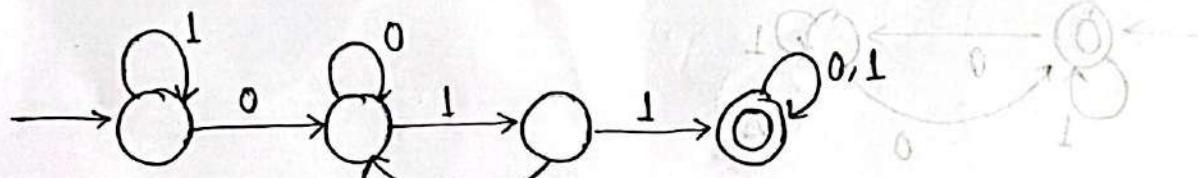


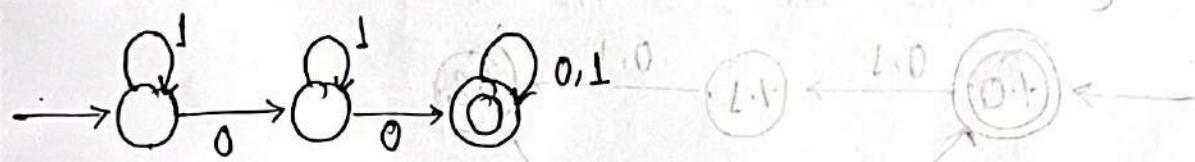
Type 1

$$L = \{ w \in \{0,1\}^*: w \text{ contains } 011 \}$$



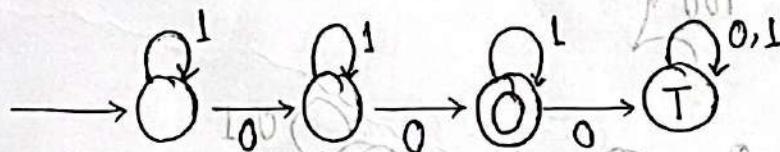
Type 2

$$L = \{ w | w \text{ contains at least two } 0's \} \quad [\text{two or more zeros}]$$



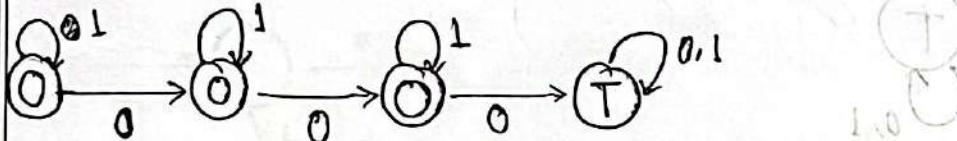
Type 3

$$L = \{ w | w \text{ contains exactly two } 0's \} \quad [\text{neither 1 nor 3}]$$



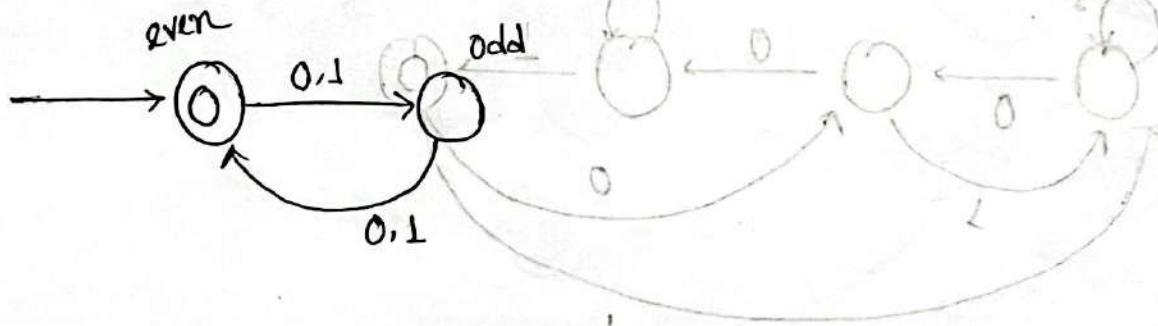
Type 4

$$L = \{ w | w \text{ contains at most two } 0's \} \quad [\text{Not up to 2}]$$



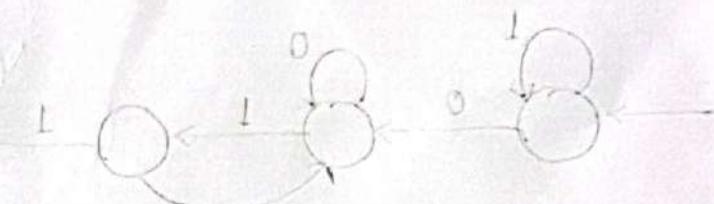
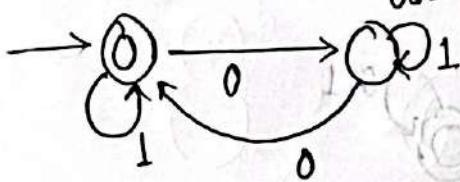
Type 5

$$L = \{ w | w \text{ length of } w \text{ is even} \}$$



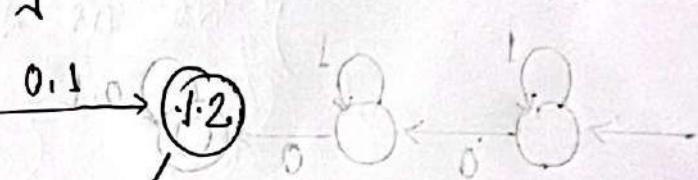
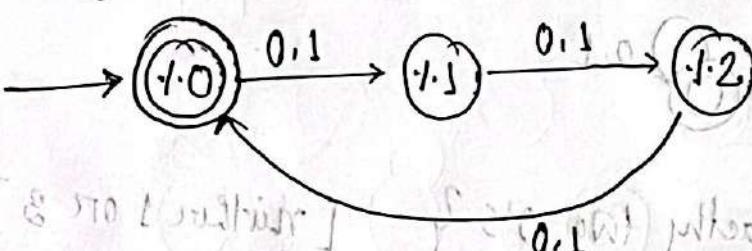
Type 6

$L = \{ w | w \text{ count of } 0's \text{ is even} \}$



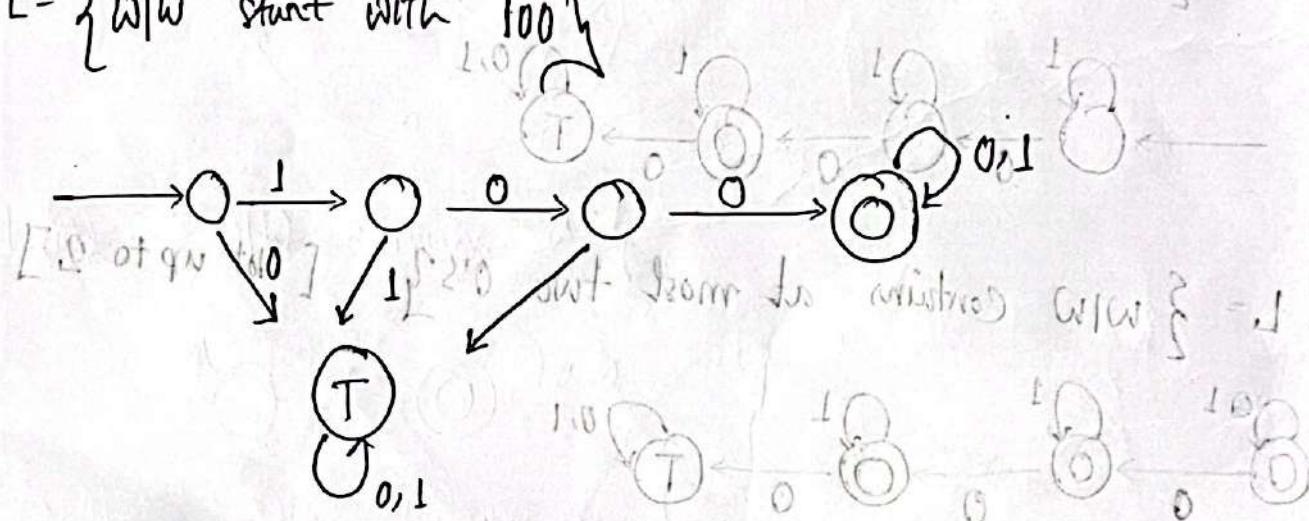
Type 7

$L = \{ w | w \text{ multiple of } 3 \}$



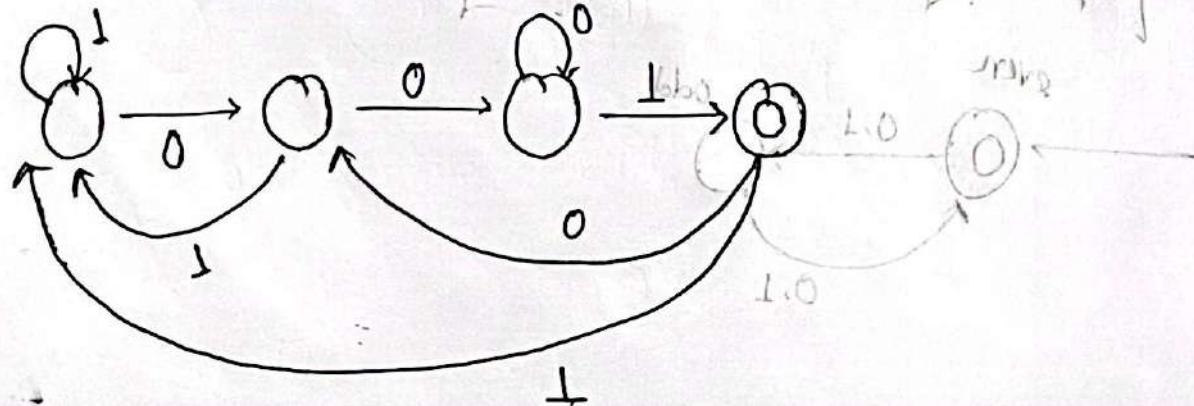
Type 8

$L = \{ w | w \text{ starts with } 100 \}$

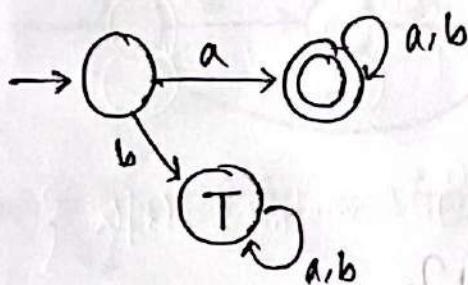


Type 9

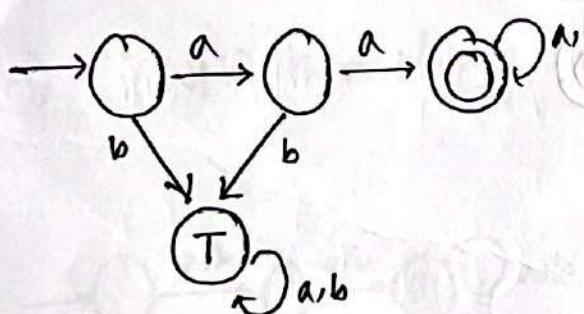
$L = \{ w | w \text{ ends with } 001 \}$



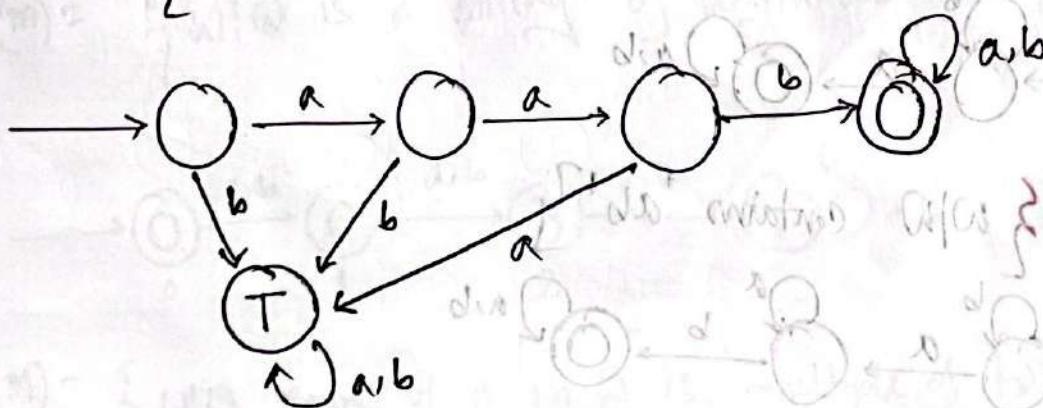
* $L(m) = \{ w | w \text{ starts with an 'a'} \}$



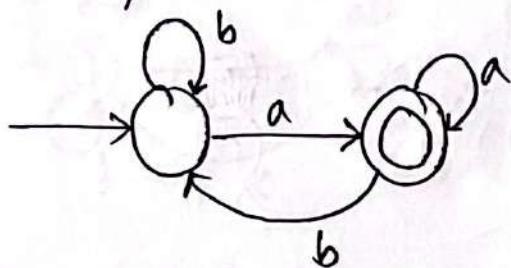
* $L(m) = \{ w | w \text{ starts with 'aa'} \}$



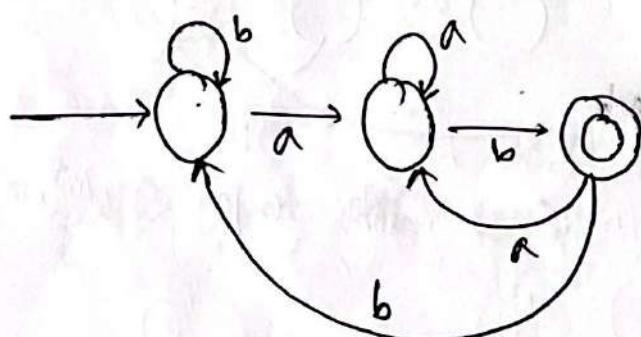
* $L(m) = \{ w | w \text{ starts with 'aab'} \}$



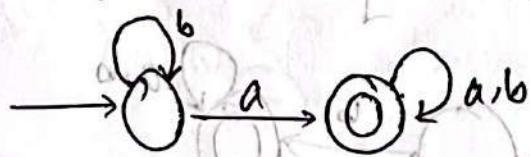
* $L(M) = \{ w | w \text{ ends with an } 'a' \}$



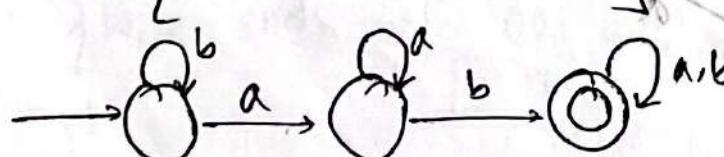
* $L(M) = \{ w | w \text{ ends with an } 'ab' \}$



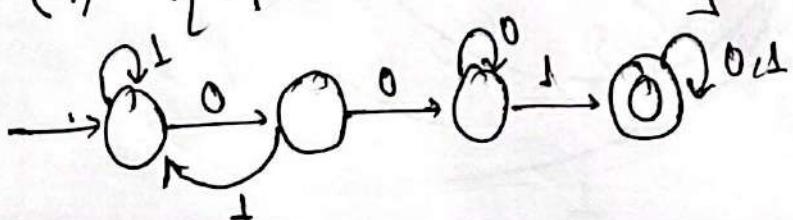
* $L(M) = \{ w | w \text{ contains } 'a' \}$



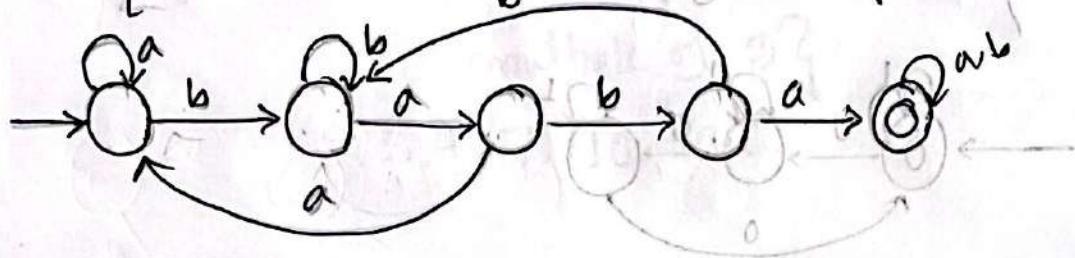
* $L(M) = \{ w | w \text{ contains } 'ab' \}$



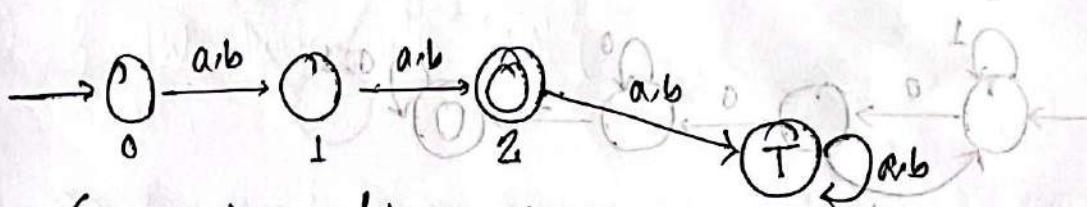
* $L(M) = \{ w | w \text{ contains } '001' \}$



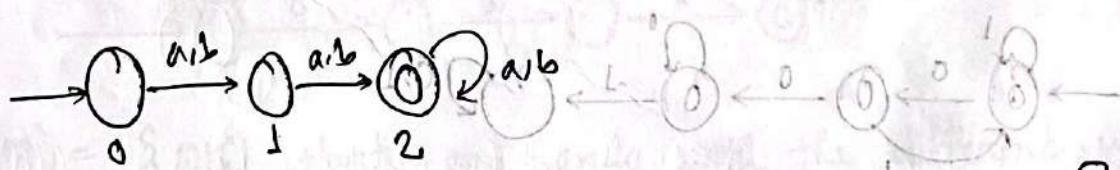
* $L(m) = \{w \mid w \text{ contains substring } 'babab'\}$



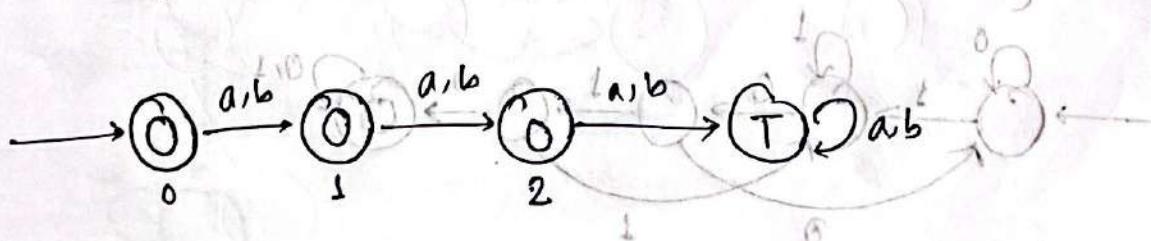
* $L(m) = \{w \mid w \text{ is a string of length } 2\}$



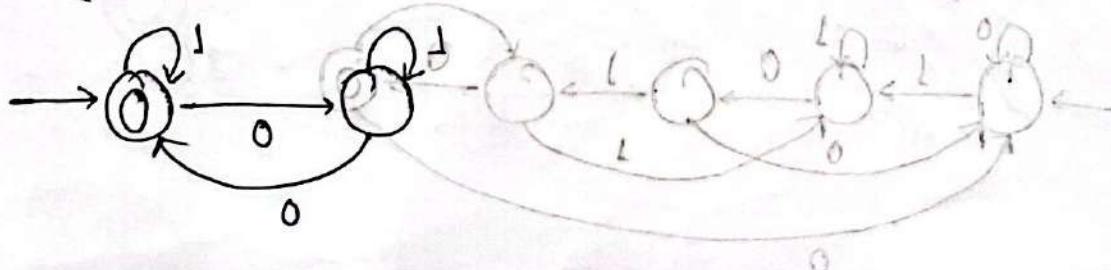
* $L(m) = \{w \mid w \text{ is a string of length at least } 2\}$



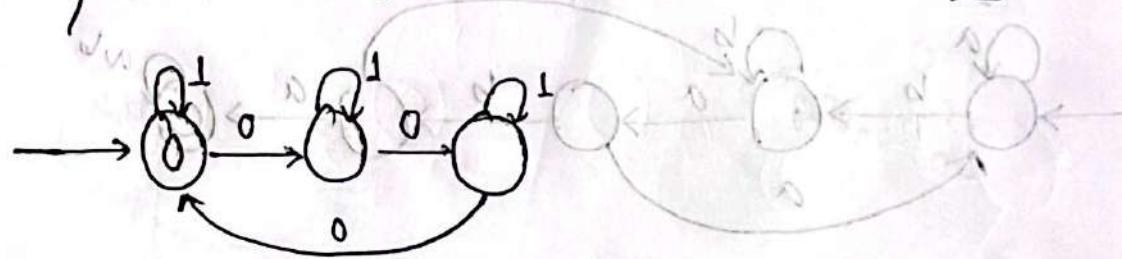
* $L(m) = \{w \mid w \text{ is a string of length at most } 2\}$



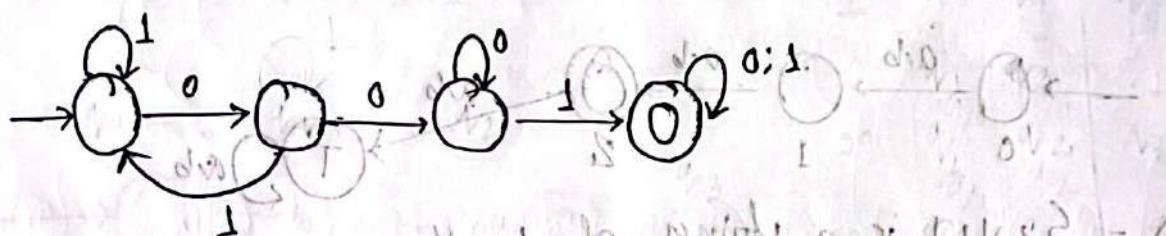
* $L(m) = \{w \mid \text{count of } 0's \text{ in } w \text{ is multiple of two}\}$



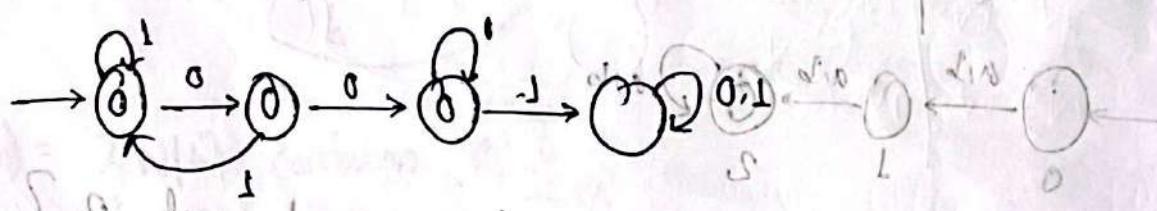
* $L(m) = \{ w | w \text{ count of } 2\text{ zeros is multiple of } 3 \}$



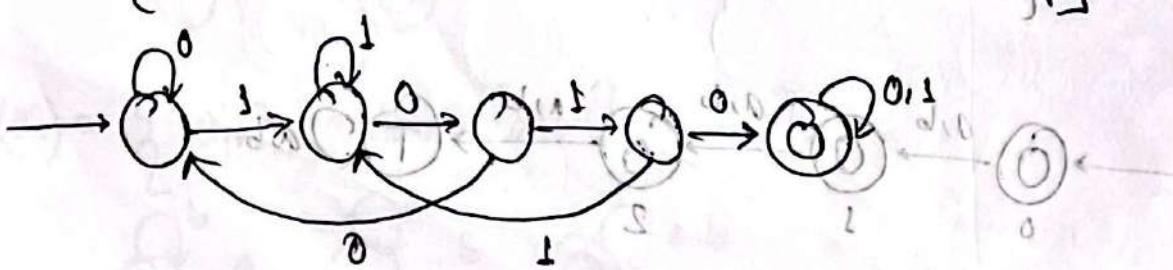
* $L(m) = \{ w | w \text{ contains } 001 \}$



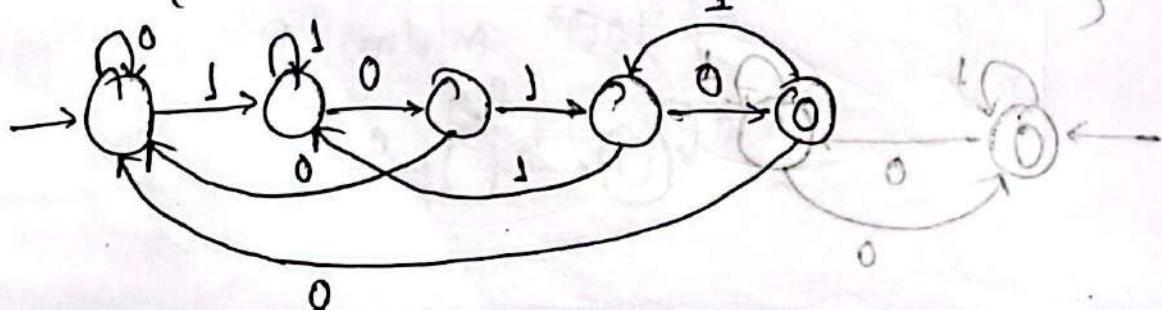
* $L(m) = \{ w | w \text{ does not contain } 001 \}$



* $L(m) = \{ w | w \text{ contains } 1010 \text{ as substring} \}$



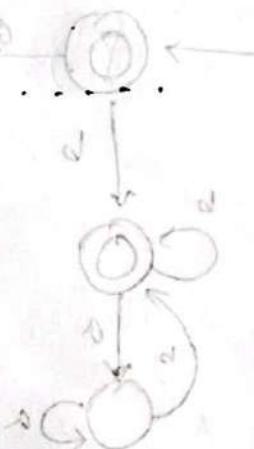
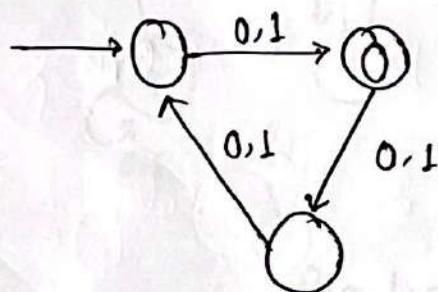
* $L(m) = \{ w | w \text{ ends with } 1010 \}$



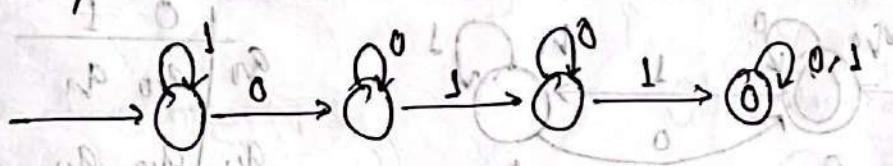
$L(m) = \{w | w \text{ length of the string is one more than multiple of } 3\}$

$\{3, 6, 9, 12, 15, \dots\}$

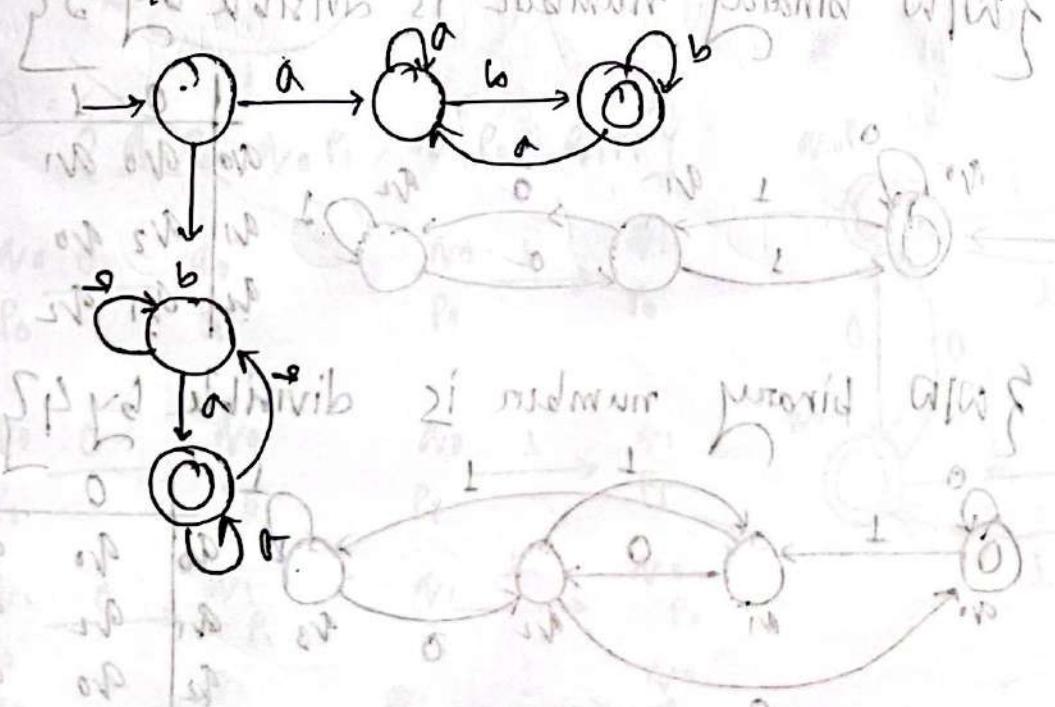
$1, 4, 7, 10, 13, \dots$



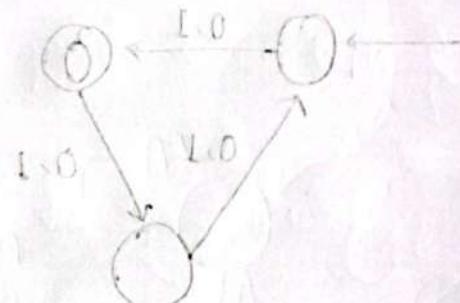
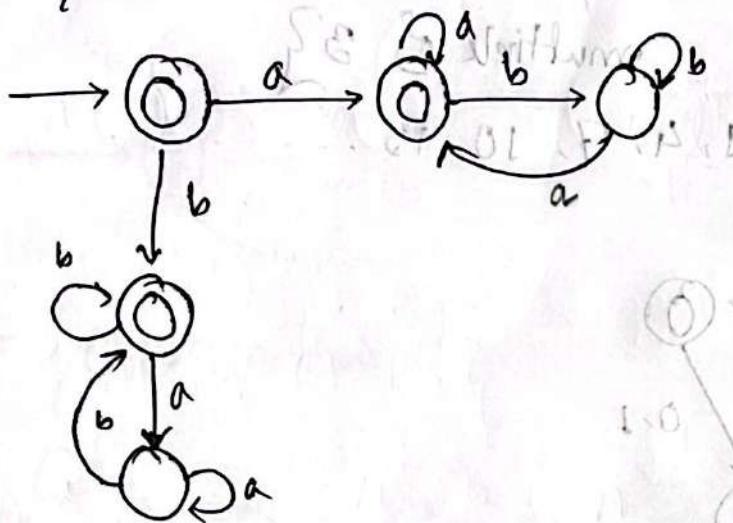
$L(m) = \{w | w \text{ have 011 as subsequence}\}$



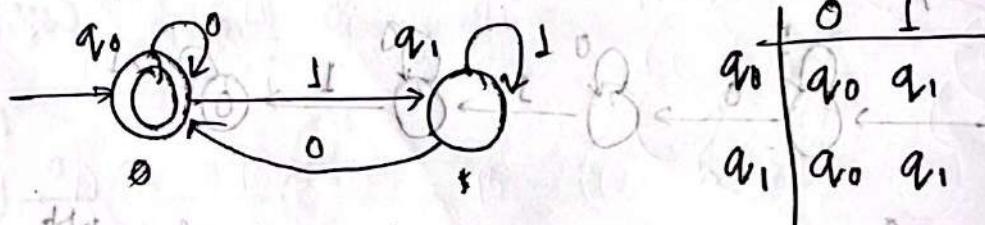
$L(m) = \{w | w \text{ starts and ends with the different symbol}\}$



* $L(m) = \{ w | w \text{ starts and ends with same symbol} \}$



* $L(m) = \{ w | w \text{ binary number is divisible by } 2 \}$

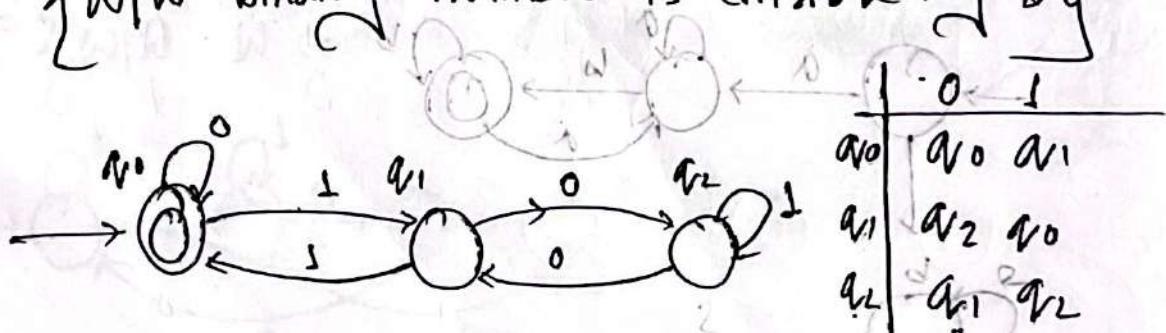


$$\begin{array}{r} 0 \\ \hline 2 \mid 0 \end{array}$$

$$\begin{array}{r} 0 \\ \hline 2 \mid 0 \quad 0 \\ \hline 0 \end{array}$$

\downarrow always starts with 0 binary number starts with 0?

