Greedy double block extended Kaczmarz method for solving inconsistent tensor linear systems under t-product.

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Abstract

The randomized Kaczmarz method is a widely adopted iterative method for solving linear systems of equations. To solve large-scale inconsistent linear systems of tensor equations under the t-product, we propose a tensor greedy double block extended Kaczmarz (TGDBEK) method. We prove theoretically the convergence guarantees and showed that the proposed method converges linearly to the minimum-norm least-square solution of the tensor system. Moreover, we assess the performance of the proposed method through several numerical experiments. Compared to existing methods, the TGDBEK does not require predefined partitions of the tensor system and reduces the running time for solving large inconsistent system and requires less iteration.

Keywords. Tensor equations, randomized extended Kaczmarz, tensor greedy randomized Kaczmarz, t-product.

1 Introduction

In this work, we focus on solving the tensor inconsistent linear system of the form

$$\mathcal{A} * \mathcal{X} = \mathcal{B},\ \mathcal{B} = \overline{\mathcal{B}} + \epsilon \tag{1}$$

where $\mathcal{A} \in \mathbb{R}^{N_1 \times N_2 \times N_3}$, $\mathcal{B} \in \mathbb{R}^{N_1 \times K \times N_3}$, $\mathcal{X} \in \mathbb{R}^{N_2 \times K \times N_3}$, are third-order tensors, ϵ the contamination, and $*$ the t-product proposed by Kilmer and collaborators [1, 2]. The t-product provides a matrix-like algebra for third-order tensors, including analogues of inverse, transpose, orthogonal projections, and pseudo-inverse, etc. [1, 2]. One can see [3, 5, 4] for applications of the t-product in various topics like Singular Value Decomposition, Image recognition, neural network [?]. The problem eq.(1) arises in many applications such color image recovery, MRI images deblurring with multiple channels or multiple slices, where the tensor $\mathcal{X}$ is the unknown clean tensor image to be found, $\mathcal{A}$, is a blur operator tensor, and $\mathcal{B}$ is the observed blurred and noisy data. When the noise is relatively higher, it becomes difficult to solve this problem through direct methods. Moreover, this setting makes typically an inconsistent system where the problem is a least-square problem and a natural targeted solution is the minimum-norm solution $\mathcal{A}^\dagger * \mathcal{X}$.

In the matrix framework where $\mathcal{A}$ is a matrix and $\mathcal{X}$ and $\mathcal{B}$ are vectors, iterative methods like the randomized Kaczmarz (RK) method can be used to solve the linear system eq. (1). The RK method selects rows at each iteration with probability proportional to their Euclidean norm square. It was proved that this row selection strategy makes the RK to converge linearly in expectation for consistent and over determined linear systems [28]. To solve inconsistent linear systems, Zouzias and Freris [29] enhanced the randomized Kaczmarz by introducing an additional iterative dual variable and proposed the randomized extended Kaczmarz (REK) which converges linearly in expectation to the minimum-norm least square solution $A^\dagger x$, where $A^\dagger$ is the Moore-Penrose pseudoinverse of the system matrix A. It is worth mentioning that the RK

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