# Secant Method

Finding the derivative is always not possible.

Consider the equation $f(x) = 0$.

**Formula:** $x_{n+1} = \dfrac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}$

Algorithm:

1. Choose two initial approximate $x_{-1}$ and $x_0$
2. Compute $x_1 = \dfrac{x_{-1} f(x_0) - x_0 f(x_{-1})}{f(x_0) - f(x_{-1})}$
3. $x_{-1} \leftarrow x_0$ and $x_0 \leftarrow x_1$
4. Repeat 2 and 3 until $|x_1 - x_0| < \epsilon$, where $\epsilon$ is the error tolerance.

Find a real root of the equation $x^3 - 2x - 5 = 0$ using secant method.

$f(x) = x^3 - 2x - 5$ and let $x_{-1} = 2$ and $x_0 = 3$.

1. Take $x_0 = 2$. Then $f(x_{-1}) = f(2) = -1$ and $f(x_0) = f(3) = 16$. Hence

$$x_1 = \frac{2 \times 16 - 3 \times -1}{16 - -1} = \frac{35}{17} = 2.058823529$$

2. $f(x_0) = f(3) = 16$ and $f(x_1) = f(2.058823529) = -0.390799923$. Then

$$x_2 = \frac{3 \times -0.390799923 - 2.058823529 \times 16}{-0.390799923 - 16} = 2.08126366$$

3. $x_3 = 2.094824145$

$x_3$ is correct to 3 decimal places.

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# Problems

- Use the secant method to determine the root, lying between 5 and 8, of the equation $x^{2.2} = 69$.
- Determine the root of the equation $x^3 - 10x^2 + 5 = 0$ lying between 0.6 and 0.8 using the secant method.

Rate of convergence measures how fast the successive approximates converges to the root.

Let $x_0, x_1, x_2, \ldots$ be the iterates, and $r$ be the root of the equation $f(x) = 0$.

Define $\epsilon_n = |r - x_n|$.

If there exists real numbers $p$ and $k$ such that $|\epsilon_{n+1}| = k|\epsilon_n|^p$, then $p$ is called the order of convergence.

Larger the value of $p$, faster the method will be.

If $p = 1$ and $k < 1$, then $k$ is called the rate of convergence.

Bisection method & Regula Falsi method: p=1

Secant method : p=1.618

Newton-Raphson method: p=2.