* $L(m) = \{ w | w \text{ binary number is divisible by } 3 \}$

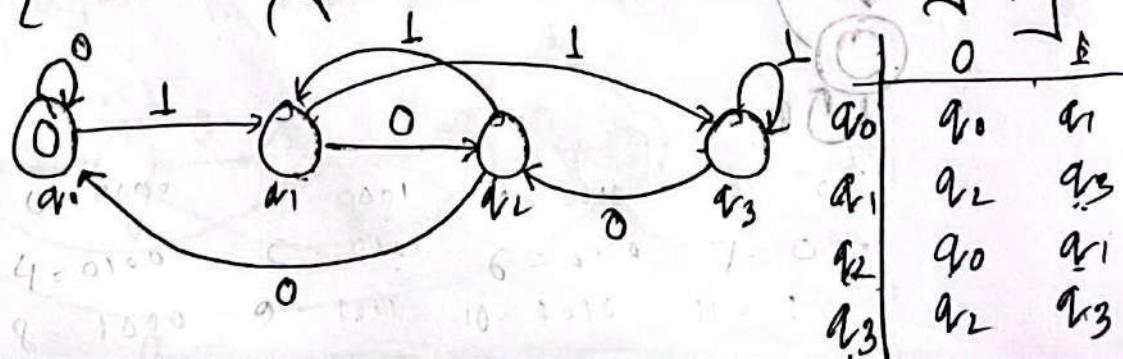


$$\begin{array}{r} 1 \cdot 0 \cdot 1 \\ \hline 3 \mid 1 \quad 0 \quad 1 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 1 \cdot 0 \cdot 1 \\ \hline 3 \mid 1 \quad 0 \quad 1 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 1 \cdot 0 \cdot 1 \\ \hline 3 \mid 1 \quad 0 \quad 1 \\ \hline 0 \end{array}$$

* $L(m) = \{ w | w \text{ binary number is divisible by } 4 \}$

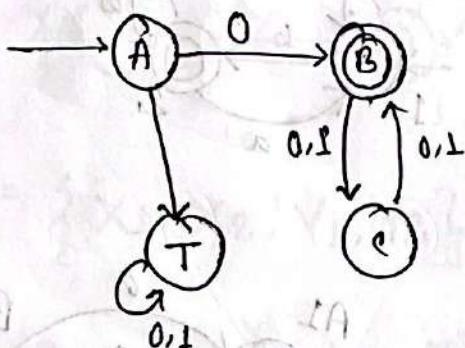


$$\begin{array}{r} 1 \cdot 0 \cdot 1 \\ \hline 4 \mid 1 \quad 0 \quad 1 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 1 \cdot 0 \cdot 1 \\ \hline 4 \mid 1 \quad 0 \quad 1 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 1 \cdot 0 \cdot 1 \\ \hline 4 \mid 1 \quad 0 \quad 1 \\ \hline 0 \end{array}$$

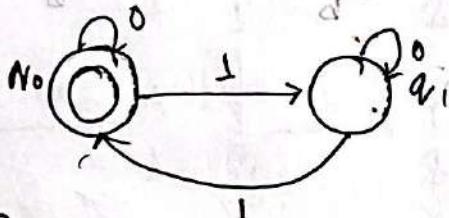
$L = \{w | w \text{ starts with } 0 \text{ and has odd length}\}$



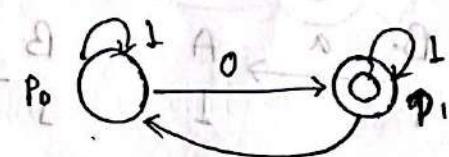
Regular Operation

$L = \{w | w \text{ has even number of 1's or odd number of 0's}\}$

Even number of 1's



Odd number of 0's



$Q = \{q_0 p_0, q_0 p_1, q_1 p_0, q_1 p_1\}$

$$q_0 \xrightarrow{0} q_0 \\ p_0$$

$$q_0 \xrightarrow{1} q_1 \\ p_0$$

$$q_0 \xrightarrow{0} q_0 \\ p_1$$

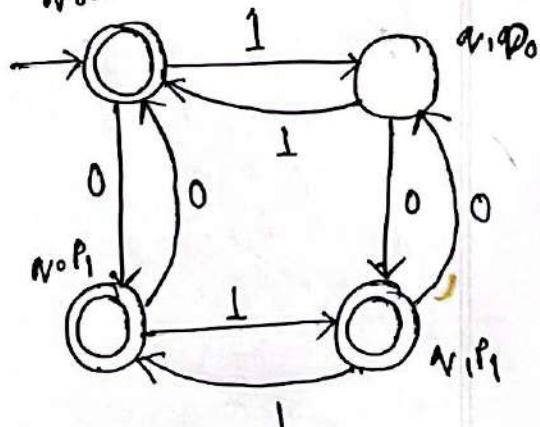
$$q_0 \xrightarrow{1} q_1 \\ p_1$$

$$q_1 \xrightarrow{0} q_1 \\ p_0$$

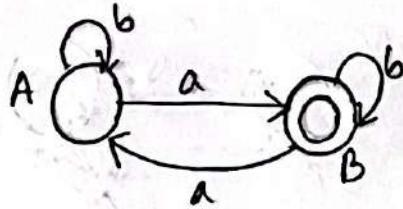
$$q_1 \xrightarrow{1} q_0 \\ p_0$$

$$q_1 \xrightarrow{0} q_0 \\ p_1$$

$$q_1 \xrightarrow{1} q_1 \\ p_1$$

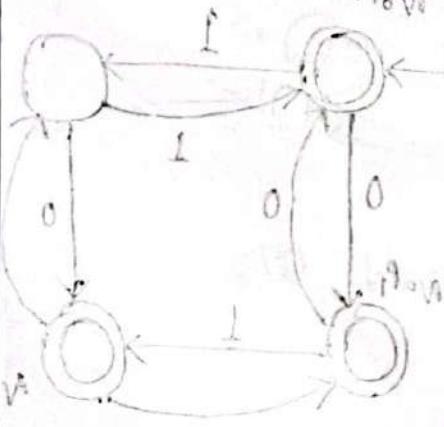
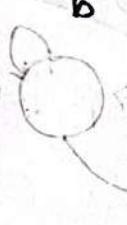
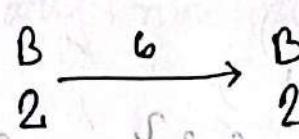
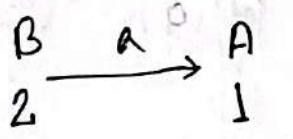
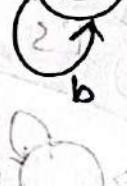
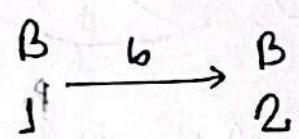
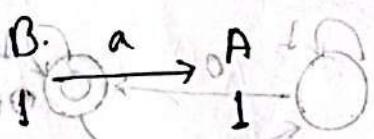
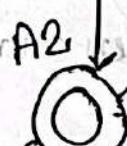
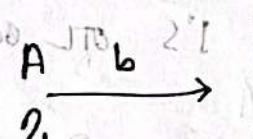
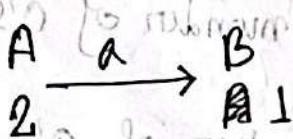
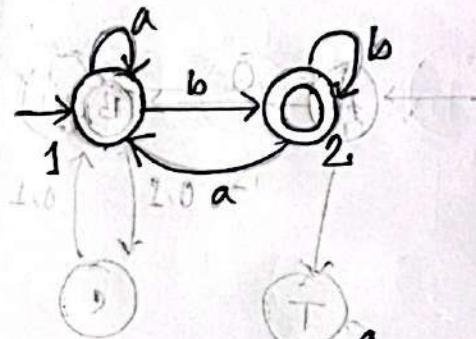
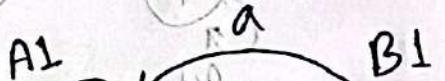
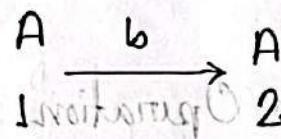
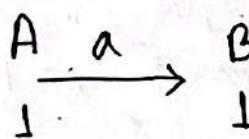


$L = \{ w \mid w \text{ has an odd number of } a's \text{ or ends with } b \}$

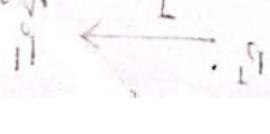
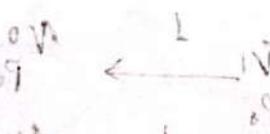


$ab \rightarrow \text{Initial stage}$

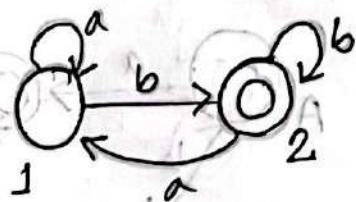
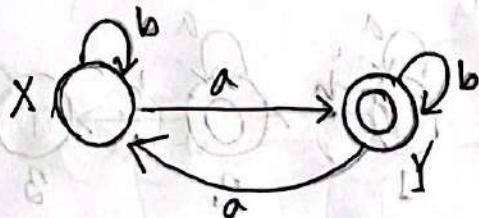
$$Q = \{ A_1, A_2, B_1, B_2 \}$$



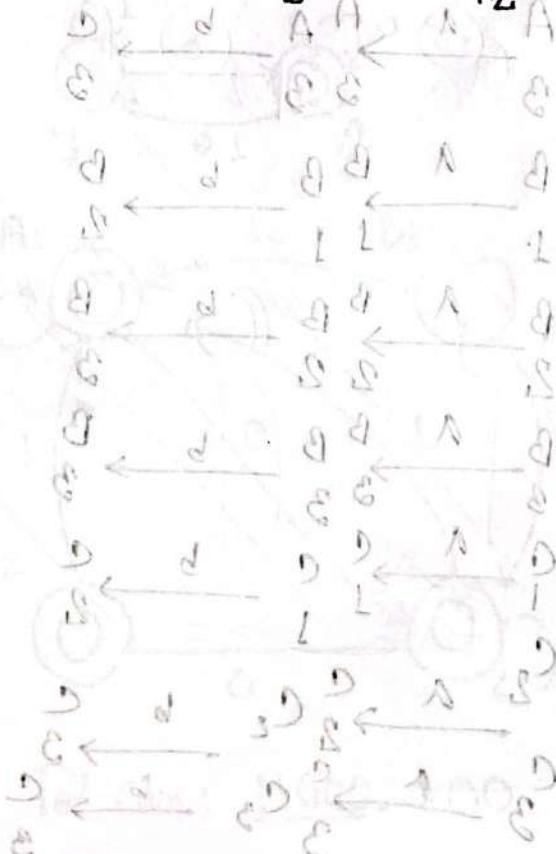
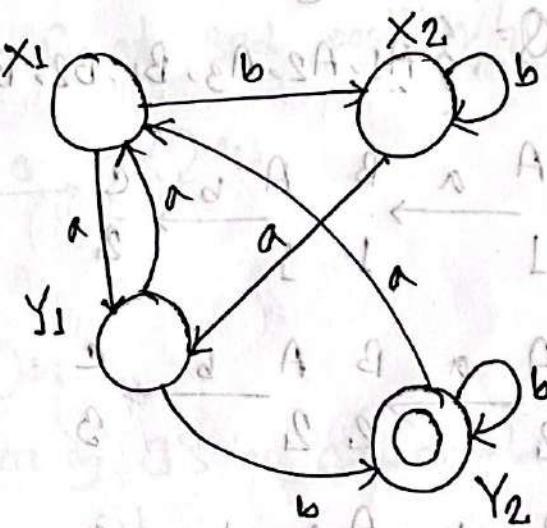
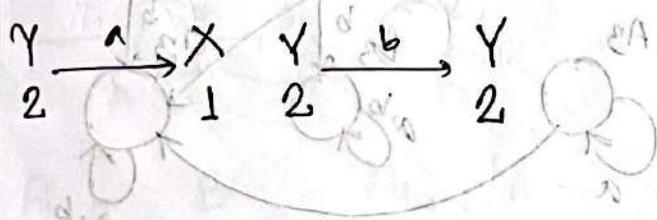
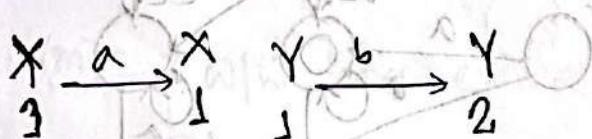
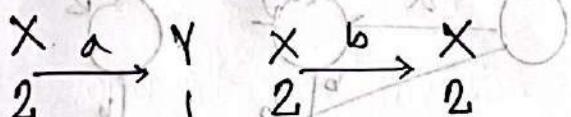
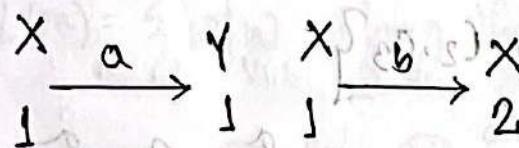
$L = \{ ab, aabb, bba, \dots \}$



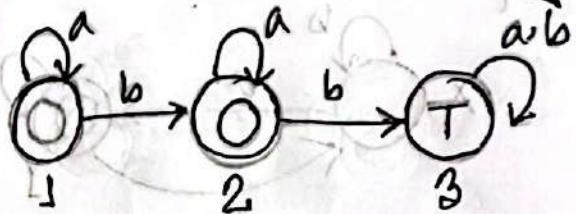
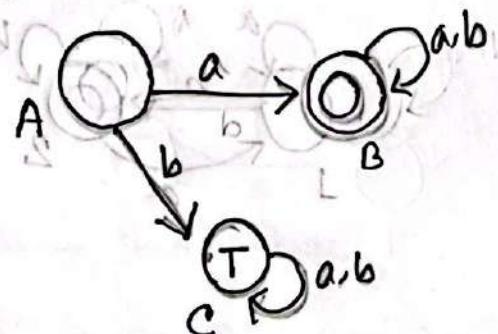
$L = \{w | w \text{ has an odd number of } a's \text{ and ends with } b\}$



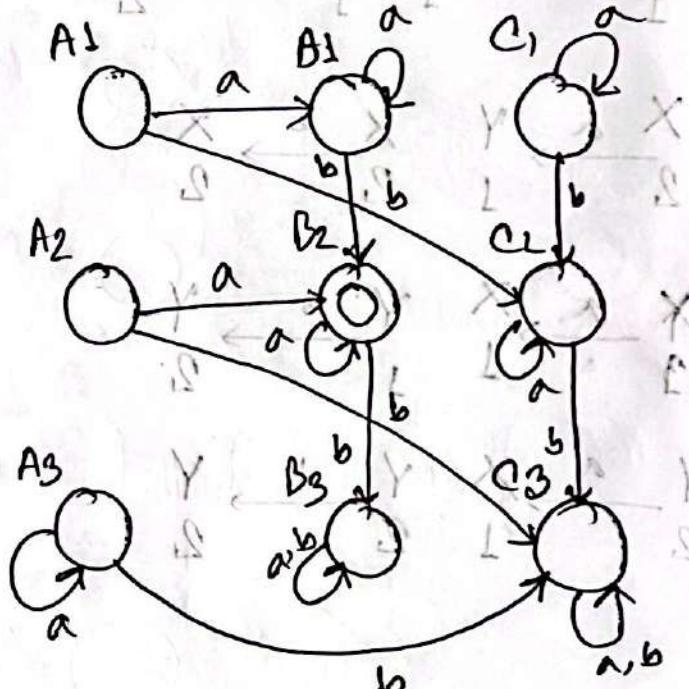
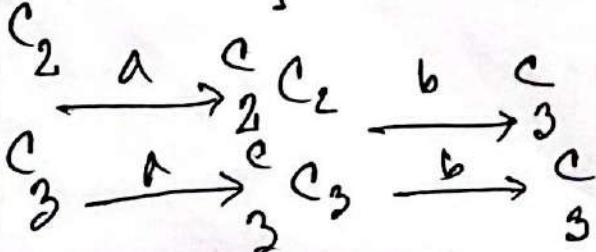
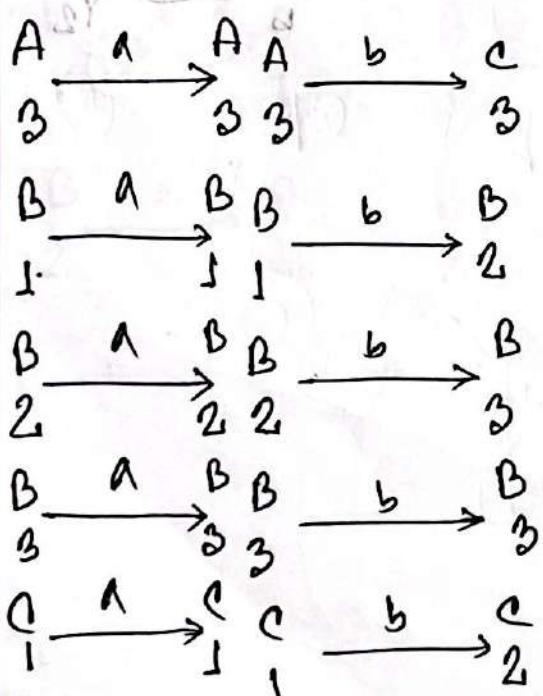
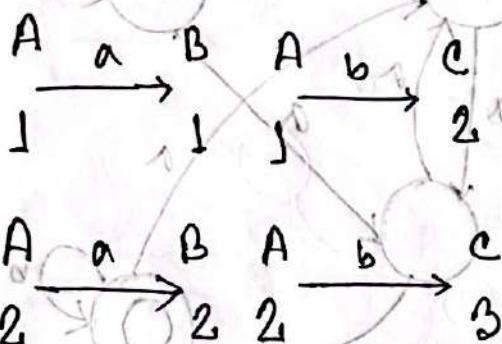
$$\mathcal{Q} = \{x_1, x_2, y_1, y_2\}$$



$L = \{ \text{w} \mid \text{w} \text{ starts with an } a \text{ and has at most one } b \}$



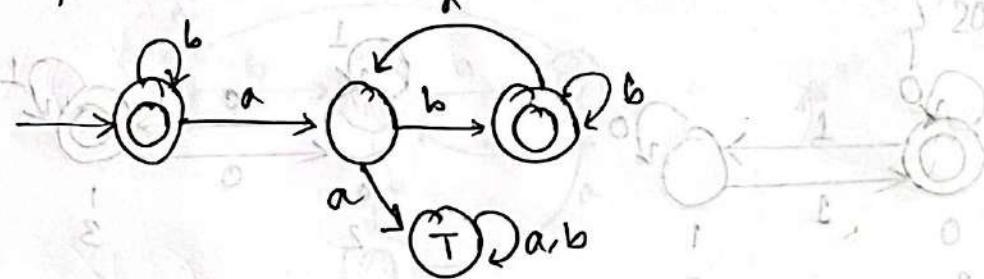
$Q = \{ A_1, A_2, A_3, B_1, B_2, B_3, C_1, C_2, C_3 \}$



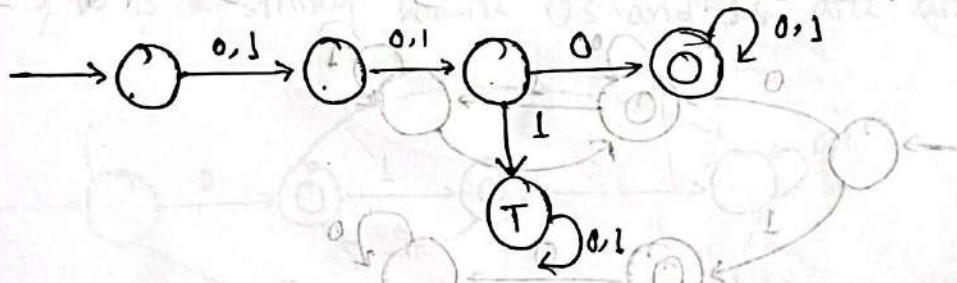
Subject :

Date :

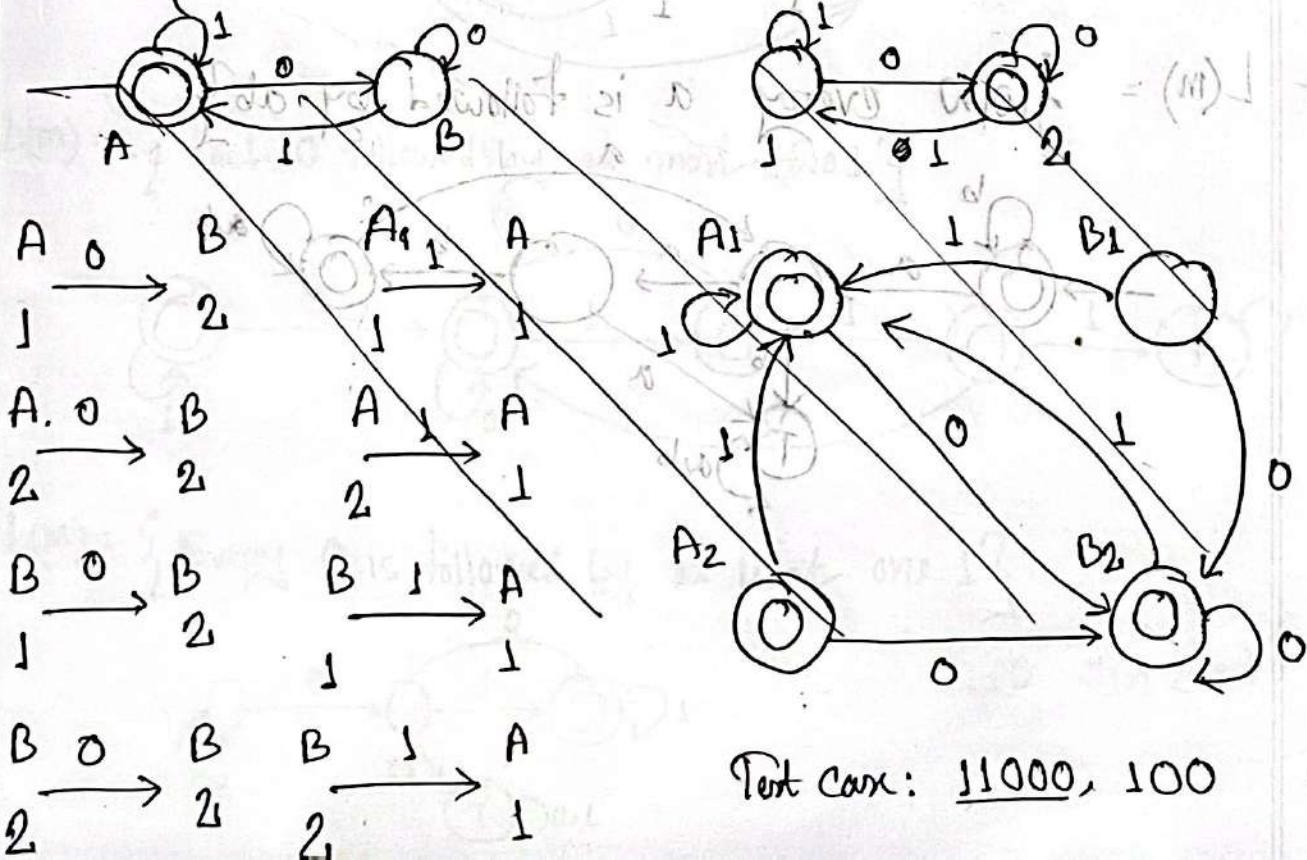
- * $L(M) = \{w | w \text{ every } a \text{ in } w \text{ is followed by one } b\}$



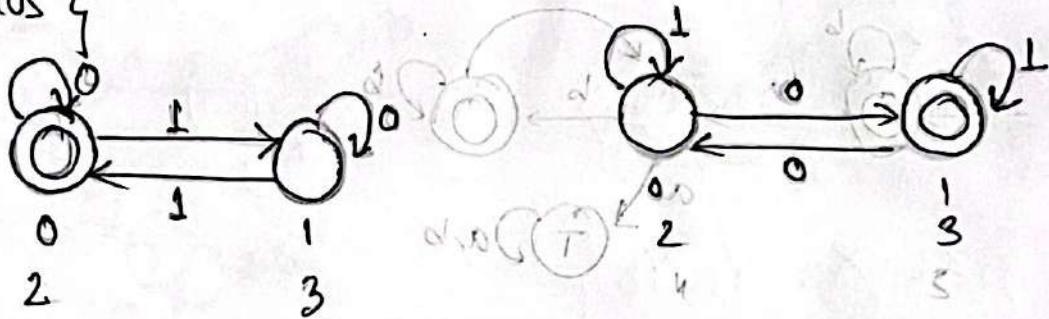
- * $L(M) = \{w | w \text{ string that has } 0 \text{ at 3rd position}\}$



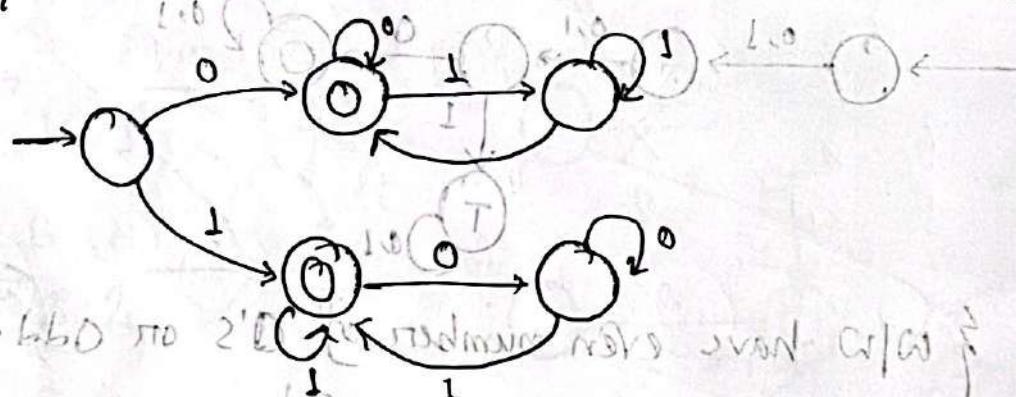
- $L(M) = \{w | w \text{ have even number of } 0's \text{ on odd numbers of } 0's\}$



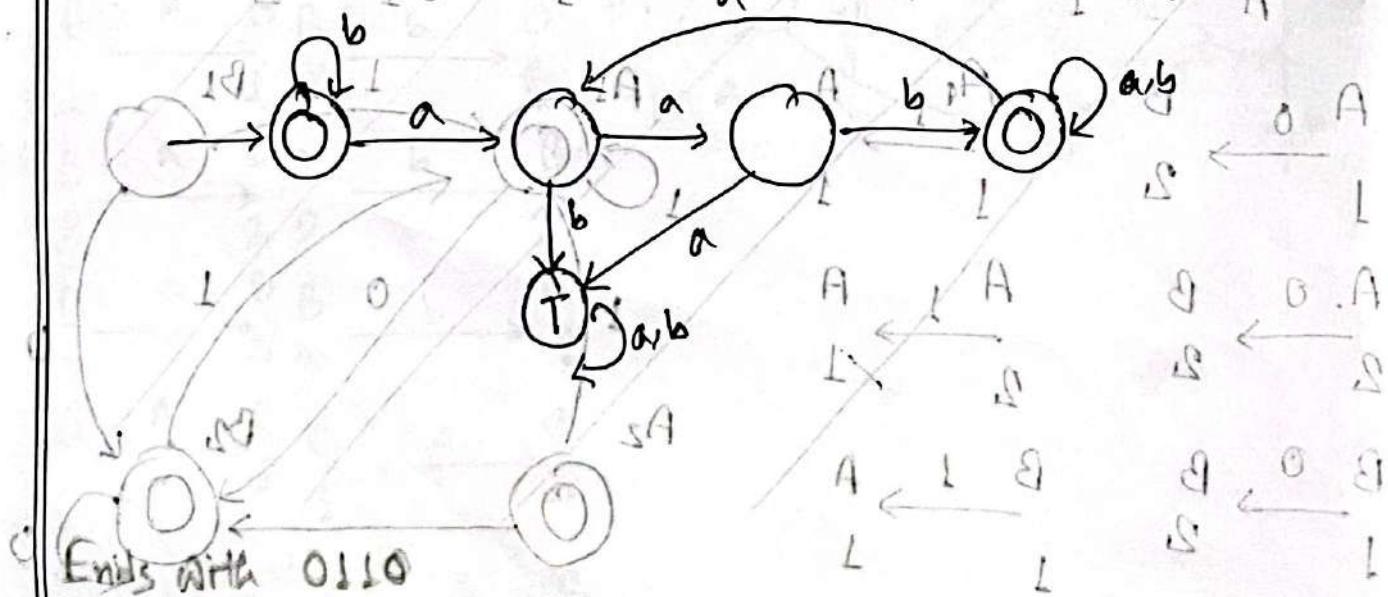
SWID have an even number of 1's or odd number of zeros?



- * $L(m)$ { writing here for O and don't write $O/C \in \mathcal{L}(m)$ }
} W/W starts and ends with same symbol }

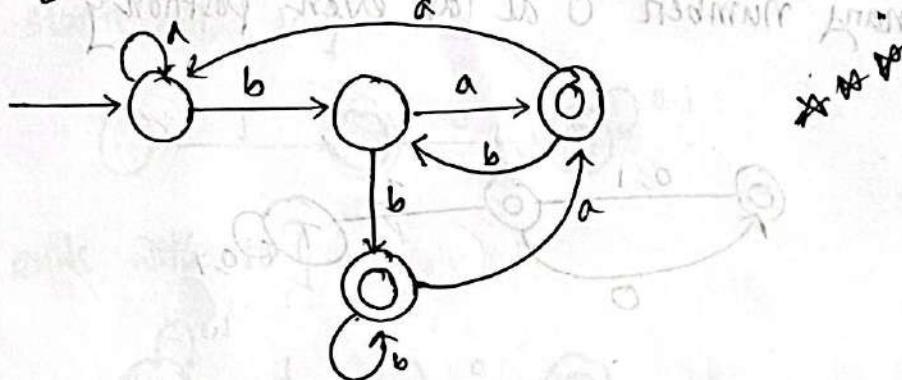


- * $L(m) = \{w | w \text{ every } a \text{ is followed by } ab\}$

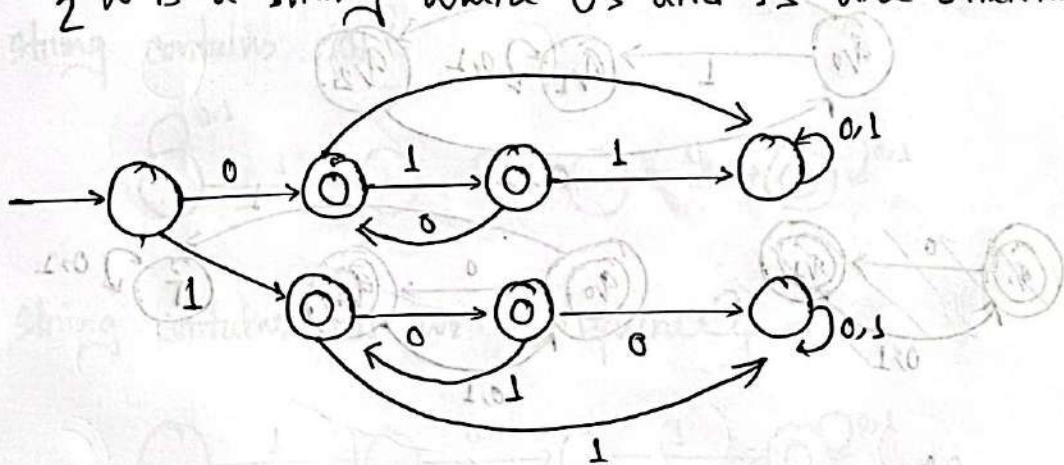


001,000L: and for

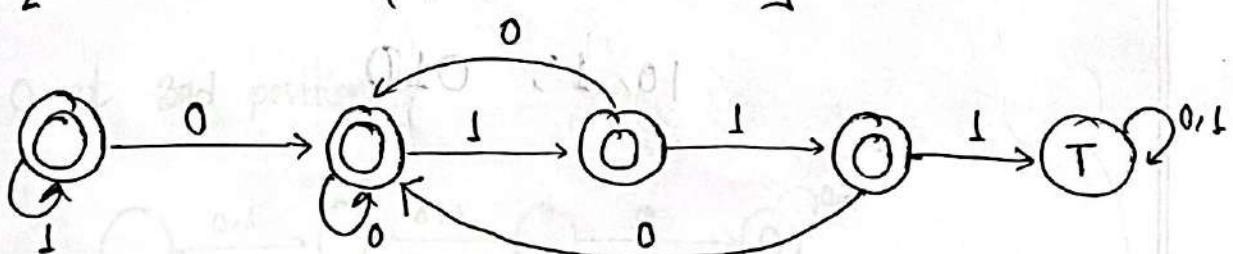
* $L(m) = \{ \text{second last symbol is } b \}$



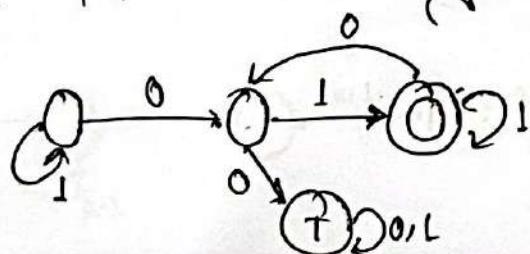
* $L(m) = \{ w \text{ is a string where 0's and 1's are alternate} \}$



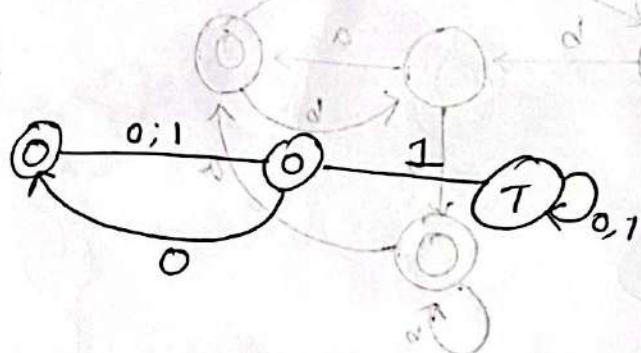
* $L(m) = \{ \text{each 0 followed by at most two 1's} \}$



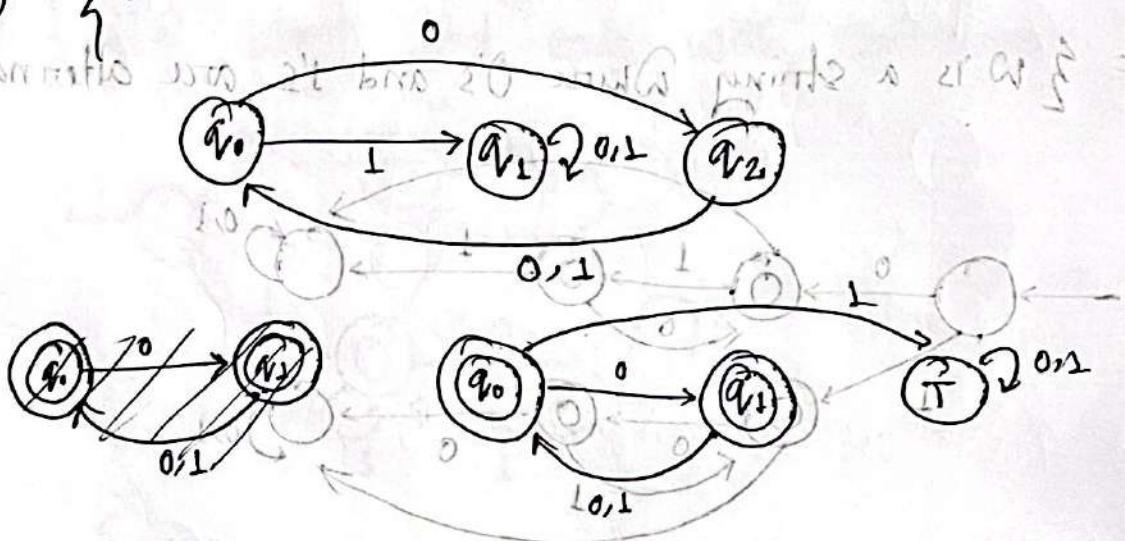
* $L(m) = \{ \text{Every 0 is followed by at least one 1} \}$



$L(M) = \{ \text{binary number } 0 \text{ at all even position} \}$



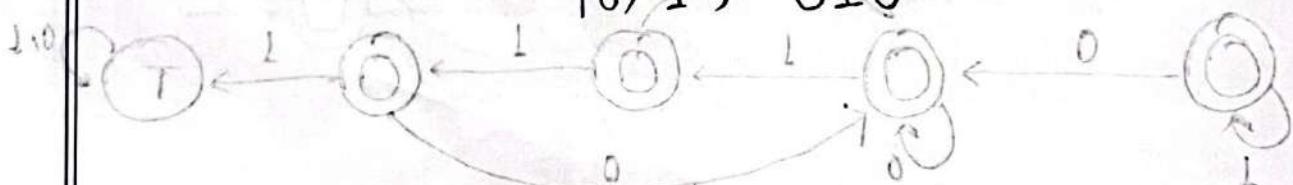
$L(M) = \{ \dots \}$



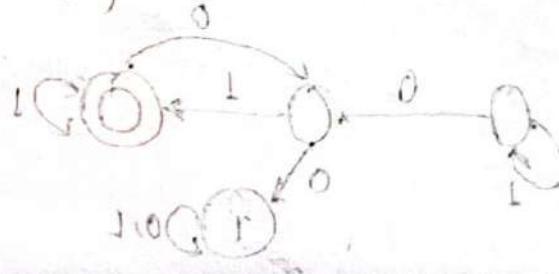
0101
001
01

00

$\{ \text{L}(0|10)^*, (1+\epsilon) \text{ binary 0 digit} \} = (M)$

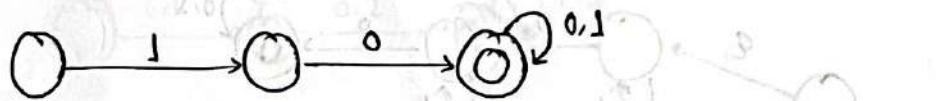


$\{ \text{L}(0|10)^*, (1+\epsilon) \text{ binary 0 digit} \} = (M)$



NFA

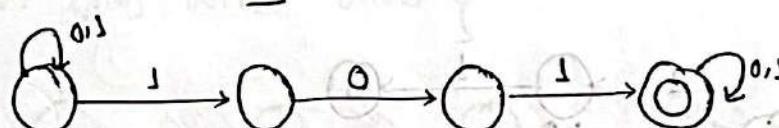
- ① $L = \{ \text{ starts with } 10 \}$



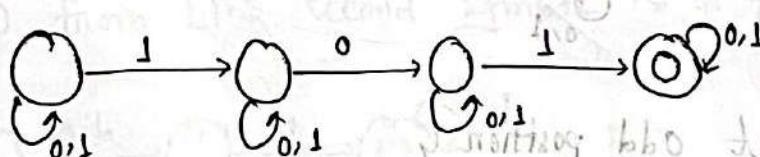
- ② $L = \{ \text{ ends with } 10 \}$



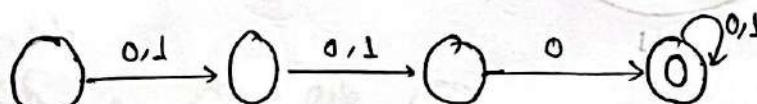
- ③ $L = \{ \text{ string contains } 101 \}$



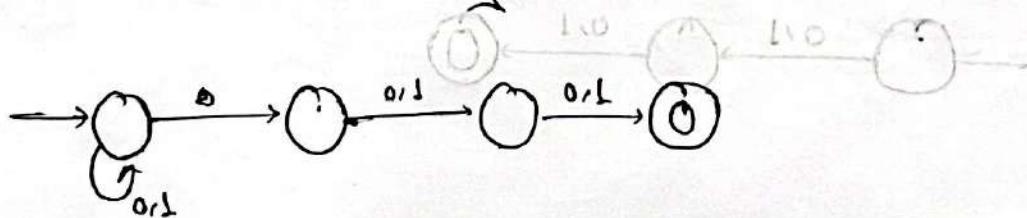
- ④ $L = \{ \text{ string contains } 101 \text{ as subsequence} \}$



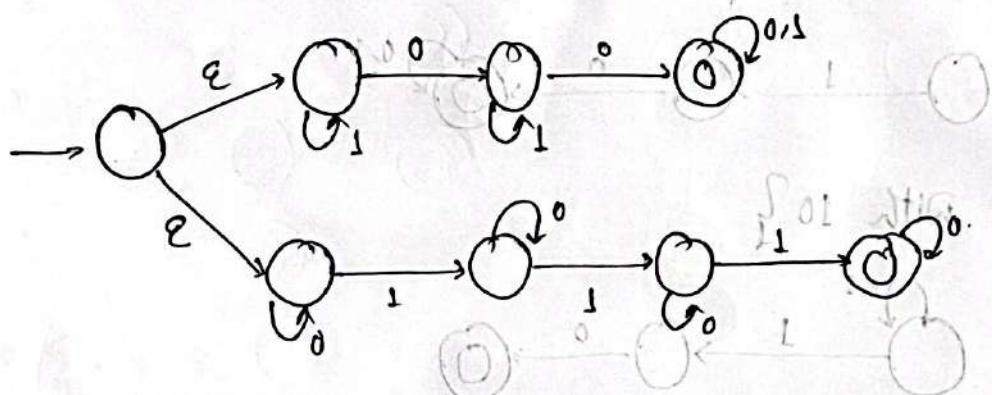
- ⑤ $L = \{ 0 \text{ at 3rd position} \}$



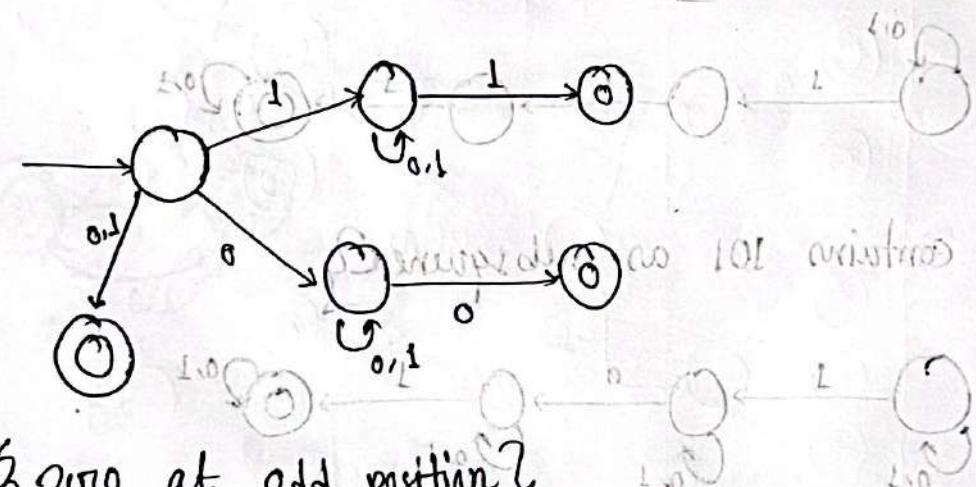
- ⑥ $L = \{ 0 \text{ at 3rd last position} \}$



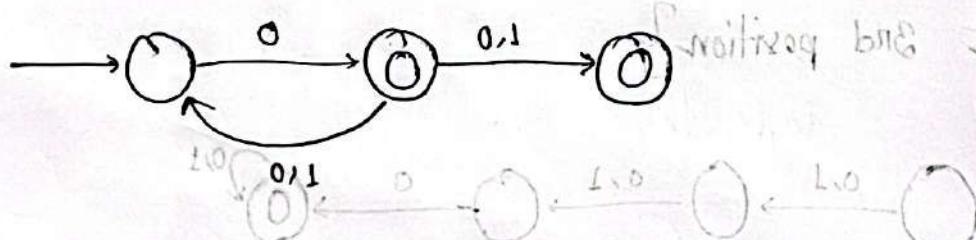
(7)

 $L = \{ \text{At least two } 0's \text{ on exactly 3 } 1's \}$


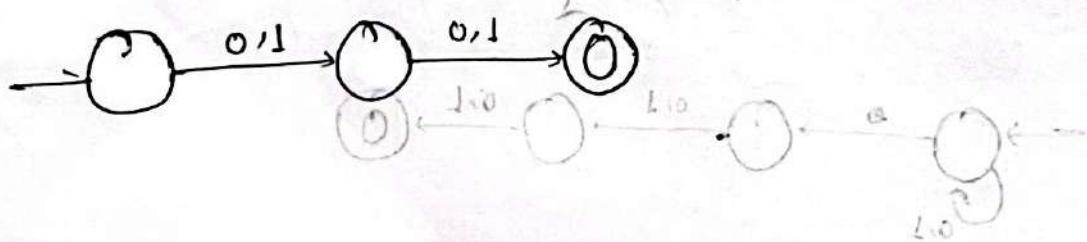
(8)

 $L = \{ \text{start and ends with same symbol} \}$


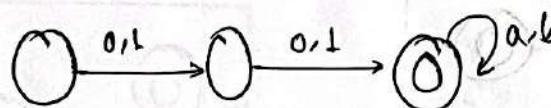
(9)

