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WESTFÄLISCHE
WILHELMS-UNIVERSITÄT
MÜNSTER

Econometrics II

– Exercise Book –

Exercise 1

It is assumed that the connection between consumption c_t and income y_t is linear:

$$c_t = \alpha + \beta y_t + u_t$$

For a study the following data is collected:

t	1	2	3	4	5	6	7	8	9	10	11	12	13
c_t	55	65	70	80	79	84	98	95	90	75	74	113	108
y_t	80	100	85	110	120	115	130	140	125	90	105	150	145
t	14	15	16	17	18	19	20	21	22	23	24	25	26
c_t	140	120	145	152	144	175	180	135	140	178	191	137	189
y_t	225	200	240	220	210	245	260	190	205	265	270	230	250

a) Test the presumption of heteroscedasticity by performing a Goldfeld-Quandt-Test. The OLS regression using first and second half of all observations yield a sum of squared residuals $S^1_{\hat{u}\hat{u}} = 377.17$ and $S^2_{\hat{u}\hat{u}} = 1536.8$ respectively.

b) It is assumed that the variance in error terms is constant in both groups, namely σ_1^2 and σ_2^2. Denoting the vector of error terms $\boldsymbol{u} := (u_1, ..., u_{26})'$, give the covariance matrix $\text{Cov}(\boldsymbol{u})$.

c) What estimation procedure can be applied in above situation? Explain the steps.

Exercise 2

a) Describe the idea of the White-Test in two sentences.

b) In which situations is a White-Test applicable, when a Goldfeld-Quandt-Test?

c) Consider the CAPM model with MAN returns as dependent variable x_t and DAX returns y_t as only exogenous variable with a total of $T = 120$ observations. For the model $y_t = \alpha + \beta x_t + u_t$ you gather the following results:

Constant: $\hat{\alpha} = 0.25$ $\quad\widehat{\text{se}}(\hat{\alpha}) = 0.24$ $\qquad$ DAX returns: $\widehat{\text{se}}(\hat{\beta}) = 0.08$ $\quad$ t statistic: 10.18

$$R^2 = 0.467 \qquad \sum_{t=1}^{T} \hat{u}_t^2 = 2.619^2 \qquad \text{mean MAN return: } 0.174$$

i) Calculate the coefficient of DAX returns.

ii) For the intercept α, test $\mathrm{H}_0 : \alpha \geq 0$ vs. $\mathrm{H}_1 : \alpha < 0$.

iii) The 120 observations are split in 2 subgroups A: $t = 1, ..., 98$ and B: $t = 99, ..., 120$. Two separate regressions yield

$$\sum_{t=1}^{98} \hat{u}_t^2 = 680 \qquad \sum_{t=99}^{120} \hat{u}_t^2 = 188$$

Test the hypothesis of heteroscedasticity with a Goldfeld-Quandt-Test.
Hint: To find the critical value, round degrees of freedom to the nearest tabulated.

Exercise 3

```
Dependent Variable: LOG(WK)
Method: Least Squares
Sample: 1979:04 2003:12
Included Observations 297
-----------------------------------------------------------------------
Variable      Coefficient      Std. Error      t-Statistic      Prob.
C             0.593957         0.625574         0.949459        0.3432
LOG(P)        -1.426313        0.439096        -3.248291        0.0013
LOG(P*)       1.286741         0.303944         4.233479        0.0000
-----------------------------------------------------------------------
R-squared            0.185941   Mean dependent var       0.021826
Adjusted R-Squared   0.180403   S.D. dependent var       0.194720
S.E. of regression   0.176283   Akaike info criterion   -0.623400
Sum squared resid    9.136280   Schwarz criterion       -0.586089
Log likelihood       95.57487   F-statistic              33.57664
Durbin-Watson stat   0.024413   Prob(F-statistic)        0.000000
```

Decide whether the following statements are true or false for the data given above.

	True	False
1. The estimated intercept is significantly different from zero.	☐	☐
2. The F statistic test the hypothesis that every estimated parameter is zero.	☐	☐
3. A value in column "Prob." lower than 0.05 is equivalent to rejecting the null hypothesis that the corresponding parameter is zero.	☐	☐
4. The value "Log likelihood = 95.57487" indicates that the parameters are true with a probability of 95.57%.	☐	☐

5. The t statistic always has the same sign as the corresponding estimator.	☐	☐
6. The adjusted coefficient of determination R^2 is always smaller that the unadjusted R^2.	☐	☐
7. Maximum likelihood estimates maximize the sum of squared residuals.	☐	☐
8. To obtain unbiasedness of OLS estimates, normally distributed error terms need to be assumed.	☐	☐
9. The variance of OLS estimates falls below the Cramer-Rao bound.	☐	☐
10. The variance in OLS estimation is larger in case of multicollinearity.	☐	☐
11. Theoretical variances of OLS estimates are identical in case of homoscedasticity.	☐	☐
12. R^2 gives the proportion of variance that is explained by error terms.	☐	☐
13. The t-test statistic for the null hypothesis $\beta_j > 0$ needs to be positive.	☐	☐

Exercise 4

Consider a regression model $\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{u}$ with an intercept and one exogenous variable. The $T = 3$ observations are given by

$$\boldsymbol{x} = \begin{pmatrix} 3 & 4 & 7 \end{pmatrix}' \qquad \boldsymbol{y} = \begin{pmatrix} 2 & 1 & 4 \end{pmatrix}'$$

a) Perform the OLS regression under the assumptions $\mathrm{E}(u) = 0$ and $\mathrm{Var}(u) = \sigma^2 I$. Calculate the sum of residuals and squared residuals.

b) It is now assumed that heteroscedasticity is present in form of $\sigma_t^2 = x_t^2\sigma^2$, where $\sigma_t^2 = \mathrm{Var}(u_t)$. Calculate the GLS estimates, the sum of residuals and the sum of squared residuals.
Hint: $\mathrm{Var}(u) = \sigma^2\Omega$ and $P'P = \Omega^{-1}$

c) How would the sum of residuals and squared residuals change if the heteroscedasticity was of form $\sigma_t^2 = x_t\sigma^2$?

d) Now assume the form of heteroscedasticity is unknown. Calculate unbiased estimates of regression parameters and White's covariance estimator (sandwich estimator).

