HarvardX Data Science Capstone: MovieLens Report

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1. Introduction

The HarvardX Data Science Capstone MovieLens project is a movie recommendation system using machine learning algorithms made famous by the one million dollar Netflix competition launched on october 2006.

At the end of the challenge in September 2009, Netflix awarded the Grand Prize to a developer team "BellKor's Pragmatic Chaos" with a winning solution with a Root Mean Squared Error(RMSE) of about 0.857 that increased the accuracy of the company's recommendation engine by 10%.

Using the 10M version of MovieLens data split into edx training set and 10% validation set, I used some of the machine learning techniques that went into the winning solution for the Netflix competition. The ML model used with the smallest RMSE was a multiple linear regression model using regularized movie and user effects at 0.8641362.

2. Overview

This report contains sections for data exploration, visualization, preprocessing, evaluated machine learning algorithms, and RMSE analysis sections including methods that were used to transform the data to create the best predicive model.

The results and conclusion sections and the end includes final thoughts on the MovieLens project and suggestions to improve the predictive model using more advanced machine learning algorithms and matrix factorization methods.

2.1. Loading libraries and data

```
# Loading packages for data exploration, visualization, preprocessing,
# machine learning algorithms, and RMSE analysis
if(!require(tidyverse)) install.packages("tidyverse", repos = "http://cran.us.r-project.org")
```

```
## Loading required package: tidyverse
## Registered S3 methods overwritten by 'ggplot2':
##
    method
                  from
##
    [.quosures
                  rlang
##
    c.quosures
                  rlang
##
    print.quosures rlang
## Registered S3 method overwritten by 'rvest':
    method
                     from
##
    read_xml.response xml2
## -- Attaching packages ------ tidyverse 1.2.1 -
## v ggplot2 3.1.1
                      v purrr 0.3.2
## v tibble 2.1.1
                      v dplyr 0.8.0.1
## v tidyr 0.8.3
                      v stringr 1.4.0
          1.3.1
## v readr
                      v forcats 0.4.0
## -- Conflicts ----- tidyverse conflicts() -
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()
                   masks stats::lag()
if(!require(caret)) install.packages("caret", repos = "http://cran.us.r-project.org")
## Loading required package: caret
## Loading required package: lattice
##
## Attaching package: 'caret'
## The following object is masked from 'package:purrr':
##
##
      lift
if(!require(data.table)) install.packages("data.table", repos = "http://cran.us.r-project.org")
## Loading required package: data.table
## Attaching package: 'data.table'
## The following objects are masked from 'package:dplyr':
##
##
      between, first, last
## The following object is masked from 'package:purrr':
##
##
      transpose
```

```
if(!require(lubridate)) install.packages("lubridate", repos = "http://cran.us.r-project.org")
## Loading required package: lubridate
##
## Attaching package: 'lubridate'
## The following objects are masked from 'package:data.table':
##
##
       hour, isoweek, mday, minute, month, quarter, second, wday,
##
       week, yday, year
## The following object is masked from 'package:base':
##
##
       date
if(!require(ggplot2)) install.packages("ggplot2", repos = "http://cran.us.r-project.org")
if(!require(knitr)) install.packages("knitr", repos = "http://cran.us.r-project.org")
## Loading required package: knitr
if(!require(rmarkdown)) install.packages("rmarkdown", repos = "http://cran.us.r-project.org")
## Loading required package: rmarkdown
if(!require(tinytex)) install.packages("tinytex", repos = "http://cran.us.r-project.org")
## Loading required package: tinytex
library(tidyverse)
library(caret)
library(data.table)
library(lubridate)
library(ggplot2)
library(knitr)
library(rmarkdown)
library(tinytex)
# https://grouplens.org/data tables/movielens/10m/
# http://files.grouplens.org/data tables/movielens/ml-10m.zip
movielens <- readRDS("edx.rds", refhook = NULL)</pre>
validation <- readRDS("validation.rds", refhook = NULL)</pre>
# if using R 3.6.0: set.seed(1, sample.kind = "Rounding")
set.seed(1, sample.kind = "Rounding")
```

```
## Warning in set.seed(1, sample.kind = "Rounding"): non-uniform 'Rounding'
## sampler used
```

```
test_index <- createDataPartition(y = movielens$rating, times = 1, p = 0.1, list = FALSE)
edx <- movielens[-test_index,]
temp <- movielens[test_index,]

# Make sure userId and movieId in validation set are also in edx set

validation <- temp %>%
    semi_join(edx, by = "movieId") %>%
    semi_join(edx, by = "userId")

# Add rows removed from validation set back into edx set

removed <- anti_join(temp, validation)</pre>
```

