## **BDA Project: Hurricane forecasting in Stan**

José Miguel Ramírez & Jonas Lindblad Aalto University

Hurricane introduction	Intensity change predictive model	Forecasting: Model Comparison	Concluding section

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- They can cause extreme levels of flooding and destroy many buildings.
- Monetary damages and loss of lives increase with an almost exponential character as a function of storm intensity.



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- This project: a statistical model for intensity

The US government forecasting agency, the National Hurricane Center (NHC), uses a large number of models operationally. The models (together: the *model ensemble*) are used together with experienced meteorologists' judgment to provide the official forecast.

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- ... but the documentation is terrible
- SHIPS: only a point estimate; our project: a predictive distribution

The SHIPS developmental data is confusing!

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We are making synoptic models.

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		67	76	69	63	70	83	67	60	65	62	60	51	41	32	22	18	16	9999	9999	9999	9999	TWAC	
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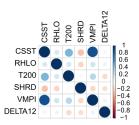
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$$T = 0$$

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			548	31	23	24	13	28	16	40	23	64	82	77	89	92	85	208	200	79	9999	9999	9999	PSLV	
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			100	104	106	110	110	138	114	102	109	101	98	84	73	62	42	32	34	9999	9999	9999	9999	TWXC	

We have not done statistical variable selection. Choice of variable subset is based on theory.

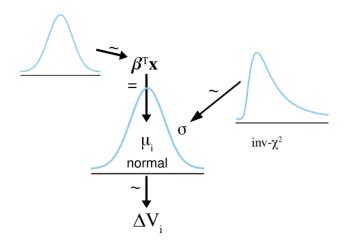
- CSST: (climatological) sea surface temperature
- RHLO: low-altitude relative humidity
- **T200**: air temperature at 200 mb (very high altitude)
- SHRD: wind shear between 850 and 200 mb
- VMPI: maximum potential intensity



- for testing, we have variable sets A, B, C
- A: LAT/LON, VMAX, CSST, SHRD
- B: LAT/LON, VMAX, CSST, SHRD, VMPI
- C: LAT/LON, VMAX, CSST, SHRD, VMPI, RHLO, T200

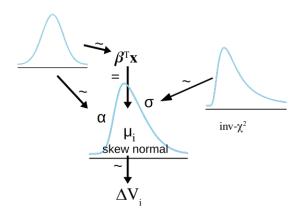
# Intensity change predictive model

The SHIPS Blunder: a simple linear regression



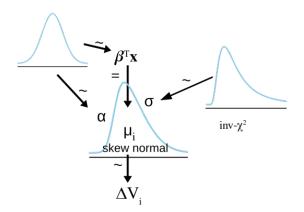
#### Model 2: regression with skewness

• errors not symmetric around the mean prediction!



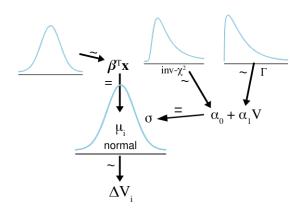
#### Model 2: regression with skewness

- errors not symmetric around the mean prediction!
- rapid intensification!



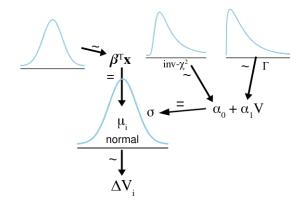
Model 3: regression with a linear model for standard deviation

fewer storms reach higher values of VMAX



Model 3: regression with a linear model for standard deviation

- fewer storms reach higher values of VMAX
- allow for higher variance to account for larger historical uncertainty



#### • SHIPS: predict $V_{max}$ ; our models: predict $\Delta V_{max}$

#### Models: some remarks

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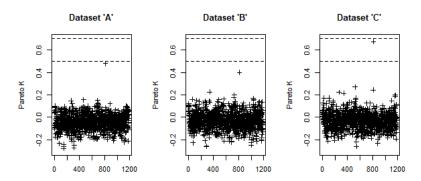
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- poor problem setup! True model is the laws of physics, but we are fitting a regression
- models were programmed in Stan; sampling with rstan using default parameters; resulted in no divergences or issues except for the skew model and it was solved by increasing max tree depth to 15

# Forecasting: Model Comparison

### **Forecasting: Model Comparison**

Dataset comparison for the linear regression model (LOOCV)

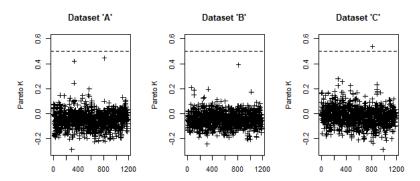
Dataset	elpd_diff	se_diff
С	0.0	0.0
В	-25.0	6.5
Α	-27.4	6.3



# Forecasting: Model Comparison (2)

Dataset comparison for the skewed regression model (LOOCV)

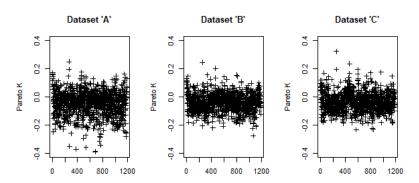
Dataset	elpd_diff	se_diff
С	0.0	0.0
В	-23.2	6.2
Α	-28.7	6.2



# Forecasting: Model Comparison (3)

Dataset comparison for the **Changing variance model** (LOOCV)