Exercise 5

a) Consider a regression model with T observations, k exogenous variables $x_{1,t}, ..., x_{k,t}$ and homoscedastic error terms u_t. Show the unbiasedness of White's Sandwich estimator $\widehat{\text{Var}}(\hat{\beta}) = (X'X)^{-1} X'\widehat{W}X (X'X)^{-1}$

b) Consider now a regression model with autocorrelated error terms, i.e.

$$y_t = \alpha + \beta x_t + u_t$$
$$u_t = \rho u_{t-1} + \epsilon_t$$

where $|\rho| < 1$ and $\epsilon_t \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_\epsilon^2)$.

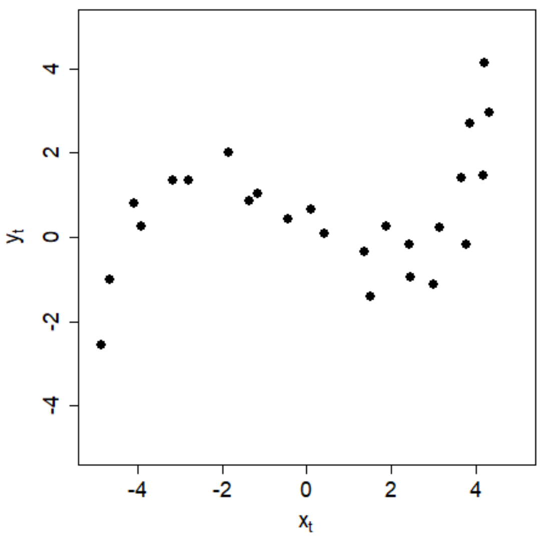
Derive expectation $\text{E}(u_t)$, variance $\text{Var}(u_t)$ and covariance $\text{Cov}(u_t, u_{t+h})$ of the error terms u_t.

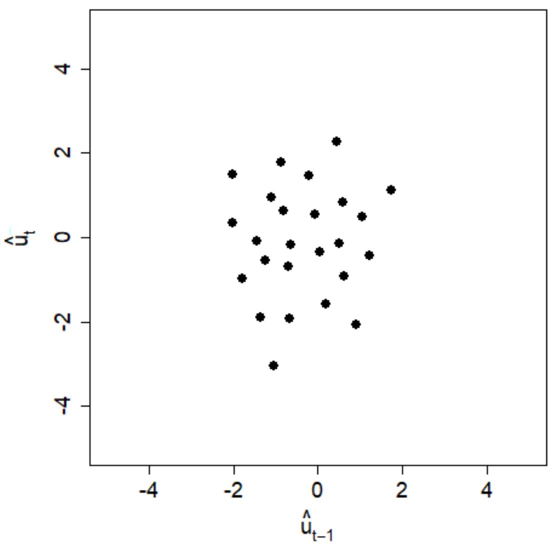
Hints:

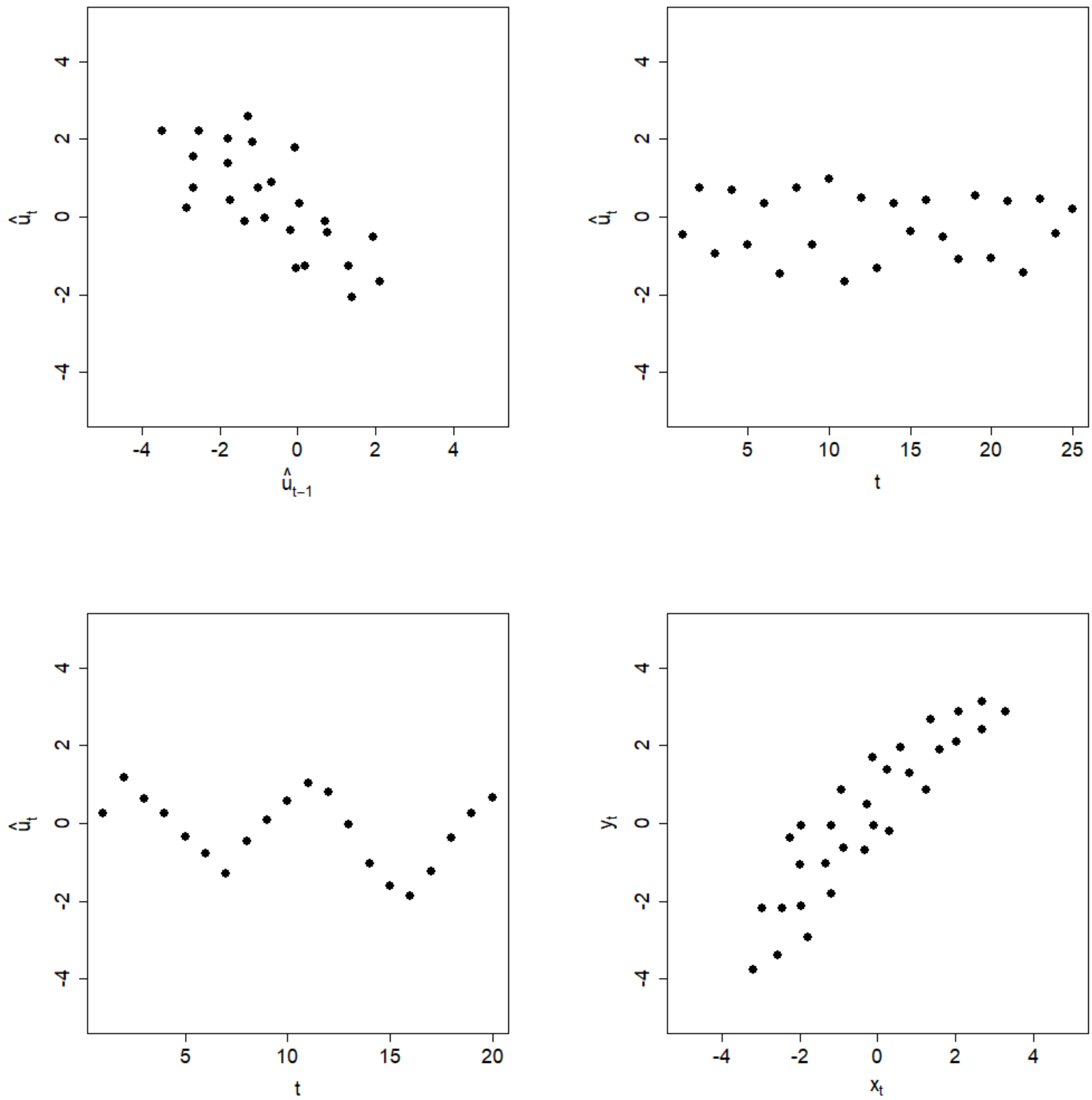
- With $|\gamma| < 1$ it holds that $\sum_{i=0}^{\infty} \gamma^i = \frac{1}{1-\gamma}$.
- u_t is a stationary AR(1) process, which implies
  - $\text{E}(u_1) = ... = \text{E}(u_T)$
  - $\text{Var}(u_1) = ... = \text{Var}(u_t) = \sigma_u^2 < \infty$
  - $\text{Cov}(u_t, u_{t+h}) = \text{Cov}(u_{t+s}, u_{t+h+s}) = \gamma_h$ for suitable indices

Exercise 6

Look at the following plots and determine if autocorrelation in error terms may be present and, if yes, in what form.







