E47: Network effects in green vehicle adoption

Energy Economics of the Green Transition

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1 Network Effects in Electrical Vehicle Adoption

In this section you are asked to consider a static model of electric vehicle (EV) adoption with indirect network effects. You may interpret the static model as a representation of a long-run steady state. Specifically, let $L > 0$ denote the total number of potential EV users in the economy and assume that any given consumer can at most hold one EV. The total stock of EVs is denoted by $Q \in [0, L]$ and the sales price of the vehicle is $p > 0$. Finally, let $N \geq 0$ denote the number of charging stations and let $c > 0$ denote the fixed costs of investing in and installing a charging station. The two equations that make up the model are

$$L - Q = N^{-\beta_1} p^{\beta_2}, \quad \beta_1, \beta_2 > 0. \tag{1}$$

$$N = Q^{\gamma_1} c^{-\gamma_2}, \quad \gamma_1, \gamma_2 > 0, \tag{2}$$

where $L - Q$ is the number of consumers not adopting an EV. Note that Q and N are the only endogenous variables.

Q1. Provide an economic interpretation of the sign of the parameters β_1, β_2, γ_1, and γ_2. Furthermore, what does γ_1 and γ_2 measure, specifically?

Q2. Show that there are always diminishing returns to Q with respect to N for any $N > 0$.

Hint: You need to show that

$$\frac{\partial^2 Q}{\partial N^2} < 0. \tag{3}$$

Q3.

In the rest of this section, you will make the two following assumptions: First, when consumers make the adoption decision they cannot perfectly observe the size of the charging network. We capture this by introducing the expectation operator $\mathbb{E}[\cdot]$ in (1) as follows:

$$L - Q = \mathbb{E}[N]^{-\beta_1} p^{\beta_2}. \tag{4}$$

In other words, when consumers make the adoption decision, their expectations for the charging network are wrong whenever $\mathbb{E}[N] \neq N$, where N is determined by (2).

The second assumption is similar to the first as firms do not perfectly observe the number of EV users when they decide to invest in new charging stations. Hence, we replace (2) by

$$N = \mathbb{E}[Q]^{\gamma_1} c^{-\gamma_2} \tag{5}$$

Show that when $\mathbb{E}[N] \leq \underline{N}$, no consumer adopts an EV where

$$\underline{N} \equiv \left(\frac{p^{\beta_2}}{L}\right)^{\frac{1}{\beta_1}}. \tag{6}$$

Hint: You need to find for which range of $\mathbb{E}[N]$ that $Q^* = 0$ where superscript '$*$' indicates an equilibrium.

Q4. Provide an intuition for the result in Q3.

Q5. Show that firms choose $N \leq \underline{N}$ when $\mathbb{E}[Q] \leq \underline{Q}$, where

$$\underline{Q} \equiv \left(\left(\frac{p^{\beta_2}}{L} \right)^{\frac{1}{\beta_1}} c^{\gamma_2} \right)^{\frac{1}{\gamma_1}} \tag{7}$$

Q6.

Having established there is a potential degenerate equilibrium where $Q^* = N^* = 0$, we will now study any potential interior equilibria (i.e. where $Q^* > 0$) by assuming that expectations are correct, i.e. $\mathbb{E}[N] = N$ and $\mathbb{E}[Q] = Q$.

Show that any interior equilibria are determined by

$$(L - Q^*)(Q^*)^{\beta_1 \gamma_1} = c^{\beta_1 \gamma_2} p^{\beta_2} \tag{8}$$

Q7.

Denote $F(Q)$ as the excess EV demand function defined by

$$F(Q) = c^{\beta_1 \gamma_2} p^{\beta_2} - (L - Q) Q^{\beta_1 \gamma_1} \tag{9}$$

In equilibrium, Q will adjust to ensure that $F(Q^*)$=0.

Show that there is only one solution to

$$\frac{\partial F(Q)}{\partial Q} = \mathbf{0} \tag{10}$$

defined by

$$\hat{Q} \equiv \frac{\beta_1 \gamma_1}{1 + \beta_1 \gamma_1} L. \tag{11}$$

Finally, show that $\hat{Q}$ is a global minimum of $F(Q)$.

Q8. Show that for any interior equilibrium to exist, it must be the case that

$$c^{\beta_1\gamma_2} p^{\beta_2} \leq \left(\frac{L}{1+\beta_1\gamma_1}\right)^{1+\beta_1\gamma_1} (\beta_1\gamma_1)^{\beta_1\gamma_1}. \tag{12}$$

Q9.

Assume in the following that the condition in (12) holds with strict inequality, such that there are two potential equilibria.

