New York University
Applied Statistics and Econometrics II
Fall 2016

Problem Set 4

(I) Consider a linear model $y_t = x_t\beta + \epsilon_t$ with autocorrelated disturbances $\epsilon_t = \rho\epsilon_{t-1} + u_t$. Can we reduce the autocorrelation by first-differencing the data: $\Delta y_t = \Delta x_t \beta + \nu_t$?

(II) Let $\{X_t\}$ be an ARMA(1,1) process defined by:

$$X_t = \phi X_{t-1} + \epsilon_t + \theta\epsilon_{t-1} \tag{1}$$

where $\epsilon \sim N(0, \sigma^2)$, and the parameters satisfy: $|\phi| < 1$, $|\theta| < 1$. Find the autocorrelation function of $\{X_t\}$. Explain the usefulness of this function.

(III) The time series for $\{S_t\}$ is a process defined by:

$$S_t = \mu_0 + \mu_1 \cdot t + \epsilon_t \tag{2}$$

Here, μ_0 and μ_1 are constant trend parameters and $\epsilon \sim N(0, \sigma^2)$ is an independently distributed random disturbance. Prove that:

1. $\{S_t\}$ is a non-stationary process.
2. The first-difference series defined by $\Delta S_t := S_t - S_{t-1}$ is stationary.

Find the order of stationarity for the process ΔS_t and calculate its autocorrelation function.

(IV) Consider example 21.3 in Greene, where he reviews the estimation by Dickey and Fuller of a unit root for the output in the US. We will re-estimate the model with an augmented sample in this problem. The model is:

$$y_t = \mu + \beta \cdot t + \gamma y_{t-1} + \phi(y_{t-1} - y_{t-2}) + \epsilon_t \tag{3}$$

1. Download data for the Federal Reserve Board Production Index in `http://research.stlouisfed.org/fred2/series/INDPRO/` until the last available data point. Our dependent variable will be the logarithm of the quarterly series of output.
2. Determine the optimal lag length in the augmented regression. Estimate:

$$y_t = \mu + \beta \cdot t + \gamma y_{t-1} + \sum_{j=1}^{p} \gamma_j \Delta y_{t-j} + \epsilon_t \tag{4}$$

   Examine which coefficients are significant and use an information criterion to decide. Explain the logic of this procedure.

3. Compute the relevant DF statistics for this regression.
4. Conclude: is there enough evidence to say that the output has a unit root?

(V) Download data for ALCOA from January 2000 until today from `finance.google.com`. Are the dialy log-returns of ALCOA predictable? Test this hypothesis:

1. Using the first 5 lags of the autocorrelation function.
2. Using the first 10 lags of the autocorrelation function

Draw your conclusion using a 5% significance level.

(VI) Solve application 2 in chapter 21 of Greene's textbook.

Carry out an ADF test for a unit root in the rate of inflation using the subset of the data in Appendix Table F5.2 since 1974.1. (This is the first quarter after the oil shock of 1973.)

(VII) Consider a Gaussian $MA(1)$ process:

$$Y_t = \mu + \epsilon + \theta\epsilon_{t-1} \tag{5}$$

Let $\boldsymbol{\theta} = (\mu, \theta, \sigma^2)$. Show that the log-conditional likelihood is given by:

$$\mathcal{L}(\boldsymbol{\theta}) = \log f_{Y_T,\ldots,Y_1|\epsilon_0=0}(y_T, \ldots, y_1|\epsilon_0 = 0; \boldsymbol{\theta}) = -\frac{T}{2}\log(2\pi) - \frac{T}{2}\log(\sigma^2) - \sum_{t=1}^{T}\frac{\epsilon_t^2}{2\sigma^2} \tag{6}$$