

# Method of False Position (Regula Falsi)

## Nonlinear equation $f(x) = 0$

- 1 Choose two real numbers  $a$  and  $b$  such that  $f(a)f(b) < 0$
- 2 Set  $x_r = \frac{af(b) - bf(a)}{f(b) - f(a)}$ .
- 3
  - ▶ If  $f(a)f(x_r) < 0$ , the root lies in the interval  $(a, x_r)$ . Then, set  $b = x_r$  and go to step 2.
  - ▶ If  $f(a)f(x_r) > 0$ , the root lies in the interval  $(x_r, b)$ . Then, set  $a = x_r$  and go to step 2.
  - ▶ If  $f(a)f(x_r) = 0$ , then  $x_r$  is a root of the equation  $f(x) = 0$  and the computation may be terminated.

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Find a real root of the equation  $f(x) = x^3 - 2x - 5 = 0$ .

- 1 Take  $a = 2$ ,  $b = 3$ . Then  $f(a) = f(2) = -1$  and  $f(b) = f(3) = 16$ .

Hence

$$x_1 = \frac{2 \times 16 - 3 \times -1}{16 \times -(-1)} = \frac{35}{17} = 2.058823529$$

Now,  $f(x_1) = -0.390799917$ .

- 2 Root is in between 2.058823529 and 3. Then

$$x_2 = \frac{2.058823529 \times 16 - 3 \times -0.390799917}{16 - (-0.390799917)} = 2.08126366$$

and  $f(x_2) = -0.147204057$ .

- 3 Root is in between 2.08126366 and 3. Then

$$x_3 = \frac{2.08126366 \times 16 - 3 \times -0.147204057}{16 - (-0.147204057)} = 2.089639211$$

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$$4 \quad x_4 = 2.092739575$$

$$5 \quad x_5 = 2.09388371$$

$$6 \quad x_6 = 2.094305452$$

$$7 \quad x_7 = 2.094460846$$

Root is 2.0945

The equation  $x^{2.2} = 69$  has a root between 5 and 8. Use the method of regula falsi to determine it.

Let  $f(x) = x^{2.2} - 69$ .  $a=5$  and  $b=8$ . Then,  $f(a) = f(5) = -34.50675846$  and  $f(b) = f(8) = 28.00586026$ .

1

$$x_1 = \frac{5 \times 28.00586026 - 8 \times -34.50675846}{28.00586026 - (-34.50675846)} = 6.655990062$$

$$f(x_1) = -4.275625415.$$

2 Root lies between 6.655990062 and 8.  $x_2 = 6.83400179$

3  $x_3 = 6.850669653$

Root is 6.85

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# Problems

Use the method of regula falsi to obtain a root, correct to three decimal places, of each of the following equations

- $x^3 + x^2 + x + 7 = 0$

- $x^3 - x - 4 = 0$

- $x^3 + x - 1 = 0.$