Draw $F(Q)$ in a diagram with the value of $F(Q)$ on the Y-axis and Q on the X-axis. Show which one of the two equilibria is unstable, and which one is locally stable by denoting the unstable by $\underline{Q}^*$ and the locally stable equilibrium by $\overline{Q}^*$.

Hint: For excess demand functions, equilibria crossing the X-axis from above are unstable, and equilibria crossing the X-axis from below are stable.

Q10. To the graph in Q9, add whether $\underline{Q}$ is located to the left or to the right of $\underline{Q}^*$ and argue why.

Q11. Based on your graphs in Q9 and Q10, specify the range of Q for which $Q^* = 0$ is locally stable (i.e. the only equilibrium) and the range for which $\overline{Q}^* > 0$ is locally stable.

Q12. Show that in an interior equilibrium the own-price elasticity of Q^* is

$$\epsilon_{Q,p}(Q^*) \equiv \frac{dQ}{dp}(Q^*)\frac{p}{Q^*} = -\beta_2 \frac{L-Q^*}{Q^* - \gamma_1\beta_1(L-Q^*)}. \tag{13}$$

Hint: Note the "hard d" in $\frac{dQ}{dP}(Q^*)$; $\epsilon_{Q,p}(Q^*)$ therefore takes equilibrium effects into account. Furthermore, you should use the Implicit Function Theorem to derive (13), which *states that:*

$$\frac{dQ}{dp}(Q^*) = -\frac{\left(\frac{\partial F(Q^*)}{\partial p}\right)}{\left(\frac{\partial F(Q^*)}{\partial Q}\right)} \tag{14}$$

Q213. Show that $\epsilon_{Q,p}(\underline{Q}^*) > 0$ and $\epsilon_{Q,p}(\overline{Q}^*) < 0$.

Q14. Comment on how the presence of indirect network effects (i.e. $\beta_1\gamma_1 > 0$) affects $\epsilon_{Q,p}(Q^*)$ at the two interior equilibria. Provide intuition for your results.

2 Suggested answers

Q2.1.

The signs of β_1 and γ_1 capture the direction of the indirect network effects on the EV adoption decision and the charging stations investment decision, respectively. Given they are both positive, the model features a positive indirect network effect on both sides of the market. $\beta_2 < 0$ implies that EV demand is falling in the sales price, and $\gamma_2 > 0$ implies that the size of the charging network is falling in the fixed costs. Finally, γ_1 and γ_2 measure the elasticity of N with respect to Q and c, respectively.

Q2.

First, we isolate for Q in (1):

$$Q = L - N^{-\beta_1} p^{\beta_2}. \tag{15}$$

Using the expression for Q in (15), we then have

$$\frac{\partial Q}{\partial N} = \beta_1 N^{-\beta_1 - 1} p^{\beta_2}. \tag{16}$$

It is fine to not show (16) as this is related to the answer to Q1. Differentiating (16) once more, we now have that

$$\frac{\partial^2 Q}{\partial N^2} = -(\beta_1 + 1)\beta_1 N^{-\beta_1 - 2} p^{\beta_2} < 0,$$

for any $\beta_1 > 0$ and $N > 0$.

Q3.

From (1), we see that $Q^* = 0$ for

$$\begin{aligned} L &\leq \mathbb{E}[N]^{-\beta_1} p^{\beta_2} \Rightarrow \\ \mathbb{E}[N] &\leq \left(\frac{p^{\beta_2}}{L}\right)^{\frac{1}{\beta_1}}, \end{aligned} \tag{17}$$

where the right-hand side of (17) is the definition of $\underline{N}$ provided in (6).

Q4.

If consumers expect the charging network is below $\underline{N}$, then the inconvenience costs of charging the vehicle is too high. Consequently, no one adopts an EV. While not necessary for a correct answer, the very good answer explains that the underlying assumption must be, that the marginal willingness to pay for an EV is always finite in this framework. Even when the charging network is small – and the indirect network effect therefore is high – there is a limit as to how much consumers are willing to pay. $\underline{N}$ is therefore falling in p.

Q5.

From (2) we see that $N \leq \underline{N}$ for

$$\begin{aligned} \mathbb{E}[Q]^{\gamma_1} c^{-\gamma_2} &\leq \underline{N} \Rightarrow \\ \mathbb{E}[Q] &\leq (\underline{N} c^{\gamma_2})^{\frac{1}{\gamma_1}} \end{aligned}$$

By inserting the expression for $\underline{N}$ in (6), we get the condition in (7).

Q6.

Assuming $\mathbb{E}[N] = N$ and $\mathbb{E}[Q] = Q$, inserting (2) into (1) yields

$$(L - Q^*) = (Q^*)^{-\beta_1\gamma_1} c^{\beta_1\gamma_2} p^{\beta_2} \tag{18}$$

It is possible to work with (18), but given we know that we are only searching for any $Q^* > 0$, we can impose this assumption directly in (18) by multiplying with $Q^{\beta_1\gamma_1}$:

$$(L - Q^*)(Q^*)^{\beta_1\gamma_1} = c^{\beta_1\gamma_2} p^{\beta_2},$$

which is identical to (8).