 $L = \{ \text{2020 at odd position} \}$


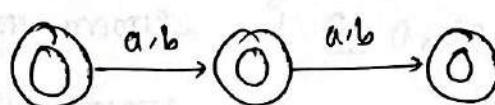
(10)

 $L = \{ w | \text{the length of the string is exactly 23} \}$


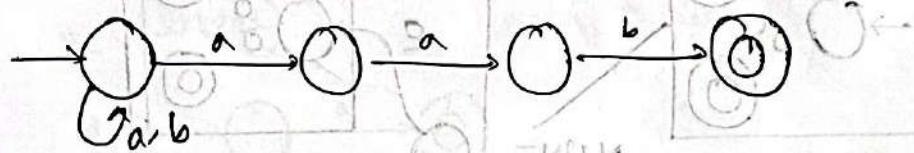
(11) $L = \{ w \mid \text{the length of the string is at least } 2 \}$



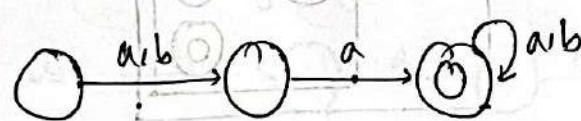
(12) $L = \{ w \mid \text{the length of the string is atmost } 2 \}$



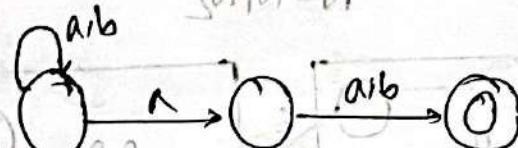
(13) $L = \{ w \mid w \text{ ends with } aab \}$



(14) $L = \{ w \mid w \text{ from LHS second symbol is } a \}$



(15) $L = \{ w \mid w \text{ from RHS second symbol is } a \}$

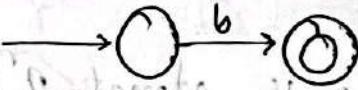


Regular Expression to NFA

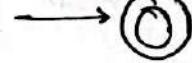
(a) $R = a$



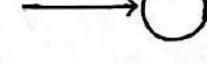
(b) $R = b$



(c) $R = \epsilon$

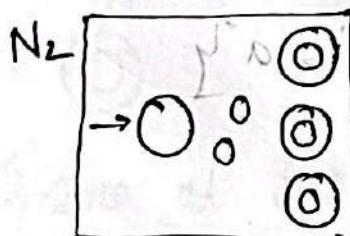
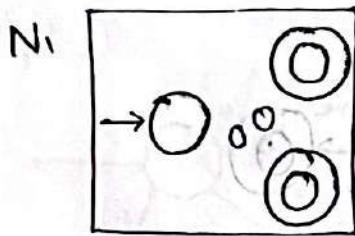


(d) $R = \emptyset$

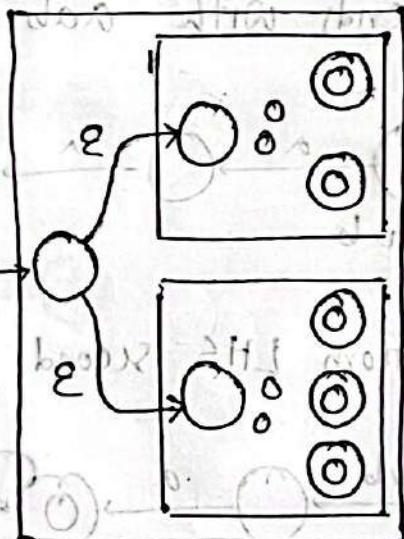


(e) $R = R_1 \cup R_2 = R_1 / R_2$

$$N = N_1 \cup N_2$$

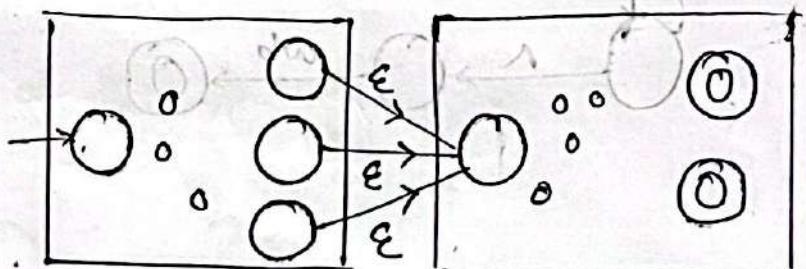
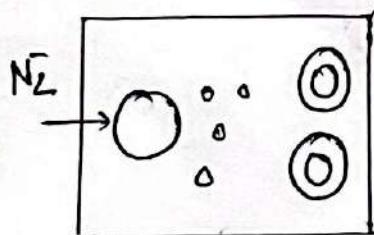
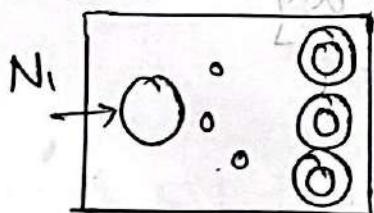


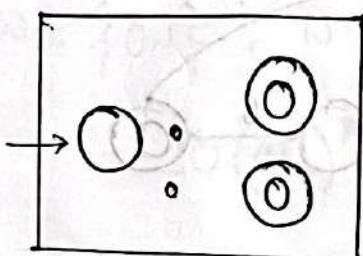
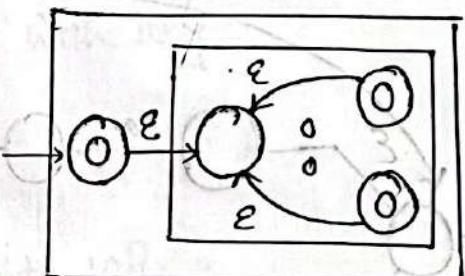
$$N_1 \cup N_2$$



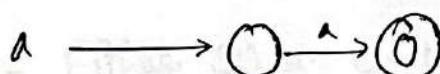
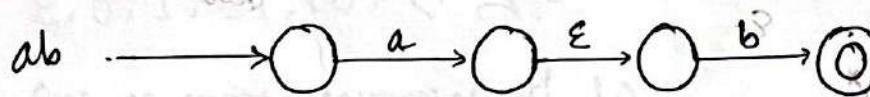
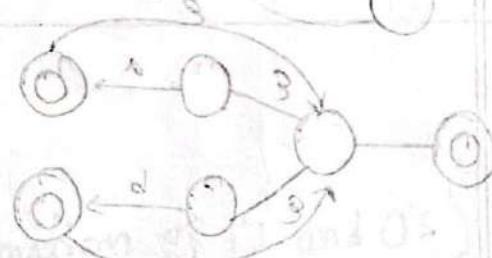
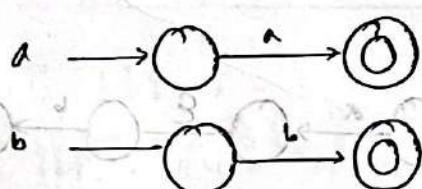
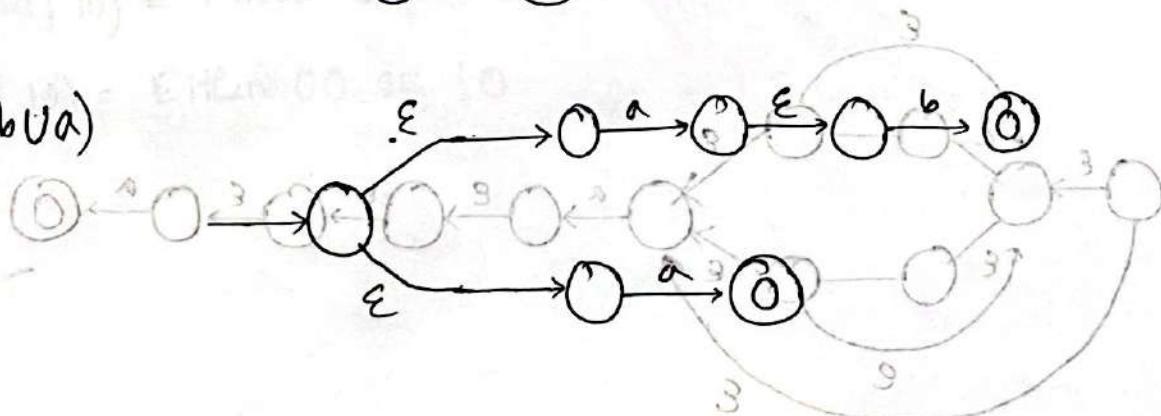
(f) $R = R_1 \circ R_2 = R_1 R_2$

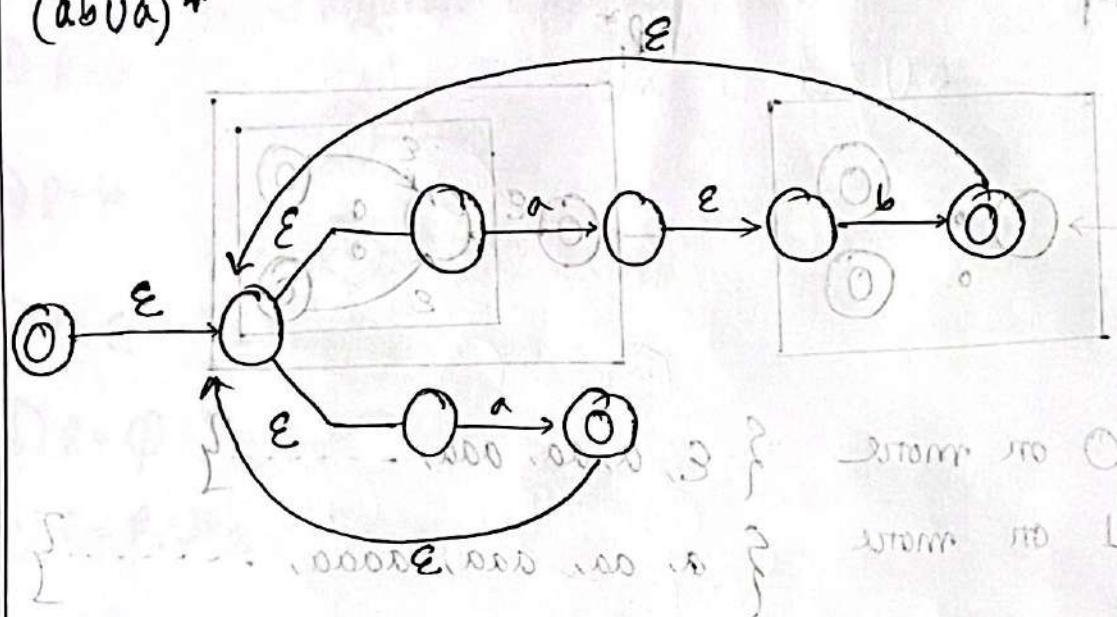
$$N = N_1 N_2$$



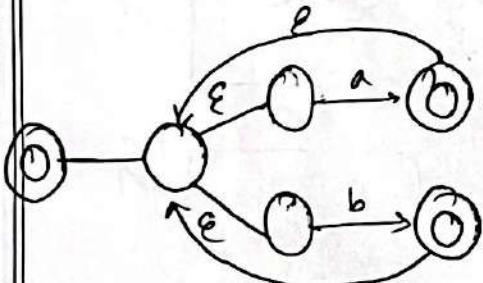
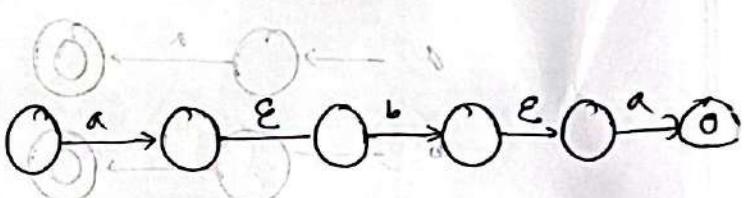
(2) $R = R_1^*$ N_1  R_1^* 
 $a^* = 0 \text{ or more}$ $\{ \epsilon, a, aa, aaa, \dots \}$
 $a^+ = 1 \text{ or more}$ $\{ a, aa, aaa, aaaa, \dots \}$

Problem 1

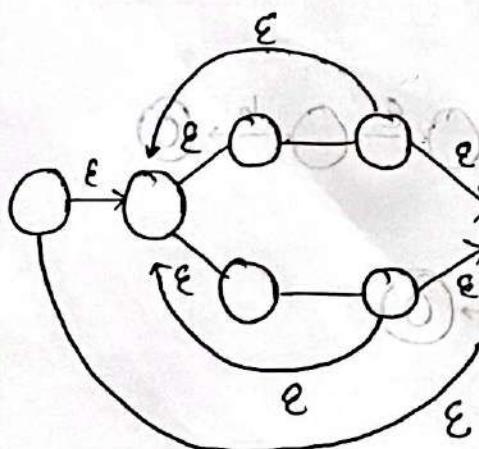
 $(ab \cup a)^*$  $(ab \cup a)$ 

$(abua)^*$ 

Problem 2:

 $(a \cup b)^* aba$  $^*(a \cup b)$ 

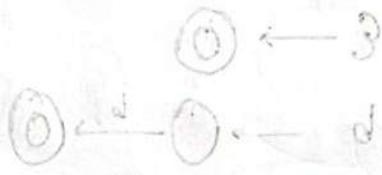
aba

 $(a \cup b)^*$  $(a \cup b)$

Problem 3:

(a) $L = \{ w \in \{0,1\}^*: w \text{ starts with } 10 \}$

$$10(01)^*$$



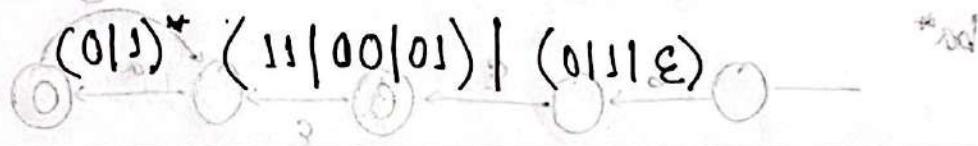
(b) $L = \{ w \in \{0,1\}^*: w \text{ ends with } 10 \}$

$$(01)^* 10$$



(c) $L = \{ w \in \{0,1\}^*: w \text{ does not end with } 10 \}$

$$(01)^* \cdot (11|00|01) | (0111 \epsilon)$$



$$J^* = " ", 1, 11, 111, \dots$$

$$J^+ = 1, 11, 111, \dots$$

$$(110)^* = \epsilon, 0, 1, 01, 10, (\text{Any combinations of } 1's \text{ and } 0's)$$

$(10)^+$ = One or more occurrence of 10

$(00|01|10)$ = Either 00 or 01 or 10

$(00|10)$ = Either 00 or 10



Problem 3:

Q. no. 1

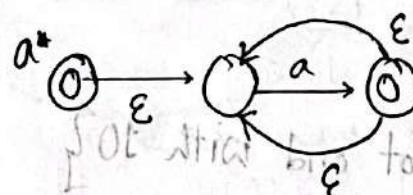
$$ba^* + \epsilon$$

Same as $ba^* \cup \epsilon$

$$\epsilon \rightarrow \textcircled{0}$$

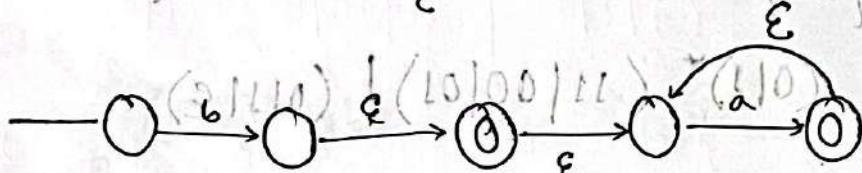
$$b \rightarrow \textcircled{0} \xrightarrow{b} \textcircled{0}$$

$$a \rightarrow \textcircled{0} \xrightarrow{a} \textcircled{0}$$

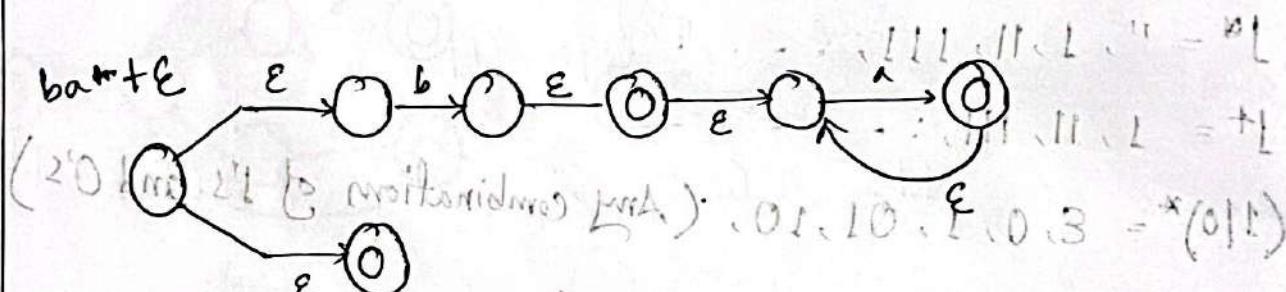


$$OL \xrightarrow{*} (1|0)$$

$$ba^*$$



$$ba^* + \epsilon$$



Problem 4:

Q. no. 2

$$a^+ \cup (ab)^+ = aa^* \cup ab(ab)^*$$

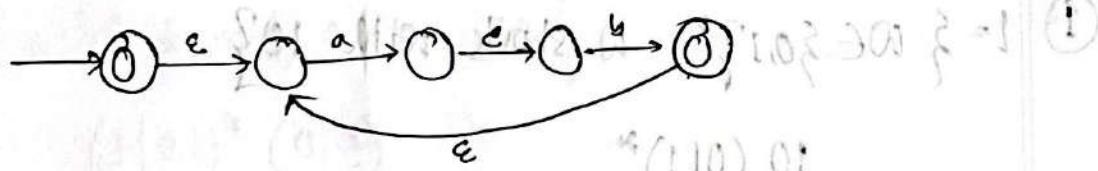
$$a \rightarrow \textcircled{0} \xrightarrow{a} \textcircled{0}$$

$$OL \xrightarrow{*} (01|10|00) = (01|10|00)$$

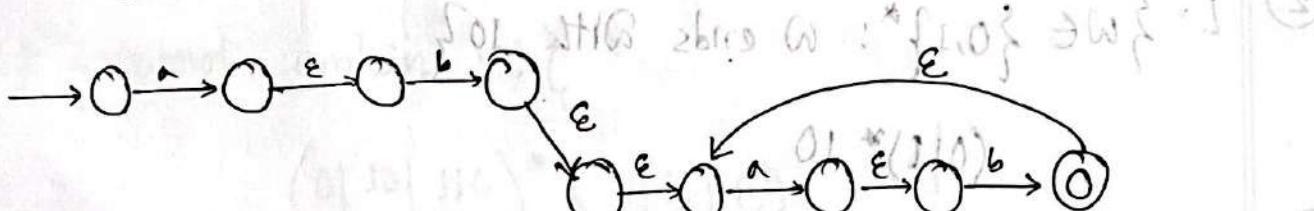
$$b \rightarrow \textcircled{0} \xrightarrow{b} \textcircled{0}$$

$$ab \rightarrow \textcircled{0} \xrightarrow{a} \textcircled{0} \xrightarrow{\epsilon} \textcircled{0} \xrightarrow{b} \textcircled{0}$$

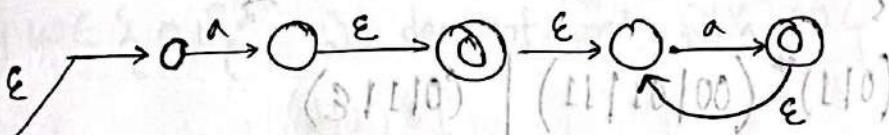
$$aa^* \rightarrow \textcircled{0} \xrightarrow{a} \textcircled{0} \xrightarrow{\epsilon} \textcircled{0} \xrightarrow{a} \textcircled{0} \xrightarrow{\epsilon} \textcircled{0}$$



$$ab(ab)^*$$

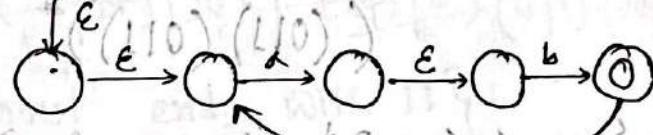


01 2016 bis Januar 01 : 1.033.363



$$(S\bar{1}10) \rightarrow (\bar{1}1\bar{1}0\bar{1}00) \rightarrow (\bar{1}10)$$

Curves of G to \mathbb{R}^n : $\{1, 0\} \rightarrow \mathbb{R}^2 = \{1, 0\}$



$$\{ \text{bbo}, \text{2i}, \text{W}, \text{g}, \text{N} \}_{\text{new}} \xrightarrow{\epsilon} \{ \text{S1}, \text{03}, \text{2}, \text{C} \}_{\text{old}} = \{ \dots \}$$

$$(110)^* \cap ((110) \cup (110)^*)$$

printed as 101 address as $\{101 \rightarrow 0\}$

$\pi(110)$ 101 $\pi(110)$

Si los existentes tienen $\{1, 0\} \rightarrow \omega^2 = 1$

(311)¹⁰(0610)

① $L = \{ w \in \{0,1\}^*: w \text{ starts with } 10 \}$

$$10(011)^*$$

② $L = \{ w \in \{0,1\}^*: w \text{ ends with } 10 \}$

$$(011)^* 10$$

③ $L = \{ w \in \{0,1\}^*: w \text{ does not end with } 10 \}$

$$(011)^* (00|01111) | (0111\epsilon)$$

④ $L = \{ w \in \{0,1\}^*: \text{length of } w \text{ is even} \}$

$$((011)(011))^*$$

⑤ $L = \{ w \in \{0,1\}^*: \text{length of } w \text{ is odd} \}$

$$\{(011)(011)\}^* (011)$$

⑥ $L = \{ w \in \{0,1\}^*: w \text{ contains } 101 \text{ as substrings} \}$

$$(011)^* 101 (011)^*$$

⑦ $L = \{ w \in \{0,1\}^*: \text{does not contain } 11 \}$

$$(0110)^* (1|\epsilon)$$

- ⑧ $L = \{ w \in \{0,1\}^* : w \text{ does not contain } 00 \}$ (21)
 $(1|0)^* (0|1)^* ((1|0) (1|0) (1|0))$
- ⑨ $L = \{ w \in \{0,1\}^* : w \text{ does not contain } 111 \}$ (21)
 $(0|10|110)^* (1|11|1)^* ((1|0) (1|0) (1|0))$
- ⑩ $L = \{ w \in \{0,1\}^* : w \text{ does not ends with } 00 \}$ (21)
 $(1|(0|1)|1(1|0)) ((1|0) 00 | (1|0))$
- ⑪ $L = \{ w \in \{0,1\}^* : w \text{ does not ends with } 01 \}$ (21)
 ~~$(0|1)^* (00|11|10)^* | (0|1|1)^* (0|1)^* (00|11|10|0)$~~
- ⑫ $L = \{ w \in \{0,1\}^* : w \text{ does not ends with } 11 \}$ (22)
 ~~$(0|1)^* (0|10)^* 0 | (0|1)^* 0$~~
- ⑬ $L = \{ w \in \{0,1\}^* : w \text{ does not contains } 10 \}$ (22)
 ~~$0^* 1^*$~~
 ~~$L \text{ does not contains } w \}$~~
- ⑭ $L = \{ w \in \{0,1\}^* : w \text{ does not contains } 01 \}$ (22)
 ~~$1^* 0^*$~~
 ~~$L \text{ does not contains } w \}$~~
- ⑮ $L = \{ w \in \{0,1\}^* : w \text{ does not contains } 00 \}$ (22)
 $(1|0)^* ((1|0) 10 | 10 ((1|0) 10 | (1|0)))$

(16) $L = \{ \text{length expressed as } 3k+2 \}$

$$((0|1)(0|1)(0|1))^* (0|1)(0|1)|1)$$

(17) $L = \{ \text{length expressed as } 2k+1 \}$

$$((0|1)(0|1))^* (0|1)$$

(18) $L = \{ w \text{ contains } 00 \text{ or } 11 \}$

$$(0|1)^* 00 (0|1)^* | (0|1)^* 11 (0|1)^*$$

(19) $L = \{ w \text{ contains at least two 1's} \}$

$$(0|1) | 11|00 | (1|0)(0|1) | (0|1)(0|1)|00 | (1|0)$$

(20) $L = \{ w \text{ contains at least two 0's} \}$

$$(0|1)^* 0 (0|1)^* 0 (0|1)^*$$

(21) $L = \{ w \text{ contains at least 1 zeros} \}$

$$(0|1)^* 0 (0|1)^*$$

(22) $L = \{ w \text{ contains at least 2 '0's} \}$

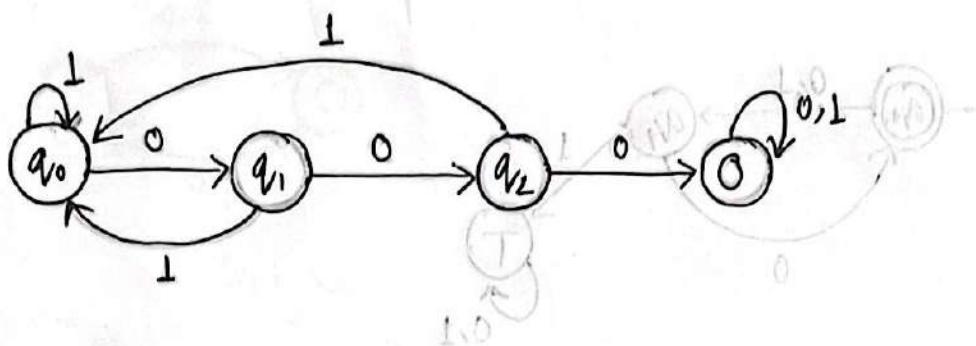
$$(0|1)^* 01 (0|1)^* 01 (0|1)^* | (10|1)$$

Subject:

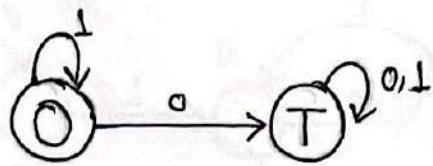
Date:

- (23) $L = \{ \omega \text{ contains exactly two } 1's \}$
 $0^* 1 0^* 1 0^*$
- (24) $L = \{ \omega \text{ contains exactly two } 0's \}$
 $1^* 0 1^* 0 1^*$
- (25) $L = \{ \omega \text{ contains exactly one } 0 \}$
 $1^* 0 1^*$
- (26) $L = \{ \omega \text{ contains exactly three } 0's \}$
 $1^* 0 1^* 0 1^* 0 1^*$
- (27) $L = \{ \omega \text{ contains at most two } 1's \}$
 $0^* + 0^* 1 0^* + 0^* 1 0^* 1 0^*$
- (28) $L = \{ \omega \text{ contains at most three } 0's \}$
 $1^* + 1^* 0 1^* + 1^* 0 1^* 0 1^* + 1^* 0 1^* 0 1^* 0 1^*$
- (29) $L = \{ \omega \text{ contains at most one } 1 \}$
 $0^* + 0^* 1 0^*$
- (30) $L = \{ \text{length of } \omega \text{ is not multiple of } 3 \}$
 $((0|1)(0|1)(0|1))^* (0|1) (011|\epsilon)$
- (31) $L = \{ \text{length is multiple of } 3 \}$
 $((0|1)(0|1)(0|1))^*$

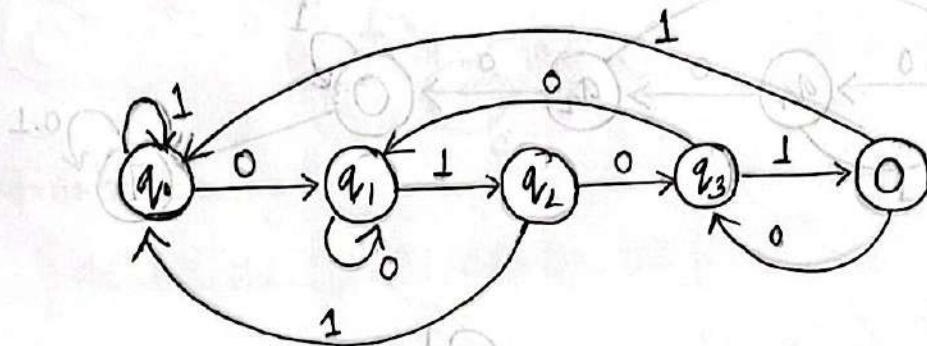
1. (a)



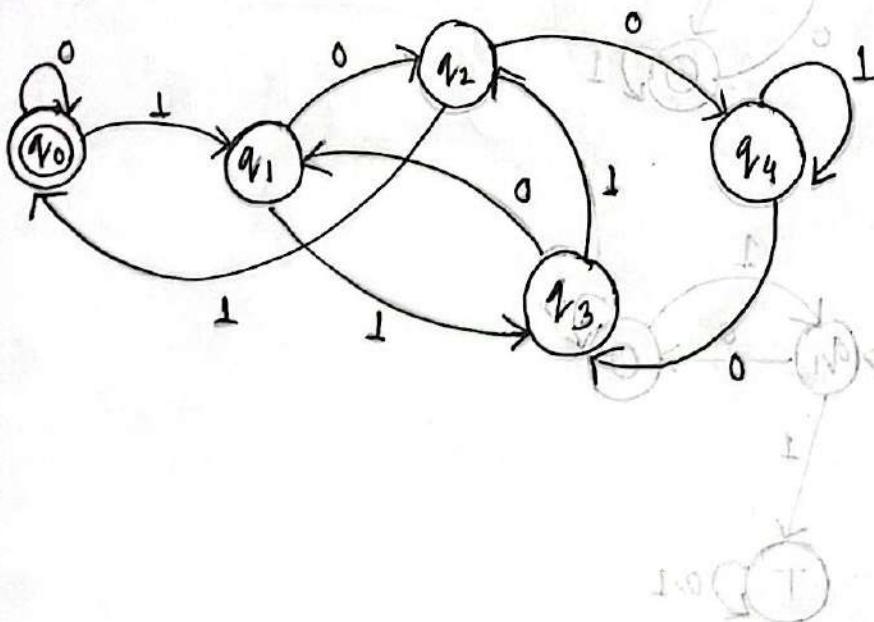
(b)



(c)

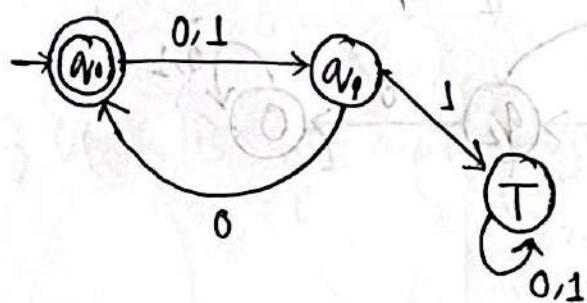


(d)



	0	1
0	q_0	q_1
1	q_2	q_3
0	q_4	q_0
1	q_1	q_2
0	q_3	q_4
1	q_2	q_3

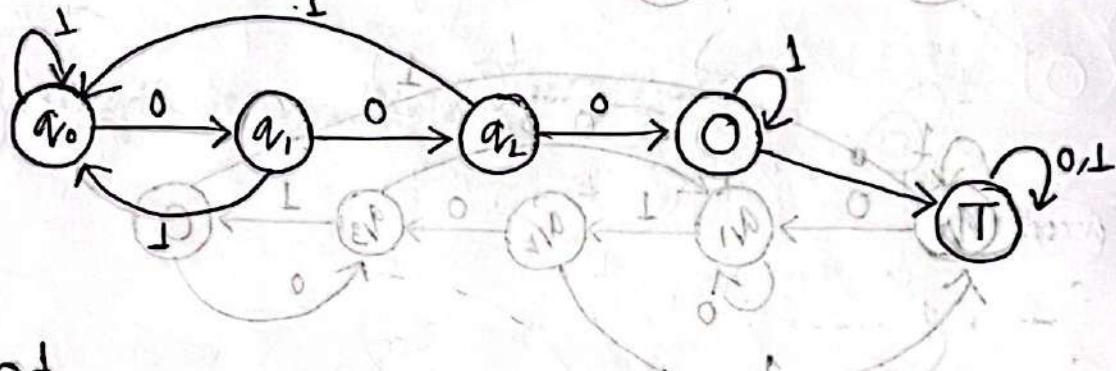
(e)



101 1010

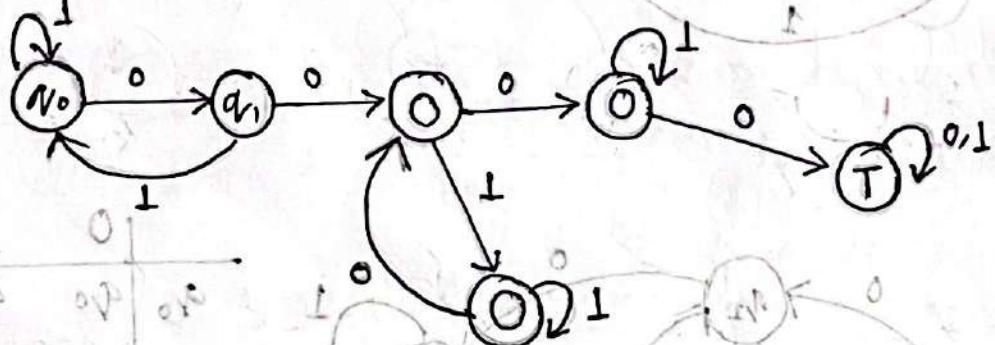
(b) L

(f)

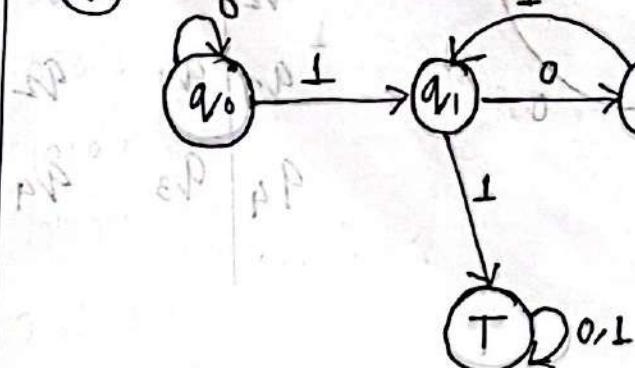


Hamid Book Brainding

(g)



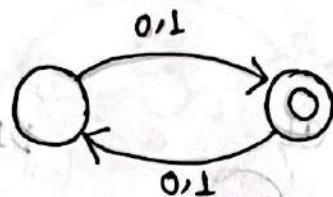
(h)



Subject:

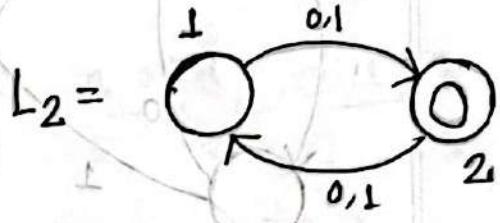
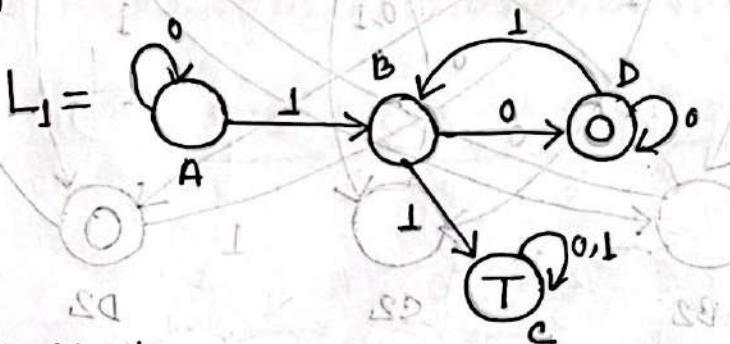
Date:

(b)



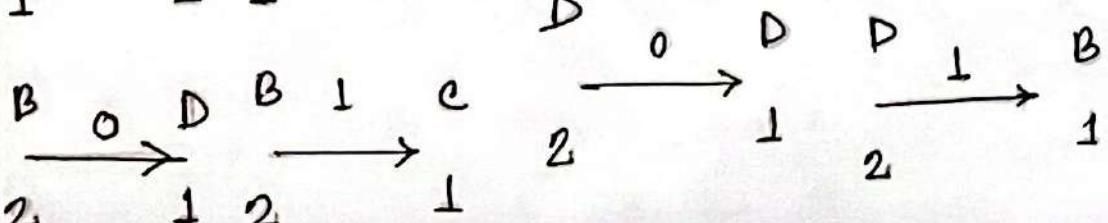
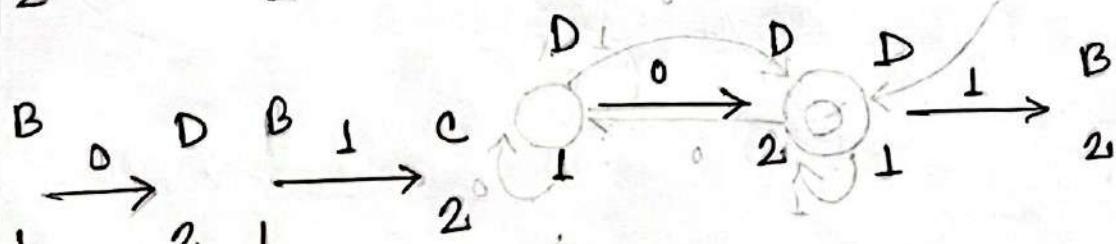
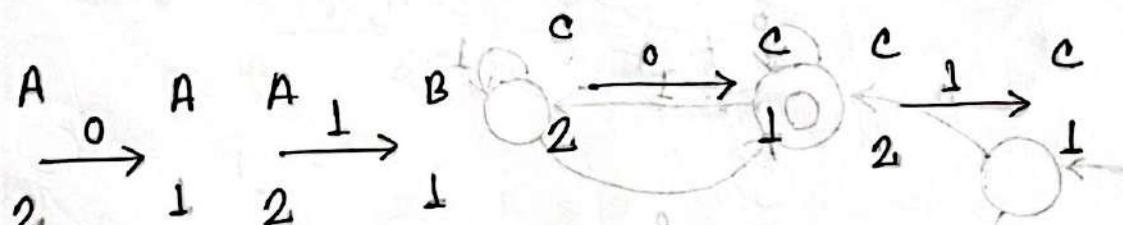
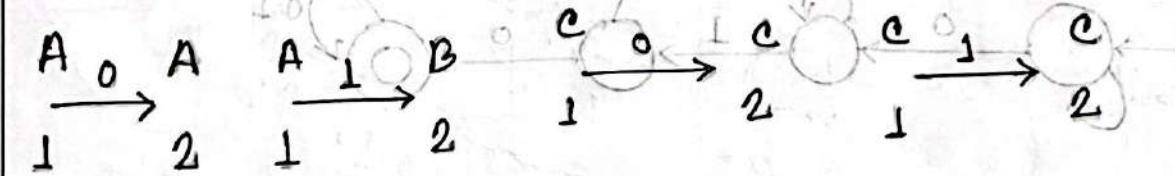
$$\Delta \cap \Gamma = \emptyset$$

(c)

