Homework 0: Basic Probability Review Problems **Name:**
ECE/CS 498 DS Spring 2020 **NetID:**
Issued: 01/27/2020
Due: 02/03/2020 (11:59 PM)
Please submit your homework on Compass 2G.

Please submit Problem 1, 4, 5, 7, 9, 10, 12 for grading. The rest are for your practice.

Problem 1 (Basic Concepts)

(a) **(5 points)** Write down the Probability Axioms.
Solution:

I For any event A, $P(A) \geq 0$.

II $P(\Omega) = 1$. Ω is the sample space.

III If $\{A_i, i \geq 1\}$ are disjoint events, then $P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$.

(b) **(5 points)** Explain the differences between a probability mass function (pmf) at a point and a probability density function (pdf) at a point.
Solution: The term pmf and pdf refer, respectively, to the continuous and discrete cases. In the discrete case, a pmf gives us "point probabilities". $P(X = x)$ measures the probability the random variable X equals x. We can sum over values of the pmf to get the cdf. In the continuous case, the analogous procedure is to substitute integrals for sums. The probability of a point in the pdf is 0.

(c) **(5 points)** If A and B are independent events with $P(A) = 0.8$, and $P(B) = 0.5$, find $P(A \bigcup B)$.
Solution: $P(A \bigcup B) = P(A) + P(B) - P(A \bigcap B)$.
Given A and B are independent events, $P(A \bigcap B) = P(A)P(B) = 0.8 \times 0.5 = 0.4$.
$P(A \bigcup B) = 0.8 + 0.5 - 0.4 = 0.9$.

(d) **(5 points)** Prove $P(A, B|C) = P(A|B, C) \times P(B|C)$.
Hint: Start from $P(A, B|C) = \frac{P(A,B,C)}{P(C)}$.
Solution:

$$\begin{aligned} P(A, B|C) &= \frac{P(A, B, C)}{P(C)} \\ &= \frac{P(A|B, C)P(B, C)}{P(C)} \\ &= P(A|B, C) \times P(B|C) \end{aligned}$$

Problem 2 (Counting)

(a) Find the number of solutions of $x + y + z = 15$ where x, y, z are all positive integers.
Solution: This is equivalent to the number of different ways you can put 2 bars in

between 15 stars to separate them into 3 groups, which will be $\binom{14}{2} = 91$.
(`https://en.wikipedia.org/wiki/Stars_and_bars_%28combinatorics%29`)

(b) Find the number of solutions of $x+y+z < 15$ where x, y, z are all positive integers.
Solution: This is equivalent to the number of solutions of $x+y+z+w = 15$ where x, y, z, w are all positive integers, which will be $\binom{14}{3} = 364$.

(c) Find the number of solutions of $x + y + z = 15$ where x, y, z are all nonnegative integers.
Solution: This is equivalent to the number of different ways you can put 2 bars in between $15 + 3 = 18$ stars to separate them into 3 groups, which will be $\binom{17}{2} = 136$.

Problem 3 (Independence)

There are 9 identical balls in an urn. 2 balls are marked "none", 2 balls are marked "1", 2 balls are marked "2", 2 balls are marked "3", and 1 ball is marked "123". "none" means no number is marked on that ball. Suppose a ball is taken from the urn at random, event $A_i = \{\text{"i" is on the ball}\}$. For example, A_1 occurs when ball "1" is picked or when ball "123" is picked. We can find $P(A_i) = \frac{1}{3}$.

(a) What is the difference between pairwise independence and mutual independence? Illustrate your answer with respect to three random variables X, Y, and Z.
Solution: `https://en.wikipedia.org/wiki/Independence_(probability_theory)#Mutual_independence`.
A finite set of events $\{E_i\}_{i=1}^n$ is pairwise independent if every pair of events is independent. That is, if and only if for all distinct pairs of indices m, k, $\mathrm{P}(E_m \cap E_k) = \mathrm{P}(E_m)\mathrm{P}(E_k)$.
A finite set of events $\{E_i\}_{i=1}^n$ is mutually independent if every event is independent of any intersection of the other events. That is, if and only if for every $k \le n$ and for every k-element subset of events $\{B_i\}_{i=1}^k$ of $\{E_i\}_{i=1}^n$, $\mathrm{P}\left(\bigcap_{i=1}^k B_i\right) = \prod_{i=1}^k \mathrm{P}(B_i)$.
Pairwise independence: $P(X,Y) = P(X)P(Y)$, $P(X,Z) = P(X)P(Z)$, $P(Y,Z) = P(Y)P(Z)$.
Mutual independence: $P(X,Y) = P(X)P(Y)$, $P(X,Z) = P(X)P(Z)$, $P(Y,Z) = P(Y)P(Z)$, $P(X,Y,Z) = P(X)P(Y)P(Z)$.

