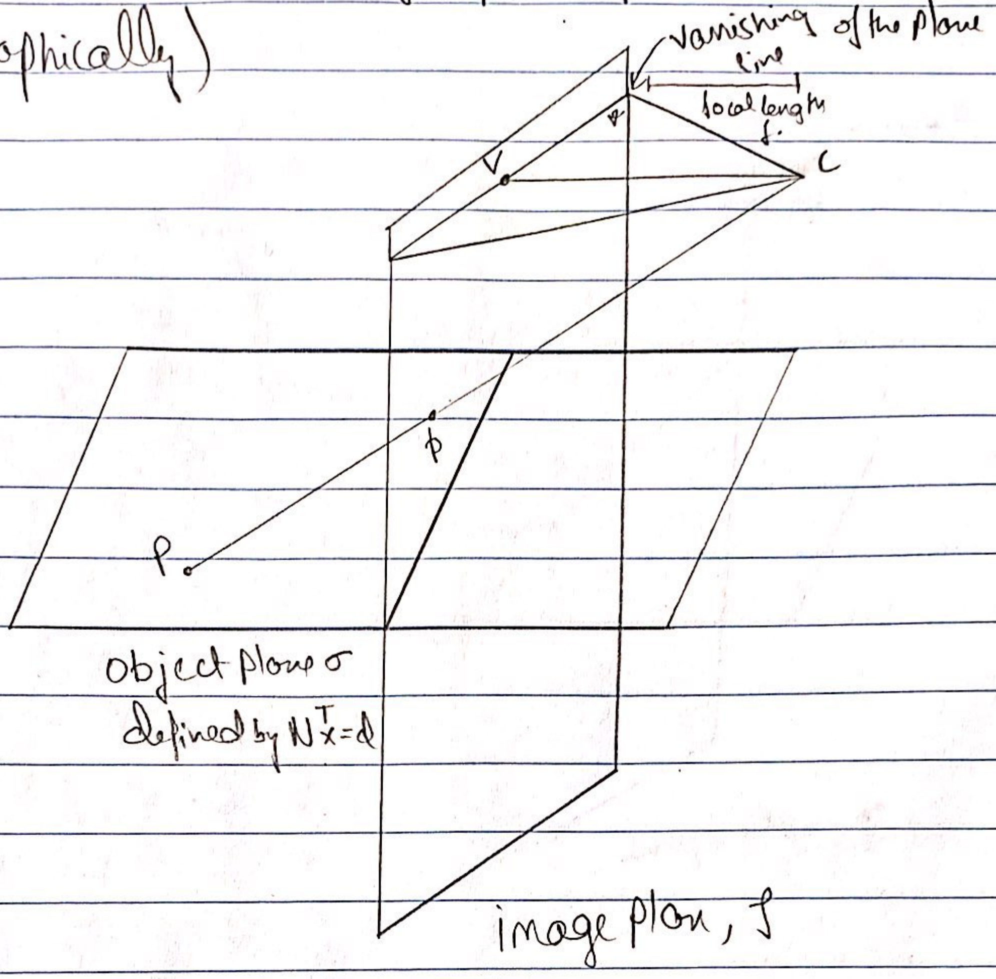
This Question is proved by 2 methods
Graphically & Analytically.

Q1. (Graphically)



Lets say we have a plane σ (object plane) defined by $N^TX = d$. going under propective projection which is perpendicular to image plane $\mathcal{J}$.

