Problem Set 1
MFE 403: Stochastic Calculus
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Question 1

Given that the Risk free rate is 10%, the pay off diagram and risk-free asset are:

(a).

$$\$50 = \begin{cases} \$65 = 50 \times 1.3 \\ \$40 = 50 \times 0.8 \end{cases} \tag{1}$$

$$C^{k=55} = \begin{cases} (\$65 - \$55)^+ = \$10(\Phi(u)) \\ (\$40 - \$55)^+ = \$0(\Phi(d)) \end{cases} \tag{2}$$

We need a portfolios that solves

$$1.10x + 65y = 10$$
$$1.10x + 40y = 0$$

Solves the equation and answer is:

$$\begin{cases} y = 0.4 \\ x = -14.55 \end{cases} \tag{3}$$

To perfectly replicate the portfolio

- Buy 0.4 shares of the stock
- Borrow \$14.55 at the risk free rate

(b).

$$C^{k=55}(50) = x + y \times 100 = -14.55 + 0.4 \times 50 = \$5.45$$

(c).

$$q_u = \frac{(1+R) - d}{u - d} = 0.60$$
$$q_d = 1 - 0.6 = 0.40$$

(d).

$$C^{k=55}(50) = \frac{q_u \times \Phi(u) + q_d \times \Phi(d)}{1+R} = \frac{0.6 \times 10 + 0.4 \times 0}{1.1} = \$5.45$$