(b) Are A_1, A_2, A_3 pairwise independent? (Show your calculation.)
Solution: $P(A_1) = P(A_2) = P(A_3) = \frac{3}{9} = \frac{1}{3}$. $A_1 \bigcap A_2$ occurs when ball "1,2,3" is picked. Thus, $P(A_1, A_2) = P(A_1, A_3) = P(A_2, A_3) = \frac{1}{9}$. It is easy to see A_1, A_2, A_3 are pairwise independent.

(c) Are A_1, A_2, A_3 mutually independent? (Show your calculation.)
Solution: $P(A_1)P(A_2)P(A_3) = \frac{1}{27}$. $P(A_1, A_2, A_3) = \frac{1}{9} \neq P(A_1)P(A_2)P(A_3)$. A_1, A_2, A_3 are not mutually independent.

Problem 4 (Exponential Distributions and Poisson Distributions)

(a) Exponential distribution is often used to model the lifetime of electronic components in autonomous vehicles. An exponential random variable X can be parameterized by its *rate* $\lambda\,(\lambda > 0)$ via the probability density function (pdf):

$$f(x) = \lambda e^{-\lambda x}, \qquad x > 0$$

(i) **(5 points)** Derive the cumulative distribution function (cdf) of the exponential distribution.
Solution: $F_X(x) = \int_0^x f(x)dx = \int_0^x \lambda e^{-\lambda x}dx = -e^{-\lambda x}\big|_0^x = -e^{-\lambda x} + 1.$

(ii) **(5 points)** Explain the memoryless property of the exponential distribution and provide the mathematical expression.
Solution: $P(X > s+t|X > s) = \frac{P(X>s+t,X>s)}{P(X>s)} = \frac{P(X>s+t)}{P(X>s)} = \frac{1-F_X(s+t)}{1-F_X(s)} = \frac{e^{-\lambda(s+t)}}{e^{-\lambda s}} = e^{-\lambda t} = 1 - F_X(t) = P(X > t).$

(iii) **(5 points)** Derive the mean and variance of the exponential distribution.
$E(X) = \int_0^\infty xf(x)dx = \int_0^\infty x\lambda e^{-\lambda x}dx = \left(-xe^{-\lambda x} - \frac{1}{\lambda}e^{-\lambda x}\right)\Big|_0^\infty = \frac{1}{\lambda}.$
$E(X^2) = \int_0^\infty x^2 f(x)dx = \int_0^\infty x^2\lambda e^{-\lambda x}dx = \left(-x^2e^{-\lambda x} - \frac{2x}{\lambda}e^{-\lambda x} - \frac{2}{\lambda^2}e^{-\lambda x}\right)\Big|_0^\infty = \frac{2}{\lambda^2}.$
$Var(X) = E(X^2) - (E(X))^2 = \frac{1}{\lambda^2}.$

(b) **(10 points)** The Poisson distribution can be seen as a limiting case of the binomial distribution as the number of trials goes to infinity and the expected number of successes remains fixed. Derive the Poisson distribution from the Binomial distribution.
Solution: Let X follow a Binomial distribution where the number of trails n goes to infinity and the expected number of successes $\lambda = np$ is fixed.

$$\begin{aligned}
P(X = k) &= \lim_{n\to\infty} \binom{n}{k} p^k (1-p)^{n-k} \\
&= \lim_{n\to\infty} \frac{n!}{(n-k)!k!}\left(\frac{\lambda}{n}\right)^k \left(1 - \frac{\lambda}{n}\right)^{n-k} \\
&= \frac{\lambda^k}{k!} \lim_{n\to\infty} \frac{n!}{(n-k)!n^k}\left(1 - \frac{\lambda}{n}\right)^n \left(1 - \frac{\lambda}{n}\right)^{-k} \\
&= \frac{\lambda^k}{k!} \cdot 1 \cdot e^{-\lambda} \cdot 1 \\
&= \frac{\lambda^k}{k!} e^{-\lambda}
\end{aligned}$$

Hence, X also follows a Poisson distribution with rate λ.