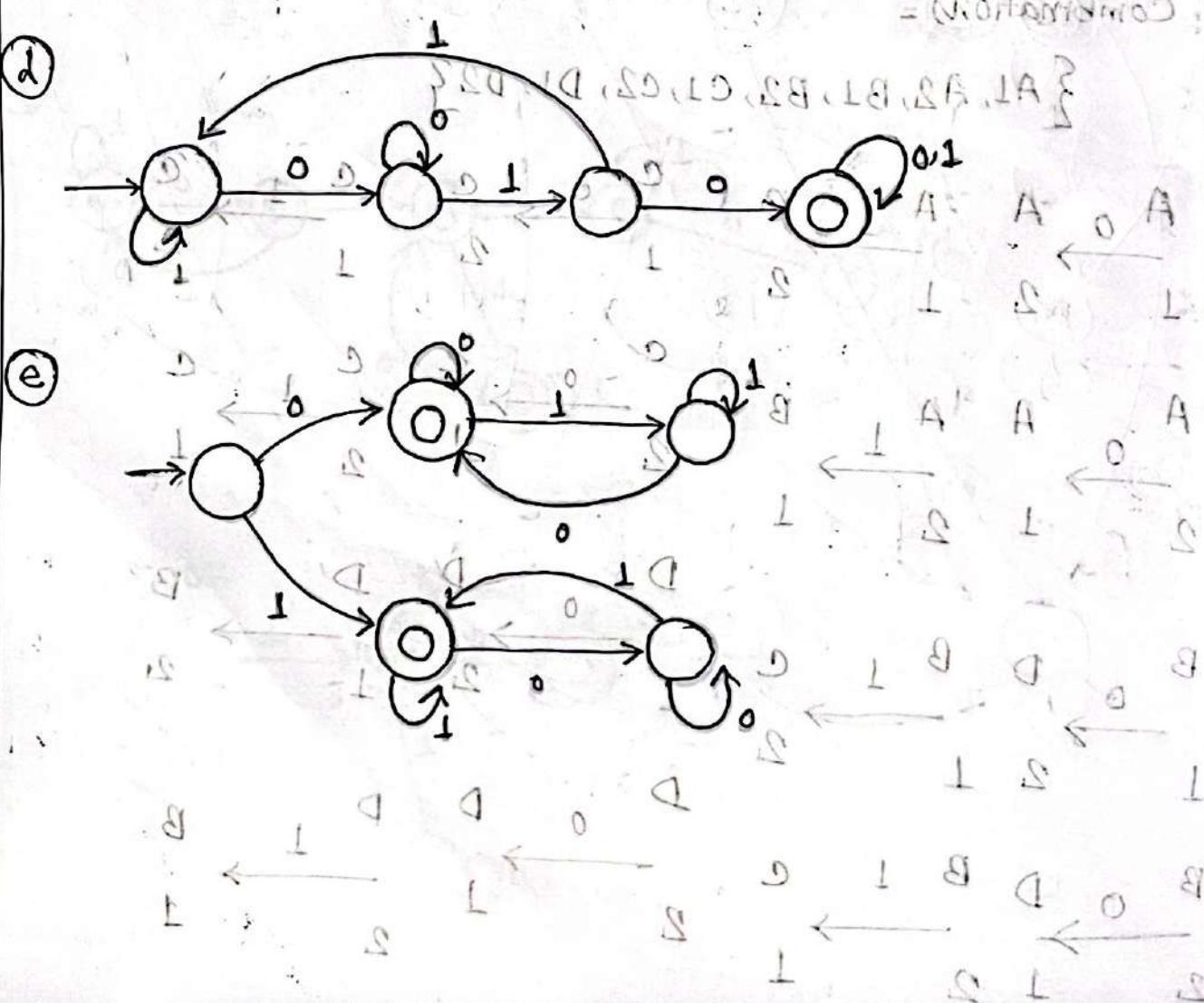
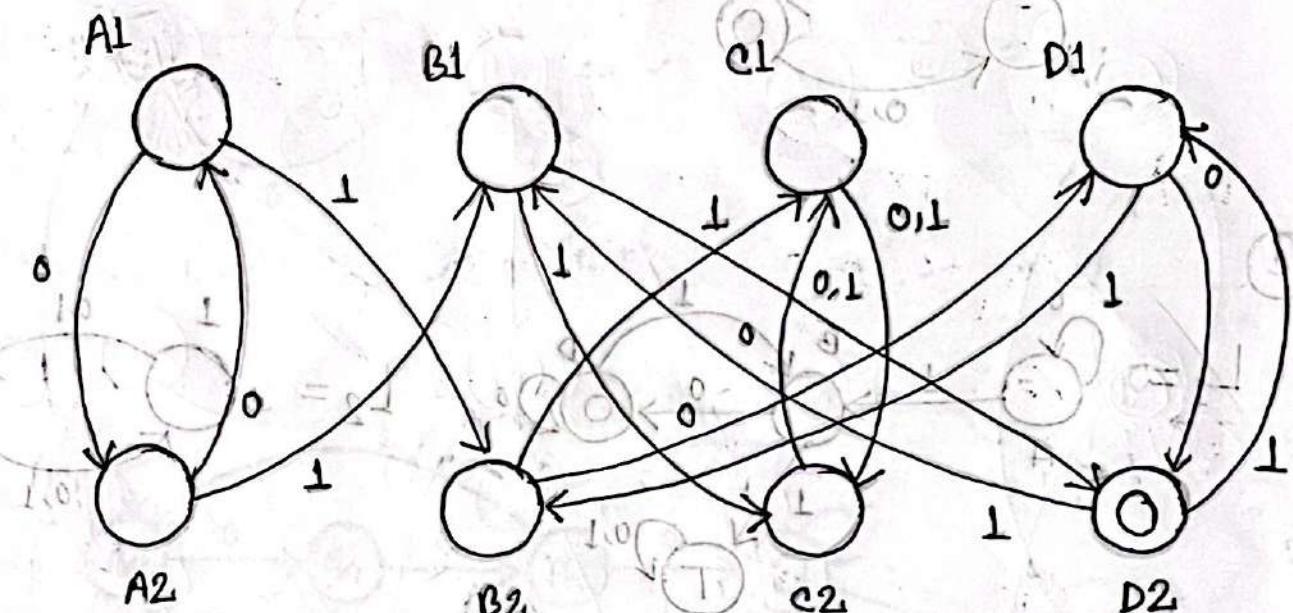


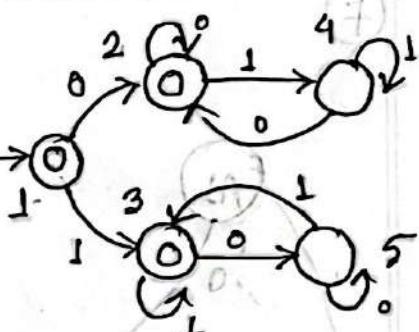
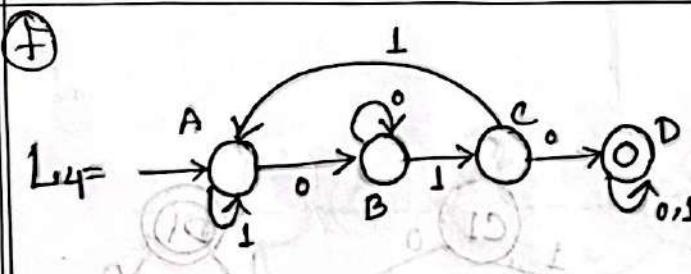
Combinations =

$$\{A1, A2, B1, B2, C1, C2, D1, D2\}$$



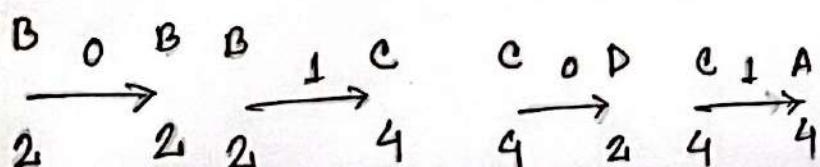
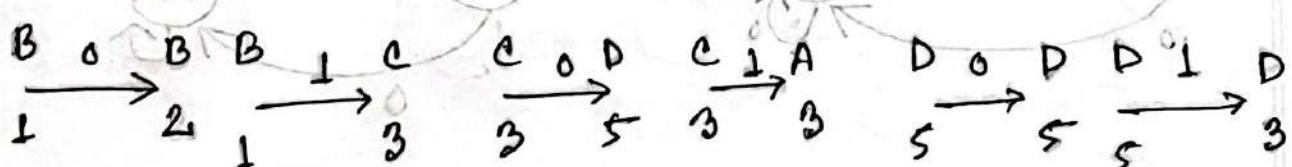
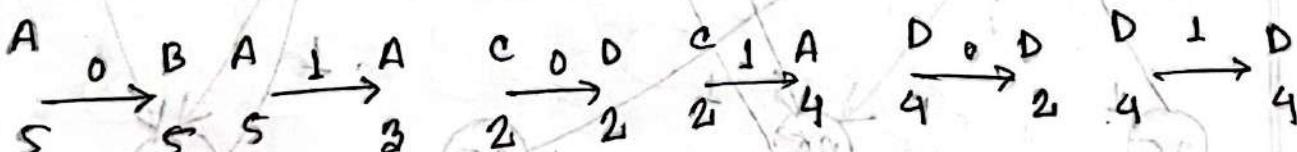
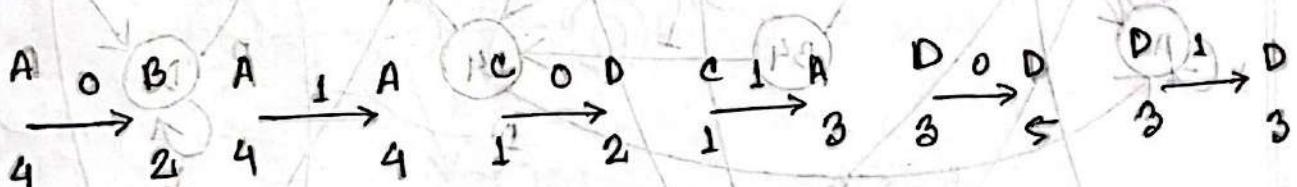
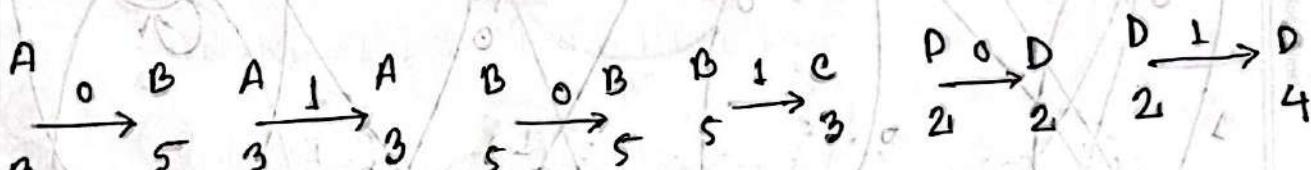
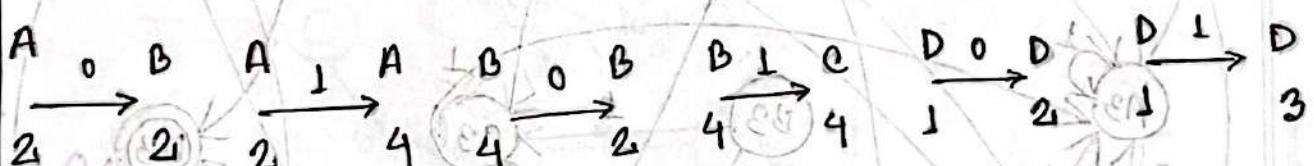
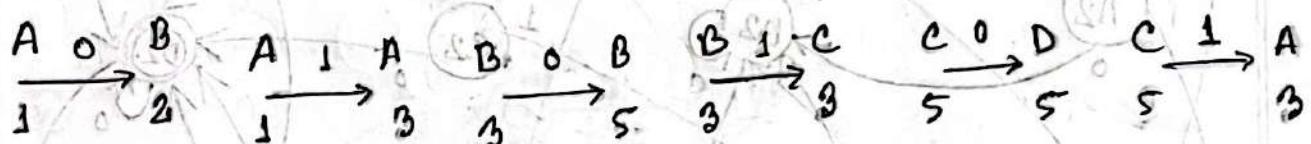
$$L_3 = L_1 \cap L_2$$

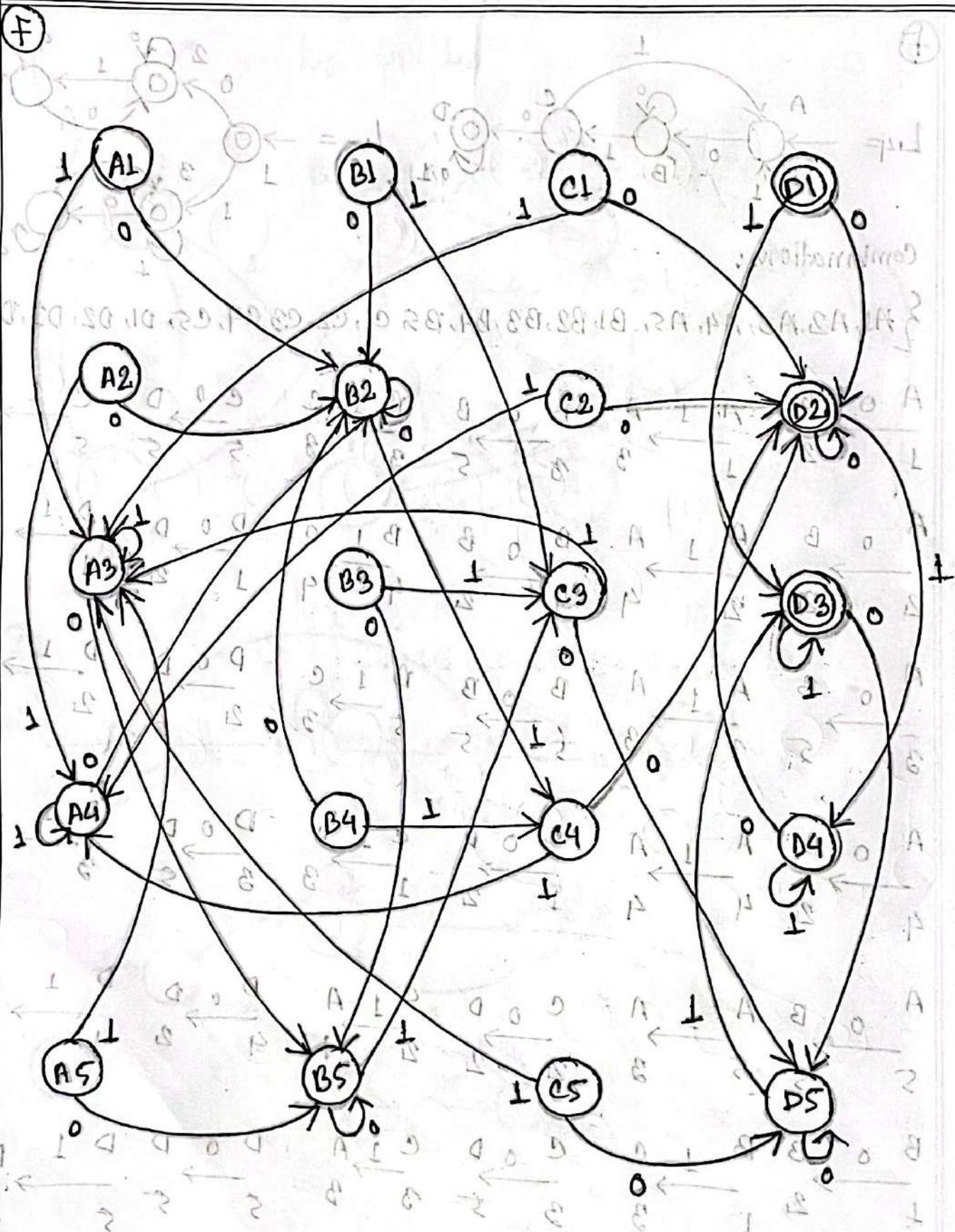




Combinations:

$\{A_1, A_2, A_3, A_4, A_5, B_1, B_2, B_3, B_4, B_5, C_1, C_2, C_3, C_4, C_5, D_1, D_2, D_3, D_4, D_5\}$



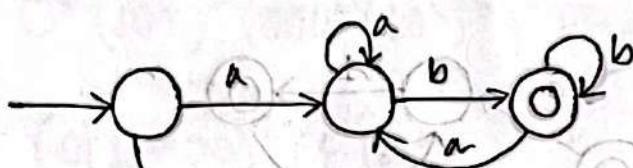


Subject : 10108

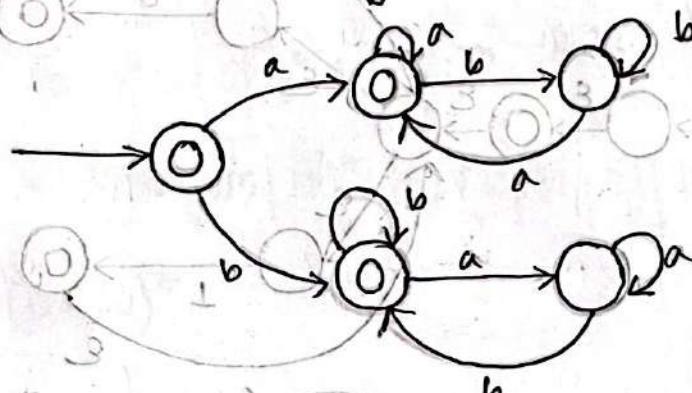
Date : 11/10/2022

3.

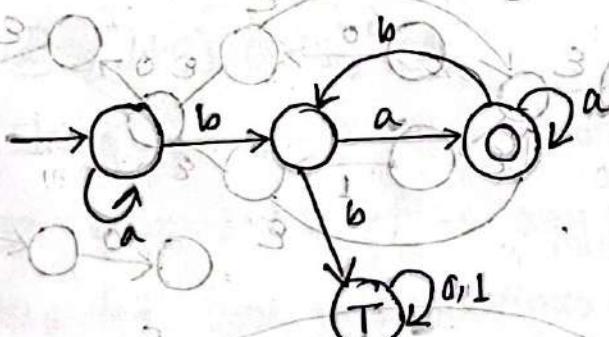
(a)



(b)



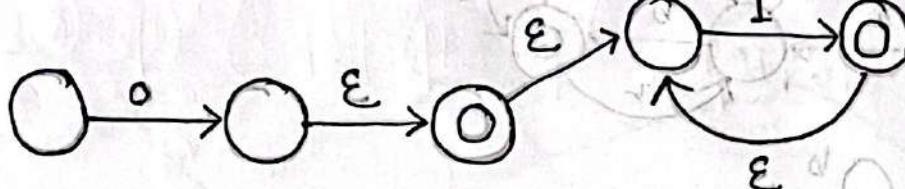
(c)



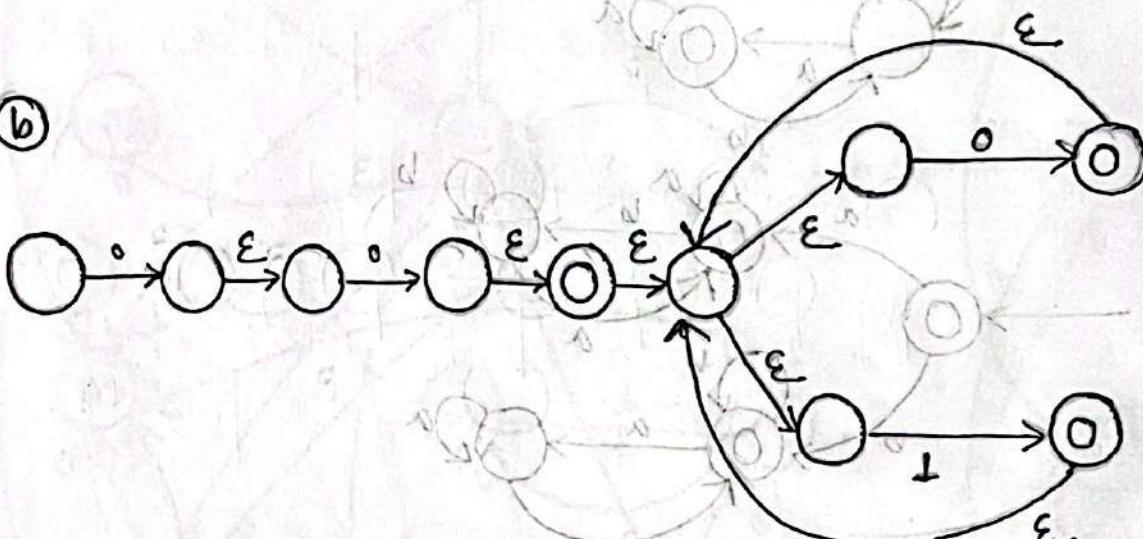
b2

Hamid Book Balinding

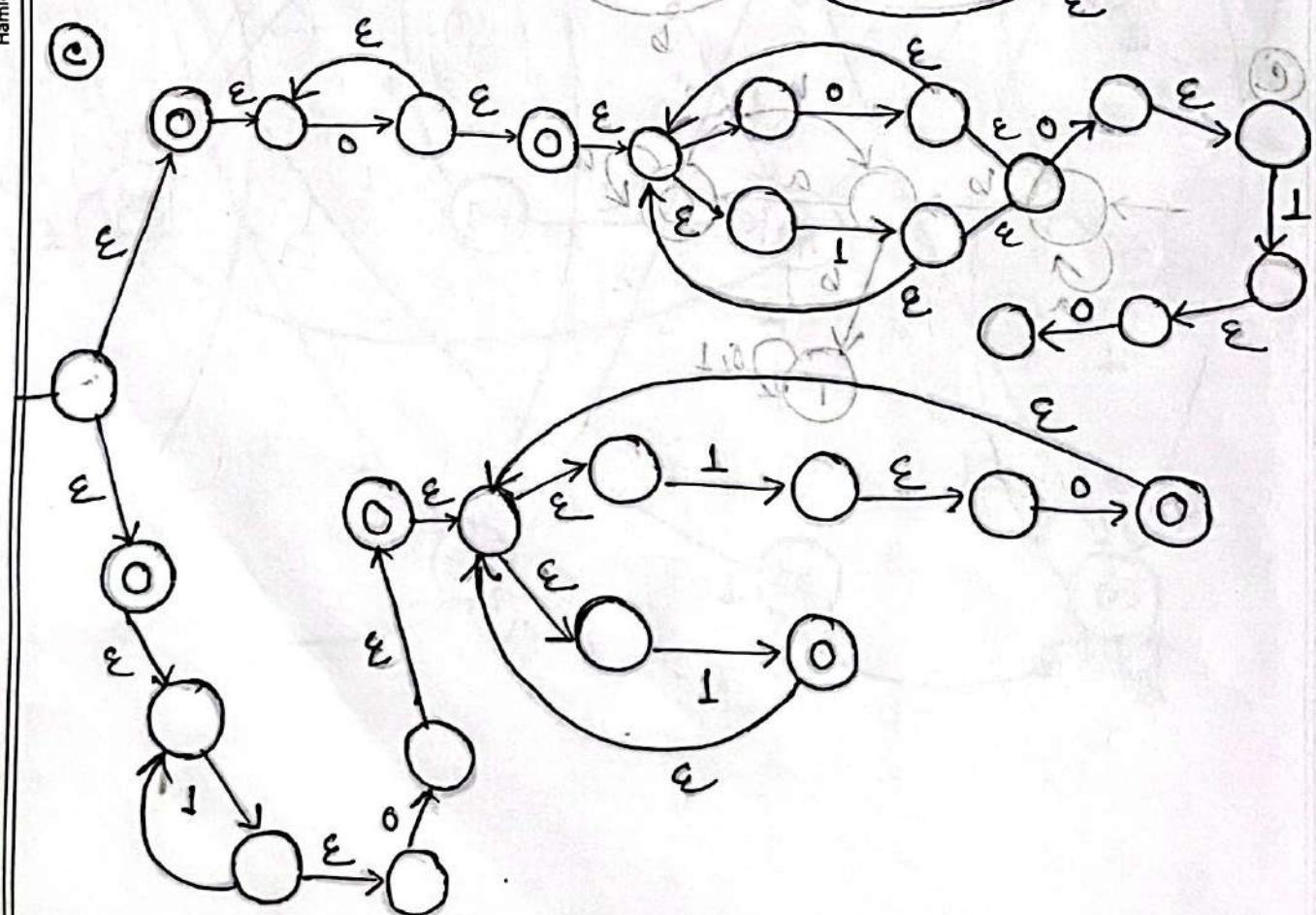
6



b



2



4.

- (a) $0^* (10^*)^* (000^* (10^*)^*)^* (111^*)^*$
- (b) $(0|11^* 00)^* (\epsilon|11^*)$
- (c) $(b^* (ab)^+ b^*)^*$
- (d) $b^* (ab^* | \epsilon)$
- (e) $(0|1)^* \sqcup (0|1)^* \sqcup (0|1)^*$
- (f) $0(00|01|10|11)^* \sqcup (00|11|01|10)^* \sqcup (01|10)^*$
- (g) $(01|10)^*$

5.

- a) $(1+\epsilon)(00^* 1)^* 0^*$

The string can start either nothing or with 1. Zero can be assign before 1 or before any number of zeros, which might be repeated zero or more times. At the end, any number of zeros can be appear.

- b) $(0+10)^* 1^*$

A sequence of single zero or '10' can be appear either zero or more times before any number of 1's.

4. a) $(0+10)^*$ $(1+01)^*$ $(0+\epsilon)$
- (0111) (0101) (0000) (0101) ①
- b) $(0|1)^*$ $((00|01|10|11) (0|1)^*)^*$
- (11|13) (001110) ②
- c) $(b^* (ab)^* b^*)^*$
- (a + (ab)^* a) ③
- d) $b^* (a^* b^* \{ \epsilon \})$
- (b^* b^*)^* a ④
- e) $(0|1)^* 1 (0|1)^* 1 (0|1)^*$
- (1|0) 1 (1|0) 1 (1|0) ⑤
- f) $0((0|1) (0|1))^*$ $1 (0|1) ((0|1) (0|1))^*$
- (01|10) 1 (11|00) 1 (11|01) (10|00) 0 ⑥
- g) $(01|10)^*$
- (01|10) ⑦

$$*0^*(1^*00)(3+1) \quad \textcircled{8}$$

and now only 1 will go to phantom states since no prints left
 before state 2010 so we stated and I stated we have
 to validate that there is no carry over so one behavior of

the last group of digits

$$*L^*(0L+0) \quad \textcircled{9}$$

the state must merge with 01 so will write 000000 A
 21 D. number has ended writing

1. $L = \{ w \in \{0,1\}^* \mid w \text{ starts with } 10 \}$
- $$(0|1)^* 10 \quad \left(\begin{matrix} ((1|0) \\ (1|0)) \end{matrix} \right)$$
- Dont start with 00
 $(0+1)(0+1)^* + 0(1+0(0+1)^*)$
- Dont start with 11
 $(0+1)(0+1)^* + 1(0+0(0+1)^*)$
2. $L = \{ w \in \{0,1\}^* \mid w \text{ ends with } 10 \}$
- $$(0|1)^* 10 \quad \left(\begin{matrix} (1|0) \\ ((1|0) \\ (1|0)) \end{matrix} \right)$$
- Dont start with 01
 $(0+1)(0+1)^* + 0(0+0(0+1)^*)$
3. $L = \{ w \in \{0,1\}^* \mid \text{doesn't end with } 00 \}$
- $$(0|1)^* 1 \quad \left(\begin{matrix} ((0|1)^* 1) \\ ((1|0) \\ (1|0)) \end{matrix} \right)$$
- Dont start with 10
 $(0+1)(0+1)^* + 1(1+0(0+1)^*)$
4. $L = \{ w \in \{0,1\}^* \mid \text{doesn't end with } 01 \}$
- $$(0|1)^* (00|10|11) \quad \left(\begin{matrix} (0|1)^* (00|10|11) \\ ((1|0) \\ ((1|0) \\ (1|0))) \end{matrix} \right)$$
5. $L = \{ w \in \{0,1\}^* \mid \text{doesn't end with } 10 \}$
- $$(0|1)^* (00|01|11) \quad \left(\begin{matrix} (0|1)^* (00|01|11) \\ ((1|0) \\ ((1|0) \\ (1|0))) \end{matrix} \right)$$
6. $L = \{ w \in \{0,1\}^* \mid \text{doesn't end with } 11 \}$
- $$(0|1)^* 0 \quad \left(\begin{matrix} (0|1)^* 0 \\ ((1|0) \\ ((1|0) \\ (1|0))) \end{matrix} \right)$$
7. $L = \{ w \in \{0,1\}^* \mid \text{doesn't end with } 000 \}$
- $$(0|1)^* (001|010|011|100|101|110|111|0|1) \quad \left(\begin{matrix} (0|1)^* (001|010|011|100|101|110|111|0|1) \\ ((1|0) \\ ((1|0) \\ ((1|0) \\ (1|0))) \end{matrix} \right)$$
8. $L = \{ w \in \{0,1\}^* \mid \text{doesn't end with } 001 \}$
- $$(0|1)^* (000|010|011|100|101|110|111|0|1) \quad \left(\begin{matrix} (0|1)^* (000|010|011|100|101|110|111|0|1) \\ ((1|0) \\ ((1|0) \\ ((1|0) \\ (1|0))) \end{matrix} \right)$$

9. $L = \{ w \in \{0,1\}^* \mid \text{length of } w \text{ is even} \}$
- $$(0|1)(0|1)^*$$
10. $L = \{ w \in \{0,1\}^* \mid \text{length of } w \text{ is odd} \}$
- $$(0|1)(0|1)^*(0|1)$$
11. $L = \{ \text{length expressed as } 3k+2 \}$
- $$(0|1)(0|1)(0|1)^*(0|1)(0|1)$$
12. $L = \{ \text{length expressed as } 2k+1 \}$
- $$(0|1)(0|1)^*(0|1)$$
13. $L = \{ \text{count of } 1's/0's \text{ is even} \}$
- $$(0+1)^*$$
14. $L = \{ \text{count of } 0's \text{ is odd} \}$
- $$(1^*01^*)^*(1^*01^*)$$
15. $L = \{ w \in \{0,1\}^* \mid w \text{ contains } 101 \text{ as substring} \}$
- $$(0|1)^*101(0|1)^*$$
16. $L = \{ w \in \{0,1\}^* \mid w \text{ contains } 11 \text{ as substring} \}$
- $$(0|1)^*11(0|1)^*$$

17. $L = \{ w \in \{0,1\}^* \text{ does not contain } 00 \text{ as substring} \}$
- $$\underline{(0|1)^*} \quad 0 \underline{(0|1)^*} \quad 1 \underline{(0|1)^*} \quad (1|01)^* \quad (0|1)^*$$
18. $L = \{ w \in \{0,1\}^* \text{ does not contain } 01 \text{ as substring} \}$
- $$(0|1)^* \quad 0^* \underline{(0|1)^*} \quad / \quad 1^* \quad 0^* \quad 1^* \quad 0^* \quad 1^* \quad 1^*$$
19. $L = \{ w \in \{0,1\}^* \text{ does not contain } 10 \text{ as substring} \}$
- $$\underline{(0|1)^*} \quad 1^* \underline{(0|1)^*} \quad / \quad 0^* \quad 1^*$$
20. $L = \{ w \in \{0,1\}^* \text{ does not contain } 11 \text{ as substring} \}$
- $$\underline{(0|1)^*} \quad 0^* \underline{(0|1)^*} \quad (0|10)^* \quad (1|0)$$
21. $L = \{ \text{contains } 00 \text{ or } 11 \text{ as substrings} \}$
- $$(0|1)^* \quad 00 \underline{(0|1)^*} \quad | \quad (0|1)^* \quad 11 \underline{(0|1)^*} \quad (1|0)$$
22. $L = \{ w \text{ contains at least two } 0's \}$
- $$(0|1)^* \quad 0 \quad (0|1)^* \quad 0 \quad (0|1)^* \quad (1|0) \quad (1|0) \quad (2|0)$$
23. $L = \{ w \text{ contains at least one } 0 \}$
- $$(0|1)^* \quad 0 \quad (0|1)^* \quad (1|0) \quad (1|0) \quad (1|0) \quad (1|0)$$
24. $L = \{ w \text{ contains at least two } 01 \}$
- $$3 \quad (0|1)^* \quad 01 \quad (0|1)^* \quad (1|0) \quad (0|1)^* \quad (1|0) \quad (1|0) \quad (1|0)$$

Subject:

Date:

17. $L = \{ w \in \{0,1\}^* \text{ does not contain } 00 \text{ as substring} \}$
- $$\underline{(0|1)^*} \ 0 \underline{(0|1)^*} \ 1 \underline{(0|1)^*} \ (1|01)^* (0|1^*) \epsilon$$
18. $L = \{ w \in \{0,1\}^* \text{ does not contain } 01 \text{ as substring} \}$
- $$\underline{(0|1)^*} 0^* \underline{(0|1)^*} / 1^* 0^*$$
19. $L = \{ w \in \{0,1\}^* \text{ does not contain } 10 \text{ as substring} \}$
- $$\underline{(0|1)^*} 1^* \underline{(0|1)^*} / 0^* 1^*$$
20. $L = \{ w \in \{0,1\}^* \text{ does not contain } 11 \text{ as substring} \}$
- $$\underline{(0|1)^*} 0^* \underline{(0|1)^*} / (0|1^*) 1^* (1|0)$$
21. $L = \{ w \text{ contains } 00 \text{ or } 11 \text{ as substrings} \}$
- $$(0|1)^* 00 \underline{(0|1)^*} / (0|1)^* 11 \underline{(0|1)^*} (1|0)$$
22. $L = \{ w \text{ contains at least two } 0's \}$
- $$(0|1)^* 0 \ (0|1)^* 0 \ (0|1)^* 0 \ (0|1)^* 0$$
23. $L = \{ w \text{ contains at least one } 0 \}$
- $$(0|1)^* 0 \ (0|1)^* 1 | 0 \ (0|1)^* 1 | 1 \ (0|1)^* 1 | 0$$
24. $L = \{ w \text{ contains at least one } 1 \}$
- $$(0|1)^* 0 \ 1 | (0|1)^* 1 | 0 \ (0|1)^* 1 | 1 | 0$$

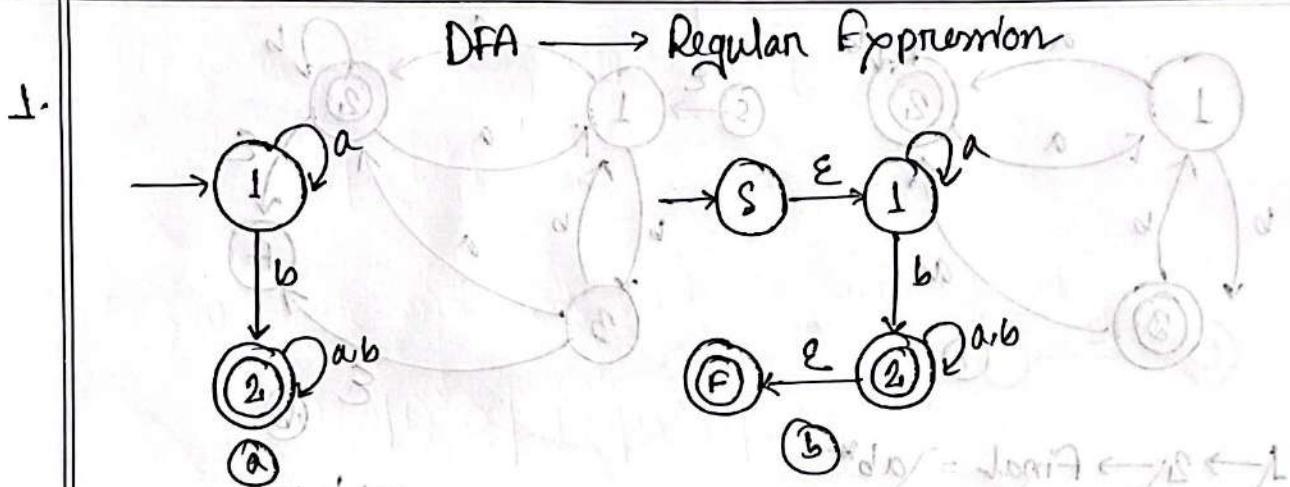
25. $L = \{ \omega \text{ contains at most two 1's} \}$ tornish $\{1, 0\}^*$
- $$(0^* + 0^* 1 0^* + 0^* 1 0^* 1 0^*)^*$$
26. $L = \{ \omega \text{ contains at most three 0's} \}$ tornish $\{1, 0\}^*$
- $$1^* + 1^* 0 1^* + 1^* 0 1^* 0 1^* + 1^* 0 1^* 0 1^* 0 1^*$$
27. $L = \{ \omega \text{ contains exactly two 1's} \}$ tornish $\{1, 0\}^*$
- $$0^* 1 0^* 1 0^*$$
28. $L = \{ \omega \text{ contains exactly two 0's} \}$ tornish $\{1, 0\}^*$
- $$1^* 0 1^* 0 1^*$$
29. $L = \{ \text{length of } \omega \text{ is multiple of 3} \}$ length of 000 is 3
- $$((0|1)(0|1)(0|1))^*$$
30. $L = \{ \text{length of } \omega \text{ is not multiple of 3} \}$ length of 011 is 3
- $$((0|1)(0|1)(0|1))^* (0|1) ((0|1) \cup)$$
31. $L = \{ \text{length of } \omega \text{ is multiple of 5} \}$ length of 00000 is 5
- $$((0|1)(0|1)(0|1)(0|1)(0|1))^*$$
32. $L = \{ \text{length of } \omega \text{ is not multiple of 5} \}$ length of 01111 is 5
- $$((0|1)(0|1)(0|1)(0|1)(0|1))^* (0|1) ((0|1) \cup)$$

33. $L = \{ \text{count of } 1 \text{ is multiple of 3} \}$
- $$0^* | (0^* 1 0^* 1 0^*)^*$$
34. $L = \{ w \text{ starts and ends with the same symbol} \}$
- $$0 (0|1)^* 0 | 1 (0|1)^* 1 | 0 | 1$$
35. $L = \{ w \text{ starts and ends with different symbols} \}$
- $$0 (0|1)^* 1 | 1 (0|1)^* 0$$
36. $L = \{ w \text{ contains 0 at every 3rd position} \}$
- $$((0|1) (0|1) 0)^* (0|1) (0|1) | ((0|1) (0|1) 0)^* (0|1) | ((0|1) (0|1) 0)^*$$
37. $L = \{ w \text{ have 0's and 1's in alternates} \}$
- $$(1|\varepsilon) (0|1)^* (0|\varepsilon) | (0|\varepsilon) (10)^* (1|\varepsilon)$$
38. $L = \{ w \text{ divisible by 3} \}$
- $$((0|1) (0|1) (0|1))^*$$
39. $L = \{ \text{Every second letter is zero} \}$
- $$((0|1) 0)^* (0|1|\varepsilon)$$

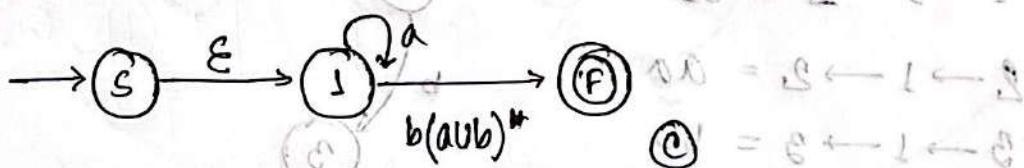
40. $L = \{ \text{string that have neither consecutive 1's, nor consecutive 0's} \}$
- $$(0|\varepsilon) (10)^* (1|\varepsilon)$$
41. $L = \{ \text{string that might have consecutive 1's, or consecutive 0's but not both} \}$
- $$(1|01)^* (0|\varepsilon)$$
42. $L = \{ 1 \text{ does not appear at any even position in } \omega \}$
43. $L = \{ \text{every 1 is followed by at least one 0} \}$
- $$(0(10)(10)) | ((10) (0(10)) | (10)(10) | (0(10))(10))$$
44. $L = \{ \text{every 1 is followed by at most one 0} \}$
- $$(3|1)^*(01) (3|0) | (3|0)^*(10) (3|1)$$
45. $L = \{ \text{every 1 is followed by exactly one 0} \}$
46. $L = \{ \text{every second letter is zero} \}$
- $$(0|10)(0|10)(1|0)$$
47. $L = \{ 01 \text{ appears even a number of times} \}$
- $$(3|10)^*(0(10))$$

Subject:

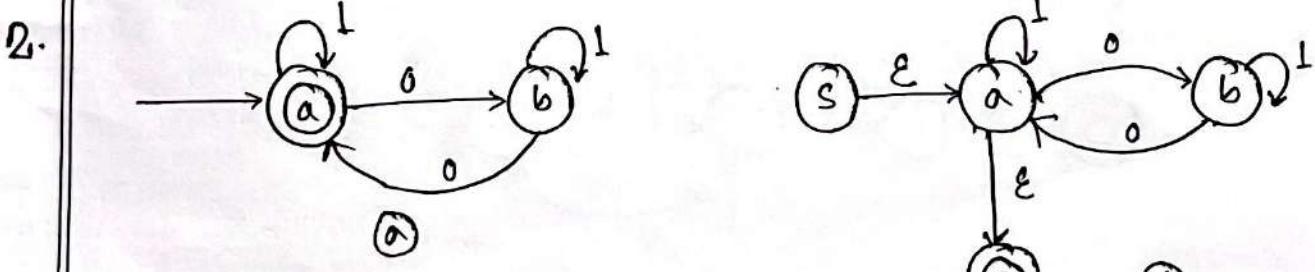
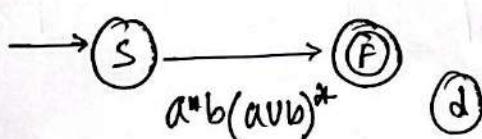
Date:



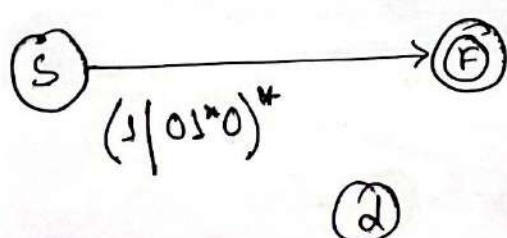
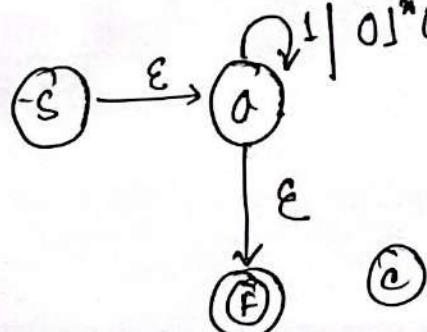
$$1 \rightarrow 2 \rightarrow \text{final} = a^* b (a \cup b)^*$$



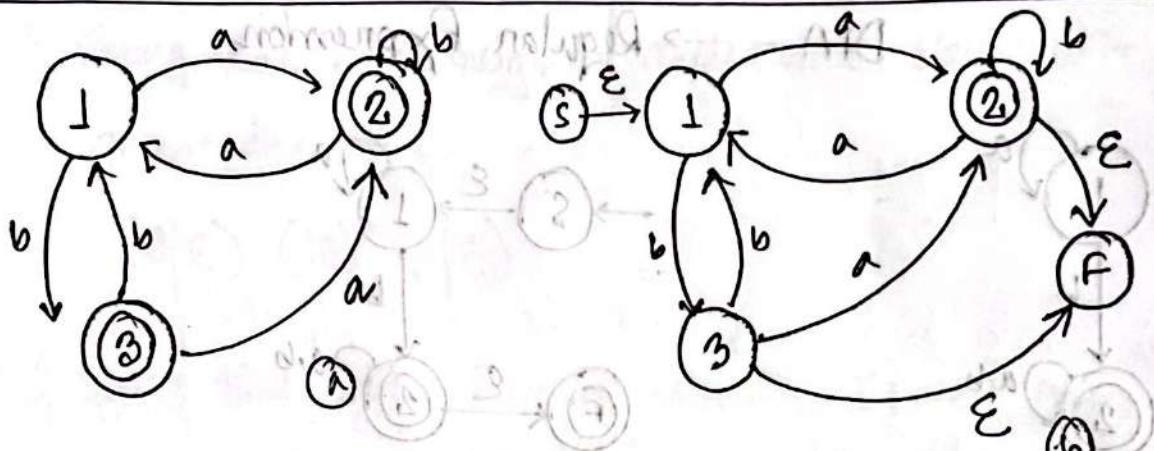
$$\text{Start} \rightarrow 1 \rightarrow \text{finish} = a^* b (a \cup b)^*$$



$$a \rightarrow b \rightarrow a = 01^* 0$$



3.



