

2. Design a C program to implement R-K method.

Runge-Kutta method : To solve ordinary differential Equations.

To solve $\frac{dy}{dx} = f(x, y) \Rightarrow y' = \dot{y}$

given $y(x_0) = y_0$ - y_1, y_2 $y_1 = y(x_1)$ & $y_2 = y(x_2)$

$x_1 = x_0 + h$ h - Step size - interval length.

$$h = x_1 - x_0 = x_2 - x_1 = x_3 - x_2 \dots$$

The fourth order R-K method algorithm:

$$k_1 = h f(x, y)$$

$$y(x+h) = y(x) + k$$

$$k_2 = h f(x + h/2, y + k_1/2)$$

$$k_3 = h f(x + h/2, y + k_2/2)$$

$$k_4 = h f(x + h, y + k_3)$$

$$k = (k_1 + 2k_2 + 2k_3 + k_4)/6$$

x_0 and y_0 To find $y_1 = y(x_1)$ Where $x_1 = x_0 + h$.

$$k_1 = h f(x_0, y_0)$$

$$k_2 = h f(x_0 + h/2, y_0 + k_1/2)$$

$$k_3 = h f(x_0 + h/2, y_0 + k_2/2)$$

$$k_4 = h f(x_0 + h, y_0 + k_3)$$

$$k = (k_1 + 2k_2 + 2k_3 + k_4)/6.$$

$$y_1 = y_0 + k$$

—

Code :

1. Start
2. Define function $f(x, y)$
3. Read the values of (x, y) at the starting point or initial condition of x & y i.e., (x_0, y_0) .
Number of steps (n) and calculated point or end point of x .
 x_n .

4. calculate the step size $h = (x_n - x_0) / n$

5. Set $i = 0$.

6. Loop

$$k_1 = h * f(x_0, y_0);$$

$$k_2 = h * f(x_0 + h/2, y_0 + k_1/2);$$

$$k_3 = h * f(x_0 + h/2, y_0 + k_2/2);$$

$$k_4 = h * f(x_0 + h, y_0 + k_3);$$

$$k = (k_1 + 2*k_2 + 2*k_3 + k_4) / 6;$$

$$y_n = y_0 + k;$$

$$i = i + 1$$

$$x_0 = x_0 + h;$$

$$y_0 = y_n$$

while $i < n$

7. Display y_n as a result.

8. Stop.

Find $y(1.1)$, $y(1.2)$ if $\frac{dy}{dx} = x^3 + \frac{y}{2}$ using R-K method
of 4th-order where $y(1) = 2$ through C-programming.

```
#include <stdio.h>
#include <conio.h>
#include <math.h>
/* define f(x,y) (pow(x,3) + (y/2)) */
float x, y;
#define f(x,y) (pow(x,3) + (y/2))
int main ()
{
    float x0, y0, h, xn, yn, k1, k2, k3, k4, k;
    int i, n;
    printf("Enter initial Condition\n");
    printf("x0 = ");
    scanf("%f", &x0);
```

```
printf ("y0 = ");
scanf ("%f", &y0);
printf ("Enter the calculation point x n = ");
scanf ("%f", &xn);
printf ("Enter the number of steps: ");
scanf ("%d", &n);
printf ("The Step-size h = ");
scanf ("%f", &h);
/* Runge Kutta method */
for ( i=0 ; i<n ; i++ )
{
    k1 = h * f(x0, y0);
```

$$k_2 = h * f((x_0 + h/2), (y_0 + k_1/2));$$

$$k_3 = h * f((x_0 + h/2), (y_0 + k_2/2));$$

$$k_4 = h * f((x_0 + h), (y_0 + k_3));$$

$$k = (k_1 + 2 * k_2 + 2 * k_3 + k_4) / 6;$$

$$y_n = y_0 + k$$

```
printf("%d %d\n", i, i);
```

```
printf("%.4f %.4f\n", x0, y0);
```

$$x_0 = x_0 + h;$$

$$y_0 = y_n$$

```
} printf("%d %d\n", i, i);
```

```
printf("%.4f %.4f\n", x0, y0);
```

```
getch();
```

Enter initial condition

$$x_0 = 1$$

$$y_0 = 2$$

Enter calculation point $x_n = 1.2$

Enter number of steps : 2

Enter step size 0.1

x_0	y_0
1.0000	2.0000

x_1	y_1
1.1000	2.2214

x_2	y_2
1.2000	2.4913

2) Find $y(0.7)$, $y(0.8)$ if $\frac{dy}{dx} = y - x^2$ using R-K method
Where $y(0.6) = 1.7379$ through C-programming

$$x_1 \quad 0.7000 \quad y_1 \quad 1.8763$$

$$x_2 \quad 0.8000 \quad y_2 \quad 2.0145$$

$$f(x, y) = y - x^2$$

$$f(x, y) = \underline{y - x^2}$$

3) obtain the values of y at $x = 0.1, 0.2$ using R-K method of 4th order for the differential equation

$$y' = -y \cdot \text{ given } y(0) = 1 \quad f(x, y) = -y$$

4) Evaluate $y(1.2)$ and $y(1.4)$ given $y' = \frac{2xy + e^x}{x^2 + xe^x}$

$$y(0) = 0$$

$$f(x, y) = \frac{(2x^2y + e^{2x})}{(x^2 + xe^x)}$$

$$y(1.2) = 0.1402 \quad ; \quad y(1.4) = 0.2705$$

5) Determine y at $x = 0.2(0.2)0.6$ by R-K method
given $\frac{dy}{dx} = xy + 1$, given $y(0) = 0$

$$x_0 = 0$$

$$y(x_0) = y_0 = 0 = 0$$

$$x_1 = 0.2$$

$$y_1 = y(0.2) = 2.243$$

$$x_2 = 0.4$$

$$y_2 = y(0.4) = 2.589$$

$$x_3 = 0.6$$

$$y_3 = y(0.6) = 3.072$$

$$f(x, y)$$

$$xy + 1$$

—