Q7.

Using (9), and given that $Q > 0$, we have that

$$\begin{aligned}
\frac{\partial F(Q)}{\partial Q} = Q^{\beta_1\gamma_1} - (L - Q)\beta_1\gamma_1 Q^{\beta_1\gamma_1 - 1} \leq 0 \Leftrightarrow \\
1 - (L - Q)Q^{-1}\beta_1\gamma_1 \leq 0 \Leftrightarrow \\
1 + \beta_1\gamma_1 \leq \beta_1\gamma_1 \frac{L}{Q} \Leftrightarrow \\
(1 + \beta_1\gamma_1)Q \leq \beta_1\gamma_1 L \Leftrightarrow \\
Q \leq \frac{\beta_1\gamma_1}{1 + \beta_1\gamma_1} L
\end{aligned} \tag{19}$$

Hence, $\partial F(Q)/\partial Q = 0$, when $Q = \beta_1\gamma_1 L/(1 + \beta_1\gamma_1)$, which is the definition of $\hat{Q}$ in (11). Furthermore, from (19) we see that $F(Q)$ is decreasing for $Q < \hat{Q}$ and increasing for $Q > \hat{Q}$. Therefore, $\hat{Q}$ is a global minimum of $F(Q)$.

Q8.

First, note from (9) that $F(0) = c^{\beta_1\gamma_2} p^{\beta_2} > 0$. Because $F(Q)$ has a global minimum in Q, the existence of any interior equilibrium is ensured by $F(\hat{Q}) \leq 0$. When $F(\hat{Q}) = 0$ there is exactly one interior equilibrium and for $F(\hat{Q}) < 0$ there are two interior equilibria. We see that

$$\begin{aligned}
F(\hat{Q}) = c^{\beta_1\gamma_2} p^{\beta_2} - (L - \frac{\beta_1\gamma_1}{1 + \beta_1\gamma_1} L) \left(\frac{\beta_1\gamma_1}{1 + \beta_1\gamma_1} L \right)^{\beta_1\gamma_1} \leq 0 \Leftrightarrow \\
c^{\beta_1\gamma_2} p^{\beta_2} \leq \left(\frac{1}{1 + \beta_1\gamma_1} \right) L^{1 + \beta_1\gamma_1} \left(\frac{\beta_1\gamma_1}{1 + \beta_1\gamma_1} \right)^{\beta_1\gamma_1} \Leftrightarrow \\
c^{\beta_1\gamma_2} p^{\beta_2} \leq \left(\frac{L}{1 + \beta_1\gamma_1} \right)^{1 + \beta_1\gamma_1} (\beta_1\gamma_1)^{\beta_1\gamma_1},
\end{aligned}$$

which is identical to the condition in (12). The intuition for this condition is that the

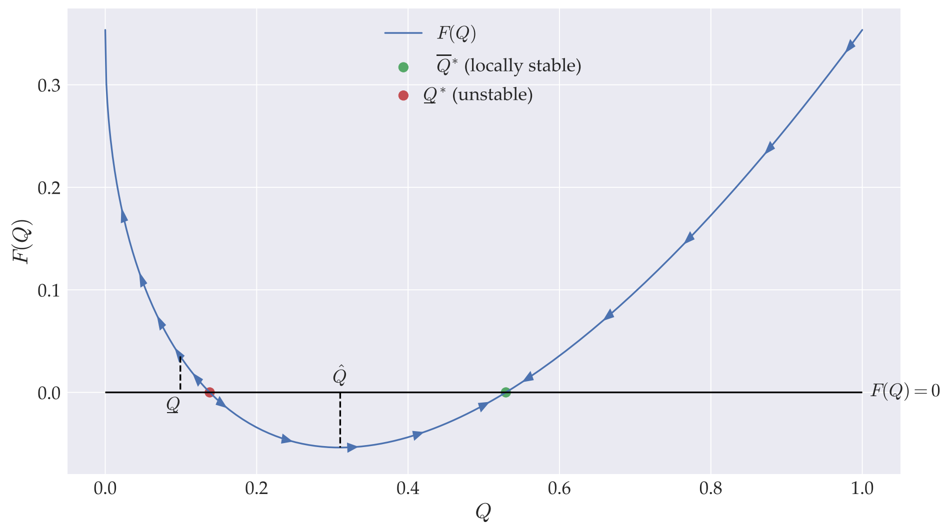

Figure 1: Excess EV demand function

Notes: Arrows indicate the stability of the interior equilibria.

negative effect of the investment costs in charging stations and EVs cannot be too large. Otherwise, no interior equilibrium exist.