Joining, by = c("userId", "movieId", "rating", "timestamp", "title", "genres")

```
edx <- rbind(edx, removed)

rm(dl, ratings, movies, test_index, temp, movielens, removed)

## Warning in rm(dl, ratings, movies, test_index, temp, movielens, removed):

## Warning in rm(dl, ratings, movies, test_index, temp, movielens, removed):

## object 'ratings' not found

## Warning in rm(dl, ratings, movies, test_index, temp, movielens, removed):

## object 'movies' not found</pre>
```

3. Executive Summary

Since I amd using R version 3.6.0, there were issues reading the MovieLens files directly from GroupLens so a RDS copy of the edx and validation datasets were provided in a HarvardX_Capstone_MovieLens Google Drive. Link is below. https://drive.google.com/drive/folders/1IZcBBX0OmL9wu9AdzMBFUG8GoPbGQ38D

The edX dataset is a subset of the MovieLens 10M data table made of 6 variables and a total of 8,100,065 observations and the validation dataset represents approximately 10% or 899,990 observations and contains the same 6 variables.

Each observations or row contains the variables or columns "userid", "movieId", "rating", "timestamp", "title", "genres".

Quantitative or numeric variables include the following:

- * userId the unique identifier for the user.
- * movieId the unique identifier for the movie.

Table 1: Head of edx dataset

	userId	movieId	rating	timestamp	title	genres
1	1	122	5	838985046	Boomerang (1992)	Comedy Romance
4	1	292	5	838983421	Outbreak (1995)	Action Drama Sci-Fi Thriller
5	1	316	5	838983392	Stargate (1994)	Action Adventure Sci-Fi
6	1	329	5	838983392	Star Trek: Generations (1994)	Action Adventure Drama Sci-Fi
7	1	355	5	838984474	Flintstones, The (1994)	Children Comedy Fantasy
8	1	356	5	838983653	Forrest Gump (1994)	${\bf Comedy} {\bf Drama} {\bf Romance} {\bf War}$

Table 2: Summary stats for edx dataset

userId	movieId	rating	timestamp	title	genres
Min.: 1 1st Qu.:18127 Median:35732 Mean:35870 3rd Qu.:53607	Min.: 1 1st Qu.: 648 Median: 1834 Mean: 4120 3rd Qu.: 3624	Min. :0.500 1st Qu.:3.000 Median :4.000 Mean :3.512 3rd Qu.:4.000	Min. :7.897e+08 1st Qu.:9.468e+08 Median :1.035e+09 Mean :1.033e+09 3rd Qu.:1.127e+09	Length:8100065 Class :character Mode :character NA NA	Length:8100065 Class :character Mode :character NA NA
Max. :71567	Max. :65133	Max. :5.000	Max. :1.231e+09	NA	NA

^{*} timestamp - unix timestamp when the user rating was provided in seconds since Unix Epoch on January 1st, 1970 at UTC.

Qualitative or non-numeric variables include the following:

The variable or column "rating" is the outcome we want to predict, y.

* rating: a numeric rating between 0 and 5 for the movie in 0.5 increments.