Exercise 7

We estimate the parameters of a regression model

$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + u_t$$

by OLS using $T = 15$ observations and obtain residuals $\hat{u}_t$:

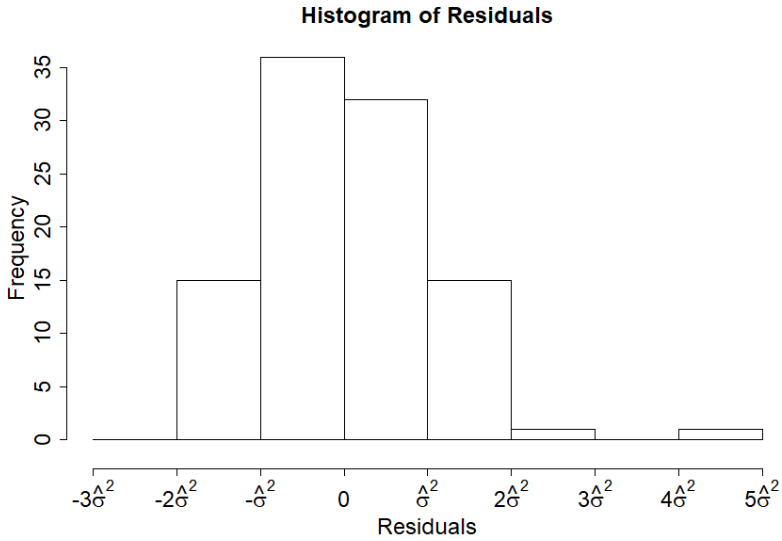
$$4 \quad 3.5 \quad 3.5 \quad 2 \quad -1 \quad -4 \quad -3 \quad -2 \quad -3 \quad -1 \quad 1 \quad 3 \quad 1 \quad -2 \quad -2$$

a) Plot pairs of residuals $(\hat{u}_{t-1}; \hat{u}_t)$. Is there any evidence for positive, negative or no autocorrelation?

b) Test your presumption in a) with a test for autocorrelation, assuming an AR(1) structure of error terms.

Exercise 8

The figures below shows a histogram of $T = 100$ residuals $\hat{u}_t$ obtained from an OLS regression. Additionally, some sums are given.



$$\frac{1}{100}\sum_{t=1}^{100}\hat{u}_t^2 = 0.2071 \qquad \frac{1}{100}\sum_{t=1}^{100}\hat{u}_t^3 = 0.0747 \qquad \frac{1}{100}\sum_{t=1}^{100}\hat{u}_t^4 = 0.1841$$

a) Is there any evidence for the violation of assumption B4?

b) Test your presumption in a) with a suitable test.

Exercise 9

Proof the law of iterated expectation

$$\mathrm{E}\left(\mathrm{E}(X|Y)\right) = \mathrm{E}(X) ,$$

where X and Y are continuous random variables with densities $f_X(x)$ and $f_Y(y)$ respectively.

Exercise 10

Consider the following macroeconomic model for $t = 1, ..., T$:

$$\begin{aligned} C_t &= \beta_0 + \beta_1 Y_t + u_t \\ Y_t &= C_t + I_t \\ u_t &\overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_u^2) \\ \mathrm{Cov}(I_t, u_t) &= 0 \qquad \forall t = 1, ..., T \end{aligned}$$

where $C = (C_1, ..., C_T)'$ denotes consumption, $Y = (Y_1, ..., Y_T)'$ the income and $I = (I_1, ..., I_T)'$ investments. With $\mathbb{1}$, a vector of ones, the following matrix is calculated:

$$\begin{pmatrix} \mathbb{1}' \\ C' \\ Y' \\ I' \end{pmatrix} \begin{pmatrix} \mathbb{1} & C & Y & I \end{pmatrix} = \begin{pmatrix} 9 & 3905.7 & 4007.6 & 101.9 \\ 3905.7 & 1695841.58 & 1740366.94 & 44525.36 \\ 4007.6 & 1740366.94 & 1786149.92 & 45782.98 \\ 101.9 & 44525.36 & 45782.98 & 1257.62 \end{pmatrix}$$

a) Why is an OLS estimation not justified in this situation?

b) Perform an IV estimation of the parameter vector. Use I_t as an instrument for Y_t.

c) What requirements are needed for an instrument to achieve a consistent estimation?

Exercise 11

To investigate if a price increase can reduce cigarette consumption, you have yearly data of 48 states of the US for 1985 to 1995. You are interested in estimating the model

$$\ln\left(\frac{y_{i;1995}}{y_{i;1985}}\right) = \alpha + \beta_1 \ln\left(\frac{x_{i;1995}}{x_{i;1985}}\right) + \beta_2 \ln\left(\frac{w_{i;1995}}{w_{i;1985}}\right) + u_i$$

where $y_{i;j}$ denotes cigarette consumption of state i in the year j (in packs per person), $x_{i;j}$ the price of cigarettes (including taxes) and $w_{i;j}$ the real income per person.
Further you have data for mean real tobacco tax load $z1_{i;j}$ and mean real excise duties $z2_{i;j}$[1]. Both variables $z1$ and $z2$ are measured in cents per pack.
Income $w_{i;j}$ is assumed to be contemporarily uncorrelated to the error term and can thus be used as exogenous variable.

a) Calculate OLS estimates for the model above.

$$(\mathbf{X}'\mathbf{X})^{-1} = \begin{bmatrix} 1.4665 & -1.4744 & -1.0659 \\ -1.4744 & 2.6917 & -0.082 \\ -1.0659 & -0.082 & 1.9366 \end{bmatrix}, \qquad \mathbf{X}'\mathbf{y} = \begin{bmatrix} -12.0695 \\ -7.2261 \\ -7.0925 \end{bmatrix}, \qquad \hat{\mathbf{u}}'\hat{\mathbf{u}} = 0.34884.$$

b) Discuss if OLS is a justified method for the estimation of parameters in this case.

