Econometrics 2

Exam

Dr. Willi Mutschler

- Answer **all** of the following exercises in either German or English.
- Explain your answers and derivations. All your computations and intermediate steps need to be verifiable and understandable.
- Formulas which we covered in the lecture and class need not to be derived again.
- If you prefer a notation different from the one used in the course, define it.
- Always use significance level $a = 5\%$.
- Please report 3 decimal places in numerical answers.
- If not otherwise stated, assume the validity of the assumption A, B and C given in the lecture.
- Permissible aids:
  - non-programmable pocket calculator
  - cheat sheet: one-sided A4 white sheet of paper with annotations, formulas, texts, sketches, etc.

1 Understanding

(a) Briefly describe the intuition and result of the usual Central Limit Theorem for the sample mean of iid random variables. What does this imply for the distribution of $\frac{1}{\sqrt{T}}\sum_t^T u_t$ if $u_t \sim iid(0,\sigma^2)$?

(b) The estimation of a dynamic model yields

$$y_t = 0.6y_{t-1} + 0.3x_t + \hat{u}_t$$

where y_t and x_t are both measured in logs. Compute the dynamic effect (multiplier) of a 1% increase in x on y (i) in the same period and (ii) in the long run.

(c) What are the effects on the OLS estimator when X is stochastic, but contemporary uncorrelated with u, i.e. $Cov(x_{k,t}, u_t) = 0$ for all t and k? What about the reliability of hypothesis tests and interval estimators? (No proof required).

2 Violation of B1

Assume that the true model is given by

$$y_t^* = \alpha + \beta x_t + u_t$$

with $u_t \sim N(0, \sigma_u^2)$. However, y_t^* is only observable with a measurement error

$$y_t^* = y_t + \lambda$$

where $\lambda \in \mathbb{R}$. The observed model is hence given by

$$y_t = \alpha + \beta x_t + \underbrace{u_t - \lambda}_{\varepsilon_t}$$

(a) Compute the (i) expectation, (ii) variance and (iii) autocorrelation function of ε_t.

(b) Compute the bias of the OLS estimators for α and β.

(c) What happens to the standard errors of the OLS estimators for α and β? Are hypothesis tests and interval estimators still valid?

(d) Provide intuition whether (or not) the OLS estimator remains consistent. A formal proof is not necessary.

3 Testing violations

Overall consumption for the period 1962 to 2001 (annual data) is determined by the regression model

$$c_t = \alpha + \beta_1 y_t + \beta_2 i_t + u_t$$

where c_t denotes the expenditures for consumption (in logs), y_t the income (in logs) and i_t the interest rate (in %). We have the following intermediate result:

$$X'X = \begin{bmatrix} 40 & 0 & 10 \\ 0 & 100 & 10 \\ 10 & 10 & 20 \end{bmatrix}$$

Furthermore, an OLS estimation yields an estimated error variance of

$$\hat{\sigma}^2 = 6$$

(a) It is assumed that the consumption function changed at the time of reunification in 1990. Hence, the model is estimated for both periods, before and after 1990, and the corresponding sums of squared residuals are given by

$$\sum_{t=1}^{28} \hat{u}_t^2 = 120 \text{ and } \sum_{t=29}^{40} \hat{u}_t^2 = 40$$

Test the hypothesis that the consumption function has not changed using the Chow Break test. What is the underlying intuition of the test?

(b) It should be checked that the error terms are free of (positive) first-order autocorrelation. The following information is determined for this:

$$\sum_{t=2}^{T} (\hat{u}_t - \hat{u}_{t-1})^2 = 160, \quad \frac{1}{T}\sum_{t=1}^{T} (y_t - \bar{y})^2 = 10, \quad R^2 = 0.445$$

Perform a Durbin-Watson-Test and briefly provide intution about the critical values one uses in the test.

