

# 1 BGG Model Equations

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## Production

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$$\begin{aligned}
K_t &= (1 - \delta)K_{t-1} + (1 - S(X_t))I_t \\
X_t &\equiv \frac{I_t}{I_{t-1}} \\
S(X_t) &= \phi_X(X_t - 1)^2 \\
S'(X_t) &= 2\phi_X(X_t - 1) \\
Q_t(1 - S(X_t) - X_t S'(X_t)) + \mathbb{E}_t[\Lambda_{t,t+1} Q_{t+1} S'(X_{t+1}) X_{t+1}^2] &= 1 \\
Y_t &= A_t K_t^\alpha L_t^{1-\alpha} \\
W_t &= (1 - \alpha) \left( \frac{K_t}{L b_t} \right)^\alpha \\
r_{K,t} &= \alpha \left( \frac{L b_t}{K_t} \right)^{1-\alpha} \\
R_{k,t+1} &= \frac{[r_{K,t+1} + (1 - \delta)Q_{t+1}]}{Q_t}
\end{aligned}$$

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## Households

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$$\begin{aligned}
Uc &= (C_t - \gamma C_{t-1})^{-1} - \beta \gamma (C_{t+1} - \gamma C_t)^{-1} \\
\Lambda_{t,t+1} &\equiv \beta \frac{u_{c,t+1}}{u_{c,t}} \\
\Lambda_{t,t+1} R_{t+1} &= 1 \\
u_{c,t} W_t &= \chi L b_t^\epsilon \\
u_{c,t} (1 - \alpha) Y / L b &= \chi (L b^\epsilon)
\end{aligned}$$

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## Entrepreneurs & Debt Contract

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$$\begin{aligned}
Q_t K_t &= L_t + N_t^E \\
N_t^E &= R_{k,t} Q_t K_{i,t} - R_{l,t} L_t \\
R_{l,t} L_t &= \bar{\psi}_t R_{k,t} Q_t K_t \\
N_t^E &= (\sigma_{E,t} + \xi^e) (1 - \Gamma(\bar{\psi}_{t+1})) R_{k,t+1} Q_t K_{t+1} \\
N_t^E \phi_t^E &= Q_t K_t \\
R_{k,t} Q_t K_t [\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1})] &\geq R_{t+1} (Q_t K_t - N_t^E) \\
R_{k,t+1} &= \mathbb{E}_t \rho(\bar{\psi}_{t+1}) R_{t+1}
\end{aligned}$$

$$\begin{aligned}
p(\bar{\psi}_t) &= \int_0^{\bar{\psi}_t} f(\psi, -0.5(\sigma_\psi)^2, \sigma_\psi^2) d\psi \\
\Gamma(\bar{\psi}_t) &= G(\bar{\psi}_t) + \bar{\psi}_t(1 - p) \\
G(\bar{\psi}_t) &= \int_0^{\bar{\psi}_t} \psi f(\psi, -0.5(\sigma_\psi)^2, \sigma_\psi^2) d\psi \\
\Gamma'(\bar{\psi}_t) &= (1 - p(\bar{\psi}_t)) \\
G'(\bar{\psi}_t) &= \frac{1}{\sigma_\psi \sqrt{\pi}} \exp \left[ -\frac{(\log(\bar{\psi}) + 0.5\sigma_\psi^2)^2}{2\sigma_\psi^2} \right] \\
\rho(\bar{\psi}_{t+1}) &= \frac{\Gamma'(\bar{\psi}_{t+1})}{[(\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1}))\Gamma'(\bar{\psi}_{t+1}) + (1 - \Gamma(\bar{\psi}_{t+1})(\Gamma'(\bar{\psi}_{t+1}) - \mu G'(\bar{\psi}_{t+1})))]} \\
L_t &= D_t
\end{aligned}$$

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**Resource Constraint**

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$$\begin{aligned}
Y_t &= C_t + I_t + G_t + \mu G(\psi_t) R_{k,t} Q_t K_t \\
G_t &= \eta Y_t
\end{aligned}$$