$

And using our result of Eq.11 we get:

$$N_{HI} = \frac{3\beta KT}{2G\mu_n m_p r_c^2}\int_0^b \left(\frac{3+(r/r_c)^2}{(1+(r/r_c)^2)^2}\right)rdr \tag{13}$$

This integral can be done and we get:

$$N_{HI} = \frac{3\beta KT}{2G\mu_n m_p}\left(\frac{b}{b+r_c} + \frac{2tan^{-1}(b/r_c)}{r_c}\right) \tag{14}$$