Dataset	elpd_diff	se_diff
С	0.0	0.0
В	-32.6	8.2
Α	-37.1	8.2

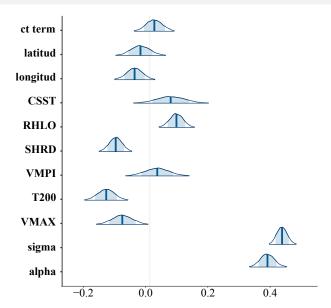


# Forecasting: Model Comparison (4)

Model comparison using the Dataset C (LOOCV)

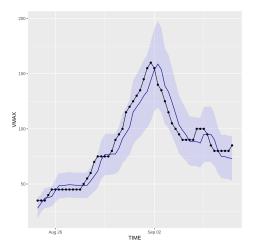
Model	elpd_diff	se_diff
Variance	0.0	0.0
Skew	-176.3	27.8
Linear	-205.5	34.9

# Marginal posteriors



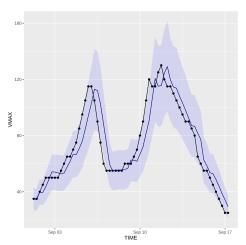
### Forecasting: checking predictions

Hurricane Dorian 2019. The image shows a 90% credible interval. Black dotted line: true VMAX



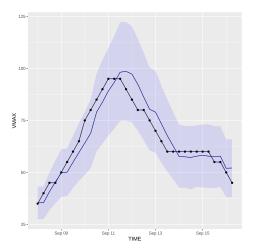
## Forecasting: checking predictions

Hurricane Florence 2018. The image shows a 90% credible interval. Black dotted line: true VMAX



### Forecasting: checking predictions

Hurricane Helene 2018. The image shows a 90% credible interval. Black dotted line: true VMAX



#### Further development ideas:

variable selection in full SHIPS dataset

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- variable selection in full SHIPS dataset
- more time series autoregressive components

#### Further development ideas:

- variable selection in full SHIPS dataset
- more time series autoregressive components
- use LGEM model (will explain)

### A final takeaway

Talking about the 2017 category 5 hurricane Irma:

"The hurricane will be moving through an environment of low vertical wind shear, a moist mid-level atmosphere, and increasing upper-ocean heat content, and this is ideal for some additional intensification. However, given that the SHIPS models do not show any significant change in the intensity, the NHC forecast keeps Irma a powerful hurricane through five days."

— NHC 5AM advisory Sep. 5th

While the NHC published the advisory, Irma was undergoing rapid intensification from 130 to 175 knots over a span of only a few hours.

### **Conclusions & contact info**

- Takeaway: the SHIPS model is terrible!
- Simple changes to the predictive distribution can improve the model
- Further development using Bayesian methods seems promising and there are several possible directions

More at our Github repo:

https://github.com/jnlb/bda-hurricane-modeling

The SHIPS website: http://rammb.cira.colostate.edu/research/tropical\_cyclones/ships/index.asp

Contact info:

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The SHIPS model:

$$y_i \sim \mathcal{N}(\alpha + X_i \cdot \beta_{N-1}, \sigma), i = 1, \dots, r,$$

where we let  $X_i$  denote the i:th row of the data,  $\beta_{N-1}$  is an N-1-dimensional parameter vector, and r is the number of observations (rows) in the data. Its priors were

$$\begin{vmatrix} \alpha_0 \\ \beta_{N-1,0} \end{vmatrix} \sim \mathcal{N}(\mathbf{0}_N, 10 \cdot \mathbf{I}_N), \sigma_0 \sim \text{Inv-}\chi^2(\frac{1}{10}).$$

The skew-normal regression model:

$$y_i \sim \text{SkewNormal}(\alpha + X_i \cdot \beta_{N-1}, \sigma, \psi), \quad i = 1, \dots, r,$$

with priors

$$\begin{bmatrix} \alpha_0 \\ \beta_{N-1,0} \end{bmatrix} \sim \mathcal{N}(\mathbf{0}_N, 10 \cdot \mathbf{I}_N), \sigma_0 \sim \text{Inv-}\chi^2(\frac{1}{10}), \ \psi_0 \sim \mathcal{N}(0, 1).$$

The variance model:

$$y_i \sim \mathcal{N}(\alpha + X_i \cdot \beta_{N-1}, \sigma + \gamma | V_{max,i} |), i = 1, \dots, r,$$

where, again, we use the same notation as before and let  $V_{max,i}$  denote the  $V_{max}$ -value of the i:th row. The priors that were fed into Stan were

$$\begin{bmatrix} \alpha_0 \\ \beta_{N-1,0} \end{bmatrix} \sim \mathcal{N}(\mathbf{0}_N, 10 \cdot \mathbf{I}_N), \ \sigma_0 \sim \text{Inv-}\chi^2(\frac{1}{10}), \ \gamma_0 \sim \Gamma(1,1).$$

iter=4000, seed = SEED)

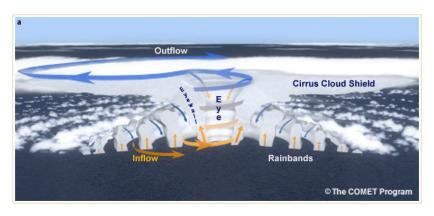


Image from Introduction to Tropical Meteorology, 2nd Ed., 2011, by A. Laing & J-L Evans.

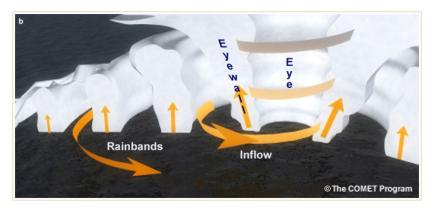


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