19MAT201: Numerical Methods

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Syllabus

- **Roots of Transcendental and Polynomial Equations:** Bisection method, Iteration methods based on first degree equations, Rate of convergence, system of nonlinear equations.
- **Interpolation and Approximation:** Lagrange, Newton's Divided Difference, Newton's Forward and Backward interpolations

Text books

- **Text book:**
  - *Numerical Methods in Engineering with Python*, Jaan Kiusalaas, Cambridge University Press, 2010.
- **References:**
  - R.L. Burden, J. D. Faires, *Numerical Analysis*, Richard Stratton, 2011, 9th edition.
  - S.D. Conte and Carl de Boor, *Elementary Numerical Analysis; An Algorithmic Approach*. International series in Pune and Applied Mathematics, McGraw Hill Book Co., 1980.

- Why?
- What?
- How?

Numerical analysis is the area of mathematics and computer science that creates, analyzes, and implements algorithms for solving numerically (numerical approximation) the problems of mathematical analysis.

Real world problems:

- Mathematical models
- Choose method/algorithm
- Programmig
- Execute
- Validate

Algorithm: Step by step logical instructions having a stopping criteria.

- Stability
- Convergence
- Error

Error

Round off error, Truncation error, Model errors

Absolute error $= |\text{Exact value} - \text{approximated value}|$

Relative error $= \dfrac{|\text{Exact value} - \text{approximated value}|}{|\text{exact value}|}$

Approximate error $=$
$\dfrac{|\text{current approximated value} - \text{past approximated value}|}{|\text{current approximated value}|}$

Root Finding

Polynomial equations: $x^5 + x^2 + 6x = 2$

Trancendental Equations: $x = e^x$

Root or Zero of the function $f(x)$: The value of x, for which $f(x) = 0$.

Example: $f(x) = x + 3$, then the root is $x = -3$.

Iterative method

- $f(x) = 0$
- $x_{n+1} = g(x_n), n = 0, 1, 2, \ldots$
- for $n = 0$, $x_1 = g(x_0)$

Algorithm

1. Choose two real numbers a and b such that $f(a)f(b) < 0$
2. Set $x_r = \dfrac{a+b}{2}$.
3.
   - If $f(a)f(x_r) < 0$, the root lies in the interval (a, x_r). Then, set $b = x_r$ and go to step 2.
   - If $f(a)f(x_r) > 0$, the root lies in the interval (x_r, b). Then, set $a = x_r$ and go to step 2.
   - If $f(a)f(x_r) = 0$, then x_r is a root of the equation $f(x) = 0$ and the computation may be terminated.

Stopping criteria

Last interval is small as desired, say ϵ.

At the end of n^{th} step:

- interval $[a_n, b_n]$ with length $\dfrac{|b-a|}{2^n}$
- $\dfrac{|b-a|}{2^n} \leq \epsilon$
- $n \geq \dfrac{log_e(|b-a|/\epsilon)}{log_e 2}$

Find a real root of the equation $x^3 - 2x - 5 = 0$.

n	a	b	x_r	$f(x_r)$
1	2	3	2.5	5.6250
2	2	2.5	2.25	1.8906
3	2	2.25	2.125	0.03457
4	2	2.125	2.0625	-0.3513
5	2.0625	2.125	2.09375	-0.0089
6	2.09375	2.125	2.10938	0.1668
7	2.09375	2.10938	2.10156	0.07856
8	2.09375	2.10156	2.09766	0.03471
9	2.09375	2.09766	2.09570	0.01286

n	a	b	x_r	$f(x)_r$
10	2.09375	2.09570	2.09473	0.00195
11	2.09375	2.09473	2.09424	-0.0035
12	2.09424	2.09473		

Root is 2.094.

Here $|b-a| = |3-2| = 1$. Let $\epsilon = 0.001$. Then $n \geq 10$.

- Use bisection to find the real root of the equation $f(x) = x^3 - x - 1 = 0$.
- Find a positive root of the equation $xe^x = 1$, which lies between o and 1.