Week 8: Turing Time

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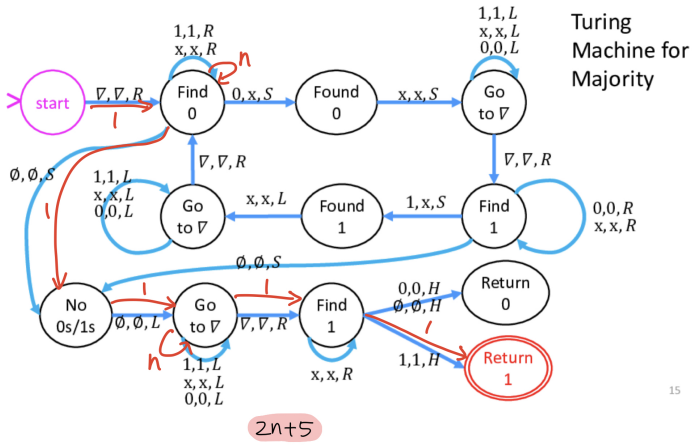
Problem 1 *Turing Machine for Majority*

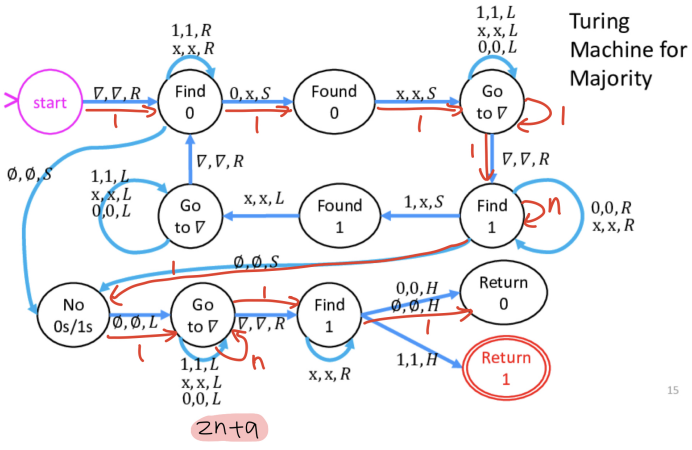
a. $\Theta(n) : f_a(n) = 2n + 5$

Since everything on the tape is 1, the machine will look for 0s for n transitions, in vain, and take n more steps to go back beginning. It will then start looking for uncrossed 0, which occurs at its first step. Several constant transitions are added.



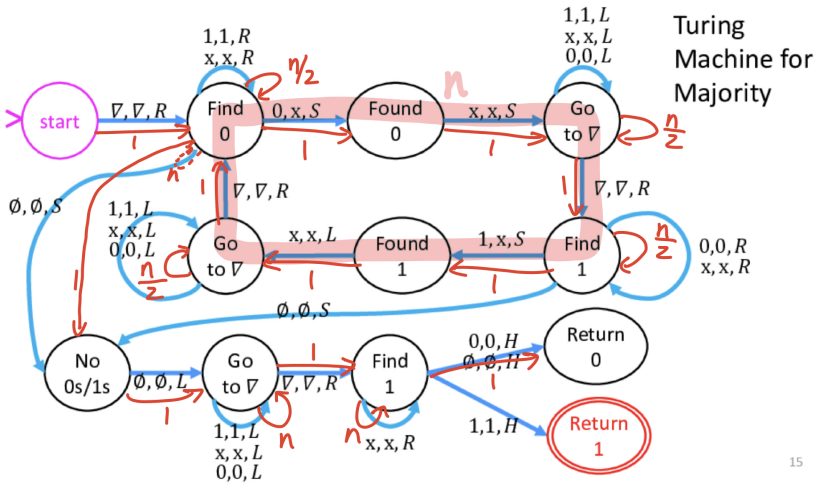
b. $\Theta(n) : f_b(n) = 2n + 9$

Since everything on the tape is 0, the machine will find a first 0 and then go back beginning. Then it starts looking for 1s for n transitions, in vain, and takes n more steps to go back beginning. It will then start looking for uncrossed 1, which occurs at its first step. Several constant transitions are added.



c. $\Theta(n^2) : f_c(n) = 2n^2 + 9n + 5$

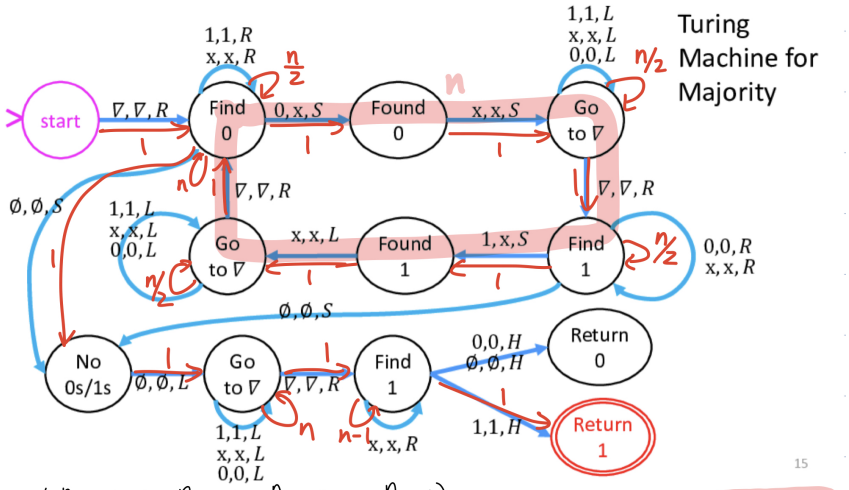
Since tape reads 010101..., the machine will look for 0s and 1s in turn, each turn averaging $\frac{n}{2}$ transitions. This number is doubled by including steps go back beginning after each turn. After n turns, it will start again looking for more 0s for n transitions, in vain, and takes n more steps back to beginning. It will then start looking for uncrossed numbers in vain, taking n more steps. Several constant transitions are added.



$1 + n(\frac{n}{2} + 1 + 1 + \frac{n}{2} + 1 + \frac{n}{2} + 1 + 1 + (1 + \frac{n}{2} + 1)) + n + 1 + 1 + (1 + n + 1 + n + 1) = 2n^2 + 9n + 5$

d. $\Theta(n^2) : f_d(n) = 2n^2 + 9n + 4$

Since tape reads 1010101..., the machine will look for 1s and 0s in turn, each turn averaging $\frac{n}{2}$ transitions. This number is doubled by including steps go back beginning after each turn. After n turns, it will start again looking for more 0s for n transitions in vain, and takes n more steps back to beginning. It will then start looking for uncrossed 1, taking n-1 more steps to find the last 1. Several constant transitions are added.



$1 + n(\frac{n}{2} + 1 + 1 + \frac{n}{2} + 1 + \frac{n}{2} + 1 + (1 + \frac{n}{2} + 1)) + n + 1 + 1 + n + 1 + n - 1 + 1 = 2n^2 + 9n + 4$