4 Instrument variables

You would like to know the influence of smoking on the annual logarithmic income $log(income)$. To this end, you use the following model

$$log(income_t) = \alpha + \beta_1 cigs_t + \beta_2 educ_t + \beta_3 age_t + \beta_4 age_t^2 + u_t, \quad (1)$$

where `cigs` denotes the number of cigarettes consumed per day, `educ` the number of educational years and `age` is the age. The following output is provided for estimating the above model:

Model 1: OLS, Observations 1–807
Endogenous Variable: log(income)

	Coefficient	Std. Error	t-statistic	p-value
const	7,79544	0,170427	45,7407	0,0000
cigs	0,00173057	0,00171372	1,0098	0,3129
educ	0,0603605	0,00789834	7,6422	0,0000
age	0,0576907	0,00764359	7,5476	0,0000
age2	−0,000630589	8,33822e–005	−7,5626	0,0000

You suspect that $cigs$ is endogenous and also estimate the following model with OLS:

$$cigs_t = \pi_0 + \pi_1 educ_t + \pi_2 age_t + \pi_3 age_t^2 + \pi_4 restaurn_t + v_t,$$

where the variable $restaurn$ is 1, if the person t lives in a state where smoking is prohibited in restaurants, and 0 else. The corresponding output is

Model 2: OLS, Observations 1–807
Endogenous variable: cigs

	Coefficient	Std. error	t-statistic	p-value
const	0,152140	3,50332	0,0434	0,9654
educ	−0,450400	0,161486	−2,7891	0,0054
age	0,822327	0,154187	5,3333	0,0000
age2	−0,00958859	0,00167790	−5,7146	0,0000
restaurn	−2,74637	1,09685	−2,5039	0,0125

Lastly, you compute the IV estimator for model (1) using $restaurn$ as an instrument for $cigs$:

Model 3: IV, Observations 1–807
Endogenous Variable: log(income)
Instruments: const educ age agesq restaurn
Instruments used for: cigs

	Ceofficient	Std. Error	t-statistic	p-value
const	7,78114	0,228129	34,1085	0,0000
cigs	−0,0413845	0,0260279	−1,5900	0,1118
educ	0,0400241	0,0161607	2,4766	0,0133
age	0,0932076	0,0236787	3,9363	0,0001
agesq	−0,00104373	0,000272324	−3,8327	0,0001

(a) Compare and interpret the estimated coefficient $\hat{\beta}_1$ in model 1 in terms of economic meaning and statistical significance. Is there a difference between the OLS and IV estimators?

(b) It is assumed that $log(income)$ and $cigs$ are jointly determined within a system of equations. What is the corresponding problem and what are the consequences for model 1?

(c) Check if $restaurn$ is a relevant instrument for $cigs$.

5 Stochastic convergence

Consider the linear regression model with a deterministic time trend

$$y_t = \alpha + \delta t + u_t$$

where t denotes the time index and $u_t \sim N(0, \sigma^2)$. Furthermore,

$$H = \begin{pmatrix} T^{1/2} & 0 \\ 0 & T^{3/2} \end{pmatrix}$$

and $\hat{\beta} = \begin{pmatrix} \hat{\alpha} \\ \hat{\delta} \end{pmatrix}$ denotes the OLS estimator of $\beta = \begin{pmatrix} \alpha \\ \delta \end{pmatrix}$.

Hints:

- $E(u_t^4) = 4\sigma^2$
- $\sum_{t=1}^{T} t = T(T+1)/2 = T^2/2 + T/2$
- $\sum_{t=1}^{T} t^2 = T(T+1)(2T+1)/6 = T^3/3 + T^2/2 + T/6$
- $\sum_{t=1}^{T} t^4 = T(T+1)(2T+1)(3T^2+3T-1)/30 = T^5/5 + T^4/2 + T^3/3 - T/30$

(a) Show that

$$\underset{T\to\infty}{plim}\left[\frac{X'X}{T}\right]$$

does not converge to a finite matrix. Hence, our usual proof of consistency of the OLS estimator does not work.

(b) Show that

$$\underset{T\to\infty}{plim}\left[H^{-1}(X'X)H^{-1}\right]$$

converges to $Q = \begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} \end{pmatrix}$.