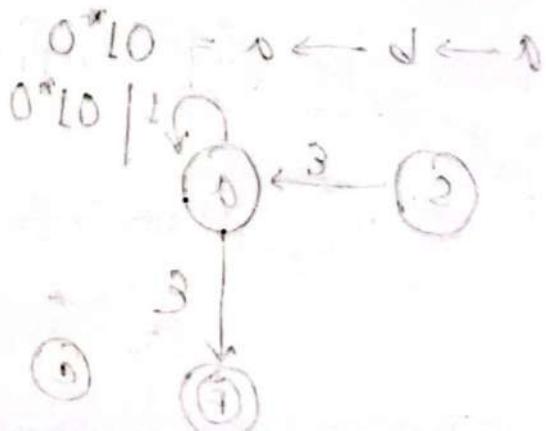
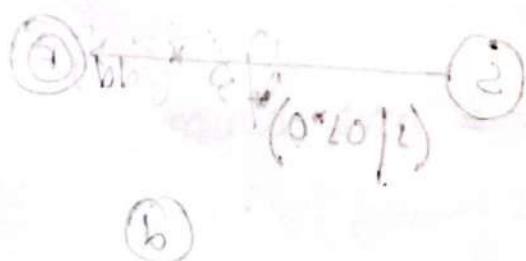
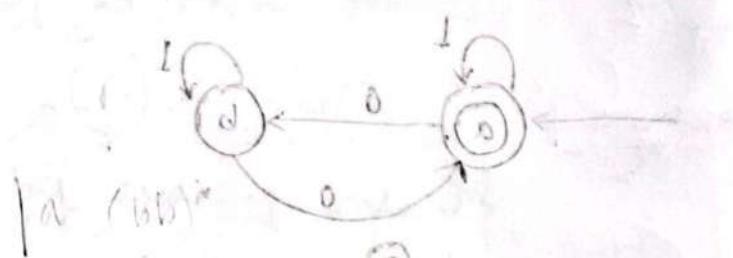
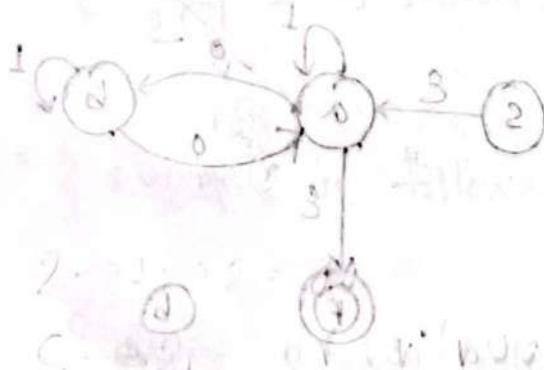
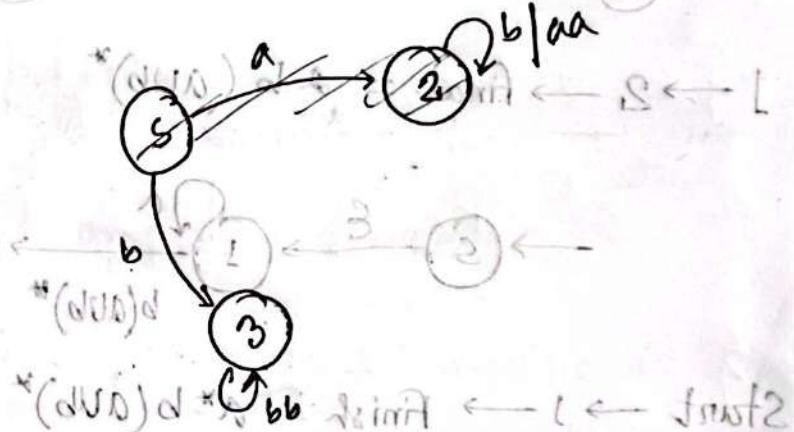
$$1 \rightarrow 2 \rightarrow \text{Final} = ab^*$$

$$1 \rightarrow 2 \rightarrow 1 = ab^*a$$

$$1 \rightarrow 3 \rightarrow 1 = bb$$

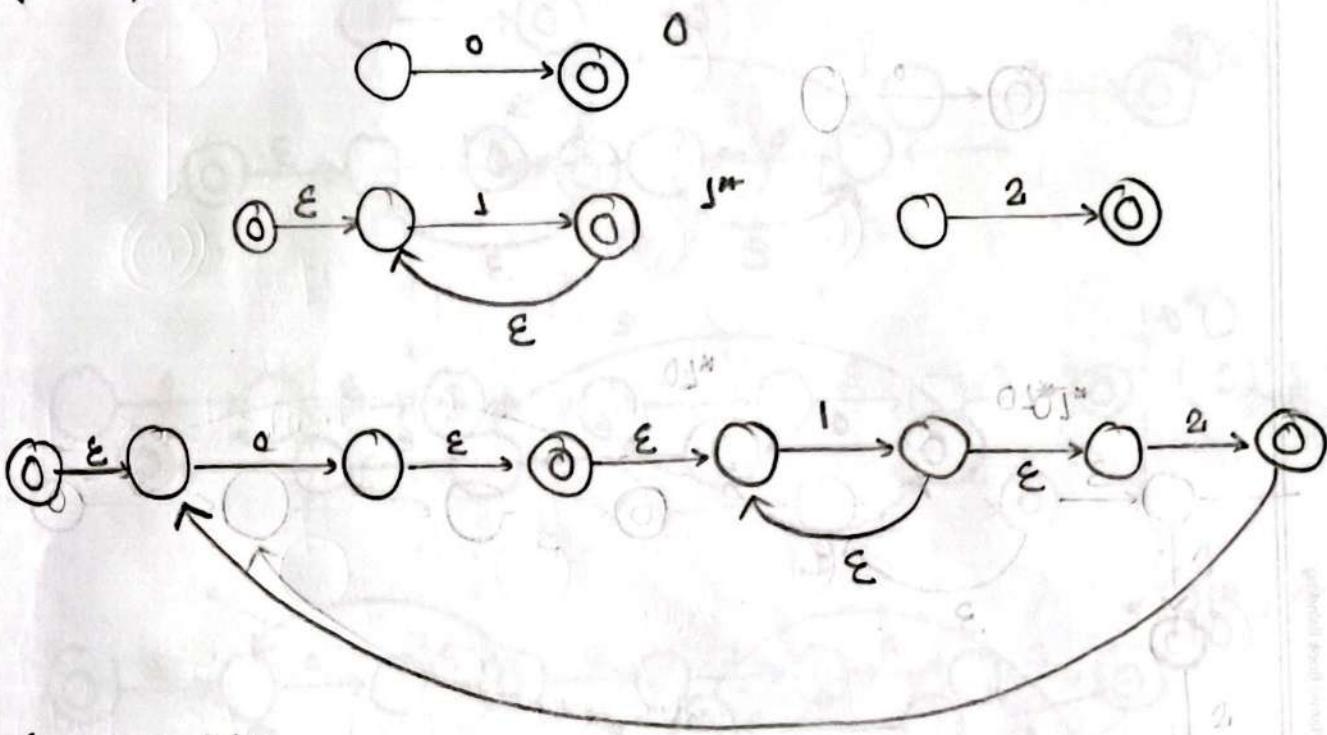
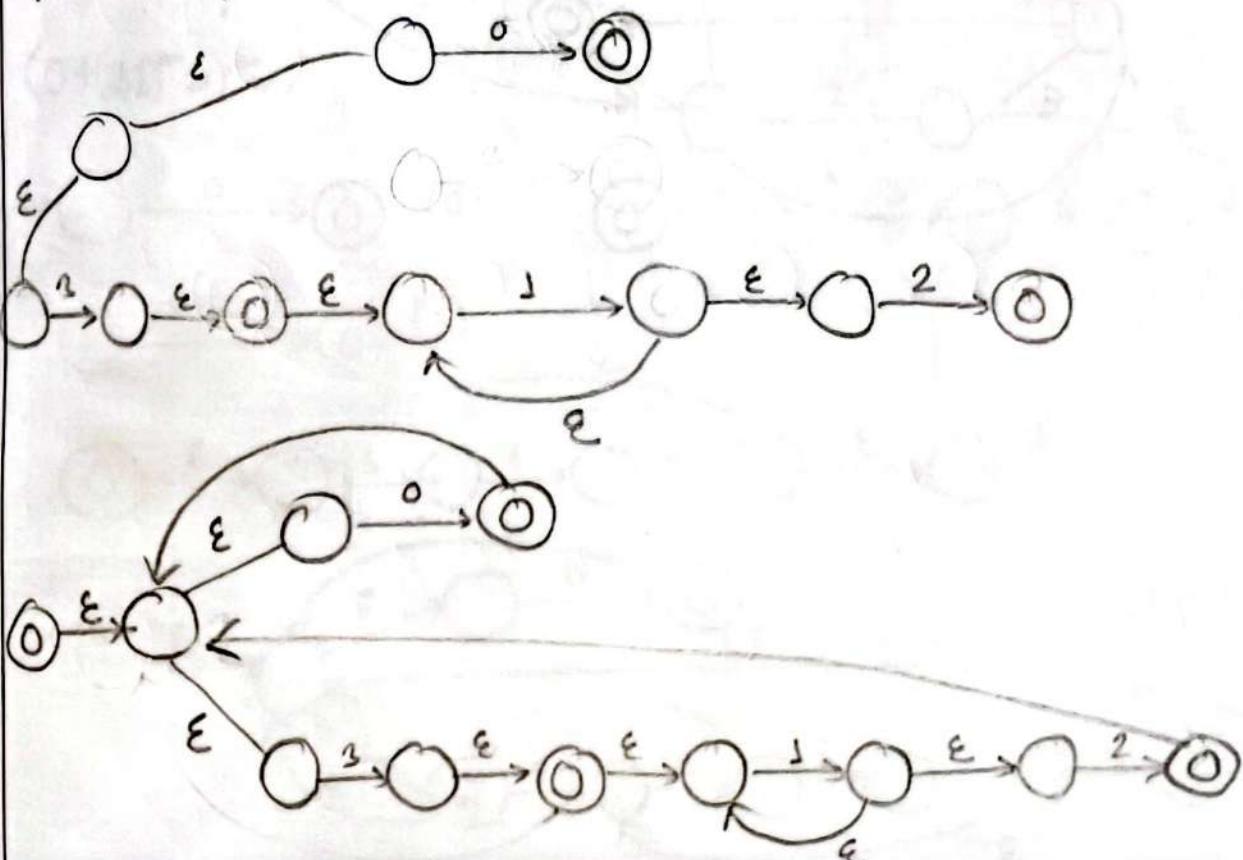
$$2 \rightarrow 1 \rightarrow 2 = aa$$

$$3 \rightarrow 1 \rightarrow 3 = bb$$



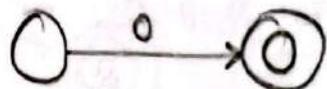
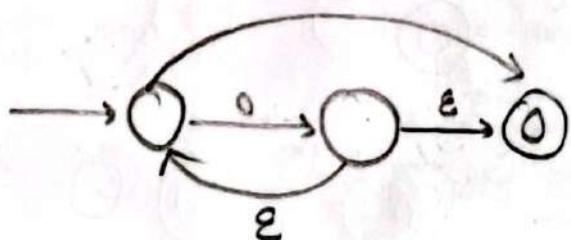
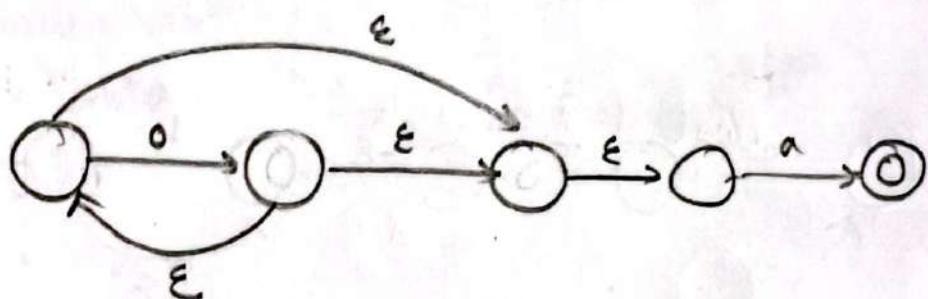
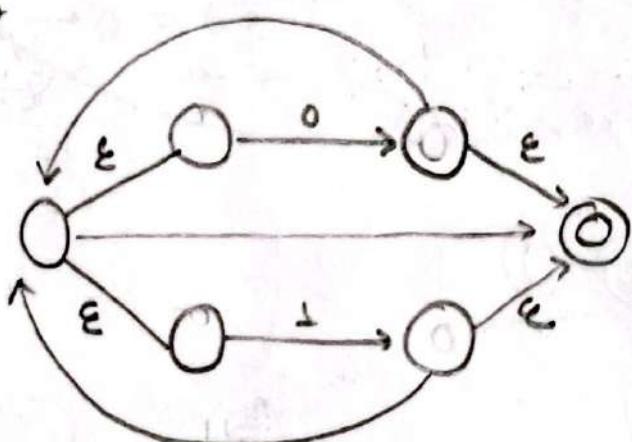
Subject : Date :

RE to NFA

 $(01^*2)^*$  $(0+31^*2)^*$ 

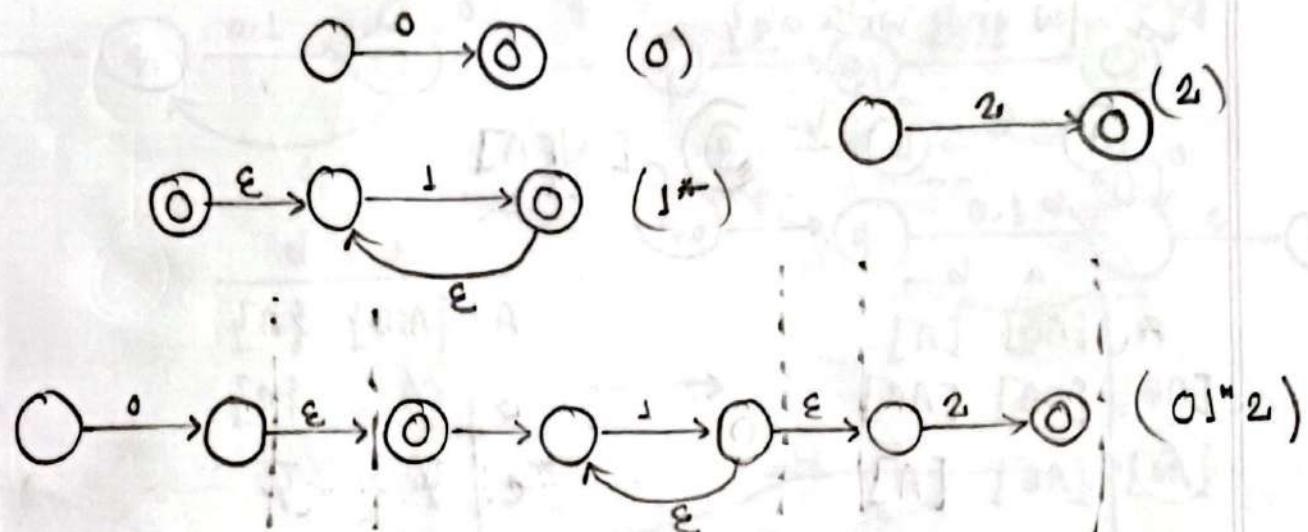
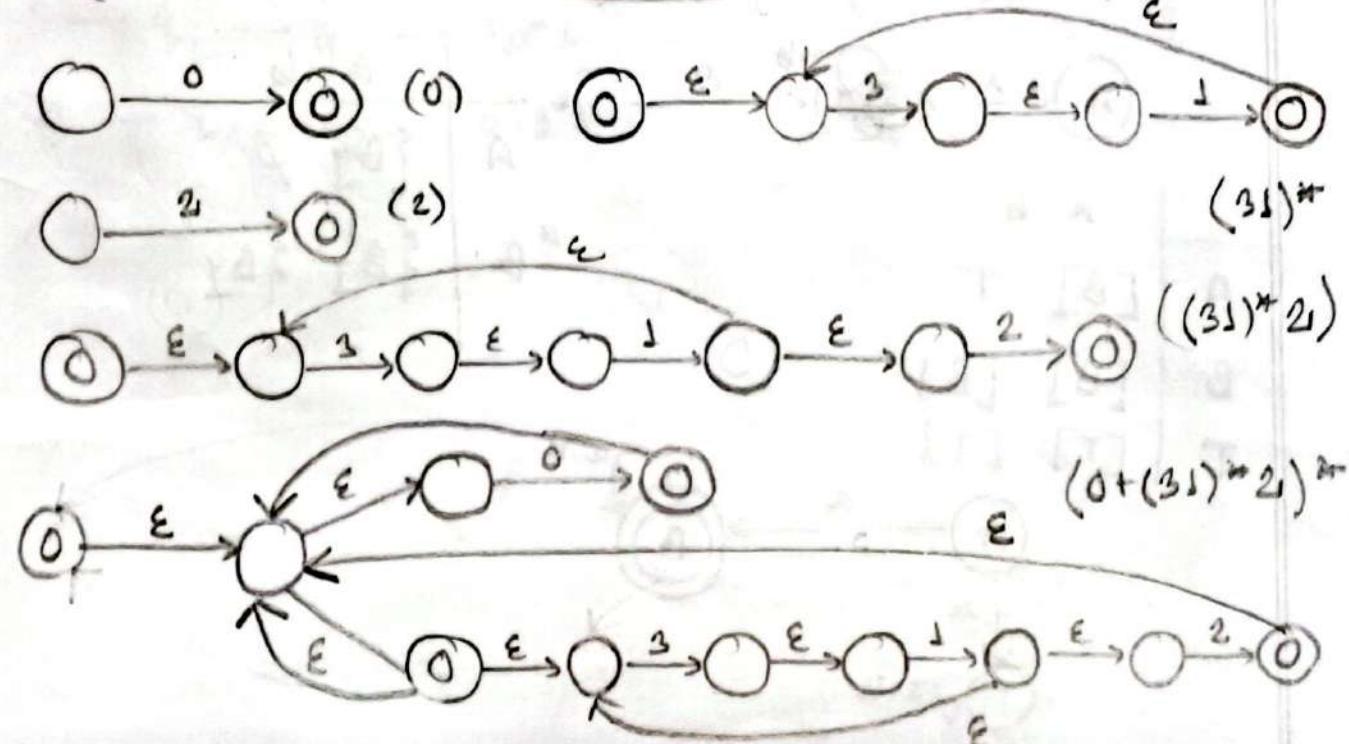
Subject:

Date:

 0^*  0^*  0^*a  $(0+1)^*$ 

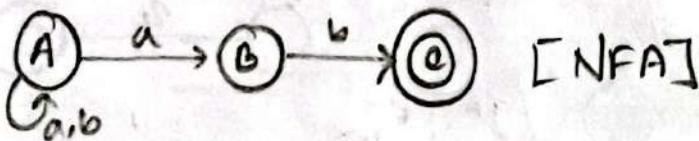
Subject: _____

Date: _____

 $(01^*2)^*$  $(0+(31)^*2)^*$ 

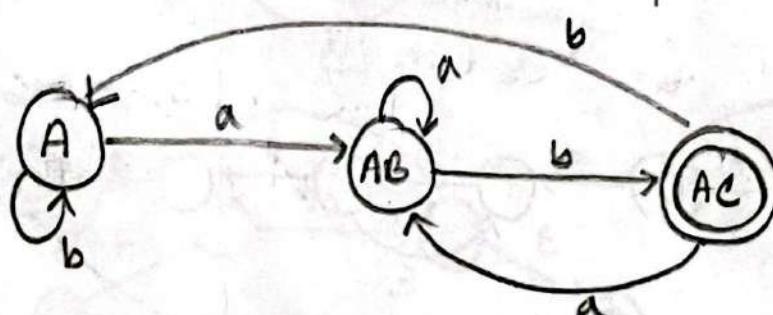
NFA TO DFA

$L = \{ w | w \text{ ends with } ab \}$

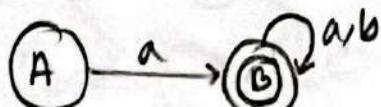


	a	b
A	[AB]	[A]
[AB]	[AB]	[AC]
[AC]	[AB]	[A]

	a + b
A	{A,B}
B	\emptyset
*c	\emptyset

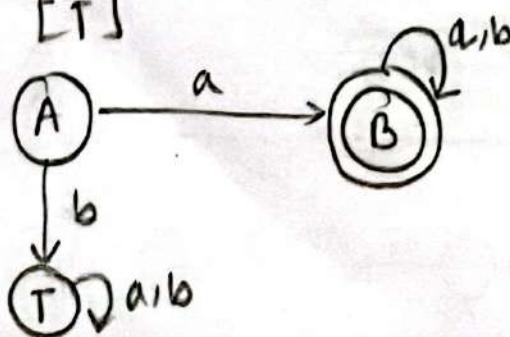


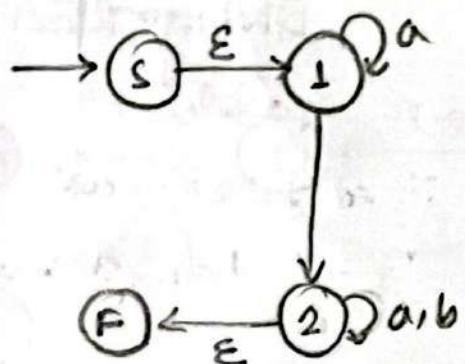
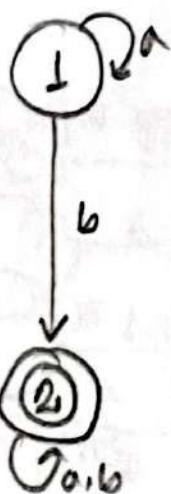
$L = \{ w | w \text{ starts with 'ab'} \}$



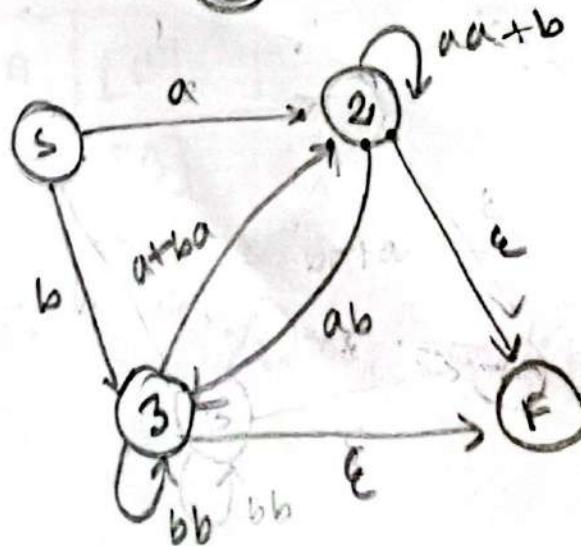
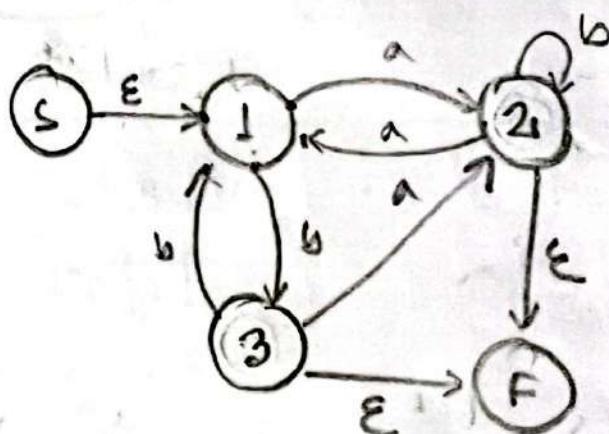
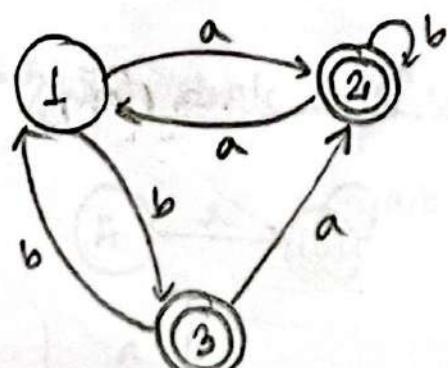
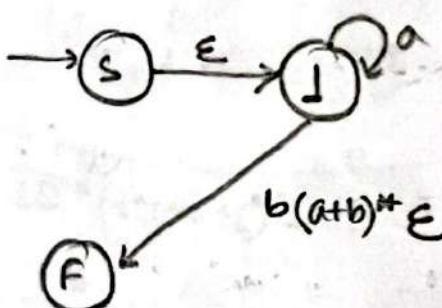
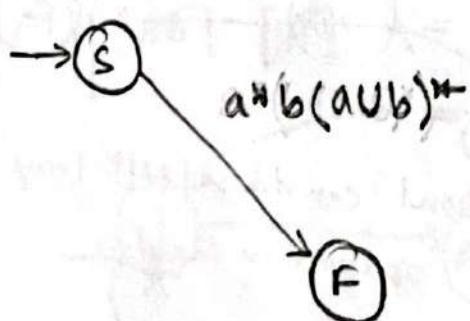
	a	b
A	[B]	T
B	[B]	[B]
T	[T]	[T]

	a	b
A	{B}	\emptyset
*B	{B}	{B}





$$1 \rightarrow 2 \rightarrow F \rightarrow b(a+b)^* \epsilon$$



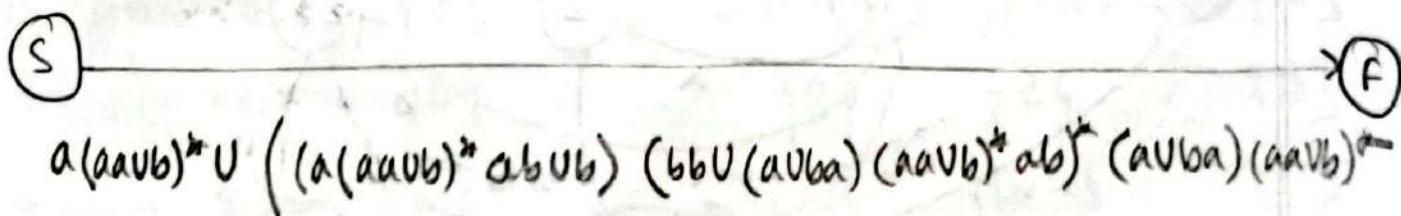
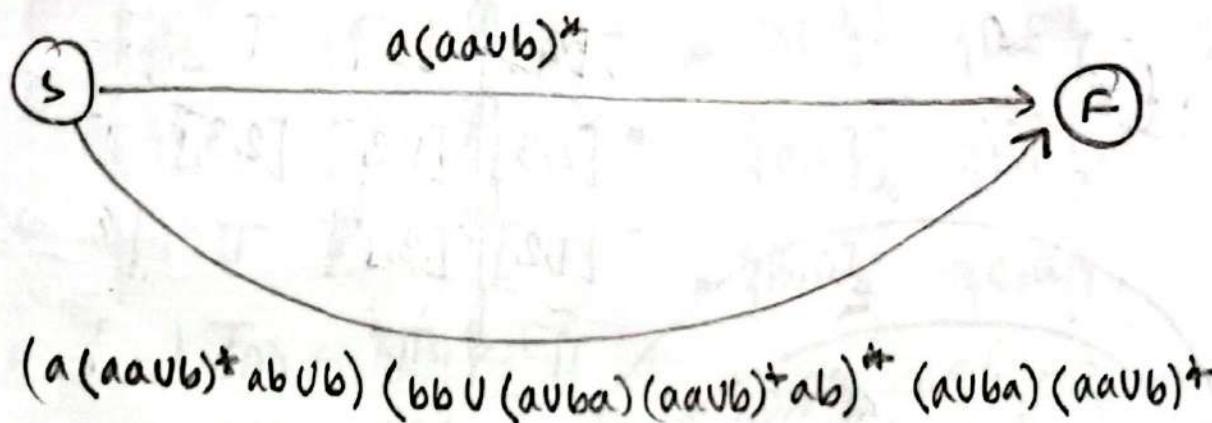
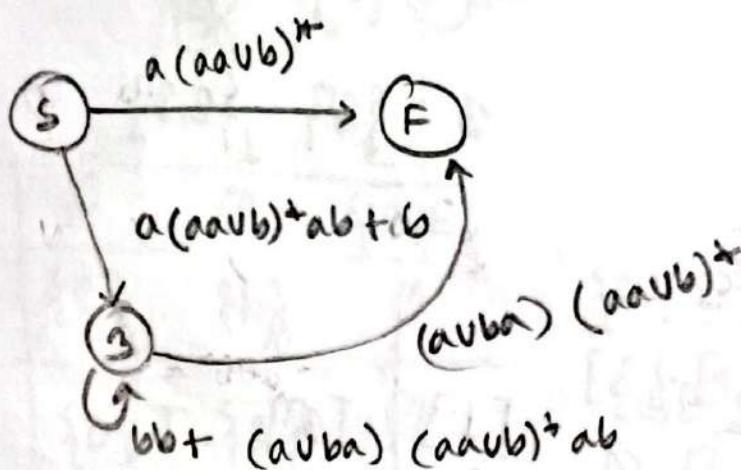
$S \rightarrow 1 \rightarrow 2 = a \checkmark$
 $S \rightarrow 1 \rightarrow 3 = b \checkmark$
 $3 \rightarrow 1 \rightarrow 2 = aba -$
 $3 \rightarrow 1 \rightarrow 3 = bbb \checkmark$
 $2 \rightarrow 1 \rightarrow 2 = aaa \checkmark$
 $2 \rightarrow 1 \rightarrow 3 = ab$

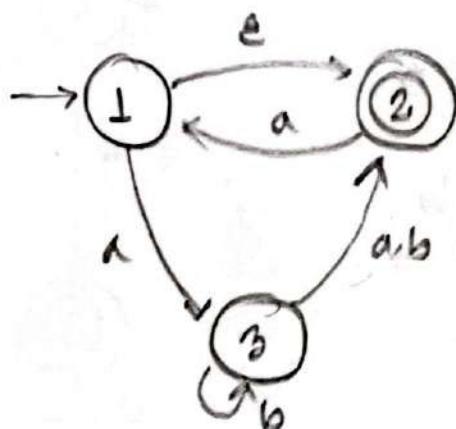
$$S \rightarrow 2 \rightarrow F = a(aaUb)^*$$

$$S \rightarrow 2 \rightarrow 3 \Leftrightarrow a(aaUb)^* ab$$

$$3 \rightarrow 2 \rightarrow F = (auba)(aaUb)^*$$

$$3 \rightarrow 2 \rightarrow 3 = (auba)(aaUb)^* ab$$

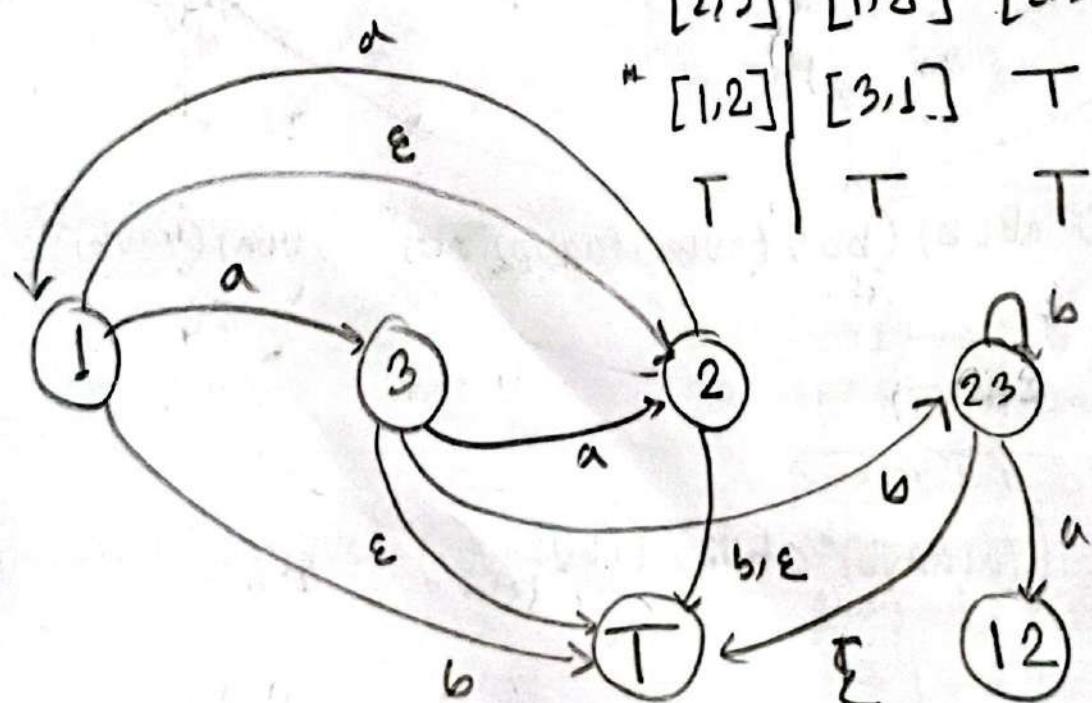


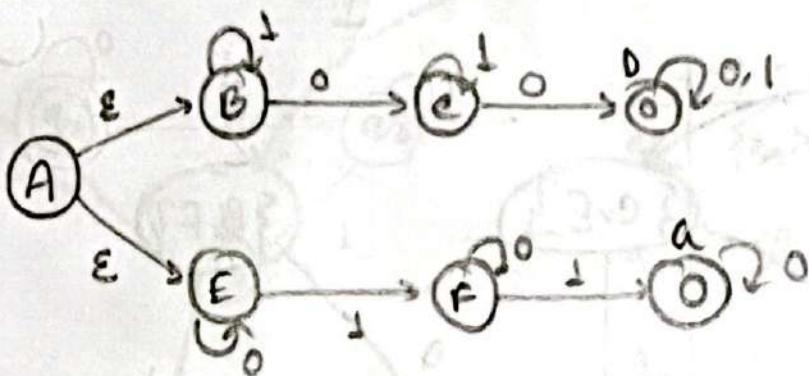
NFA \rightarrow DFA

	a	b	b
1	{3}	\emptyset	
2	{1}	\emptyset	
3	{2}	{2,3}	

	a	b	e
[3,1]	[3,2]	[2,3]	[2]
[3,2]	[1,2]	[2,3]	\emptyset

	a	b
[1]	[3]	T
[3]	[2]	[2,3]
*[2]	[1]	T
*[2,3]	[1,2]	[2,3]
*[1,2]	[3,1]	T
T	T	T

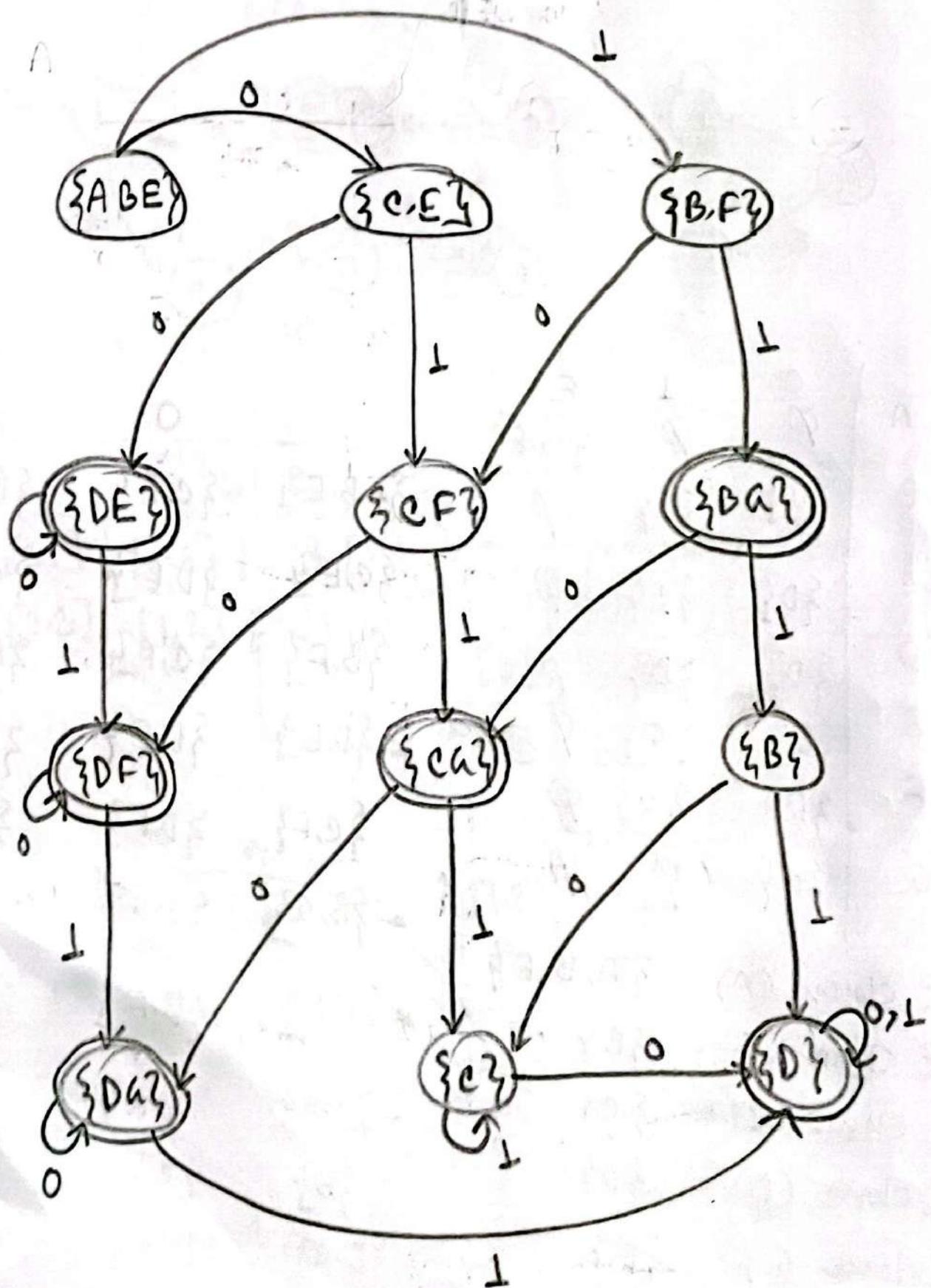




	0	1	ϵ
A	\emptyset	\emptyset	$\{B, E\}$
B	$\{C\}$	$\{B\}$	\emptyset
C	$\{D\}$	$\{C\}$	\emptyset
D	$\{D\}$	$\{D\}$	\emptyset
E	$\{E\}$	$\{F\}$	\emptyset
F	$\{F\}$	$\{G\}$	\emptyset
G	$\{G\}$	\emptyset	\emptyset