c) Can $z2$ be used to obtain a valid instrument for the cigarette price x? Give reasons for your answer.

d) Perform an IV estimation by using $\ln(z2)$ as instrument for $\ln(x)$. Estimate the variance matrix of the obtained estimator $\hat{\beta}_{\mathrm{IV1}}$.

$$(\mathbf{Z}_1'\mathbf{X})^{-1} = \begin{bmatrix} 1.0631 & -0.7733 & -1.0762 \\ -0.7379 & 1.4117 & -0.0634 \\ -1.0884 & -0.0430 & 1.9361 \end{bmatrix}, \qquad \mathbf{Z}_1'\mathbf{y} = \begin{bmatrix} -12.0695 \\ -7.5505 \\ -7.0925 \end{bmatrix}$$

$$(\mathbf{X}'\mathbf{Z}_1(\mathbf{Z}_1'\mathbf{Z}_1)^{-1}\mathbf{Z}_1'\mathbf{X})^{-1} = \begin{bmatrix} 1.7501 & -1.9921 & -1.0502 \\ -1.9921 & 3.6367 & -0.1108 \\ -1.0502 & -0.1108 & 1.9375 \end{bmatrix}, \qquad \hat{\mathbf{u}}_1'\hat{\mathbf{u}}_1 = 0.36844,$$

[1] the average tax due to the broad-based state sales tax applied to all consumption goods.

e) Test the contemporary correlation between $\ln\left(\frac{x_{i;1995}}{x_{i;1985}}\right)$ and u_i using a 5% level of significance.

f) Repeat the IV estimation, but this time also include $\ln(z1)$ as an instrument for $ln(x)$. Compute the variance matrix estimator $\widehat{\text{Var}}\left(\hat{\beta}_{\text{IV2}}\right)$.

$$(\mathbf{X}'\mathbf{P}\mathbf{X})^{-1} = \begin{bmatrix} 1.7237 & -1.9440 & -1.0516 \\ -1.9440 & 3.5489 & -0.1081 \\ -1.0516 & -0.1081 & 1.9374 \end{bmatrix}, \qquad \mathbf{X}'\mathbf{P}\mathbf{y} = \begin{bmatrix} -12.0695 \\ -7.1749 \\ -7.0925 \end{bmatrix}$$

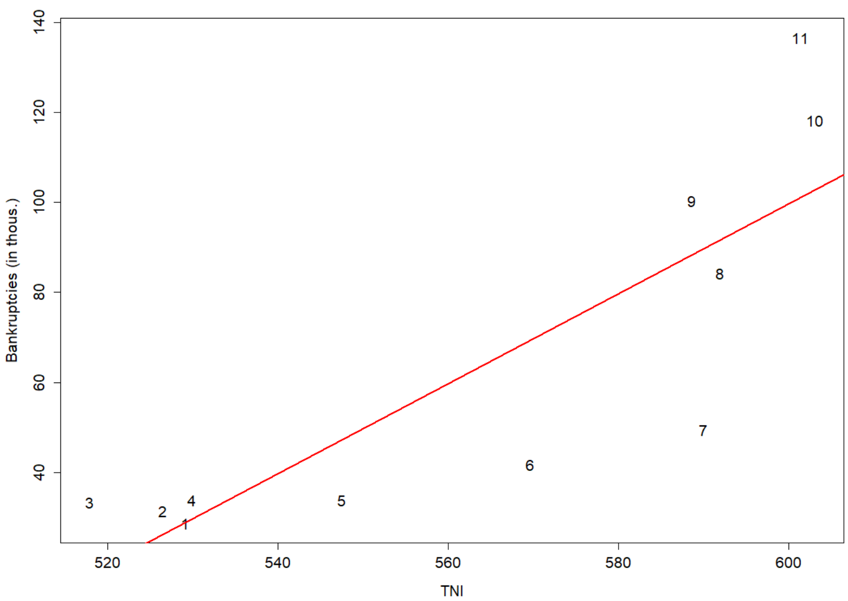
$$\hat{\mathbf{u}}'\hat{\mathbf{u}} = 0.35834. \qquad \mathbf{P} = \mathbf{Z}_2(\mathbf{Z}_2'\mathbf{Z}_2)^{-1}\mathbf{Z}_2'$$

g) Which of the two models would you prefer? Give reasons for your answer.

h) What is your conclusion regarding the question at the beginning?

Exercise 12

The following figure shows data from 1995 to 2005, including the total net income in billion TNI (x_t) and the total number of private bankruptcies (y_t) in Germany.



The parameters of the regression model are estimated as

$$\hat{y}_t = -498\ 800.2 + 997.4x_t$$

with a sum of squared residuals $S^0_{\hat{u}\hat{u}} = 4\ 540\ 767\ 881$.

a) At the end of 2001 a new law simplified the insolvency filing. Try to guess where a structural break in the data might be.

b) With the help of dummy variables, set up a regression model, that takes a structural break into account. Sketch the regressor matrix X.

c) The above model is estimated:

$$\hat{y}_t = -\,99\;604.64 - 1\;287\;453.9D_t + 249.32x_t + 2\;261.3D_t x_t$$
$$(79\;061.3) \qquad (462\;758.5) \qquad (145.1) \qquad (778.3)$$
$$S^{\text{SB}}_{\hat{u}\hat{u}} = 617\;588\;408$$

Use the F-test to test for a structural break at a 5% level.

d) Separate regression in the two subgroups yields following results. Fill in the blank spaces and compare with c).

$$\hat{y}^I_t = -\,99\;604.64 + 249.32x_t$$
$$(24\;769.34) \qquad (45.46)$$
$$S^I_{\hat{u}\hat{u}} = 43\;298\;384$$
$$\hat{y}^{II}_t = \underline{\qquad\qquad\qquad} + \underline{\qquad\qquad\qquad} x_t$$
$$(822\;568) \qquad\qquad (1\;380)$$
$$S^{II}_{\hat{u}\hat{u}} = \underline{\qquad\qquad\qquad}$$

