

Econometrics II

– Comprehension Exercises –

Exercise 1

The food expenses y_t of $n = 150$ households are regressed on household income x_t and squared household income x_t^2 . You should investigate if there is heteroskedasticity. There are three groups: The poorest third of the households, the middle third and the richest third. Regressions for the first and third group yield the following sums of squared residuals:

$$S_{\hat{u}\hat{u}}^I = 0.00838$$
$$S_{\hat{u}\hat{u}}^{III} = 0.07346.$$

- What might be a problem with classical OLS estimation in this situation?
- Perform a suitable test to check if the aforementioned problem is present.
- How would you proceed if the above test suggests the problem is present?

Exercise 2

Consider the model

$$y = X\beta + u \quad \text{with} \quad u \sim N(0, \Omega)$$

with the GLS estimator

$$\hat{\beta}_{GLS} = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}y.$$

- Show that $\hat{\beta}_{GLS}$ is still unbiased if X is stochastic but independent of u . You may assume that the matrix Ω is non-stochastic and known.
- Which conditions must be satisfied to ensure consistency of the estimator?

Exercise 3

Consider the simple linear regression model

$$y_t = \alpha + \beta x_t + u_t.$$

- Explain why an OLS estimation of the coefficients yields inconsistent estimators if the regressor is correlated with the error term.
- Is x_{t-1} a valid instrument if a large (small) error in $t - 1$ tends to be followed by a large (small) error in t ?

Exercise 4

- a) In a regression model with errors that are non-normally distributed, what concept is the justification for tests to give valid results at large samples?
- b) Describe the principle of the *Law of Large Numbers* in your own words.
- c) State the null hypothesis of the *White-Test*. What is the advantage of it compared to the *Goldfeld-Quandt-Test*?
- d) Is there a way to deal with possible heteroscedasticity without using the aforementioned tests?
- e) When not using a computer for the one-sided *Durbin-Watson-Test*: Why is a lower and upper quantile for the distribution of test statistic needed?
- f) What values of a test statistics indicate positive/negative autocorrelation at a *Durbin-Watson-Test*?