 $MovieLens\ edx\ dataset$

```
edx_head <- head(edx)
kable(edx_head, "latex", booktabs = T, caption = "Head of edx dataset")</pre>
```

```
edx_summary <- summary(edx)
kable(edx_summary, "latex", booktabs = T, caption = "Summary stats for edx dataset")</pre>
```

 $MovieLens\ validation\ dataset$

^{*} title: movie title with year of release in ()

^{*} genres: genres associated with the movie separated by

Table 3: Edx dataset - unique userIds and movieIds

users	movies
68159	9701

Table 4: Head of validation dataset

userId	movieId	rating	${\it timestamp}$	title	genres
1	185	5	838983525	Net, The (1995)	Action Crime
2	260	5	868244562	Star Wars: Episode IV - A New Hope (a.k.a. Star Wars) (1977)	Action Adver
2	590	5	868245608	Dances with Wolves (1990)	Adventure Di
2	1049	3	868245920	Ghost and the Darkness, The (1996)	Action Adver
2	1210	4	868245644	Star Wars: Episode VI - Return of the Jedi (1983)	Action Adver
3	1148	4	1133571121	Wallace & Gromit: The Wrong Trousers (1993)	Animation C

```
val_head <- head(validation)
kable(val_head, "latex", booktabs = T, caption = "Head of validation dataset")</pre>
```

```
val_summary <- summary(validation)
kable(val_summary, "latex", booktabs = T, caption = "Summary stats for validation dataset")</pre>
```

4. Methods and Analysis:

4.1. Data exploration and visualization

The MovieLens edx and validation datasets contain a unix timestamp field for when the user rating was provided in seconds since Jan. 1, 1970 at UTC. In order to use the variable as a factor for visualization and modeling, I needed to transform it into standard datatime.

I used the as_datetime function in the lubridate package to mutate in the datatime format. I then created a scatterplot of y = average ratings vs x = date grouped by month and added geom_smooth line option for improved visualization of the date trend of ratings.

As you can see from the chart below, the variability of the average monthly rating decreased over time and converged to an average monthly rating of approximately 3.5 after 2005 when there was a higher frequency of ratings and movies in the edx dataset.

```
edx <- data.table(edx)
edx$timestamp <- as_datetime(edx$timestamp)

validation <- data.table(validation)
validation$timestamp <- as_datetime(validation$timestamp)</pre>
```

Table 5: Summary stats for validation dataset

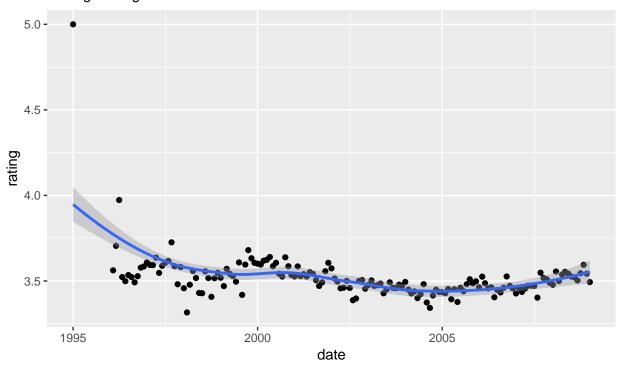
userId	movieId	rating	timestamp	title	genres
Min.: 1 1st Qu.:18105 Median:35761 Mean:35868 3rd Qu.:53598	Min.: 1 1st Qu.: 648 Median: 1834 Mean: 4133 3rd Qu.: 3638	Min. :0.500 1st Qu.:3.000 Median :4.000 Mean :3.513 3rd Qu.:4.000	Min. :7.897e+08 1st Qu.:9.468e+08 Median :1.036e+09 Mean :1.033e+09 3rd Qu.:1.127e+09	Length:899990 Class :character Mode :character NA NA	Length:899990 Class :character Mode :character NA NA
Max. :71567	Max. :65130	Max. :5.000	Max. :1.231e+09	NA	NA

Table 6: Validation dataset - unique user Ids and movie
 Ids

users	movies
68159	9701

`geom_smooth()` using method = 'loess' and formula 'y ~ x'

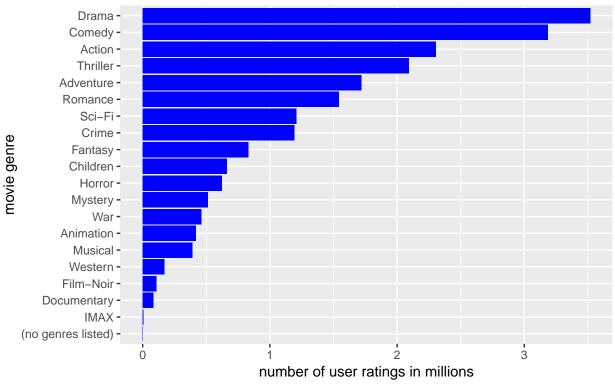
Timestamp of movie ratings by month average ratings