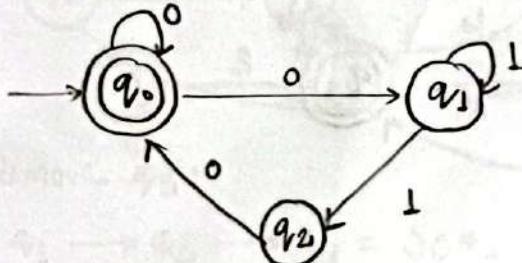
	0	1
ϵ closure (A) =	$\{A, B, E\}$	
ϵ closure (B) =	$\{B\}$	$\{B, F\}$
ϵ closure (C) =	$\{C\}$	$\{C, F\}$
ϵ closure (D) =	$\{D\}$	$\{D, F\}$
ϵ closure (E) =	$\{E\}$	$\{D, G\}$
ϵ closure (F) =	$\{F\}$	$\{G\}$
ϵ closure (G) =	$\{G\}$	$\{D\}$

Hamid Book Binding

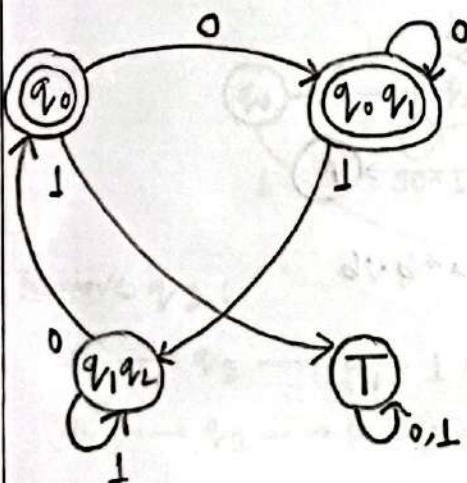


NFA TO DFA

1.

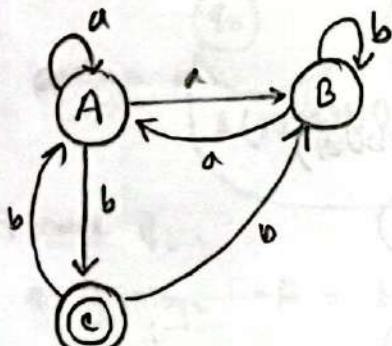


	0	1
q_0	$\{q_0, q_1\}$	\emptyset
q_1	\emptyset	$\{q_1, q_2\}$
q_2	q_0	\emptyset



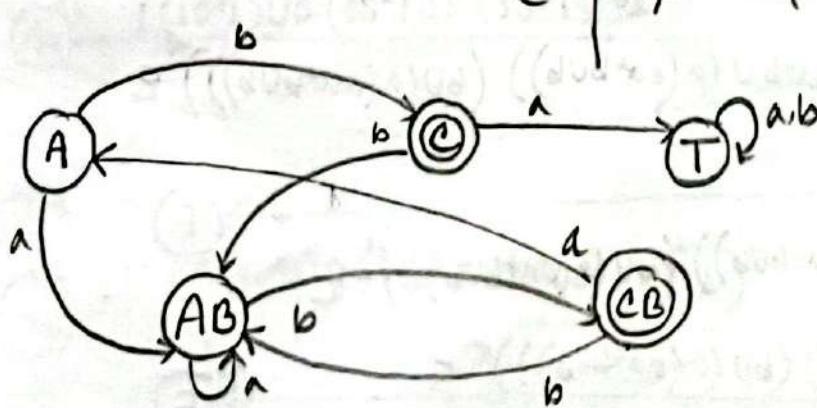
	0	1
$*[q_0]$	$[q_0, q_1]$	T
$*[q_0, q_1]$	$[q_0, q_1]$	$[q_1, q_2]$
$*[q_1, q_2]$	$[q_0]$	$[q_1, q_2]$
T	T	T

2.



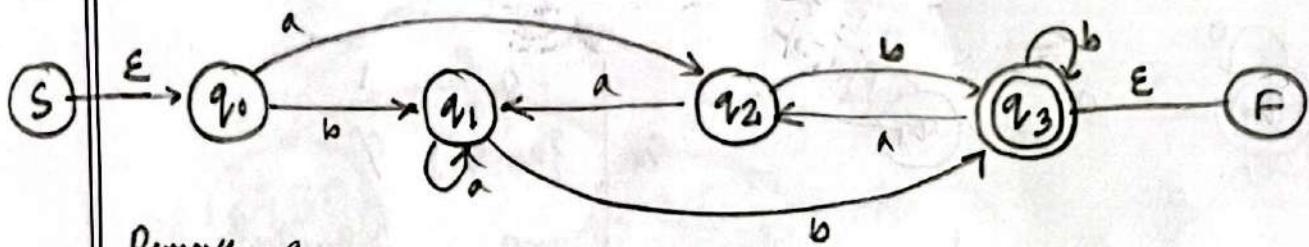
	a	b
A	$\{A, B\}$	$\{C\}$
B	$\{A\}$	$\{B\}$
C	\emptyset	$\{A, B\}$

	a	b
$*[A]$	$[A, B]$	$[C]$
$*[AB]$	$[A, B]$	$[C, B]$
$*[C]$	T	$[A, B]$
$*[C, B]$	$[A]$	$[A, B]$
T	T	T



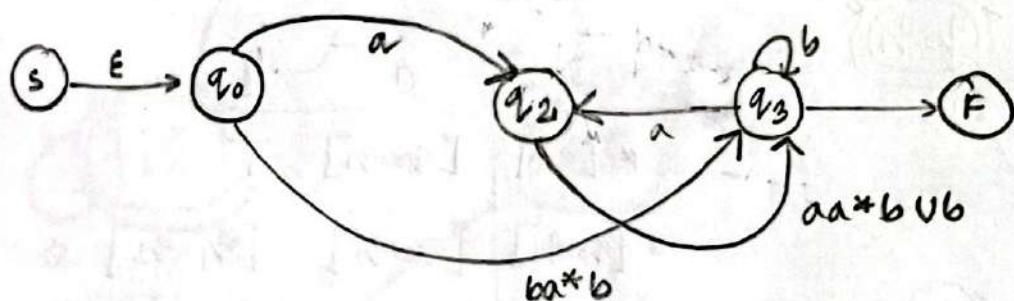
1.

DFA to RE (Mid Question)

Remove q_1

$$q_0 \rightarrow q_1 \rightarrow q_3 = ba^*b$$

$$q_2 \rightarrow q_1 \rightarrow q_3 = aa^*b$$

Remove q_2

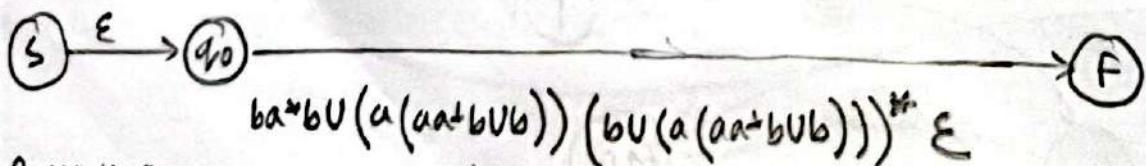
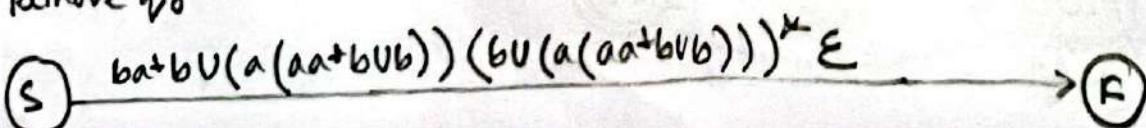
$$q_0 \rightarrow q_2 \rightarrow q_3 = a(aa^*b \cup b)$$

$$q_3 \rightarrow q_2 \rightarrow q_3 = a(aa^*b \cup b)$$

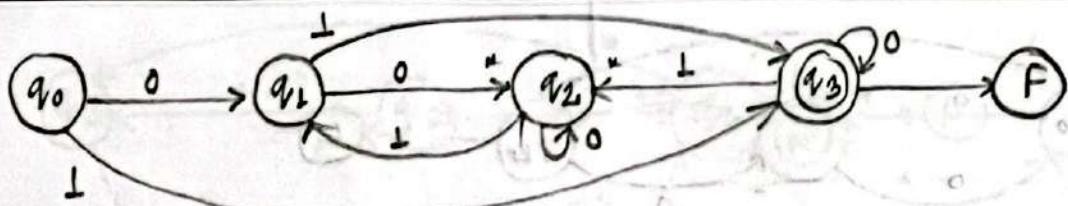
Remove q_3

$$ba^*b \cup (a(aa^*b \cup b))$$

$$q_0 \rightarrow q_3 \rightarrow F = ba^*b \cup (a(aa^*b \cup b)) (b \cup (a(aa^*b \cup b)))^* \epsilon$$

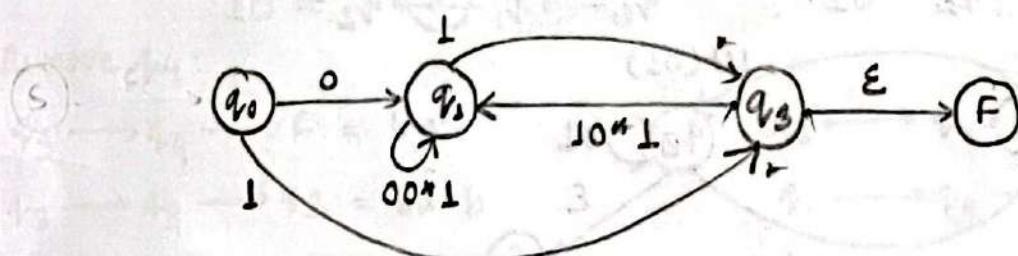
Remove q_0 

2.

Remove q_2 :

$$q_1 \rightarrow q_2 \rightarrow q_3 = 00^*1$$

$$q_3 \rightarrow q_2 \rightarrow q_1 = 10^*1$$

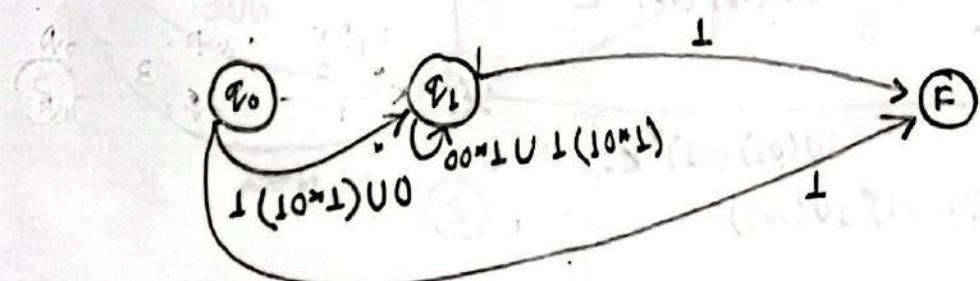
Remove q_3 :

$$q_1 \rightarrow q_3 \rightarrow q_1 = 1(10^*1)$$

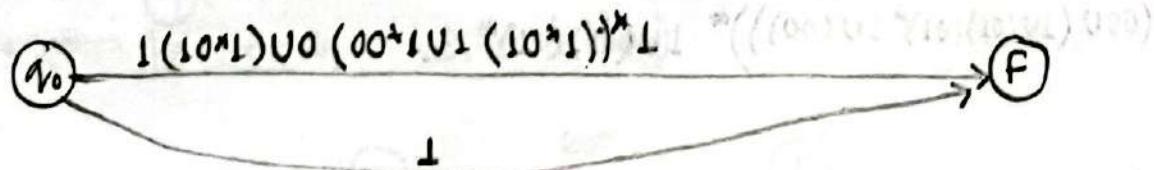
$$q_1 \rightarrow q_3 \rightarrow F = 1$$

$$q_0 \rightarrow q_3 \rightarrow q_1 = 1(10^*1)$$

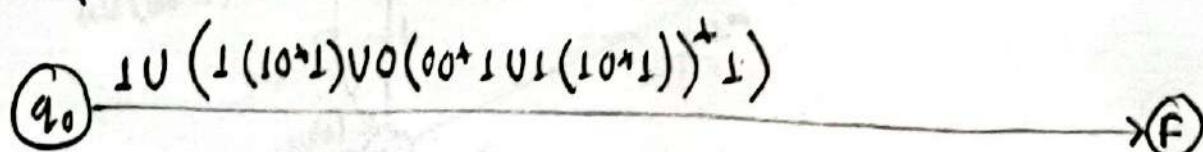
$$q_0 \rightarrow q_3 \rightarrow F = 1$$

Remove q_1 :

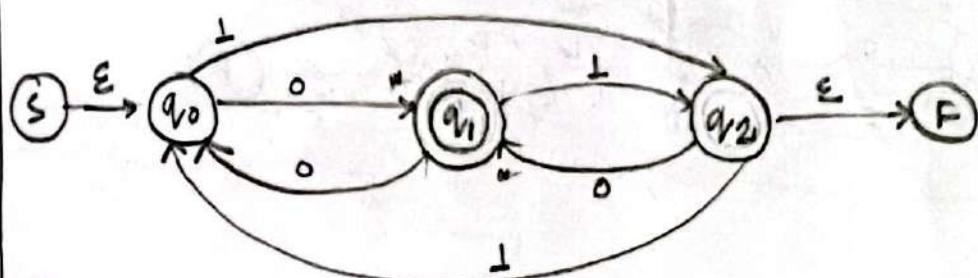
$$q_0 \rightarrow q_1 \rightarrow F = 1(10^*1)U0(00^*1U1(10^*1))^*1$$



Finally



3.

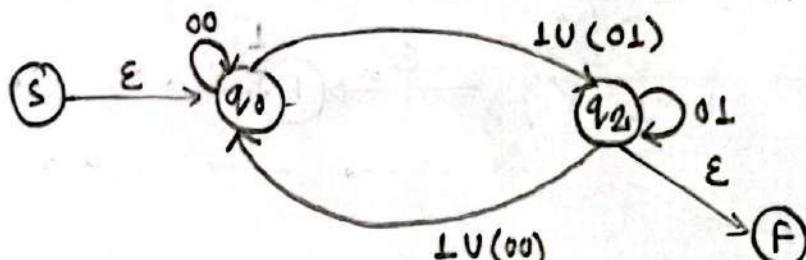
Remove q_1 :

$$q_0 \rightarrow q_1 \rightarrow q_0 = 00$$

$$q_2 \rightarrow q_1 \rightarrow q_0 = 00$$

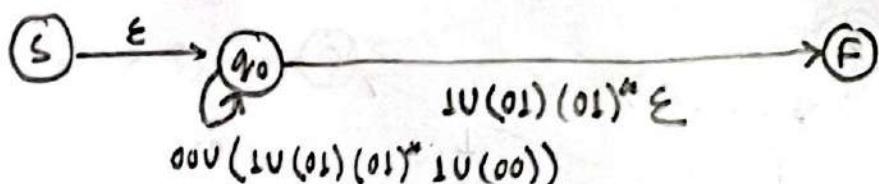
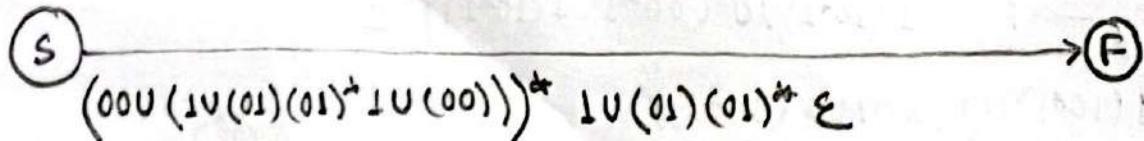
$$q_0 \rightarrow q_1 \rightarrow q_2 = 01$$

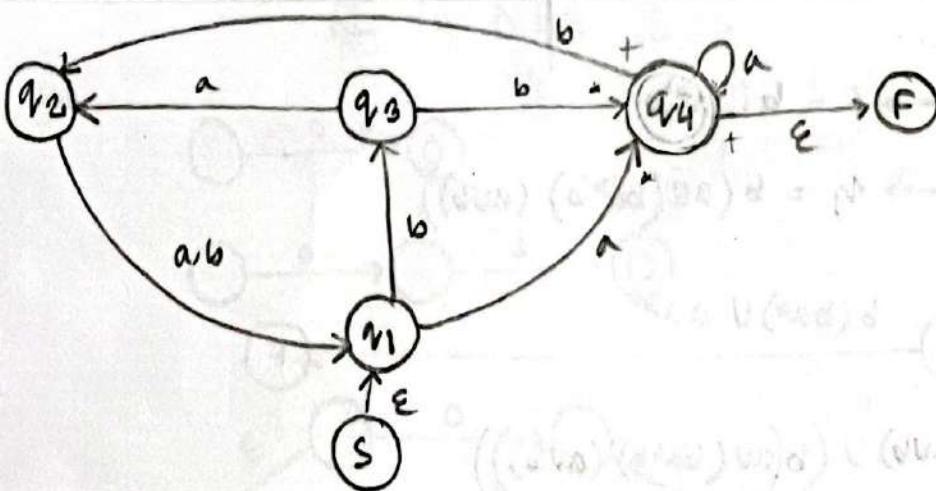
$$q_2 \rightarrow q_1 \rightarrow q_2 = 01$$

Remove q_2 :

$$q_0 \rightarrow q_2 \rightarrow q_0 = L \cup (01) (01)^* L \cup (00)$$

$$q_0 \rightarrow q_2 \rightarrow F = L \cup (01) (01)^* \epsilon$$

Remove q_0 :



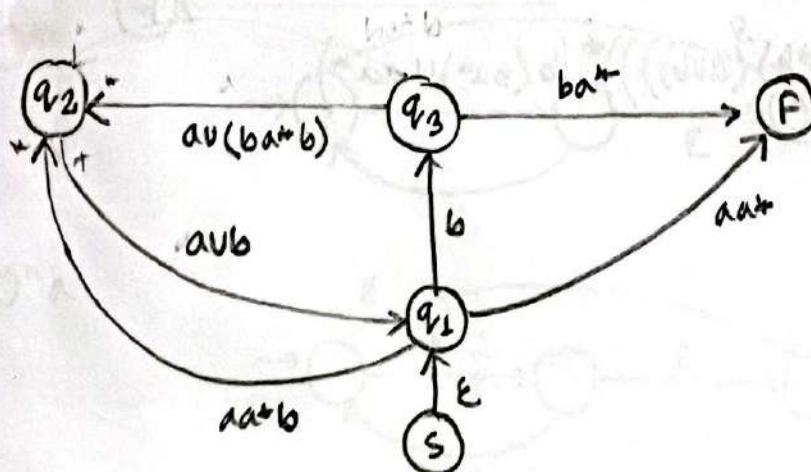
Remove q_4 :

$$q_3 \rightarrow q_4 \rightarrow F = ba^*$$

$$q_3 \rightarrow q_4 \rightarrow q_2 = ba^*b$$

$$q_1 \rightarrow q_4 \rightarrow F = aa^*$$

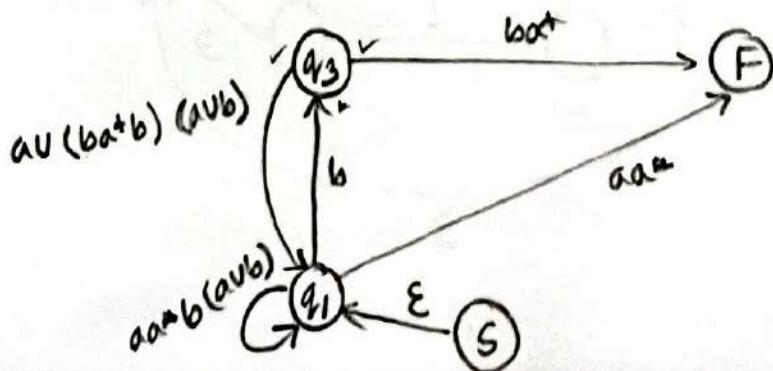
$$q_1 \rightarrow q_4 \rightarrow q_2 = aa^*b$$



Remove q_2 :

$$q_1 \rightarrow q_2 \rightarrow q_1 = aa^*b (a \cup b)$$

$$q_3 \rightarrow q_2 \rightarrow q_1 = a \cup (ba^*b) (a \cup b)$$



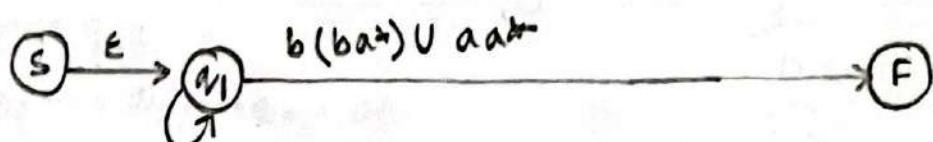
Subject:

Date:

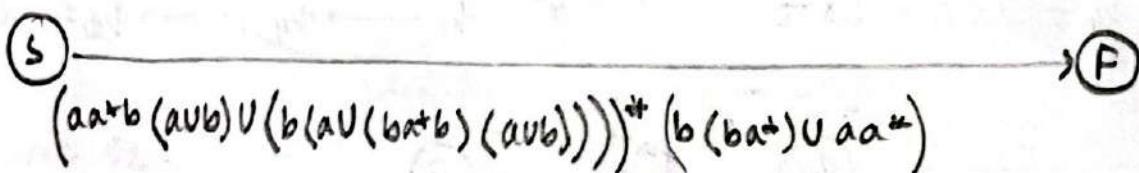
Remove q_3 :

$$q_1 \rightarrow q_3 \rightarrow f = b(ba^*)$$

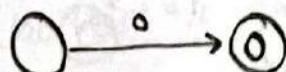
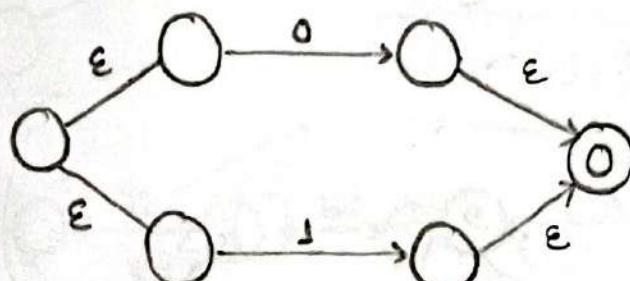
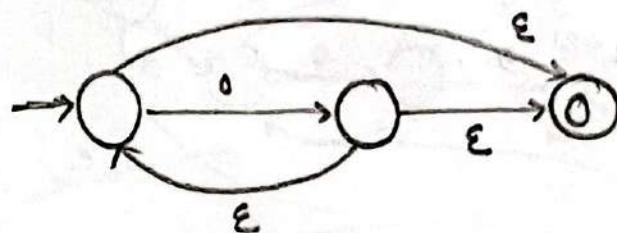
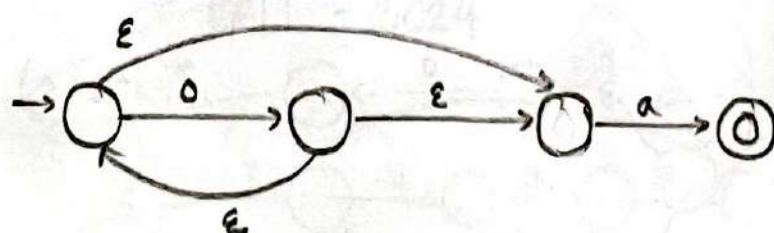
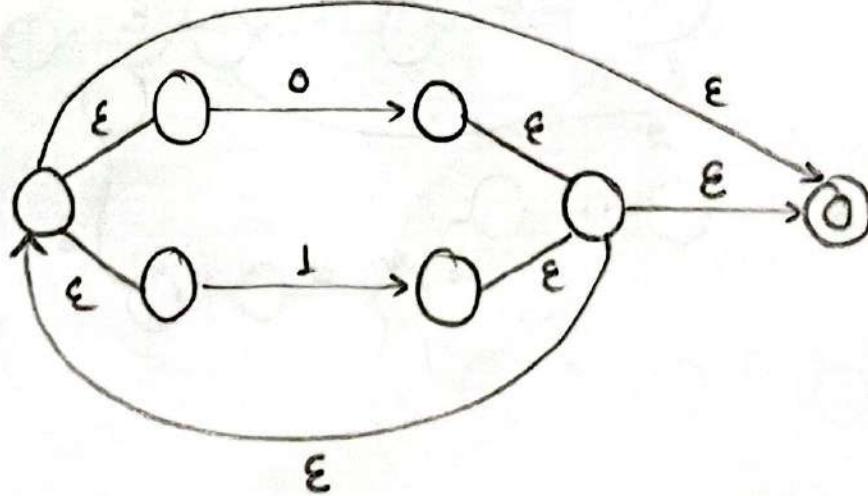
$$q_1 \rightarrow q_3 \rightarrow q_1 = b(a \cup (ba^*b) (a \cup b))$$



$$aa^*b(a \cup b) \cup (b(a \cup (ba^*b) (a \cup b)))$$

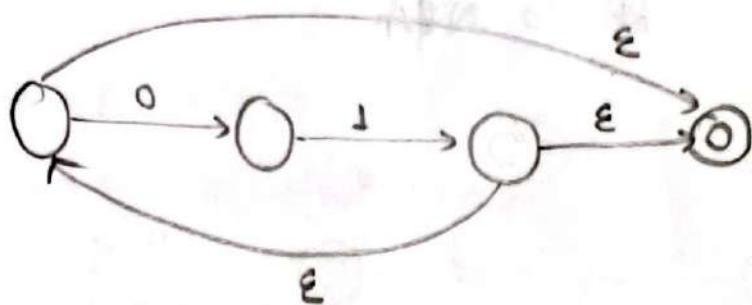
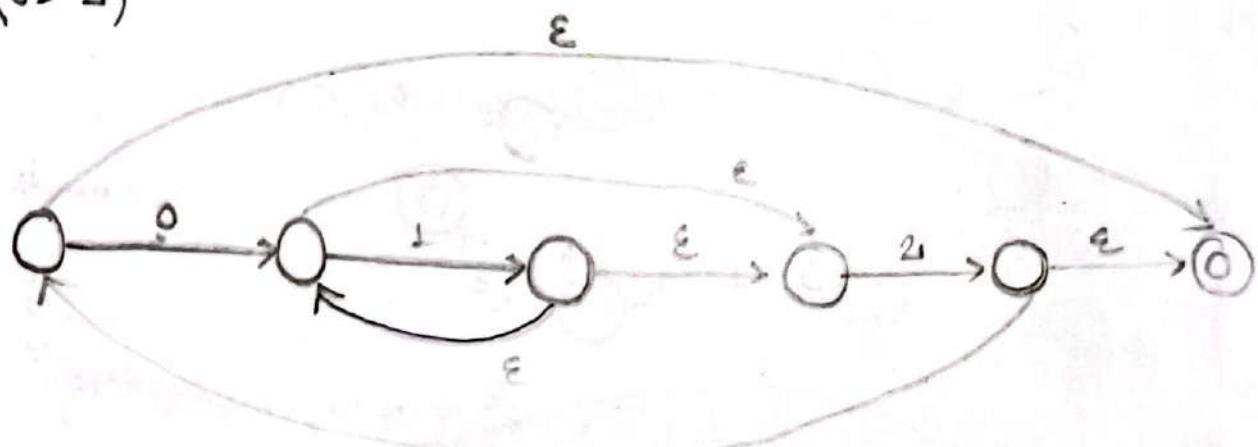
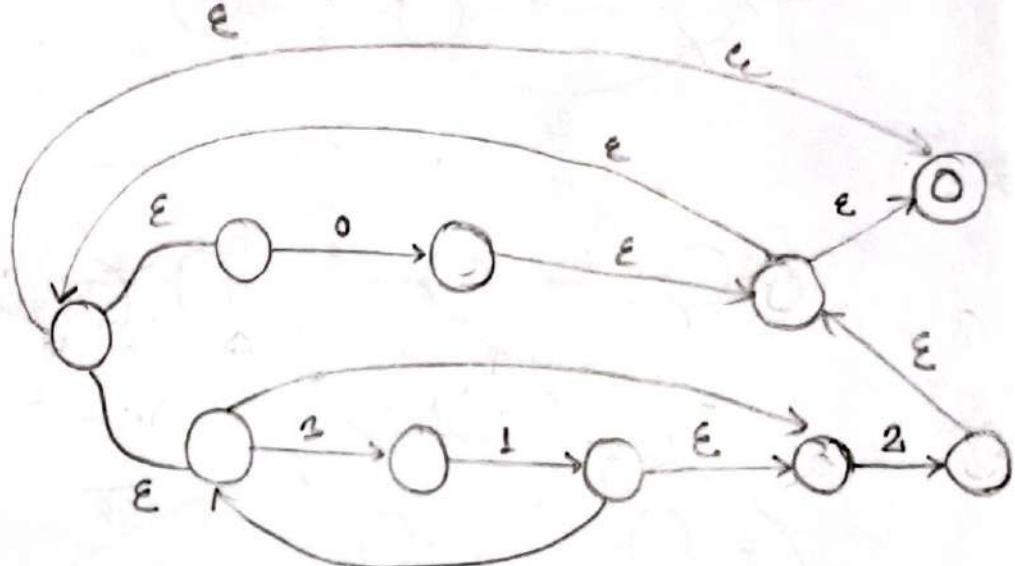
Remove q_1 :

RE to NFA

 0^*  01  $0+1$  0^*  0^*a  $(0+1)^*$ 

Subject :

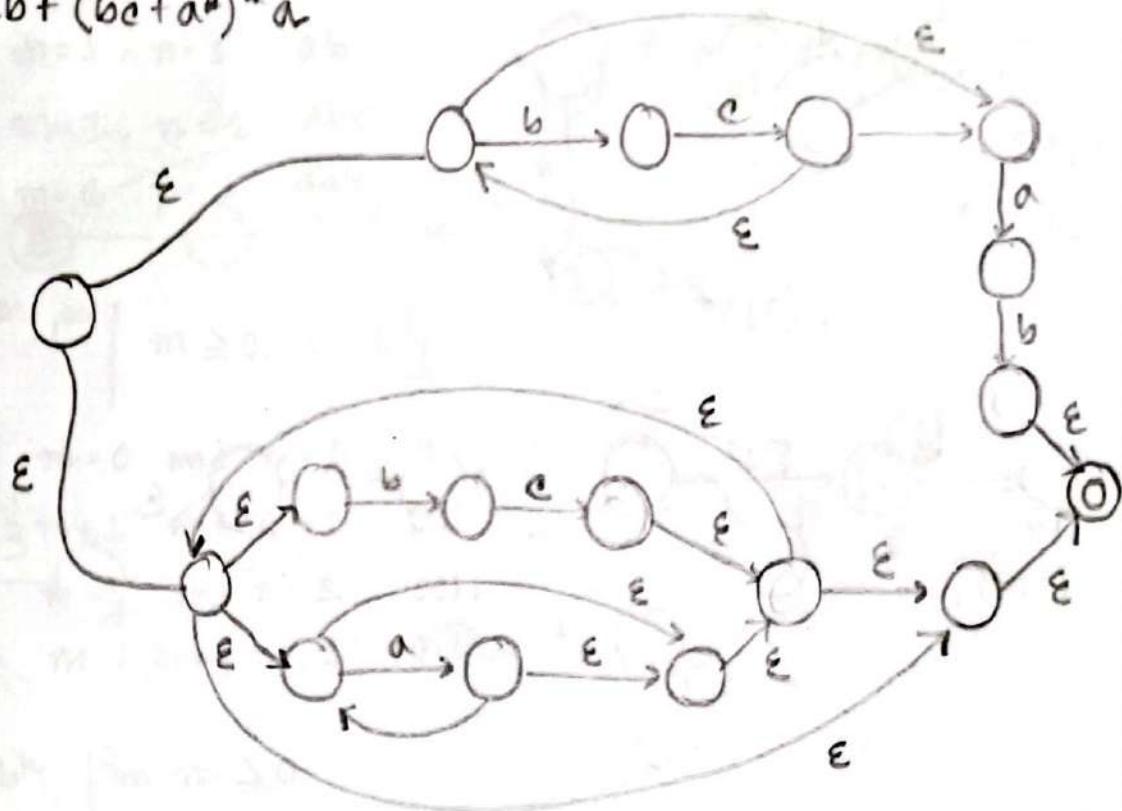
Date :