Exercise 13

The amount of ice cream sold in a month q_t shall be explained by the price p_t, the expenses for advertisement x_t and dummy variables indicating early, main and late season in an inhomogeneous regression model. The dummy variables are defined as

$$D_{1,t} = \begin{cases} 1 & \text{if } t \text{ is a month from January and April} \\ 0 & \text{else} \end{cases}$$

and accordingly $D_{2,t}$ for the months May to August and $D_{3,t}$ from September to December. Data is available for the months Jan.1997 to May 2000.

a) Formulate a suitable regression model for the amount of ice cream sold. Is it reasonable to include every dummy variable in the model? Review critically.

b) The following regression was performed

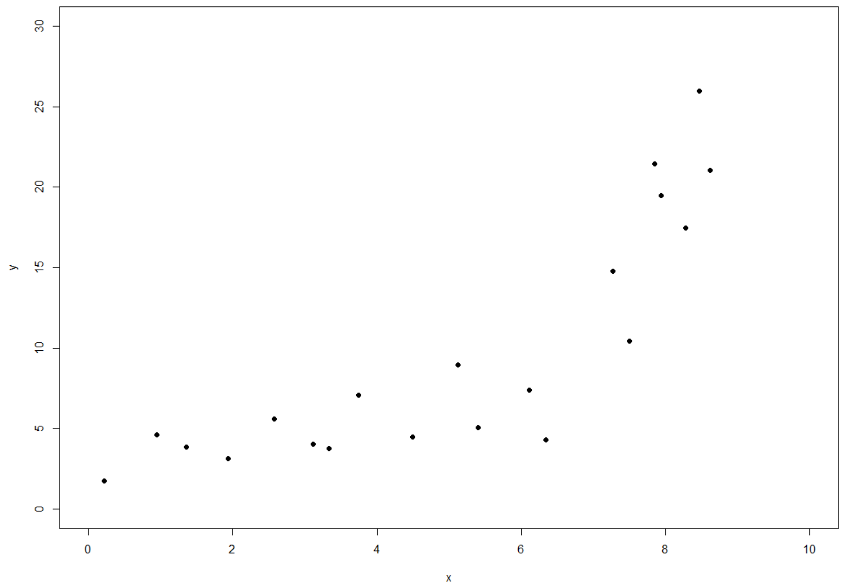
$$\hat{q}_t = 2237.58 - 575.99p_t + 0.13x_t + 440.4D_{2,t} + 22.6D_{3,t}$$
$$(591.55) \quad (218.26) \quad (0.04) \qquad (70.92) \quad (61.59)$$
$$R^2 = 0.778 \qquad\qquad \bar{R}^2 = 0.754$$

i) Comment the sign of estimated coefficients.

ii) Test if the turnover in the main season differs significantly from turnover in early season. (5% level)

Exercise 14

Consider the data from following plot



You are interested in testing for a structural break.

a) Only from visual impression, where would you suspect the break?

b) You separate the data into two groups, where T_1 shall denote the largest index belonging to group 1. You perform an OLS regression on the whole sample, giving $S_{\hat{u}\hat{u}}$ and for $T_1 = 3, ..., 17$ you perform separate OLS regressions, which yields sums of squares residuals $S^I_{\hat{u}\hat{u}}$ and $S^{II}_{\hat{u}\hat{u}}$.

T_1	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
$S^I_{\hat{u}\hat{u}}$	1.5	3.6	4.1	5.6	6.5	10.0	11.8	17.8	21.2	21.3	28.6	70.5	71.2	172.8	210.8
$S^{II}_{\hat{u}\hat{u}}$	262.9	249.6	220.5	209.8	199.1	150.1	139.4	94.7	80.8	71.0	69.5	65.3	36.0	31.1	28.3
Tstat	1.9	2.3	3.6	4.1	4.7	8.3	9.3	15.2	17.6	20.3	18.6	11.2	16.3	4.8	2.9

i) How were the test statistics in the table computed?

ii) Where would suspect a structural break now given the table?

iii) Would your suggestion(s) for structural breaks in a) and ii) be significant at a 5% level?

iv) If you wanted to complete the table for remaining possible values of T_1, how would you calculate a test statistic and what would be its distribution under the null hypothesis of no structural break?

c) Could you perform the above investigation if you hat two exogenous variables and an intercept? Sketch the steps you would take.

d) Is there an other way of dealing with above data besides modeling a structural break?

Exercise 15

Consider a bivariate dynamic model with four lags in x:

$$y_t = 0.55\,(0.02x_t + 0.15x_{t-1} + 0.43x_{t-2} + 0.23x_{t-3} + 0.17x_{t-4}) + u_t.$$

Compute the short run and long run multiplier for given model.

Exercise 16

The monthly sales turnover of a product is modeled using the dynamic model

$$S_t = \alpha + \beta_0 A_t + \beta_1 A_{t-1} + ... + u_t$$

where A_t is the advertising expenses in month t and $u_t \sim \mathcal{N}(0, \sigma^2)$. You assume that the coefficients β_i decay geometrically starting from β_2, so that $\beta_k = \lambda^{k-1}\beta_1$ for $k = 2, 3, ...$, where $0 \leq \lambda < 1$.

1. Perform the Koyck transformation and explain why an OLS estimation is not justified in this situation.
2. How would you estimate the parameters of the model?
3. The estimation gives

$$\hat{S}_t = 286.3 + 0.127A_t + 0.081A_{t-1} + 0.399S_{t-1}$$

   Compute the estimates for λ, α, β_0, β_1 and β_k (k = 2,3,...).