source data: edx data table

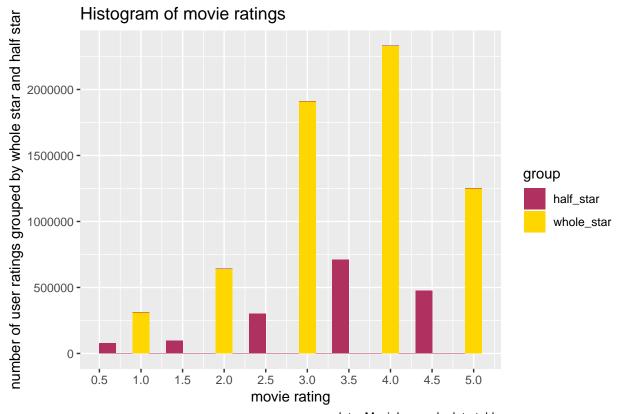
Grouping user ratings by movie genre for the edx dataset, the genres of "Drama" and "Comedy" were the most popular with over three million user ratings followed by "Action" and "Thriller" with over 2 million user ratings. Documentary and IMAX genre movies were the least rated by users in the edx dataset.





source data: MovieLens edx data table

Exploring whole and half star ratings for the edx dataset, the most popular rating is 4.0 with over two million user ratings followed by 3.0 and 5.0. Half star ratings in general are less popular than whole ratings as shown in the below histogram.



source data: MovieLens edx data table

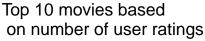
Ranking the top 10 movies by number of user ratings in the edx dataset, Pulp Fiction (1994) had the most user ratings followed by Forest Gump (1994), Silence of the Lambs, The (1991), Jurassic Park (1993), and Shawshank Redemption, The (1994). These top 5 movies are also part of the Drama, Comedy, or Action genres which ranked at the top of user ratings by genre.

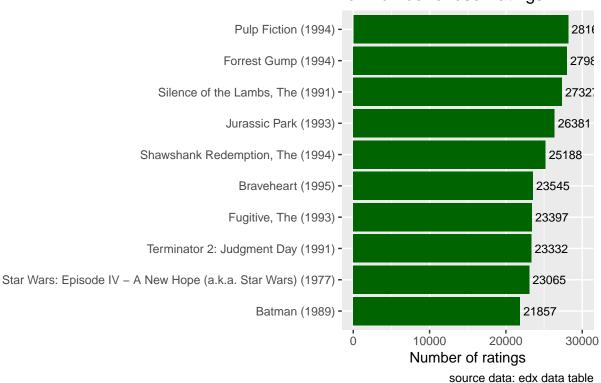
Interestingly, movies released in the 1990's dominated the top 10 movies with all but the bottom two Star Wars: Episode IV - A New Hope (a.k.a Star Wars)(1977) and Batman (1989) not in that decade.

```
# Create top_movies dataframe from edx data table which contains the top 10 movies by number of user ra
top_movies <- edx %>%
  group_by(title) %>%
  summarize(count=n()) %>%
  top_n(10,count) %>%
  arrange(desc(count))

# Bar chart of top_movies

top_movies %>%
  ggplot(aes(x=reorder(title, count), y=count)) +
  geom_bar(stat='identity', fill="dark green") + coord_flip(y=c(0, 30000)) +
  labs(x="", y="Number of ratings") +
  geom_text(aes(label= count), hjust=-0.1, size=3) +
  labs(title="Top 10 movies based \n on number of user ratings", caption = "source data: edx data table")
```





4.2. Data preprocessing and transformation

The dependent variables userId and movieId should be treated as factors for linear regression modeling purposes. To perform this transformation we make a copies of edx dataset and renamed as training_set and validation dataset and rename as test_set.

```
training_set <- edx
training_set$userId <- as.factor(training_set$userId)
training_set$movieId <- as.factor(training_set$movieId)</pre>
```