Problem 5 (Marginal/Joint Distributions)

Let X and Y be jointly continuous random variables with the following joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 10e^{-(2x+5y)} & x \geq 0, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) **(5 points)** Find the marginal distribution of the random variable X.
Solution: $f_X(x) = \int_0^\infty f_{X,Y}(x,y)dy = \int_0^\infty 10e^{-(2x+5y)}dy = 2e^{-2x}$

(b) **(5 points)** Find the marginal distribution of the random variable Y.
Solution: $f_Y(y) = \int_0^\infty f_{X,Y}(x,y)dx = \int_0^\infty 10e^{-(2x+5y)}dx = 5e^{-5y}$

(c) **(5 points)** Are X and Y independent? Explain your answer.
Solution: Yes. Continuous random variables X and Y are independent if their joint PDF is the product of their marginal PDFs.
$f_X(x)f_Y(y) = 2e^{-2x} \cdot 5e^{-5y} = 10e^{-(2x+5y)} = f_{X,Y}(x,y)$

(d) **(5 points)** What is the conditional PDF $f_{Y|X}(y|x)$. Include the values of x and y for which it is (i) well defined and (ii) zero .
Solution: $f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)} = \frac{10e^{-(2x+5y)}}{2e^{-2x}} = 5e^{-5y}$
Well-defined for $x \geq 0$ and zero for $y < 0$
OR
Since X and Y are independent, $f_{Y|X}(y|x) = f_Y(y) = 5e^{-5y}$

(e) **(5 points)** Determine $P\{Y > X\}$.
Solution: $P\{Y > X\} = \int_0^\infty \int_x^\infty 10e^{-(2x+5y)}dydx = \frac{2}{7}$

Problem 6 (Inequalities)

(a) A coin is weighted so that the probability of landing on heads is 40%. Suppose the coin is flipped a 100 times. Find the upper bound on the probability the coin lands on heads at least 80 times.
Solution: Let X be the random variable that represents the number of heads. $P(X)$ follows a binomial distribution with $n = 100$ and $p = 0.4$. Hence $E(X) = np = 100 \cdot 0.4 = 40$ Since X takes only non-negative values, we can apply the Markov's inequality to get the upper bound.
$P\{X \geq 80\} \leq \frac{E(X)}{80} = \frac{40}{80} = \frac{1}{2}$

(b) Using the same coin in (a), find the upper bound on the probability that the coin lands on heads at least 50 times or at most 30 times.
Solution: $P\{X \leq 30 \cup X \geq 50\} = P\{|X - 40| \geq 10\}$
From Chebyshev's inequality: $P\{|X - E(X)| \geq a\} \leq \frac{Var(X)}{a^2}$
From (a), we know $E(X) = 40$. $Var(X) = n \cdot p \cdot (1-p) = 100 \cdot 0.4 \cdot 0.6 = 24$
$P\{|X - 40| \geq 10\} \leq \frac{24}{10^2} = 0.24$

(c) Derive Chebyshev's Inequality from Markov's Inequality.
Solution:
Markov's Inequality: If X is a nonnegative random variable and $a > 0$, then

$P(X \geq a) \leq \frac{E(X)}{a}$.
Let $Y = (X - E(X))^2 \geq 0$ and $b^2 > 0$, then $P(Y \geq b^2) \leq \frac{E(Y)}{b^2}$.
Hence, $P(|X - E(X)| \geq b) = P(Y \geq b^2) \leq \frac{E(Y)}{b^2} = \frac{Var(X)}{b^2}$ (**Chebyshev's Inequality**).

Problem 7 (Covariance and Correlation Coefficient)

(a) Suppose X and Y are random variables, $\mathrm{Var}(X+Y) = 7$, $\mathrm{Var}(2X - 2Y) = 12$.