df	90%	92.5%	95%	97.5%	99%	df	90%	92.5%	95%	97.5%	99%
1	3.078	4.165	6.314	12.706	31.821	**51**	1.298	1.462	1.675	2.008	2.402
2	1.886	2.282	2.920	4.303	6.965	**52**	1.298	1.461	1.675	2.007	2.400
3	1.638	1.924	2.353	3.182	4.541	**53**	1.298	1.461	1.674	2.006	2.399
4	1.533	1.778	2.132	2.776	3.747	**54**	1.297	1.460	1.674	2.005	2.397
5	1.476	1.699	2.015	2.571	3.365	**55**	1.297	1.460	1.673	2.004	2.396
6	1.440	1.650	1.943	2.447	3.143	**56**	1.297	1.460	1.673	2.003	2.395
7	1.415	1.617	1.895	2.365	2.998	**57**	1.297	1.459	1.672	2.002	2.394
8	1.397	1.592	1.860	2.306	2.896	**58**	1.296	1.459	1.672	2.002	2.392
9	1.383	1.574	1.833	2.262	2.821	**59**	1.296	1.459	1.671	2.001	2.391
10	1.372	1.559	1.812	2.228	2.764	**60**	1.296	1.458	1.671	2.000	2.390
11	1.363	1.548	1.796	2.201	2.718	**61**	1.296	1.458	1.670	2.000	2.389
12	1.356	1.538	1.782	2.179	2.681	**62**	1.295	1.458	1.670	1.999	2.388
13	1.350	1.530	1.771	2.160	2.650	**63**	1.295	1.457	1.669	1.998	2.387
14	1.345	1.523	1.761	2.145	2.624	**64**	1.295	1.457	1.669	1.998	2.386
15	1.341	1.517	1.753	2.131	2.602	**65**	1.295	1.457	1.669	1.997	2.385
16	1.337	1.512	1.746	2.120	2.583	**66**	1.295	1.456	1.668	1.997	2.384
17	1.333	1.508	1.740	2.110	2.567	**67**	1.294	1.456	1.668	1.996	2.383
18	1.330	1.504	1.734	2.101	2.552	**68**	1.294	1.456	1.668	1.995	2.382
19	1.328	1.500	1.729	2.093	2.539	**69**	1.294	1.456	1.667	1.995	2.382
20	1.325	1.497	1.725	2.086	2.528	**70**	1.294	1.456	1.667	1.994	2.381
21	1.323	1.494	1.721	2.080	2.518	**71**	1.294	1.455	1.667	1.994	2.380
22	1.321	1.492	1.717	2.074	2.508	**72**	1.293	1.455	1.666	1.993	2.379
23	1.319	1.489	1.714	2.069	2.500	**73**	1.293	1.455	1.666	1.993	2.379
24	1.318	1.487	1.711	2.064	2.492	**74**	1.293	1.455	1.666	1.993	2.378
25	1.316	1.485	1.708	2.060	2.485	**75**	1.293	1.454	1.665	1.992	2.377
26	1.315	1.483	1.706	2.056	2.479	**76**	1.293	1.454	1.665	1.992	2.376
27	1.314	1.482	1.703	2.052	2.473	**77**	1.293	1.454	1.665	1.991	2.376
28	1.313	1.480	1.701	2.048	2.467	**78**	1.292	1.454	1.665	1.991	2.375
29	1.311	1.479	1.699	2.045	2.462	**79**	1.292	1.454	1.664	1.990	2.374
30	1.310	1.477	1.697	2.042	2.457	**80**	1.292	1.453	1.664	1.990	2.374
31	1.309	1.476	1.696	2.040	2.453	**81**	1.292	1.453	1.664	1.990	2.373
32	1.309	1.475	1.694	2.037	2.449	**82**	1.292	1.453	1.664	1.989	2.373
33	1.308	1.474	1.692	2.035	2.445	**83**	1.292	1.453	1.663	1.989	2.372
34	1.307	1.473	1.691	2.032	2.441	**84**	1.292	1.453	1.663	1.989	2.372
35	1.306	1.472	1.690	2.030	2.438	**85**	1.292	1.453	1.663	1.988	2.371
36	1.306	1.471	1.688	2.028	2.434	**86**	1.291	1.453	1.663	1.988	2.370
37	1.305	1.470	1.687	2.026	2.431	**87**	1.291	1.452	1.663	1.988	2.370
38	1.304	1.469	1.686	2.024	2.429	**88**	1.291	1.452	1.662	1.987	2.369
39	1.304	1.468	1.685	2.023	2.426	**89**	1.291	1.452	1.662	1.987	2.369
40	1.303	1.468	1.684	2.021	2.423	**90**	1.291	1.452	1.662	1.987	2.368
41	1.303	1.467	1.683	2.020	2.421	**91**	1.291	1.452	1.662	1.986	2.368
42	1.302	1.466	1.682	2.018	2.418	**92**	1.291	1.452	1.662	1.986	2.368
43	1.302	1.466	1.681	2.017	2.416	**93**	1.291	1.452	1.661	1.986	2.367
44	1.301	1.465	1.680	2.015	2.414	**94**	1.291	1.451	1.661	1.986	2.367
45	1.301	1.465	1.679	2.014	2.412	**95**	1.291	1.451	1.661	1.985	2.366
46	1.300	1.464	1.679	2.013	2.410	**96**	1.290	1.451	1.661	1.985	2.366
47	1.300	1.463	1.678	2.012	2.408	**97**	1.290	1.451	1.661	1.985	2.365
48	1.299	1.463	1.677	2.011	2.407	**98**	1.290	1.451	1.661	1.984	2.365
49	1.299	1.462	1.677	2.010	2.405	**99**	1.290	1.451	1.660	1.984	2.365
50	1.299	1.462	1.676	2.009	2.403	∞	1.282	1.440	1.645	1.960	2.326

