Newton-Raphson Method

Generally used to improve the results obtained by one of the previous methods.

Consider the equation $f(x) = 0$. Let $f'(x)$ be the derivative of $f(x)$

Newton-Raphson formula: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Algorithm:

1. Choose an initial approximate x
2. Put $\Delta x = -\dfrac{f(x)}{f'(x)}$
3. $x \leftarrow x + \Delta x$
4. Repeat 2 and 3 until $|\Delta x| < \epsilon$, where ϵ is the error tolerance.

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Find a real root of the equation $x^3 - 2x - 5 = 0$ using Newton-Raphson method.

$f(x) = x^3 - 2x - 5$ and $f'(x) = 3x^2 - 2$. Hence,

$$x_{n+1} = x_n - \frac{x_n^3 - 2x_n - 5}{3x_n^2 - 2}$$

1 Take $x_0 = 2$. Then $f(x_0) = f(2) = -1$ and $f'(x_0) = f'(2) = 10$. Hence

$$x_1 = 2 - (\frac{-1}{10}) = 2.1$$

2 $f(x_0) = f(2.1) = 0.061$ and $f'(x_0) = f'(2.1) = 11.23$. Then

$$x_2 = 2.1 - (\frac{0.061}{11.23}) = 2.094568$$

Root is 2.0945

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Use the Newton-Raphson method to obtain successive approximations of $\sqrt{2}$ as the ratio of two integers.

Equlavalent to finding the root of $x^2 - 2 = 0$. Now let $f(x) = x^2 - 2$. Then $f'(x) = 2x$ and

$$x_{n+1} = x_n - \frac{x_n^2 - 2}{2x_n} = \frac{x_n^2 + 2}{2x_n}$$

Let $x_0 = 1$

1. $$x_1 = \frac{1^2 + 2}{2 \times 1} = \frac{3}{2}$$
2. $x_2 = \dfrac{3(3/2) + 2}{2(3/2)} = \dfrac{17}{12}$
3. $x_3 = \frac{577}{408}$

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Problems

- Find a root of the equation $x \sin x + \cos x = 0$ (take initial approxitate as $x_0 = \pi$).
- Find a real root of the equation $x = e^{-x}$ using the Newton-Raphson method (take initial approxitate as $x_0 = 1$).
- A root of $x^3 - 10x^2 + 5 = 0$ lies close to $x = 7$. Compute this root with the Newton-Raphson method.