I added a column to the training_set for date which is the month and year for the rating and set it as a factor.

```
training_set <- training_set %>%
  mutate(date = round_date(as_datetime(timestamp), unit = "month"))
training_set$date <- as.factor(training_set$date)</pre>
```

Due to the large size of the edx dataset I selected the training_set columns with the quantitative dependent variables to by used in the linear regression modeling process userId, movieId, date, and the predictive outcome variable y, rating.

```
training_set <- training_set %>% select(userId, movieId, date, rating)
```

I transformed the validation dataset the same as the training_set with columns userId, movieId, date, and rating as the test_set.

```
test_set <- validation

test_set$userId <- as.factor(test_set$userId)
test_set$movieId <- as.factor(validation$movieId)

test_set <- test_set %>%
   mutate(date = round_date(as_datetime(timestamp), unit = "month"))

test_set$date <- as.factor(test_set$date)

test_set <- test_set %>% select(userId, movieId, date, rating)
```

4.3. Evaluated Machine Learning Algorithms

4.3.1 Baseline Naive Model

The baseline naive model that I will be trying to beat with more advanced linear regression models is generated by using the average rating (μ) of 3.51245564078807 on the training_set to be predicted into the test_set.

The baseline-naive model is calculated as follows:

$$Y_{u,i} = \mu + \varepsilon_{u,i}$$

- * is the "true" rating for all movies.
- * (epsilon) are independent errors sampled from the same distribution centered at 0.

The resulting RMSE for the baseline model is 1.060054 which is a poor model and a marginally better predictive model than random guessing at 1.177464 RMSE. I will try to improve on this model using dependent variables movieId, userId, and date in linear regression models along with regularization methods.

```
# Random guessing rating across all movies.
random_guess <- rep(3, nrow(test_set))
RMSE_random <- RMSE(test_set$rating, random_guess)

rmse_table <- data_frame(Method = "Random guessing rating across all movies" , RMSE = RMSE_random)

## Warning: `data_frame()` is deprecated, use `tibble()`.
## This warning is displayed once per session.

rmse_table %>% knitr::kable(caption = "RMSEs")
```

Table 7: RMSEs

Method	RMSE
Random guessing rating across all movies	1.177464

Table 8: RMSEs

Method	RMSE
Random guessing rating across all movies Naive model using mean rating across all movies	1.177464 1.060054

4.3.2 Simple Linear Regression Model

I followed the same approach to build the linear regression models as the simplest recommendation systems described by professor Rafa Irizarry in the dsbook sourced from the github page https://rafalab.github.io/dsbook/.

The simple linear regression model that I will generate first uses the rating by movieId bias or the movie effect b_i dependent variable on the training_set to predict the rating $(Y_{u,i})$ for the test_set. According to professor Irizarry, statistics textbooks refer to the b_s as effects. However, in the Netflix challenge papers, they refer to them as "bias", thus the b notation.

The movie-specific effect model is calculated as follows:

$$Y_i = \mu + b_i + \varepsilon_i$$

where:

- * is the "true" rating for all movies.
- * bi effects or bias, movie-specific effect.
- * (epsilon) are independent errors sampled from the same distribution centered at 0.

The resulting RMSE from this movie-specific effect model on the test_set was an improvement on the baseline naive model RMSE at 0.9429615. We can likely do better with adding more dependent variables to the model.

```
# Fitting Simple Regression Model to the edx training set.

# The average of all movie ratings mu of the training_set.
mu <- mean(training_set$rating)</pre>
```

```
# Group average ratings by movieId on the training set.
movie_avgs <- training_set %>%
  group_by(movieId) %>%
  summarize(bi = mean(rating - mu))

# Predicted ratings using movie-specific effect.
predicted_ratings_bi <- mu + test_set %>%
  left_join(movie_avgs, by='movieId') %>%
  .$bi
```

Warning: Column `movieId` joining factors with different levels, coercing
to character vector

Table 9: RMSEs

Method	RMSE
Random guessing rating across all movies	1.1774645
Naive model using mean rating across all movies	1.0600537
Predicted movie ratings regression model using movie-specific effect	0.9429615

4.3.2 Multiple Linear Regression Models

The next regression model that I will generate uses the rating by movieId bias or the movie effect (m_i) and userId bias or user effect (b_u) dependent variables on the training_set to predict the rating $Y_{u,i}$ for the test_set.