 $(01)^*$  $(01^*2)^*$  $(0+(31^*2))^*$ 

1

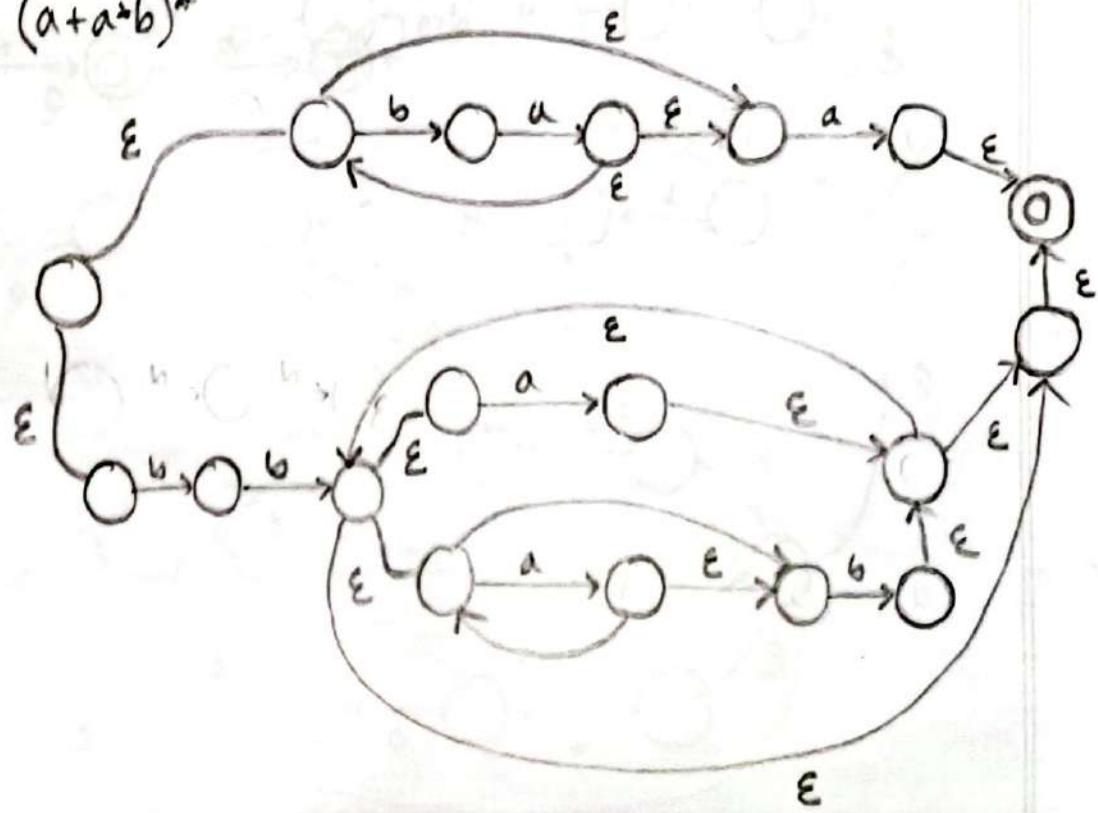
RE \rightarrow NFA

$$(bc)^* ab + (bc + a^*)^* a$$



FALL - 2024

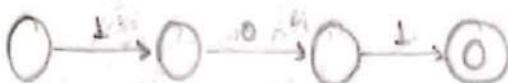
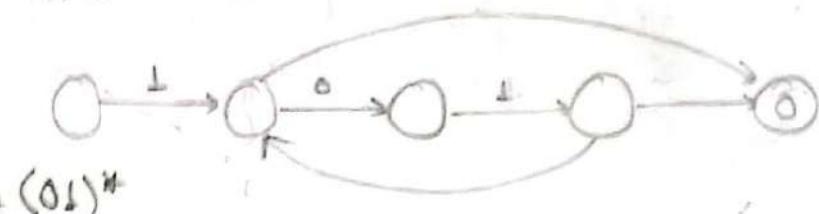
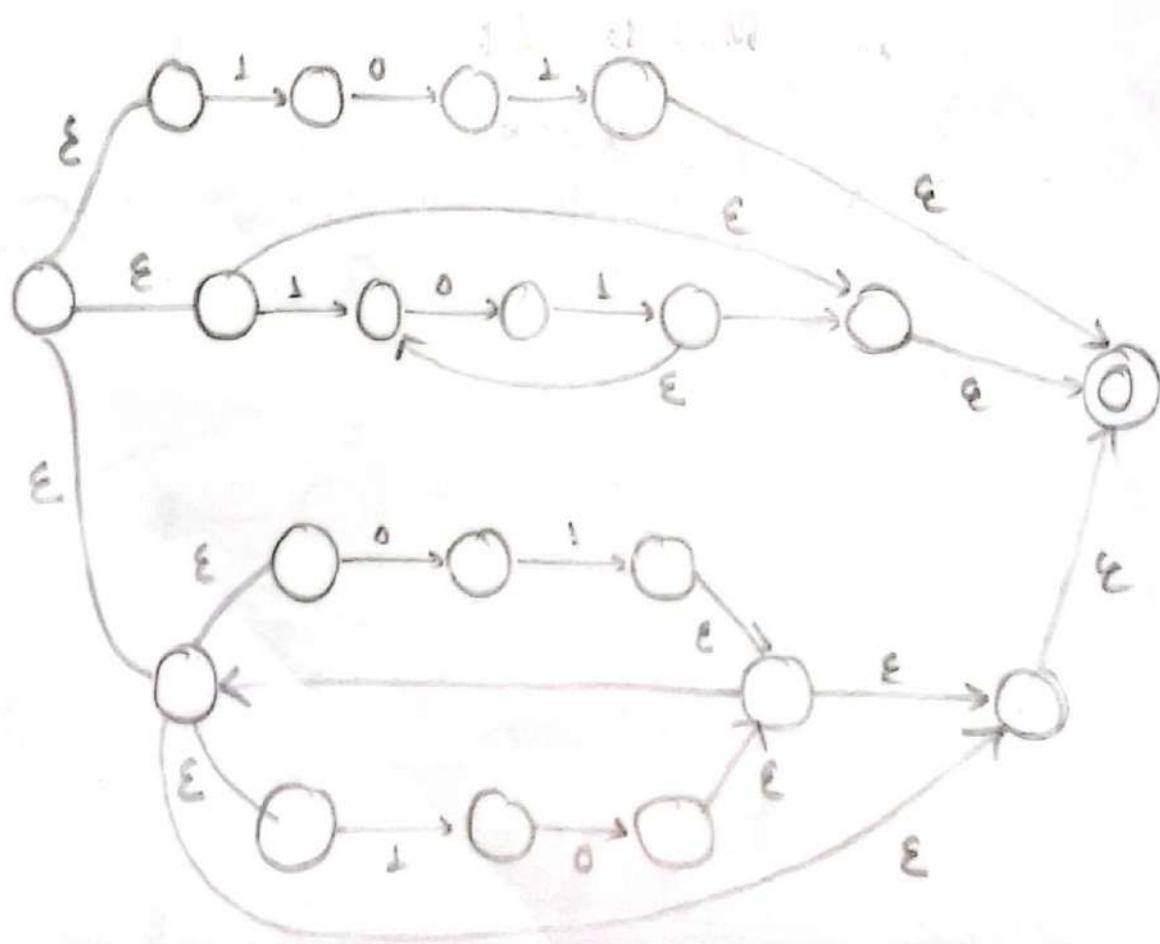
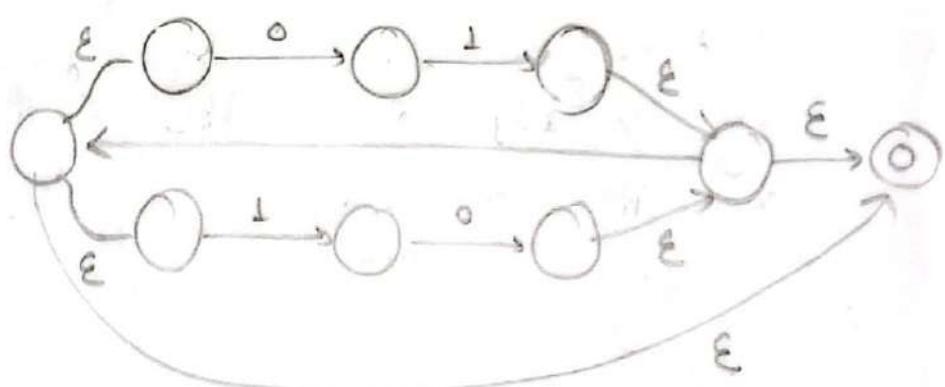
$$(ba)^* a + bbb (a+a^*b)^*$$



Subject :

Date :

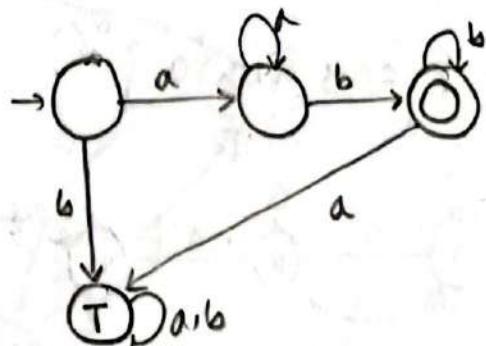
$$101 + \perp (01)^* + (01 + 10)^*$$

 (101)  $\perp (01)^*$ $(01+10)^*$ 

DFA

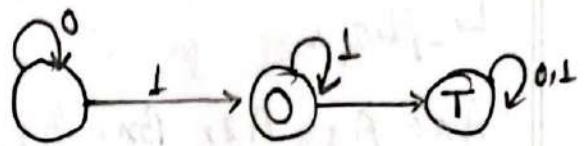
$$L = a^m b^n \mid m, n \geq 1$$

- When, $m=1, n=1$ ab
 $m=1, n=2$ abb
 $m=2, n=1$ aab



$$L = \{ 0^m, 1^m \mid m \geq 0, n \geq 1 \}$$

- When, $m=0$ and $n=1$ 1
 $m=1$ and $n=1$ 01
 $m=2$ and $n=2$ 0011
 $m=1$ and $n=2$ 011



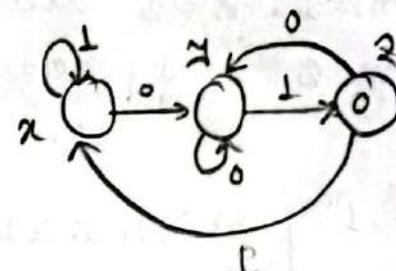
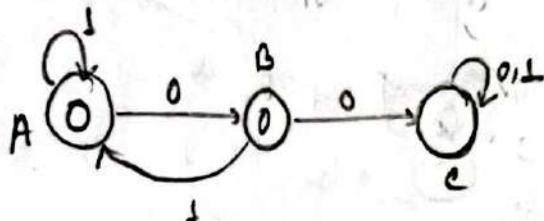
$$L = a^m b^n \mid m, n \geq 0$$

$$\{ \epsilon, a, b, ab, aab, abb, aabb, \dots \}$$



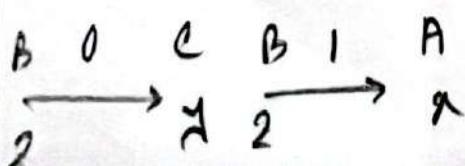
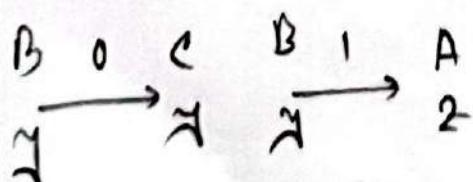
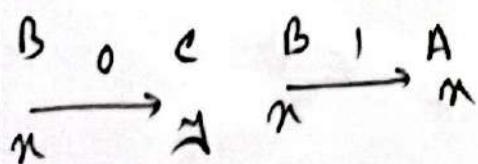
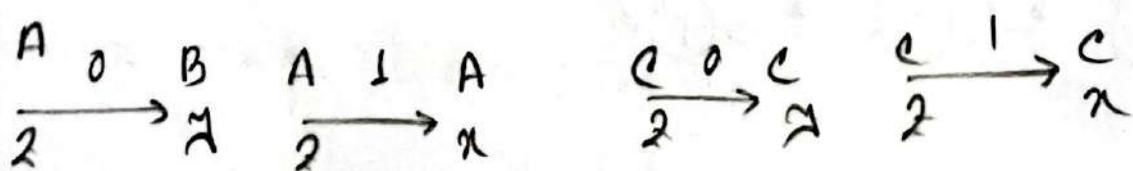
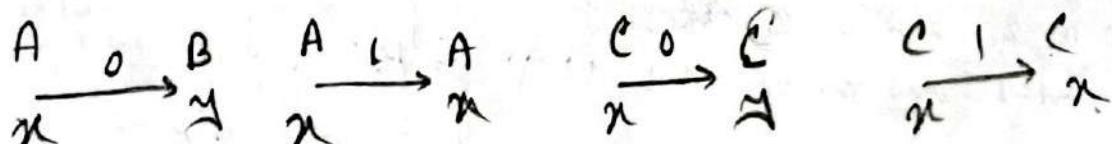
$L_1 = \{ \omega, \text{ } \omega \text{ is not a starting of } \omega \}$

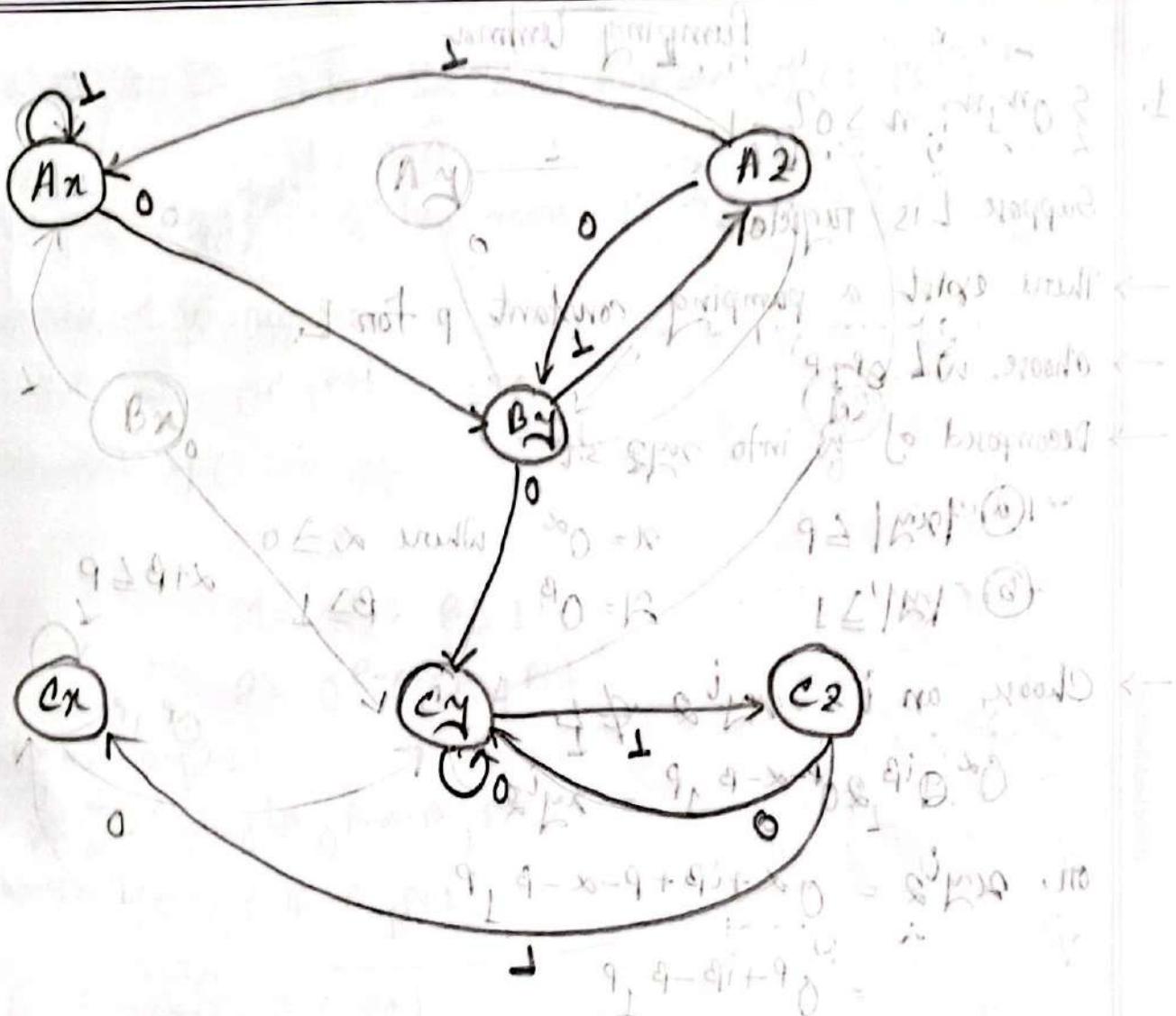
$L_2 = \{ \text{ends with } 01 \}$



$L_1 / L_2 :$

$A_x, A_y, A_z, B_x, B_y, B_z, C_x, C_y, C_z$





Pumping Lemma

$$1. \{0^n 1^n : n \geq 0\} = L$$

000111
P

Suppose L is regular

→ There exist a pumping constant p for L

→ Choose w = 0^p 1^p

→ Decomposed of w into xyz s.t.

$$(a) |xy| \leq p \quad x = 0^\alpha \quad \text{where } \alpha \geq 0$$

$$(b) |y| \geq 1 \quad y = 0^\beta \quad \beta \geq 1 \quad \alpha + \beta \leq p$$

→ Choose an i $xyz^i \notin L$

$$0^\alpha 0^{\beta+i} 1^{p-\alpha-\beta} = xyz^i$$

$$\text{or, } xyz^i = 0^{\alpha+i\beta+p-\alpha-\beta} 1^p$$

$$= 0^{p+i\beta-\beta} 1^p$$

Choose i such that $p+i\beta-\beta \neq p$

$$\Rightarrow i\beta - \beta \neq 0$$

$$\therefore \beta \neq 1$$

$\therefore L$ is not regular.

$0^p 1^p$

$0^\alpha 0^\beta 1^{p-\alpha-\beta}$

~~$0^{\alpha+\beta+p-\alpha-\beta} 1^p$~~

$0^p 1^p$

xyz^i

$0^\alpha 0^{\beta+i} 1^{p-\alpha-\beta}$

$0^{\alpha+i\beta+p-\alpha-\beta} 1^p$

$0^{p+i\beta-\beta} 1^p$

$0^{\beta(p-p+i(i-1))} 1^p$

$p+i\beta(i-1) \neq p$

2. $\{w \in \{0,1\}^*: w \text{ has the same number of } 0's, 1's\}$

3. $L = \{w \in \{0,1\}^*: w \text{ has more } 1's \text{ than } 0's\}$

Suppose L is regular

Choose $w = 0^p 1^{p+1}$

Decomposed of w into $\alpha\beta\gamma$:

$$\alpha = 0^\alpha$$

$$\beta = 0^\beta \quad \beta \geq 1$$

$$\gamma = 0^{p-\alpha-\beta} 1^{p+1}$$

Look at $\alpha\beta\gamma$:

$$\begin{aligned}\alpha\beta\gamma &= 0^\alpha 0^{i\beta} 0^{p-\alpha-\beta} 1^{p+1} \\ &= 0^{p-i\beta-\beta} 1^{p+1}\end{aligned}$$

in L iff $p+i\beta-\beta \leq p+1$

$$i\beta - \beta \leq 1$$

$$\beta(i-1) \leq 1$$

4. $L = \{0^{2n}1^n : n \geq 0\}$

Choose $\omega = 0^{2p}1^p$

Decomposed: $\omega_1\omega_2 : \alpha = 0^\alpha, \beta = 0^\beta, Q = 0^{2p-\alpha-\beta}1^p$

$$\begin{aligned}\omega_1\omega_2 &= 0^\alpha 0^{ip} 0^{2p-\alpha-\beta}1^p \\ &= 0^{2p-\beta+ip}1^p\end{aligned}$$

$$2p+ip-\beta = 2p$$

$$ip-\beta = 0$$

$$(i-1) = 0$$

$$i=1$$

$L = \{0^n10^n : n \geq 0\}$

Choose $\omega = 0^p10^p$

Decomposed: $\omega_1\omega_2 : \alpha = 0^\alpha, \beta = 0^{\beta+q}, Q = 0^{p-\alpha-\beta}1^p$

$$\begin{aligned}\omega_1\omega_2 &= 0^\alpha 0^{ip} 0^{p-\alpha-\beta}1^p \\ &= 0^{ip+\beta-p}1^p\end{aligned}$$

$$p+ip-\beta = p$$

$$ip-\beta = 0$$

$$i=1$$

Subject:

Date:

$$6. L = \{0^n 1^n : n+m\}$$

Suppose L were regular.

$$W = 0^p 1^{p+1}$$

$$\alpha = 0^\alpha, \gamma = 0^\beta, \delta = 0^{p-\alpha-\beta} 1^{p+1}$$

$$\alpha\gamma\delta^i = 0^\alpha 0^{i\beta} 0^{p-\alpha-\beta} 1^{p+1}$$

$$= 0^{\alpha+i\beta+p-\alpha-\beta} 1^{p+1}$$

$$= 0^{p+i\beta-\beta} 1^{p+1}$$

$$p+i\beta-\beta \neq p+1$$

$$\beta(i-1) \neq p!$$

$$i \neq \frac{p}{p} + 1$$

$$\begin{matrix} \alpha \\ \gamma \\ \delta \end{matrix}$$

$$\begin{matrix} 0^\alpha \\ 0^\beta \\ 0^{p-\alpha-\beta} \end{matrix}$$

$$7. L = \{0^n 1^m : n \leq 3m\}$$

Suppose L were regular

$$W = 0^p 1^{p-1} 1^p$$

$$\alpha = 0^\alpha, \gamma = 0^\beta, \delta = 0^{3p-1-\alpha-\beta}$$

$$\begin{aligned} \alpha\gamma\delta^i &= 0^\alpha 0^{i\beta} 0^{3p-1-\alpha-\beta} 1^p \\ &= 0^{3p+i\beta-1-\beta} 1^p \end{aligned}$$

$$3p+i\beta-1-\beta \leq 3p$$

$$i\beta-1-\beta \leq 0$$

$$\cdot \beta(i-1) \leq 1$$

$$9. L = \{0^{2^n} : n \geq 0\}$$

$$W = 0^{2^p}$$

$$\alpha = 0^\alpha, \gamma = 0^\beta, \delta =$$

$$\begin{aligned} \alpha\gamma\delta^i &= 0^\alpha 0^{i\beta} 0^{2^p-\alpha-\beta} \\ &= 0^{2^p+(i-1)\beta} \end{aligned}$$

$$\text{choose } i = 2$$

$$2^p \leq 2^p + \beta \leq 2 + p \leq 2^{p+1}$$

$$1120 \leftarrow 2$$

$$3 \leftarrow 2$$

$$8. L = \{0^{n^2} : n \geq 0\}$$

$$W = 0^{p^2}$$

$$\alpha = 0^\alpha, \gamma = 0^\beta, \delta = 0^{p^2-\alpha-\beta}$$

$$\alpha\gamma\delta^i = 0^\alpha 0^{i\beta} 0^{p^2-\alpha-\beta}$$

$$= 0^{p^2+\beta}$$

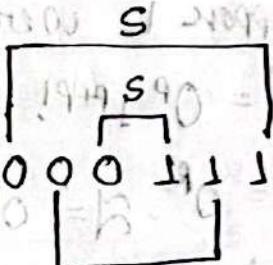
$$p^2 \leq p^2 + \beta \leq p^2 + p \leq (p+1)^2$$

CFA

Examples:

$$\textcircled{1} \quad 0^n 1^n$$

$$\begin{matrix} S \\ \Rightarrow OS1 \end{matrix}$$



$$\begin{matrix} S \rightarrow OS1 \\ S \rightarrow e \end{matrix}$$

$$\Rightarrow 0OS11$$

$$\Rightarrow 00OS111$$

$$\Rightarrow 000111$$

$$\textcircled{2} \quad 0^n 1^{2n}$$

$$\begin{matrix} S \\ \Rightarrow OS11 \end{matrix}$$

$$S \rightarrow OS11$$

$$\Rightarrow 0OS111$$

$$\textcircled{3} \quad 0^{3n} 1^{5n}$$

$$S \rightarrow 000S11111$$

$$S \rightarrow e$$

$$0 = q \quad 0 = k \quad x_0 = k$$

$$q - q^2 - q^3 - q^4 - q^5 = 0$$

$$q + q^2 + q^3 + q^4 + q^5 = 0$$

$$(1+q) - q(1-q) = q + q^2 - q^3 - q^4 = 0$$

$$19+9 + q - q^2 + q^3$$

$$19 + (1-q)q$$

$$1 + \frac{19}{q} + q$$

$$q_1 + q_2 + q_3 + q_4 + q_5 = 0$$

$$q_1 + q_2 + q_3 + q_4 + q_5 = 0$$

$$q_1 + q_2 + q_3 + q_4 + q_5 = 0$$

$$q_1 + q_2 + q_3 + q_4 + q_5 = 0$$

$$q_1 + q_2 + q_3 + q_4 + q_5 = 0$$

$$q_1 + q_2 + q_3 + q_4 + q_5 = 0$$

$$q_1 + q_2 + q_3 + q_4 + q_5 = 0$$

$$q_1 + q_2 + q_3 + q_4 + q_5 = 0$$

$$1 + (1-q)q$$

Subject:

Date:

1. $\{w | w \text{ contains at least three } 1's\}$

$$(0|1)^* 1 (0|1)^* 1 (0|1)^* 1 (0|1)^*$$

$$S \rightarrow A 1 A 1 A 1 A$$

$$A \rightarrow 0A | 1A | \epsilon$$

2. $\{w | w \text{ starts and ends with same symbol}\}$

$$0 (0|1)^* 0 \mid 1 (0|1)^* 1$$

$$S \rightarrow 0A0 \mid 1A1$$

$$A \rightarrow 0A \mid 1A \mid \epsilon$$

3. $\{w | w \text{ the length of } w \text{ is odd}\}$

$$((0|1)^* (0|1)^*)^* (0|1)$$

$$S \rightarrow AB$$

$$A \rightarrow BBA \mid \epsilon$$

$$B \rightarrow 01 \# / 120 / 02 / 21 \leftarrow 2$$

4. $\{w | \text{the length of } w \text{ is odd and its middle symbol is } 0\}$

$$S \rightarrow 0S0 \mid 1S1 \mid 0S1 \mid 1S0 \mid 0$$

5. $\{w | w = w^R, w \text{ is a palindrome}\}$

$$S \rightarrow 0S0 \mid 1S1 \mid 1 \mid 0 \mid \epsilon$$

6. $\{w \mid 0^i 1^j 2^k, i=j \text{ for } i \neq k\} \not\in \Sigma^*$ witness CS/GP

for $i=i$

$S \rightarrow A|B$

$A \rightarrow PQ$

$Q \rightarrow 2Q|\epsilon$ pumping across $2Q$ gives both states with ϵ

$P \rightarrow OP1|\epsilon$

for $i \neq k$

$D \rightarrow WX$

$W \rightarrow 0W|\epsilon$

$X \rightarrow Y|2$

$Y \rightarrow 1Y2|M$

$M \rightarrow 2M|\epsilon$

$Z \rightarrow 1Z2|N$

$N \rightarrow 1N|\epsilon$

$\begin{matrix} \leftarrow 2 \\ 1ZL \end{matrix}$

$\begin{matrix} \leftarrow 1Z2L \\ 10Z0L \end{matrix}$

$\begin{matrix} \leftarrow 1000L \\ 10000L \end{matrix}$

$\frac{0^i 1^j 2^k}{P} \frac{1^j 2^k}{Q} \frac{i=j}{A} \frac{i \neq k}{B} \text{ ALKAIA} \leftarrow 2$

$3|110|110 \leftarrow A3$

$1^i(1|0)1 | 0^j(1|0)0$

$\frac{0^i 1^j 2^k}{W} \frac{i=j}{X} \frac{i \neq k}{A} \frac{i \neq k}{B} \text{ S|AI|AD} \leftarrow A$

$\begin{matrix} \leftarrow 2 \\ \text{both } 2 \text{ in } W \text{ legal in } W|W \end{matrix} \leftarrow A$

7. $w_1 \# w_2$

number of 0's in $w_1 = \text{no of } 1's \text{ in } w_2$

$S \rightarrow 1S|S0|0S1|\#1|0 \leftarrow g$

8. $A = \{w \in \{0,1\}^*: w \text{ contains odd no of } 1's\}$

$(0^* 1 0^* 1)^* 0^* 1 0^* 1 | 020 \leftarrow 2$

$A \rightarrow BC|C$

$C \rightarrow 0C|\epsilon$

$B \rightarrow DB|\epsilon$

$D \rightarrow C1C1$

Subject:

Date:

$$9. L_1 = \{ 1^i 0^j 2^k \mid i, j, k \geq 0, i = k \}$$

$$S \rightarrow 1S1 | A1A | A0 \leftarrow A$$

$$A \rightarrow 0B$$

$$B \rightarrow 2B | \varepsilon$$

$$10. \{ 1^i 0^j 2^k \mid i, j, k \geq 0, k = i+2j \}$$

$$S \rightarrow S | 1S1 | C \leftarrow A$$

$$C \rightarrow 0Cn$$

$$a \rightarrow 2a11 | \varepsilon$$

$$11. L_3 = \{ 1^i 0^j 2^k \mid i, j, k \geq 0, k = 2i + 3j \}$$

$$S \rightarrow 1S22 | 0S222 | \varepsilon$$

$$12. L_4 = \{ 1^i 0^j 2^k \mid i, j, k \geq 0, k = 3i + j \}$$

$$S \rightarrow 1S222 | 0S2 | \varepsilon$$

$$13. L_5 = \{ 0^i 2^j 1^k \mid i, j, k \geq 0, k = 2i + 2j \}$$

$$S \rightarrow 0S21 | 2S11 | \varepsilon$$

$$14. L_6 = \{ 2^i 0^j 1^k \mid i, j, k \geq 0, k = 3i + 2j \}$$

$$S \rightarrow 2S11 | 0S11 | \varepsilon$$

15. Starts with 10

$$S \rightarrow 10A$$

$$A \rightarrow 0A \mid 1A \mid \epsilon$$

16. Ends with 10

$$S \rightarrow A10$$

$$A \rightarrow 0A \mid 1A \mid \epsilon$$

17. Contains 101 as substring

$$S \rightarrow A101B$$

$$A \rightarrow 0A \mid 1A \mid \epsilon$$

$$B \rightarrow 0B \mid 1B \mid \epsilon$$

19. Even number of zeros

$$S \rightarrow 0A \mid 1S \mid \epsilon$$

$$A \rightarrow 0S \mid 1A$$

18. zero at 3rd position

$$S \rightarrow XX0A$$

$$X \rightarrow 0 \mid 1$$

$$A \rightarrow 0A \mid 1A \mid \epsilon$$

21. Length is Even

$$S \rightarrow 0A \mid 1A \mid \epsilon$$

$$A \rightarrow 0S \mid 1S$$

23. At most 2 zeros

$$S \rightarrow A \mid 0A \mid 0AOA$$

$$A \rightarrow 1A \mid \epsilon$$

25. At least 2 zeros

$$S \rightarrow 00AO$$

$$A \rightarrow 0A \mid 1A \mid \epsilon$$

22. Length is odd $\leftarrow 2\rightleftharpoons$

$$S \rightarrow 1E \mid 0S$$

$$E \rightarrow 1S \mid 0E$$

24. Exactly 3 ones

$$S \rightarrow A1A1A1A$$

$$A \rightarrow 0A \mid \epsilon$$

26. Number of zeros is multiple of 2

$$S \rightarrow$$

$$S \mid 1120 \mid 11122 \leftarrow 2$$

27. $a^n b^n, L = \{ \}$ No regular recognizable

$$S \rightarrow aAb$$

$$A \rightarrow aNb | \epsilon$$

28. $A = \{ w \in \{0,1\}^*: w \text{ contains at least two } 0s \}$

$$S \rightarrow 0A0$$

$$A \rightarrow 0A | 1A | \epsilon$$

$L = \{ w \in \{0,1\}^*: w = 0^{3i} A 1^{2i} \text{ and } i \geq 0 \}$

$$S \rightarrow 000 S 11 | A$$

$$A \rightarrow 0B0$$

$$B \rightarrow 0B | 1B | \epsilon$$

29. $L = \{ w \in \Sigma^*: w \text{ is an odd length palindrome} \}$

$$S \rightarrow asa | bsb | a | b$$

30. $L = \{ w \in \Sigma^*: w \text{ is an even length palindrome} \}$

$$S \rightarrow asa | bsb | \epsilon$$

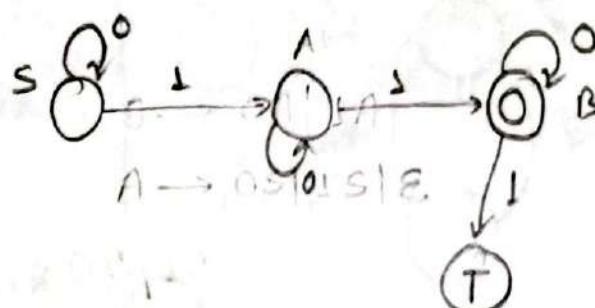
31. $S \rightarrow A \perp \perp B$

$$A \rightarrow 0C | 1C$$

$$C \rightarrow 0A | 1A | \epsilon$$

$$B \rightarrow 0D | 1D$$

$$D \rightarrow 0B | 1B | \epsilon$$



$$S \rightarrow 0S | 1A$$

$$A \rightarrow 0A | 1B$$

$$B \rightarrow 0B | \epsilon$$

Subject :

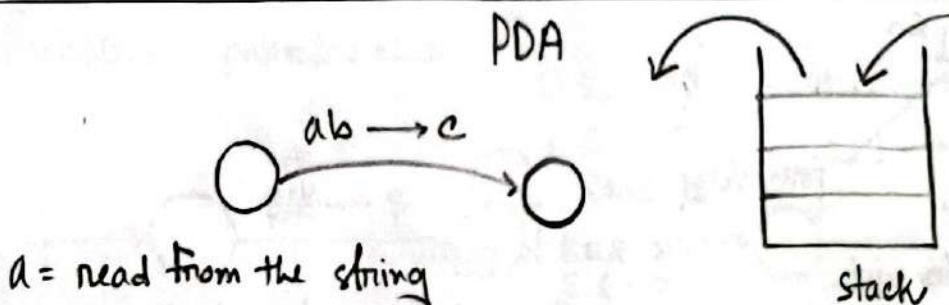
Date :

1. ① $L = \{w \in \{a, b, c\}^* \mid w = a^i b^j c^k, i = 2i+k, j = 2i+2k, i, j, k \geq 0\}$

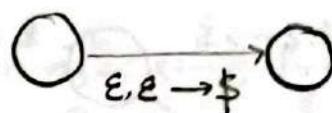
 $a^i b^{2i+k} c^k$
 $a^i b^{2i} b^k c^k$
 $a^i b^j c^{2i+2k}$
 $a^i b^j c^{2i} c^{2k}$
 $S \rightarrow XY$
 $S \rightarrow XY$
 $X \rightarrow aXbb|\epsilon$
 $X \rightarrow axcc|Y$
 $Y \rightarrow bYc$
 $Y \rightarrow bYcc|\epsilon$

Subject:

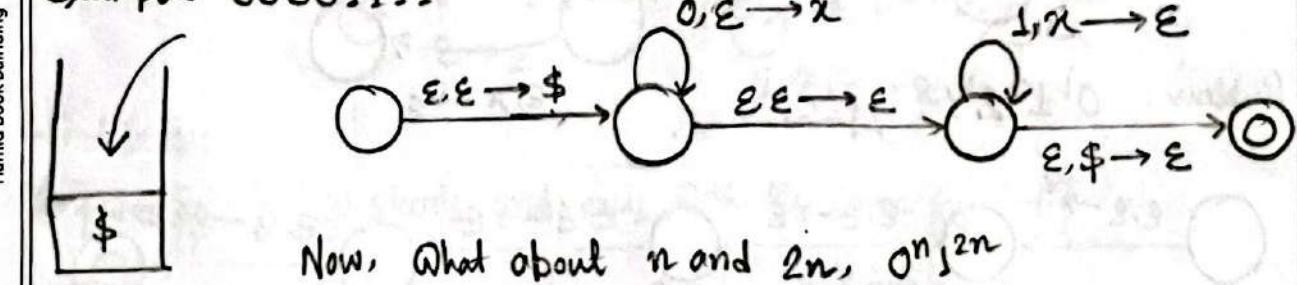
Date:



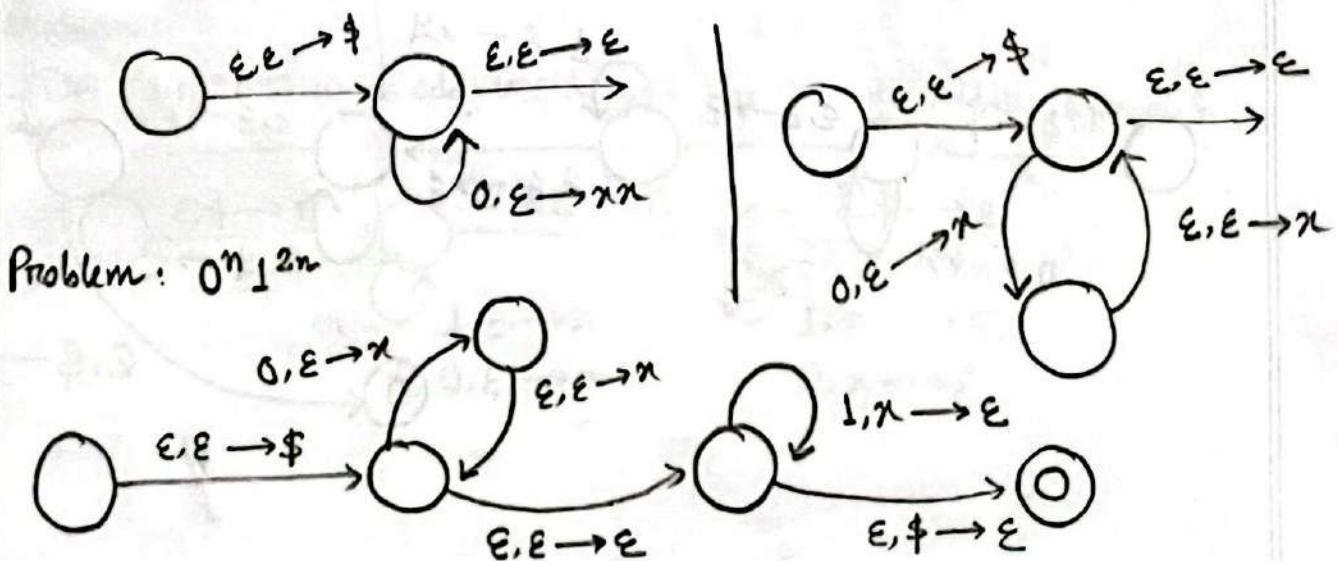
Default state

 $\$ = \text{when to stop}$ Problem: $0^n 1^n$ (equal 0 and 1)

example: 00001111



Now, What about n and $2n$, $0^n 1^{2n}$
for every zero we need to push two x in the stack
and for every one we need to pop only one x.