Tabelle 2: Quantiles of t-distribution

Z →	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	40	50	60	70	80	90	100
N↓ 2	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.40	19.41	19.42	19.42	19.43	19.43	19.44	19.44	19.44	19.45	19.45	19.45	19.45	19.45	19.46	19.46	19.46	19.46	19.46	19.46	19.47	19.48	19.48	19.48	19.48	19.48	19.49
3	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74	8.73	8.71	8.70	8.69	8.68	8.67	8.67	8.66	8.65	8.65	8.64	8.64	8.63	8.63	8.63	8.62	8.62	8.62	8.59	8.58	8.57	8.57	8.56	8.56	8.55
4	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91	5.89	5.87	5.86	5.84	5.83	5.82	5.81	5.80	5.79	5.79	5.78	5.77	5.77	5.76	5.76	5.75	5.75	5.75	5.72	5.70	5.69	5.68	5.67	5.67	5.66
5	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.70	4.68	4.66	4.64	4.62	4.60	4.59	4.58	4.57	4.56	4.55	4.54	4.53	4.53	4.52	4.52	4.51	4.50	4.50	4.50	4.46	4.44	4.43	4.42	4.41	4.41	4.41
6	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00	3.98	3.96	3.94	3.92	3.91	3.90	3.88	3.87	3.86	3.86	3.85	3.84	3.83	3.83	3.82	3.82	3.81	3.81	3.77	3.75	3.74	3.73	3.72	3.72	3.71
7	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57	3.55	3.53	3.51	3.49	3.48	3.47	3.46	3.44	3.43	3.43	3.42	3.41	3.40	3.40	3.39	3.39	3.38	3.38	3.34	3.32	3.30	3.29	3.29	3.28	3.27
8	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28	3.26	3.24	3.22	3.20	3.19	3.17	3.16	3.15	3.14	3.13	3.12	3.12	3.11	3.10	3.10	3.09	3.08	3.08	3.04	3.02	3.01	2.99	2.99	2.98	2.97
9	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07	3.05	3.03	3.01	2.99	2.97	2.96	2.95	2.94	2.93	2.92	2.91	2.90	2.89	2.89	2.88	2.87	2.87	2.86	2.83	2.80	2.79	2.78	2.77	2.76	2.76
10	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91	2.89	2.86	2.85	2.83	2.81	2.80	2.79	2.77	2.76	2.75	2.75	2.74	2.73	2.72	2.72	2.71	2.70	2.70	2.66	2.64	2.62	2.61	2.60	2.59	2.59
11	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79	2.76	2.74	2.72	2.70	2.69	2.67	2.66	2.65	2.64	2.63	2.62	2.61	2.60	2.59	2.59	2.58	2.58	2.57	2.53	2.51	2.49	2.48	2.47	2.46	2.46
12	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69	2.66	2.64	2.62	2.60	2.58	2.57	2.56	2.54	2.53	2.52	2.51	2.51	2.50	2.49	2.48	2.48	2.47	2.47	2.43	2.40	2.38	2.37	2.36	2.36	2.35
13	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60	2.58	2.55	2.53	2.51	2.50	2.48	2.47	2.46	2.45	2.44	2.43	2.42	2.41	2.41	2.40	2.39	2.39	2.38	2.34	2.31	2.30	2.28	2.27	2.27	2.26
14	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53	2.51	2.48	2.46	2.44	2.43	2.41	2.40	2.39	2.38	2.37	2.36	2.35	2.34	2.33	2.33	2.32	2.31	2.31	2.27	2.24	2.22	2.21	2.20	2.19	2.19
15	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48	2.45	2.42	2.40	2.38	2.37	2.35	2.34	2.33	2.32	2.31	2.30	2.29	2.28	2.27	2.27	2.26	2.25	2.25	2.20	2.18	2.16	2.15	2.14	2.13	2.12
16	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.46	2.42	2.40	2.37	2.35	2.33	2.32	2.30	2.29	2.28	2.26	2.25	2.24	2.24	2.23	2.22	2.21	2.21	2.20	2.19	2.15	2.12	2.11	2.09	2.08	2.07	2.07
17	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.41	2.38	2.35	2.33	2.31	2.29	2.27	2.26	2.24	2.23	2.22	2.21	2.20	2.19	2.18	2.17	2.17	2.16	2.15	2.15	2.10	2.08	2.06	2.05	2.03	2.03	2.02
18	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.37	2.34	2.31	2.29	2.27	2.25	2.23	2.22	2.20	2.19	2.18	2.17	2.16	2.15	2.14	2.13	2.13	2.12	2.11	2.11	2.06	2.04	2.02	2.00	1.99	1.98	1.98
19	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.34	2.31	2.28	2.26	2.23	2.21	2.20	2.18	2.17	2.16	2.14	2.13	2.12	2.11	2.11	2.10	2.09	2.08	2.08	2.07	2.03	2.00	1.98	1.97	1.96	1.95	1.94
20	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28	2.25	2.22	2.20	2.18	2.17	2.15	2.14	2.12	2.11	2.10	2.09	2.08	2.07	2.07	2.06	2.05	2.05	2.04	1.99	1.97	1.95	1.93	1.92	1.91	1.91
21	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.28	2.25	2.22	2.20	2.18	2.16	2.14	2.12	2.11	2.10	2.08	2.07	2.06	2.05	2.05	2.04	2.03	2.02	2.02	2.01	1.96	1.94	1.92	1.90	1.89	1.88	1.88
22	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23	2.20	2.17	2.15	2.13	2.11	2.10	2.08	2.07	2.06	2.05	2.04	2.03	2.02	2.01	2.00	2.00	1.99	1.98	1.94	1.91	1.89	1.88	1.86	1.86	1.85
23	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.24	2.20	2.18	2.15	2.13	2.11	2.09	2.08	2.06	2.05	2.04	2.02	2.01	2.01	2.00	1.99	1.98	1.97	1.97	1.96	1.91	1.88	1.86	1.85	1.84	1.83	1.82
24	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.22	2.18	2.15	2.13	2.11	2.09	2.07	2.05	2.04	2.03	2.01	2.00	1.99	1.98	1.97	1.97	1.96	1.95	1.95	1.94	1.89	1.86	1.84	1.83	1.82	1.81	1.80
25	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.20	2.16	2.14	2.11	2.09	2.07	2.05	2.04	2.02	2.01	2.00	1.98	1.97	1.96	1.96	1.95	1.94	1.93	1.93	1.92	1.87	1.84	1.82	1.81	1.80	1.79	1.78
26	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15	2.12	2.09	2.07	2.05	2.03	2.02	2.00	1.99	1.98	1.97	1.96	1.95	1.94	1.93	1.92	1.91	1.91	1.90	1.85	1.82	1.80	1.79	1.78	1.77	1.76
27	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.17	2.13	2.10	2.08	2.06	2.04	2.02	2.00	1.99	1.97	1.96	1.95	1.94	1.93	1.92	1.91	1.90	1.90	1.89	1.88	1.84	1.81	1.79	1.77	1.76	1.75	1.74
28	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15	2.12	2.09	2.06	2.04	2.02	2.00	1.99	1.97	1.96	1.95	1.93	1.92	1.91	1.91	1.90	1.89	1.88	1.88	1.87	1.82	1.79	1.77	1.75	1.74	1.73	1.73
29	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.14	2.10	2.08	2.05	2.03	2.01	1.99	1.97	1.96	1.94	1.93	1.92	1.91	1.90	1.89	1.88	1.88	1.87	1.86	1.85	1.81	1.77	1.75	1.74	1.73	1.72	1.71
30	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09	2.06	2.04	2.01	1.99	1.98	1.96	1.95	1.93	1.92	1.91	1.90	1.89	1.88	1.87	1.86	1.85	1.85	1.84	1.79	1.76	1.74	1.72	1.71	1.70	1.70
40	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00	1.97	1.95	1.92	1.90	1.89	1.87	1.85	1.84	1.83	1.81	1.80	1.79	1.78	1.77	1.77	1.76	1.75	1.74	1.69	1.66	1.64	1.62	1.61	1.60	1.59
50	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.99	1.95	1.92	1.89	1.87	1.85	1.83	1.81	1.80	1.78	1.77	1.76	1.75	1.74	1.73	1.72	1.71	1.70	1.69	1.69	1.63	1.60	1.58	1.56	1.54	1.53	1.52
60	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92	1.89	1.86	1.84	1.82	1.80	1.78	1.76	1.75	1.73	1.72	1.71	1.70	1.69	1.68	1.67	1.66	1.66	1.65	1.59	1.56	1.53	1.52	1.50	1.49	1.48
70	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02	1.97	1.93	1.89	1.86	1.84	1.81	1.79	1.77	1.75	1.74	1.72	1.71	1.70	1.68	1.67	1.66	1.65	1.65	1.64	1.63	1.62	1.57	1.53	1.50	1.49	1.47	1.46	1.45
80	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95	1.91	1.88	1.84	1.82	1.79	1.77	1.75	1.73	1.72	1.70	1.69	1.68	1.67	1.65	1.64	1.63	1.63	1.62	1.61	1.60	1.54	1.51	1.48	1.46	1.45	1.44	1.43
90	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94	1.90	1.86	1.83	1.80	1.78	1.76	1.74	1.72	1.70	1.69	1.67	1.66	1.65	1.64	1.63	1.62	1.61	1.60	1.59	1.59	1.53	1.49	1.46	1.44	1.43	1.42	1.41
100	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.89	1.85	1.82	1.79	1.77	1.75	1.73	1.71	1.69	1.68	1.66	1.65	1.64	1.63	1.62	1.61	1.60	1.59	1.58	1.57	1.52	1.48	1.45	1.43	1.41	1.40	1.39