(i) **(5 points)** Find the covariance $\mathrm{Cov}(X, Y)$.
Solutions:
Based on the given equations in the questions, we have
$Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y) = 7 \cdots ①$
$Var(2X - 2Y) = 4(Var(X) + Var(Y) - 2Cov(X,Y)) = 12 \cdots ②$
By solving the equations ①② above, we can get
$Cov(X,Y) = \frac{7-12/4}{4} = 1$

(ii) **(5 points)** In addition to (i), given Var(X)=1, find the correlation coefficient $\rho_{X,Y}$. **Solutions:**

Combining Var(X)=1 with ① ②, we can get Var(Y)=4
$\rho_{X,Y} = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}} = \frac{1}{2} = 0.5$

(b) **(10 points)** Suppose random variables $X_1, X_2, \cdots, X_{10}$ are uncorrelated. For each i in $\{1, 2, \cdots, 10\}$, $\mathrm{E}[X_i] = i$ and $\mathrm{Var}(X_i)$=5. Find $\mathrm{Var}(\frac{S_{10}}{\sqrt{10}})$, where $S_{10} = X_1 + X_2 + \cdots + X_{10}$.
Solutions:
Note $X_1, X_2, \cdots, X_{10}$ are uncorrelated, $Cov(X_i, X_j) = 0$ $(\forall i \neq j)$.
$Var(\frac{S_{10}}{\sqrt{10}}) = \frac{1}{10} \cdot Var(S_{10}) = \frac{1}{10}(\sum_{i=1}^{10} Var(X_i) + 0) = \frac{10 \cdot 5}{10} = 5$

Problem 8 (Continuous Random Variable)

Here we define a probability density function $f(x)$. $f(x) = \frac{x^3}{\alpha}$ when $0 < x < 6$, $f(x) = 0$ otherwise. $X_1, ..., X_{50}$ are independent, continuous random variables and each one has probability density function $f(x)$.

(a) Find the valid value of α.

Solutions:
$\int_{-\infty}^{+\infty} f(x) = \int_0^6 \frac{x^3}{\alpha} = \frac{x^4}{4\alpha}\Big|_0^6 = \frac{324}{\alpha}$
According to the Probability Axioms, this integration should be 1. Accordingly, α should be 324.

(b) Find the expectation $\mathrm{E}[X_i]$.

Solutions:
$E[X_i] = \int_{-\infty}^{+\infty} x f(x) dx = \int_0^6 \frac{x^4}{324} dx = \frac{x^5}{5 \times 324}\Big|_0^6 = \frac{24}{5}$

(c) Find the variance $\text{Var}(X_i)$.

Solutions:

$E[X_i^2] = \int_0^6 \frac{x^5}{324} dx = \frac{x^6}{6\times 324}\Big|_0^6 = 24$

$Var(X_i) = E\left[X_i^2\right] - (E[X_i])^2 = 24 - (24/5)^2 = \frac{24}{25}$

(d) Find a good estimation for $P(X_1 + X_2 + \cdots + X_{50} < 230)$. (Hint: Central Limit Theorem)

Solutions:

$P(X_1+X_2\cdots+X_{50} < 230) = P(\frac{X_1 + X_2 \cdots + X_{50} - 50\cdot(24/5)}{\sqrt{(24/25)}\cdot\sqrt{50}} < \frac{230 - 50\cdot(24/5)}{\sqrt{(24/25)}\cdot\sqrt{50}})$

$\approx P(Z < -1.4434) = 1 - P(Z < 1.4434) = 0.07445$

Problem 9 (Central Limit Theorem)

An autonomous vehicle consists of 400 independent components. Assume the probability that each component functions properly is 0.98.

1. **(5 points)** Random variable X is the number of properly functioning components. Find the distribution of X.
   Solution: X follows a Binomial distribution with $n = 400$ and $p = 0.98$.

2. **(5 points)** The vehicle requires at least 390 properly functioning components to work. Use the Central Limit Theorem to find the probability that the system works.
   Solution:
   We can view X as the summation of 400 independent and identically distributed random variables, each of which follows a Bernoulli distribution with $p = 0.98$.
   $P(X \geq 390) = P\left(\frac{X - 400\cdot 0.98}{\sqrt{0.98 * (1-0.98)}\cdot\sqrt{400}} \geq \frac{390 - 400\cdot 0.98}{\sqrt{0.98*(1-0.98)}\cdot\sqrt{400}}\right)$
   $\approx 1 - \Phi(-0.7143) = 0.7625.$

Problem 10 (Bayes Theorem and Conditional Probabilities)

(10 points) When autonomous vehicles have malfunctions, the probability of a disengagement is 0.85. When autonomous vehicles do not have malfunctions, the probability of a disengagement is 0.002. If the probability of a malfunction is 0.0002, evaluate the probability that a given disengagement is due to a malfunction.