The movie-user-specific effect model is calculated as follows:

$$Y_{u,i} = \mu + b_i + b_u + \varepsilon_{u,i}$$

where:

- * is the "true" rating for all movies.
- * bi effects or bias, movie-specific effect.
- * bu effects or bias, user-specific effect.
- * (epsilon) are independent errors sampled from the same distribution centered at 0.

The resulting RMSE from this movie-user-specific effect model on the test_set was an improvement on the movie-specific effect model RMSE at 0.8646843. Let's see if we can do better with adding the last dependent variable date to the model.

```
# Fitting Multiple Regression Models to the edx training set.

# Predicted movie ratings regression model using both movieId and userId averages.

user_avgs <- training_set %>%
  left_join(movie_avgs, by='movieId') %>%
  group_by(userId) %>%
  summarize(bu = mean(rating - mu - bi))

predicted_ratings_bi_bu <- test_set %>%
  left_join(movie_avgs, by='movieId') %>%
  left_join(user_avgs, by='userId') %>%
  mutate(pred = mu + bi + bu) %>%
  .$pred
```

```
## Warning: Column `movieId` joining factors with different levels, coercing
## to character vector
```

Warning: Column `userId` joining factors with different levels, coercing to ## character vector

Table 10: RMSEs

Method	RMSE
Random guessing rating across all movies	1.1774645
Naive model using mean rating across all movies	1.0600537
Predicted movie ratings regression model using movie-specific effect	0.9429615
Predicted movie ratings regression model using movie-user-specific effect	0.8646843

The last linear regression model that I will generate uses the rating by movieId bias or the movie effect (b_i) , userId bias or user effect (b_u) , and date bias or date effect (b_d) dependent variables on the training_set to predict the rating $Y_{i,u,d}$ for the test_set.

The movie-user-date-specific effect model is calculated as follows:

$$Y_{i,u,d} = \mu + b_i + b_u + b_d + \varepsilon_{i,u,d}$$

where:

- * is the "true" rating for all movies.
- * bi effects or bias, movie-specific effect.

- * bu effects or bias, user-specific effect.
- * bd effects or bias, date-specific effect.
- * (epsilon) are independent errors sampled from the same distribution centered at 0.

The resulting RMSE from this movie-user-date-specific effect model on the test_set was a neligible improvement on the movie-user-specific effect model RMSE at 0.8646637. Next step is if regularization method improves the model RMSE.

```
# Predicted movie ratings regression model using movieId, userId, and date averages.

date_avgs <- training_set %>%
  left_join(movie_avgs, by='movieId') %>%
  left_join(user_avgs, by='userId') %>%
  group_by(date) %>%
  summarize(bd = mean(rating - mu - bi - bu))

predicted_ratings_bi_bu_bd <- test_set %>%
  left_join(movie_avgs, by='movieId') %>%
  left_join(user_avgs, by='userId') %>%
  left_join(date_avgs, by='date') %>%
  mutate(pred = mu + bi + bu + bd) %>%
  .$pred
```

```
## Warning: Column `movieId` joining factors with different levels, coercing
## to character vector
```

Warning: Column `userId` joining factors with different levels, coercing to

```
## character vector

movie_user_date_model <- RMSE(predicted_ratings_bi_bu_bd, test_set$rating)</pre>
```

Table 11: RMSEs

Method	RMSE
Random guessing rating across all movies	1.1774645
Naive model using mean rating across all movies	1.0600537
Predicted movie ratings regression model using movie-specific effect	0.9429615
Predicted movie ratings regression model using movie-user-specific effect	0.8646843
Predicted movie ratings regression model using movie-user-date-specific effect	0.8646637

4.3.3 Regularization

I followed the same approach for the regularization method for recommendation systems described by pro-

fessor Irizarry in the dsbook sourced from the github page https://rafalab.github.io/dsbook/.

