Rotation Matrices used in Nibauer & Bonaca 2025

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Let $\boldsymbol{x} = (x, y, z)^{\mathsf{T}}$ denote Cartesian coordinates in the Galactocentric frame and $\boldsymbol{x}' = (x', y', z')^{\mathsf{T}}$ the same position expressed in the rotated (halo) frame. We define the coordinate transformation as the passive rotation

$$\boldsymbol{x}' = R\,\boldsymbol{x}, \qquad \boldsymbol{x} = R^{\mathsf{T}}\boldsymbol{x}'. \tag{1}$$

Elementary rotations. We parameterize the rotation by a yaw angle ϕ (rotation about the z axis) and a pitch angle θ (rotation about the y axis). Using rotation matrices,

$$R_z(\phi) = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad R_y(\theta) = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}. \tag{2}$$

Total rotation. The total rotation is taken to be "yaw then pitch" (rightmost acts first),

$$R_{\text{tot}} = R_y(\theta)\, R_z(\phi). \tag{3}$$

In the implementation, however, we adopt the sign convention $\phi \mapsto -\phi$ (i.e. the input ϕ is negated before constructing the matrix). Equivalently,

$$R \;=\; R_y(\theta)\, R_z(-\phi). \tag{4}$$

Multiplying out, this yields

$$R = \begin{pmatrix} \cos\theta\cos\phi & -\cos\theta\sin\phi & -\sin\theta \\ \sin\phi & \cos\phi & 0 \\ \sin\theta\cos\phi & -\sin\theta\sin\phi & \cos\theta \end{pmatrix}. \tag{5}$$

Remarks. With this choice, the rotated z' axis lies in the (x, z) plane and is obtained by pitching the frame by θ about the y axis after the (signed) yaw about z. Angle ranges are $\phi \in [0, \pi]$ and $\theta \in [-\pi/2, \pi/2]$.

Coordinate transform. Given a position $\boldsymbol{x}$ in the Galactocentric frame, we first transform it into the halo principal-axis frame via

$$\boldsymbol{x}' = R\,\boldsymbol{x}, \qquad R = R_y(\theta)\, R_z(-\phi), \qquad R^{\mathsf{T}}R = I. \tag{6}$$

(Equivalently, $\boldsymbol{x} = R^{\mathsf{T}}\boldsymbol{x}'$.)

Density. Let $\rho_{\text{halo}}(\boldsymbol{x}')$ denote the halo density expressed in the halo frame. The halo density at the Galactocentric location $\boldsymbol{x}$ is then

$$\rho_{\text{halo}}(\boldsymbol{x}'(\boldsymbol{x})) = \rho_{\text{halo}}(R\boldsymbol{x}). \tag{7}$$