FYI: A disengagement is a failure that causes the control of the vehicle to switch from the software to the human driver.

Solution: M=Malfunction, D=Disengagement.

$$\begin{aligned} P(M|D) = \frac{P(M,D)}{P(D)} &= \frac{P(D|M)P(M)}{P(D|M)P(M) + P(D|\bar{M})P(\bar{M})} \\ &= \frac{0.85\cdot 0.0002}{0.85\cdot 0.0002 + 0.002\cdot(1-0.0002)} \approx 0.07835. \end{aligned}$$

Problem 11 (Bayes Theorem and Conditional Probabilities)

Timely patching is important for server security. Suppose an organization has 3 servers. Two of them have been patched, while one of them still remains unpatched. The probability an unpatched server gets compromised is $\frac{1}{2}$, and the probability that a patched server get compromised is p. If an attacker randomly attacks one of the three servers, the probability he compromises the server is $\frac{2}{3}$.

(a) What is the value of p?
Solution: A=Patched, C=Compromised.
$P(C) = P(C|A)P(A) + P(C|\overline{A})P(\overline{A}) = p \cdot \frac{2}{3} + \frac{1}{2} \cdot \frac{1}{3} = \frac{2}{3}$
$\Rightarrow p = \frac{3}{4}$

(b) This time the attacker randomly chose two serves to attack, and observed only one of them got compromised. What is the conditional probability that both attacked servers are patched?
Solution:
Note: Set $\{C, \overline{C}\}$ means one server gets compromised, while another server remains uncompromised. Sequence $C\overline{C}$ means the first server gets compromised, while the second server remains uncompromised.

The conditional probability we need is $P(\{A, A\}|\{C, \overline{C}\})$.

We can calculate it through Bayes Theorem: $P(\{A, A\}|\{C, \overline{C}\}) = \dfrac{P(\{C, \overline{C}\}|\{A, A\})P(\{A, A\})}{P(\{C, \overline{C}\})}$

The calculation flows are as follows:

$P(\{C, \overline{C}\}|\{A, A\}) = P(C\overline{C}|\{A, A\}) + P(\overline{C}C|\{A, A\}) = \frac{3}{4} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{3}{4} = \frac{3}{8}$

$P(\{C, \overline{C}\}|\{A, \overline{A}\}) = P(C\overline{C}|\{A, \overline{A}\}) + P(\overline{C}C|\{A, \overline{A}\}) = \frac{1}{2} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{3}{4} = \frac{1}{2}$

Then we use the theorem of total probability to get $P(\{C, \overline{C}\})$.

$P(\{C, \overline{C}\}) = P(\{C, \overline{C}\}|\{A, A\})P(\{A, A\}) + P(\{C, \overline{C}\}|\{A, \overline{A}\})P(\{A, \overline{A}\}) = \frac{3}{8} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{2}{3} = \frac{11}{24}$

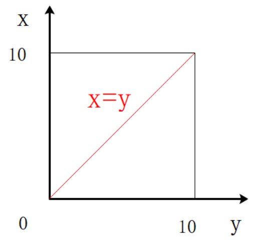
$$P(\{A, A\}|\{C, \overline{C}\}) = \frac{P(\{C, \overline{C}\}|\{A, A\})P(\{A, A\})}{P(\{C, \overline{C}\})} = \frac{\frac{3}{8} \cdot \frac{1}{3}}{\frac{11}{24}} = \frac{3}{11}$$

Problem 12 (Uniform Distribution)

Suppose you are waiting for buses at a bus stop. In the next 10 minutes, both bus A and bus B are expected to arrive, and their arrival time are independent to each other. We use X to denote the arrival time of bus A, and Y to denote the arrival time of bus B. X and Y are continuous variables, and each follows the uniform distribution over [0,10].

(a) **(5 points)** Find the probability that bus A and bus B arrive at exactly the same time.