(c) Show that

$$H^{-1}X'u = \begin{pmatrix} \frac{1}{\sqrt{T}} \sum_{t=1}^{T} u_t \\ \frac{1}{\sqrt{T}} \sum_{t=1}^{T} \frac{t}{T} u_t \end{pmatrix}$$

(d) Show that

(i) $\lim\limits_{T\to\infty} E\left[\frac{1}{\sqrt{T}} \sum_{t=1}^{T} u_t\right] = 0$

(ii) $\lim\limits_{T\to\infty} V\left[\frac{1}{\sqrt{T}} \sum_{t=1}^{T} u_t\right] = \sigma^2$

(e) Show that

(i) $\lim\limits_{T\to\infty} E\left[\frac{1}{\sqrt{T}} \sum_{t=1}^{T} \frac{t}{T} u_t\right] = 0$

(ii) $\lim\limits_{T\to\infty} V\left[\frac{1}{\sqrt{T}} \sum_{t=1}^{T} \frac{t}{T} u_t\right] = \sigma^2/3$

(f) Show that $\underset{T\to\infty}{plim}\left[\frac{H^{-1}X'u}{T}\right] = 0$ by checking the sufficient conditions of consistency.

(g) Use the previous results to show that the OLS estimator $\hat{\beta}$ is a consistent estimator for β.

Table of the quantiles of the F_{ν_1,ν_2}-distribution, given are the 0.95 -quantiles (i.e. $a = 0.05$)

	ν_1										
ν_2	1	2	3	4	5	10	15	20	25	50	∞
1	161.45	199.50	215.71	224.58	230.16	241.88	245.95	248.01	249.26	251.77	254
2	18.51	19.00	19.16	19.25	19.30	19.40	19.43	19.45	19.46	19.48	19.5
3	10.13	9.55	9.28	9.12	9.01	8.79	8.70	8.66	8.63	8.58	8.53
4	7.71	6.94	6.59	6.39	6.26	5.96	5.86	5.80	5.77	5.70	5.63
5	6.61	5.79	5.41	5.19	5.05	4.74	4.62	4.56	4.52	4.44	4.37
6	5.99	5.14	4.76	4.53	4.39	4.06	3.94	3.87	3.83	3.75	3.67
7	5.59	4.74	4.35	4.12	3.97	3.64	3.51	3.44	3.40	3.32	3.23
8	5.32	4.46	4.07	3.84	3.69	3.35	3.22	3.15	3.11	3.02	2.93
9	5.12	4.26	3.86	3.63	3.48	3.14	3.01	2.94	2.89	2.80	2.71
10	4.96	4.10	3.71	3.48	3.33	2.98	2.85	2.77	2.73	2.64	2.54
15	4.54	3.68	3.29	3.06	2.90	2.54	2.40	2.33	2.28	2.18	2.07
20	4.35	3.49	3.10	2.87	2.71	2.35	2.20	2.12	2.07	1.97	1.84
25	4.24	3.39	2.99	2.76	2.60	2.24	2.09	2.01	1.96	1.84	1.71
30	4.17	3.32	2.92	2.69	2.53	2.16	2.01	1.93	1.88	1.76	1.62
35	4.12	3.27	2.87	2.64	2.49	2.11	1.96	1.88	1.82	1.70	1.56
40	4.08	3.23	2.84	2.61	2.45	2.08	1.92	1.84	1.78	1.66	1.51
45	4.06	3.20	2.81	2.58	2.42	2.05	1.89	1.81	1.75	1.63	1.47
50	4.03	3.18	2.79	2.56	2.40	2.03	1.87	1.78	1.73	1.60	1.44
60	4.00	3.15	2.76	2.53	2.37	1.99	1.84	1.75	1.69	1.56	1.39
70	3.98	3.13	2.74	2.50	2.35	1.97	1.81	1.72	1.66	1.53	1.35
80	3.96	3.11	2.72	2.49	2.33	1.95	1.79	1.70	1.64	1.51	1.32
90	3.95	3.10	2.71	2.47	2.32	1.94	1.78	1.69	1.63	1.49	1.30
100	3.94	3.09	2.70	2.46	2.31	1.93	1.77	1.68	1.62	1.48	1.28
∞	3.84	3.00	2.60	2.37	2.21	1.83	1.67	1.57	1.49	1.35	1.01