New York University
Applied Statistics and Econometrics II
Fall 2016

Problem Set 5

(I) Suppose that $r_1, \ldots, r_n$ are observations of return series that follows the AR(1)-GARCH(1,1) model:

$$r_t = \mu + \phi_1 r_{t_1} + a_t \ , \qquad a_t = \sigma_t \epsilon_t \tag{1}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{2}$$

Here, ϵ is a standard Gaussian white noise.

1. Calculate the unconditional variance of r_t.
2. Derive the conditional log-likelihood of the data.
3. Estimate the standard error of a your estimate for the unconditional variance and the one- and two-step ahead forecast of the conditional variance.

(II) We have data for stocks in the file *paribas.xlsx*. We will estimate several GARCH models using these returns.

1. Calculate the daily returns for this share. Take a look at your data. Are there any obvious data errors? Correct if you find any.
2. Estimate a GARCH(1,1). Calculate and plot the annualized volatility.
3. Test for residual autocorrelation and residual heteroskedasticity.
4. Test the hypothesis that this is correctly specified vs having one more lag in ARCH or GARCH.

(III) Co-integration theory can be used to detect arbitrage opportunities in trading. Consider the monthly 1-year and 10-year Treasury constant maturity rates from October 1968 to October 2015.

1. Are the two series cointegrated?
2. Are the two series threshold-cointegrated? Use the interest spread $s_t := r_{10,t} - r_{1,t}$ as the threshold variable, where $r_{i,t}$ is the $i-$year Treasury constant maturity rate.
3. If they are threshold-cointegrated, build a multivariate model for the two series.

(IV) In this problem we work with Clark (1987) and his model of unobserved components:

$$\begin{aligned}
y_t =& n_t + x_t \\
n_t =& g_{t-1} + n_{t-1} + v_t, \qquad v_t \sim \mathbf{N}\left(0, \sigma_v^2\right) \\
g_t =& g_{t-1} + w_t, \qquad w_t \sim \mathbf{N}\left(0, \sigma_w^2\right) \\
x_t =& \phi_1 x_{t-1} + \phi_2 x_{t-2} + e_t, \qquad \sim \mathbf{N}\left(0, \sigma_e^2\right)
\end{aligned}$$

1. Write down the model in state-space representation.
2. Estimate this model using macroeconomic data from the FRED database: `https://research.stlouisfed.org/fred2/`.

(V) Consider Hamilton's growth model (1989) described by the relation:

$$(\Delta y_t - \mu_{s_t}) = \phi_1 \left(\Delta y_{t-1} - \mu_{s_{t-1}}\right) + \cdots + \phi_4 \left(\Delta y_{t-4} - \mu_{s_{t-4}}\right) + e_t$$

with:

$$e_t \sim \mathbf{N}\left(0, \sigma^2\right)$$

$$\mu_{s_t} = \mu_0 (1 - S_t) + \mu_1 S_t$$

$$\mathbb{P}(S_t = 1 | S_{t-1} = 1) = p, \qquad \mathbb{P}(S_t = 0 | S_{t-1} = 0) = q$$

1. Estimate the model for real GDP using the sample period 1952:II - 1984:IV.
2. Extend the sample to 1952:II - 1995:III. Are the parameters obtained reasonable? Explain.
3. The model now is replaced by:

$$\mu_{s_t} = (\mu_0 + \mu_0^* D_t)(1 - S_t) + (\mu_1 + \mu_1^* D_t) S_t$$

The inclusion of a dummy variable tries to capture the change in mean growth during expansions versus contractions. Estimate this modified model and report the differences with the previous approach.