The formula for the regularization method on the movie-user-specific effect model is as follows.

$$\frac{1}{N} \sum_{u,i} (y_{u,i} - \mu - b_i - b_u)^2 + \lambda \left(\sum_i b_i^2 + \sum_u b_u^2 \right)$$

The movie-user-specific effect model with the regularization method on the test_set to seeks to minimize the RMSE using cross validation to pick a

 λ

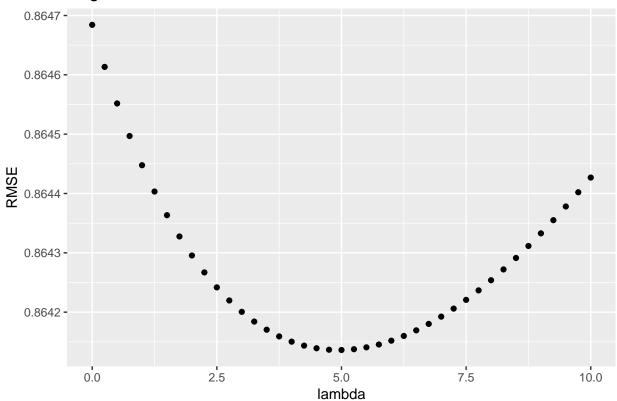
. In this case, the

 λ

that optimized (minimized) the RMSE was 5.

```
lambdas <- seq(0, 10, 0.25)
rmses <- sapply(lambdas, function(1){</pre>
  mu <- mean(training_set$rating)</pre>
  b_i <- training_set %>%
    group_by(movieId) %>%
    summarize(b_i = sum(rating - mu)/(n()+1))
  b_u <- training_set %>%
    left_join(b_i, by="movieId") %>%
    group_by(userId) %>%
    summarize(b_u = sum(rating - b_i - mu)/(n()+1))
  predicted_ratings <-</pre>
    test_set %>%
    left_join(b_i, by = "movieId") %>%
    left_join(b_u, by = "userId") %>%
    mutate(pred = mu + b_i + b_u) %>%
    pull(pred)
    return(RMSE(predicted_ratings, test_set$rating))
qplot(lambdas, rmses,
      main = "Regularization",
      xlab = "lambda", ylab = "RMSE") # lambda vs RMSE
```

Regularization



```
lambda_opt <- lambdas[which.min(rmses)]
lambda_opt # lambda which optimizes the model (minimizes RMSE) which is 5 in this case</pre>
```

[1] 5

Table 12: RMSEs

Method	RMSE
Random guessing rating across all movies	1.1774645
Naive model using mean rating across all movies	1.0600537
Predicted movie ratings regression model using movie-specific effect	0.9429615
Predicted movie ratings regression model using movie-user-specific effect	0.8646843
Predicted movie ratings regression model using movie-user-date-specific effect	0.8646637
Regularization of ratings regression prediction model using movieId and userid averages	0.8641362

5. Results:

The minimized RMSE is obtained from the movie-user-specific effect model using the regularization method with a $\lambda = 5$. The resulting minimized RMSE was 0.8641362, which was close to the Netflix challenge winning solution.

More advanced ML algorithms with matrix factorization, dimension reduction, ensamble methods could improve upon the RMSE although computational resources were limited to my personal laptop. Also, there was limited time to research different analytic modeling techniques such as random forest, decision tree, support vector machine, gbm, neural network, ensamble, and others. The large size of the MovieLens dataset made most of the ML libraries unusable unless a smaller subset of the dataset was used which was not allowed for this particular project.

rmse_table %>% knitr::kable(caption = "RMSEs")

Table 13: RMSEs

Method	RMSE
Random guessing rating across all movies	1.1774645
Naive model using mean rating across all movies	1.0600537
Predicted movie ratings regression model using movie-specific effect	0.9429615
Predicted movie ratings regression model using movie-user-specific effect	0.8646843
Predicted movie ratings regression model using movie-user-date-specific effect	0.8646637
Regularization of ratings regression prediction model using movieId and userid averages	0.8641362

6. Conclusion:

The main learning objective of the HarvardX: Introduction to Data Science program was to give aspiring data scientists like myself the tools in R to run analytic models using machine learning to make predictions and solve real world problems. This wapurls a fascinating journey over the past six months and I look forward to improving my data science skills in R and Python through my work and personally through Kaggle competitions that I have an interest in.

My "Harvard Data Science MovieLens Github repository" is in this link

References

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