Problem: $0^{3n} 1^{2n}$

for every 2010,

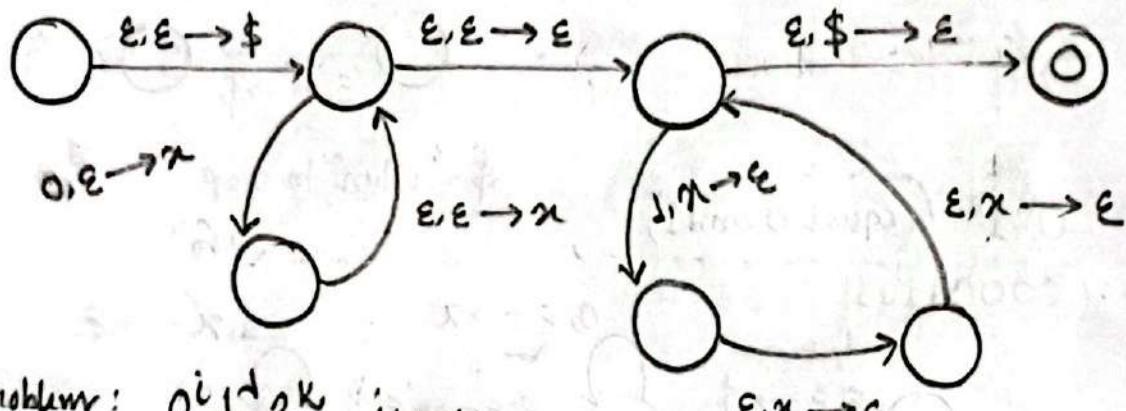
we need to push

2 one into the stack

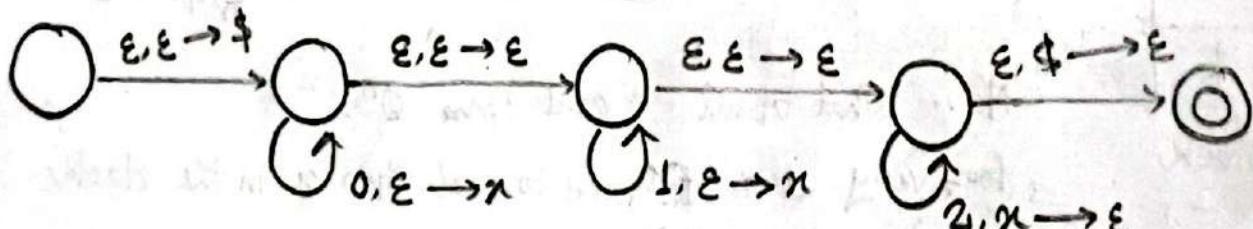
for every one,

we need to pop

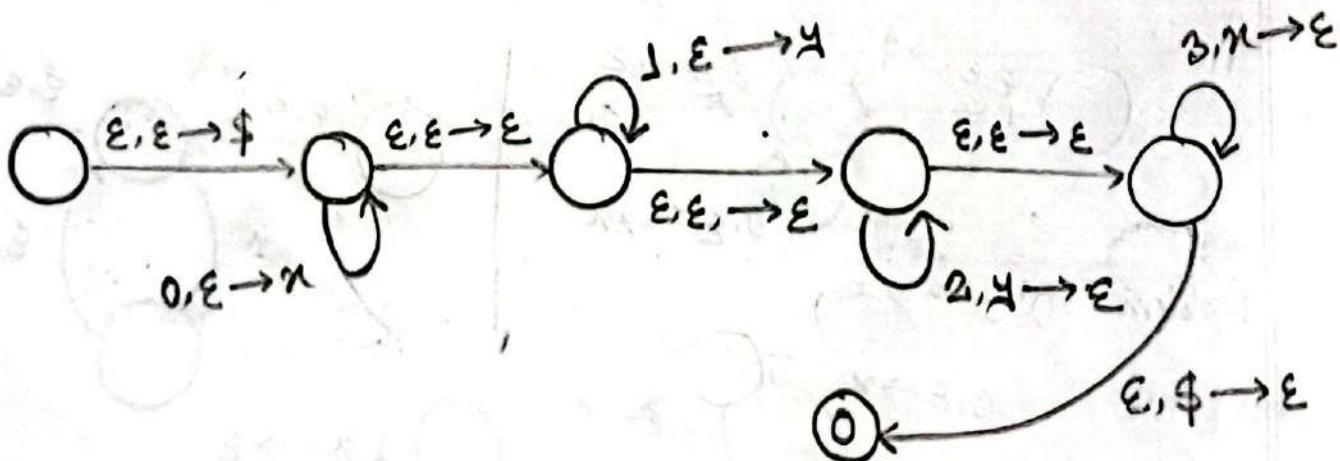
3 zeros from the stack



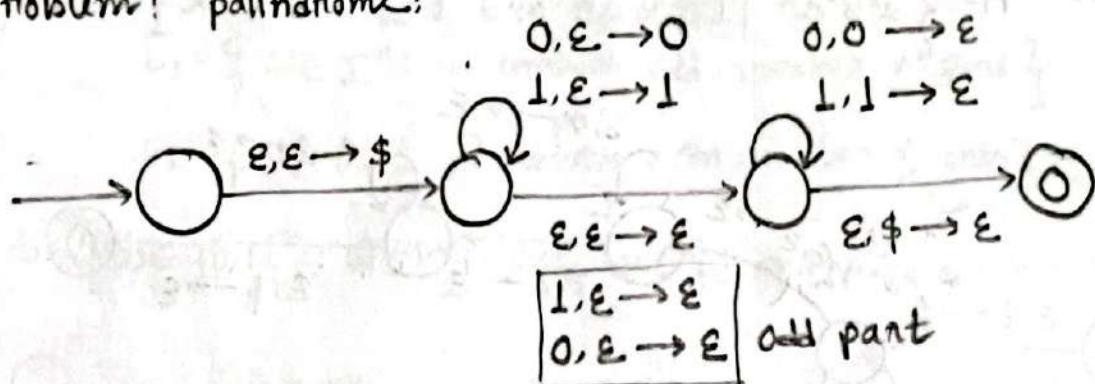
Problem: $0^i 1^j 2^k$, $i+j=k$



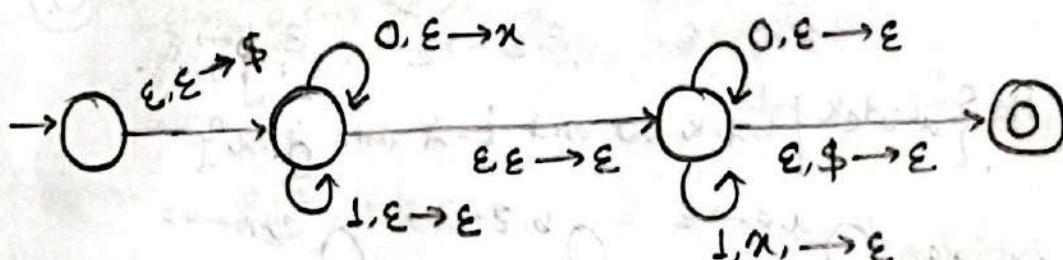
Problem: $0^i 1^j 2^k 3^l$



Problem: palindrome:

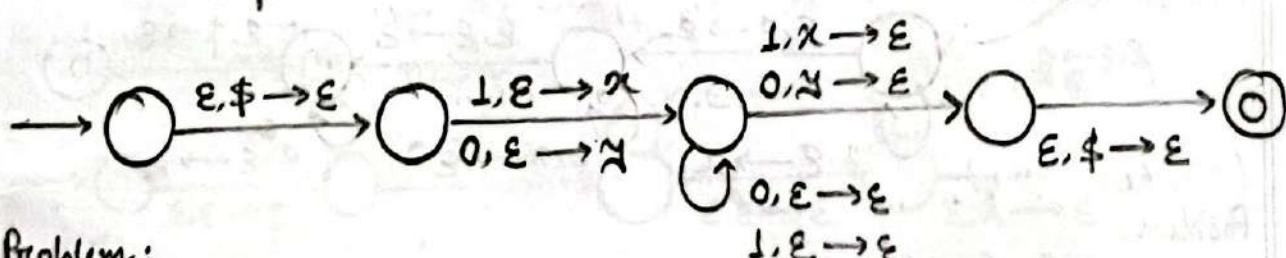


Problem: $W_1 \# W_2$, where, no. of 0's in W_1 = no. of 1's in W_2 .



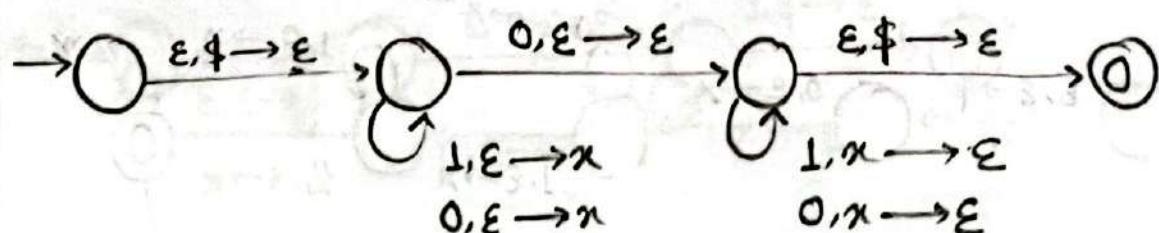
Problem:

$\{w \in \{0, 1\}^* \mid w \text{ starts and ends with the same symbol}\}$

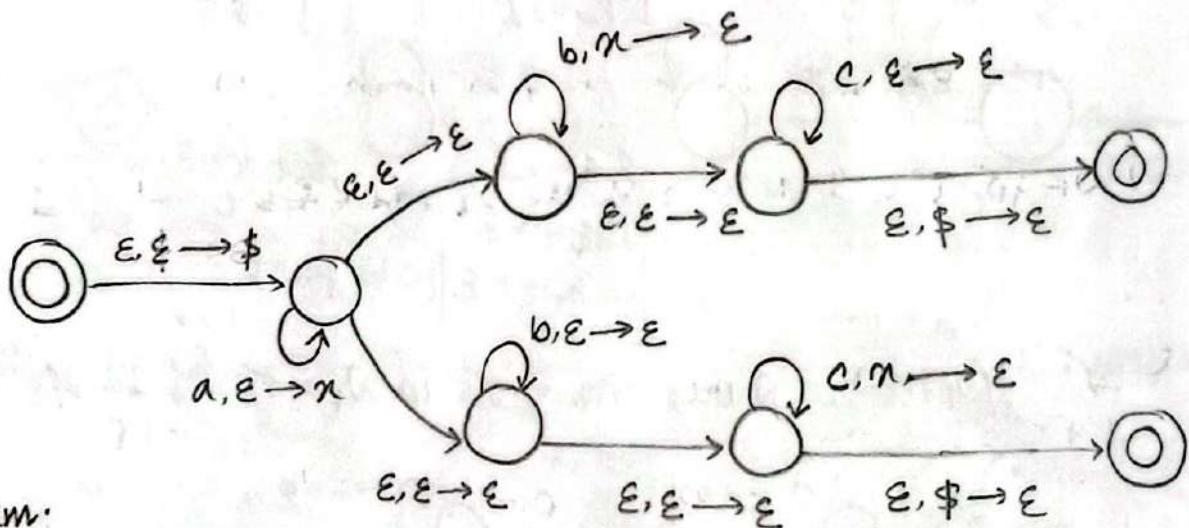


Problem:

$\{w \in \{0, 1\}^* \mid w \text{ is of odd length and has } 0 \text{ as its middle symbol}\}$

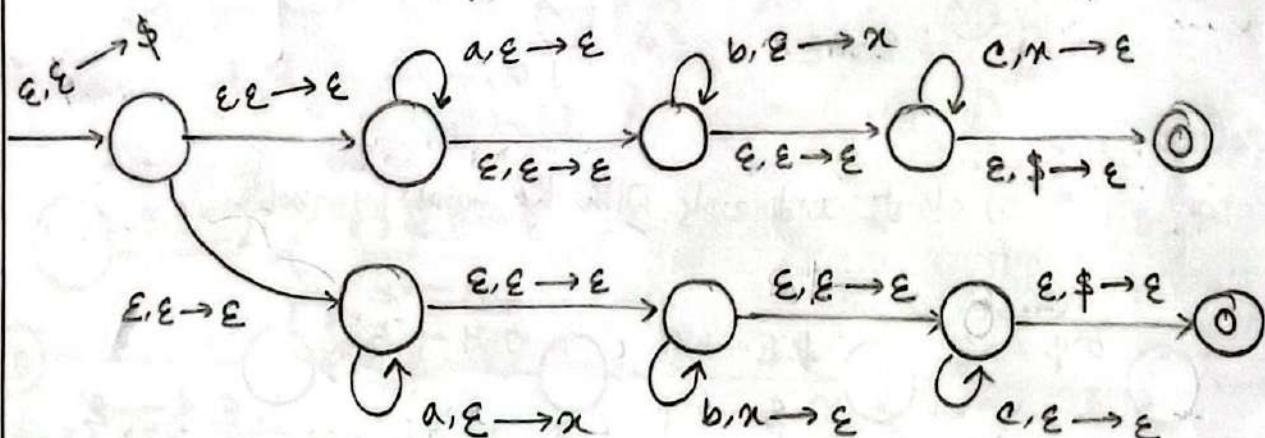


Problem: $A = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i=j \text{ and } i=k\}$



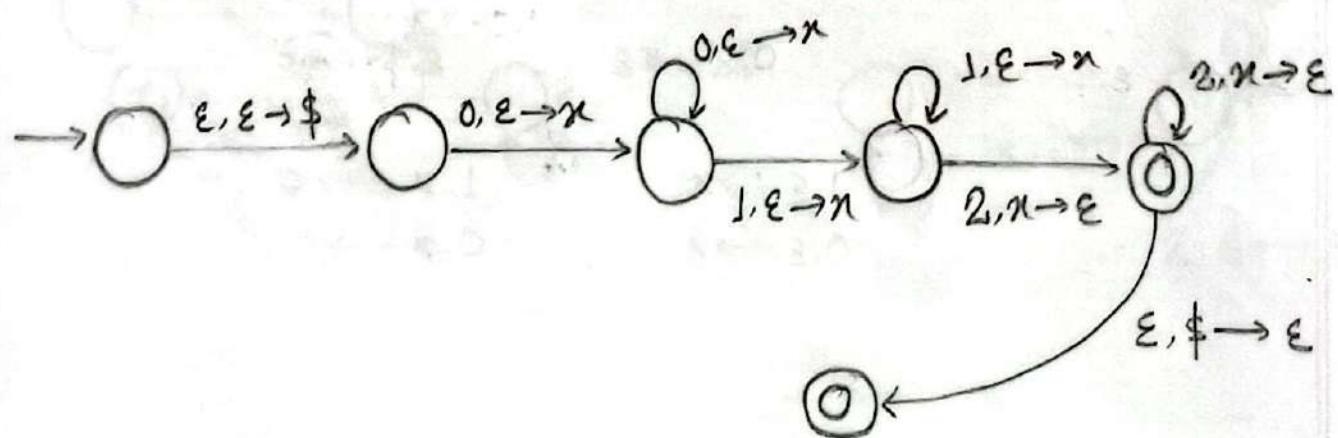
Problem:

$A = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i=j \text{ and } j=k\}$

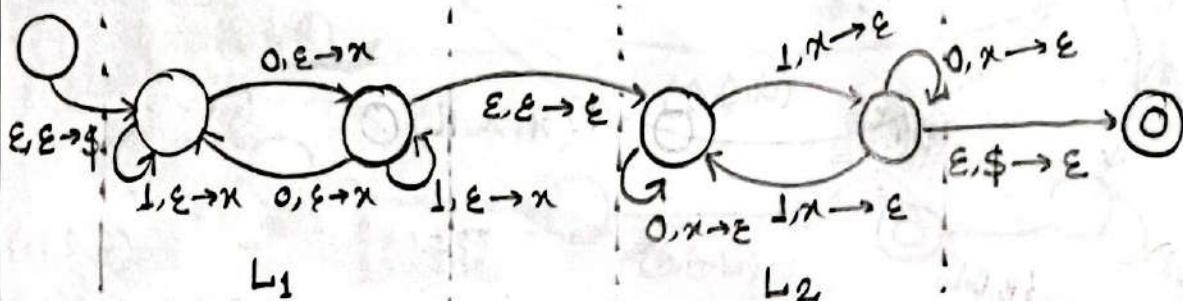
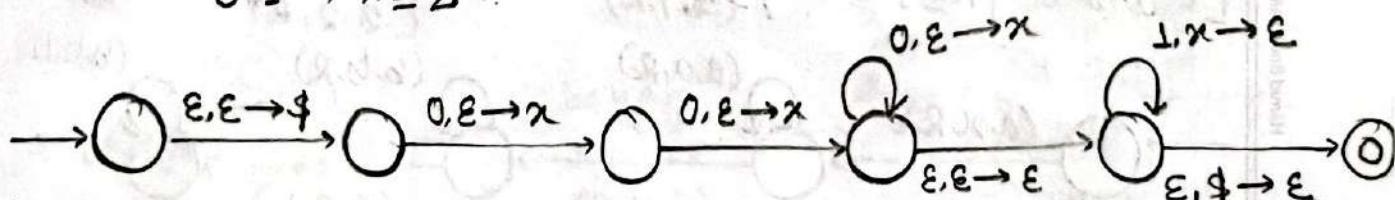
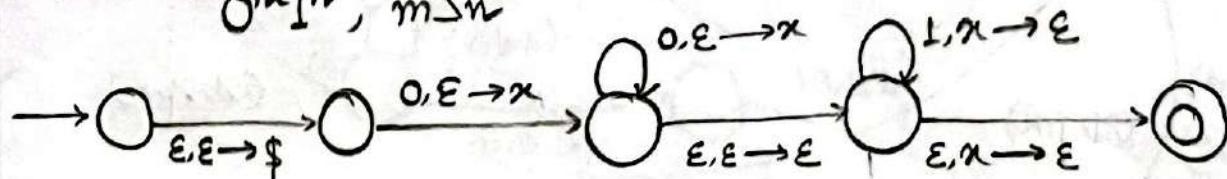
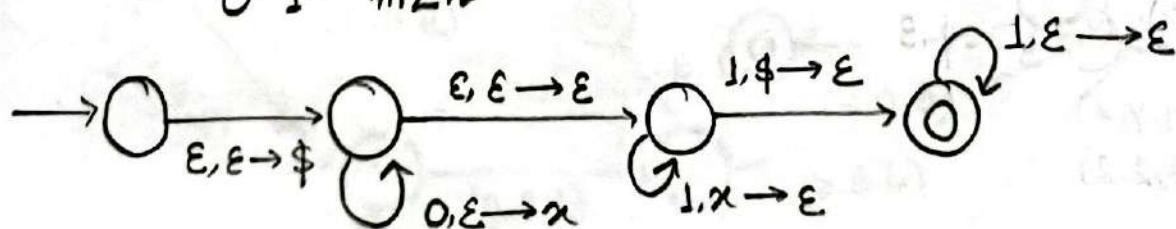


Problem:

$A = \{0^i 1^j 2^k \mid i+j \geq k \text{ and } i, j, k \geq 0\}$

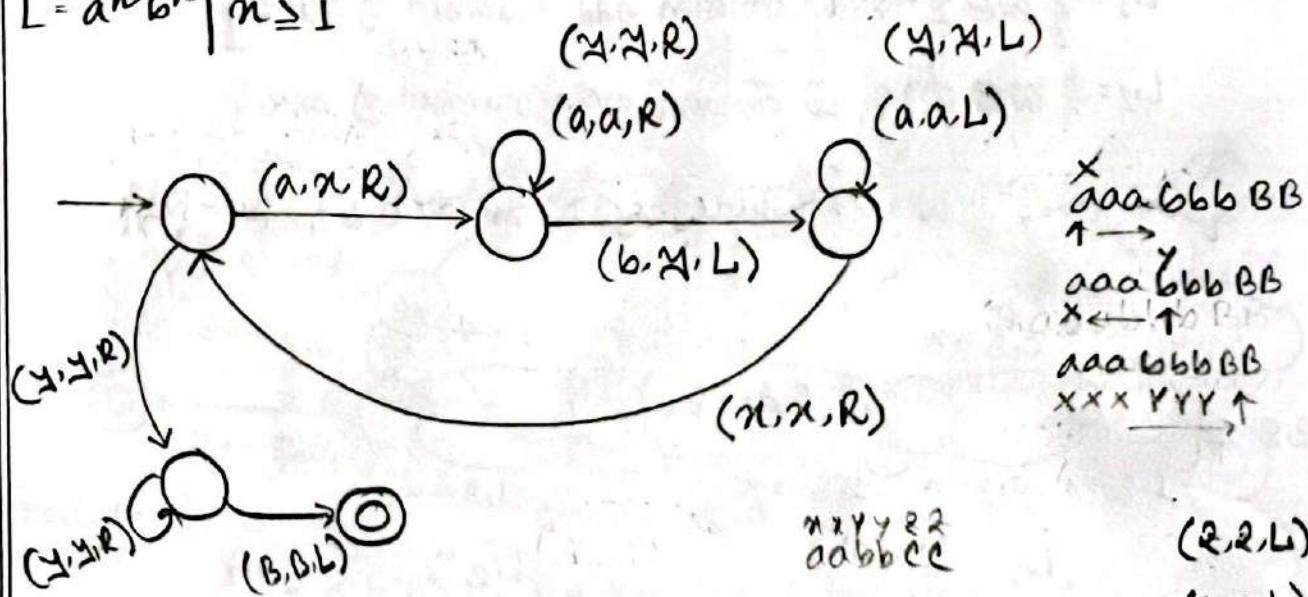


Problem:

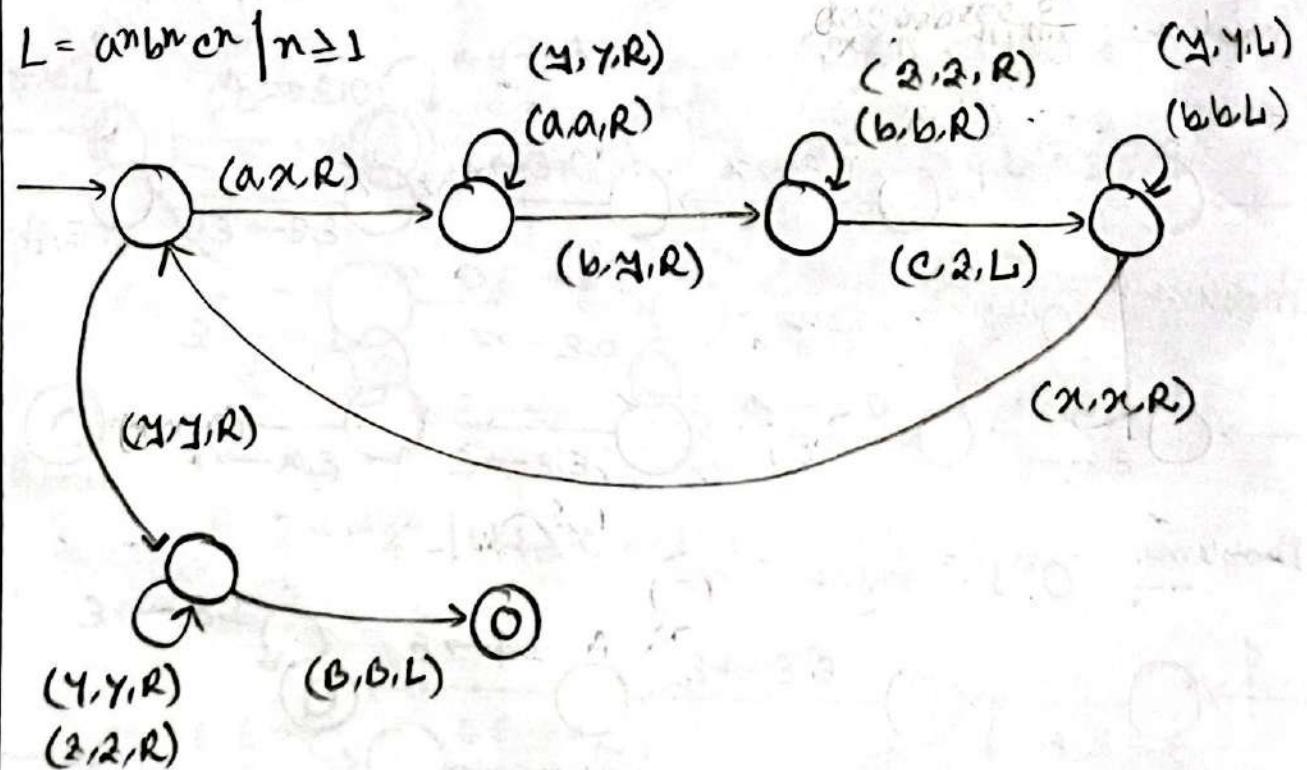
 $L_1 = \{ \omega \in \Sigma^*: \omega \text{ contains odd number of zeros} \}$
 $L_2 = \{ \omega \in \Sigma^*: \omega \text{ contains even number of ones} \}$
 $L = \{ \omega \in \{0,1\}^*: \omega = uv, \text{ where } u \in L_1 \text{ and } v \in L_2, |u| = |v| \}$
Problem: $0^n 1^n, n \geq 2$ Problem: $0^m 1^n, m \leq n$ Problem: $0^m 1^n, m < n$ 

Turing Machine

$$L = a^n b^n \mid n \geq 1$$



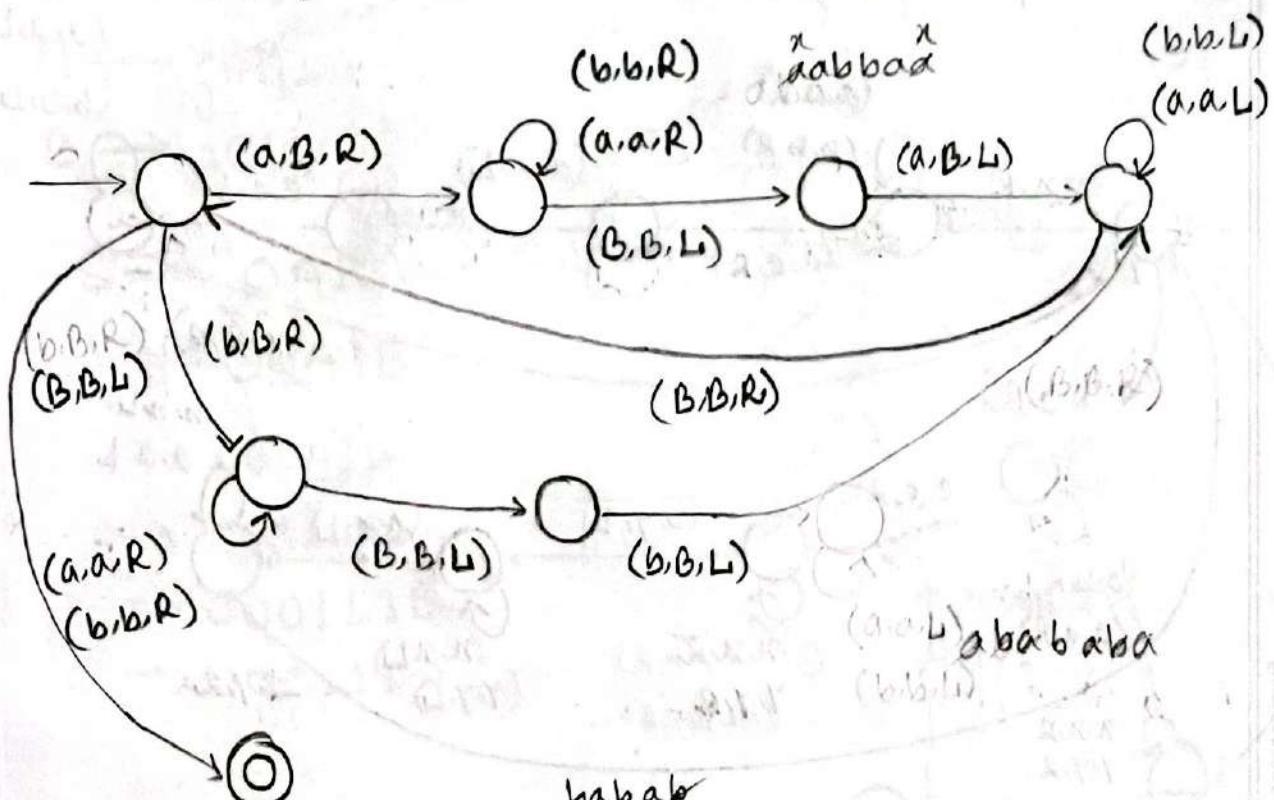
$$L = a^n b^n c^n \mid n \geq 1$$



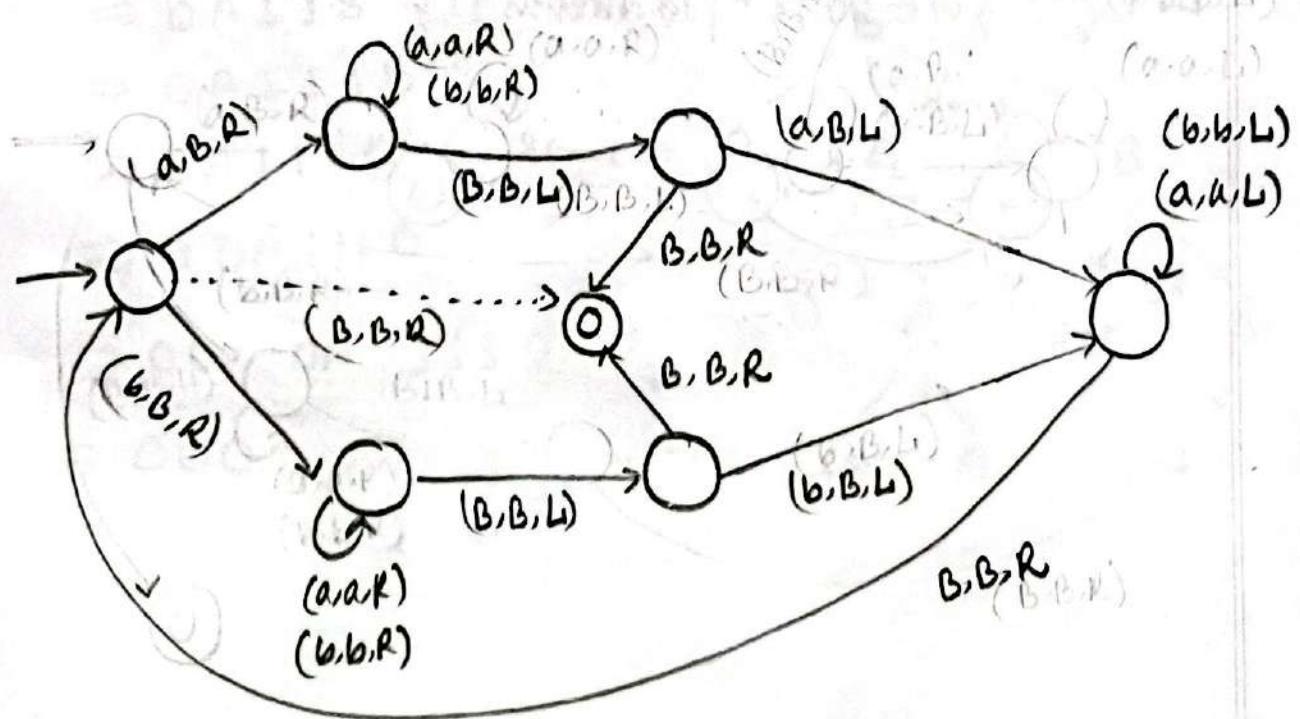
Subject :

Date :

$L = WWR$ $W \in \{a, b\}$ Even



$L = W_a W_R$ [Odd]

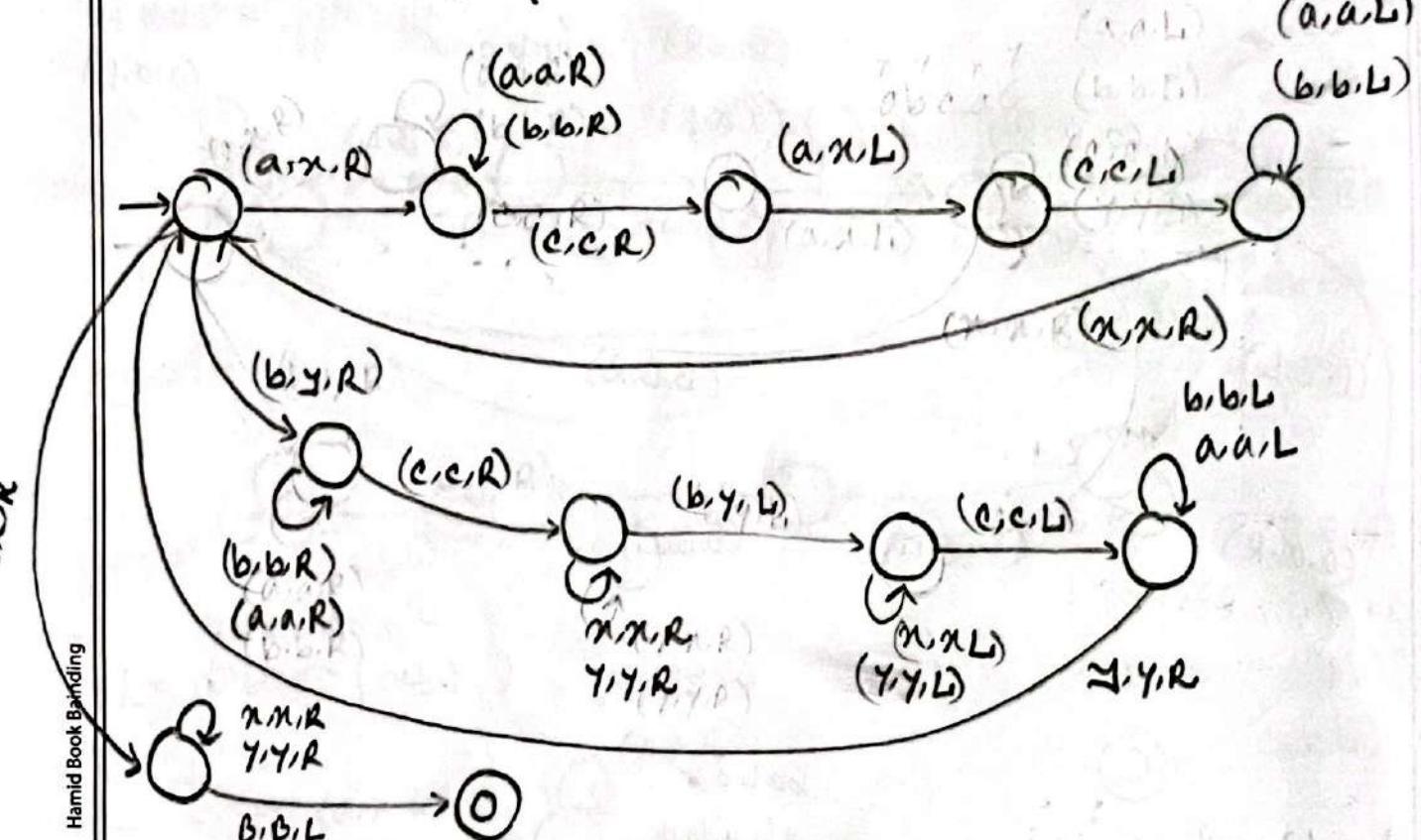


Subject:

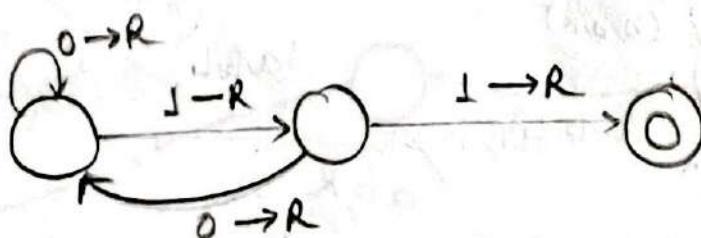
Date:

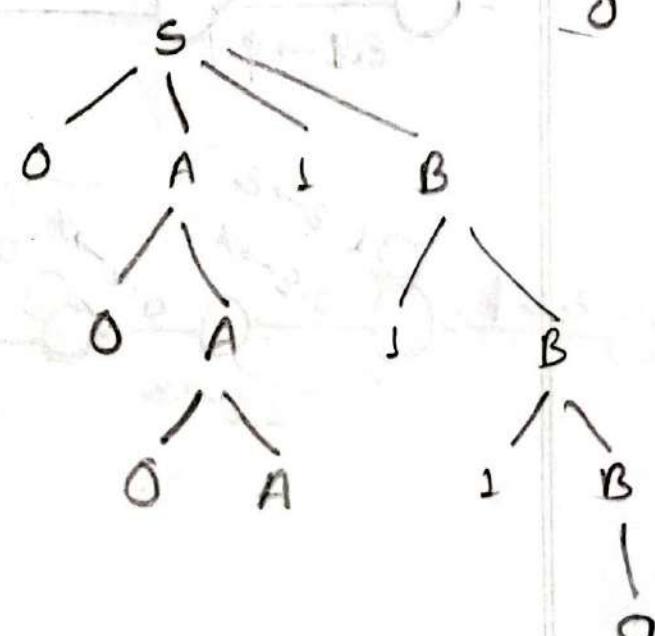
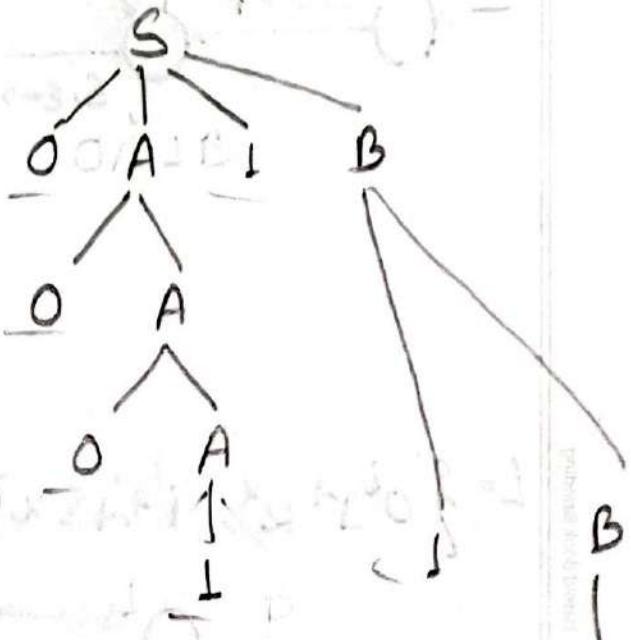
abc ab
γ

$$L = \{w \in \{a,b\}^* \mid w \text{ contains } \gamma\}$$



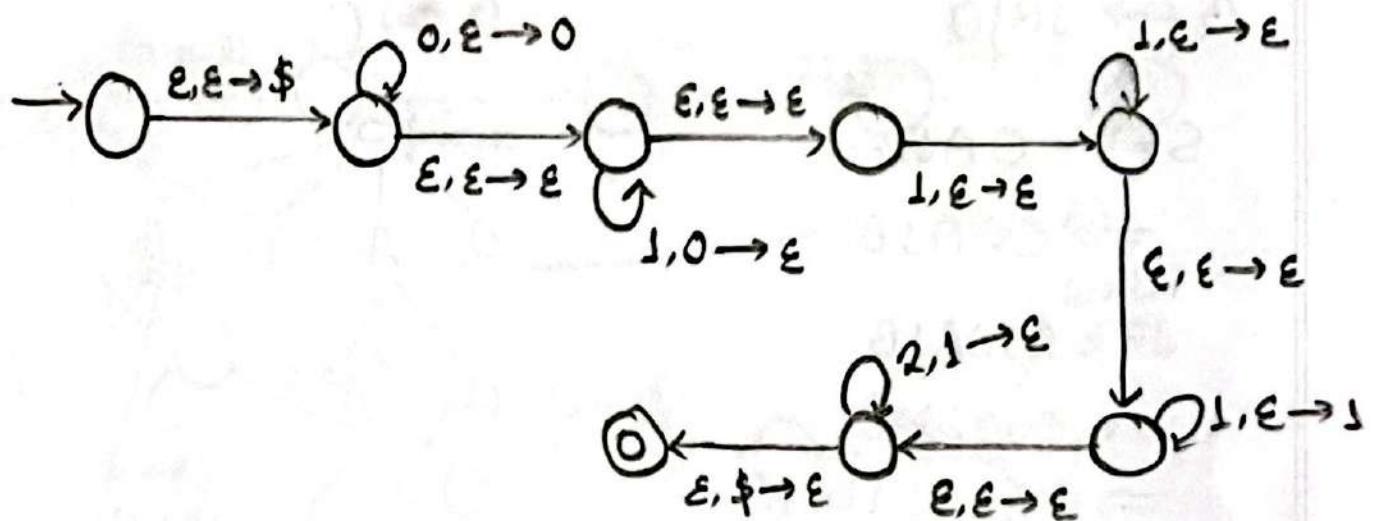
$$L(M) \rightarrow \{w \in \{0,1\}^* \mid w \text{ contains } 11\}$$



$S \rightarrow OAIB$
 $A \rightarrow OA|I$
 $B \rightarrow IB|O$
 $S \rightarrow OAIB$
 $\Rightarrow OOAIB$
 $\Rightarrow OOOAIB$
 $\Rightarrow OOOIB$
 $\Rightarrow OOOIIB$
 $\Rightarrow OOOIIIB$
 $\Rightarrow OOOIIIIB$
 $S \Rightarrow OAIB$
 $OAIB$
 $\Rightarrow OAIBB$
 $\Rightarrow OAIBIB$
 $\Rightarrow OAIIIO$
 $\Rightarrow OOAIIIO$
 $\Rightarrow OO OA IIIIO$
 $\Rightarrow OOO$
 011100


PDA

$$L = \{ 0^i 1^n 2^k; i \leq n+k \}$$



$$L = \{ 0^i 1^n 2^k; i + n \leq k \}$$

