New York University
Applied Statistics and Econometrics II
Spring 2016

Problem Set 1

(I) We want to estimate the parameters of the joint distribution of two random variables X and Y. X is normally distributed with unknown mean μ and unit variance. Y is an exponential with unknown parameter λ. We model the join distribution with the copula:

$$\mathfrak{C}(u_1, u_2) = \left(u_1^{-\theta} + u_2^{-\theta} - 1\right)^{-\frac{1}{\theta}} \tag{1}$$

Write down the log-likelihood function of the copula-likelihood procedure.

(II) You have two sets of data A and B, with uncorrelated observations. Observations in A are drawn from a distribution with mean μ. Observations in B come from a distribution with mean $\mu^2 + 10$. The sample means are $\bar{y}_A = 7$, $\bar{y}_B = 25$, and the estimated variances are $s_A^2 = 1$, $s_B^2 = 4$.

1. Write the quadratic form of μ that is minimized at the asymptotically efficient GMM estimate of μ.
2. Calculate the numerical value of this estimate.
3. Calculate the value of the GMM test statistic testing the assumption about the relationship between the means of samples A and B. State the asymptotic null distribution of this test statistic, including degrees of freedom.

(III) Use the data included in *klein.xls* to estimate the following linear model using GMM:

$$C_t = \alpha + \beta_1 P_t + \beta_2 P_{t-1} + \beta_3 W_p g$$

Your code should be able to estimate using the optimal weighting matrix, and provide estimators for the residual variance and the significance tests. Test also for the validity of the moment conditions.

(IV) In this problem we dig more deeply into GMM estimation. We follow the methodology in Hansen and Singleton (1983).

In this model, investors maximize lifetime utility:

$$\max \mathbb{E}_t \left(\sum_{i=0}^{\infty} \beta^i u(C_{t+i}) \right) \tag{2}$$

This gives the following FOC for a CRRA specification:

$$C_t^{-\gamma} = \beta \mathbb{E}_t \left((1 + \mathfrak{R}_{i,t+1}) C_{t+1}^{-\gamma} \right) \tag{3}$$

Alternatively:

$$\mathbb{E}_t \left(1 - \beta(1 + \mathfrak{R}_{i,t+1})\frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}}\right) = 0 \tag{4}$$

This implies that an econometrician who observes M instruments in the vector x_t can conjecture the following orthogonality conditions:

$$\mathbb{E}_t \left(\left(1 - \beta(1 + \mathfrak{R}_{i,t+1})\frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}}\right) x_t\right) = 0 \tag{5}$$

We can define accordingly:

$$h(\theta, w_t) = \begin{pmatrix} \left(1 - \beta(1 + \mathfrak{R}_{1,t+1})\frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}}\right) x_t \\ \left(1 - \beta(1 + \mathfrak{R}_{2,t+1})\frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}}\right) x_t \\ \cdots \\ \left(1 - \beta(1 + \mathfrak{R}_{N,t+1})\frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}}\right) x_t \end{pmatrix}$$

The sample moments are given by:

$$g(\theta, y_T) = \frac{1}{T}\sum_{t=1}^{T} h(\theta, w_t) \tag{6}$$

As in any GMM estimation, we minimize the objective function:

$$Q(\theta, y_T) = g(\theta, y_T)' \cdot W_T \cdot g(\theta, y_T) \tag{7}$$

The information to test this consumption model is included in the file *Returns.xls*. The description of the variables can be found in the second sheet. Observe that you can choose to estimate the model with some subset of the assets and include or not some instrumental variables in x_t (if you choose not to put any special instrument in it, it is like you are testing only against the constant as instrument). We will call the assets we put in the moment conditions *test assets*. The variables in x_t will be *instruments*.

1. Create a computer routine to estimate by GMM the parameters of the model. In all your calculations, you are allowed (and I recommend you) to specify your functions so that you just set $\beta = 1$ and optimize in one variable. The optimal value of β will not be important. Use the two-step procedure described in Greene.
2. Estimate the GMM with two test assets: *vwr* and *tbill*, and no additional instruments except the constant. Make sure you report the estimated parameter for each step (1 and 2), the estimated standard error of the last step. Include in your table as well the J stat and p-values.