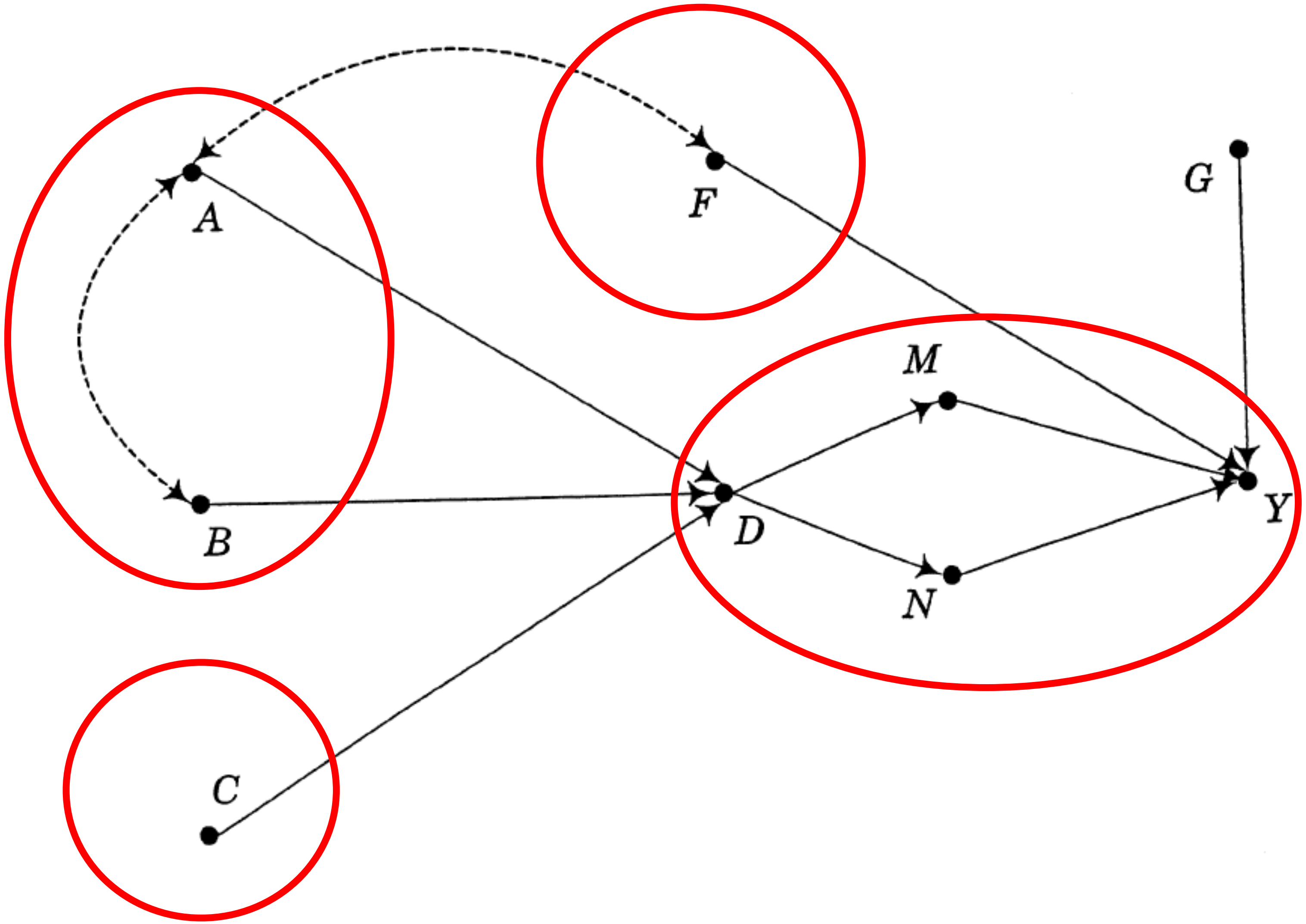
Counterfactual Analysis

Estimate D ⟶ Y



Counterfactual Analyses

- The problem of conventional analyses by using observed data

$$\mathrm{E}\,[Y^1 - Y^0]$$ the average treatment effect

ex: whether going to college influences individuals' wage

Table 2.1: The Fundamental Problem of Causal Inference

Group	Y^1	Y^0
Treatment group ($D = 1$)	Observable as Y	Counterfactual
Control group ($D = 0$)	Counterfactual	Observable as Y

Selection !!

Counterfactual Analyses

- Decomposition

Observed

$$E[\delta] = \{\pi E[Y^1|D=1] + (1-\pi)E[Y^1|D=0]\}$$
$$- \{\pi E[Y^0|D=1] + (1-\pi)E[Y^0|D=0]\}.$$

Unobserved

Counterfactual Analyses

- Decomposition

The average treatment effect

Baseline Bias = 0

$$
\begin{aligned}
E[Y^1|D=1] - E[Y^0|D=0] \quad = \quad & E[\delta] \\
& + \boxed{\{E[Y^0|D=1] - E[Y^0|D=0]\}} \\
& + \boxed{(1-\pi)\{E[\delta|D=1] - E[\delta|D=0]\}.}
\end{aligned}
$$

Differential treatment effect bias = 0

Counterfactual Analyses

Table 2.3: An Example of the Bias of the Naive Estimator

Group	$E[Y^1\|.]$	$E[Y^0\|.]$
Treatment group ($D = 1$)	10	6
Control group ($D = 0$)	8	5

Counterfactual Analyses

- The estimate from the conventional model = 10-5 = 5
- The effect of college on the wage in treatment group = 10-6 =4
- The effect of college on the wage in control group = 8-5=3
- The average treatment effect = 0.3(10-6)+0.7(8-5)= 3.3

Table 2.3: An Example of the Bias of the Naive Estimator

Group	$E[Y^1\|.]$	$E[Y^0\|.]$
Treatment group ($D = 1$)	10	6
Control group ($D = 0$)	8	5

$\pi = 0.3$

30% of population obtains college degrees

Counterfactual Analyses

- The bias of conventional models is from two assumptions

Differential treatment effect bias = 0

$$\text{Assumption 1:} \quad E[Y^1|D=1] \quad = \quad E[Y^1|D=0],$$
$$\text{Assumption 2:} \quad E[Y^0|D=1] \quad = \quad E[Y^0|D=0].$$

Baseline Bias = 0