Q9 and Q10.

The function should look something like Figure 1 but it does not need to be a convex function, as this would depend on parameter values. Regarding Q10, we see that because $\underline{Q}^*$ is an interior equilibrium and $\underline{Q}$ is not an equilibrium but leads directly to $Q^* = 0$, then $\underline{Q} < \underline{Q}^*$.

Q211.

$Q^* = 0$ is the only equilibrium for $Q \in [0, \underline{Q}^*[$ and $\overline{Q}^*$ is the only equilibrium for $Q \in]\underline{Q}^*, L]$. This is also depicted in Figure 1.

Q12.

First, note that

$$\frac{\partial F(Q)}{\partial p} = \beta_2 c^{\beta_1 \gamma_1} p^{\beta_2 - 1}$$

Second, we have derived in Q7 that

$$\frac{\partial F(Q)}{\partial Q} = Q^{\beta_1\gamma_1} - \beta_1\gamma_1(L-Q)Q^{\beta_1\gamma_1-1}$$

Using the Implicit Function Theorem in (14), we therefore have that

$$\frac{dQ}{dp}(Q^*) = -\frac{\beta_2 c^{\beta_1\gamma_1} p^{\beta_2-1}}{(Q^*)^{\beta_1\gamma_1} - \beta_1\gamma_1(L-Q^*)(Q^*)^{\beta_1\gamma_1-1}} \tag{20}$$

By multiplying the expression in (20) by p/Q^*, we get the own-price elasticity of Q at Q^*:

$$\epsilon_{Q,p}(Q^*) = -\beta_2 \frac{c^{\beta_1\gamma_1} p^{\beta_2}}{(Q^*)^{1+\beta_1\gamma_1} - \beta_1\gamma_1(L-Q^*)(Q^*)^{\beta_1\gamma_1}} \tag{21}$$

To arrive at (13), we note from (8), that the numerator in (21) is equal to $\beta_2(L - Q^*)(Q^*)^{\beta_1\gamma_1}$ and inserting into (21) implies

$$\begin{aligned}\epsilon_{Q,p}(Q^*) &= -\frac{\beta_2(L-Q^*)(Q^*)^{\beta_1\gamma_1}}{(Q^*)^{1+\beta_1\gamma_1} - \beta_1\gamma_1(L-Q^*)(Q^*)^{\beta_1\gamma_1}} \\ &= -\beta_2 \frac{(L-Q^*)}{Q^* - \beta_1\gamma_1(L-Q^*)},\end{aligned} \tag{22}$$

which is identical to the expression in (13).

Q213.

We first see that $\epsilon_{Q,p}(Q^*) < 0$ for

$$-\beta_2 \frac{(L-Q^*)}{Q^* - \beta_1\gamma_1(L-Q^*)} < 0.$$

Because $Q^* < L$ and $\beta_2 > 0$, this condition simplifies to

$$\begin{aligned}Q^* - \beta_1\gamma_1(L-Q^*) > 0 &\Leftrightarrow \\ Q^*(1+\beta_1\gamma_1) > \beta_1\gamma_1 &\Leftrightarrow \\ Q^* > \frac{\beta_1\gamma_1}{1+\beta_1\gamma_1} L&\end{aligned} \tag{23}$$

Note that the right-hand side of (23) is the definition of $\hat{Q}$. And from the graph in

Q9 and Q10, we know that $\underline{Q}^*$ is located to the left of $\hat{Q}$ and $\overline{Q}^*$ is located to the right of $\hat{Q}$. It therefore follows from the condition in (23), that $\epsilon_{Q,p}(\overline{Q}^*) < 0$ and $\epsilon_{Q,p}(\underline{Q}^*) > 0$.

Q2.14

From the expression of the own-price elasticity of Q in (13), it follows that a larger value of the product $\beta_1\gamma_1$ makes the own-price elasticity larger in absolute terms. The intuition for the price elasticity can be decomposed into two effects: First, the direct/partial effect of a percentage increase in the sales price of the EV is a percentage change in demand of the magnitude

$$-\beta_2 \frac{L - Q^*}{Q^*}.$$

But with indirect network effects ($\beta_1\gamma_1 > 0$) there is an additional equilibrium effect. When the individual consumer changes demand, the size of the the charging network changes as well. In the case of e.g. $\overline{Q}^*$, the direct/partial effect of a higher price causes demand to decrease, which lowers the size of the charging network. In turn, this lowers aggregate EV demand even further.

In other words, in equilibrium, the impact of an increase in the sales prices is amplified due to positive indirect network effects, and the magnitude of the amplification depends on the strength of the network effects ($\beta_1\gamma_1$).