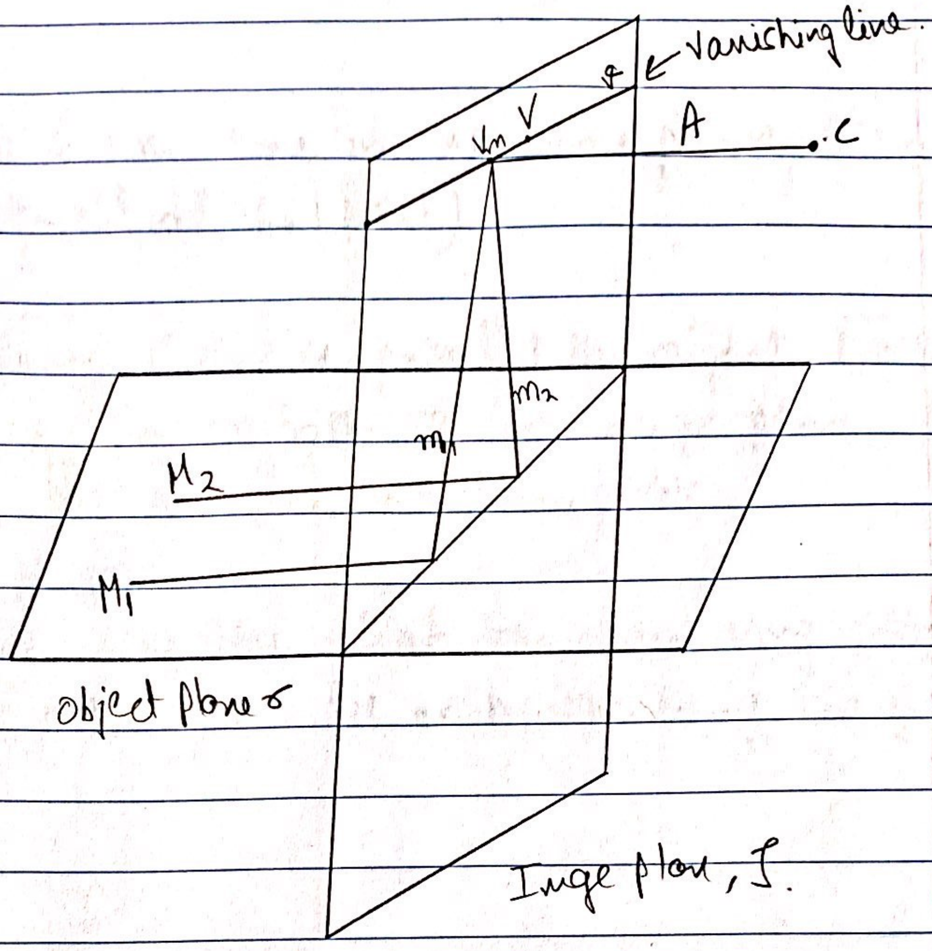
To each point P on σ we have point p on $\mathcal{J}$ corresponding to the intersection of the line CP with the plane $\mathcal{J}$.

Scanned with CamScanner

C is called centre of projection & CV is the focal length f.

→ The point of intersection of the line through C, that is ⊥ to image plane (and so parallel to the object planes) is called principal vanishing point V.

→ The line vr, which is intersection of the picture plane & through P ∥ to the object plane is called horizontal line.



Let M1 & M_2 be lines on object plane.

- The image of M_i is the line m_i where the plane π, containing C & M_i intersects the image plane.

There is line CV_m which is // to M_i & so this line is also on π.

Since this line is Horizontal, it also lies in plane determined by C and v′, the vanishing line.

And V line & CVm line in the same plane, then they must intersect at Vm (point)

This plane of CVm is parallel to object plane.
This places Vm on both π & image plane, so it must lie on the intersection. mi

Hence we can say that the vanishing points of lines on a plane lie on the vanishing line of the plane

Q1. Analytically

For the plane

$$N^T X = d$$

$$\Rightarrow \begin{bmatrix} N_x \\ N_y \\ N_z \end{bmatrix} \begin{bmatrix} x & y & z \end{bmatrix} = d.$$

we have $x = f\frac{x}{z}$, $y = f\left(\frac{y}{z}\right)$, $z = f$.

$$\begin{bmatrix} N_x \\ N_y \\ N_z \end{bmatrix} \begin{bmatrix} f\frac{x}{z} & f\frac{y}{z} & f \end{bmatrix} = f\frac{d}{z}$$

for vanishing plane $z \to \infty$.

$$\therefore N_x + N_y + N_z f = 0 \quad \text{——— } (1)$$

for Line

$$L_1(\lambda) = A + \lambda D$$

$$= \left[A_x + \lambda D_x,\ A_y + \lambda D_y,\ A_z + \lambda D_z\right]$$

$$= \left(f \frac{A_x + \lambda D_x}{A_z + \lambda D_z} , f \frac{A_y + \lambda D_y}{A_z + \lambda D_z} \right)$$

for Vanishing line $\lambda \Rightarrow \infty$

$$= \left(f \frac{D_x}{D_z} , f \frac{D_y}{D_z} \right)$$

$$\Rightarrow N_x D_x + N_y D_y + N_z D_z = 0 . \quad — 2$$

Looking at 1 & 2 we can say that.
Vanishing points of lines lie on vanishing line of plane.