**Numerical Methods for Ordinary Differential Equations (ODEs) - Study Guide**

**1. Introduction to Numerical Methods for ODEs**

Solving an ordinary differential equation (ODE) numerically means finding an **approximate** solution using stepwise calculations. These methods are particularly useful when analytical solutions are difficult or impossible to obtain.

**Types of Numerical Methods**

1. **Single-Step Methods**: Use only the most recent point to compute the next value.
   * Taylor’s Method
   * Euler’s Method
   * Modified Euler’s Method
   * Runge-Kutta Methods (RK4, RK2, etc.)
2. **Multi-Step Methods**: Use previous computed values to predict new values.
   * Adams-Bashforth Method
   * Adams-Moulton Method
   * Milne’s Method
3. **Predictor-Corrector Methods**:
   * Predictor step estimates a solution.
   * Corrector step refines it.
   * Examples: **Adams-Bashforth & Adams-Moulton**, **Milne’s Method**

**2. Overview of Each Method**

**(A) Taylor’s Method**

* Uses Taylor series expansion to approximate the solution.
* Needs higher derivatives of the function.
* Good for accuracy but computationally expensive.
* **Category**: Single-step method
* **Key Question:** "\_\_\_\_ method needs former calculations of upper derivatives." → *Taylor's Method*

**(B) Euler’s Method**

* Simplest numerical method for solving ODEs.
* Uses the formula: yn+1=yn+hf(xn,yn)y\_{n+1} = y\_n + h f(x\_n, y\_n)
* **Category**: Single-step method
* **Key Question:** "The first category method for solving ODEs is \_\_\_\_." → *Euler’s Method*

**(C) Modified Euler’s Method**

* Also called the **Improved Euler’s Method** or **Heun’s Method**.
* Uses an initial estimate and then refines it by averaging slopes.
* More accurate than Euler’s Method.
* **Category**: Single-step method

**(D) Runge-Kutta Methods (RK4)**

* One of the most widely used methods for ODEs.
* Uses multiple slope evaluations for higher accuracy.
* **Formula (RK4):** k1=hf(xn,yn)k\_1 = h f(x\_n, y\_n) k2=hf(xn+h/2,yn+k1/2)k\_2 = h f(x\_n + h/2, y\_n + k\_1/2) k3=hf(xn+h/2,yn+k2/2)k\_3 = h f(x\_n + h/2, y\_n + k\_2/2) k4=hf(xn+h,yn+k3)k\_4 = h f(x\_n + h, y\_n + k\_3) yn+1=yn+16(k1+2k2+2k3+k4)y\_{n+1} = y\_n + \frac{1}{6} (k\_1 + 2k\_2 + 2k\_3 + k\_4)
* **Category**: Single-step method
* **Key Question:** "What is the most widely used practical method for solving ODEs?" → *Runge-Kutta (RK4)*

**(E) Adams-Bashforth Method**

* Explicit multi-step method using previous function values.
* Does not require solving algebraic equations.
* **Category**: Multi-step method (Explicit Predictor)

**(F) Adams-Moulton Method**

* Implicit multi-step method that corrects the predicted values.
* Used together with Adams-Bashforth in predictor-corrector schemes.
* **Category**: Multi-step method (Implicit Corrector)
* **Key Question:** "A commonly used predictor-corrector pair is \_\_\_\_." → *Adams-Bashforth & Adams-Moulton*

**(G) Milne’s Method**

* Uses **Simpson’s rule** for integration to predict values.
* Uses **trapezoidal rule** for correction.
* **Category**: Multi-step predictor-corrector method
* **Key Question:** "Which method is based on Simpson’s rule?" → *Milne’s Method*

**3. Important Definitions & Quick Answers**

* **Numerical solutions for ODEs are called?** → *Initial Value Problems (IVP)*
* **Which method requires former calculations of upper derivatives?** → *Taylor’s Method*
* **Which method is a predictor-corrector method?** → *Adams-Moulton, Milne’s Method*
* **Which method is the most commonly used single-step method?** → *Runge-Kutta (RK4)*

**4. Summary Table of Methods**

| **Method** | **Type** | **Predictor-Corrector?** | **Key Feature** |
| --- | --- | --- | --- |
| Taylor’s Method | Single-step | No | Uses higher derivatives |
| Euler’s Method | Single-step | No | Simple, first-order |
| Modified Euler’s | Single-step | No | Improved accuracy |
| Runge-Kutta (RK4) | Single-step | No | Uses weighted slopes |
| Adams-Bashforth | Multi-step (Explicit) | Yes (Predictor) | Uses past values |
| Adams-Moulton | Multi-step (Implicit) | Yes (Corrector) | Refines predictions |
| Milne’s Method | Multi-step | Yes | Uses Simpson’s rule |

**5. Sample Questions for Practice**

1. **Which method requires previous function values to compute the next value?** (*Multi-step methods like Adams-Bashforth and Adams-Moulton*)
2. **Which method is based on Simpson’s rule?** (*Milne’s Method*)
3. **What is the most commonly used numerical method for solving ODEs?** (*Runge-Kutta (RK4)*)
4. **Which method uses a predictor-corrector approach?** (*Adams-Moulton, Milne’s Method*)
5. **Which method needs former calculations of upper derivatives?** (*Taylor’s Method*)

**6. Conclusion**

* Single-step methods are simple but require small step sizes for accuracy.
* Multi-step methods like Adams and Milne improve efficiency by reusing past values.
* Predictor-corrector methods enhance accuracy by refining estimates.
* Runge-Kutta (RK4) is widely used due to its balance between accuracy and simplicity.

This guide provides a structured overview of key numerical methods for solving ODEs, including their properties, advantages, and typical exam-style questions. Reviewing this should help with both conceptual understanding and problem-solving!