Tabelle 3: 95% Quantiles F(n, m) Distribution

df	0.1%	0.5%	1%	2.5%	5%	10%	90%	95%	97.5%	99%	99.5%	99.9%
1	0.00	0.00	0.00	0.00	0.00	0.02	2.71	3.84	5.02	6.63	7.88	10.83
2	0.00	0.01	0.02	0.05	0.10	0.21	4.61	5.99	7.38	9.21	10.60	13.82
3	0.02	0.07	0.12	0.22	0.35	0.58	6.25	7.82	9.35	11.35	12.84	16.27
4	0.09	0.21	0.30	0.48	0.71	1.06	7.78	9.49	11.14	13.28	14.86	18.47
5	0.21	0.41	0.55	0.83	1.14	1.61	9.24	11.07	12.83	15.09	16.75	20.52
6	0.38	0.68	0.87	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55	22.46
7	0.60	0.99	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28	24.32
8	0.86	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.54	20.09	21.95	26.12
9	1.15	1.74	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59	27.88
10	1.48	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19	29.59
11	1.83	2.60	3.05	3.82	4.58	5.58	17.27	19.68	21.92	24.73	26.76	31.26
12	2.21	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30	32.91
13	2.62	3.56	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82	34.53
14	3.04	4.08	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32	36.12
15	3.48	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80	37.70
16	3.94	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.84	32.00	34.27	39.25
17	4.42	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72	40.79
18	4.91	6.26	7.01	8.23	9.39	10.87	25.99	28.87	31.53	34.80	37.16	42.31
19	5.41	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58	43.82
20	5.92	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00	45.31
21	6.45	8.03	8.90	10.28	11.59	13.24	29.61	32.67	35.48	38.93	41.40	46.80
22	6.98	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80	48.27
23	7.53	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18	49.73
24	8.09	9.89	10.86	12.40	13.85	15.66	33.20	36.41	39.36	42.98	45.56	51.18
25	8.65	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93	52.62
26	9.22	11.16	12.20	13.84	15.38	17.29	35.56	38.88	41.92	45.64	48.29	54.05
27	9.80	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.20	46.96	49.65	55.48
28	10.39	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99	56.89
29	10.99	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34	58.30
30	11.59	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67	59.70
31	12.20	14.46	15.65	17.54	19.28	21.43	41.42	44.98	48.23	52.19	55.00	61.10
32	12.81	15.13	16.36	18.29	20.07	22.27	42.59	46.19	49.48	53.49	56.33	62.49
33	13.43	15.81	17.07	19.05	20.87	23.11	43.74	47.40	50.73	54.78	57.65	63.87
34	14.06	16.50	17.79	19.81	21.66	23.95	44.90	48.60	51.97	56.06	58.96	65.25
35	14.69	17.19	18.51	20.57	22.46	24.80	46.06	49.80	53.20	57.34	60.27	66.62
36	15.32	17.89	19.23	21.34	23.27	25.64	47.21	51.00	54.44	58.62	61.58	67.98
37	15.96	18.59	19.96	22.11	24.07	26.49	48.36	52.19	55.67	59.89	62.88	69.35
38	16.61	19.29	20.69	22.88	24.88	27.34	49.51	53.38	56.90	61.16	64.18	70.70
39	17.26	20.00	21.43	23.65	25.70	28.20	50.66	54.57	58.12	62.43	65.48	72.06
40	17.92	20.71	22.16	24.43	26.51	29.05	51.80	55.76	59.34	63.69	66.77	73.40
41	18.57	21.42	22.91	25.21	27.33	29.91	52.95	56.94	60.56	64.95	68.05	74.75
42	19.24	22.14	23.65	26.00	28.14	30.77	54.09	58.12	61.78	66.21	69.34	76.08
43	19.91	22.86	24.40	26.79	28.96	31.62	55.23	59.30	62.99	67.46	70.62	77.42
44	20.58	23.58	25.15	27.57	29.79	32.49	56.37	60.48	64.20	68.71	71.89	78.75
45	21.25	24.31	25.90	28.37	30.61	33.35	57.51	61.66	65.41	69.96	73.17	80.08
46	21.93	25.04	26.66	29.16	31.44	34.22	58.64	62.83	66.62	71.20	74.44	81.40
47	22.61	25.77	27.42	29.96	32.27	35.08	59.77	64.00	67.82	72.44	75.70	82.72
48	23.30	26.51	28.18	30.75	33.10	35.95	60.91	65.17	69.02	73.68	76.97	84.04
49	23.98	27.25	28.94	31.55	33.93	36.82	62.04	66.34	70.22	74.92	78.23	85.35
50	24.67	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49	86.66

Tabelle 4: Quantiles χ^2_{df} Distribution