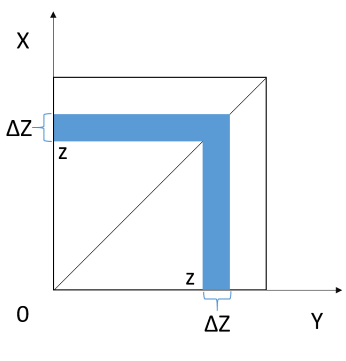
Solution: P(X=Y)=0
Because the arrival time is only an instant on [0,10] time interval, the probability that these two instants (bus A's arrival time and bus B's arrival time) overlap is 0. The following diagram also helps to illustrate. Note red line x=y represents the case bus A and bus B arrive at the same time. $P(X = Y) =$ Area of red line/Area of square = 0.



(b) **(5 points)** Find the probability that bus A arrives earlier than bus B.

Solution: From (a) we know that bus A and bus B can not arrive at exactly the same time. This means bus A arrives either earlier or later than bus B. These two cases are symmetric because A and B are equally defined. $P(X < Y) = P(Y < X) = 1/2$.
This can also be illustrated by the above diagram. The triangle below the line x=y stands for the case that bus A arrives earlier than B. $P(X < Y)$=Area of triangle/Area of square=1/2.

(c) **(5 points)** Denote Z as the arrival time of the later of the two. Find the pdf of Z and the expectation E[Z]. (Use minute as the unit, and leave the answer in decimal or fraction)
Solution 1:
For any $z \in (0, 10)$, $P(Z \leq z) = P(X \leq z, Y \leq z) = P(X \leq z)P(Y \leq z) = (\frac{z}{10})^2$.
Then we take the derivative of $P(Z \leq z)$ and get the pdf: $f_Z(z) = \begin{cases} \frac{z}{50} & 0 < z < 10 \\ 0 & otherwise \end{cases}$
$E[Z] = \int_0^{10} \frac{z^2}{50} dz = \frac{20}{3}$
Solution 2:
$f_Z(z) = \frac{dF(Z)}{dZ}\Big|_{Z=z} = \lim_{\Delta Z \to 0} \frac{F(Z \leq z+\Delta Z) - F(Z \leq z)}{\Delta Z} = \lim_{\Delta Z \to 0} \frac{p(z < Z \leq z+\Delta Z)}{\Delta Z}$
When $0 < Z < 10$, $\lim_{\Delta Z \to 0} p(z < Z \leq z + \Delta Z)$ is the ratio of blue area to square area in the diagram below.



$\lim\limits_{\Delta Z \to z} P(z < Z \leq z + \Delta Z) = \frac{Blue\_area}{Total\_area} = \frac{\Delta Z \cdot z \cdot 2}{10 \cdot 10} = \frac{\Delta Z \cdot z}{50}$

Therefore, when $0 < Z < 10$, $f_Z(z) = \lim\limits_{\Delta Z \to 0} \frac{p(z < Z \leq z + \Delta Z)}{\Delta Z} = \frac{\left(\frac{\Delta Z \cdot z}{50}\right)}{\Delta Z} = \frac{z}{50}$

The final answer is $f_Z(z) = \begin{cases} \frac{z}{50} & 0 < z < 10 \\ 0 & otherwise \end{cases}$

$E[Z] = \int_0^{10} \frac{z^2}{50} dz = \frac{20}{3}$

(Verification)

The pdf $f_Z(z) = \begin{cases} \frac{z}{50} & 0 < z < 10 \\ 0 & otherwise \end{cases}$ can be verified with Monte Carlo sampling. Click here to see.

(d) **(5 points)** Denote W as the arrival time of the earlier of the two. Find the expectation E[W].

Solution:

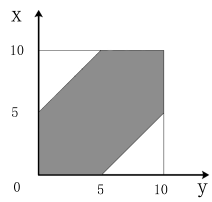
Since $Z = max(X, Y)$, $W = min(X, Y)$, we have $X + Y = Z + W$. Taking their expectations, we have $E[W] = E[X] + E[Y] - E[Z] = \frac{10}{3}$

The answer $E[W] = \frac{10}{3}$ can also be calculated with the methods in (c).

(e) **(5 points)** Suppose both bus A and bus B will wait for 5 minutes at the bus stop. Find the probability that bus A and bus B are together at the bus stop.

Solution:

The region $\{(x, y) : 0 < x, y < 10, |x - y| \leq 5\}$ is drawn as follows.



Then P(bus A and bus B meet) $= P(|X - Y| \leq 5) =$ Shadow area/Total area $= \frac{3}{4}$